

# Financial Instruments Toolbox™

## User's Guide



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*Financial Instruments Toolbox™ User's Guide*

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# Getting Started

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## **Financial Instruments Toolbox Product Description**

### **Design, price, and hedge complex financial instruments**

Financial Instruments Toolbox provides functions for pricing, modeling, and analyzing fixed-income, credit, and equity instrument portfolios. You can use the toolbox to perform cash-flow modeling and yield curve fitting analysis, compute prices and sensitivities, view price evolutions, and perform hedging analyses using common equity and fixed-income modeling methods. The toolbox lets you create new financial instrument types, fit yield curves to market data using parametric fitting models and bootstrapping, and construct dual curve-based pricing models.

You can price and analyze fixed-income and equity instruments. For fixed-income modeling, you can calculate price, yield, spread, and sensitivity values for several types of securities and derivatives, including convertible bonds, mortgage-backed securities, treasury bills, bonds, swaps, caps, floors, and floating-rate notes. For equities, you can compute price, implied volatility, and greek values of vanilla options and several exotic derivatives.

Financial Instruments Toolbox contains functions to model counterparty credit risk and CVA exposure. For credit derivatives, the toolbox includes credit default swap pricing and default probability curve modeling functions. For energy derivatives, you can model exotic and vanilla options. The toolbox also provides connectivity to Numerix® CrossAsset Integration Layer.

### **Key Features**

- Yield curve fitting with bootstrapping and parametric fitting models, and term-structure analysis with dual curve construction and pricing of swaps, caps, floors, and swaptions (using LIBOR-OIS and other curves)
- Black Scholes, Black, Garman-Kohlhagen, Roll-Geske-Whaley, Bjerksund-Stensland, Nengjiu Ju, Stulz, Levy jump diffusion, Longstaff-Schwartz, SABR, and tree models and Monte Carlo simulation
- Fixed-income and equity derivative calculations for price, yield, discount rate, cash-flow schedule, spread, implied volatility, option adjusted spread (OAS), and greeks
- Counterparty credit risk, CVA modeling, and credit instruments for mortgage pools, balloon mortgages, and credit default swaps
- Interest-rate instruments: bonds, stepped-coupon bonds, futures, vanilla options, Bermudan options, bonds with embedded options, vanilla swaps, forward swaps,



amortizing swaps, swaptions, caps, floors, range notes, floating-rate notes, and collared floating-rate notes

- Equity instruments: stocks, vanilla options, Bermudan options, Asian options, lookback options, barrier options, digital options, rainbow options, basket options, compound options, and chooser options
- Energy and commodity instruments: Asian options, Bermudan options, lookback options, swing options, spread options, and vanilla European/American options

## Interest-Rate-Based Derivatives

The toolbox provides functionality that supports the creation on page 1-8 and management on page 1-20 of these interest-rate-based instruments:

- Bonds
- Bond options (puts and calls)
- Bond with embedded options
- Caps
- Convertible bonds
- Fixed-rate notes
- Floating-rate notes
- Floors
- Swaps
- Swaption

Additionally, the toolbox provides functions to create *arbitrary cash flow instruments*. The toolbox provides pricing and sensitivity routines for these instruments. For more information, see “Pricing Using Interest-Rate Term Structure” on page 2-72, “Pricing Using Interest-Rate Tree Models” on page 2-99, and “Interest-Rate Derivatives Using Closed-Form Solutions” on page 2-121.

### See Also

`instbond` | `instcap` | `instcbond` | `instcf` | `instfixed` | `instfloat` | `instfloor` | `instoptbnd` | `instoptembnd` | `instoptemfloat` | `instoptfloat` | `instrangefloat` | `instswap` | `instswaption`

### Related Examples

- “Creating Instruments or Properties” on page 1-21

### More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24

- “Supported Energy Derivatives” on page 3-44

## Equity-Based Derivatives

The toolbox also provides functions to create and manage various equity-based derivatives, including the following:

- Asian options
- Barrier options
- Basket options
- Compound options
- Convertible bonds
- Digital options
- Lookback options
- Rainbow options
- Vanilla stock options (put and call options)

The toolbox also provides pricing and sensitivity routines for these instruments. (See “Pricing Equity Derivatives Using Trees” on page 3-130, “Equity Derivatives Using Closed-Form Solutions” on page 3-150, and “Basket Option” on page 3-28.)

### See Also

`instasian` | `instbarrier` | `instcbond` | `instcompound` | `instlookback` | `instoptstock`

### Related Examples

- “Creating Instruments or Properties” on page 1-21
- “Pricing Equity Derivatives Using Trees” on page 3-130

### More About

- “Supported Equity Derivatives” on page 3-24
- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Energy Derivatives” on page 3-44

## Expected Users

In general, this guide assumes experience working with financial derivatives and some familiarity with the underlying models.

In designing Financial Instruments Toolbox documentation, we assume that your title is similar to one of these:

- Analyst, quantitative analyst
- Risk manager
- Portfolio manager
- Fund manager, asset manager
- Financial engineer
- Trader
- Student, professor, or other academic

We also assume that your background, education, training, and responsibilities match some aspects of this profile:

- Finance, economics, perhaps accounting
- Engineering, mathematics, physics, other quantitative sciences
- Bachelor's degree minimum; MS or MBA likely; Ph.D. perhaps; CFA
- Comfortable with probability theory, statistics, and algebra
- Understand linear or matrix algebra, calculus, and differential equations
- Previously done traditional programming (C, Fortran, and so on)
- Responsible for instruments or analyses involving large sums of money
- Perhaps new to MATLAB®

## Portfolio Creation

In this section...
“Introduction” on page 1-8
“Interest-Rate-Based Derivatives” on page 1-8
“Equity Derivatives” on page 1-9

### Introduction

The `instadd` function creates a set of instruments (portfolio) or adds instruments to an existing instrument collection. The `TypeString` argument specifies the type of the investment instrument. For interest-rate-based derivatives, the types are: `Bond`, `OptBond`, `CashFlow`, `Fixed`, `Float`, `Cap`, `Floor`, and `Swap`. For equity derivatives, the types are `Asian`, `Barrier`, `Compound`, `Lookback`, and `OptStock`.

The input arguments following `TypeString` are specific to the type of investment instrument. Thus, the `TypeString` argument determines how the remainder of the input arguments is interpreted. For example, `instadd` with the type character vector for `Bond` creates a portfolio of bond instruments.

```
InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period,  
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate,  
StartDate, Face)
```

### Interest-Rate-Based Derivatives

In addition to the bond instrument already described, the toolbox can create portfolios containing the following set of interest-rate-based derivatives:

- Bond option

```
InstSet = instadd('OptBond', BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)
```

- Arbitrary cash flow instrument

```
InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)
```

- Fixed-rate note instrument

```
InstSet = instadd('Fixed', CouponRate, Settle, Maturity, FixedReset, Basis, Principal)
```

- Floating-rate note instrument

```
InstSet = instadd('Float', Spread, Settle, Maturity, FloatReset, Basis, Principal)
```

- Cap instrument

```
InstSet = instadd('Cap', Strike, Settle, Maturity, CapReset, Basis, Principal)
```

- Convertible bond instrument

```
InstSet = instcbond(CouponRate,Settle,Maturity,ConvRatio)
```

- Floor instrument

```
InstSet = instadd('Floor', Strike, Settle, Maturity, FloorReset, Basis, Principal)
```

- Swap instrument

```
InstSet = instadd('Swap', LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)
```

- Swaption instrument

```
InstSet = instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, ...  
Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal)
```

- Bond with embedded option instrument

```
InstSet = instadd('OptEmBond', CouponRate, Settle, Maturity, OptSpec, Strike, ...  
ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period', Period, 'Basis', Basis, ...  
'EndMonthRule', EndMonthRule, 'Face', Face, 'IssueDate', IssueDate, 'FirstCouponDate', ...  
FirstCouponDate, 'LastCouponDate', LastCouponDate, 'StartDate', StartDate)
```

## Equity Derivatives

The toolbox can create portfolios containing the following set of equity derivatives:

- Asian instrument

```
InstSet = instadd('Asian', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, ...  
AvgType, AvgPrice, AvgDate)
```

- Barrier instrument

```
InstSet = instadd('Barrier', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, ...  
BarrierType, Barrier, Rebate)
```

- Compound instrument

```
InstSet = instadd('Compound', UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, ...  
COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)
```

- Convertible bond instrument

```
InstSet = instcbond(CouponRate,Settle,Maturity,ConvRatio)
```

- Lookback instrument

```
InstSet = instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)
```

- Stock option instrument

```
InstSet = instadd('OptStock', OptSpec, Strike, Settle, Maturity, AmericanOpt)
```

## **See Also**

hedgeopt | hedgeslf | instadd | instaddfield | instdelete | instdisp |  
instfields | instfind | instget | instgetcell | instlength | instselect |  
instsetfield | insttypes | intenvset

## **Related Examples**

- “Creating Instruments or Properties” on page 1-21
- “Adding Instruments to an Existing Portfolio” on page 1-11
- “Instrument Construction and Portfolio Management” on page 1-20

## **More About**

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44



## Adding Instruments to an Existing Portfolio

To use the `instadd` function to add additional instruments to an existing instrument portfolio, provide the name of an existing portfolio as the first argument to the `instadd` function.

Consider, for example, a portfolio containing two cap instruments only:

```
Strike = [0.06; 0.07];
Settle = '08-Feb-2000';
Maturity = '15-Jan-2003';

Port_1 = instadd('Cap', Strike, Settle, Maturity);
```

These commands create a portfolio containing two cap instruments with the same settlement and maturity dates, but with different strikes. In general, the input arguments describing an instrument can be either a scalar, or a number of instruments (`NumInst`)-by-1 vector in which each element corresponds to an instrument. Using a scalar assigns the same value to all instruments passed in the call to `instadd`.

Use the `instdisp` command to display the contents of the instrument set:

```
instdisp(Port_1)

Index Type Strike Settle      Maturity    CapReset Basis Principal
1     Cap  0.06  08-Feb-2000 15-Jan-2003 1         0      100
2     Cap  0.07  08-Feb-2000 15-Jan-2003 1         0      100
```

Now add a single bond instrument to `Port_1`. The bond has a 4.0% coupon and the same settlement and maturity dates as the cap instruments.

```
CouponRate = 0.04;
Port_1 = instadd(Port_1, 'Bond', CouponRate, Settle, Maturity);
```

Use `instdisp` again to see the resulting instrument set:

```
instdisp(Port_1)

Index Type Strike Settle      Maturity    CapReset Basis Principal
1     Cap  0.06  08-Feb-2000 15-Jan-2003 1         0      100
2     Cap  0.07  08-Feb-2000 15-Jan-2003 1         0      100

Index Type CouponRate Settle      Maturity    Period Basis EndMonthRule IssueDate ... Face
3     Bond  0.04      08-Feb-2000 15-Jan-2003 2         0      1           NaN      ... 100
```

## See Also

hedgeopt | hedgeslf | instadd | instaddfield | instdelete | instdisp |  
instfields | instfind | instget | instgetcell | instlength | instselect |  
instsetfield | insttypes | intenvset

## Related Examples

- “Portfolio Creation” on page 1-8
- “Instrument Construction and Portfolio Management” on page 1-20

## More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Pricing a Portfolio Using the Black-Derman-Toy Model

This example illustrates how the Financial Instruments Toolbox™ is used to create a Black-Derman-Toy (BDT) tree and price a portfolio of instruments using the BDT model.

### Create the Interest Rate Term Structure

The structure `RateSpec` is an interest-rate term structure that defines the initial forward-rate specification from which the tree rates are derived. Use the information of annualized zero coupon rates in the table below to populate the `RateSpec` structure.

From To Rate

01 Jan 2005 01 Jan 2006 0.0275

01 Jan 2005 01 Jan 2007 0.0312

01 Jan 2005 01 Jan 2008 0.0363

01 Jan 2005 01 Jan 2009 0.0415

01 Jan 2005 01 Jan 2010 0.0458

```
StartDates = ['01 Jan 2005'];
```

```
EndDates = ['01 Jan 2006';
            '01 Jan 2007';
            '01 Jan 2008';
            '01 Jan 2009';
            '01 Jan 2010'];
```

```
ValuationDate = ['01 Jan 2005'];
```

```
Rates = [0.0275; 0.0312; 0.0363; 0.0415; 0.0458];
```

```
Compounding = 1;
```

```
RateSpec = intenvset('Compounding',Compounding,'StartDates', StartDates,...
                    'EndDates', EndDates, 'Rates', Rates, 'ValuationDate', ValuationDate);
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
```

```
StartTimes: [5x1 double]
EndDates: [5x1 double]
StartDates: 732313
ValuationDate: 732313
Basis: 0
EndMonthRule: 1
```

## Specify the Volatility Model

Create the structure VolSpec that specifies the volatility process with the following data.

```
Volatility = [0.005; 0.0055; 0.006; 0.0065; 0.007];
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)
```

```
BDTVolSpec = struct with fields:
    FinObj: 'BDTVolSpec'
    ValuationDate: 732313
    VolDates: [5x1 double]
    VolCurve: [5x1 double]
    VolInterpMethod: 'linear'
```

## Specify the Time Structure of the Tree

The structure TimeSpec specifies the time structure for an interest-rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

```
Maturity = EndDates;
BDTTimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)
```

```
BDTTimeSpec = struct with fields:
    FinObj: 'BDTTimeSpec'
    ValuationDate: 732313
    Maturity: [5x1 double]
    Compounding: 1
    Basis: 0
    EndMonthRule: 1
```

## Create the BDT Tree

Use the previously computed values for RateSpec, VolSpec and TimeSpec to create the BDT tree.

```
BDTTree = bdttree(BDTVolspec, RateSpec, BDTTimeSpec)
```

```
BDTTree = struct with fields:
```

```
  FinObj: 'BDTFwdTree'
  VolSpec: [1x1 struct]
  TimeSpec: [1x1 struct]
  RateSpec: [1x1 struct]
  tObs: [0 1 2 3 4]
  dObs: [732313 732678 733043 733408 733774]
  TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
  CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
  FwdTree: {1x5 cell}
```

### Observe the Interest Rate Tree

Visualize the interest-rate evolution along the tree by looking at the output structure BDTTree. BDTTree returns an inverse discount tree, which you can convert into an interest-rate tree with the `cvtree` function.

```
BDTTreeR = cvtree(BDTTree);
```

Look at the upper branch and lower branch paths of the tree:

```
%Rate at root node:
```

```
RateRoot = treepath(BDTTreeR.RateTree, [0])
```

```
RateRoot = 0.0275
```

```
%Rates along upper branch:
```

```
RatePathUp = treepath(BDTTreeR.RateTree, [1 1 1 1])
```

```
RatePathUp = 5x1
```

```
0.0275
0.0347
0.0460
0.0560
0.0612
```

```
%Rates along lower branch:
```

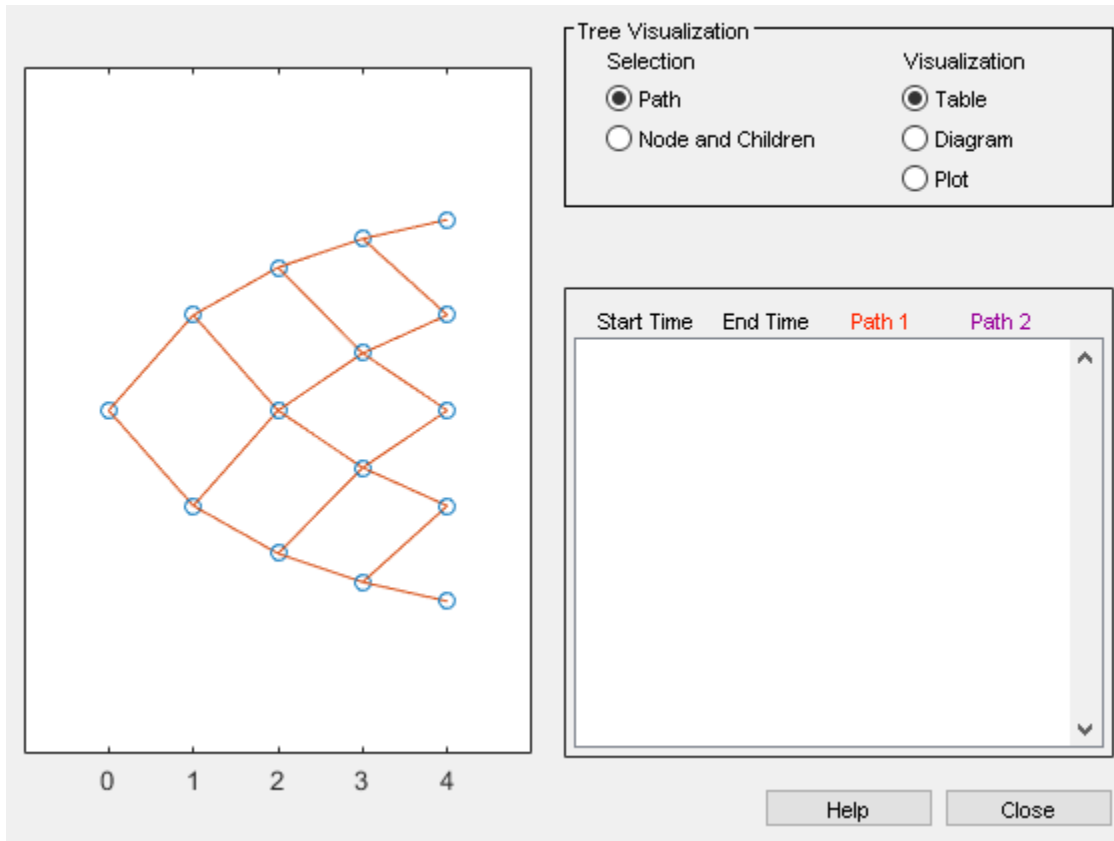
```
RatePathDown = treepath(BDTTreeR.RateTree, [2 2 2 2])
```

```
RatePathDown = 5×1
```

```
0.0275  
0.0351  
0.0472  
0.0585  
0.0653
```

You can also display a graphical representation of the tree to examine interactively the rates on the nodes of the tree until maturity. The function `treeviewer` displays the structure of the rate tree in the left pane. The tree visualization in the right pane is blank, but by selecting `Diagram` and clicking on the nodes you can examine the rates along the paths.

```
treeviewer(BDTreeR)
```



### Create an Instrument Portfolio

Create a portfolio consisting of two bond instruments and a option on the 5% Bond.

```
% Bonds
CouponRate = [0.04;0.05];
Settle = '01 Jan 2005';
Maturity = ['01 Jan 2009';'01 Jan 2010'];
Period = 1;

% Option
OptSpec = {'call'};
Strike = 98;
ExerciseDates = ['01 Jan 2010'];
```

```
AmericanOpt = 1;
```

```
InstSet = instadd('Bond',CouponRate, Settle, Maturity, Period);  
InstSet = instadd(InstSet,'OptBond', 2, OptSpec, Strike, ExerciseDates, AmericanOpt);
```

Examine the set of instruments contained in the variable InstSet.

```
instdisp(InstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.04	01-Jan-2005	01-Jan-2009	1	0	1	NaN
2	Bond	0.05	01-Jan-2005	01-Jan-2010	1	0	1	NaN

Index	Type	UnderInd	OptSpec	Strike	ExerciseDates	AmericanOpt
3	OptBond	2	call	98	01-Jan-2010	1

## Price the Portfolio Using a BDT Tree

Calculate the price of each instrument in the instrument set.

```
Price = bdtprice(BDTree, InstSet)
```

```
Price = 3×1
```

```
99.6374  
102.2460  
4.2460
```

The prices in the output vector Price correspond to the prices at observation time zero (tObs = 0), which is defined as the Valuation Date of the interest-rate tree.

In the Price vector, the first element, 99.6374, represents the price of the first instrument (4% Bond); the second element, 102.2460, represents the price of the second instrument (5% Bond), and 4.2460 represents the price of the Option.

## See Also

hedgeopt | hedgeself | instadd | instaddfield | instdelete | instdisp |  
instfields | instfind | instget | instgetcell | instlength | instselect |  
instsetfield | insttypes | intenvset



## **Related Examples**

- “Portfolio Creation” on page 1-8
- “Instrument Construction and Portfolio Management” on page 1-20

## **More About**

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Instrument Construction and Portfolio Management

In this section...
"Instrument Constructors" on page 1-20
"Creating Instruments or Properties" on page 1-21
"Searching or Subsetting a Portfolio" on page 1-23

### Instrument Constructors

The toolbox provides constructors for the most common financial instruments. A *constructor* is a function that builds a structure dedicated to a certain type of object; in this toolbox, an *object* is a type of market instrument.

The instruments and their constructors in this toolbox are listed below.

Instrument	Constructor
Asian option	instasian
Barrier option	instbarrier
Bond	instbond
Bond option	instoptbnd
Arbitrary cash flow	instcf
Compound option	instcompound
Convertible bond	instcbond
Fixed-rate note	instfixed
Floating-rate note	instfloat
Cap	instcap
Floor	instfloor
Lookback option	instlookback
Stock option	instoptstock
Swap	instswap
Swaption	instswaption

Each instrument has parameters (fields) that describe the instrument. The toolbox functions let you do the following:

- Create an instrument or portfolio of instruments.
- Enumerate stored instrument types and information fields.
- Enumerate instrument field data.
- Search and select instruments.

The instrument structure consists of various fields according to instrument type. A *field* is an element of data associated with the instrument. For example, a bond instrument contains the fields: `CouponRate`, `Settle`, `Maturity`. Also, each instrument has a field that identifies the investment type (bond, cap, floor, and so on).

In reality, the set of parameters for each instrument is not fixed. You have the ability to add additional parameters. These additional fields are ignored by the toolbox functions. They may be used to attach additional information to each instrument, such as an internal code describing the bond.

Parameters not specified when *creating* an instrument default to NaN, which, in general, means that the functions using the instrument set (such as `intenvprice` or `hjmprice`) will use default values. At the time of *pricing*, an error occurs if any of the required fields is missing, such as `Strike` in a cap or `CouponRate` in a bond.

## Creating Instruments or Properties

Use the `instaddfield` function to create a kind of instrument or to add new properties to the instruments in an existing instrument collection.

To create a kind of instrument with `instaddfield`, you must specify three arguments:

- `Type`
- `FieldName`
- `Data`

`Type` defines the type of the new instrument, for example, `Future`. `FieldName` names the fields uniquely associated with the new type of instrument. `Data` contains the data for the fields of the new instrument.

An optional fourth argument is `ClassList`. `ClassList` specifies the data types of the contents of each unique field for the new instrument.

Use either syntax to create a kind of instrument using `instaddfield`:

```
InstSet = instaddfield('FieldName', FieldList, 'Data', DataList,...  
'Type', TypeString)  
InstSet = instaddfield('FieldName', FieldList, 'FieldClass',...  
ClassList, 'Data', DataList, 'Type', TypeString)
```

To add new instruments to an existing set, use:

```
InstSetNew = instaddfield(InstSetOld, 'FieldName', FieldList,...  
'Data', DataList, 'Type', TypeString)
```

As an example, consider a futures contract with a delivery date of July 15, 2000, and a quoted price of \$104.40. Since Financial Instruments Toolbox software does not directly support this instrument, you must create it using the function `instaddfield`. Use these parameters to create instruments:

- Type: Future
- Field names: Delivery and Price
- Data: Delivery is July 15, 2000, and price is \$104.40.

Enter the data into MATLAB software:

```
Type = 'Future';  
FieldName = {'Delivery', 'Price'};  
Data = {'Jul-15-2000', 104.4};
```

Finally, create the portfolio with a single instrument:

```
Port = instaddfield('Type', Type, 'FieldName', FieldName,...  
'Data', Data);
```

Now use the function `instdisp` to examine the resulting single-instrument portfolio:

```
instdisp(Port)  
  
Index Type    Delivery    Price  
1      Future Jul-15-2000 104.4
```

Because your portfolio `Port` has the same structure as those created using the function `instadd`, you can combine portfolios created using `instadd` with portfolios created using `instaddfield`. For example, you can now add two cap instruments to `Port` with `instadd`.

```
Strike = [0.06; 0.07];  
Settle = '08-Feb-2000';
```

```
Maturity = '15-Jan-2003';
Port = instadd(Port, 'Cap', Strike, Settle, Maturity);
```

View the resulting portfolio using `instdisp`.

```
instdisp(Port)
```

Index	Type	Delivery	Price
1	Future	15-Jul-2000	104.4

Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal
2	Cap	0.06	08-Feb-2000	15-Jan-2003	1	0	100
3	Cap	0.07	08-Feb-2000	15-Jan-2003	1	0	100

## Searching or Subsetting a Portfolio

Financial Instruments Toolbox provides functions that enable you to:

- Find specific instruments within a portfolio.
- Create a subset portfolio consisting of instruments selected from a larger portfolio.

The `instfind` function finds instruments with a specific parameter value; it returns an instrument index (position) in a large instrument set. The `instselect` function, on the other hand, subsets a large instrument set into a portfolio of instruments with designated parameter values; it returns an instrument set (portfolio) rather than an index.

### **instfind**

The general syntax for `instfind` is

```
IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data', ...
DataList, 'Index', IndexSet, 'Type', TypeList)
```

`InstSet` is the instrument set to search. Within `InstSet` instruments categorized by type, each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

The `FieldList`, `DataList`, and `TypeList` arguments indicate values to search for in the `FieldName`, `Data`, and `Type` data fields of the instrument set. `FieldList` is a cell array of field names specific to the instruments. `DataList` is a cell array or matrix of acceptable values for one or more parameters specified in `FieldList`. `FieldName` and `Data` (therefore, `FieldList` and `DataList`) parameters must appear together or not at all.

`IndexSet` is a vector of integer indexes designating positions of instruments in the instrument set to check for matches; the default is all indices available in the instrument set. `TypeList` is a character vector or cell array of character vectors restricting instruments to match one of the `TypeList` types; the default is all types in the instrument set.

`IndexMatch` is a vector of positions of instruments matching the input criteria. Instruments are returned in `IndexMatch` if all the `FieldName`, `Data`, `Index`, and `Type` conditions are met. An instrument meets an individual field condition if the stored `FieldName` data matches any of the rows listed in the `DataList` for that `FieldName`.

## instfind Examples

The examples use the provided MAT-file `deriv.mat`.

The MAT-file contains an instrument set, `HJMInstSet`, that contains eight instruments of seven types.

```
load deriv.mat
instdisp(HJMInstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	...	Name	Quantity
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	...	4% bond	100
2	Bond	0.04	01-Jan-2000	01-Jan-2004	2	NaN	...	4% bond	50

Index	Type	UnderInd	OptSpec	Strike	ExerciseDates	AmericanOpt	Name	Quantity
3	OptBond	2	call	101	01-Jan-2003	NaN	Option 101	-50

Index	Type	CouponRate	Settle	Maturity	FixedReset	Basis	Principal	Name	Quantity
4	Fixed	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	4% Fixed	80

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	Name	Quantity
5	Float	20	01-Jan-2000	01-Jan-2003	1	NaN	NaN	20BP Float	8

Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal	Name	Quantity
6	Cap	0.03	01-Jan-2000	01-Jan-2004	1	NaN	NaN	3% Cap	30

Index	Type	Strike	Settle	Maturity	FloorReset	Basis	Principal	Name	Quantity
7	Floor	0.03	01-Jan-2000	01-Jan-2004	1	NaN	NaN	3% Floor	40

Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	Name	Quantity
8	Swap	[0.06 20]	01-Jan-2000	01-Jan-2003	[1 1]	NaN	NaN	[NaN]	6%/20BP Swap	10

Find all instruments with a maturity date of January 01, 2003.

```
Mat2003 = ...
instfind(HJMInstSet, 'FieldName', 'Maturity', 'Data', '01-Jan-2003')
Mat2003 =
```

```
1
4
5
8
```

Find all cap and floor instruments with a maturity date of January 01, 2004.

```
CapFloor = instfind(HJMInstSet,...
'FieldName','Maturity','Data','01-Jan-2004','Type',...
{'Cap';'Floor'})
```

```
CapFloor =
```

```
6
7
```

Find all instruments where the portfolio is long or short a quantity of 50.

```
Pos50 = instfind(HJMInstSet,'FieldName',...
'Quantity','Data',{'50';'-50'})
```

```
Pos50 =
```

```
2
3
```

### instselect

The syntax for `instselect` is the same syntax as for `instfind`. `instselect` returns a full portfolio instead of indexes into the original portfolio. Compare the values returned by both functions by calling them equivalently.

Previously you used `instfind` to find all instruments in `HJMInstSet` with a maturity date of January 01, 2003.

```
Mat2003 = ...
instfind(HJMInstSet,'FieldName','Maturity','Data','01-Jan-2003')
```

```
Mat2003 =
```

```
1
4
5
8
```

Now use the same instrument set as a starting point, but execute the `instselect` function instead, to produce a new instrument set matching the identical search criteria.

```
Select2003 = ...
instselect(HJMInstSet, 'FieldName', 'Maturity', 'Data', ...
'01-Jan-2003')
```

```
instdisp(Select2003)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate	FirstCouponDate	LastCouponDate	Sta
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	NaN	NaN	NaN	NaN

Index	Type	CouponRate	Settle	Maturity	FixedReset	Basis	Principal	Name	Quantity
2	Fixed	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	4% Fixed	80

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	Name	Quantity
3	Float	20	01-Jan-2000	01-Jan-2003	1	NaN	NaN	20BP Float	8

Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	Name	Quantity
4	Swap	[0.06 20]	01-Jan-2000	01-Jan-2003	[1 1]	NaN	NaN	[NaN]	6%/20BP Swap	10

## instselect Examples

These examples use the portfolio `ExampleInst` provided with the MAT-file `InstSetExamples.mat`.

```
load InstSetExamples.mat
instdisp(ExampleInst)
```

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	100	9.2	Call	0
3	Option	105	6.8	Call	1000

Index	Type	Delivery	F	Contracts
4	Futures	01-Jul-1999	104.4	-1000

Index	Type	Strike	Price	Opt	Contracts
5	Option	105	7.4	Put	-1000
6	Option	95	2.9	Put	0

Index	Type	Price	Maturity	Contracts
7	TBill	99	01-Jul-1999	6

The instrument set contains three instrument types: `Option`, `Futures`, and `TBill`. Use `instselect` to make a new instrument set containing only options struck at 95. In other words, select all instruments containing the field `Strike` and with the data value for that field equal to 95.



```
InstSet = instselect(ExampleInst, 'FieldName', 'Strike', 'Data', 95);
instdisp(InstSet)
```

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	95	2.9	Put	0

You can use all the various forms of `instselect` and `instfind` to locate specific instruments within this instrument set.

## See Also

`hedgeopt` | `hedgeslf` | `instadd` | `instaddfield` | `instdelete` | `instdisp` | `instfields` | `instfind` | `instget` | `instgetcell` | `instlength` | `instselect` | `instsetfield` | `insttypes` | `intenvset`

## Related Examples

- “Portfolio Creation” on page 1-8
- “Hedging Functions” on page 4-4
- “Hedging with hedgeopt” on page 4-5
- “Self-Financing Hedges with hedgeslf” on page 4-12
- “Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13
- “Pricing and Hedging a Portfolio Using the Black-Karasinski Model”
- “Specifying Constraints with ConSet” on page 4-32
- “Portfolio Rebalancing” on page 4-34
- “Hedging with Constrained Portfolios” on page 4-37

## More About

- “Hedging” on page 4-2
- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44



# Interest-Rate Derivatives

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## Supported Interest-Rate Instruments

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- “Convertible Bond” on page 2-3
- “Stepped Coupon Bonds” on page 2-5
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- “Bonds with an Amortization Schedule” on page 2-6
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- “Fixed-Rate Note” on page 2-10
- “Floating-Rate Note” on page 2-10
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- “Floating-Rate Note with Caps, Collars, and Floors” on page 2-11
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- “Swap” on page 2-15
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- “Swaption” on page 2-17
- “Bond Futures” on page 2-18

### Bond

A bond is a long-term debt security with a preset interest-rate and maturity. At maturity, you must pay the principal and interest.

The price or value of a bond is determined by discounting the expected cash flows of the bond to the present, using the appropriate discount rate. The following equation represents the relationship of the expected cash flows and discount rate:

$$B_0 = \frac{C}{2} \left[ \frac{1 - \left(1 + \frac{r}{2}\right)^{-2t}}{\frac{r}{2}} \right] + \frac{F}{\left(1 + \frac{r}{2}\right)^{2t}}$$

where:

$B_0$  is the bond value.

$C$  is the annual coupon payment.

$F$  is the face value of the bond.

$r$  is the required return on the bond.

$t$  is the number of years remaining until maturity.

Financial Instruments Toolbox supports the following for pricing and specifying a bond.

Function	Purpose
<code>bondbybdt</code>	Price a bond using a BDT interest-rate tree.
<code>bondbyhw</code>	Price a bond using an HW interest-rate tree.
<code>bondbybk</code>	Price a bond using a BK interest-rate tree.
<code>bondbyhjm</code>	Price a bond using an HJM interest-rate tree.
<code>bondbycir</code>	Price bonds using a CIR tree model.
<code>bondbyzero</code>	Price a bond using a set of zero curves.
<code>instbond</code>	Construct a bond instrument.

## Convertible Bond

A convertible bond is a financial instrument that combines equity and debt features. It is a bond with the embedded option to turn it into a fixed number of shares. The holder of a convertible bond has the right, but not the obligation, to exchange the convertible security for a predetermined number of equity shares at a preset price. The debt

component is derived from the coupon payments and the principal. The equity component is provided by the conversion feature.

Convertible bonds have several defining features:

- **Coupon** — The coupon in convertible bonds are typically lower than coupons in vanilla bonds since investors are willing to take the lower coupon for the opportunity to participate in the company's stock via the conversion.
- **Maturity** — Most convertible bonds are issued with long-stated maturities. Short-term maturity convertible bonds usually do not have call or put provisions.
- **Conversion ratio** — Conversion ratio is the number of shares that the holder of the convertible bond receives from exercising the call option of the convertible bond:  

$$\text{Conversion ratio} = \frac{\text{par value convertible bond}}{\text{conversion price of equity}}$$

For example, a conversion ratio of 25 means a bond can be exchanged for 25 shares of stock. This also implies a conversion price of \$40 (1000/25). This, \$40, would be the price at which the owner would buy the shares. This can be expressed as a ratio or as the conversion price and is specified in the contract along with other provisions.

- **Option type:**
  - **Callable Convertible:** a convertible bond that is callable by the issuer. The issuer of the bond forces conversion, removing the advantage that conversion is at the discretion of the bondholder. Upon call, the bondholder can either convert the bond or redeem at the call price. This option enables the issuer to control the price of the convertible bond and if necessary refinance the debt with a new cheaper one.
  - **Puttable Convertible:** a convertible bond with a put feature that allows the bondholder to sell back the bond at a premium on a specific date. This option protects the holder against rising interest rates by reducing the year to maturity.

Function	Purpose
cbondbycrr	Price convertible bonds using a CRR binomial tree with the Tsiveriotis and Fernandes model.
cbondbyeqp	Price convertible bonds using an EQP binomial tree with the Tsiveriotis and Fernandes model.
cbondbyitt	Price convertible bonds using an implied trinomial tree with the Tsiveriotis and Fernandes model.

Function	Purpose
cbondbystt	Price convertible bonds using a standard trinomial tree with the Tsiveriotis and Fernandes model.
instcbond	Construct a cbond instrument for a convertible bond.

## Stepped Coupon Bonds

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. For more information on options features (call and puts), see “Stepped Coupon Bonds with Calls and Puts” on page 2-8. The following functions have a modified `CouponRate` argument to support a new variable coupon schedule allowing pricing of stepped coupon bonds.

Function	Purpose
bondbyzero	Price bonds using a term structure model.
bondbybdt	Price bonds using a BDT tree model.
bondbyhjm	Price bonds using an HJM tree model.
bondbyhw	Price bonds using an HW tree model.
bondbybk	Price bonds using a BK tree model.
bondbycir	Price bonds using a CIR tree model.
instbond	Construct a bond instrument.
instoptbnd	Construct a bond option instrument.
instdisp	Display instruments stored in a variable.

## Sinking Fund Bonds

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal before maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity. For more information on options support for sinking fund bonds, see “Sinking Fund Bonds with an Embedded Option” on page 2-9. The following

functions have a modified Face argument to support a variable face schedule for pricing bonds with a sinking provisions.

<b>Function</b>	<b>Purpose</b>
bondbyzero	Price bonds using a term structure model.
bondbybdt	Price bonds using a BDT tree model.
bondbyhjm	Price bonds using an HJM tree model.
bondbyhw	Price bonds using an HW tree model.
bondbybk	Price bonds using a BK tree model.
bondbycir	Price bonds using a CIR tree model.
instoptbnd	Construct a bond option instrument.
instbond	Construct a bond instrument.
instdisp	Display instruments stored in a variable.

## **Bonds with an Amortization Schedule**

A bond with an amortization schedule repays part of the principal (face value) along with the coupon payments. An amortizing bond is a special case of a sinking fund bond when there is no market purchase option and no call provision. The following functions have a modified Face argument to support an amortization schedule.

<b>Function</b>	<b>Purpose</b>
bondbyzero	Price bonds using a term structure model.
bondbybdt	Price bonds using a BDT tree model.
bondbyhjm	Price bonds using an HJM tree model.
bondbyhw	Price bonds using an HW tree model.
bondbybk	Price bonds using a BK tree model.
bondbycir	Price bonds using a CIR tree model.

## **Bond Options**

Financial Instruments Toolbox supports three types of put and call options on bonds:



- American option: An option that you exercise any time until its expiration date.
- European option: An option that you exercise only on its expiration date.
- Bermuda option: A Bermuda option resembles a hybrid of American and European options. You can exercise it on predetermined dates only, usually monthly.

Financial Instruments Toolbox supports the following for pricing and specifying a bond option.

Function	Purpose
optbndbybdt	Price a bond option price using a BDT interest-rate tree.
optbndbyhw	Price a bond option price using an HW interest-rate tree.
optbndbybk	Price a bond option price using a BK interest-rate tree.
optbndbyhjm	Price a bond option price using an HJM interest-rate tree.
optbndbycir	Price a bond option price using a CIR interest-rate tree.
instoptbnd	Construct a bond option instrument.

## Bond with Embedded Options

A bond with embedded options allows the issuer to buy back or redeem the bond at a predetermined price at specified future dates. Financial Instruments Toolbox supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:

- For a callable bond:  $PriceCallableBond = BondPrice - BondCallOption$
- For a puttable bond:  $PricePuttableBond = PriceBond + PricePutOption$

In addition, Option Adjusted Spread (OAS) is a useful way to value and compare securities with embedded options, like callable or puttable bonds. For more information on OAS, see “OAS for Callable and Puttable Bonds” on page 2-76.

Financial Instruments Toolbox supports the following for pricing and specifying a bond with embedded options.

<b>Function</b>	<b>Purpose</b>
optembndbybdt	Price a bond with embedded options using a BDT interest-rate tree.
optembndbyhw	Price a bond with embedded options using an HW interest rate tree.
optembndbybk	Price a bond with embedded options using a BK interest-rate tree.
optembndbyhjm	Price a bond with embedded options using an HJM interest-rate tree.
optembndbycir	Price a bond with embedded options using a CIR interest-rate tree.
instoptembnd	Construct a bond-with-embedded-options instrument.
oasbybdt	Determine an option adjusted spread using Black-Derman-Toy model.
oasbybk	Determine an option adjusted spread using Black-Karasinski model.
oasbyhjm	Determine an option adjusted spread using Heath-Jarrow-Morton model.
oasbyhw	Determine an option adjusted spread using Hull-White model.
oasbycir	Determine an option adjusted spread using Cox-Ingersoll-Ross model.
agencyoas	Compute the OAS of the callable bond using the Agency OAS model.
agencyprice	Price the callable bond OAS using the Agency OAS model.

## **Stepped Coupon Bonds with Calls and Puts**

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. For more information on stepped coupon bonds, see “Stepped Coupon Bonds” on page 2-5. Stepped coupon bonds can have options features (call and puts). The following functions have a modified `CouponRate` argument to support a new variable

coupon schedule allowing pricing stepped coupon bonds with callable and puttable features:

Function	Purpose
optembndbybdt	Price bonds with embedded options using a BDT model tree.
optembndbyhjm	Price bonds with embedded options using an HJM model tree.
optembndbybk	Price bonds with embedded options using a BK model tree.
optembndbyhw	Price bonds with embedded options using an HW model tree.
optembndbycir	Price bonds with embedded options using a CIR model tree.
instbond	Construct a bond instrument.
instoptbnd	Construct a bond option instrument.
instoptembnd	Construct a bond with an embedded option instrument.
instdisp	Display instruments stored in a variable.

## Sinking Fund Bonds with an Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. For more information on sinking fund bonds, see “Sinking Fund Bonds” on page 2-5. The sinking fund bond can have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper.

If interest rates are high, then the issuer buys back the required amount of bonds from the market since bonds are cheap. But if interest rates are low (bond prices are high), then most likely the issuer buys the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a nonsinking fund bond. The following functions have a modified Face argument to support a variable face schedule for pricing bonds with a sinking fund option provision.

Function	Purpose
optembndbybdt	Price bonds with embedded options using a BDT model tree.
optembndbyhjm	Price bonds with embedded options using an HJM model tree.
optembndbybk	Price bonds with embedded options using a BK model tree.

Function	Purpose
optembndbyhw	Price bonds with embedded options using an HW model tree.
optembndbycir	Price bonds with embedded options using a CIR model tree.
instbond	Construct a bond instrument.
instoptbond	Construct a bond option instrument.
instdisp	Display instruments stored in a variable.

### Fixed-Rate Note

A fixed-rate note is a long-term debt security with a preset interest rate and maturity, by which the interest must be paid. The principal may or may not be paid at maturity. In Financial Instruments Toolbox, the principal is always paid at maturity.

Function	Purpose
fixedbybdt	Price a fixed-rate note using a BDT interest-rate tree.
fixedbyhw	Price a fixed-rate note using an HW interest-rate tree.
fixedbybk	Price a fixed-rate note using a BK interest-rate tree.
fixedbyhjm	Price a fixed-rate note using an HJM interest-rate tree.
fixedbycir	Price a fixed-rate note using a CIR interest-rate tree.
fixedbyzero	Price a fixed-rate note using a set of zero curves.
instfixed	Construct a fixed-rate instrument.

### Floating-Rate Note

A floating-rate note is a security like a bond, but the interest rate of the note is reset periodically, relative to a reference index rate, to reflect fluctuations in market interest rates.

Function	Purpose
floatbybdt	Price a floating-rate note using a BDT interest-rate tree.
floatbyhw	Price a floating-rate note using an HW interest-rate tree.

Function	Purpose
floatbybk	Price a floating-rate note using a BK interest-rate tree.
floatbyhjm	Price a floating-rate note using an HJM interest-rate tree.
floatbycir	Price a floating-rate note using a CIR interest-rate tree.
floatbyzero	Price a floating-rate note using a set of zero curves.
instfloat	Construct a floating-rate note instrument.

## Floating-Rate Note with an Amortization Schedule

A floating-rate note with an amortization schedule repays part of the principal (face value) along with the coupon payments. The following functions have a `Principal` argument to support an amortization schedule.

Function	Purpose
floatbyzero	Price floating-rate note from set of zero curves.
floatbybdt	Price floating-rate note from Black-Derman-Toy interest-rate tree.
floatbyhjm	Price floating-rate note from Heath-Jarrow-Morton interest-rate tree.
floatbyhw	Price floating-rate note from Hull-White interest-rate tree.
floatbybk	Price floating-rate note from Black-Karasinski interest-rate tree.
floatbycir	Price a floating-rate note using a CIR interest-rate tree.

## Floating-Rate Note with Caps, Collars, and Floors

A floating-rate note with caps, collars, and floors. This type of instrument can carry restrictions on the maximum (cap) or minimum (floor) coupon rate paid. A cap is an unattractive feature for an investor, since they constrain the coupon rates from increasing. A floor is an attractive feature, since it allows investors to get a minimum coupon rate when market rates decrease below a certain level. Also, a floating-rate note can have a collar which is a combination of a cap and a floor together. The following functions have a `CapRate` and `FloorRate` argument to support a capped, collared, or floored floating-rate note.

Function	Purpose
floatbybdt	Price a capped floating-rate note from a Black-Derman-Toy interest-rate tree.
floatbyhjm	Price a capped floating-rate note from a Heath-Jarrow-Morton interest-rate tree.
floatbyhw	Price a capped floating-rate note from a Hull-White interest-rate tree.
floatbybk	Price a capped floating-rate note from a Black-Karasinski interest-rate tree.
floatbycir	Price a floating-rate note using a CIR interest-rate tree.
instfloat	Create a capped floating-rate note instrument.
instadd	Add a capped floating-rate note instrument to a portfolio.

## Floating-Rate Note Options

Financial Instruments Toolbox supports three types of put and call options on floating-rate notes:

- American option — An option that you exercise any time until its expiration date.
- European option — An option that you exercise only on its expiration date.
- Bermuda option — A Bermuda option resembles a hybrid of American and European options; you can only exercise it on predetermined dates, usually monthly.

Financial Instruments Toolbox supports the following for pricing and specifying a floating-rate note option:

Function	Purpose
optfloatbybdt	Price an option for floating-rate note using a Black-Derman-Toy interest-rate tree.
optfloatbyhjm	Price an option for floating-rate note using a Heath-Jarrow-Morton interest-rate tree.
optfloatbyhw	Price an option for floating-rate note using a Hull-White interest-rate tree.

Function	Purpose
optfloatbycir	Price an option for floating-rate note using a Cox-Ingersoll-Ross interest-rate tree.
optfloatbybk	Price an option for floating-rate note using a Black-Karasinski interest-rate tree.
instoptfloat	Define the option instrument for floating-rate note.

## Floating-Rate Note with Embedded Options

A floating-rate note with an embedded option enables floating-rate notes to have early redemption features. An FRN with an embedded option gives investors or issuers the option to retire the outstanding principal prior to maturity. An embedded call option gives the right to retire the note prior to the maturity date (callable floater), and an embedded put option gives the right to sell the note back at a specific price (puttable floater).

Financial Instruments Toolbox supports the following for pricing and specifying a floating-rate note with an embedded option:

Function	Purpose
optemfloatbybdt	Price an embedded option for floating-rate note using a Black-Derman-Toy interest-rate tree.
optemfloatbybk	Price an embedded option for floating-rate note using a Black-Karasinski interest-rate tree.
optemfloatbyhjm	Price an embedded option for floating-rate note using a Heath-Jarrow-Morton interest-rate tree.
optemfloatbyhw	Price an embedded option for floating-rate note using a Hull-White interest-rate tree.
optemfloatbycir	Price an embedded option for floating-rate note using a Cox-Ingersoll-Ross interest-rate tree.
instoptemfloat	Define the floating-rate note with embedded option instrument.

## Cap

A cap is a contract that includes a guarantee that sets the maximum interest rate to be paid by the holder, based on an otherwise floating interest rate. The payoff for a cap is:

$$\max(\text{CurrentRate} - \text{CapRate}, 0)$$

Function	Purpose
capbybdt	Price a cap instrument using a BDT interest-rate tree.
capbyhw	Price a cap instrument using an HW interest-rate tree.
capbybk	Price a cap instrument using a BK interest-rate tree.
capbyhjm	Price a cap instrument using an HJM interest-rate tree.
capbycir	Price a cap instrument using a CIR interest-rate tree.
capbyblk	Price a cap instrument using the Black option pricing model.
capbylg2f	Price a cap using Linear Gaussian two-factor model.
capbynormal	Price a cap instrument with negative rates using the Normal (Bachelier) option pricing model.
capvolstrip	Strip caplet volatilities from flat cap volatilities.
instcap	Construct a cap instrument.

## Floor

A floor is a contract that includes a guarantee setting the minimum interest rate to be received by the holder, based on an otherwise floating interest rate. The payoff for a floor is:

$$\max(\text{FloorRate} - \text{CurrentRate}, 0)$$

Function	Purpose
floorbybdt	Price a floor instrument using a BDT interest-rate tree.
floorbyhw	Price a floor instrument using an HW interest-rate tree.
floorbybk	Price a floor instrument using a BK interest-rate tree.
floorbyhjm	Price a floor instrument using an HJM interest-rate tree.
floorbycir	Price a floor instrument using a CIR interest-rate tree.
floorbyblk	Price a floor instrument using the Black option pricing model.



Function	Purpose
floorbylg2f	Price a floor using Linear Gaussian two-factor model.
floorbynormal	Price a floor instrument with negative rates using the Normal (Bachelier) option pricing model.
floorvolstrip	Strip floorlet volatilities from flat floor volatilities.
instfloor	Construct a floor instrument.

## Range Note

A range note is a structured (market-linked) security whose coupon-rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon-rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest-rate that is floored to be positive and gives the holder of the note direct exposure to the reference rate. This type of instrument is useful for cases where you believe that interest rates will stay within a certain range. In return for the drawback that no interest is paid for the time the range is left, a range note offers higher coupon rates than comparable standard products, like vanilla floating notes.

Function	Purpose
instrangefloat	Create a range note instrument.
rangefloatbybdt	Price range floating note using a BDT tree.
rangefloatbybk	Price range floating note using a BK tree.
rangefloatbyhjm	Price range floating note using an HJM tree.
rangefloatbyhw	Price range floating note using an HW tree.
rangefloatbycir	Price range floating note using a CIR tree.

## Swap

A swap is contract between two parties obligating the parties to exchange future cash flows. This toolbox version handles only the vanilla swap, which is composed of a floating-rate leg and a fixed-rate leg.

Function	Purpose
swapbybdt	Price a swap instrument using a BDT interest-rate tree.
swapbyhw	Price a swap instrument using an HW interest-rate tree.
swapbybk	Price a swap instrument using a BK interest-rate tree.
swapbyhjm	Price a swap instrument using an HJM interest-rate tree.
swapbycir	Price a swap instrument using a CIR interest-rate tree.
swapbyzero	Price a swap instrument using a set of zero curves and price cross currency swaps.
instswap	Construct a swap instrument.

## Swap with an Amortization Schedule

A swap with an amortization schedule repays part of the principal (face value) along with the coupon payments. A swap with an amortization schedule is used to manage interest rate risk and serve as a cash flow management tool. For this particular type of swap, the notional amount decreases over time. This means that interest payments decrease not only on the floating leg but also on the fixed leg. The following swap functions have a `Principal` argument to support an amortization schedule.

Function	Purpose
swapbyzero	Price swap instrument from set of zero curves.
swapbybdt	Price swap instrument from Black-Derman-Toy interest-rate tree.
swapbyhjm	Price swap instrument from Heath-Jarrow-Morton interest-rate tree.
swapbyhw	Price swap instrument from Hull-White interest-rate tree.
swapbybk	Price swap instrument from Black-Karasinski interest-rate tree.
swapbycir	Price a swap instrument using a CIR interest-rate tree.
instswap	Construct swap instrument.

## Forward Swap

In a forward interest-rate swap, a fixed interest-rate loan is exchanged for a floating interest-rate loan at a future specified date. The following functions have a `StartDate` argument to support the future date for the forward swap.

Function	Purpose
<code>swapbyzero</code>	Price a forward swap from a zero curve.
<code>swapbybdt</code>	Price a forward swap from a Black-Derman-Toy interest-rate tree.
<code>swapbyhjm</code>	Price a forward swap from a Heath-Jarrow-Morton interest-rate tree.
<code>swapbyhw</code>	Price a forward swap from a Hull-White interest-rate tree.
<code>swapbybk</code>	Price a forward swap from a Black-Karasinski interest-rate tree.
<code>swapbycir</code>	Price a swap instrument using a CIR interest-rate tree.
<code>instswap</code>	Create a forward swap instrument.
<code>instadd</code>	Add a capped floating-rate note instrument to a portfolio.

## Swaption

A swaption is an option to enter into an interest-rate swap contract. A call swaption allows the option buyer to enter into an interest-rate swap where the buyer of the option pays the fixed-rate and receives the floating-rate. A put swaption allows the option buyer to enter into an interest-rate swap where the buyer of the option receives the fixed-rate and pays the floating-rate.

Function	Purpose
<code>swaptionbybdt</code>	Price a swaption instrument using a BDT interest-rate tree.
<code>swaptionbyhw</code>	Price a swaption instrument using an HW interest-rate tree.
<code>swaptionbybk</code>	Price a swaption instrument using a BK interest-rate tree.

<b>Function</b>	<b>Purpose</b>
<code>swaptionbyhjm</code>	Price a swaption instrument using an HJM interest-rate tree.
<code>swaptionbycir</code>	Price a swaption instrument using a CIR interest-rate tree.
<code>swaptionbyblk</code>	Price swaptions using the Black model with a forward on a swap.
<code>swaptionbylg2f</code>	Price European swaptions using Linear Gaussian two-factor model.
<code>swaptionbynormal</code>	Price swaptions for negative rates using the Normal (Bachelier) model with a forward on a swap.
<code>instswaption</code>	Construct a swaption instrument.

Use `swaptionbyblk` to price a swaption using the Black model. The Black model is standard model used in the swaption market when pricing European swaptions. This type of model is widely used by when speed is important to quickly obtain a price at settlement date, even if the price is less accurate than other swaption pricing models based on interest-rate tree models.

## **Bond Futures**

Bond futures are futures contracts where the commodity for delivery is a government bond. There are established global markets for government bond futures. Bond futures provide a liquid alternative for managing interest-rate risk.

In the US market, the Chicago Mercantile Exchange (CME) offers futures on Treasury bonds and notes with maturities of 2, 5, 10, and 30 years. Typically, the following bond future contracts from the CME have maturities of 3, 6, 9, and 12 months:

- 30-year U.S. Treasury bond
- 10-year U.S. Treasury bond
- 5-year U.S. Treasury bond
- 2-year U.S. Treasury bond

The short position in a Treasury bond or note future contract must deliver to the long position in one of many possible existing Treasury bonds. For example, in a 30-year Treasury bond future, the short position must deliver a Treasury bond with at least 15

years to maturity. Because these bonds have different values, the bond future contract is standardized by computing a conversion factor. The conversion factor normalizes the price of a bond to a theoretical bond with a coupon of 6%. The price of a bond future contract is represented as:

$$\text{InvoicePrice} = \text{FutPrice} \times \text{CF} + \text{AI}$$

where:

*FutPrice* is the price of the bond future.

*CF* is the conversion factor for a bond to deliver in a futures contract.

*AI* is the accrued interest.

The short position in a futures contract has the option of which bond to deliver and, in the US bond market, when in the delivery month to deliver the bond. The short position typically chooses to deliver the bond known as the Cheapest to Deliver (CTD). The CTD bond most often delivers on the last delivery day of the month.

Financial Instruments Toolbox supports the following bond futures:

- US Treasury bonds and notes
- German Bobl, Bund, Buxl, and Schatz
- UK gilts
- Japanese government bonds (JGBs)

The functions supporting all bond futures are:

Function	Purpose
convfactor	Calculates bond conversion factors for US Treasury bonds, German Bobl, Bund, Buxl, and Schatz, UK gilts, and JGBs.
bndfutprice	Prices bond future given repo rates.
bndfutimrepo	Calculates implied repo rates for a bond future given price.

The functions supporting US Treasury bond futures are:

Function	Purpose
tfutbyprice	Calculates future prices of Treasury bonds given the spot price.

Function	Purpose
tfutbyyield	Calculates future prices of Treasury bonds given current yield.
tfutimrepo	Calculates implied repo rates for the Treasury bond future given price.
tfutpricebyrepo	Calculates Treasury bond futures price given the implied repo rates.
tfutyieldbyrepo	Calculates Treasury bond futures yield given the implied repo rates.

For more information on bond futures, see “Bond Futures” on page 7-10.

## See Also

agencyoas | agencyprice | bdtprice | bdtrens | bdttimespec | bdttree |  
 bdtvolspec | bkprice | bksens | bktimespec | bktree | bkvolspec |  
 blackvolbyrebonato | blackvolbysabr | bndfutimrepo | bndfutprice |  
 bondbybdt | bondbybk | bondbyhjm | bondbyhw | bondbyzero | capbybdt | capbybk |  
 capbyblk | capbyhjm | capbyhw | capbylg2f | cfbybdt | cfbybk | cfbyhjm | cfbyhw  
 | cfbyzero | convfactor | fixedbybdt | fixedbybk | fixedbyhjm | fixedbyhw |  
 fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw | floatbyzero |  
 floatdiscmargin | floatmargin | floorbybdt | floorbybk | floorbyblk |  
 floorbyhjm | floorbyhw | floorbylg2f | hjmprice | hjmsens | hjmtimespec |  
 hjmtree | hjmvolspec | hwalbycap | hwalbycap | hwalbyfloor | hwalbyfloor  
 | hwprice | hwsens | hwtimespec | hwtree | hwwolspec | instbond | instcap |  
 instcf | instfixed | instfloat | instfloor | instoptbnd | instoptembnd |  
 instoptemfloat | instoptfloat | instrangefloat | instswap | instswaption |  
 intenvprice | intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt |  
 oasbybk | oasbyhjm | oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm |  
 optbndbyhw | optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw |  
 optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw |  
 optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw |  
 optsensbysabr | rangefloatbybdt | rangefloatbybk | rangefloatbyhjm |  
 rangefloatbyhw | swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero |  
 swaptionbybdt | swaptionbybk | swaptionbyblk | swaptionbyhjm |  
 swaptionbyhw | swaptionbylg2f | tfutbyprice | tfutbyyield | tfutimrepo |  
 tfutpricebyrepo | tfutyieldbyrepo

## **Related Examples**

- “Overview of Interest-Rate Tree Models” on page 2-50
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Graphical Representation of Trees” on page 2-157
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Understanding Interest-Rate Tree Models” on page 2-79
- “Understanding the Interest-Rate Term Structure” on page 2-55

## **More About**

- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Work with Negative Interest Rates

<b>In this section...</b>
“Interest-Rate Modeling Options for Negative Rates” on page 2-22
“Modeling Negative Rates” on page 2-22

### Interest-Rate Modeling Options for Negative Rates

Financial Instruments Toolbox computes prices for caps, floors, and swaptions when modeling for negative interest-rates using the following:

Support the Normal volatility model (Bachelier model) for interest-rate options to handle negative rates:

- `swaptionbynormal`
- `capbynormal`
- `floorbynormal`
- `normalvolbysabr`

The following functions provide an optional `Shift` argument to support the shifted Black model and the shifted SABR model for interest-rate options to handle negative rates:

- `blackvolbysabr` (Shifted SABR)
- `optsensbysabr` (Shifted SABR)
- `swaptionbyblk` (Shifted Black)
- `capbyblk` (Shifted Black)
- `floorbyblk` (Shifted Black)
- `capvolstrip` (Shifted Black)
- `floorvolstrip` (Shifted Black)

### Modeling Negative Rates

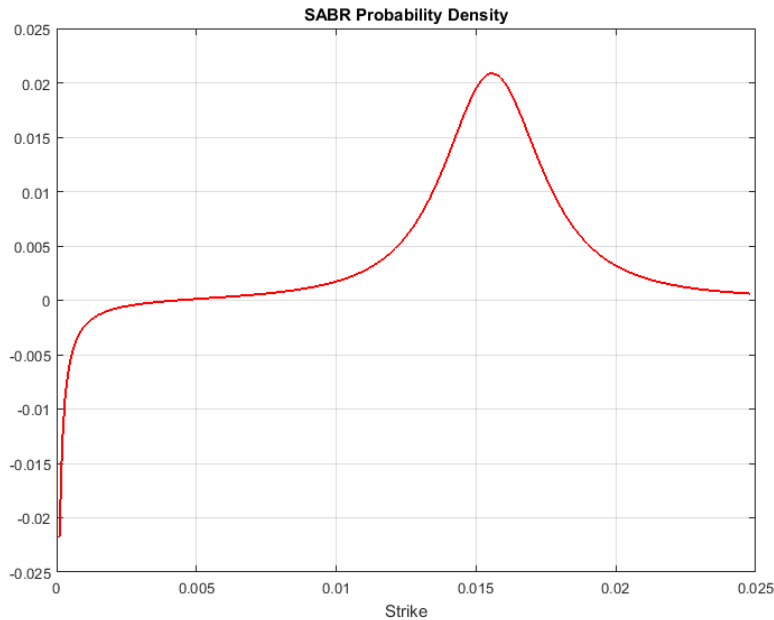
The original authors of the SABR model provided a closed form approximation of the implied Black volatility in terms of the SABR model parameters (known as “Hagan’s formula”), so that the option price could be computed by inserting the computed SABR Black volatility into the Black formula:



$$Call(K, T) = Black_{call}(F, K, r, T, \sigma_{Black}(\alpha, \beta, \rho, \nu, F, K, T))$$

However, these methods started to break down with the introduction of negative interest rates, due to the assumption of the Black model that the underlying rates are lognormally distributed (and therefore cannot be negative).

In addition, even when the underlying rate is positive, the closed form approximation of the SABR implied Black volatility (Hagan et al., 2002) is known to become increasingly inaccurate as the strike approaches zero. Even without crossing the zero strike boundary, the implied probability density of the underlying rate at option expiry can become negative at low positive strikes, although probability densities clearly should not be negative:



Options with negative strikes cannot be represented by Black volatilities. To work around this problem, the market started to quote the cap, floor, and swaption prices also in terms of either Normal volatilities or Shifted Black volatilities. Instead of the Black model, both types of volatilities come from alternative models that allow negative rates.

### Normal Model

The Normal volatilities are associated with the Normal model (also known as the Bachelier model):

$$dF = \sigma_{Normal}dW$$

where the underlying rates are assumed to be normally distributed. Unlike in a lognormal model (where rates have a lower bound), the rates in the Normal model can be both infinitely positive and infinitely negative.

### Shifted Black

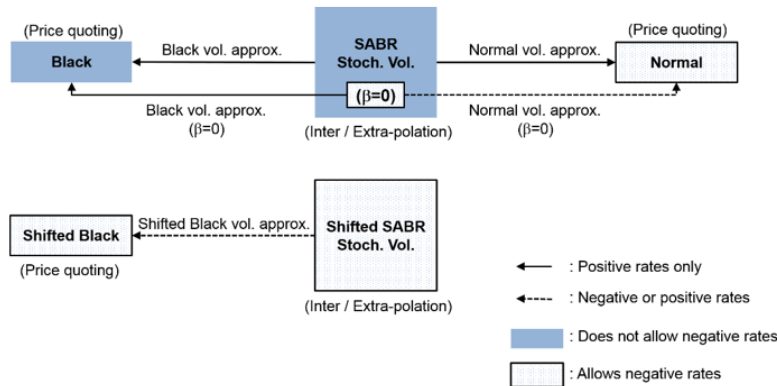
The Shifted Black volatilities are associated with the Shifted Black model (also known as “Displaced Diffusion” or “Shifted Lognormal” model):

$$dF = \sigma_{Shifted\_Black}(F + Shift)dW$$

The Shifted Black model is essentially the same as the Black model, except that it models the movements of  $(F + Shift)$  as the underlying asset, instead of  $F$  (where  $F$  is the forward swap rate in the case of swaptions, and the forward rate in the case of caplets and floorlets). So, the Shifted Black model allows negative rates, with a fixed negative lower bound defined by the amount of shift, that is, the zero lower bound of the Black model has been shifted.

### Shifted SABR

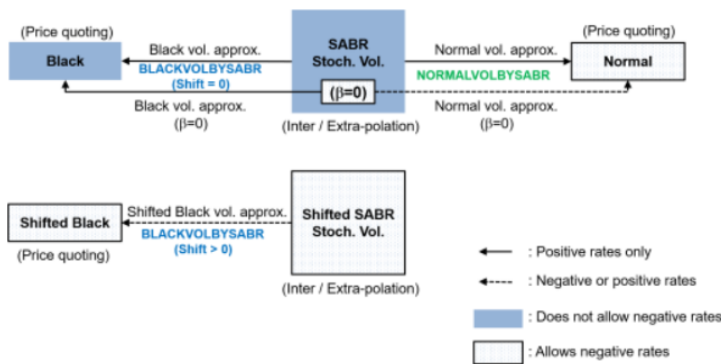
The introduction of negative interest rates also called for an update in the method for interpolating the volatilities quoted in the market. The following shows the connections between the volatilities and the SABR models:



As shown, the Black and Normal volatility approximations allow you to use the SABR model with the Black and Normal model option pricing formulas. However, although the Normal model itself allows negative rates and the SABR model has an implied Normal volatility approximation, the underlying dynamics of the SABR model do not allow negative rates, unless  $\beta = 0$ . In the Shifted SABR model, the Shifted Black volatility approximation can be used to allow negative rates with a fixed negative lower bound defined by the amount of shift.

### Implied Normal Volatility and SABR

You can compute the implied Normal volatility in terms of the SABR model parameters, for either  $\beta = 0$  (Normal SABR), or any other value of  $\beta$  allowed by the SABR model ( $0 \leq \beta \leq 1$ ) using `normalvolbysabr`.



`normalvolbysabr` computes the implied Normal volatility  $\sigma_N$  in terms of the SABR model parameters. Using `normalvolbysabr` to compute  $\sigma_N$ , you can then use this with other functions for Normal model pricing (for example, `capbynormal`, `floorbynormal`, and `swaptionbyblk`).

### See Also

`capbyblk` | `capbynormal` | `floorbyblk` | `floorbynormal` | `normalvolbysabr` | `swaptionbyblk` | `swaptionbynormal`

### Related Examples

- “Price Swaptions with Negative Strikes Using the Shifted SABR Model” on page 2-27
- “Calibrate the SABR Model using Normal (Bachelier) Volatilities with Negative Strikes”

### More About

- “Supported Interest-Rate Instruments” on page 2-2

## Price Swaptions with Negative Strikes Using the Shifted SABR Model

This example shows how to price swaptions with negative strikes by using the Shifted SABR model. The market Shifted Black volatilities are used to calibrate the Shifted SABR model parameters. The calibrated Shifted SABR model is then used to compute the Shifted Black volatilities for negative strikes.

The swaptions with negative strikes are then priced using the computed Shifted Black volatilities and the `swaptionbyblk` function with the 'Shift' parameter set to the prespecified shift. Similarly, Shifted SABR Greeks can be computed by using the `optsensbysabr` function by setting the 'Shift' parameter. Finally, from the swaption prices, the probability density of the underlying asset is computed to show that the swaption prices imply positive probability densities for some negative strikes.

### Load the market data.

First, load the market interest rates and swaption volatility data. The market swaption volatilities are quoted in terms of Shifted Black volatilities with a 0.8 percent shift.

Define RateSpec.

```
ValuationDate = '5-Apr-2016';
EndDates = datemnth(ValuationDate,[1 2 3 6 9 12*[1 2 3 4 5 6 7 8 9 10 12]]);
ZeroRates = [-0.34 -0.29 -0.25 -0.13 -0.07 -0.02 0.010 0.025 ...
             0.031 0.040 0.052 0.090 0.190 0.290 0.410 0.520]'/100;
Compounding = 1;
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',ValuationDate, ...
                    'EndDates',EndDates,'Rates',ZeroRates,'Compounding',Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [16x1 double]
    Rates: [16x1 double]
    EndTimes: [16x1 double]
    StartTimes: [16x1 double]
    EndDates: [16x1 double]
    StartDates: 736425
    ValuationDate: 736425
    Basis: 0
    EndMonthRule: 1
```

Define the swaption.

```
SwaptionSettle = '5-Apr-2016';
SwaptionExerciseDate = '5-Apr-2017';
SwapMaturity = '5-Apr-2022';
Reset = 1;
OptSpec = 'call';
TimeToExercise = yearfrac(SwaptionSettle,SwaptionExerciseDate);
```

Use `swapyzero` to compute the forward swap rate.

```
LegRate = [NaN 0]; % To compute the forward swap rate, set the fixed rate to NaN.
[~, CurrentForwardValue] = swapyzero(RateSpec,LegRate,SwaptionSettle,SwapMaturity,...
'StartDate',SwaptionExerciseDate)
```

```
CurrentForwardValue = 6.6384e-04
```

Specify amount of shift in decimals for Shifted Black and Shifted SABR models.

```
Shift = 0.008; % 0.8 percent shift
```

Load the market implied Shifted Black volatility data for swaptions.

```
MarketShiftedBlackVolatilities = [21.1; 15.3; 14.0; 14.6; 16.0; 17.7; 19.8; 23.9; 26.2];
StrikeGrid = [-0.5; -0.25; -0.125; 0; 0.125; 0.25; 0.5; 1.0; 1.5]/100;
MarketStrikes = CurrentForwardValue + StrikeGrid;
ATMShiftedBlackVolatility = MarketShiftedBlackVolatilities(StrikeGrid==0);
```

### Calibrate the Shifted SABR model parameters.

To better represent the market at-the-money volatility, the Alpha parameter value is implied by the market at-the-money volatility. This is similar to the "Method 2" in "Calibrate the SABR Model" on page 2-36. However, note the addition of `Shift` to `CurrentForwardValue` and the use of the 'Shift' parameter with `blackvolbysabr`. The Beta parameter is predetermined at 0.5.

```
Beta = 0.5;
```

This function solves the Shifted SABR at-the-money volatility equation as a polynomial of Alpha. Note the addition of `Shift` to `CurrentForwardValue`.

```
alpharoots = @(Rho,Nu) roots([...
(1 - Beta)^2*TimeToExercise/24/(CurrentForwardValue + Shift)^(2 - 2*Beta) ...
Rho*Beta*Nu*TimeToExercise/4/(CurrentForwardValue + Shift)^(1 - Beta) ...
(1 + (2 - 3*Rho^2)*Nu^2*TimeToExercise/24) ...
-ATMShiftedBlackVolatility*(CurrentForwardValue + Shift)^(1 - Beta)]);
```

This function converts at-the-money volatility into Alpha by picking the smallest positive real root.

```
atmVol2ShiftedSabrAlpha = @(Rho,Nu) min(real(arrayfun(@(x) ...
    x*(x>0) + realmax*(x<0 || abs(imag(x))>1e-6), alphasroots(Rho,Nu))));
```

Fit Rho and Nu (while converting at-the-money volatility into Alpha). Note the 'Shift' parameter of blackvolbysabr is set to the prespecified shift.

```
objFun = @(X) MarketShiftedBlackVolatilities - ...
    blackvolbysabr(atmVol2ShiftedSabrAlpha(X(1), X(2)), ...
    Beta, X(1), X(2), SwaptionSettle, SwaptionExerciseDate, CurrentForwardValue, ...
    MarketStrikes, 'Shift', Shift);
```

```
options = optimoptions('lsqnonlin','Display','none');
X = lsqnonlin(objFun, [0 0.5], [-1 0], [1 Inf], options);
Rho = X(1);
Nu = X(2);
```

Get the final Alpha from the calibrated parameters.

```
Alpha = atmVol2ShiftedSabrAlpha(Rho, Nu)
```

```
Alpha = 0.0133
```

Show the calibrated Shifted SABR parameters.

```
CalibratedParameters = array2table([Shift Alpha Beta Rho Nu],...
    'VariableNames',{'Shift' 'Alpha' 'Beta' 'Rho' 'Nu'},...
    'RowNames',{'1Y into 5Y'})
```

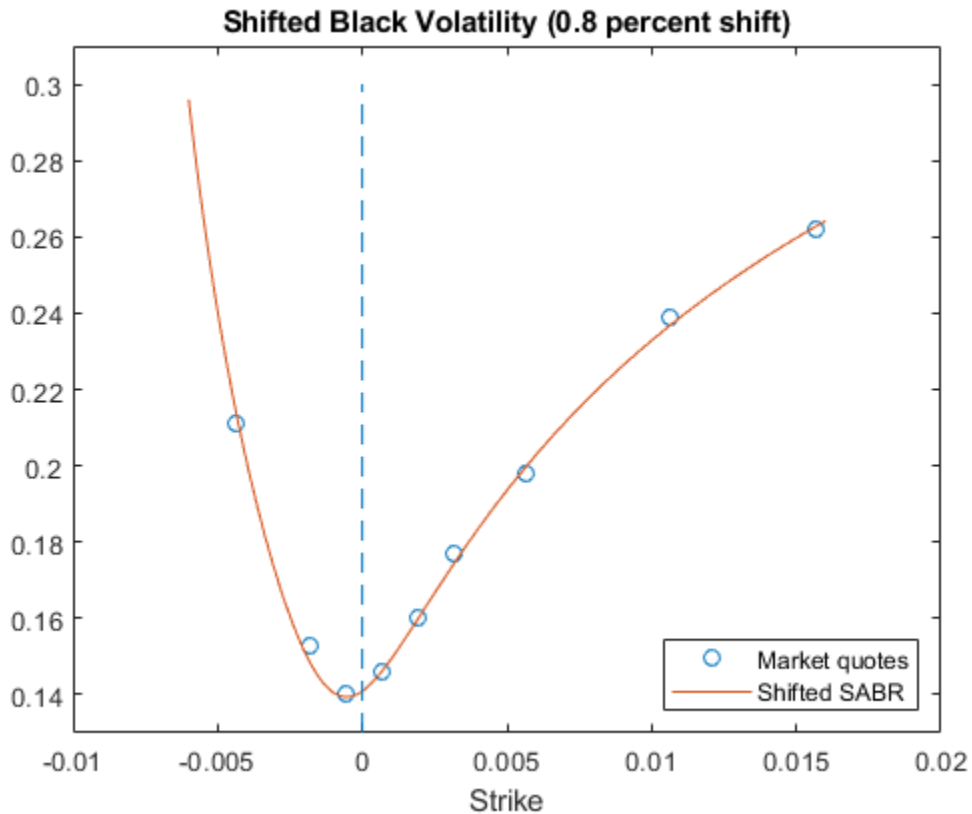
```
CalibratedParameters=1x5 table
           Shift      Alpha      Beta      Rho      Nu
           -----
1Y into 5Y  0.008    0.013345    0.5    0.46698    0.49816
```

### Compute the swaption volatilities using the calibrated Shifted SABR model.

Use blackvolbysabr with the 'Shift' parameter.

```
Strikes = (-0.6:0.01:1.6)/100; % Include negative strikes.
SABRShiftedBlackVolatilities = blackvolbysabr(Alpha, Beta, Rho, Nu, SwaptionSettle, ..
    SwaptionExerciseDate, CurrentForwardValue, Strikes, 'Shift', Shift);
```

```
figure;  
plot(MarketStrikes, MarketShiftedBlackVolatilities, 'o', ...  
     Strikes, SABRShiftedBlackVolatilities);  
h = gca;  
line([0,0],[min(h.YLim),max(h.YLim)], 'LineStyle','--');  
ylim([0.13 0.31])  
xlabel('Strike');  
legend('Market quotes','Shifted SABR', 'location','southeast');  
title(['Shifted Black Volatility (' ,num2str(Shift*100),' percent shift)']);
```

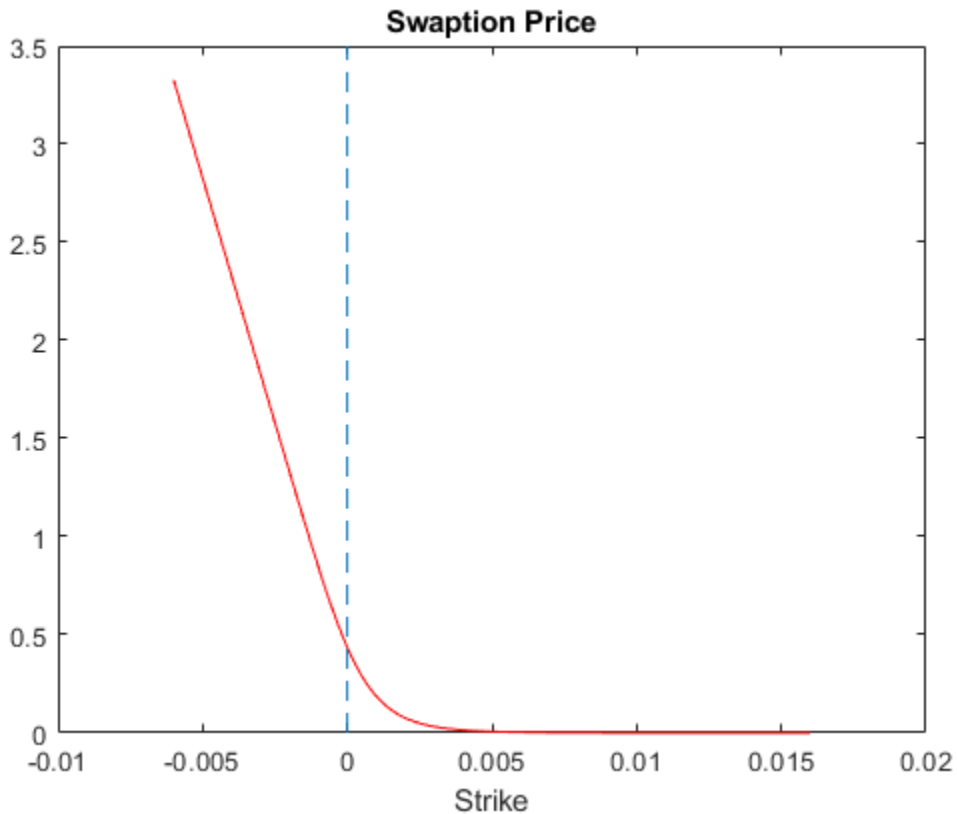




**Price the swaptions, including those with negative strikes.**

Use `swaptionbyblk` with the 'Shift' parameter to compute swaption prices using the Shifted Black model.

```
SwaptionPrices = swaptionbyblk(RateSpec, OptSpec, Strikes, SwaptionSettle, SwaptionExerciseDate, SwapMaturity, SABRShiftedBlackVolatilities, 'Reset', Reset, 'Shift', Shift);  
figure;  
plot(Strikes, SwaptionPrices, 'r');  
h = gca;  
line([0,0],[min(h.YLim),max(h.YLim)],'LineStyle','--');  
xlabel('Strike');  
title('Swaption Price');
```

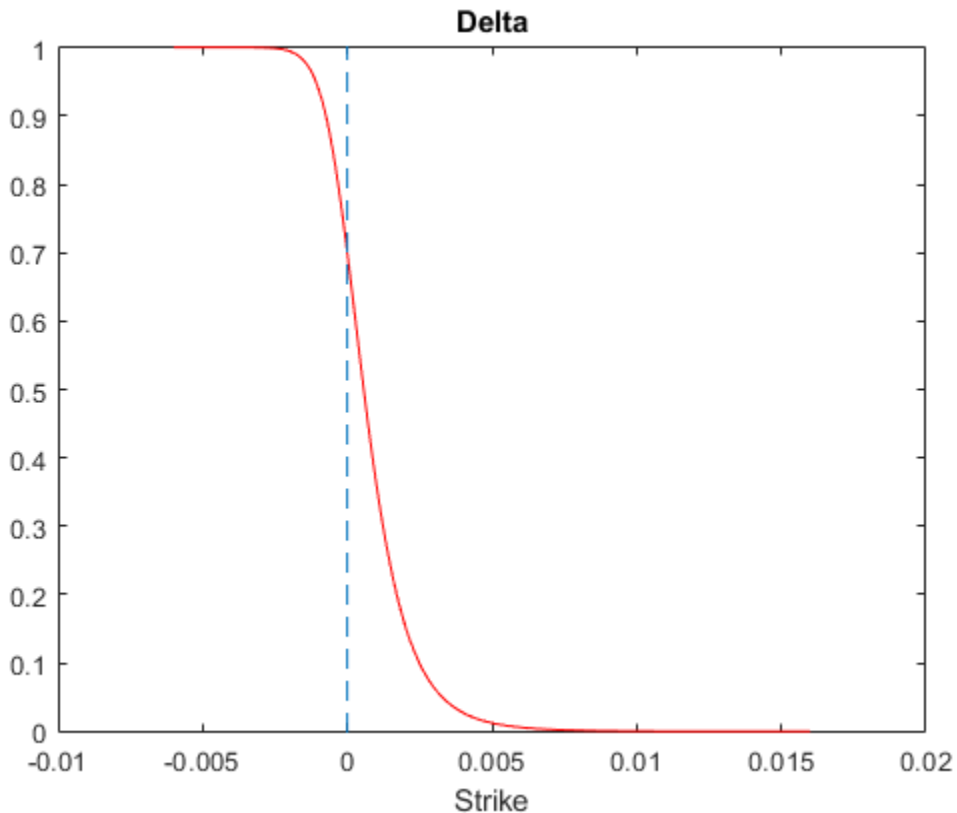


### Compute Shifted SABR Delta.

Use `optsensbysabr` with the 'Shift' parameter to compute Delta using the Shifted SABR model.

```
ShiftedSABRDelta = optsensbysabr(RateSpec, Alpha, Beta, Rho, Nu, SwaptionSettle, ...  
SwaptionExerciseDate, CurrentForwardValue, Strikes, OptSpec, 'Shift', Shift);
```

```
figure;  
plot(Strikes,ShiftedSABRDelta,'r-');  
ylim([-0.002 1.002]);  
h = gca;  
line([0,0],[min(h.YLim),max(h.YLim)],'LineStyle','--');  
xlabel('Strike');  
title('Delta');
```



### Compute the probability density.

The risk-neutral probability density of the terminal underlying asset prices can be approximated as the second derivative of swaption prices with respect to strike (Breen and Litzenberger, 1978). As can be seen in the plot below, the computed probability density is positive for some negative rates above -0.8 percent (the lower bound determined by 'Shift').

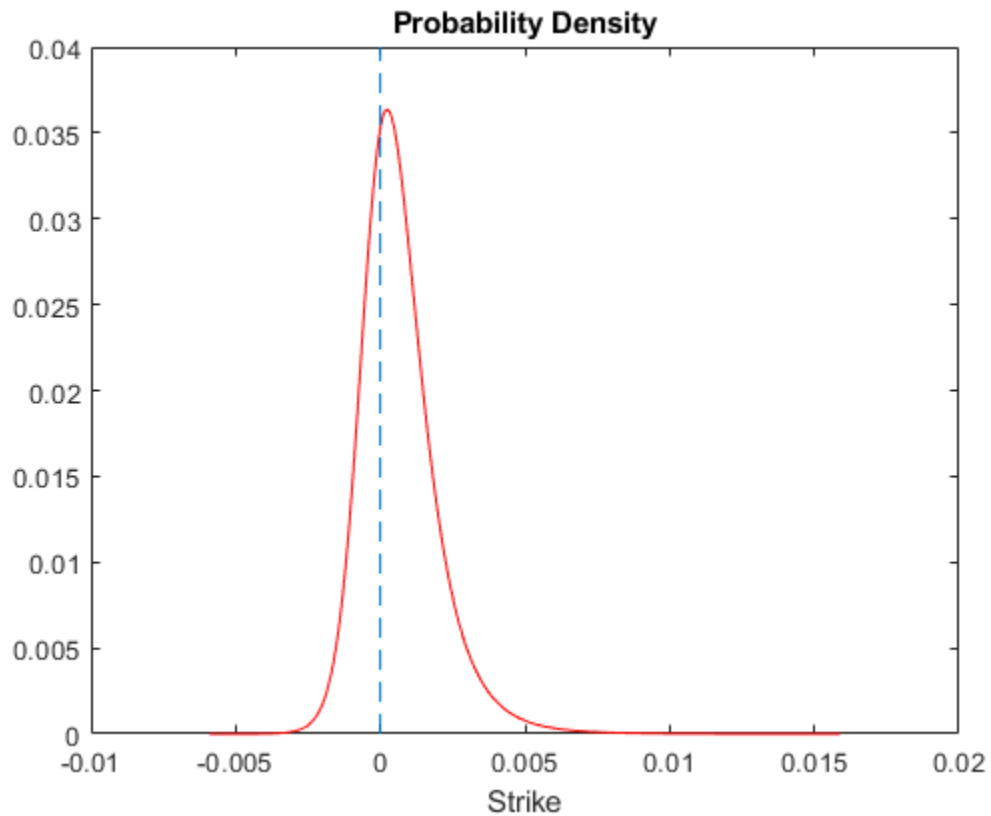
```
NumGrids = length(Strokes);
ProbDensity = zeros(NumGrids-2,1);
dStrike = mean(diff(Strokes));
```

```
for k = 2:(NumGrids-1)
    ProbDensity(k-1) = (SwaptionPrices(k-1) - 2*SwaptionPrices(k) + SwaptionPrices(k+1)) / (dStrike^2);
```

```
end
```

```
ProbDensity = ProbDensity./sum(ProbDensity);  
ProbStrikes = Strikes(2:end-1);
```

```
figure;  
plot(ProbStrikes,ProbDensity,'r-');  
h = gca;  
line([0,0],[min(h.YLim),max(h.YLim)],'LineStyle','--');  
xlabel('Strike');  
title('Probability Density');
```



## References

Hagan, P. S., Kumar, D., Lesniewski, A. S. and Woodward, D. E. "Managing Smile Risk." *Wilmott Magazine*. 2002.

Kienitz, J. *Interest Rate Derivatives Explained*. Vol. 1. Palgrave MacMillan, 2014.

Breeden, D. T. and Litzenberger, R. H. "Prices of State-Contingent Claims Implicit in Option Prices." *Journal Business*. Vol. 51. 1978.

## See Also

[capbyblk](#) | [capbynormal](#) | [capvolstrip](#) | [floorbyblk](#) | [floorbynormal](#) | [floorvolstrip](#) | [optsensbysabr](#) | [swaptionbyblk](#) | [swaptionbynormal](#)

## Related Examples

- "Calibrate the SABR Model" on page 2-36
- "Price a Swaption Using the SABR Model" on page 2-42

## More About

- "Work with Negative Interest Rates" on page 2-22

## Calibrate the SABR Model

This example shows how to use two different methods to calibrate the SABR stochastic volatility model from market implied Black volatilities. Both approaches use `blackvolbysabr`.

In this section...
“Load Market Implied Black Volatility Data” on page 2-36
“Method 1: Calibrate Alpha, Rho, and Nu Directly” on page 2-37
“Method 2: Calibrate Rho and Nu by Implying Alpha from At-The-Money Volatility” on page 2-37
“Use the Calibrated Models” on page 2-39
“References” on page 2-41

### Load Market Implied Black Volatility Data

This example shows how to set up hypothetical market implied Black volatilities for European swaptions over a range of strikes before calibration. The swaptions expire in three years from the `Settle` date and have 10-year swaps as the underlying instrument. The rates are expressed in decimals. (Changing the units affect the numerical value and interpretation of the `Alpha` input parameter to the function `blackvolbysabr`.)

Load the market implied Black volatility data for swaptions expiring in three years.

```
Settle = '12-Jun-2013';
```

```
ExerciseDate = '12-Jun-2016';
```

```
MarketStrikes = [2.0 2.5 3.0 3.5 4.0 4.5 5.0]'/100;
```

```
MarketVolatilities = [45.6 41.6 37.9 36.6 37.8 39.2 40.0]'/100;
```

At the time of `Settle`, define the underlying forward rate and the at-the-money volatility.

```
CurrentForwardValue = MarketStrikes(4)
```

```
ATMVolatility = MarketVolatilities(4)
```

```
CurrentForwardValue =
```

```
    0.0350
```

```
ATMVolatility =
    0.3660
```

## Method 1: Calibrate Alpha, Rho, and Nu Directly

This example shows how to calibrate the Alpha, Rho, and Nu input parameters directly. The value of Beta is predetermined either by fitting historical market volatility data or by choosing a value deemed appropriate for that market [1].

Define the predetermined Beta.

```
Beta1 = 0.5;
```

After fixing the value of  $\beta$  (Beta), the parameters  $\alpha$  (Alpha),  $\rho$  (Rho), and  $\nu$  (Nu) are all fitted directly. The Optimization Toolbox™ function `lsqnonlin` generates the parameter values that minimize the squared error between the market volatilities and the volatilities computed by `blackvolbysabr`.

```
% Calibrate Alpha, Rho, and Nu
objFun = @(X) MarketVolatilities - ...
    blackvolbysabr(X(1), Beta1, X(2), X(3), Settle, ...
    ExerciseDate, CurrentForwardValue, MarketStrikes);

X = lsqnonlin(objFun, [0.5 0 0.5], [0 -1 0], [Inf 1 Inf]);

Alpha1 = X(1);
Rho1 = X(2);
Nu1 = X(3);
```

Local minimum possible.

`lsqnonlin` stopped because the final change in the sum of squares relative to its initial value is less than the default value of the function tolerance.

## Method 2: Calibrate Rho and Nu by Implying Alpha from At-The-Money Volatility

This example shows how to use an alternative calibration method where the value of  $\beta$  (Beta) is again predetermined as in Method 1.

Define the predetermined Beta.

```
Beta2 = 0.5;
```

However, after fixing the value of  $\beta$  (Beta), the parameters  $\rho$  (Rho), and  $\nu$  (Nu) are fitted directly while  $\alpha$  (Alpha) is implied from the market at-the-money volatility. Models calibrated using this method produce at-the-money volatilities that are equal to market quotes. This approach is widely used in swaptions, where at-the-money volatilities are quoted most frequently and are important to match. To imply  $\alpha$  (Alpha) from market at-the-money volatility ( $\sigma_{ATM}$ ), the following cubic polynomial is solved for  $\alpha$  (Alpha), and the smallest positive real root is selected [2].

$$\frac{(1-\beta)^2 T}{24 F^{(2-2\beta)}} \alpha^3 + \frac{\rho \beta \nu T}{4 F^{(1-\beta)}} \alpha^2 + \left(1 + \frac{2-3\rho^2}{24} \nu^2 T\right) \alpha - \sigma_{ATM} F^{(1-\beta)} = 0$$

where:

- $F$  is the current forward value.
- $T$  is the year fraction to maturity.

To accomplish this, define an anonymous function as:

```
% Year fraction from Settle to option maturity
T = yearfrac(Settle, ExerciseDate, 1);

% This function solves the SABR at-the-money volatility equation as a
% polynomial of Alpha
alpharoots = @(Rho,Nu) roots([...
    (1 - Beta2)^2*T/24/CurrentForwardValue^(2 - 2*Beta2) ...
    Rho*Beta2*Nu*T/4/CurrentForwardValue^(1 - Beta2) ...
    (1 + (2 - 3*Rho^2)*Nu^2*T/24) ...
    -ATMVolatility*CurrentForwardValue^(1 - Beta2)]);

% This function converts at-the-money volatility into Alpha by picking the
% smallest positive real root
atmVol2SabrAlpha = @(Rho,Nu) min(real(arrayfun(@(x) ...
    x*(x>0) + realmax*(x<0 | | abs(imag(x))>1e-6), alpharoots(Rho,Nu))));

The function atmVol2SabrAlpha converts at-the-money volatility into  $\alpha$  (Alpha) for a
given set of  $\rho$  (Rho) and  $\nu$  (Nu). This function is then used in the objective function to fit
parameters  $\rho$  (Rho) and  $\nu$  (Nu).

% Calibrate Rho and Nu (while converting at-the-money volatility into Alpha
% using atmVol2SabrAlpha)
objFun = @(X) MarketVolatilities - ...
```



```

blackvolbysabr(atmVol2SabrAlpha(X(1), X(2)), ...
Beta2, X(1), X(2), Settle, ExerciseDate, CurrentForwardValue, ...
MarketStrikes);

X = lsqnonlin(objFun, [0 0.5], [-1 0], [1 Inf]);

Rho2 = X(1);
Nu2 = X(2);

Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the function tolerance.

The calibrated parameter  $\alpha$  (Alpha) is computed using the calibrated parameters  $\rho$  (Rho)
and  $\nu$  (Nu).

% Obtain final Alpha from at-the-money volatility using calibrated parameters
Alpha2 = atmVol2SabrAlpha(Rho2, Nu2);

% Display calibrated parameters
C = {Alpha1 Beta1 Rho1 Nu1;Alpha2 Beta2 Rho2 Nu2};
CalibratedParameters = cell2table(C,...
    'VariableNames',{'Alpha' 'Beta' 'Rho' 'Nu'},...
    'RowNames',{'Method 1';'Method 2'})

CalibratedParameters =

           Alpha           Beta           Rho           Nu
    _____    _____    _____    _____
Method 1    0.060277         0.5         0.2097         0.75091
Method 2    0.058484         0.5         0.20568         0.79647

```

## Use the Calibrated Models

This example shows how to use the calibrated models to compute new volatilities at any strike value.

Compute volatilities for models calibrated using Method 1 and Method 2 and plot the results.

```
PlottingStrikes = (1.75:0.1:5.50)'/100;
```

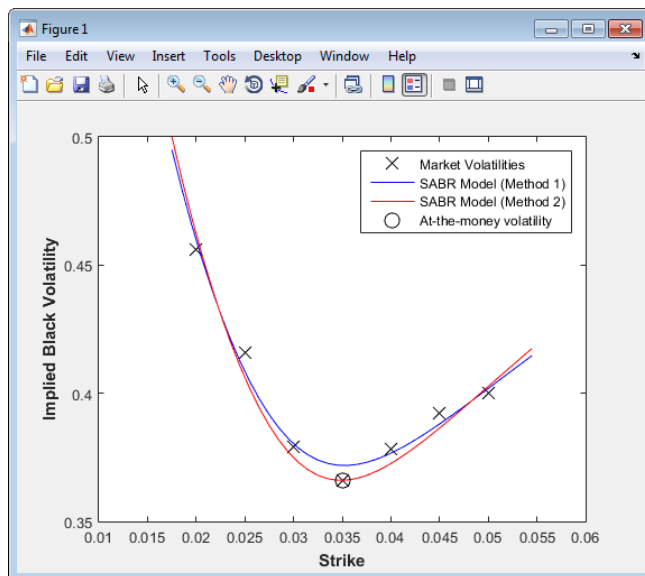
```

% Compute volatilities for model calibrated by Method 1
ComputedVols1 = blackvolbysabr(Alpha1, Beta1, Rho1, Nu1, Settle, ...
    ExerciseDate, CurrentForwardValue, PlottingStrikes);

% Compute volatilities for model calibrated by Method 2
ComputedVols2 = blackvolbysabr(Alpha2, Beta2, Rho2, Nu2, Settle, ...
    ExerciseDate, CurrentForwardValue, PlottingStrikes);

figure;
plot(MarketStrikes,MarketVolatilities,'xk',...
    PlottingStrikes,ComputedVols1,'b', ...
    PlottingStrikes,ComputedVols2,'r', ...
    CurrentForwardValue,ATMVolatility,'ok',...
    'MarkerSize',10);
xlim([0.01 0.06]);
ylim([0.35 0.5]);
xlabel('Strike', 'FontWeight', 'bold');
ylabel('Implied Black Volatility', 'FontWeight', 'bold');
legend('Market Volatilities', 'SABR Model (Method 1)',...
    'SABR Model (Method 2)', 'At-the-money volatility');

```



The model calibrated using Method 2 reproduces the market at-the-money volatility (marked with a circle) exactly.

## References

[1] Hagan, P. S., Kumar, D., Lesniewski, A. S. and Woodward, D. E., *Managing smile risk*, Wilmott Magazine, 2002.

[2] West, G., "Calibration of the SABR Model in Illiquid Markets," *Applied Mathematical Finance*, 12(4), pp. 371-385, 2004.

## See Also

`blackvolbysabr` | `optsensbysabr` | `swaptionbyblk`

## Related Examples

- "Price a Swaption Using the SABR Model" on page 2-42

## Price a Swaption Using the SABR Model

This example shows how to price a swaption using the SABR model. First, a swaption volatility surface is constructed from market volatilities. This is done by calibrating the SABR model parameters separately for each swaption maturity. The swaption price is then computed by using the implied Black volatility on the surface as an input to the `swaptionbyblk` function.

### Step 1. Load market swaption volatility data.

Load the market implied Black volatility data for swaptions.

```
Settle = '12-Jun-2013';
ExerciseDates = {'12-Sep-2013'; '12-Jun-2014'; '12-Jun-2015'; ...
    '12-Jun-2016'; '12-Jun-2017'; '12-Jun-2018'; '12-Jun-2020'; ...
    '12-Jun-2023'};

YearsToExercise = yearfrac(Settle, ExerciseDates, 1);
NumMaturities = length(YearsToExercise);

MarketVolatilities = [ ...
    57.6 53.7 49.4 45.6 44.1 41.1 35.2 32.0
    46.6 46.9 44.8 41.6 39.8 37.4 33.4 31.0
    35.9 39.3 39.6 37.9 37.2 34.7 30.5 28.9
    34.1 36.5 37.8 36.6 35.0 31.9 28.1 26.6
    41.0 41.3 39.5 37.8 36.0 32.6 29.0 26.0
    45.8 43.4 41.9 39.2 36.9 33.2 29.6 26.3
    50.3 46.9 44.0 40.0 37.5 33.8 30.2 27.3]/100;

MarketStrikes = [ ...
    1.00 1.25 1.68 2.00 2.26 2.41 2.58 2.62;
    1.50 1.75 2.18 2.50 2.76 2.91 3.08 3.12;
    2.00 2.25 2.68 3.00 3.26 3.41 3.58 3.62;
    2.50 2.75 3.18 3.50 3.76 3.91 4.08 4.12;
    3.00 3.25 3.68 4.00 4.26 4.41 4.58 4.62;
    3.50 3.75 4.18 4.50 4.76 4.91 5.08 5.12;
    4.00 4.25 4.68 5.00 5.26 5.41 5.58 5.62]/100;

CurrentForwardValues = MarketStrikes(4,:);

CurrentForwardValues = 1x8

    0.0250    0.0275    0.0318    0.0350    0.0376    0.0391    0.0408    0.0412
```

```

ATMVolatilities = MarketVolatilities(4,:);
ATMVolatilities = 1x8
    0.3410    0.3650    0.3780    0.3660    0.3500    0.3190    0.2810    0.2660

```

The current underlying forward rates and the corresponding at-the-money volatilities across the eight swaption maturities are represented in the fourth rows of the two matrices.

## Step 2. Calibrate the SABR model parameters for each swaption maturity.

Using a model implemented in the function `blackvolbysabr`, a static SABR model, where the model parameters are assumed to be constant with respect to time, the parameters are calibrated separately for each swaption maturity (years to exercise) in a `for` loop. To better represent market at-the-money volatilities, the Alpha parameter values are implied by the market at-the-money volatilities (see "Method 2" for "Calibrate the SABR Model" on page 2-36).

Define the predetermined Beta, calibrate SABR model parameters for each swaption maturity and display calibrated parameters in a table.

```

Beta = 0.5;
Betas = repmat(Beta, NumMaturities, 1);
Alphas = zeros(NumMaturities, 1);
Rhos = zeros(NumMaturities, 1);
Nus = zeros(NumMaturities, 1);

options = optimoptions('lsqnonlin','Display','none');

for k = 1:NumMaturities
    % This function solves the SABR at-the-money volatility equation as a
    % polynomial of Alpha
    alpharoots = @(Rho,Nu) roots([...
        (1 - Beta)^2*YearsToExercise(k)/24/CurrentForwardValues(k)^(2 - 2*Beta) ...
        Rho*Beta*Nu*YearsToExercise(k)/4/CurrentForwardValues(k)^(1 - Beta) ...
        (1 + (2 - 3*Rho^2)*Nu^2*YearsToExercise(k)/24) ...
        -ATMVolatilities(k)*CurrentForwardValues(k)^(1 - Beta)]);

    % This function converts at-the-money volatility into Alpha by picking the
    % smallest positive real root
    atmVol2SabrAlpha = @(Rho,Nu) min(real(arrayfun(@(x) ...
        x*(x>0) + realmax*(x<0 || abs(imag(x))>1e-6), alpharoots(Rho,Nu))));

```

```

% Fit Rho and Nu (while converting at-the-money volatility into Alpha)
objFun = @(X) MarketVolatilities(:,k) - ...
    blackvolbysabr(atmVol2SabrAlpha(X(1), X(2)), ...
    Beta, X(1), X(2), Settle, ExerciseDates(k), CurrentForwardValues(k), ...
    MarketStrikes(:,k));

X = lsqnonlin(objFun, [0 0.5], [-1 0], [1 Inf], options);
Rho = X(1);
Nu = X(2);

% Get final Alpha from the calibrated parameters
Alpha = atmVol2SabrAlpha(Rho, Nu);

Alphas(k) = Alpha;
Rhos(k) = Rho;
Nus(k) = Nu;
end

CalibratedParameters = array2table([Alphas Betas Rhos Nus], ...
    'VariableNames',{'Alpha' 'Beta' 'Rho' 'Nu'}, ...
    'RowNames',{'3M into 10Y';'1Y into 10Y';...
    '2Y into 10Y';'3Y into 10Y';'4Y into 10Y';...
    '5Y into 10Y';'7Y into 10Y';'10Y into 10Y'})

```

CalibratedParameters=8×4 table

	Alpha	Beta	Rho	Nu
3M into 10Y	0.051947	0.5	0.39572	1.4146
1Y into 10Y	0.054697	0.5	0.2955	1.1257
2Y into 10Y	0.058433	0.5	0.24175	0.93463
3Y into 10Y	0.058484	0.5	0.20568	0.79647
4Y into 10Y	0.056054	0.5	0.13685	0.76993
5Y into 10Y	0.051072	0.5	0.060285	0.73595
7Y into 10Y	0.04475	0.5	0.083385	0.66341
10Y into 10Y	0.044548	0.5	0.02261	0.49487

### Step 3. Construct a volatility surface.

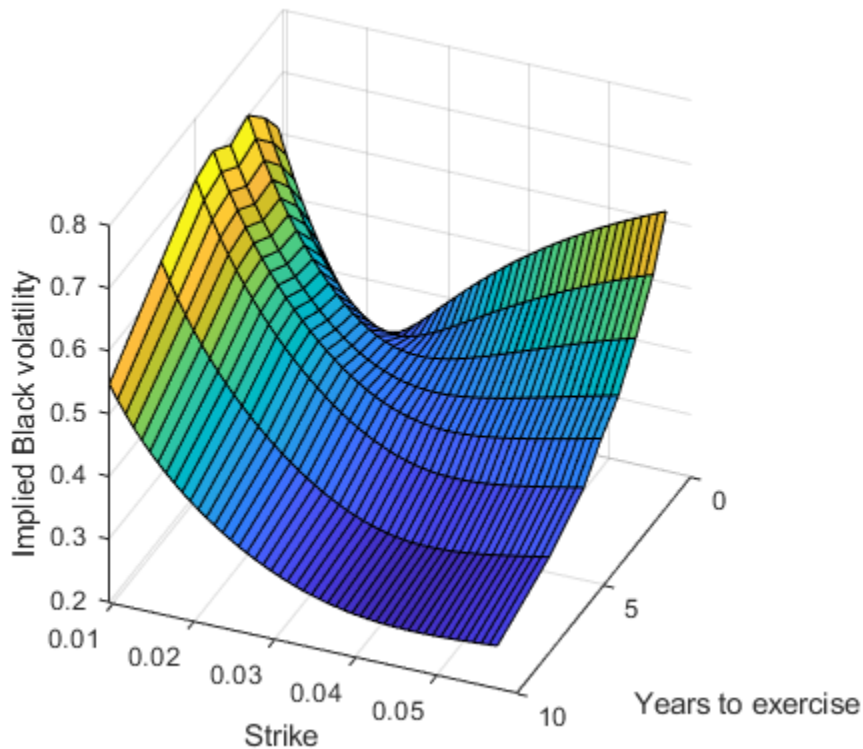
Use the calibrated model to compute new volatilities at any strike value to produce a smooth smile for a given maturity. This can be repeated for each maturity to form a volatility surface

Compute volatilities using the calibrated models for each maturity and plot the volatility surface.

```
PlottingStrikes = (0.95:0.1:5.8)'/100;
ComputedVols = zeros(length(PlottingStrikes), NumMaturities);

for k = 1:NumMaturities
    ComputedVols(:,k) = blackvolbysabr(Alphas(k), Betas(k), Rhos(k), Nus(k), Settle, ...
        ExerciseDates(k), CurrentForwardValues(k), PlottingStrikes);
end

figure;
surf(YearsToExercise, PlottingStrikes, ComputedVols);
xlim([0 10]); ylim([0.0095 0.06]); zlim([0.2 0.8]);
view(113,32);
set(gca, 'Position', [0.13 0.11 0.775 0.815], ...
    'PlotBoxAspectRatioMode', 'manual');
xlabel('Years to exercise', 'Fontweight', 'bold');
ylabel('Strike', 'Fontweight', 'bold');
zlabel('Implied Black volatility', 'Fontweight', 'bold');
```



Note, in this volatility surface, the smiles tend to get flatter for longer swaption maturities (years to exercise). This is consistent with the  $\nu$  parameter values tending to decrease with swaption maturity, as shown previously in the table for `CalibratedParameters`.

#### Step 4. Use `swaptionbyblk` to price a swaption.

Use the volatility surface to price a swaption that matures in five years. Define a swaption (for a 10-year swap) that matures in five years and use the interest-rate term structure at the time of the swaption `Settle` date to define the `RateSpec`. Use the `RateSpec` to compute the current forward swap rate using the `swapbyzero` function. Compute the SABR implied Black volatility for this swaption using the `blackvolbysabr` function (and it is marked with a red arrow in the figure that follows). Price the swaption using the SABR implied Black volatility as an input to the `swaptionbyblk` function.



```

% Define the swaption
SwaptionSettle = '12-Jun-2013';
SwaptionExerciseDate = '12-Jun-2018';
SwapMaturity = '12-Jun-2028';
Reset = 1;
OptSpec = 'call';
Strike = 0.0263;

% Define RateSpec
ValuationDate = '12-Jun-2013';
EndDates = {'12-Jul-2013'; '12-Sep-2013'; '12-Dec-2013'; '12-Jun-2014'; ...
            '12-Jun-2015'; '12-Jun-2016'; '12-Jun-2017'; '12-Jun-2018'; ...
            '12-Jun-2019'; '12-Jun-2020'; '12-Jun-2021'; '12-Jun-2022'; ...
            '12-Jun-2023'; '12-Jun-2025'; '12-Jun-2028'; '12-Jun-2033'};
Rates = [0.2 0.3 0.4 0.7 0.5 0.7 1.0 1.4 1.7 1.9 ...
         2.1 2.3 2.5 2.8 3.1 3.3]'/100;
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, ...
                    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [16x1 double]
    Rates: [16x1 double]
    EndTimes: [16x1 double]
    StartTimes: [16x1 double]
    EndDates: [16x1 double]
    StartDates: 735397
    ValuationDate: 735397
    Basis: 0
    EndMonthRule: 1

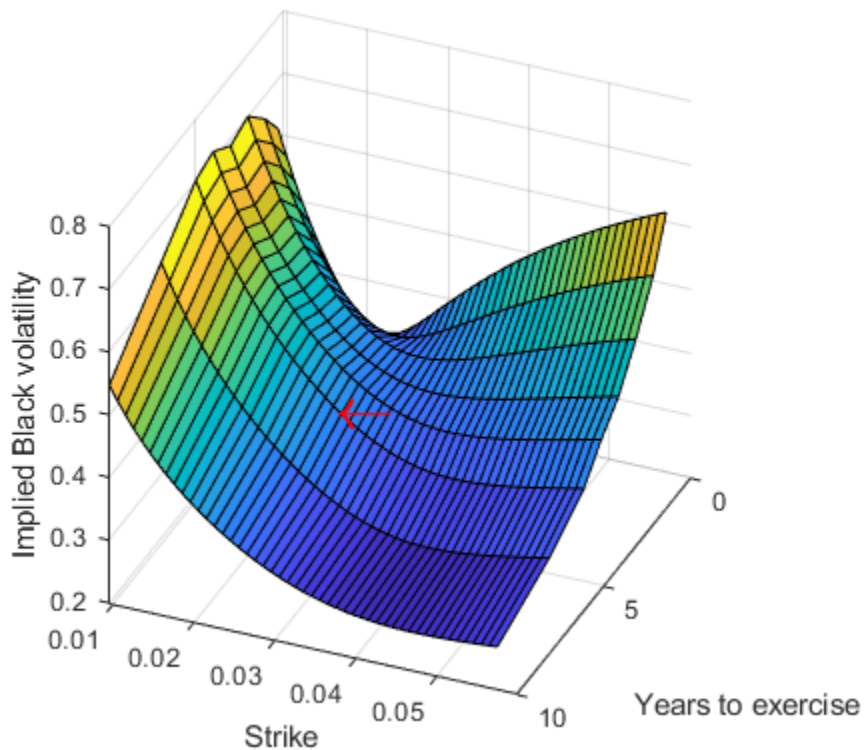
% Use swapyzero
LegRate = [NaN 0]; % To compute the forward swap rate, set the coupon rate to NaN.
[~, CurrentForwardSwapRate] = swapyzero(RateSpec, LegRate, SwaptionSettle, SwapMaturity,
    'StartDate', SwaptionExerciseDate);

% Use blackvolbysabr
SABRBlackVolatility = blackvolbysabr(Alphas(6), Betas(6), Rhos(6), Nus(6), SwaptionSettle,
    SwaptionExerciseDate, CurrentForwardSwapRate, Strike)

```

```
SABRBlackVolatility = 0.3932
```

```
text(YearsToExercise(6), Strike, SABRBlackVolatility, '\leftarrow', ...  
     'Color', 'r', 'FontWeight', 'bold', 'FontSize', 22);
```



```
% Use swaptionbyblk
```

```
Price = swaptionbyblk(RateSpec, OptSpec, Strike, SwaptionSettle, SwaptionExerciseDate,  
                     SwapMaturity, SABRBlackVolatility, 'Reset', Reset)
```

```
Price = 14.2403
```

[1] Hagan, P. S., Kumar, D., Lesniewski, A. S. and Woodward, D. E., "Managing Smile Risk," Wilmott Magazine, 2002.

[2] West, G., "Calibration of the SABR Model in Illiquid Markets," *Applied Mathematical Finance*, 12(4), pp. 371-385, 2004.

## See Also

[blackvolbysabr](#) | [swapbyzero](#) | [swaptionbyblk](#)

## Related Examples

- "Calibrate the SABR Model" on page 2-36

## Overview of Interest-Rate Tree Models

<b>In this section...</b>
“Interest-Rate Modeling” on page 2-50
“Rate and Price Trees” on page 2-51
“Viewing Rate or Price Movement” on page 2-52

### Interest-Rate Modeling

Financial Instruments Toolbox computes prices and sensitivities of interest-rate contingent claims based on several methods of modeling changes in interest rates over time:

- The interest-rate term structure

This model uses sets of zero-coupon bonds to predict changes in interest rates.

- Heath-Jarrow-Morton (HJM) model

The HJM model considers a given initial term structure of interest rates and a specification of the volatility of forward rates to build a tree representing the evolution of the interest rates, based on a statistical process.

- Black-Derman-Toy (BDT) model

In the BDT model, all security prices and rates depend on the short rate (annualized one-period interest rate). The model uses long rates and their volatilities to construct a tree of possible future short rates. The resulting tree can then be used to determine the value of interest-rate sensitive securities from this tree.

- Hull-White (HW) model

The Hull-White model incorporates the initial term structure of interest rates and the volatility term structure to build a trinomial recombining tree of short rates. The resulting tree is used to value interest-rate dependent securities. The implementation of the HW model in Financial Instruments Toolbox is limited to one factor.

- Black-Karasinski (BK) model

The BK model is a single-factor, log-normal version of the HW model.

For detailed information about interest-rate models, see:

- “Pricing Using Interest-Rate Term Structure” on page 2-72 for a discussion of price and sensitivity based on portfolios of zero-coupon bonds
- “Pricing Using Interest-Rate Tree Models” on page 2-99 for a discussion of price and sensitivity based on the HJM and BDT interest-rate models

---

**Note** Historically, the initial version of Financial Instruments Toolbox provided only the HJM interest-rate model. A later version added the BDT model. The current version adds both the HW and BK models. This section provides extensive examples of using the HJM and BDT models to compute prices and sensitivities of interest-rate based financial derivatives.

The HW and BK tree structures are similar to the BDT tree structure. To avoid needless repetition throughout this section, documentation is provided only where significant deviations from the BDT structure exist. Specifically, “HW and BK Tree Structures” on page 2-95 explains the few noteworthy differences among the various formats.

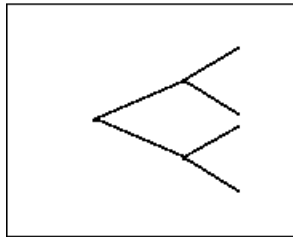
---

## Rate and Price Trees

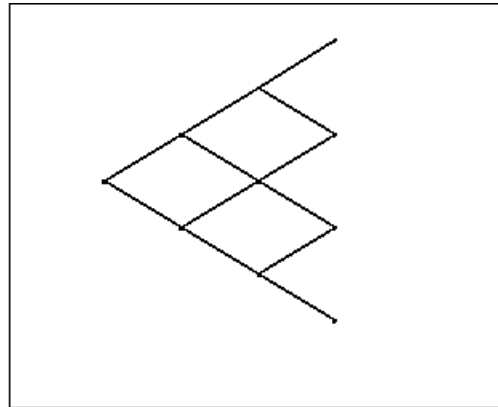
The interest-rate or price trees supported in this toolbox can be either binomial (two branches per node) or trinomial (three branches per node). Typically, binomial trees assume that underlying interest rates or prices can only either increase or decrease at each node. Trinomial trees allow for a more complex movement of rates or prices. With trinomial trees, the movement of rates or prices at each node is unrestricted (for example, up-up-up or unchanged-down-down).

### Types of Trees

Financial Instruments Toolbox trees can be classified as *bushy* or *recombining*. A bushy tree is a tree in which the number of branches increases exponentially relative to observation times; branches never recombine. In this context, a recombining tree is the opposite of a bushy tree. A recombining tree has branches that recombine over time. From any given node, the node reached by taking the path up-down is the same node reached by taking the path down-up. A bushy tree and a recombining binomial tree are illustrated next.



**Bushy Tree**



**Recombining Binomial Tree**

In this toolbox, the Heath-Jarrow-Morton model works with bushy trees. The Black-Derman-Toy model, on the other hand, works with recombining binomial trees.

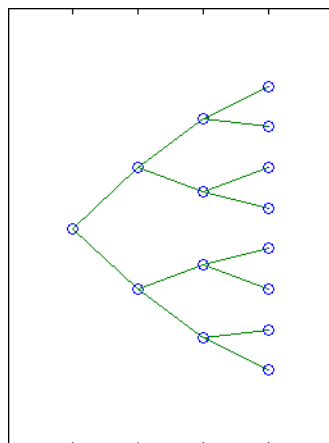
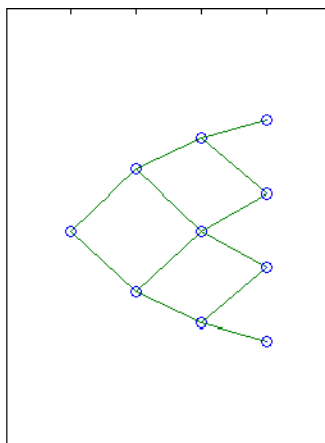
The other two interest rate models supported in this toolbox, Hull-White and Black-Karasinski, work with recombining trinomial trees.

### Viewing Rate or Price Movement

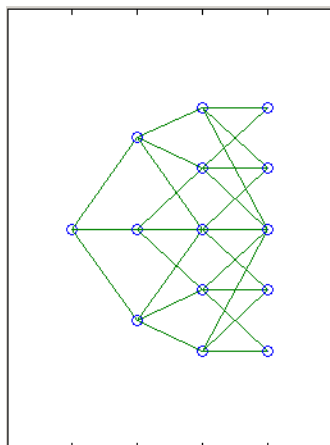
This toolbox provides the data file `deriv.mat` that contains four interest-rate based trees:

- `HJMTtree` — A bushy binomial tree
- `BDTtree` — A recombining binomial tree
- `HWTtree` and `BKTtree` — Recombining trinomial trees

The toolbox also provides the `treeviewer` function, which graphically displays the shape and data of price, interest rate, and cash flow trees. Viewed with `treeviewer`, the bushy shape of an HJM tree and the recombining shape of a BDT tree are apparent.

**HJMTree (bushy)****BDTTree (recombining)**

With `treeviewer`, you can also see the recombining shape of HW and BK trinomial trees.

**HWTree and BKTree (recombining)**

## See Also

`bdtprice` | `bdtens` | `bdttimespec` | `bdttree` | `bdtvolspec` | `bkprice` | `bksens` |  
`bktimespec` | `bktree` | `bkvolspec` | `bondbybdt` | `bondbybk` | `bondbyhjm` | `bondbyhw` |  
`bondbyzero` | `capbybdt` | `capbybk` | `capbyblk` | `capbyhjm` | `capbyhw` | `cfbybdt` |

cfbybk | cfbyhjm | cfbyhw | cfbyzero | fixedbybdt | fixedbybk | fixedbyhjm |  
fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw |  
floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk |  
floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec |  
hjmtree | hjmvolspec | hwcalbycap | hwcalbyfloor | hwprice | hwsens |  
hwtimespec | hwtree | hwwolspec | instbond | instcap | instcf | instfixed |  
instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat |  
instoptfloat | instrangefloat | instswap | instswaption | intenvprice |  
intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm  
| oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw |  
optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw |  
optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw |  
optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw |  
rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw |  
swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt |  
swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### Related Examples

- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Graphical Representation of Trees” on page 2-157
- “Understanding the Interest-Rate Term Structure” on page 2-55

### More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44



# Understanding the Interest-Rate Term Structure

## In this section...

“Introduction” on page 2-55

“Interest Rates Versus Discount Factors” on page 2-55

## Introduction

The *interest-rate term structure* represents the evolution of interest rates through time. In MATLAB, the interest-rate environment is encapsulated in a structure called `RateSpec` (*rate specification*). This structure holds all information required to completely identify the evolution of interest rates. Several functions included in Financial Instruments Toolbox software are dedicated to the creating and managing of the `RateSpec` structure. Many others take this structure as an input argument representing the evolution of interest rates.

Before looking further at the `RateSpec` structure, examine three functions that provide key functionality for working with interest rates: `disc2rate`, its opposite, `rate2disc`, and `ratetimes`. The first two functions map between discount factors and interest rates. The third function, `ratetimes`, calculates the effect of term changes on the interest rates.

## Interest Rates Versus Discount Factors

*Discount factors* are coefficients commonly used to find the current value of future cash flows. As such, there is a direct mapping between the rate applicable to a period of time, and the corresponding discount factor. The function `disc2rate` converts discount factors for a given term (period) into interest rates. The function `rate2disc` does the opposite; it converts interest rates applicable to a given term (period) into the corresponding discount factors.

### Calculating Discount Factors from Rates

As an example, consider these annualized zero-coupon bond rates.

From	To	Rate
15 Feb 2000	15 Aug 2000	0.05

From	To	Rate
15 Feb 2000	15 Feb 2001	0.056
15 Feb 2000	15 Aug 2001	0.06
15 Feb 2000	15 Feb 2002	0.065
15 Feb 2000	15 Aug 2002	0.075

To calculate the discount factors corresponding to these interest rates, call `rate2disc` using the syntax

```
Disc = rate2disc(Compounding, Rates, EndDates, StartDates,  
ValuationDate)
```

where:

- `Compounding` represents the frequency at which the zero rates are compounded when annualized. For this example, assume this value to be 2.
- `Rates` is a vector of annualized percentage rates representing the interest rate applicable to each time interval.
- `EndDates` is a vector of dates representing the end of each interest-rate term (period).
- `StartDates` is a vector of dates representing the beginning of each interest-rate term.
- `ValuationDate` is the date of observation for which the discount factors are calculated. In this particular example, use February 15, 2000 as the beginning date for all interest-rate terms.

Next, set the variables in MATLAB.

```
StartDates = ['15-Feb-2000'];  
EndDates = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001'; ...  
'15-Feb-2002'; '15-Aug-2002'];  
Compounding = 2;  
ValuationDate = ['15-Feb-2000'];  
Rates = [0.05; 0.056; 0.06; 0.065; 0.075];
```

Finally, compute the discount factors.

```
Disc = rate2disc(Compounding, Rates, EndDates, StartDates, ...  
ValuationDate)
```

```
Disc =
```

```
0.9756
0.9463
0.9151
0.8799
0.8319
```

By adding a fourth column to the rates table (see “Calculating Discount Factors from Rates” on page 2-55) to include the corresponding discounts, you can see the evolution of the discount factors.

From	To	Rate	Discount
15 Feb 2000	15 Aug 2000	0.05	0.9756
15 Feb 2000	15 Feb 2001	0.056	0.9463
15 Feb 2000	15 Aug 2001	0.06	0.9151
15 Feb 2000	15 Feb 2002	0.065	0.8799
15 Feb 2000	15 Aug 2002	0.075	0.8319

### Optional Time Factor Outputs

The function `rate2disc` optionally returns two additional output arguments: `EndTimes` and `StartTimes`. These vectors of time factors represent the start dates and end dates in discount periodic units. The scale of these units is determined by the value of the input variable `Compounding`.

To examine the time factor outputs, find the corresponding values in the previous example.

```
[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates, ...
EndDates, StartDates, ValuationDate);
```

Arrange the two vectors into a single array for easier visualization.

```
Times = [StartTimes, EndTimes]
```

```
Times =
```

```
0    1
0    2
0    3
```

```
0    4
0    5
```

Because the valuation date is equal to the start date for all periods, the `StartTimes` vector is composed of 0s. Also, since the value of `Compounding` is 2, the rates are compounded semiannually, which sets the units of periodic discount to six months. The vector `EndDates` is composed of dates separated by intervals of six months from the valuation date. This explains why the `EndTimes` vector is a progression of integers from 1 to 5.

### Alternative Syntax (`rate2disc`)

The function `rate2disc` also accommodates an alternative syntax that uses periodic discount units instead of dates. Since the relationship between discount factors and interest rates is based on time periods and not on absolute dates, this form of `rate2disc` allows you to work directly with time periods. In this mode, the valuation date corresponds to 0, and the vectors `StartTimes` and `EndTimes` are used as input arguments instead of their date equivalents, `StartDates` and `EndDates`. This syntax for `rate2disc` is:

```
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
```

Using as input the `StartTimes` and `EndTimes` vectors computed previously, you should obtain the previous results for the discount factors.

```
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
```

```
Disc =
```

```
0.9756
0.9463
0.9151
0.8799
0.8319
```

### Calculating Rates from Discounts

The function `disc2rate` is the complement to `rate2disc`. It finds the rates applicable to a set of compounding periods, given the discount factor in those periods. The syntax for calling this function is:

```
Rates = disc2rate(Compounding, Disc, EndDates, StartDates,
ValuationDate)
```

Each argument to this function has the same meaning as in `rate2disc`. Use the results found in the previous example to return the rate values you started with.

```
Rates = disc2rate(Compounding, Disc, EndDates, StartDates, ...
ValuationDate)
```

```
Rates =
```

```
0.0500
0.0560
0.0600
0.0650
0.0750
```

### Alternative Syntax (`disc2rate`)

As in the case of `rate2disc`, `disc2rate` optionally returns `StartTimes` and `EndTimes` vectors representing the start and end times measured in discount periodic units. Again, working with the same values as before, you should obtain the same numbers.

```
[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc, ...
EndDates, StartDates, ValuationDate);
```

Arrange the results in a matrix convenient to display.

```
Result = [StartTimes, EndTimes, Rates]
```

```
Result =
```

```
0    1.0000    0.0500
0    2.0000    0.0560
0    3.0000    0.0600
0    4.0000    0.0650
0    5.0000    0.0750
```

As with `rate2disc`, the relationship between rates and discount factors is determined by time periods and not by absolute dates. So, the alternate syntax for `disc2rate` uses time vectors instead of dates, and it assumes that the valuation date corresponds to time = 0. The time-based calling syntax is:

```
Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes);
```

Using this syntax, you again obtain the original values for the interest rates.

```
Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes)
```

Rates =

```
0.0500
0.0560
0.0600
0.0650
0.0750
```

### See Also

bdtprice | bdtens | bdttimespec | bdttree | bdtvolspec | bkprice | bksens |  
bktimespec | bktree | bkvolspec | bondbybdt | bondbybk | bondbyhjm | bondbyhw |  
bondbyzero | capbybdt | capbybk | capbyblk | capbyhjm | capbyhw | cfbybdt |  
cfbybk | cfbyhjm | cfbyhw | cfbyzero | fixedbybdt | fixedbybk | fixedbyhjm |  
fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw |  
floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk |  
floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec |  
hjmtree | hjmvolspec | hwalbycap | hwalbyfloor | hwprice | hwsens |  
hwtimespec | hwtree | hwwolspec | instbond | instcap | instcf | instfixed |  
instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat |  
instoptfloat | instrangefloat | instswap | instswaption | intenvprice |  
intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm  
| oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw |  
optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw |  
optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw |  
optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw |  
rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw |  
swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt |  
swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### Related Examples

- “Modeling the Interest-Rate Term Structure” on page 2-67
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Graphical Representation of Trees” on page 2-157

## **More About**

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Interest-Rate Term Conversions

Interest-rate evolution is typically represented by a set of interest rates, including the beginning and end of the periods the rates apply to. For zero rates, the start dates are typically at the valuation date, with the rates extending from that valuation date until their respective maturity dates.

### Spot Curve to Forward Curve Conversion

Frequently, given a set of rates including their start and end dates, you may be interested in finding the rates applicable to different terms (periods). This problem is addressed by the function `ratetimes`. This function interpolates the interest rates given a change in the original terms. The syntax for calling `ratetimes` is

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates,  
RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate);
```

where:

- `Compounding` represents the frequency at which the zero rates are compounded when annualized.
- `RefRates` is a vector of initial interest rates representing the interest rates applicable to the initial time intervals.
- `RefEndDates` is a vector of dates representing the end of the interest rate terms (period) applicable to `RefRates`.
- `RefStartDates` is a vector of dates representing the beginning of the interest rate terms applicable to `RefRates`.
- `EndDates` represent the maturity dates for which the interest rates are interpolated.
- `StartDates` represent the starting dates for which the interest rates are interpolated.
- `ValuationDate` is the date of observation, from which the `StartTimes` and `EndTimes` are calculated. This date represents time = 0.

The input arguments to this function can be separated into two groups:

- The initial or reference interest rates, including the terms for which they are valid
- Terms for which the new interest rates are calculated

As an example, consider the rate table specified in “Calculating Discount Factors from Rates” on page 2-55.



From	To	Rate
15 Feb 2000	15 Aug 2000	0.05
15 Feb 2000	15 Feb 2001	0.056
15 Feb 2000	15 Aug 2001	0.06
15 Feb 2000	15 Feb 2002	0.065
15 Feb 2000	15 Aug 2002	0.075

Assuming that the valuation date is February 15, 2000, these rates represent zero-coupon bond rates with maturities specified in the second column. Use the function `ratetimes` to calculate the forward rates at the beginning of all periods implied in the table. Assume a compounding value of 2.

```
% Reference Rates.
```

```
RefStartDates = ['15-Feb-2000'];
RefEndDates = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
Compounding = 2;
ValuationDate = ['15-Feb-2000'];
RefRates = [0.05; 0.056; 0.06; 0.065; 0.075];
```

```
% New Terms.
```

```
StartDates = ['15-Feb-2000'; '15-Aug-2000'; '15-Feb-2001';...
'15-Aug-2001'; '15-Feb-2002'];
EndDates = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
```

```
% Find the new rates.
```

```
Rates = ratetimes(Compounding, RefRates, RefEndDates,...
RefStartDates, EndDates, StartDates, ValuationDate)
```

```
Rates =
```

```
0.0500
0.0620
0.0680
0.0801
0.1155
```

Place these values in a table like the previous one. Observe the evolution of the forward rates based on the initial zero-coupon rates.

From	To	Rate
15 Feb 2000	15 Aug 2000	0.0500
15 Aug 2000	15 Feb 2001	0.0620
15 Feb 2001	15 Aug 2001	0.0680
15 Aug 2001	15 Feb 2002	0.0801
15 Feb 2002	15 Aug 2002	0.1155

### Alternative Syntax (ratetimes)

The `ratetimes` function can provide the additional output arguments `StartTimes` and `EndTimes`, which represent the time factor equivalents to the `StartDates` and `EndDates` vectors. The `ratetimes` function uses time factors for interpolating the rates. These time factors are calculated from the start and end dates, and the valuation date, which are passed as input arguments. `ratetimes` can also use time factors directly, assuming time = 0 as the valuation date. This alternate syntax is:

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates,
RefEndTimes, RefStartTimes, EndTimes, StartTimes);
```

Use this alternate version of `ratetimes` to find the forward rates again. In this case, you must first find the time factors of the reference curve. Use `date2time` for this.

```
RefEndTimes = date2time(ValuationDate, RefEndDates, Compounding)
```

```
RefEndTimes =
```

```
1
2
3
4
5
```

```
RefStartTimes = date2time(ValuationDate, RefStartDates, ...
Compounding)
```

```
RefStartTimes =
```

```
0
```

These are the expected values, given semiannual discounts (as denoted by a value of 2 in the variable `Compounding`), end dates separated by six-month periods, and the valuation date equal to the date marking beginning of the first period (time factor = 0).

Now call `ratetimes` with the alternate syntax.

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding,...
RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes);
```

```
Rates =
```

```
    0.0500
    0.0620
    0.0680
    0.0801
    0.1155
```

`EndTimes` and `StartTimes` have, as expected, the same values they had as input arguments.

```
Times = [StartTimes, EndTimes]
```

```
Times =
```

```
    0     1
    1     2
    2     3
    3     4
    4     5
```

## See Also

`bdtprice` | `bdtens` | `bdttimespec` | `bdttree` | `bdtvolspec` | `bkprice` | `bksens` | `bktimespec` | `bktree` | `bkvolspec` | `bondbybdt` | `bondbybk` | `bondbyhjm` | `bondbyhw` | `bondbyzero` | `capbybdt` | `capbybk` | `capbyblk` | `capbyhjm` | `capbyhw` | `cfbybdt` | `cfbybk` | `cfbyhjm` | `cfbyhw` | `cfbyzero` | `fixedbybdt` | `fixedbybk` | `fixedbyhjm` | `fixedbyhw` | `fixedbyzero` | `floatbybdt` | `floatbybk` | `floatbyhjm` | `floatbyhw` | `floatbyzero` | `floatdiscmargin` | `floatmargin` | `floorbybdt` | `floorbybk` | `floorbyblk` | `floorbyhjm` | `floorbyhw` | `hjmprice` | `hjmsens` | `hjmtimespec` | `hjmtree` | `hjmvolspec` | `hwalbycap` | `hwalbyfloor` | `hwprice` | `hwsens` | `hwtimespec` | `hwtree` | `hvwolspec` | `instbond` | `instcap` | `instcf` | `instfixed` | `instfloat` | `instfloor` | `instoptbnd` | `instoptembnd` | `instoptemfloat` | `instoptfloat` | `instrangefloat` | `instswap` | `instswaption` | `intenvprice` |

intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm  
| oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw |  
optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw |  
optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw |  
optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw |  
rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw |  
swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt |  
swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### Related Examples

- “Modeling the Interest-Rate Term Structure” on page 2-67
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Graphical Representation of Trees” on page 2-157

### More About

- “Understanding the Interest-Rate Term Structure” on page 2-55
- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Modeling the Interest-Rate Term Structure

Financial Instruments Toolbox includes a set of functions to encapsulate interest-rate term information into a single structure. These functions present a convenient way to package all information related to interest-rate terms into a common format, and to resolve interdependencies when one or more of the parameters is modified. For information, see:

- “Creating or Modifying (`intenvset`)” on page 2-67 for a discussion of how to create or modify an interest-rate term structure (`RateSpec`) using the `intenvset` function
- “Obtaining Specific Properties (`intenvget`)” on page 2-69 for a discussion of how to extract specific properties from a `RateSpec`

### Creating or Modifying (`intenvset`)

The main function to create or modify an interest-rate term structure `RateSpec` (rates specification) is `intenvset`. If the first argument to this function is a previously created `RateSpec`, the function modifies the existing rate specification and returns a new one. Otherwise, it creates a `RateSpec`.

When using `RateSpec` to specify the rate term structure to price instruments based on yields (zero coupon rates) or forward rates, specify zero rates or forward rates as the input argument. However, the `RateSpec` structure is not limited or specific to this problem domain. `RateSpec` is an encapsulation of rates-times relationships; `intenvset` acts as either a constructor or a modifier, and `intenvget` as an accessor. The interest rate models supported by the Financial Instruments Toolbox software work either with zero coupon rates or forward rates.

The other `intenvset` arguments are name-value pairs. The name-value pair arguments that can be specified or modified are:

- `Basis`
- `Compounding`
- `Disc`
- `EndDates`
- `EndMonthRule`
- `Rates`

- StartDates
- ValuationDate

For more information on Basis, see **basis** on page Glossary-0 .

Consider again the original table of interest rates (see “Calculating Discount Factors from Rates” on page 2-55).

From	To	Rate
15 Feb 2000	15 Aug 2000	0.05
15 Feb 2000	15 Feb 2001	0.056
15 Feb 2000	15 Aug 2001	0.06
15 Feb 2000	15 Feb 2002	0.065
15 Feb 2000	15 Aug 2002	0.075

Use the information in this table to populate the RateSpec structure.

```

StartDates = [ '15-Feb-2000' ];
EndDates =   [ '15-Aug-2000' ;
               '15-Feb-2001' ;
               '15-Aug-2001' ;
               '15-Feb-2002' ;
               '15-Aug-2002' ];
Compounding = 2;
ValuationDate = [ '15-Feb-2000' ];
Rates = [0.05; 0.056; 0.06; 0.065; 0.075];

rs = intenvset('Compounding',Compounding,'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates,...
'ValuationDate', ValuationDate)

rs =

    FinObj:    'RateSpec'
  Compounding:    2
        Disc:    [5x1 double]
        Rates:    [5x1 double]
    EndTimes:    [5x1 double]
  StartTimes:    [5x1 double]
    EndDates:    [5x1 double]
  StartDates:    730531
    
```

```
ValuationDate: 730531
              Basis: 0
              EndMonthRule: 1
```

Some of the properties filled in the structure were not passed explicitly in the call to `RateSpec`. The values of the automatically completed properties depend on the properties that are explicitly passed. Consider for example the `StartTimes` and `EndTimes` vectors. Since the `StartDates` and `EndDates` vectors are passed in, and the `ValuationDate`, `intenvset` has all the information required to calculate `StartTimes` and `EndTimes`. Hence, these two properties are read-only.

## Obtaining Specific Properties (`intenvget`)

The complementary function to `intenvset` is `intenvget`, which gets function-specific properties from the interest-rate term structure. Its syntax is:

```
ParameterValue = intenvget(RateSpec, 'ParameterName')
```

To obtain the vector `EndTimes` from the `RateSpec` structure, enter:

```
EndTimes = intenvget(rs, 'EndTimes')
```

```
EndTimes =
```

```
1
2
3
4
5
```

To obtain `Disc`, the values for the discount factors that were calculated automatically by `intenvset`, type:

```
Disc = intenvget(rs, 'Disc')
```

```
Disc =
```

```
0.9756
0.9463
0.9151
0.8799
0.8319
```

These discount factors correspond to the periods starting from `StartDates` and ending in `EndDates`.

---

**Caution** Although you can directly access these fields within the structure instead of using `intenvget`, it is advised not to do so. The format of the interest-rate term structure could change in future versions of the toolbox. Should that happen, any code accessing the `RateSpec` fields directly would stop working.

---

Now use the `RateSpec` structure with its functions to examine how changes in specific properties of the interest-rate term structure affect those depending on it. As an exercise, change the value of `Compounding` from 2 (semiannual) to 1 (annual).

```
rs = intenvset(rs, 'Compounding', 1);
```

Since `StartTimes` and `EndTimes` are measured in units of periodic discount, a change in `Compounding` from 2 to 1 redefines the basic unit from semiannual to annual. This means that a period of six months is represented with a value of 0.5, and a period of one year is represented by 1. To obtain the vectors `StartTimes` and `EndTimes`, enter:

```
StartTimes = intenvget(rs, 'StartTimes');  
EndTimes = intenvget(rs, 'EndTimes');  
Times = [StartTimes, EndTimes]
```

```
Times =
```

```
0    0.5000  
0    1.0000  
0    1.5000  
0    2.0000  
0    2.5000
```

Since all the values in `StartDates` are the same as the valuation date, all `StartTimes` values are 0. On the other hand, the values in the `EndDates` vector are dates separated by six-month periods. Since the redefined value of compounding is 1, `EndTimes` becomes a sequence of numbers separated by increments of 0.5.

## See Also

`bdtprice` | `bdtrens` | `bdttimespec` | `bdttree` | `bdtvolspec` | `bkprice` | `bksens` | `bktimespec` | `bktree` | `bkvolspec` | `bondbybdt` | `bondbybk` | `bondbyhjm` | `bondbyhw` | `bondbyzero` | `capbybdt` | `capbybk` | `capbyblk` | `capbyhjm` | `capbyhw` | `cfbybdt` |



cfbybk | cfbyhjm | cfbyhw | cfbyzero | fixedbybdt | fixedbybk | fixedbyhjm | fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw | floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk | floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec | hjmtree | hjmvolspec | hwcabycap | hwcabylfloor | hwprice | hwsens | hwtimespec | hwtree | hwvolspec | instbond | instcap | instcf | instfixed | instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat | instoptfloat | instrangefloat | instswap | instswaption | intenvprice | intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm | oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw | optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw | optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw | optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw | rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw | swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt | swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

## Related Examples

- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Graphical Representation of Trees” on page 2-157

## More About

- “Understanding the Interest-Rate Term Structure” on page 2-55
- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Pricing Using Interest-Rate Term Structure

In this section...
“Introduction” on page 2-72
“Computing Instrument Prices” on page 2-73
“Computing Instrument Sensitivities” on page 2-74
“OAS for Callable and Puttable Bonds” on page 2-76
“Agency OAS” on page 2-76

### Introduction

The instruments can be presented to the functions as a portfolio of different types of instruments or as groups of instruments of the same type. The current version of the toolbox can compute price and sensitivities for five instrument types of using interest-rate curves:

- Bonds
- Fixed-rate notes
- Floating-rate notes
- Swaps
- OAS for callable and puttable bonds
- Agency OAS

In addition to these instruments, the toolbox also supports the calculation of price and sensitivities of arbitrary sets of cash flows.

Options and interest-rate floors and caps are absent from the above list of supported instruments. These instruments are not supported because their pricing and sensitivity function require a stochastic model for the evolution of interest rates. The interest-rate term structure used for pricing is treated as deterministic, and as such is not adequate for pricing these instruments.

Financial Instruments Toolbox also contains functions that use the Heath-Jarrow-Morton (HJM) and Black-Derman-Toy (BDT) models to compute prices and sensitivities for financial instruments. These models support computations involving options and interest-rate floors and caps. See “Pricing Using Interest-Rate Tree Models” on page 2-99 for

information on computing price and sensitivities of financial instruments using the HJM and BDT models.

## Computing Instrument Prices

The main function used for pricing portfolios of instruments is `intenvprice`. This function works with the family of functions that calculate the prices of individual types of instruments. When called, `intenvprice` classifies the portfolio contained in `InstSet` by instrument type, and calls the appropriate pricing functions. The map between instrument types and the pricing function `intenvprice` calls is

<code>bondbyzero:</code>	Price a bond by a set of zero curves
<code>fixedbyzero:</code>	Price a fixed-rate note by a set of zero curves
<code>floatbyzero:</code>	Price a floating-rate note by a set of zero curves
<code>swapbyzero:</code>	Price a swap by a set of zero curves

You can use each of these functions individually to price an instrument. Consult the reference pages for specific information on using these functions.

`intenvprice` takes as input an interest-rate term structure created with `intenvset`, and a portfolio of interest-rate contingent derivatives instruments created with `instadd`.

The syntax for using `intenvprice` to price an entire portfolio is

```
Price = intenvprice(RateSpec, InstSet)
```

where:

- `RateSpec` is the interest-rate term structure.
- `InstSet` is the name of the portfolio.

### Example: Pricing a Portfolio of Instruments

Consider this example of using the `intenvprice` function to price a portfolio of instruments supplied with Financial Instruments Toolbox software.

The provided MAT-file `deriv.mat` stores a portfolio as an instrument set variable `ZeroInstSet`. The MAT-file also contains the interest-rate term structure `ZeroRateSpec`. You can display the instruments with the function `instdisp`.

```
load deriv.mat;
instdisp(ZeroInstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis...
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN...
2	Bond	0.04	01-Jan-2000	01-Jan-2004	2	NaN...

Index	Type	CouponRate	Settle	Maturity	FixedReset	Basis...
3	Fixed	0.04	01-Jan-2000	01-Jan-2003	1	NaN...

Index	Type	Spread	Settle	Maturity	FloatReset	Basis...
4	Float	20	01-Jan-2000	01-Jan-2003	1	NaN...

Index	Type	LegRate	Settle	Maturity	LegReset	Basis...
5	Swap	[0.06 20]	01-Jan-2000	01-Jan-2003	[1 1]	NaN...

Use `intenvprice` to calculate the prices for the instruments contained in the portfolio `ZeroInstSet`.

```
format bank
Prices = intenvprice(ZeroRateSpec, ZeroInstSet)
```

```
Prices =
```

```
    98.72
    97.53
    98.72
   100.55
     3.69
```

The output `Prices` is a vector containing the prices of all the instruments in the portfolio in the order indicated by the `Index` column displayed by `instdisp`. So, the first two elements in `Prices` correspond to the first two bonds; the third element corresponds to the fixed-rate note; the fourth to the floating-rate note; and the fifth element corresponds to the price of the swap.

## Computing Instrument Sensitivities

In general, you can compute sensitivities either as dollar price changes or as percentage price changes. The toolbox reports all sensitivities as dollar sensitivities.

Using the interest-rate term structure, you can calculate two types of derivative price sensitivities, delta and gamma. *Delta* represents the dollar sensitivity of prices to shifts in the observed forward yield curve. *Gamma* represents the dollar sensitivity of delta to shifts in the observed forward yield curve.

The `intenvsens` function computes instrument sensitivities and instrument prices. If you need both the prices and sensitivity measures, use `intenvsens`. A separate call to `intenvprice` is not required.

Here is the syntax

```
[Delta, Gamma, Price] = intenvsens(RateSpec, InstSet)
```

where, as before:

- `RateSpec` is the interest-rate term structure.
- `InstSet` is the name of the portfolio.

### Example: Sensitivities and Prices

Here is an example that uses `intenvsens` to calculate both sensitivities and prices.

```
format bank
load deriv.mat;
[Delta, Gamma, Price] = intenvsens(ZeroRateSpec, ZeroInstSet);
```

Display the results in a single matrix in bank format.

```
All = [Delta Gamma Price]
```

```
All =
```

-272.64	1029.84	98.72
-347.44	1622.65	97.53
-272.64	1029.84	98.72
-1.04	3.31	100.55
-282.04	1059.62	3.69

To view the per-dollar sensitivity, divide the first two columns by the last one.

```
[Delta./Price, Gamma./Price, Price]
```

```
ans =
```

-2.76	10.43	98.72
-3.56	16.64	97.53
-2.76	10.43	98.72
-0.01	0.03	100.55
-76.39	286.98	3.69

## OAS for Callable and Puttable Bonds

Option Adjusted Spread (OAS) is a useful way to value and compare securities with embedded options, like callable or puttable bonds. Basically, when the constant or flat spread is added to the interest-rate curve/rates in the tree, the pricing model value equals the market price. Financial Instruments Toolbox supports pricing American, European, and Bermuda callable and puttable bonds using different interest rate models. The pricing for a bond with embedded options is:

- For a callable bond, where the holder has bought a bond and sold a call option to the issuer:

$$\text{Price callable bond} = \text{Price Option free bond} - \text{Price call option}$$

- For a puttable bond, where the holder has bought a bond and a put option:

$$\text{Price puttable bond} = \text{Price Option free bond} + \text{Price put option}$$

There are two additional sensitivities related to OAS for bonds with embedded options: Option Adjusted Duration and Option Adjusted Convexity. These are similar to the concepts of modified duration and convexity for option-free bonds. The measure Duration is a general term that describes how sensitive a bond's price is to a parallel shift in the yield curve. Modified Duration and Modified Convexity assume that the bond's cash flows do not change when the yield curve shifts. This is not true for OA Duration or OA Convexity because the cash flows may change due to the option risk component of the bond.

Function	Purpose
oasbybdt	Compute OAS using a BDT model.
oasbybk	Compute OAS using a BK model.
oasbyhjm	Compute OAS using an HJM model.
oasbyhw	Compute OAS using an HW model.

## Agency OAS

Often bonds are issued with embedded options, which then makes standard price/yield or spread measures irrelevant. For example, a municipality concerned about the chance that interest rates may fall in the future might issue bonds with a provision that allows the bond to be repaid before the bond's maturity. This is a call option on the bond and must be incorporated into the valuation of the bond. Option-adjusted spread (OAS), which

adjusts a bond spread for the value of the option, is the standard measure for valuing bonds with embedded options. Financial Instruments Toolbox supports computing option-adjusted spreads for bonds with single embedded options using the agency model.

The Securities Industry and Financial Markets Association (SIFMA) has a simplified approach to compute OAS for agency issues (Government Sponsored Entities like Fannie Mae and Freddie Mac) termed “Agency OAS.” In this approach, the bond has only one call date (European call) and uses Black’s model (see *The BMA European Callable Securities Formula* at <https://www.sifma.org>) to value the bond option. The price of the bond is computed as follows:

$$\text{Price}_{\text{Callable}} = \text{Price}_{\text{NonCallable}} - \text{Price}_{\text{Option}}$$

where

$\text{Price}_{\text{Callable}}$  is the price of the callable bond.

$\text{Price}_{\text{NonCallable}}$  is the price of the noncallable bond, that is, price of the bond using `bndspread`.

$\text{Price}_{\text{Option}}$  is the price of the option, that is, price of the option using Black’s model.

The Agency OAS is the spread, when used in the previous formula, yields the market price. Financial Instruments Toolbox supports these functions:

### Agency OAS

Agency OAS Functions	Purpose
<code>agencyoas</code>	Compute the OAS of the callable bond using the Agency OAS model.
<code>agencyprice</code>	Price the callable bond OAS using the Agency OAS model.

For more information on agency OAS, see “Agency Option-Adjusted Spreads” on page 6-2.

## See Also

`bdtprice` | `bdtens` | `bdttimespec` | `bdttree` | `bdtvolspec` | `bkprice` | `bksens` | `bktimespec` | `bktree` | `bkvolspec` | `bondbybdt` | `bondbybk` | `bondbyhjm` | `bondbyhw` | `bondbyzero` | `capbybdt` | `capbybk` | `capbyblk` | `capbyhjm` | `capbyhw` | `cfbybdt` | `cfbybk` | `cfbyhjm` | `cfbyhw` | `cfbyzero` | `fixedbybdt` | `fixedbybk` | `fixedbyhjm` |

fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw |  
floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk |  
floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec |  
hjmtree | hjmvolspec | hwalbycap | hwalbyfloor | hwprice | hwsens |  
hwtimespec | hwtree | hwwolspec | instbond | instcap | instcf | instfixed |  
instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat |  
instoptfloat | instrangefloat | instswap | instswaption | intenvprice |  
intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm  
| oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw |  
optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw |  
optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw |  
optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw |  
rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw |  
swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt |  
swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### Related Examples

- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Understanding the Interest-Rate Term Structure” on page 2-55

### More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44



# Understanding Interest-Rate Tree Models

## In this section...

“Introduction” on page 2-79

“Building a Tree of Forward Rates” on page 2-80

“Specifying the Volatility Model (VolSpec)” on page 2-82

“Specifying the Interest-Rate Term Structure (RateSpec)” on page 2-84

“Specifying the Time Structure (TimeSpec)” on page 2-85

“Creating Trees” on page 2-87

“Examining Trees” on page 2-88

## Introduction

Financial Instruments Toolbox supports the following interest-rate trees:

- Black-Derman-Toy (BDT)
- Black-Karasinski (BK)
- Heath-Jarrow-Morton (HJM)
- Hull-White (HW)
- Cox-Ingersoll-Ross (CIR)

The Heath-Jarrow-Morton model is one of the most widely used models for pricing interest-rate derivatives. The model considers a given initial term structure of interest rates and a specification of the volatility of forward rates to build a tree representing the evolution of the interest rates, based on a statistical process. For further explanation, see the book *Modelling Fixed Income Securities and Interest Rate Options* by Robert A. Jarrow.

The Black-Derman-Toy model is another analytical model commonly used for pricing interest-rate derivatives. The model considers a given initial zero rate term structure of interest rates and a specification of the yield volatilities of long rates to build a tree representing the evolution of the interest rates. For further explanation, see the paper “A One Factor Model of Interest Rates and its Application to Treasury Bond Options” by Fischer Black, Emanuel Derman, and William Toy.

The Hull-White model incorporates the initial term structure of interest rates and the volatility term structure to build a trinomial recombining tree of short rates. The resulting

tree is used to value interest rate-dependent securities. The implementation of the Hull-White model in Financial Instruments Toolbox software is limited to one factor.

The Black-Karasinski model is a single factor, log-normal version of the Hull-White model.

For further information on the Hull-White and Black-Karasinski models, see the book *Options, Futures, and Other Derivatives* by John C. Hull.

### Building a Tree of Forward Rates

The tree of forward rates is the fundamental unit representing the evolution of interest rates in a given period of time. This section explains how to create a forward-rate tree using Financial Instruments Toolbox.

---

**Note** To avoid needless repetition, this document uses the HJM and BDT models to illustrate the creation and use of interest-rate trees. The HW and BK models are similar to the BDT model. Where specific differences exist, they are documented in “HW and BK Tree Structures” on page 2-95.

---

The MATLAB functions that create rate trees are `hjmtree` and `bdttree`. The `hjmtree` function creates the structure, `HJMTree`, containing time and forward-rate information for a bushy tree. The `bdttree` function creates a similar structure, `BDTTree`, for a recombining tree.

This structure is a self-contained unit that includes the tree of rates (found in the `FwdTree` field of the structure) and the volatility, rate, and time specifications used in building this tree.

These functions take three structures as input arguments:

- The volatility model `VolSpec`. (See “Specifying the Volatility Model (`VolSpec`)” on page 2-82.)
- The interest-rate term structure `RateSpec`. (See “Specifying the Interest-Rate Term Structure (`RateSpec`)” on page 2-84.)
- The tree time layout `TimeSpec`. (See “Specifying the Time Structure (`TimeSpec`)” on page 2-85.)

An easy way to visualize any trees you create is with the `treeviewer` function, which displays trees in a graphical manner. See “Graphical Representation of Trees” on page 2-157 for information about `treeviewer`.

### Calling Sequence

The calling syntax for `hjmtree` is `HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)`.

Similarly, the calling syntax for `bdttree` is `BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)`.

Each of these functions requires `VolSpec`, `RateSpec`, and `TimeSpec` input arguments:

- `VolSpec` is a structure that specifies the forward-rate volatility process. You create `VolSpec` using either of the functions `hjmvolspec` or `bdtvolspec`.

The `hjmvolspec` function supports the specification of up to three factors. It handles these models for the volatility of the interest-rate term structure:

- Constant
- Stationary
- Exponential
- Vasicek
- Proportional

A one-factor model assumes that the interest term structure is affected by a single source of uncertainty. Incorporating multiple factors allows you to specify different types of shifts in the shape and location of the interest-rate structure. See `hjmvolspec` for details.

The `bdtvolspec` function supports only a single volatility factor. The volatility remains constant between pairs of nodes on the tree. You supply the input volatility values in a vector of decimal values. See `bdtvolspec` for details.

- `RateSpec` is the interest-rate specification of the initial rate curve. You create this structure with the function `intenvset`. (See “Modeling the Interest-Rate Term Structure” on page 2-67.)
- `TimeSpec` is the tree time layout specification. You create this variable with the functions `hjmtimespec` or `bdttimespec`. It represents the mapping between level

times and level dates for rate quoting. This structure indirectly determines the number of levels in the tree.

## Specifying the Volatility Model (VolSpec)

Because HJM supports multifactor (up to 3) volatility models while BDT (also, BK and HW) supports only a single volatility factor, the `hjmvolspec` and `bdtvolspec` functions require different inputs and generate slightly different outputs. For examples, see “Creating an HJM Volatility Model” on page 2-82. For BDT examples, see “Creating a BDT Volatility Model” on page 2-83.

### Creating an HJM Volatility Model

The function `hjmvolspec` generates the structure `VolSpec`, which specifies the volatility process  $\sigma(t, T)$  used in the creation of the forward-rate trees. In this context capital  $T$  represents the starting time of the forward rate, and  $t$  represents the observation time. The volatility process can be constructed from a combination of factors specified sequentially in the call to function that creates it. Each factor specification starts with a character vector specifying the name of the factor, followed by the pertinent parameters.

### HJM Volatility Specification Example

Consider an example that uses a single factor, specifically, a constant-sigma factor. The constant factor specification requires only one parameter, the value of  $\sigma$ . In this case, the value corresponds to 0.10.

```
HJMVolSpec = hjmvolspec('Constant', 0.10)
```

```
HJMVolSpec =
```

```
    FinObj: 'HJMVolSpec'  
  FactorModels: {'Constant'}  
    FactorArgs: {{1x1 cell}}  
    SigmaShift: 0  
    NumFactors: 1  
    NumBranch: 2  
      PBranch: [0.5000 0.5000]  
    Fact2Branch: [-1 1]
```

The `NumFactors` field of the `VolSpec` structure, `VolSpec.NumFactors = 1`, reveals that the number of factors used to generate `VolSpec` was one. The `FactorModels` field indicates that it is a `Constant` factor, and the `NumBranches` field indicates the number

of branches. As a consequence, each node of the resulting tree has two branches, one going up, and the other going down.

Consider now a two-factor volatility process made from a proportional factor and an exponential factor.

```
% Exponential factor
Sigma_0 = 0.1;
Lambda = 1;
% Proportional factor
CurveProp = [0.11765; 0.08825; 0.06865];
CurveTerm = [ 1 ; 2 ; 3 ];
% Build VolSpec
HJMVolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm,...
1e6,'Exponential', Sigma_0, Lambda)
```

```
HJMVolSpec =
```

```
      FinObj: 'HJMVolSpec'
FactorModels: {'Proportional' 'Exponential'}
FactorArgs: {{1x3 cell} {1x2 cell}}
SigmaShift: 0
NumFactors: 2
NumBranch: 3
PBranch: [0.2500 0.2500 0.5000]
Fact2Branch: [2x3 double]
```

The output shows that the volatility specification was generated using two factors. The tree has three branches per node. Each branch has probabilities of 0.25, 0.25, and 0.5, going from top to bottom.

### Creating a BDT Volatility Model

The function `bdtvolspec` generates the structure `VolSpec`, which specifies the volatility process. The function requires three input arguments:

- The valuation date `ValuationDate`
- The yield volatility end dates `VolDates`
- The yield volatility values `VolCurve`

An optional fourth argument `InterpMethod`, specifying the interpolation method, can be included.

The syntax used for calling `bdtvolspec` is:

```
BDTVolSpec = bdtvolspec(ValuationDate, VolDates, VolCurve,...  
InterpMethod)
```

where:

- `ValuationDate` is the first observation date in the tree.
- `VolDates` is a vector of dates representing yield volatility end dates.
- `VolCurve` is a vector of yield volatility values.
- `InterpMethod` is the method of interpolation to use. The default is `linear`.

### BDT Volatility Specification Example

Consider the following example:

```
ValuationDate = datenum('01-01-2000');  
EndDates = datenum(['01-01-2001'; '01-01-2002'; '01-01-2003';  
'01-01-2004'; '01-01-2005']);  
Volatility = [.2; .19; .18; .17; .16];
```

Use `bdtvolspec` to create a volatility specification. Because no interpolation method is explicitly specified, the function uses the `linear` default.

```
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)
```

```
BDTVolSpec =  
    FinObj: 'BDTVolSpec'  
    ValuationDate: 730486  
    VolDates: [5x1 double]  
    VolCurve: [5x1 double]  
    VolInterpMethod: 'linear'
```

## Specifying the Interest-Rate Term Structure (RateSpec)

The structure `RateSpec` is an interest term structure that defines the initial forward-rate specification from which the tree rates are derived. “Modeling the Interest-Rate Term Structure” on page 2-67 explains how to create these structures using the function `intenvset`, given the interest rates, the starting and ending dates for each rate, and the compounding value.

### Rate Specification Creation Example

Consider the following example:

```

Compounding = 1;
Rates = [0.02; 0.02; 0.02; 0.02];
StartDates = [ '01-Jan-2000';
               '01-Jan-2001';
               '01-Jan-2002';
               '01-Jan-2003'];
EndDates = [ '01-Jan-2001';
             '01-Jan-2002';
             '01-Jan-2003';
             '01-Jan-2004'];
ValuationDate = '01-Jan-2000';

RateSpec = intenvset('Compounding',1,'Rates', Rates,...
                    'StartDates', StartDates, 'EndDates', EndDates,...
                    'ValuationDate', ValuationDate)

RateSpec =

    FinObj: 'RateSpec'
  Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
    EndTimes: [4x1 double]
  StartTimes: [4x1 double]
    EndDates: [4x1 double]
  StartDates: [4x1 double]
ValuationDate: 730486
          Basis: 0
    EndMonthRule: 1

```

Use the function `datedisp` to examine the dates defined in the variable `RateSpec`. For example:

```

datedisp(RateSpec.ValuationDate)
01-Jan-2000

```

## Specifying the Time Structure (TimeSpec)

The structure `TimeSpec` specifies the time structure for an interest-rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

`TimeSpec` is built using either the `hjmtimespec` or `bdttimespec` function. These functions require three input arguments:

- The valuation date `ValuationDate`
- The maturity date `Maturity`
- The compounding rate `Compounding`

For example, the syntax used for calling `hjmtimespec` is

```
TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)
```

where:

- `ValuationDate` is the first observation date in the tree.
- `Maturity` is a vector of dates representing the cash flow dates of the tree. Any instrument cash flows with these maturities fall on tree nodes.
- `Compounding` is the frequency at which the rates are compounded when annualized.

### Creating a Time Specification

Calling the time specification creation functions with the same data used to create the interest-rate term structure, `RateSpec` on page 2-84 builds the structure that specifies the time layout for the tree.

#### HJM Time Specification Example

Consider the following example:

```
Maturity = EndDates;  
HJMTimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)
```

```
HJMTimeSpec =
```

```
    FinObj: 'HJMTimeSpec'  
ValuationDate: 730486  
    Maturity: [4x1 double]  
    Compounding: 1  
            Basis: 0  
    EndMonthRule: 1
```

Maturities specified when building `TimeSpec` need not coincide with the `EndDates` of the rate intervals in `RateSpec`. Since `TimeSpec` defines the time-date mapping of the tree, the rates in `RateSpec` are interpolated to obtain the initial rates with maturities equal to those in `TimeSpec`.



### Creating a BDT Time Specification

Consider the following example:

```
Maturity = EndDates;
BDTTimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)
```

```
BDTTimeSpec =
```

```
    FinObj: 'BDTTimeSpec'
  ValuationDate: 730486
    Maturity: [4x1 double]
  Compounding: 1
        Basis: 0
  EndMonthRule: 1
```

### Creating Trees

Use the `VolSpec`, `RateSpec`, and `TimeSpec` you have previously created as inputs to the functions used to create HJM and BDT trees.

#### Creating an HJM Tree

```
% Reset the volatility factor to the Constant case
HJMVolSpec = hjmvolspec('Constant', 0.10);

HJMTree = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec)
```

```
HJMTree =
```

```
    FinObj: 'HJMFwdTree'
  VolSpec: [1x1 struct]
  TimeSpec: [1x1 struct]
  RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
  FwdTree: {[4x1 double][3x1x2 double][2x2x2 double][1x4x2 double]}
```

#### Creating a BDT Tree

Now use the previously computed values for `VolSpec`, `RateSpec`, and `TimeSpec` as input to the function `bdttree` to create a BDT tree.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
```

```
BDTTree =  
  
    FinObj: 'BDTFwdTree'  
    VolSpec: [1x1 struct]  
    TimeSpec: [1x1 struct]  
    RateSpec: [1x1 struct]  
        tObs: [0 1.00 2.00 3.00]  
        TFwd: {[4x1 double] [3x1 double] [2x1 double] [3.00]}  
        CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4.00]}  
        FwdTree: {[1.02] [1.02 1.02] [1.01 1.02 1.03] [1.01 1.02 1.02 1.03]}
```

### Examining Trees

When working with the models, Financial Instruments Toolbox uses trees to represent forward rates, prices, and so on. At the highest level, these trees have structures wrapped around them. The structures encapsulate information required to interpret completely the information contained in a tree.

Consider this example, which uses the interest rate and portfolio data in the MAT-file `deriv.mat` included in the toolbox.

Load the data into the MATLAB workspace.

```
load deriv.mat
```

Display the list of the variables loaded from the MAT-file.

```
whos
```

Name	Size	Bytes	Class	Attributes
BDTInstSet	1x1	15956	struct	
BDTTree	1x1	5138	struct	
BKInstSet	1x1	15946	struct	
BKTree	1x1	5904	struct	
CRRInstSet	1x1	12434	struct	
CRRTree	1x1	5058	struct	
EQPInstSet	1x1	12434	struct	
EQPTree	1x1	5058	struct	
HJMInstSet	1x1	15948	struct	
HJMTree	1x1	5838	struct	
HWInstSet	1x1	15946	struct	
HWTTree	1x1	5904	struct	
ITTInstSet	1x1	12438	struct	
ITTTTree	1x1	8862	struct	
ZeroInstSet	1x1	10282	struct	
ZeroRateSpec	1x1	1580	struct	

## HJM Tree Structure

You can now examine in some detail the contents of the `HJMtree` structure contained in this file.

### HJMtree

```
HJMtree =
    FinObj: 'HJMfwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    FwdTree: {[4x1 double][3x1x2 double][2x2x2 double][1x4x2 double]}
```

`FwdTree` contains the actual forward-rate tree. MATLAB represents it as a cell array with each cell array element containing a tree level.

The other fields contain other information relevant to interpreting the values in `FwdTree`. The most important are `VolSpec`, `TimeSpec`, and `RateSpec`, which contain the volatility, time structure, and rate structure information respectively.

### First Node

Observe the forward rates in `FwdTree`. The first node represents the valuation date, `tObs = 0`.

```
HJMtree.FwdTree{1}
```

```
ans =
    1.0356
    1.0468
    1.0523
    1.0563
```

---

**Note** Financial Instruments Toolbox uses *inverse discount* notation for forward rates in the tree. An inverse discount represents a factor by which the current value of an asset is multiplied to find its future value. In general, these forward factors are reciprocals of the discount factors.

---

Look closely at the `RateSpec` structure used in generating this tree to see where these values originate. Arrange the values in a single array.

```
[HJMTree.RateSpec.StartTimes HJMTree.RateSpec.EndTimes...  
HJMTree.RateSpec.Rates]
```

```
ans =
```

```
      0      1.0000      0.0356  
1.0000  2.0000  0.0468  
2.0000  3.0000  0.0523  
3.0000  4.0000  0.0563
```

If you find the corresponding inverse discounts of the interest rates in the third column, you have the values at the first node of the tree. You can turn interest rates into inverse discounts using the function `rate2disc`.

```
Disc = rate2disc(HJMTree.TimeSpec.Compounding,...  
HJMTree.RateSpec.Rates, HJMTree.RateSpec.EndTimes,...  
HJMTree.RateSpec.StartTimes);  
FRates = 1./Disc
```

```
FRates =  
    1.0356  
    1.0468  
    1.0523  
    1.0563
```

### Second Node

The second node represents the first-rate observation time, `tObs = 1`. This node displays two states: one representing the branch going up and the other representing the branch going down.

Note that `HJMTree.VolSpec.NumBranch = 2`.

```
HJMTree.VolSpec
```

```
ans =
```

```
      FinObj: 'HJMVolSpec'  
      FactorModels: {'Constant'}  
      FactorArgs: {{1x1 cell}}  
      SigmaShift: 0  
      NumFactors: 1
```

```

    NumBranch: 2
      PBranch: [0.5000 0.5000]
    Fact2Branch: [-1 1]

```

Examine the rates of the node corresponding to the up branch.

```
HJMTree.FwdTree{2}(:, :, 1)
```

```
ans =
```

```

    1.0364
    1.0420
    1.0461

```

Now examine the corresponding down branch.

```
HJMTree.FwdTree{2}(:, :, 2)
```

```
ans =
```

```

    1.0574
    1.0631
    1.0672

```

### Third Node

The third node represents the second observation time,  $t_{Obs} = 2$ . This node contains a total of four states, two representing the branches going up and the other two representing the branches going down. Examine the rates of the node corresponding to the up states.

```
HJMTree.FwdTree{3}(:, :, 1)
```

```
ans =
```

```

    1.0317    1.0526
    1.0358    1.0568

```

Next examine the corresponding down states.

```
HJMTree.FwdTree{3}(:, :, 2)
```

```
ans =
```

```

    1.0526    1.0738
    1.0568    1.0781

```

### Isolating a Specific Node

Starting at the third level, indexing within the tree cell array becomes complex, and isolating a specific node can be difficult. The function `bushpath` isolates a specific node by specifying the path to the node as a vector of branches taken to reach that node. As an example, consider the node reached by starting from the root node, taking the branch up, then the branch down, and then another branch down. Given that the tree has only two branches per node, branches going up correspond to a 1, and branches going down correspond to a 2. The path up-down-down becomes the vector `[1 2 2]`.

```
FRates = bushpath(HJMTree.FwdTree, [1 2 2])
```

```
FRates =
```

```
    1.0356  
    1.0364  
    1.0526  
    1.0674
```

`bushpath` returns the spot rates for all the nodes tapped by the path specified in the input argument, the first one corresponding to the root node, and the last one corresponding to the target node.

Isolating the same node using direct indexing obtains

```
HJMTree.FwdTree{4}(:, 3, 2)
```

```
ans =
```

```
    1.0674
```

As expected, this single value corresponds to the last element of the rates returned by `bushpath`.

You can use these techniques with any type of tree generated with Financial Instruments Toolbox, such as forward-rate trees or price trees.

### BDT Tree Structure

You can now examine in some detail the contents of the `BDTTree` structure.

```
BDTTree
```

```
BDTTree =
```

```

FinObj: 'BDTFwdTree'
VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
  tObs: [0 1.00 2.00 3.00]
  TFwd: {[4x1 double] [3x1 double] [2x1 double] [3.00]}
  CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4.00]}
  FwdTree: {[1.10] [1.10 1.14] [1.10 1.14 1.19] [1.09 1.12 1.16 1.22]}

```

`FwdTree` contains the actual rate tree. MATLAB represents it as a cell array with each cell array element containing a tree level.

The other fields contain other information relevant to interpreting the values in `FwdTree`. The most important are `VolSpec`, `TimeSpec`, and `RateSpec`, which contain the volatility, time structure, and rate structure information respectively.

Look at the `RateSpec` structure used in generating this tree to see where these values originate. Arrange the values in a single array.

```
[BDTTree.RateSpec.StartTimes BDTTree.RateSpec.EndTimes...
BDTTree.RateSpec.Rates]
```

```
ans =
```

```

    0    1.0000    0.1000
    0    2.0000    0.1100
    0    3.0000    0.1200
    0    4.0000    0.1250

```

Look at the rates in `FwdTree`. The first node represents the valuation date, `tObs = 0`. The second node represents `tObs = 1`. Examine the rates at the second, third, and fourth nodes.

```
BDTTree.FwdTree{2}
```

```
ans =
```

```
1.0979    1.1432
```

The second node represents the first observation time, `tObs = 1`. This node contains a total of two states, one representing the branch going up (1.0979) and the other representing the branch going down (1.1432).

---

**Note** The convention in this document is to display *prices* going up on the upper branch. So, when displaying *rates*, rates are falling on the upper branch and increasing on the lower branch.

---

```
BDTTree.FwdTree{3}
```

```
ans =
```

```
1.0976    1.1377    1.1942
```

The third node represents the second observation time,  $t_{Obs} = 2$ . This node contains a total of three states, one representing the branch going up (1.0976), one representing the branch in the middle (1.1377) and the other representing the branch going down (1.1942).

```
BDTTree.FwdTree{4}
```

```
ans =
```

```
1.0872    1.1183    1.1606    1.2179
```

The fourth node represents the third observation time,  $t_{Obs} = 3$ . This node contains a total of four states, one representing the branch going up (1.0872), two representing the branches in the middle (1.1183 and 1.1606), and the other representing the branch going down (1.2179).

### Isolating a Specific Node

The function `treepath` isolates a specific node by specifying the path to the node as a vector of branches taken to reach that node. As an example, consider the node reached by starting from the root node, taking the branch up, then the branch down, and finally another branch down. Given that the tree has only two branches per node, branches going up correspond to a 1, and branches going down correspond to a 2. The path up-down-down becomes the vector `[1 2 2]`.

```
FRates = treepath(BDTTree.FwdTree, [1 2 2])
```

```
FRates =
```

```
1.1000  
1.0979  
1.1377  
1.1606
```

`treepath` returns the short rates for all the nodes tapped by the path specified in the input argument, the first one corresponding to the root node, and the last one corresponding to the target node.



## HW and BK Tree Structures

The HW and BK tree structures are similar to the BDT tree structure. You can see this if you examine the sample HW tree contained in the file `deriv.mat`.

```
load deriv.mat;
HWTtree
```

```
HWTtree =
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1.00 2.00 3.00]
    dObs: [731947.00 732313.00 732678.00 733043.00]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4.00]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2.00] [2.00 3.00 4.00] [2.00 2.00 3.00 4.00 4.00]}
    FwdTree: {[1.03] [1.05 1.04 1.02] [1.08 1.07 1.05 1.03 1.01] [1.09 1.08 1.06 1.04 1.02]}
```

All fields of this structure are similar to their BDT counterparts. There are two additional fields not present in BDT: **Probs** and **Connect**. The **Probs** field represents the occurrence probabilities at each branch of each node in the tree. The **Connect** field describes the connectivity of the nodes of a given tree level to nodes to the next tree level.

### Probs Field

While BDT and one-factor HJM models have equal probabilities for each branch at a node, HW and BK do not. For HW and BK trees, the **Probs** field indicates the likelihood that a particular branch will be taken in moving from one node to another node on the next level.

The **Probs** field consists of a cell array with one cell per tree level. Each cell is a 3-by-**NUMNODES** array with the top row representing the probability of an up movement, the middle row representing the probability of a middle movement, and the last row the probability of a down movement.

As an illustration, consider the first two elements of the **Probs** field of the structure, corresponding to the first (root) and second levels of the tree.

```
HWTtree.Probs{1}
0.166666666666667
0.666666666666667
0.166666666666667
```

```
HWTtree.Probs{2}
```

```
0.12361333418768  0.16666666666667  0.21877591615172
0.65761074966060  0.66666666666667  0.65761074966060
0.21877591615172  0.16666666666667  0.12361333418768
```

Reading from top to bottom, the values in `HWTtree.Probs{1}` correspond to the up, middle, and down probabilities at the root node.

`HWTtree.Probs{2}` is a 3-by-3 matrix of values. The first column represents the top node, the second column represents the middle node, and the last column represents the bottom node. As with the root node, the first, second, and third rows hold the values for up, middle, and down branching off each node.

As expected, the sum of all the probabilities at any node equals 1.

```
sum(HWTtree.Probs{2})
```

```
1.0000  1.0000  1.0000
```

### Connect Field

The other field that distinguishes HW and BK tree structures from the BDT tree structure is `Connect`. This field describes how each node in a given level connects to the nodes of the next level. The need for this field arises from the possibility of nonstandard branching in a tree.

The `Connect` field of the HW tree structure consists of a cell array with one cell per tree level.

```
HWTtree.Connect
```

```
ans =
```

```
[2]    [1x3 double]    [1x5 double]
```

Each cell contains a 1-by-`NUMNODES` vector. Each value in the vector relates to a node in the corresponding tree level and represents the index of the node in the next tree level that the middle branch of the node connects to.

If you subtract 1 from the values contained in `Connect`, you reveal the index of the nodes in the next level that the up branch connects to. If you add 1 to the values, you reveal the index of the corresponding down branch.

As an illustration, consider `HWTtree.Connect{1}`:

`HWTree.Connect{1}`

ans =

2

This indicates that the middle branch of the root node connects to the second (from the top) node of the next level, as expected. If you subtract 1 from this value, you obtain 1, which tells you that the up branch goes to the top node. If you add 1, you obtain 3, which points to the last node of the second level of the tree.

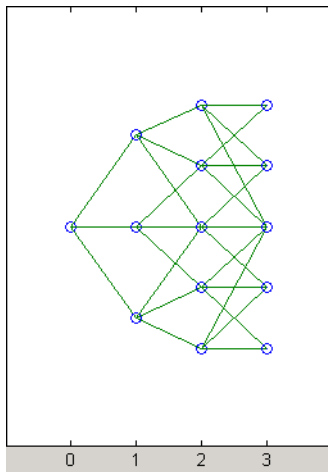
Now consider level 3 in this example:

`HWTree.Connect{3}`

2      2      3      4      4

On this level, there is nonstandard branching. This can be easily recognized because the middle branch of two nodes is connected to the same node on the next level.

To visualize this, consider the following illustration of the tree.



Here it becomes apparent that there is nonstandard branching at the third level of the tree, on the top and bottom nodes. The first and second nodes connect to the same trio of nodes on the next level. Similar branching occurs at the bottom and next-to-bottom nodes of the tree.

### See Also

bdtprice | bdtens | bdttimespec | bdttree | bdtvolspec | bkprice | bksens | bktimespec | bktree | bkvolspec | bondbybdt | bondbybk | bondbyhjm | bondbyhw | bondbyzero | capbybdt | capbybk | capbyblk | capbyhjm | capbyhw | cfbybdt | cfbybk | cfbyhjm | cfbyhw | cfbyzero | fixedbybdt | fixedbybk | fixedbyhjm | fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw | floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk | floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec | hjmtree | hjmvolspec | hwcalbycap | hwcalbyfloor | hwprice | hwsens | hwtimespec | hwtree | hwvolspec | instbond | instcap | instcf | instfixed | instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat | instoptfloat | inststrangefloat | instswap | instswaption | intenvprice | intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm | oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw | optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw | optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw | optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw | rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw | swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt | swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### Related Examples

- “Overview of Interest-Rate Tree Models” on page 2-50
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Graphical Representation of Trees” on page 2-157

### More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

# Pricing Using Interest-Rate Tree Models

**In this section...**

“Introduction” on page 2-99

“Computing Instrument Prices” on page 2-99

## Introduction

For purposes of illustration, this section relies on the HJM and BDT models. The HW and BK functions that perform price and sensitivity computations are not explicitly shown here. Functions that use the HW and BK models operate similarly to the BDT model.

## Computing Instrument Prices

The portfolio pricing functions `hjmprice` and `bdtprice` calculate the price of any set of supported instruments, based on an interest-rate tree. The functions are capable of pricing these instrument types:

- Bonds
- Bond options
- Bond with embedded options
- Arbitrary cash flows
- Fixed-rate notes
- Floating-rate notes
- Floating-rate notes with options or embedded options
- Caps
- Floors
- Range Notes
- Swaps
- Swaptions

For example, the syntax for calling `hjmprice` is:

```
[Price, PriceTree] = hjmprice(HJMTree, InstSet, Options)
```

Similarly, the calling syntax for `bdtpprice` is:

```
[Price, PriceTree] = bdtpprice(BDTree, InstSet, Options)
```

Each function requires two input arguments: the interest-rate tree and the set of instruments, `InstSet`. An optional argument, `Options`, further controls the pricing and the output displayed. (See “Pricing Options Structure” on page B-2 for information about the `Options` argument.)

`HJMTree` is the Heath-Jarrow-Morton tree sampling of a forward-rate process, created using `hjmtree`. `BDTree` is the Black-Derman-Toy tree sampling of an interest-rate process, created using `bdttree`. See “Building a Tree of Forward Rates” on page 2-80 to learn how to create these structures.

`InstSet` is the set of instruments to be priced. This structure represents the set of instruments to be priced independently using the model.

`Options` is an options structure created with the function `derivset`. This structure defines how the tree is used to find the price of instruments in the portfolio, and how much additional information is displayed in the command window when calling the pricing function. If this input argument is not specified in the call to the pricing function, a default `Options` structure is used. The pricing options structure is described in “Pricing Options Structure” on page B-2.

The portfolio pricing functions classify the instruments and call the appropriate instrument-specific pricing function for each of the instrument types. The HJM instrument-specific pricing functions are `bondbyhjm`, `cfbyhjm`, `fixedbyhjm`, `floatbyhjm`, `optbndbyhjm`, `rangefloatbyhjm`, `swapbyhjm`, and `swaptionbyhjm`. A similarly named set of functions exists for BDT models. You can also use these functions directly to calculate the price of sets of instruments of the same type.

### HJM Pricing Example

Consider the following example, which uses the portfolio and interest-rate data in the MAT-file `deriv.mat` included in the toolbox. Load the data into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

```
whos
```

Name	Size	Bytes	Class	Attributes
BDTInstSet	1x1	15956	struct	
BDTTree	1x1	5138	struct	
BKInstSet	1x1	15946	struct	
BKTree	1x1	5904	struct	
CRRInstSet	1x1	12434	struct	
CRRTree	1x1	5058	struct	
EQPInstSet	1x1	12434	struct	
EQPTree	1x1	5058	struct	
HJMInstSet	1x1	15948	struct	
HJMTree	1x1	5838	struct	
HWInstSet	1x1	15946	struct	
HWTTree	1x1	5904	struct	
ITTInstSet	1x1	12438	struct	
ITTree	1x1	8862	struct	
ZeroInstSet	1x1	10282	struct	
ZeroRateSpec	1x1	1580	struct	

HJMTree and HJMInstSet are the input arguments required to call the function `hjmprice`.

Use the function `instdisp` to examine the set of instruments contained in the variable `HJMInstSet`.

`instdisp(HJMInstSet)`

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate	FirstCouponDate	LastCouponDate	StartDate	Face
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	Bond	0.04	01-Jan-2000	01-Jan-2004	2	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Index	Type	UnderInd	OptSpec	Strike	ExerciseDates	AmericanOpt	Name	Quantity				
3	OptBond	2	call	101	01-Jan-2003	NaN	Option 101	-50				
Index	Type	CouponRate	Settle	Maturity	FixedReset	Basis	Principal	Name	Quantity			
4	Fixed	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	4% Fixed	80			
Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	Name	Quantity			
5	Float	20	01-Jan-2000	01-Jan-2003	1	NaN	NaN	20BP Float	8			
Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal	Name	Quantity			
6	Cap	0.03	01-Jan-2000	01-Jan-2004	1	NaN	NaN	3% Cap	30			
Index	Type	Strike	Settle	Maturity	FloorReset	Basis	Principal	Name	Quantity			
7	Floor	0.03	01-Jan-2000	01-Jan-2004	1	NaN	NaN	3% Floor	40			
Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	Name	Quantity		
8	Swap	[0.06 20]	01-Jan-2000	01-Jan-2003	[1 1]	NaN	NaN	[NaN]	6%/20BP Swap	10		
Index	Type	CouponRate	Settle	Maturity	Period	Basis	...	Name	Quantity			
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	...	4% bond	100			
2	Bond	0.04	01-Jan-2000	01-Jan-2004	2	NaN	...	4% bond	50			

There are eight instruments in this portfolio set: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap. Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `hjmprice`.

Now use `hjmprice` to calculate the price of each instrument in the instrument set.

```
Price = hjmprice(HJMTree, HJMInstSet)
```

```
Warning: Not all cash flows are aligned with the tree. Result will  
be approximated.
```

```
Price =  
    98.7159  
    97.5280  
     0.0486  
    98.7159  
   100.5529  
     6.2831  
     0.0486  
     3.6923
```

---

**Note** The warning shown above appears because some of the cash flows for the second bond do not fall exactly on a tree node.

---

### BDT Pricing Example

Load the MAT-file `deriv.mat` into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

```
whos
```

Name	Size	Bytes	Class	Attributes
BDTInstSet	1x1	15956	struct	
BDTTree	1x1	5138	struct	
BKInstSet	1x1	15946	struct	
BKTree	1x1	5904	struct	
CRRInstSet	1x1	12434	struct	
CRRTree	1x1	5058	struct	
EQPInstSet	1x1	12434	struct	
EQPTree	1x1	5058	struct	



```
HJMInstSet      1x1      15948  struct
HJMTree         1x1      5838   struct
HWInstSet       1x1      15946  struct
HWTTree         1x1      5904   struct
ITTInstSet      1x1      12438  struct
ITTTTree        1x1      8862   struct
ZeroInstSet     1x1      10282  struct
ZeroRateSpec    1x1      1580   struct
```

BDTTree and BDTInstSet are the input arguments required to call the function `bdtprice`.

Use the function `instdisp` to examine the set of instruments contained in the variable `BDTInstSet`.

`instdisp(BDTInstSet)`

```
Index Type CouponRate Settle      Maturity      Period Basis EndMonthRule IssueDate FirstCouponDate LastCouponDate StartDate Face
1 Bond 0.1 01-Jan-2000 01-Jan-2003 1 NaN NaN NaN NaN NaN NaN NaN NaN
2 Bond 0.1 01-Jan-2000 01-Jan-2004 2 NaN NaN NaN NaN NaN NaN NaN NaN

Index Type UnderInd OptSpec Strike ExerciseDates AmericanOpt Name Quantity
3 OptBond 1 call 95 01-Jan-2002 NaN Option 95 -50

Index Type CouponRate Settle      Maturity      FixedReset Basis Principal Name Quantity
4 Fixed 0.1 01-Jan-2000 01-Jan-2003 1 NaN NaN 10% Fixed 80

Index Type Spread Settle      Maturity      FloatReset Basis Principal Name Quantity
5 Float 20 01-Jan-2000 01-Jan-2003 1 NaN NaN 20BP Float 8

Index Type Strike Settle      Maturity      CapReset Basis Principal Name Quantity
6 Cap 0.15 01-Jan-2000 01-Jan-2004 1 NaN NaN 15% Cap 30

Index Type Strike Settle      Maturity      FloorReset Basis Principal Name Quantity
7 Floor 0.09 01-Jan-2000 01-Jan-2004 1 NaN NaN 9% Floor 40

Index Type LegRate Settle      Maturity      LegReset Basis Principal LegType Name Quantity
8 Swap [0.15 10] 01-Jan-2000 01-Jan-2003 [1 1] NaN NaN [NaN] 15%/10BP Swap 10
```

There are eight instruments in this portfolio set: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap. Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `bdtprice`.

Now use `bdtprice` to calculate the price of each instrument in the instrument set.

`Price = bdtprice(BDTTree, BDTInstSet)`

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

Price =

```
95.5030
93.9079
 1.7657
95.5030
100.4865
 1.4863
 0.0245
 7.4222
```

### Price Vector Output

The prices in the output vector `Price` correspond to the prices at observation time zero ( $t_{Obs} = 0$ ), which is defined as the valuation date of the interest-rate tree. The instrument indexing within `Price` is the same as the indexing within `InstSet`.

In the HJM example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(HJMInstSet, 'FieldName', 'Name')
```

```
InstNames =
```

```
4% bond
4% bond
Option 101
4% Fixed
20BP Float
3% Cap
3% Floor
6%/20BP Swap
```

So, in the `Price` vector, the fourth element, 98.7159, represents the price of the fourth instrument (4% fixed-rate note); the sixth element, 6.2831, represents the price of the sixth instrument (3% cap).

In the BDT example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(BDTInstSet, 'FieldName', 'Name')
```

```
InstNames =
```

```
10% Bond
10% Bond
Option 95
10% Fixed
```

```

20BP Float
15% Cap
9% Floor
15%/10BP Swap

```

So, in the `Price` vector, the fourth element, 95.5030, represents the price of the fourth instrument (10% fixed-rate note); the sixth element, 1.4863, represents the price of the sixth instrument (15% cap).

### Price Tree Structure Output

If you call a pricing function with two output arguments, for example,

```
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet)
```

you generate a price tree along with the price information.

The optional output price tree structure `PriceTree` holds all the pricing information.

### HJM Price Tree

In the HJM example, the first field of this structure, `FinObj`, indicates that this structure represents a price tree. The second field, `PBush`, is the tree holding the price of the instruments in each node of the tree. The third field, `AIBush`, is the tree holding the accrued interest of the instruments in each node of the tree. Finally, the fourth field, `tObs`, represents the observation time of each level of `PBush` and `AIBush`, with units in terms of compounding periods.

In this example, the price tree looks like

```
PriceTree =
```

```

FinObj: 'HJMPriceTree'
PBush: {[8x1 double] [8x1x2 double] ... [8x8 double]}
AIBush: {[8x1 double] [8x1x2 double] ... [8x8 double]}
tObs: [0 1 2 3 4]

```

Both `PBush` and `AIBush` are 1-by-5 cell arrays, consistent with the five observation times of `tObs`. The data display has been shortened here to fit on a single line.

Using the command-line interface, you can directly examine `PriceTree.PBush`, the field within the `PriceTree` structure that contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
PriceTree.PBush{1}
```

```
ans =  
  
98.7159  
97.5280  
0.0486  
98.7159  
100.5529  
6.2831  
0.0486  
3.6923
```

With this interface, you can observe the prices for *all* instruments in the portfolio at a *specific time*.

### **BDT Price Tree**

The BDT output price tree structure `PriceTree` holds all the pricing information. The first field of this structure, `FinObj`, indicates that this structure represents a price tree. The second field, `Ptree`, is the tree holding the price of the instruments in each node of the tree. The third field, `AITree`, is the tree holding the accrued interest of the instruments in each node of the tree. The fourth field, `tObs`, represents the observation time of each level of `Ptree` and `AITree`, with units in terms of compounding periods.

You can directly examine the field within the `PriceTree` structure, which contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
[Price, PriceTree] = bdtprice(BDTree, BDTInstSet)
```

```
PriceTree.Ptree{1}
```

```
ans =  
  
95.5030  
93.9079  
1.7657  
95.5030  
100.4865  
1.4863  
0.0245  
7.4222
```

## See Also

bdtprice | bdtens | bdttimespec | bdttree | bdtvolspec | bkprice | bksens |  
 bktimespec | bktree | bkvolspec | bondbybdt | bondbybk | bondbyhjm | bondbyhw |  
 bondbyzero | capbybdt | capbybk | capbyblk | capbyhjm | capbyhw | cfbybdt |  
 cfbybk | cfbyhjm | cfbyhw | cfbyzero | fixedbybdt | fixedbybk | fixedbyhjm |  
 fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw |  
 floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk |  
 floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec |  
 hjmtree | hjmvolspec | hwcalbycap | hwcalbyfloor | hwprice | hwsens |  
 hwtimespec | hwtree | hwvolspec | instbond | instcap | instcf | instfixed |  
 instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat |  
 instoptfloat | instrangefloat | instswap | instswaption | intenvprice |  
 intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm  
 | oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw |  
 optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw |  
 optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw |  
 optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw |  
 rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw |  
 swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt |  
 swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

## Related Examples

- “Overview of Interest-Rate Tree Models” on page 2-50
- “Computing Instrument Sensitivities” on page 2-108
- “Graphical Representation of Trees” on page 2-157
- “Understanding Interest-Rate Tree Models” on page 2-79
- “Understanding the Interest-Rate Term Structure” on page 2-55
- “Pricing Using Interest-Rate Term Structure” on page 2-72

## More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Computing Instrument Sensitivities

Sensitivities can be reported either as dollar price changes or percentage price changes. The delta, gamma, and vega sensitivities that the toolbox computes are dollar sensitivities.

The functions `hjmsens` and `bdtsens` compute the delta, gamma, and vega sensitivities of instruments using an interest-rate tree. They also optionally return the calculated price for each instrument. The sensitivity functions require the same two input arguments used by the pricing functions (`HJMTree` and `HJMInstSet` for HJM; `BDTTree` and `BDTInstSet` for BDT).

Sensitivity functions calculate the dollar value of delta and gamma by shifting the observed forward yield curve by 100 basis points in each direction, and the dollar value of vega by shifting the volatility process by 1%. To obtain the per-dollar value of the sensitivities, divide the dollar sensitivity by the price of the corresponding instrument.

### HJM Sensitivities Example

The calling syntax for the function is:

```
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet)
```

Use the previous example data to calculate the price of instruments.

```
load deriv.mat
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet);

Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
```

---

**Note** The warning appears because some of the cash flows for the second bond do not fall exactly on a tree node.

---

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

```
All = [Delta, Gamma, Vega, Price]
```

```
All =
```

```
    -272.65    1029.90     0.00     98.72
```

-347.43	1622.69	-0.04	97.53
-8.08	643.40	34.07	0.05
-272.65	1029.90	0.00	98.72
-1.04	3.31	0	100.55
294.97	6852.56	93.69	6.28
-47.16	8459.99	93.69	0.05
-282.05	1059.68	0.00	3.69

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in `HJMIInstSet`. To view the *per-dollar sensitivities*, divide each dollar sensitivity by the corresponding instrument price.

## BDT Sensitivities Example

The calling syntax for the function is:

```
[Delta, Gamma, Vega, Price] = bdtSENS(BDTree, BDTInstSet);
```

Arrange the sensitivities and prices into a single matrix.

```
All = [Delta, Gamma, Vega, Price]
```

```
All =
```

-232.67	803.71	-0.00	95.50
-281.05	1181.93	-0.01	93.91
-50.54	246.02	5.31	1.77
-232.67	803.71	0	95.50
0.84	2.45	0	100.49
78.38	748.98	13.54	1.49
-4.36	382.06	2.50	0.02
-253.23	863.81	0	7.42

To view the *per-dollar sensitivities*, divide each dollar sensitivity by the corresponding instrument price.

```
All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]
```

```
All =
```

-2.44	8.42	-0.00	95.50
-2.99	12.59	-0.00	93.91
-28.63	139.34	3.01	1.77
-2.44	8.42	0	95.50
0.01	0.02	0	100.49
52.73	503.92	9.11	1.49
-177.89	15577.42	101.87	0.02
-34.12	116.38	0	7.42

### See Also

bdtprice | bdtens | bdttimespec | bdttree | bdtvolspec | bkprice | bksens | bktimespec | bktree | bkvolspec | bondbybdt | bondbybk | bondbyhjm | bondbyhw | bondbyzero | capbybdt | capbybk | capbyblk | capbyhjm | capbyhw | cfbybdt | cfbybk | cfbyhjm | cfbyhw | cfbyzero | fixedbybdt | fixedbybk | fixedbyhjm | fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw | floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk | floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec | hjmtree | hjmvolspec | hwcalbycap | hwcalbyfloor | hwprice | hwsens | hwtimespec | hwtree | hwvolspec | instbond | instcap | instcf | instfixed | instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat | instoptfloat | instrangefloat | instswap | instswaption | intenvprice | intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm | oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw | optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw | optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw | optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw | rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw | swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt | swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### Related Examples

- “Overview of Interest-Rate Tree Models” on page 2-50
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Graphical Representation of Trees” on page 2-157
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### More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44



## Calibrating Hull-White Model Using Market Data

The pricing of interest-rate derivative securities relies on models that describe the underlying process. These interest rate models depend on one or more parameters that you must determine by matching the model predictions to the existing data available in the market. In the Hull-White model, there are two parameters related to the short rate process: mean reversion and volatility. Calibration is used to determine these parameters, such that the model can reproduce, as close as possible, the prices of caps or floors observed in the market. The calibration routines find the parameters that minimize the difference between the model price predictions and the market prices for caps and floors.

For a Hull-White model, the minimization is two dimensional, with respect to mean reversion ( $\alpha$ ) and volatility ( $\sigma$ ). That is, calibrating the Hull-White model minimizes the difference between the model's predicted prices and the observed market prices of the corresponding caplets or floorlets.

### Hull-White Model Calibration Example

Use market data to identify the implied volatility ( $\sigma$ ) and mean reversion ( $\alpha$ ) coefficients needed to build a Hull-White tree to price an instrument. The ideal case is to use the volatilities of the caps or floors used to calculate  $\alpha$  and  $\sigma$ . This will most likely not be the case, so market data must be interpolated to obtain the required values.

Consider a cap with these parameters:

```
Settle = ' Jan-21-2008';
Maturity = 'Mar-21-2011';
Strike = 0.0690;
Reset = 4;
Principal = 1000;
Basis = 0;
```

The caplets for this example would fall in:

```
capletDates = cfdates(Settle, Maturity, Reset, Basis);
datestr(capletDates')

ans =

21-Mar-2008
21-Jun-2008
21-Sep-2008
```

21-Dec-2008  
 21-Mar-2009  
 21-Jun-2009  
 21-Sep-2009  
 21-Dec-2009  
 21-Mar-2010  
 21-Jun-2010  
 21-Sep-2010  
 21-Dec-2010  
 21-Mar-2011

In the best case, look up the market volatilities for caplets with a  $\text{Strike} = 0.0690$ , and maturities in each reset date listed, but the likelihood of finding these exact instruments is low. As a consequence, use data that is available in the market and interpolate to find appropriate values for the caplets.

Based on the market data, you have the cap information for different dates and strikes. Assume that instead of having the data for  $\text{Strike} = 0.0690$ , you have the data for  $\text{Strike1} = 0.0590$  and  $\text{Strike2} = 0.0790$ .

<b>Maturity</b>	<b>Strike1 = 0.0590</b>	<b>Strike2 = 0.0790</b>
21-Mar-2008	0.1533	0.1526
21-Jun-2008	0.1731	0.1730
21-Sep-2008	0.1727	0.1726
21-Dec-2008	0.1752	0.1747
21-Mar-2009	0.1809	0.1808
21-Jun-2009	0.1809	0.1792
21-Sep-2009	0.1805	0.1797
21-Dec-2009	0.1802	0.1794
21-Mar-2010	0.1802	0.1733
21-Jun-2010	0.1757	0.1751
21-Sep-2010	0.1755	0.1750
21-Dec-2010	0.1755	0.1745
21-Mar-2011	0.1726	0.1719

The nature of this data lends itself to matrix nomenclature, which is perfect for MATLAB. `hwcalbycap` requires that the dates, the strikes, and the actual volatility be separated into three variables: `MarketStrike`, `MarketMat`, and `MarketVol`.

```
MarketStrike = [0.0590; 0.0790];
MarketMat = {'21-Mar-2008';
'21-Jun-2008';
'21-Sep-2008';
'21-Dec-2008';
'21-Mar-2009';
'21-Jun-2009';
'21-Sep-2009';
'21-Dec-2009';
'21-Mar-2010';
'21-Jun-2010';
'21-Sep-2010';
'21-Dec-2010';
'21-Mar-2011'};

MarketVol = [0.1533 0.1731 0.1727 0.1752 0.1809 0.1800 0.1805 0.1802 0.1735 0.1757 ...
0.1755 0.1755 0.1726; % First row in table corresponding to Strike1
0.1526 0.1730 0.1726 0.1747 0.1808 0.1792 0.1797 0.1794 0.1733 0.1751 ...
0.1750 0.1745 0.1719]; % Second row in table corresponding to Strike2
```

Complete the input arguments using this data for `RateSpec`:

```
Rates= [0.0627;
0.0657;
0.0691;
0.0717;
0.0739;
0.0755;
0.0765;
0.0772;
0.0779;
0.0783;
0.0786;
0.0789;
0.0792;
0.0793];

ValuationDate = '21-Jan-2008';
EndDates = {'21-Mar-2008'; '21-Jun-2008'; '21-Sep-2008'; '21-Dec-2008'; ...
'21-Mar-2009'; '21-Jun-2009'; '21-Sep-2009'; '21-Dec-2009'; ...
'21-Mar-2010'; '21-Jun-2010'; '21-Sep-2010'; '21-Dec-2010'; ...
'21-Mar-2011'; '21-Jun-2011'};

Compounding = 4;
Basis = 0;

RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec =
```

```
    FinObj: 'RateSpec'  
    Compounding: 4  
        Disc: [14x1 double]  
        Rates: [14x1 double]  
        EndTimes: [14x1 double]  
        StartTimes: [14x1 double]  
        EndDates: [14x1 double]  
        StartDates: 733428  
    ValuationDate: 733428  
        Basis: 0  
    EndMonthRule: 1
```

### Call the calibration routine to find values for volatility parameters Alpha and Sigma

Use `hwcalbycap` to calculate the values of Alpha and Sigma based on market data. Internally, `hwcalbycap` calls the Optimization Toolbox function `lsqnonlin`. You can customize `lsqnonlin` by passing an optimization options structure created by `optimoptions` and then this can be passed to `hwcalbycap` using the name-value pair argument for `OptimOptions`. For example, `optimoptions` defines the target objective function tolerance as `100*eps` and then calls `hwcalbycap`:

```
o=optimoptions('lsqnonlin','TolFun',100*eps);  
  
[Alpha, Sigma] = hwcalbycap(RateSpec, MarketStrike, MarketMat, MarketVol,...  
    Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal, 'Basis',...  
    Basis, 'OptimOptions', o)  
  
Local minimum possible.  
  
lsqnonlin stopped because the size of the current step is less than  
the default value of the step size tolerance.  
  
Warning: LSQNONLIN did not converge to an optimal solution. It exited with exitflag = 2.  
  
> In hwcalbycapfloor at 93  
    In hwcalbycap at 75  
  
Alpha =  
  
    1.0000e-06  
  
Sigma =  
  
    0.0127
```

The previous warning indicates that the conversion was not optimal. The search algorithm used by the Optimization Toolbox™ function `lsqnonlin` did not find a solution that conforms to all the constraints. To discern whether the solution is acceptable, look at the results of the optimization by specifying a third output (`OptimOut`) for `hwcalbycap`:

```
[Alpha, Sigma, OptimOut] = hwalbycap(RateSpec, MarketStrike, MarketMat,...
MarketVol, Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal,...
'Basis', Basis, 'OptimOptions', o);
```

The `OptimOut.residual` field of the `OptimOut` structure is the optimization residual. This value contains the difference between the Black caplets and those calculated during the optimization. You can use the `OptimOut.residual` value to calculate the percentual difference (error) compared to Black caplet prices and then decide whether the residual is acceptable. There is almost always some residual, so decide if it is acceptable to parameterize the market with a single value of Alpha and Sigma.

### Price caplets using market data and Black's formula to obtain reference caplet values

To determine the effectiveness of the optimization, calculate reference caplet values using Black's formula and the market data. Note, you must first interpolate the market data to obtain the caplets for calculation:

```
MarketMatNum = datenum(MarketMat);
[Mats, Strikes] = meshgrid(MarketMatNum, MarketStrike);
FlatVol = interp2(Mats, Strikes, MarketVol, datenum(Maturity), Strike, 'spline');
```

Compute the price of the cap using the Black model:

```
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, FlatVol,...
'Reset', Reset, 'Basis', Basis, 'Principal', Principal);
Caplets = Caplets(2:end)';
```

Caplets =

```
0.3210
1.6355
2.4863
3.1903
3.4110
3.2685
3.2385
3.4803
3.2419
3.1949
3.2991
3.3750
```

### Compare optimized values and Black values and display graphically

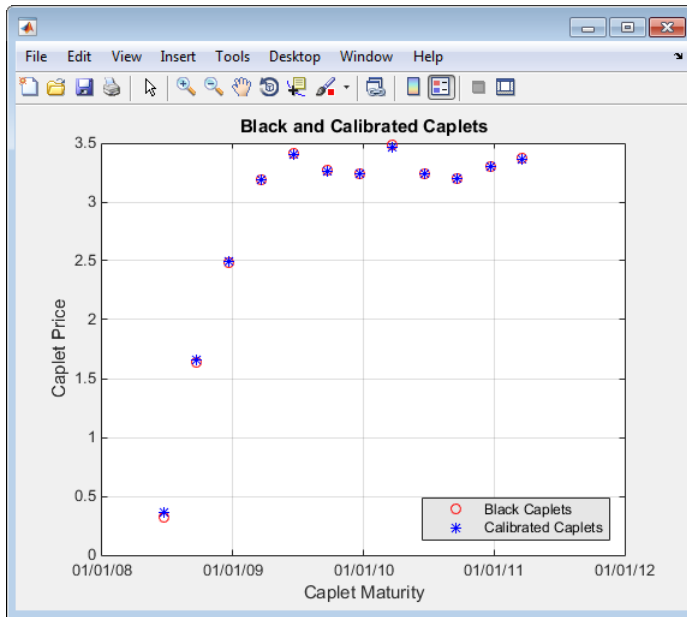
After calculating the reference values for the caplets, compare the values, analytically and graphically, to determine whether the calculated single values of Alpha and Sigma provide an adequate approximation:

```
OptimCaplets = Caplets+OptimOut.residual;

disp(' ');
disp('      Black76      Calibrated Caplets');
disp([Caplets              OptimCaplets])

plot(MarketMatNum(2:end), Caplets, 'or', MarketMatNum(2:end), OptimCaplets, '*b');
datetick('x', 2)
xlabel('Caplet Maturity');
ylabel('Caplet Price');
title('Black and Calibrated Caplets');
h = legend('Black Caplets', 'Calibrated Caplets');
set(h, 'color', [0.9 0.9 0.9]);
set(h, 'Location', 'SouthEast');
set(gcf, 'NumberTitle', 'off')
grid on
```

Black76	Calibrated Caplets
0.3210	0.3636
1.6355	1.6603
2.4863	2.4974
3.1903	3.1874
3.4110	3.4040
3.2685	3.2639
3.2385	3.2364
3.4803	3.4683
3.2419	3.2408
3.1949	3.1957
3.2991	3.2960
3.3750	3.3663



### Compare cap prices using the Black, HW analytical, and HW tree models

Using the calculated caplet values, compare the prices of the corresponding cap using the Black model, Hull-White analytical, and Hull-White tree models. To calculate a Hull-White tree based on Alpha and Sigma, use these calibration routines:

- Black model:

$$\text{CapPriceBLK} = \text{CapPrice};$$

- HW analytical model:

$$\text{CapPriceHWAAnalytical} = \text{sum}(\text{OptimCaplets});$$

- HW tree model to price the cap derived from the calibration process:

- 1 Create VolSpec from the calibration parameters Alpha and Sigma:

```

VolDates    = EndDates;
VolCurve    = Sigma*ones(14,1);
AlphaDates  = EndDates;
AlphaCurve  = Alpha*ones(14,1);
HWVolSpec  = hwwolSpec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve);

```

- 2 Create the TimeSpec:

```
HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding);
```

- 3 Build the HW tree using the HW2000 method:

```
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec, 'Method', 'HW2000');
```

- 4 Price the cap:

```
Price = capbyhw(HWTree, Strike, Settle, Maturity, Reset, Basis, Principal);

disp(' ');
disp([' CapPrice Black76 .....: ', num2str(CapPriceBLK,'%15.5f')]);
disp([' CapPrice HW analytical.....: ', num2str(CapPriceHWAAnalytical,'%15.5f')]);
disp([' CapPrice HW from capbyhw ..: ', num2str(Price,'%15.5f')]);
disp(' ');

CapPrice Black76 .....: 34.14220
CapPrice HW analytical.....: 34.18008
CapPrice HW from capbyhw ..: 34.14192
```

### Price a portfolio of instruments using the calibrated HW tree

After building a Hull-White tree, based on parameters calibrated from market data, use HWTtree to price a portfolio of these instruments:

- Two bonds

```
CouponRate = [0.07; 0.09];
Settle= ' Jan-21-2008';
Maturity = {'Mar-21-2010'; 'Mar-21-2011'};
Period = 1;
Face = 1000;
Basis = 0;
```

- Bond with an embedded American call option

```
CouponRateOEB = 0.08;
SettleOEB = ' Jan-21-2008';
MaturityOEB = 'Mar-21-2011';
OptSpec = 'call';
StrikeOEB = 950;
ExerciseDatesOEB = 'Mar-21-2011';
AmericanOpt= 1;
Period =1;
Face = 1000;
Basis =0;
```

To price this portfolio of instruments using the calibrated HWTtree:



**1** Use `instadd` to create the portfolio `InstSet`:

```
InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period, Basis, [], [], [], [], [], Face);
InstSet = instadd(InstSet, 'OptEmBond', CouponRateOEB, SettleOEB, MaturityOEB, OptSpec, ...
StrikeOEB, ExerciseDatesOEB, 'AmericanOpt', AmericanOpt, 'Period', Period, ...
'Face', Face, 'Basis', Basis);
```

**2** Add the cap instrument used in the calibration:

```
SettleCap = 'Jan-21-2008';
MaturityCap = 'Mar-21-2011';
StrikeCap = 0.0690;
Reset = 4;
Principal = 1000;

InstSet = instadd(InstSet, 'Cap', StrikeCap, SettleCap, MaturityCap, Reset, Basis, Principal);
```

**3** Assign names to the portfolio instruments:

```
Names = {'7% Bond'; '8% Bond'; 'BondEmbCall'; '6.9% Cap'};
InstSet = instsetfield(InstSet, 'Index', 1:4, 'FieldName', {'Name'}, 'Data', Names);
```

**4** Examine the set of instruments contained in `InstSet`:

```
instdisp(InstSet)
```

```
IdxType  CoupRate  Settle  Mature  Period  Basis  EOMRule  IssueDate  1stCoupDate  LastCoupDate  StartDate  Face  Name
1 Bond  0.07      21-Jan-2008  21-Mar-2010  1 0 NaN NaN      NaN NaN NaN 1000  7% Bond
2 Bond  0.09      21-Jan-2008  21-Mar-2011  1 0 NaN NaN      NaN NaN NaN 1000  8% Bond
```

```
IdxType  CoupRate  Settle  Mature  OptSpec  Stke  ExDate  Per  Basis  EOMRule  IssDate  1stCoupDate  LstCoupDate  StrtDate  Face  An
3 OptEmBond 0.08  21-Jan-2008  21-Mar-2011  call  950  21-Jan-2008  21-Mar-2011  1 0 1 NaN NaN NaN NaN 1000 1 Bond
```

```
Index Type  Strike  Settle      Maturity  CapReset  Basis  Principal  Name
4 Cap  0.069  21-Jan-2008  21-Mar-2011  4 0 1000  6.9% Cap
```

**5** Use `hwprice` to price the portfolio using the calibrated `HWTtree`:

```
format bank
PricePortfolio = hwprice(HWTtree, InstSet)
```

```
PricePortfolio =
    980.45
   1023.05
    945.73
    34.14
```

## See Also

`bdtprice` | `bdtens` | `bdttimespec` | `bdttree` | `bdtvolspec` | `bkprice` | `bksens` | `bktimespec` | `bktree` | `bkvolspec` | `bondbybdt` | `bondbybk` | `bondbyhjm` | `bondbyhw` | `bondbyzero` | `capbybdt` | `capbybk` | `capbyblk` | `capbyhjm` | `capbyhw` | `cfbybdt` | `cfbybk` | `cfbyhjm` | `cfbyhw` | `cfbyzero` | `fixedbybdt` | `fixedbybk` | `fixedbyhjm` | `fixedbyhw` | `fixedbyzero` | `floatbybdt` | `floatbybk` | `floatbyhjm` | `floatbyhw`

floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk | floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec | hjmtree | hjmvolspec | hwalbycap | hwalbyfloor | hwprice | hwsens | hwtimespec | hwtree | hwwolspec | instbond | instcap | instcf | instfixed | instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat | instoptfloat | instrangefloat | instswap | instswaption | intenvprice | intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm | oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw | optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw | optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw | optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw | rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw | swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt | swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### Related Examples

- “Overview of Interest-Rate Tree Models” on page 2-50
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Graphical Representation of Trees” on page 2-157
- “Understanding Interest-Rate Tree Models” on page 2-79
- “Understanding the Interest-Rate Term Structure” on page 2-55
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### More About

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

# Interest-Rate Derivatives Using Closed-Form Solutions

## Pricing Caps and Floors Using the Black Option Model

Caps and floors are contracts that allow the holder to be protected if interest rates rise or decrease. The Black model uses a forward price as an underlier in place of a spot price. The assumption is that the forward price at maturity of the option is log-normally distributed.

Closed-form solutions for pricing caps and floors using the Black model support the following tasks:

Task	Function
Price the interest rate caps using the Black option pricing model.	capbyblk
Price the interest rate floors using the Black option pricing model.	floorbyblk

## See Also

agencyoas | agencyprice | blackvolbyrebonato | blackvolbysabr | bndfutimprepo | bndfutprice | capbyblk | capbylg2f | convfactor | floorbyblk | floorbylg2f | hwcalbycap | hwcalbyfloor | optsensbysabr | swaptionbyblk | swaptionbylg2f | tfutbyprice | tfutbyyield | tfutimprepo | tfutpricebyrepo | tfutyieldbyrepo

## Related Examples

- “Calibrate the SABR Model” on page 2-36
- “Price a Swaption Using the SABR Model” on page 2-42
- “Computing the Agency OAS for Bonds” on page 6-3
- “Analysis of Bond Futures” on page 7-13
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17
- “Fitting the Diebold Li Model” on page 7-25
- “Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

- “Pricing Bermudan Swaptions with Monte Carlo Simulation” on page 2-141

### **More About**

- “Managing Present Value with Bond Futures” on page 7-16
- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

# Price Swaptions with Interest-Rate Models Using Simulation

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“Construct a Zero Curve” on page 2-124

“Define Swaption Parameters” on page 2-126

“Compute the Black Model and the Swaption Volatility Matrix” on page 2-126

“Select Calibration Instruments” on page 2-126

“Compute Swaption Prices Using Black's Model” on page 2-127

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“Simulate Interest-Rate Paths Using the Hull-White One-Factor Model” on page 2-128

“Simulate Interest-Rate Paths Using the Linear Gaussian Two-Factor Model” on page 2-131

“Simulate Interest-Rate Paths Using the LIBOR Market Model” on page 2-134

“Compare Interest-Rate Modeling Results” on page 2-139

“References” on page 2-140

## Introduction

This example shows how to price European swaptions using interest-rate models in Financial Instruments Toolbox. Specifically, a Hull-White one factor model, a Linear Gaussian two-factor model, and a LIBOR Market Model are calibrated to market data and then used to generate interest-rate paths using Monte Carlo simulation.

The following sections set up the data that is then used with examples for “Simulate Interest-Rate Paths Using the Hull-White One-Factor Model” on page 2-128, “Simulate Interest-Rate Paths Using the Linear Gaussian Two-Factor Model” on page 2-131, and “Simulate Interest-Rate Paths Using the LIBOR Market Model” on page 2-134:

- “Construct a Zero Curve” on page 2-124
- “Define Swaption Parameters” on page 2-126
- “Compute the Black Model and the Swaption Volatility Matrix” on page 2-126

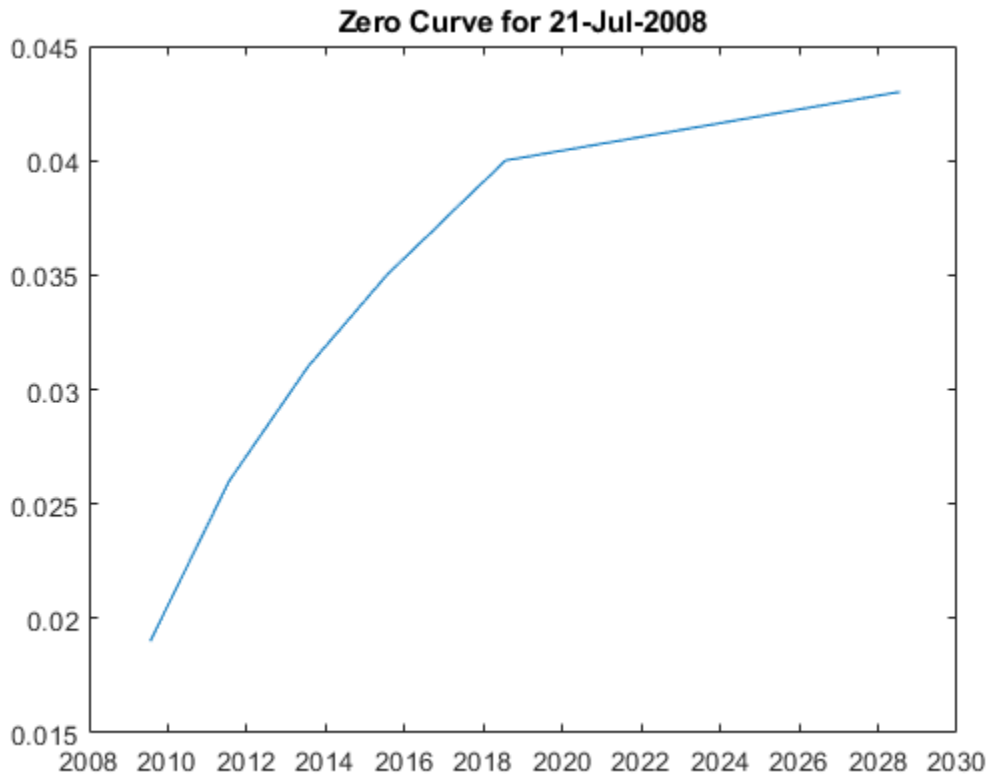
- “Select Calibration Instruments” on page 2-126
- “Compute Swaption Prices Using Black's Model” on page 2-127
- “Define Simulation Parameters” on page 2-127

### Construct a Zero Curve

This example shows how to use `ZeroRates` for a zero curve that is hard-coded. You can also create a zero curve by bootstrapping the zero curve from market data (for example, deposits, futures/forwards, and swaps)

The hard-coded data for the zero curve is defined as:

```
Settle = datenum('21-Jul-2008');  
  
% Zero Curve  
CurveDates = daysadd(Settle,360*([1 3 5 7 10 20]),1);  
ZeroRates = [1.9 2.6 3.1 3.5 4 4.3]'/100;  
  
plot(CurveDates,ZeroRates)  
datetick  
title(['Zero Curve for ' datestr(Settle)]);
```



Construct an IRCurve object.

```
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
```

Create the RateSpec.

```
RateSpec = intenvset('Rates',ZeroRates,'EndDates',CurveDates,'StartDate',Settle)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
    Disc: [6x1 double]
    Rates: [6x1 double]
    EndTimes: [6x1 double]
    StartTimes: [6x1 double]
```

```
EndDates: [6x1 double]
StartDates: 733610
ValuationDate: 733610
Basis: 0
EndMonthRule: 1
```

### Define Swaption Parameters

While Monte Carlo simulation is typically used to value more sophisticated derivatives (for example, Bermudan swaptions), in this example, the price of a European swaption is computed with an exercise date of five years and an underlying swap of five years.

```
InstrumentExerciseDate = datenum('21-Jul-2013');
InstrumentMaturity = datenum('21-Jul-2018');
InstrumentStrike = .045;
```

### Compute the Black Model and the Swaption Volatility Matrix

Black's model is often used to price and quote European exercise interest-rate options, that is, caps, floors and swaptions. In the case of swaptions, Black's model is used to imply a volatility given the current observed market price. The following matrix shows the Black implied volatility for a range of swaption exercise dates (columns) and underlying swap maturities (rows).

```
SwaptionBlackVol = [22 21 19 17 15 13 12
 21 19 17 16 15 13 11
 20 18 16 15 14 12 11
 19 17 15 14 13 12 10
 18 16 14 13 12 11 10
 15 14 13 12 12 11 10
 13 13 12 11 11 10 9]/100;
ExerciseDates = [1:5 7 10];
Tenors = [1:5 7 10];

EurExDatesFull = repmat(daysadd(Settle,ExerciseDates*360,1)',...
 length(Tenors),1);
EurMatFull = reshape(daysadd(EurExDatesFull,...
 repmat(360*Tenors,1,length(ExerciseDates)),1),size(EurExDatesFull));
```

### Select Calibration Instruments

Selecting the instruments to calibrate the model to is one of the tasks in calibration. For Bermudan swaptions, it is typical to calibrate to European swaptions that are co-terminal



with the Bermudan swaption to be priced. In this case, all swaptions having an underlying tenor that matures before the maturity of the swaption to be priced (21-Jul-2018) are used in the calibration.

```
% Find the swaptions that expire on or before the maturity date of the
% sample swaption
relidx = find(EurMatFull <= InstrumentMaturity);
```

## Compute Swaption Prices Using Black's Model

This example shows how to compute swaption prices using Black's Model. The swaption prices are then used to compare the model's predicted values that are obtained from the calibration process.

To compute the swaption prices using Black's model:

```
SwaptionBlackPrices = zeros(size(SwaptionBlackVol));
SwaptionStrike = zeros(size(SwaptionBlackVol));

for iSwaption=1:length(ExerciseDates)
    for iTenor=1:length(Tenors)
        [~,SwaptionStrike(iTenor,iSwaption)] = swapbyzero(RateSpec,[NaN 0], Settle, EurMatFull(iTenor,iSwaption),...
            'StartDate',EurExDatesFull(iTenor,iSwaption),'LegReset',[1 1]);
        SwaptionBlackPrices(iTenor,iSwaption) = swaptionbyblk(RateSpec, 'call', SwaptionStrike(iTenor,iSwaption),Settle,
            EurExDatesFull(iTenor,iSwaption), EurMatFull(iTenor,iSwaption), SwaptionBlackVol(iTenor,iSwaption));
    end
end
```

## Define Simulation Parameters

This example shows how to use the `simTermStructs` method with `HullWhite1F`, `LinearGaussian2F`, and `LiborMarketModel` objects.

To demonstrate using the `simTermStructs` method with `HullWhite1F`, `LinearGaussian2F`, and `LiborMarketModel` objects, use the following simulation parameters:

```
nPeriods = 5;
DeltaTime = 1;
nTrials = 1000;

Tenor = (1:10)';

SimDates = daysadd(Settle,360*DeltaTime*(0:nPeriods),1)
SimTimes = diff(yearfrac(SimDates(1),SimDates))

% For 1 year periods and an evenly spaced tenor, the exercise row will be
% the 5th row and the swaption maturity will be the 5th column
exRow = 5;
endCol = 5;
```

```
SimDates =  
    733610  
    733975  
    734340  
    734705  
    735071  
    735436
```

```
SimTimes =  
    1.0000  
    1.0000  
    1.0000  
    1.0027  
    1.0000
```

### Simulate Interest-Rate Paths Using the Hull-White One-Factor Model

This example shows how to simulate interest-rate paths using the Hull-White one-factor model. Before beginning this example that uses a `HullWhite1F` model, make sure that you have set up the data as described in:

- “Construct a Zero Curve” on page 2-124
- “Define Swaption Parameters” on page 2-126
- “Compute the Black Model and the Swaption Volatility Matrix” on page 2-126
- “Select Calibration Instruments” on page 2-126
- “Compute Swaption Prices Using Black's Model” on page 2-127
- “Define Simulation Parameters” on page 2-127

The Hull-White one-factor model describes the evolution of the short rate and is specified using the zero curve,  $\alpha$ , and  $\sigma$  parameters for the equation

$$dr = [\theta(t) - a(t)r]dt + \sigma(t)dW$$

where:

$dr$  is the change in the short-term interest rate over a small interval,  $dt$ .

$r$  is the short-term interest rate.

$\theta(t)$  is a function of time determining the average direction in which  $r$  moves, chosen such that movements in  $r$  are consistent with today's zero coupon yield curve.

$\alpha$  is the mean reversion rate.

$dt$  is a small change in time.

$\sigma$  is the annual standard deviation of the short rate.

$W$  is the Brownian motion.

The Hull-White model is calibrated using the function `swaptionbyhw`, which constructs a trinomial tree to price the swaptions. Calibration consists of minimizing the difference between the observed market prices (computed above using the Black's implied swaption volatility matrix, see “Compute the Black Model and the Swaption Volatility Matrix” on page 2-126) and the model's predicted prices.

In this example, the Optimization Toolbox function `lsqnonlin` is used to find the parameter set that minimizes the difference between the observed and predicted values. However, other approaches (for example, simulated annealing) may be appropriate. Starting parameters and constraints for  $\alpha$  and  $\sigma$  are set in the variables `x0`, `lb`, and `ub`; these could also be varied depending upon the particular calibration approach.

Calibrate the set of parameters that minimize the difference between the observed and predicted values using `swaptionbyhw` and `lsqnonlin`.

```
TimeSpec = hwtimespec(Settle,daysadd(Settle,360*(1:11),1), 2);
HW1Fobjfun = @(x) SwaptionBlackPrices(releidx) - ...
    swaptionbyhw(hwtree(hwvolspec(Settle,'11-Aug-2015',x(2),'11-Aug-2015',x(1)), RateSpec, TimeSpec), 'call', SwaptionStr
    EurExDatesFull(releidx), 0, EurExDatesFull(releidx), EurMatFull(releidx));
options = optimset('disp','iter','MaxFunEvals',1000,'TolFun',1e-5);

% Find the parameters that minimize the difference between the observed and
% predicted prices
x0 = [.1 .01];
lb = [0 0];
ub = [1 1];
HW1Fparams = lsqnonlin(HW1Fobjfun,x0,lb,ub,options);

HW_alpha = HW1Fparams(1)
HW_sigma = HW1Fparams(2)
```

Iteration	Func-count	f(x)	Norm of step	First-order optimality
0	3	0.953772		20.5
1	6	0.142828	0.0169199	1.53
2	9	0.123022	0.0146705	2.31
3	12	0.122222	0.0154098	0.482
4	15	0.122217	0.00131297	0.00409

Local minimum possible.

`lsqnonlin` stopped because the final change in the sum of squares relative to its initial value is less than the selected value of the function tolerance.

```
HW_alpha =  
    0.0967
```

```
HW_sigma =  
    0.0088
```

Construct the HullWhite1F model using the HullWhite1F constructor.

```
HW1F = HullWhite1F(RateSpec,HW_alpha,HW_sigma)
```

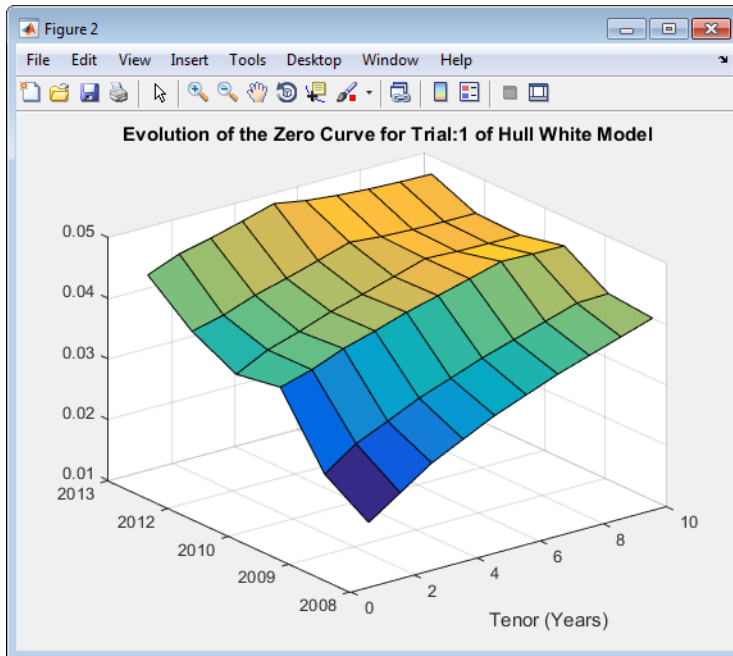
```
HW1F =
```

HullWhite1F with properties:

```
ZeroCurve: [1x1 IRDataCurve]  
Alpha: @(t,V)inAlpha  
Sigma: @(t,V)inSigma
```

Use Monte Carlo simulation to generate the interest-rate paths with HullWhite1F.simTermStructs.

```
HW1FSimPaths = HW1F.simTermStructs(nPeriods,'NTRIALS',nTrials,...  
    'DeltaTime',DeltaTime,'Tenor',Tenor,'antithetic',true);  
trialIdx = 1;  
figure  
surf(Tenor,SimDates,HW1FSimPaths(:,:,trialIdx))  
datetick y keep_ticks keep_limits  
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of Hull White Model'])  
xlabel('Tenor (Years)')
```



Price the European swaption.

```
DF = exp(bsxfun(@times,-HW1FSimPaths, repmat(Tenor', [nPeriods+1 1])));
SwapRate = (1 - DF(exRow,endCol,:))./sum(bsxfun(@times,1,DF(exRow,1:endCol,:)));
PayoffValue = 100*max(SwapRate-InstrumentStrike,0).*sum(bsxfun(@times,1,DF(exRow,1:endCol,:)));
RealizedDF = prod(exp(bsxfun(@times,-HW1FSimPaths(1:exRow,1,:), SimTimes(1:exRow))),1);
HW1F_SwaptionPrice = mean(RealizedDF.*PayoffValue)
```

```
HW1F_SwaptionPrice =
```

```
2.1839
```

## Simulate Interest-Rate Paths Using the Linear Gaussian Two-Factor Model

This example shows how to simulate interest-rate paths using the Linear Gaussian two-factor model. Before beginning this example that uses a `LinearGaussian2F` model, make sure that you have set up the data as described in:

- “Construct a Zero Curve” on page 2-124
- “Define Swaption Parameters” on page 2-126

- “Compute the Black Model and the Swaption Volatility Matrix” on page 2-126
- “Select Calibration Instruments” on page 2-126
- “Compute Swaption Prices Using Black's Model” on page 2-127
- “Define Simulation Parameters” on page 2-127

The Linear Gaussian two-factor model (called the G2++ by Brigo and Mercurio, see “Interest-Rate Modeling Using Monte Carlo Simulation” on page C-5) is also a short rate model, but involves two factors. Specifically:

$$r(t) = x(t) + y(t) + \phi(t)$$

$$dx(t) = -a(t)x(t)dt + \sigma(t)dW_1(t), x(0) = 0$$

$$dy(t) = -b(t)y(t)dt + \eta(t)dW_2(t), y(0) = 0$$

where  $dW_1(t)dW_2(t) = \rho dt$  is a two-dimensional Brownian motion with correlation  $\rho$ , and  $\phi$  is a function chosen to match the initial zero curve.

The function `swaptionbylg2f` is used to compute analytic values of the swaption price for model parameters, and therefore can be used to calibrate the model. Calibration consists of minimizing the difference between the observed market prices (computed above using the Black's implied swaption volatility matrix, see “Compute the Black Model and the Swaption Volatility Matrix” on page 2-126) and the model's predicted prices.

In this example, the approach is similar to “Simulate Interest-Rate Paths Using the Hull-White One-Factor Model” on page 2-128 and the Optimization Toolbox function `lsqnonlin` is used to minimize the difference between the observed swaption prices and the predicted swaption prices. However, other approaches (for example, simulated annealing) may also be appropriate. Starting parameters and constraints for  $a$ ,  $b$ ,  $\eta$ ,  $\rho$ , and  $\sigma$  are set in the variables `x0`, `lb`, and `ub`; these could also be varied depending upon the particular calibration approach.

Calibrate the set of parameters that minimize the difference between the observed and predicted values using `swaptionbylg2f` and `lsqnonlin`.

```
G2PPobjfun = @(x) SwaptionBlackPrices(releidx) - swaptionbylg2f(ircdc,x(1),x(2),x(3),x(4),x(5),SwaptionStrike(releidx),...
    EurExDatesFull(releidx),EurMatFull(releidx),'Reset',1);

options = optimset('disp','iter','MaxFunEvals',1000,'TolFun',1e-5);
x0 = [.2 .1 .02 .01 -.5];
lb = [0 0 0 0 -1];
ub = [1 1 1 1 1];
LG2Fparams = lsqnonlin(G2PPobjfun,x0,lb,ub,options)
```

Iteration	Func-count	f(x)	Norm of step	First-order optimality
0	6	12.3547		67.6
1	12	1.37984	0.0979743	8.59
2	18	1.37984	0.112847	8.59
3	24	0.445202	0.0282118	1.31
4	30	0.236746	0.0564236	3.02
5	36	0.134678	0.0843366	7.78
6	42	0.0398816	0.015084	6.34
7	48	0.0287731	0.038967	0.732
8	54	0.0273025	0.112847	0.881
9	60	0.0241689	0.213033	1.06
10	66	0.0241689	0.125602	1.06
11	72	0.0239103	0.0314005	9.78
12	78	0.0234246	0.0286685	1.21
13	84	0.0234246	0.0491135	1.21
14	90	0.023304	0.0122784	1.67
15	96	0.0231931	0.0245568	5.92
16	102	0.0230898	0.00785421	0.434
17	108	0.0230898	0.0245568	0.434
18	114	0.023083	0.00613919	0.255

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the selected value of the function tolerance.

LG2Fparams =

```
0.5752    0.1181    0.0146    0.0119   -0.7895
```

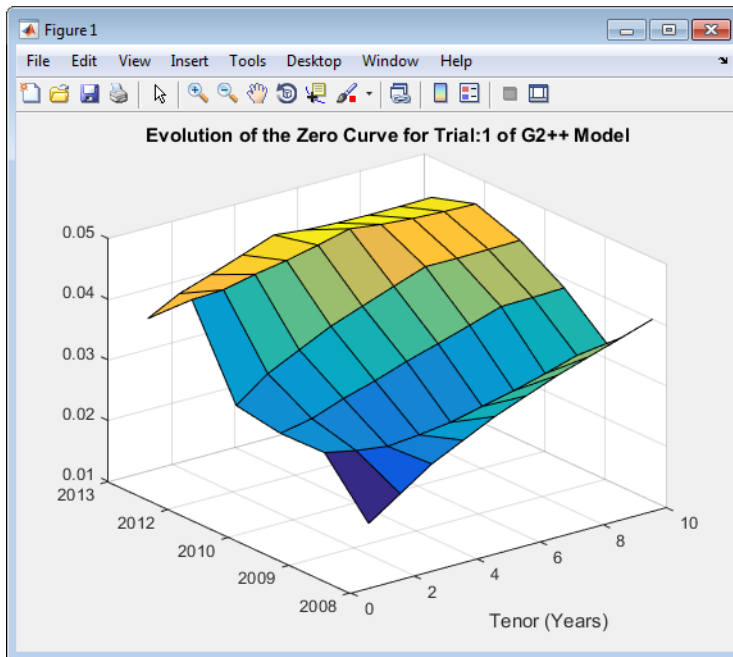
Create the G2PP object and use Monte Carlo simulation to generate the interest-rate paths with `LinearGaussian2F.simTermStructs`.

```
LG2f_a = LG2Fparams(1);
LG2f_b = LG2Fparams(2);
LG2f_sigma = LG2Fparams(3);
LG2f_eta = LG2Fparams(4);
LG2f_rho = LG2Fparams(5);

G2PP = LinearGaussian2F(RateSpec, LG2f_a, LG2f_b, LG2f_sigma, LG2f_eta, LG2f_rho);

G2PPSimPaths = G2PP.simTermStructs(nPeriods, 'NTRIALS', nTrials, ...
    'DeltaTime', DeltaTime, 'Tenor', Tenor, 'antithetic', true);

trialIdx = 1;
figure
surf(Tenor, SimDates, G2PPSimPaths(:, :, trialIdx))
datetick y keep ticks keep limits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of G2++ Model'])
xlabel('Tenor (Years)')
```



Price the European swaption.

```
DF = exp(bsxfun(@times, -G2PPSimPaths, repmat(Tenor', [nPeriods+1 1])));
SwapRate = (1 - DF(exRow, endCol, :)) ./ sum(bsxfun(@times, 1, DF(exRow, 1: endCol, :)));
PayoffValue = 100 * max(SwapRate - InstrumentStrike, 0) * sum(bsxfun(@times, 1, DF(exRow, 1: endCol, :)));
RealizedDF = prod(exp(bsxfun(@times, -G2PPSimPaths(1: exRow, 1, :), SimTimes(1: exRow))), 1);
G2PP_SwaptionPrice = mean(RealizedDF .* PayoffValue)
```

G2PP\_SwaptionPrice =

2.0988

### Simulate Interest-Rate Paths Using the LIBOR Market Model

This example shows how to simulate interest-rate paths using the LIBOR market model. Before beginning this example that uses a `LiborMarketModel`, make sure that you have set up the data as described in:

- “Construct a Zero Curve” on page 2-124
- “Define Swaption Parameters” on page 2-126



- “Compute the Black Model and the Swaption Volatility Matrix” on page 2-126
- “Select Calibration Instruments” on page 2-126
- “Compute Swaption Prices Using Black's Model” on page 2-127
- “Define Simulation Parameters” on page 2-127

The LIBOR Market Model (LMM) differs from short rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate

$$\frac{dF_i(t)}{F_i} = -\mu_i dt + \sigma_i(t) dW_i$$

where:

$W$  is an  $N$ -dimensional geometric Brownian motion with

$$dW_i(t)dW_j(t) = \rho_{ij}$$

The LMM relates the drifts of the forward rates based on no-arbitrage arguments. Specifically, under the Spot LIBOR measure, the drifts are expressed as

$$\mu_i(t) = -\sigma_i(t) \sum_{j=q(t)}^i \frac{\tau_j \rho_{i,j} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)}$$

where:

$\tau_i$  is the time fraction associated with the  $i$ th forward rate

$q(t)$  is an index defined by the relation

$$T_{q(t)-1} < t < T_{q(t)}$$

and the Spot LIBOR numeraire is defined as

$$B(t) = P(t, T_{q(t)}) \prod_{n=0}^{q(t)-1} (1 + \tau_n F_n(T_n))$$

The choice with the LMM is how to model volatility and correlation and how to estimate the parameters of these models for volatility and correlation. In practice, you may use a combination of historical data (for example, observed correlation between forward rates)

and current market data. For this example, only swaption data is used. Further, many different parameterizations of the volatility and correlation exist. For this example, two relatively straightforward parameterizations are used.

One of the most popular functional forms in the literature for volatility is:

$$\sigma_i(t) = \phi_i(a(T_i - t) + b)e^{c(T_i - t)} + d$$

where  $\phi$  adjusts the curve to match the volatility for the  $i$  th forward rate. For this example, all of the  $\phi$ 's are taken to be 1. For the correlation, the following functional form is used:

$$\rho_{i,j} = e^{-\beta|i-j|}$$

Once the functional forms have been specified, these parameters must be estimated using market data. One useful approximation, initially developed by Rebonato, is the following, which relates the Black volatility for a European swaption, given a set of volatility functions and a correlation matrix

$$(v_{\alpha,\beta}^{LFM})^2 = \sum_{i,j=\alpha+1}^{\beta} \frac{w_i(0)w_j(0)F_i(0)F_j(0)\rho_{i,j}}{S_{\alpha,\beta}(0)^2} \int_0^{T_\alpha} \sigma_i(t)\sigma_j(t)dt$$

where:

$$w_i(t) = \frac{\tau_i P(t, T_i)}{\sum_{k=\alpha+1}^{\beta} \tau_k P(t, t_k)}$$

This calculation is done using the function `blackvolbyrebonato` to compute analytic values of the swaption price for model parameters, and therefore, is then used to calibrate the model. Calibration consists of minimizing the difference between the observed implied swaption Black volatilities and the predicted Black volatilities.

In this example, the approach is similar to “Simulate Interest-Rate Paths Using the Hull-White One-Factor Model” on page 2-128 and “Simulate Interest-Rate Paths Using the Linear Gaussian Two-Factor Model” on page 2-131 where the Optimization Toolbox function `lsqnonlin` is used to minimize the difference between the observed swaption prices and the predicted swaption prices. However, other approaches (for example, simulated annealing) may also be appropriate. Starting parameters and constraints for  $a$ ,

$b$ ,  $d$ , and  $\beta$  are set in the variables  $x_0$ ,  $lb$ , and  $ub$ ; these could also be varied depending upon the particular calibration approach.

Calibrate the set of parameters that minimize the difference between the observed and predicted values using `blackvolbyrebonato` and `lsqnonlin`.

```
nRates = 10;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
objfun = @(x) SwaptionBlackVol(reIidx) - blackvolbyrebonato(RateSpec,...
    repmat({@t) ones(size(t)).*(x(1)*t + x(2)).*exp(-x(3)*t) + x(4)},nRates-1,1),...
    CorrFunc(meshgrid(1:nRates-1)',meshgrid(1:nRates-1),x(5)),...
    EurExDatesFull(reIidx),EurMatFull(reIidx),'Period',1);
options = optimset('disp','iter','MaxFunEvals',1000,'TolFun',1e-5);
x0 = [.2 .05 1 .05 .2];
lb = [0 0 .5 0 .01];
ub = [1 1 2 .3 1];
LMMparams = lsqnonlin(objfun,x0,lb,ub,options)
```

Iteration	Func-count	f(x)	Norm of step	First-order optimality
0	6	0.156251		0.483
1	12	0.00870177	0.188164	0.0339
2	18	0.00463441	0.165527	0.00095
3	24	0.00331055	0.351017	0.0154
4	30	0.00294775	0.0892617	7.47e-05
5	36	0.00281565	0.385779	0.00917
6	42	0.00278988	0.0145632	4.15e-05
7	48	0.00278522	0.115042	0.00116

Local minimum possible.

`lsqnonlin` stopped because the final change in the sum of squares relative to its initial value is less than the selected value of the function tolerance.

```
LMMparams =
    0.0781    0.1656    0.5121    0.0617    0.0100
```

Calculate `VolFunc` for the LMM object.

```
a = LMMparams(1);
b = LMMparams(2);
c = LMMparams(3);
d = LMMparams(4);

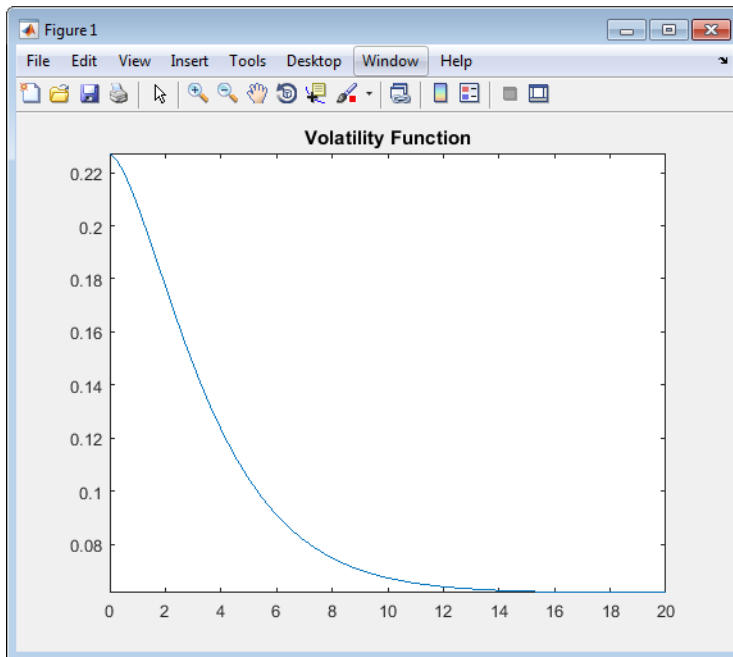
Beta = LMMparams(5);

VolFunc = repmat({@t) ones(size(t)).*(a*t + b).*exp(-c*t) + d},nRates-1,1);
```

Plot the volatility function.

```
figure
fplot(VolFunc{1},{0 20})
title('Volatility Function')
```

```
CorrelationMatrix = CorrFunc(meshgrid(1:nRates-1)',meshgrid(1:nRates-1),Beta);
```



Inspect the correlation matrix.

```
disp('Correlation Matrix')
fprintf(['%1.3f ',1,length(CorrelationMatrix)] '\n'],CorrelationMatrix)

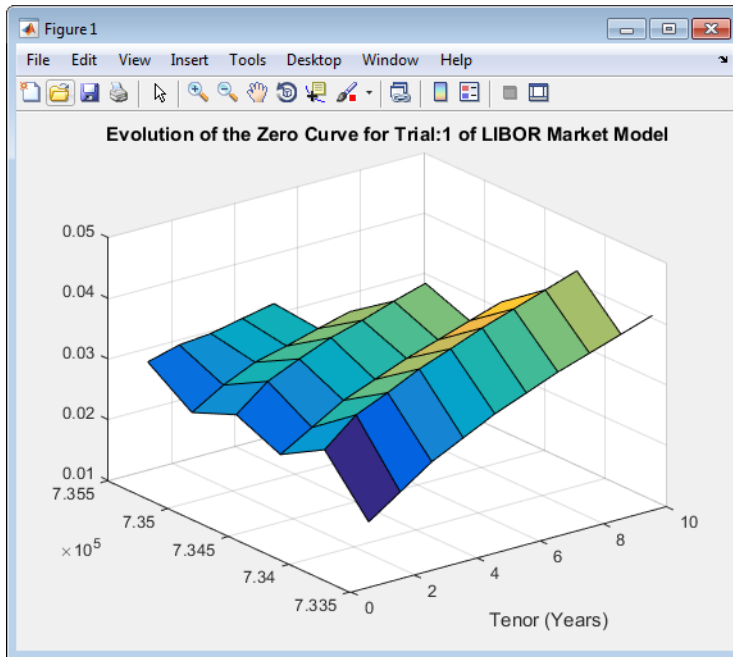
Correlation Matrix
1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932 0.923
0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932
0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942
0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951
0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961
0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970
0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980
0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990
0.923 0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000
```

Create the LMM object and use Monte Carlo simulation to generate the interest-rate paths with `LiborMarketModel.simTermStructs`.

```
LMM = LiborMarketModel(irdc,VolFunc,CorrelationMatrix,'Period',1);
[LMMZeroRates, ForwardRates] = LMM.simTermStructs(nPeriods,'nTrials',nTrials);

trialIdx = 1;
figure
tmpPlotData = LMMZeroRates(:, :, trialIdx);
tmpPlotData(tmpPlotData == 0) = NaN;
```

```
surf(Tenor,SimDates,tmpPlotData)
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of LIBOR Market Model'])
xlabel('Tenor (Years)')
```



Price the European swaption.

```
DF = exp(bsxfun(@times,-LMMZeroRates, repmat(Tenor', [nPeriods+1 1])));
SwapRate = (1 - DF(exRow,endCol,:))./sum(bsxfun(@times,1,DF(exRow,1:endCol,:)));
PayoffValue = 100*max(SwapRate-InstrumentStrike,0).*sum(bsxfun(@times,1,DF(exRow,1:endCol,:)));
RealizedDF = prod(exp(bsxfun(@times,-LMMZeroRates(2:exRow+1,1,:), SimTimes(1:exRow))),1);
LMM_SwaptionPrice = mean(RealizedDF.*PayoffValue)
```

LMM\_SwaptionPrice =

1.9915

## Compare Interest-Rate Modeling Results

This example shows how to compare the results for pricing a European swaption with different interest-rate models.

Compare the results for pricing a European swaption with interest-rate models using Monte Carlo simulation.

```
disp(' ')
fprintf('      # of Monte Carlo Trials: %8d\n'      , nTrials)
fprintf('      # of Time Periods/Trial: %8d\n\n'    , nPeriods)
fprintf('HW1F European Swaption Price: %8.4f\n', HW1F_SwaptionPrice)
fprintf('LG2F European Swaption Price: %8.4f\n', LG2F_SwaptionPrice)
fprintf('LMM European Swaption Price: %8.4f\n', LMM_SwaptionPrice)

# of Monte Carlo Trials:      1000
# of Time Periods/Trial:      5

HW1F European Swaption Price:  2.1839
LG2F European Swaption Price:  2.0988
LMM European Swaption Price:   1.9915
```

### References

Brigo, D. and F. Mercurio, *Interest Rate Models - Theory and Practice with Smile, Inflation and Credit*, Springer Finance, 2006.

Andersen, L. and V. Piterbarg, *Interest Rate Modeling*, Atlantic Financial Press. 2010.

Hull, J, *Options, Futures, and Other Derivatives*, Springer Finance, 2003.

Glasserman, P, *Monte Carlo Methods in Financial Engineering*, Prentice Hall, 2008.

Rebonato, R., K. McKay, and R. White, *The Sabr/Libor Market Model: Pricing, Calibration and Hedging for Complex Interest-Rate Derivatives*, John Wiley & Sons, 2010.

### See Also

[HullWhite1F](#) | [LiborMarketModel](#) | [LinearGaussian2F](#) | [blackvolbyrebonato](#) | [capbylg2f](#) | [floorbylg2f](#) | [lsqnonlin](#) | [simTermStructs](#) | [simTermStructs](#) | [simTermStructs](#) | [swaptionbyhw](#) | [swaptionbylg2f](#)

### Related Examples

- “Pricing Bermudan Swaptions with Monte Carlo Simulation” on page 2-141

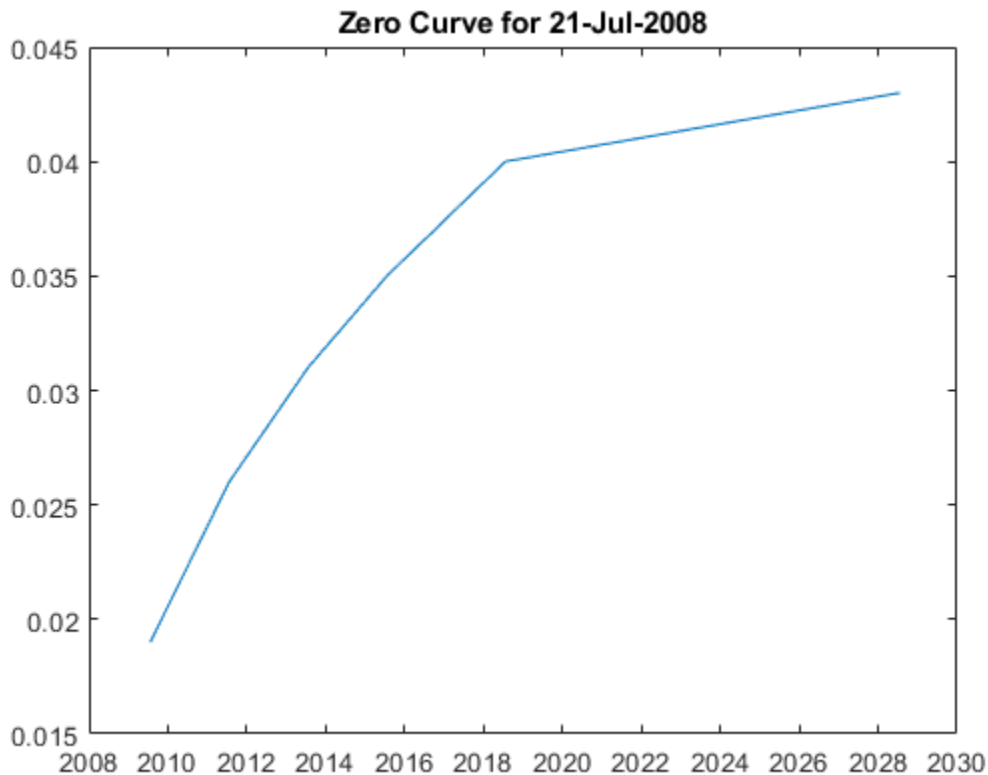
## Pricing Bermudan Swaptions with Monte Carlo Simulation

This example shows how to price Bermudan swaptions using interest-rate models in Financial Instruments Toolbox™. Specifically, a Hull-White one factor model, a Linear Gaussian two-factor model, and a LIBOR Market Model are calibrated to market data and then used to generate interest-rate paths using Monte Carlo simulation.

### Zero Curve

In this example, the `ZeroRates` for a zero curve is hard-coded. You can also create a zero curve by bootstrapping the zero curve from market data (for example, deposits, futures/forwards, and swaps). The hard-coded data for the zero curve is defined as:

```
Settle = datenum('21-Jul-2008');  
  
% Zero Curve  
CurveDates = daysadd(Settle,360*([1 3 5 7 10 20]),1);  
ZeroRates = [1.9 2.6 3.1 3.5 4 4.3]'/100;  
  
plot(CurveDates,ZeroRates)  
datetick  
title(['Zero Curve for ' datestr(Settle)]);
```



```
RateSpec = intenvset('Rates',ZeroRates,'EndDates',CurveDates,'StartDate',Settle);
```

### Define Swaption Parameters

For this example, we compute the price of a 10-no-call-1 Bermudan swaption.

```
BermudanExerciseDates = daysadd(Settle,360*(1:9),1);  
BermudanMaturity = datenum('21-Jul-2018');  
BermudanStrike = .045;
```

### Black's Model and the Swaption Volatility Matrix

Black's model is often used to price and quote European exercise interest-rate options, that is, caps, floors and swaptions. In the case of swaptions, Black's model is used to



imply a volatility given the current observed market price. The following matrix shows the Black implied volatility for a range of swaption exercise dates (columns) and underlying swap maturities (rows).

```
SwaptionBlackVol = [22 21 19 17 15 13 12
    21 19 17 16 15 13 11
    20 18 16 15 14 12 11
    19 17 15 14 13 12 10
    18 16 14 13 12 11 10
    15 14 13 12 12 11 10
    13 13 12 11 11 10 9]/100;
ExerciseDates = [1:5 7 10];
Tenors = [1:5 7 10];

EurExDatesFull = repmat(daysadd(Settle,ExerciseDates*360,1)',...
    length(Tenors),1);
EurMatFull = reshape(daysadd(EurExDatesFull,...
    repmat(360*Tenors,1,length(ExerciseDates)),1),size(EurExDatesFull));
```

### Selecting the Calibration Instruments

Selecting the instruments to calibrate the model to is one of the tasks in calibration. For Bermudan swaptions, it is typical to calibrate to European swaptions that are co-terminal with the Bermudan swaption to be priced. In this case, all swaptions having an underlying tenor that matures before the maturity of the swaption to be priced are used in the calibration.

```
% Find the swaptions that expire on or before the maturity date of the
% sample swaption
reidx = find(EurMatFull <= BermudanMaturity);
```

### Compute Swaption Prices Using Black's Model

Swaption prices are computed using Black's Model. The swaption prices are then used to compare the model's predicted values. To compute the swaption prices using Black's model:

```
% Compute Swaption Prices using Black's model
SwaptionBlackPrices = zeros(size(SwaptionBlackVol));
SwaptionStrike = zeros(size(SwaptionBlackVol));

for iSwaption=1:length(ExerciseDates)
    for iTenor=1:length(Tenors)
        [~,SwaptionStrike(iTenor,iSwaption)] = swapbyzero(RateSpec,[NaN 0], Settle, Eur
```

```

        'StartDate',EurExDatesFull(iTenor,iSwaption),'LegReset',[1 1]);
    SwaptionBlackPrices(iTenor,iSwaption) = swaptionbyblk(RateSpec, 'call', SwaptionBlk,
        EurExDatesFull(iTenor,iSwaption), EurMatFull(iTenor,iSwaption), SwaptionBlk);
end
end

```

### Simulation Parameters

The following parameters will be used; each exercise date is a simulation date.

```

nPeriods = 9;
DeltaTime = 1;
nTrials = 1000;

Tenor = (1:10)';

SimDates = daysadd(Settle,360*DeltaTime*(0:nPeriods),1);
SimTimes = diff(yearfrac(SimDates(1),SimDates));

```

### Hull White 1 Factor Model

The Hull-White one-factor model describes the evolution of the short rate and is specified by the following:

$$dr = [\theta(t) - \alpha r]dt + \sigma dW$$

The Hull-White model is calibrated using the function `swaptionbyhw`, which constructs a trinomial tree to price the swaptions. Calibration consists of minimizing the difference between the observed market prices (computed above using the Black's implied swaption volatility matrix) and the model's predicted prices.

In this example, the Optimization Toolbox™ function `lsqnonlin` is used to find the parameter set that minimizes the difference between the observed and predicted values. However, other approaches (for example, simulated annealing) may be appropriate. Starting parameters and constraints for  $\alpha$  and  $\sigma$  are set in the variables `x0`, `lb`, and `ub`; these could also be varied depending upon the particular calibration approach.

```

warning('off')
TimeSpec = hwtimespec(Settle,daysadd(Settle,360*(1:11),1), 2);
HW1Fobjfun = @(x) SwaptionBlackPrices(releidx) - ...
    swaptionbyhw(hwtree(hwvolspec(Settle,'11-Aug-2015',x(2),'11-Aug-2015',x(1)), RateSpec,
    EurExDatesFull(releidx), 0, EurExDatesFull(releidx), EurMatFull(releidx)));
options = optimset('disp','iter','MaxFunEvals',1000,'TolFun',1e-5);

```

```

% Find the parameters that minimize the difference between the observed and
% predicted prices
x0 = [.1 .01];
lb = [0 0];
ub = [1 1];
warning('off');
HW1Fparams = lsqnonlin(HW1Fobjfun,x0,lb,ub,options);

```

Iteration	Func-count	f(x)	Norm of step	First-order optimality
0	3	0.953772		20.5
1	6	0.142828	0.0169199	1.53
2	9	0.123022	0.0146705	2.31
3	12	0.122222	0.0154099	0.482
4	15	0.122217	0.00131282	0.00409

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the value of the function tolerance.

```

HW_alpha = HW1Fparams(1);
HW_sigma = HW1Fparams(2);

```

```

% Construct the HullWhite1F model using the HullWhite1F constructor.
HW1F = HullWhite1F(RateSpec,HW_alpha,HW_sigma);

```

```

% Use Monte Carlo simulation to generate the interest-rate paths with
% HullWhite1F.simTermStructs.

```

```

HW1FSimPaths = HW1F.simTermStructs(nPeriods,'NTRIALS',nTrials,...
    'DeltaTime',DeltaTime,'Tenor',Tenor,'antithetic',true);

```

```

% Examine one simulation

```

```

trialIdx = 1;

```

```

figure

```

```

surf(Tenor,SimDates,HW1FSimPaths(:,:,trialIdx))

```

```

datetick y keepticks keeplimits

```

```

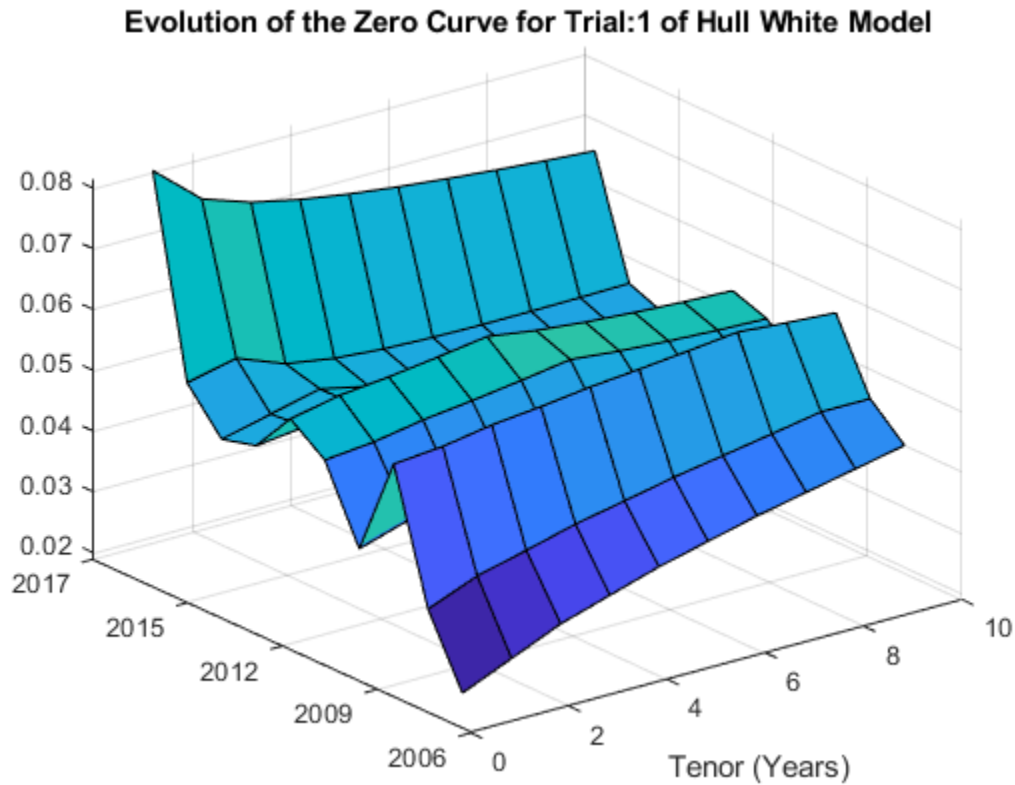
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of Hull White Model'])

```

```

xlabel('Tenor (Years)')

```



```
% Price the swaption using the helper function hBermudanSwaption
HW1FBermPrice = hBermudanSwaption(HW1FSimPaths, SimDates, Tenor, BermudanStrike, ...
    BermudanExerciseDates, BermudanMaturity);
```

### Linear Gaussian 2 Factor Model

The Linear Gaussian two-factor model (called the G2++ by Brigo and Mercurio) is also a short rate model, but involves two factors. Specifically:

$$r(t) = x(t) + y(t) + \varphi(t)$$

$$dx(t) = -ax(t)dt + \sigma dW_1(t)$$

$$dy(t) = -by(t)dt + \eta dW_2(t)$$

where  $dW_1(t)dW_2(t)$  is a two-dimensional Brownian motion with correlation  $\rho$

$$dW_1(t)dW_2(t) = \rho$$

and  $\varphi$  is a function chosen to match the initial zero curve.

The function `swaptionbylg2f` is used to compute analytic values of the swaption price for model parameters, and consequently can be used to calibrate the model. Calibration consists of minimizing the difference between the observed market prices and the model's predicted prices.

```
% Calibrate the set of parameters that minimize the difference between the
% observed and predicted values using swaptionbylg2f and lsqnonlin.
G2PPobjfun = @(x) SwaptionBlackPrices(reidx) - ...
    swaptionbylg2f(RateSpec,x(1),x(2),x(3),x(4),x(5),SwaptionStrike(reidx),...
    EurExDatesFull(reidx),EurMatFull(reidx),'Reset',1);
x0 = [.2 .1 .02 .01 -.5];
lb = [0 0 0 0 -1];
ub = [1 1 1 1 1];
LG2Fparams = lsqnonlin(G2PPobjfun,x0,lb,ub,options);
```

Iteration	Func-count	f(x)	Norm of step	First-order optimality
0	6	12.1928		67.1
1	12	1.36663	0.0974259	8.54
2	18	1.36663	0.112377	8.54
3	24	0.442322	0.0280943	1.3
4	30	0.236944	0.0561887	3.23
5	36	0.130779	0.0840413	7.66
6	42	0.0394584	0.0145002	6.79
7	48	0.0275889	0.0372418	0.755
8	54	0.0261953	0.112377	0.693
9	60	0.0234048	0.206005	0.142
10	66	0.0225717	0.140343	0.116
11	72	0.02254	0.0245609	1.45
12	78	0.0225305	0.0188091	1.36

Local minimum possible.

`lsqnonlin` stopped because the final change in the sum of squares relative to its initial value is less than the value of the function tolerance.

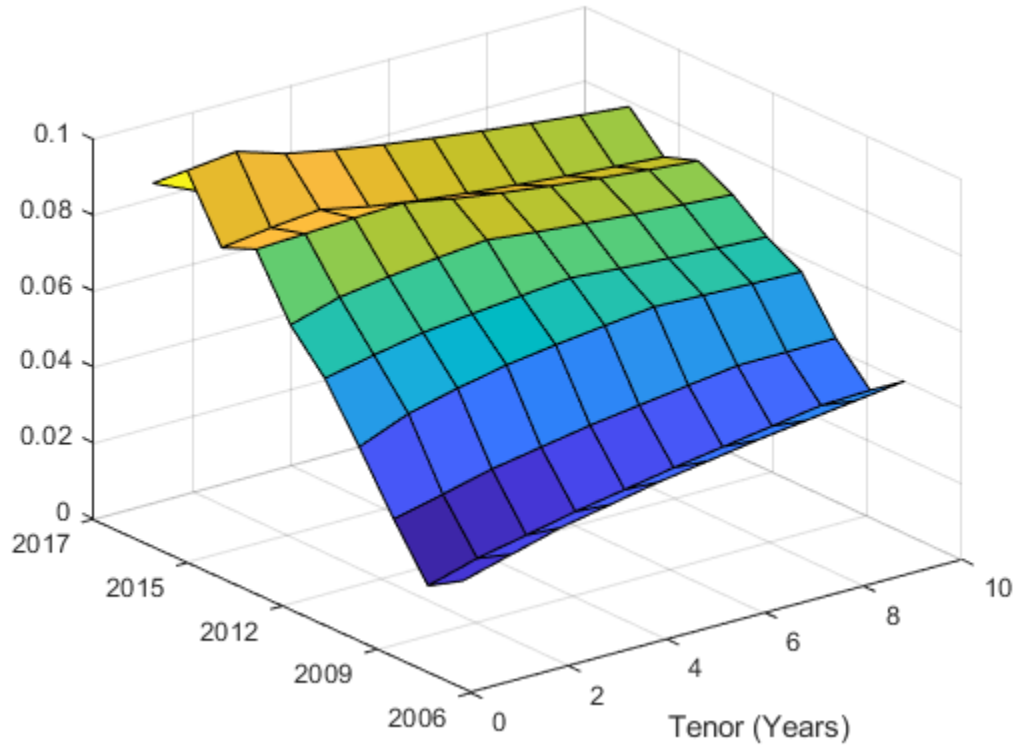
```
LG2f_a = LG2Fparams(1);
LG2f_b = LG2Fparams(2);
LG2f_sigma = LG2Fparams(3);
LG2f_eta = LG2Fparams(4);
LG2f_rho = LG2Fparams(5);

% Create the G2PP object and use Monte Carlo simulation to generate the
% interest-rate paths with LinearGaussian2F.simTermStructs.
G2PP = LinearGaussian2F(RateSpec, LG2f_a, LG2f_b, LG2f_sigma, LG2f_eta, LG2f_rho);

G2PPSimPaths = G2PP.simTermStructs(nPeriods, 'NTRIALS', nTrials, ...
    'DeltaTime', DeltaTime, 'Tenor', Tenor, 'antithetic', true);

% Examine one simulation
trialIdx = 1;
figure
surf(Tenor, SimDates, G2PPSimPaths(:, :, trialIdx))
datetick y kepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of G2++ Model'])
xlabel('Tenor (Years)')
```

Evolution of the Zero Curve for Trial:1 of G2++ Model



```
% Price the swaption using the helper function hBermudanSwaption
```

```
LG2FBermPrice = hBermudanSwaption(G2PPSimPaths, SimDates, Tenor, BermudanStrike, BermudanEx
```

### LIBOR Market Model

The LIBOR Market Model (LMM) differs from short rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate

$$\frac{dF_i(t)}{F_i} = -\mu_i dt + \sigma_i(t) dW_i$$

where

$\sigma_i$  is the volatility function for each rate and  $dW$  is an N dimensional geometric Brownian motion with:

$$dW_i(t)dW_j(t) = \rho_{i,j}$$

The LMM relates the drifts of the forward rates based on no-arbitrage arguments.

The choice with the LMM is how to model volatility and correlation and how to estimate the parameters of these models for volatility and correlation. In practice, you may use a combination of historical data (for example, observed correlation between forward rates) and current market data. For this example, only swaption data is used. Further, many different parameterizations of the volatility and correlation exist. For this example, two relatively straightforward parameterizations are used.

One of the most popular functional forms in the literature for volatility is:

$$\sigma_i(t) = \phi_i(a(T_i - t) + b)e^{c(T_i - t)} + d$$

where  $\phi$  adjusts the curve to match the volatility for the  $i^{th}$  forward rate. For this example, all of the Phi's will be taken to be 1.

For the correlation, the following functional form will be used:

$$\rho_{i,j} = e^{-\beta|i-j|}$$

Once the functional forms have been specified, these parameters need to be estimated using market data. One useful approximation, initially developed by Rebonato, is the following, which computes the Black volatility for a European swaption, given an LMM with a set of volatility functions and a correlation matrix.

$$(v_{\alpha,\beta}^{LFM})^2 = \sum_{i,j=\alpha+1}^{\beta} \frac{w_i(0)w_j(0)F_i(0)F_j(0)\rho_{i,j}}{S_{\alpha,\beta}(0)^2} \int_0^{T_\alpha} \sigma_i(t)\sigma_j(t)dt$$

where

$$w_i(t) = \frac{\tau_i P(t, T_i)}{\sum_{k=\alpha+1}^{\beta} \tau_k P(t, t_k)}$$

This calculation is done using `blackvolbyrebonato` to compute analytic values of the swaption price for model parameters, and consequently, is then used to calibrate the



model. Calibration consists of minimizing the difference between the observed implied swaption Black volatilities and the predicted Black volatilities.

```
nRates = 10;

CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));

objfun = @(x) SwaptionBlackVol(releidx) - blackvolbyrebonato(RateSpec,...
    repmat({@(t) ones(size(t)).*(x(1)*t + x(2)).*exp(-x(3)*t) + x(4)},nRates-1,1),...
    CorrFunc(meshgrid(1:nRates-1)',meshgrid(1:nRates-1),x(5)),...
    EurExDatesFull(releidx),EurMatFull(releidx),'Period',1);

x0 = [.2 .05 1 .05 .2];
lb = [0 0 .5 0 .01];
ub = [1 1 2 .3 1];
LMMparams = lsqnonlin(objfun,x0,lb,ub,options);
```

Iteration	Func-count	f(x)	Norm of step	First-order optimality
0	6	0.156251		0.483
1	12	0.00870177	0.188164	0.0339
2	18	0.00463441	0.165527	0.00095
3	24	0.00331055	0.351017	0.0154
4	30	0.00294775	0.0892616	7.47e-05
5	36	0.00281565	0.385779	0.00917
6	42	0.00278988	0.0145632	4.15e-05
7	48	0.00278522	0.115042	0.00116

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the value of the function tolerance.

```
% Calculate VolFunc for the LMM object.
```

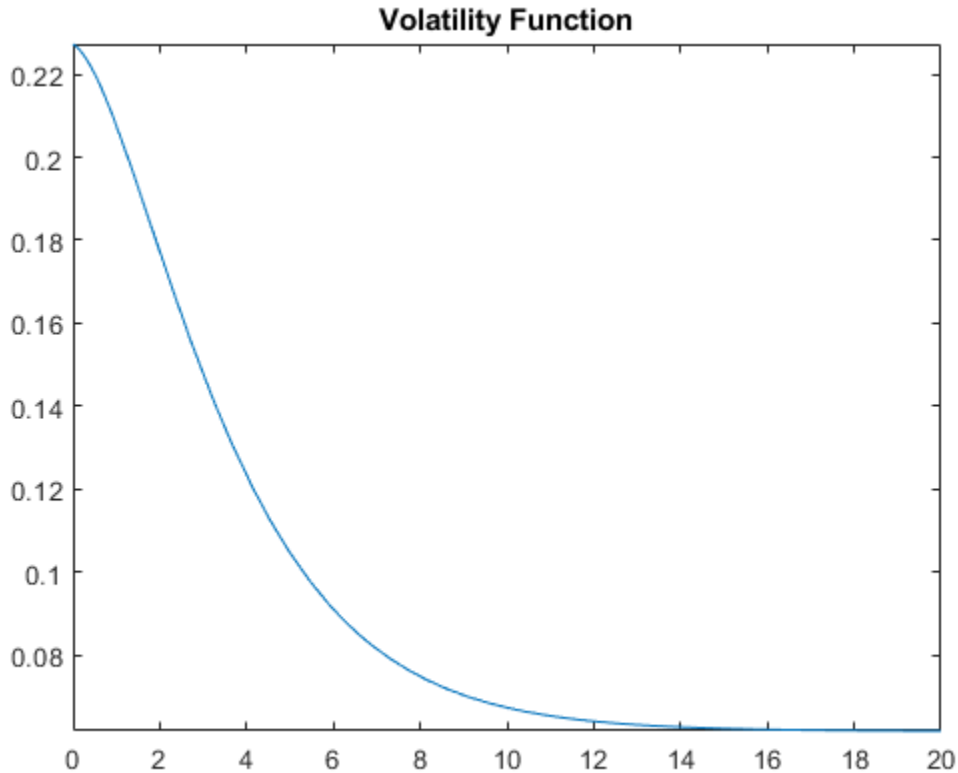
```
a = LMMparams(1);
b = LMMparams(2);
c = LMMparams(3);
d = LMMparams(4);
```

```
Beta = LMMparams(5);
```

```
VolFunc = repmat({@(t) ones(size(t)).*(a*t + b).*exp(-c*t) + d},nRates-1,1);
```

```
% Plot the volatility function
```

```
figure
fplot(VolFunc{1},[0 20])
title('Volatility Function')
```



```
% Inspect the correlation matrix
CorrelationMatrix = CorrFunc(meshgrid(1:nRates-1)', meshgrid(1:nRates-1), Beta);
displayCorrelationMatrix(CorrelationMatrix);
```

```
Correlation Matrix
1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932 0.923
0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942 0.932
0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951 0.942
0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961 0.951
```

```
0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970 0.961
0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980 0.970
0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990 0.980
0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000 0.990
0.923 0.932 0.942 0.951 0.961 0.970 0.980 0.990 1.000
```

```
% Create the LMM object and use Monte Carlo simulation to generate the
% interest-rate paths with LiborMarketModel.simTermStructs.
```

```
LMM = LiborMarketModel(RateSpec,VolFunc,CorrelationMatrix,'Period',1);
```

```
[LMMZeroRates, ForwardRates] = LMM.simTermStructs(nPeriods,'nTrials',nTrials);
```

```
% Examine one simulation
```

```
trialIdx = 1;
```

```
figure
```

```
tmpPlotData = LMMZeroRates(:, :, trialIdx);
```

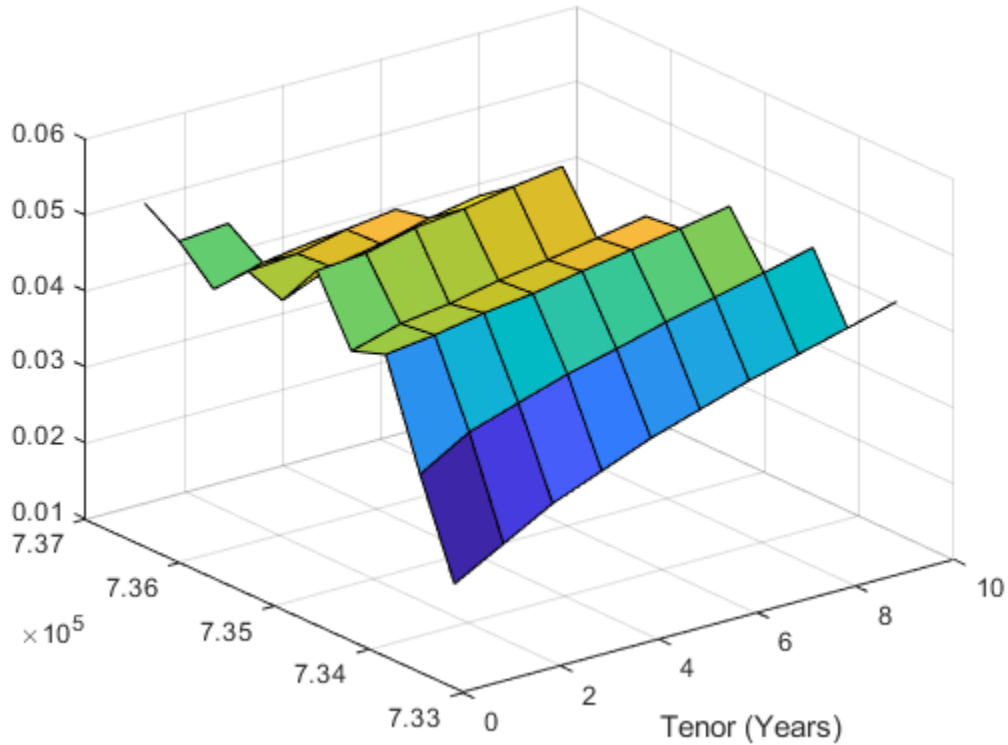
```
tmpPlotData(tmpPlotData == 0) = NaN;
```

```
surf(Tenor, SimDates, tmpPlotData)
```

```
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of LIBOR Market Mo
```

```
xlabel('Tenor (Years)')
```

Evolution of the Zero Curve for Trial:1 of LIBOR Market Model



```
% Price the swaption using the helper function hBermudanSwaption
LMMTenor = 1:10;
LMMBermPrice = hBermudanSwaption(LMMZeroRates, SimDates, LMMTenor, .045, BermudanExercisedD
```

**Results**

```
displayResults(nTrials, nPeriods, HW1FBermPrice, LG2FBermPrice, LMMBermPrice);
```

```
# of Monte Carlo Trials:    1000
# of Time Periods/Trial:    9
```

```
HW1F Bermudan Swaption Price:  3.7629
LG2F Bermudan Swaption Price:  3.5496
LMM Bermudan Swaption Price:   3.4911
```

## Bibliography

This example is based on the following books, papers and journal articles:

- 1 Andersen, L. and V. Piterbarg (2010). Interest Rate Modeling, Atlantic Financial Press.
- 2 Brigo, D. and F. Mercurio (2001). Interest Rate Models - Theory and Practice with Smile, Inflation and Credit (2nd ed. 2006 ed.). Springer Verlag.
- 3 Glasserman, P. (2003). Monte Carlo Methods in Financial Engineering. Springer.
- 4 Hull, J. (2008). Options, Futures, and Other Derivatives. Prentice Hall.
- 5 Rebonato, R., K. McKay, and R. White (2010). The Sabr/Libor Market Model: Pricing, Calibration and Hedging for Complex Interest-Rate Derivatives. John Wiley & Sons.

## Utility Functions

```
function displayCorrelationMatrix(CorrelationMatrix)
fprintf('Correlation Matrix\n');
fprintf([repmat('%1.3f ',1,length(CorrelationMatrix)) '\n'],CorrelationMatrix);
end
```

```
function displayResults(nTrials, nPeriods, HW1FBermPrice, LG2FBermPrice, LMMBermPrice)
fprintf(' # of Monte Carlo Trials: %8d\n' , nTrials);
fprintf(' # of Time Periods/Trial: %8d\n\n' , nPeriods);
fprintf('HW1F Bermudan Swaption Price: %8.4f\n', HW1FBermPrice);
fprintf('LG2F Bermudan Swaption Price: %8.4f\n', LG2FBermPrice);
fprintf(' LMM Bermudan Swaption Price: %8.4f\n', LMMBermPrice);
end
```

## See Also

agencyoas | agencyprice | blackvolbyrebonato | blackvolbysabr |  
 bndfutimprepo | bndfutprice | capbyblk | capbylg2f | convfactor | floorbyblk  
 | floorbylg2f | hwcalbycap | hwcalbyfloor | optsensbysabr | swaptionbyblk |  
 swaptionbylg2f | tfutbyprice | tfutbyyield | tfutimprepo | tfutpricebyrepo  
 | tfutyieldbyrepo

## Related Examples

- “Calibrate the SABR Model” on page 2-36
- “Price a Swaption Using the SABR Model” on page 2-42

- “Computing the Agency OAS for Bonds” on page 6-3
- “Analysis of Bond Futures” on page 7-13
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17
- “Fitting the Diebold Li Model” on page 7-25
- “Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

### **More About**

- “Managing Present Value with Bond Futures” on page 7-16
- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

# Graphical Representation of Trees

## In this section...

“Introduction” on page 2-157

“Observing Interest Rates” on page 2-157

“Observing Instrument Prices” on page 2-161

## Introduction

You can use the function `treeviewer` to display a graphical representation of a tree, allowing you to examine interactively the prices and rates on the nodes of the tree until maturity. To get started with this process, first load the data file `deriv.mat` included in this toolbox.

```
load deriv.mat
```

---

**Note** `treeviewer` price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, consequently, decreasing prices appear on the lower branch. Conversely, for interest rate displays, *decreasing* interest rates appear on the upper branch (prices are rising) and *increasing* interest rates on the lower branch (prices are falling).

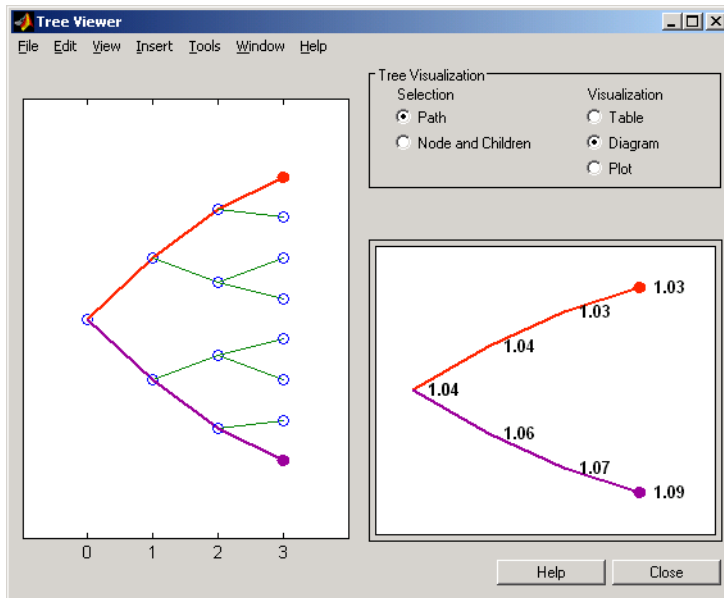
---

For information on the use of `treeviewer` to observe interest rate movement, see “Observing Interest Rates” on page 2-157. For information on using `treeviewer` to observe the movement of prices, see “Observing Instrument Prices” on page 2-161.

## Observing Interest Rates

If you provide the name of an interest rate tree to the `treeviewer` function, it displays a graphical view of the path of interest rates. For example, here is the `treeviewer` representation of all the rates along both the up and down branches of `HJMTree`.

```
treeviewer(HJMTree)
```



The example in “Isolating a Specific Node” on page 2-92 used `bushpath` to find the path of forward rates along an HJM tree by taking the first branch up and then two branches down the rate tree.

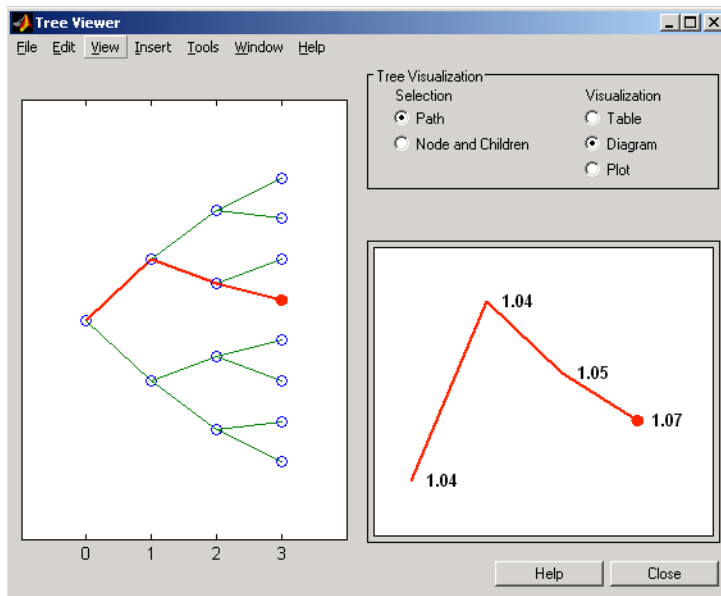
```
FRates = bushpath(HJMTree.FwdTree, [1 2 2])
```

```
FRates =
```

```
1.0356
1.0364
1.0526
1.0674
```

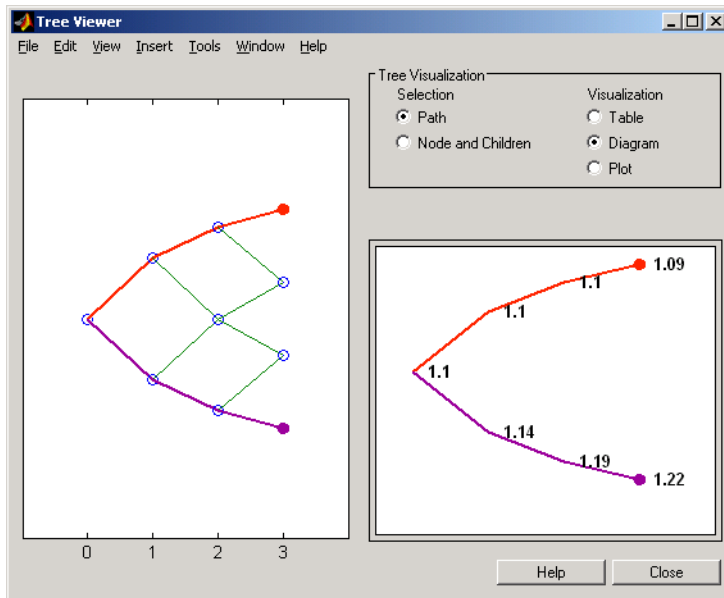
With the `treeview` function you can display the identical information by clicking along the same sequence of nodes, as shown next.





Next is a treeviewer representation of interest rates along several branches of BDTTree.

```
treeviewer(BDTTree)
```




---

**Note** When using treeviewer with recombining trees, such as BDT, BK, and HW, you must click each node in succession from the beginning to the end. Because these trees can recombine, treeviewer is unable to complete the path automatically.

---

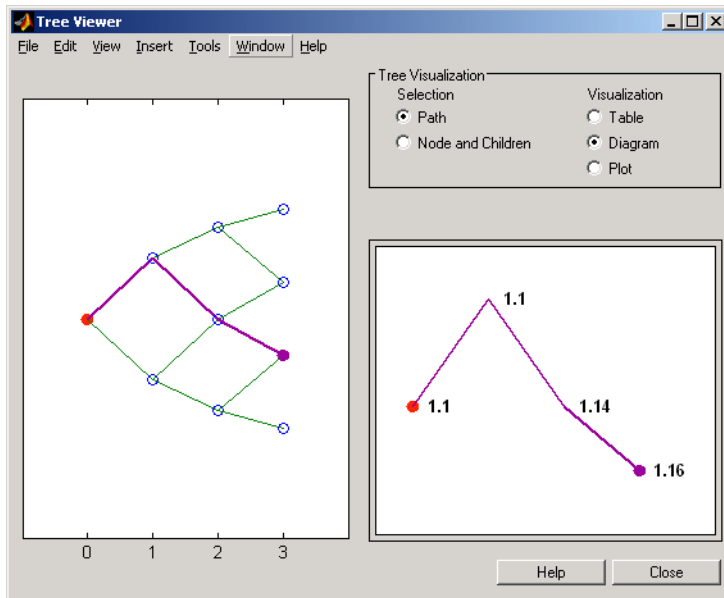
The example in “Isolating a Specific Node” on page 2-92 used `treepath` to find the path of interest rates taking the first branch up and then two branches down the rate tree.

```
FRates = treepath(BDTree.FwdTree, [1 2 2])
```

```
FRates =
```

```
1.1000
1.0979
1.1377
1.1606
```

You can display the identical information by clicking along the same sequence of nodes, as shown next.

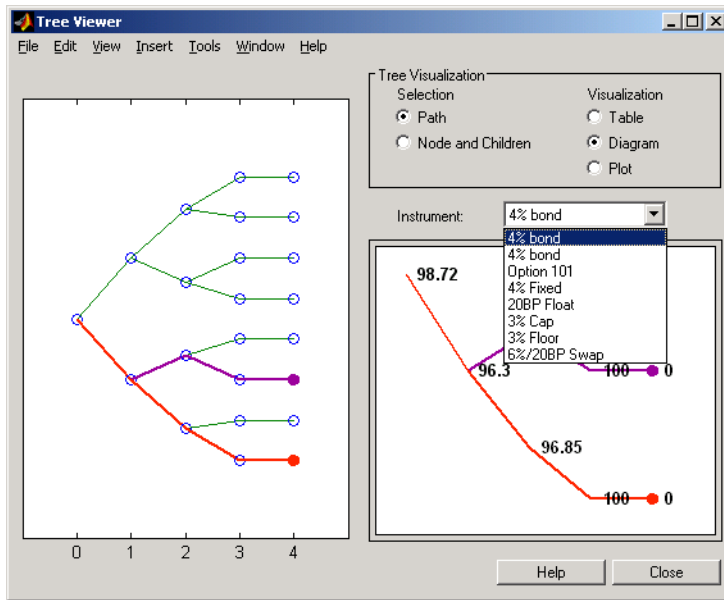


## Observing Instrument Prices

To use `treeviewer` to display a tree of instrument prices, provide the name of an instrument set along with the name of a price tree in your call to `treeviewer`, for example:

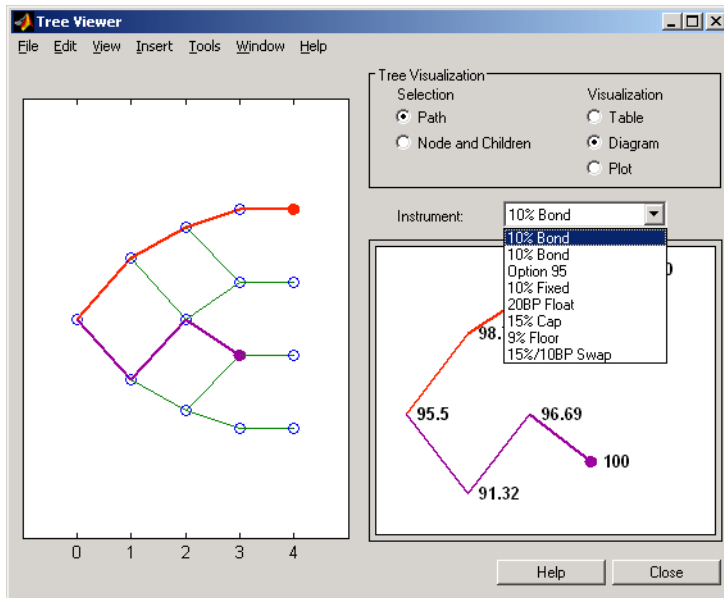
```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
treeviewer(PriceTree, HJMInstSet)
```

With `treeviewer` you select *each instrument individually* in the instrument portfolio for display.



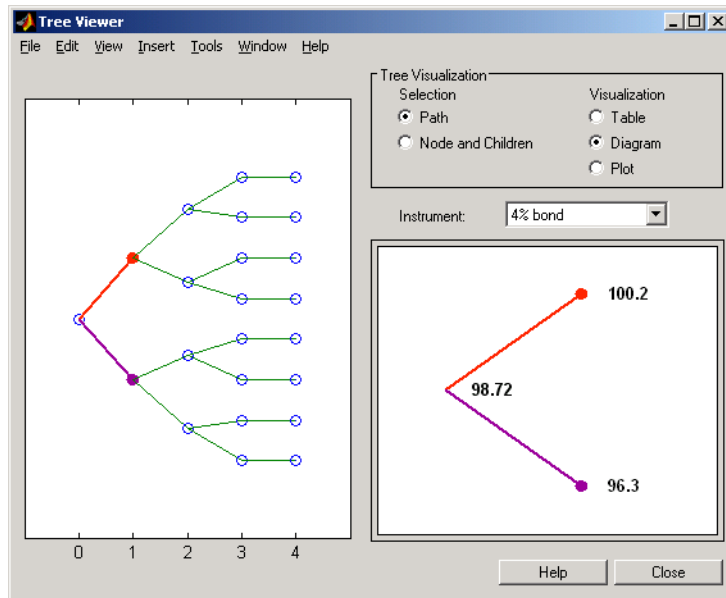
You can use an analogous process to view instrument prices based on the BDT interest rate tree included in `deriv.mat`.

```
load deriv.mat
[BDTPrice, BDTPriceTree] = bdtprice(BDTree, BDTInstSet);
treeviewer(BDTPriceTree, BDTInstSet)
```

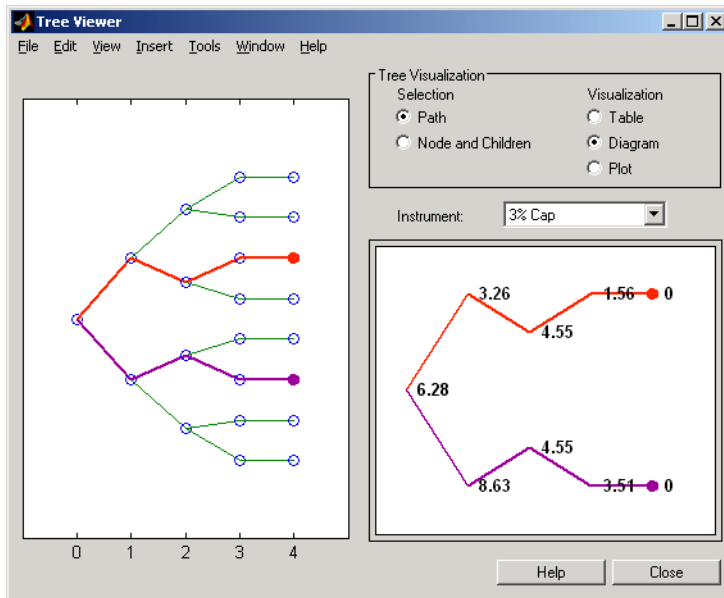


### Valuation Date Prices

You can use `treeviewer` instrument-by-instrument to observe instrument prices through time. For the first 4% bond in the HJM instrument portfolio, `treeviewer` indicates a valuation date price of 98.72, the same value obtained by accessing the `PriceTree` structure directly.



As a further example, look at the sixth instrument in the price vector, the 3% cap. At the valuation date, its value obtained directly from the structure is 6.2831. Use `treeviewer` on this instrument to confirm this price.



### Additional Observation Times

The second node represents the first-rate observation time,  $t_{0bs} = 1$ . This node displays two states, one representing the branch going up and the other one representing the branch going down.

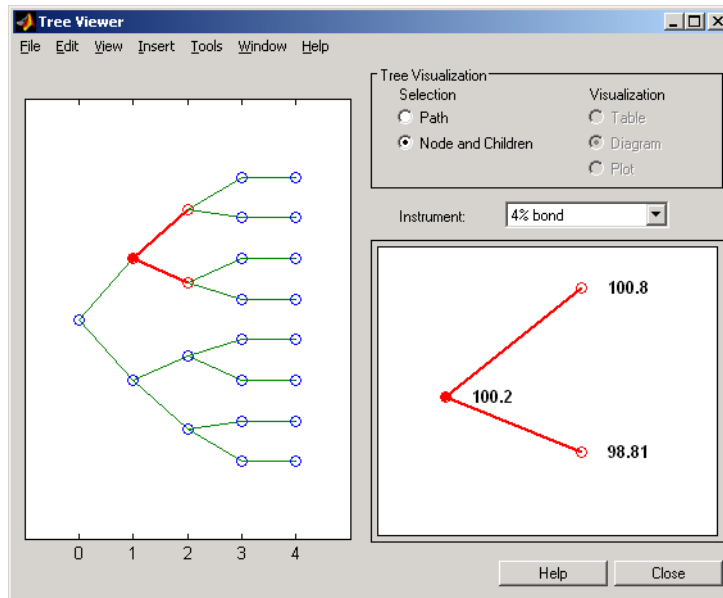
Examine the prices of the node corresponding to the up branch.

```
PriceTree.PBush{2}(:, :, 1)
```

```
ans =
```

```
100.1563
 99.7309
  0.1007
100.1563
100.3782
  3.2594
  0.1007
  3.5597
```

As before, you can use `treeviewer`, this time to examine the price for the 4% bond on the up branch. `treeviewer` displays a price of 100.2 for the first node of the up branch, as expected.



Now examine the corresponding down branch.

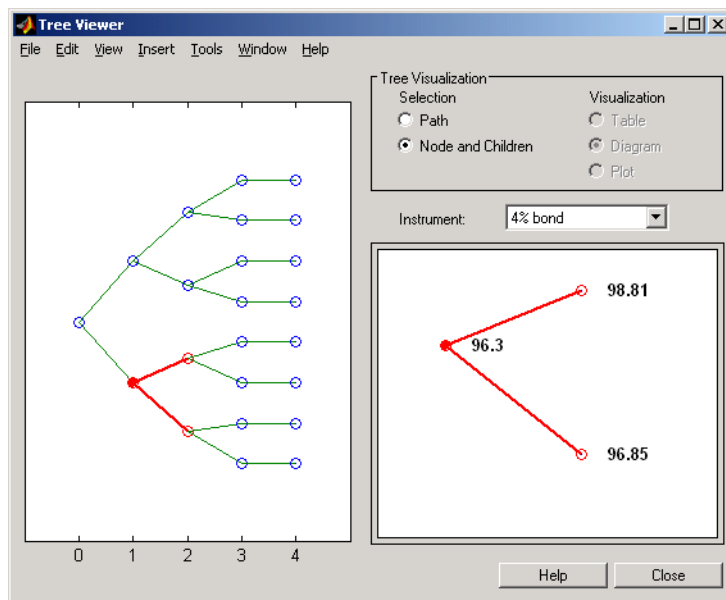
```
PriceTree.PBush{2}(:, :, 2)
```

```
ans =
```

```
96.3041
94.1986
    0
96.3041
100.3671
 8.6342
    0
-0.3923
```

Use `treeviewer` once again, now to observe the price of the 4% bond on the down branch. The displayed price of 96.3 conforms to the price obtained from direct access of the `PriceTree` structure. You may continue this process as far along the price tree as you want.





## See Also

bdtprice | bdtrens | bdttimespec | bdttree | bdtvolspec | bkprice | bksens |  
 bktimespec | bktree | bkvolspec | bondbybdt | bondbybk | bondbyhjm | bondbyhw |  
 bondbyzero | capbybdt | capbybk | capbyblk | capbyhjm | capbyhw | cfbybdt |  
 cfbybk | cfbyhjm | cfbyhw | cfbyzero | fixedbybdt | fixedbybk | fixedbyhjm |  
 fixedbyhw | fixedbyzero | floatbybdt | floatbybk | floatbyhjm | floatbyhw |  
 floatbyzero | floatdiscmargin | floatmargin | floorbybdt | floorbybk |  
 floorbyblk | floorbyhjm | floorbyhw | hjmprice | hjmsens | hjmtimespec |  
 hjmtree | hjmvolspec | hwalbycap | hwalbyfloor | hwprice | hwsens |  
 hwtimespec | hwtree | hwwolspec | instbond | instcap | instcf | instfixed |  
 instfloat | instfloor | instoptbnd | instoptembnd | instoptemfloat |  
 instoptfloat | instrangefloat | instswap | instswaption | intenvprice |  
 intenvsens | intenvset | mmktbybdt | mmktbyhjm | oasbybdt | oasbybk | oasbyhjm |  
 oasbyhw | optbndbybdt | optbndbybk | optbndbyhjm | optbndbyhw |  
 optembndbybdt | optembndbybk | optembndbyhjm | optembndbyhw |  
 optemfloatbybdt | optemfloatbybk | optemfloatbyhjm | optemfloatbyhw |  
 optfloatbybdt | optfloatbybk | optfloatbyhjm | optfloatbyhw |  
 rangefloatbybdt | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw |

swapbybdt | swapbybk | swapbyhjm | swapbyhw | swapbyzero | swaptionbybdt | swaptionbybk | swaptionbyblk | swaptionbyhjm | swaptionbyhw

### **Related Examples**

- “Overview of Interest-Rate Tree Models” on page 2-50
- “Pricing Using Interest-Rate Term Structure” on page 2-72
- “Pricing Using Interest-Rate Tree Models” on page 2-99
- “Understanding Interest-Rate Tree Models” on page 2-79
- “Understanding the Interest-Rate Term Structure” on page 2-55

### **More About**

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

# Equity Derivatives

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- “Understanding Equity Trees” on page 3-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44
- “Pricing European and American Spread Options” on page 3-57
- “Hedging Strategies Using Spread Options” on page 3-76
- “Pricing Swing Options using the Longstaff-Schwartz Method” on page 3-84
- “Simulating Electricity Prices with Mean-Reversion and Jump-Diffusion” on page 3-98
- “Pricing Asian Options” on page 3-113
- “Pricing Equity Derivatives Using Trees” on page 3-130
- “Computing Equity Instrument Sensitivities” on page 3-144
- “Equity Derivatives Using Closed-Form Solutions” on page 3-150
- “Pricing European Call Options Using Different Equity Models” on page 3-163
- “Compute the Option Price on a Future” on page 3-172

## Understanding Equity Trees

In this section...
“Introduction” on page 3-2
“Building Equity Binary Trees” on page 3-3
“Building Implied Trinomial Trees” on page 3-8
“Building Standard Trinomial Trees” on page 3-14
“Examining Equity Trees” on page 3-18
“Differences Between CRR and EQP Tree Structures” on page 3-22

### Introduction

Financial Instruments Toolbox supports five types of recombining tree models to represent the evolution of stock prices:

- Cox-Ross-Rubinstein (CRR) model
- Equal probabilities (EQP) model
- Leisen-Reimer (LR) model
- Implied trinomial tree (ITT) model
- Standard trinomial tree (STT) model

For a discussion of recombining trees, see “Rate and Price Trees” on page 2-51.

The CRR, EQP, LR, STT, and ITT models are examples of discrete time models. A discrete time model divides time into discrete bits; prices can only be computed at these specific times.

The CRR model is one of the most common methods used to model the evolution of stock processes. The strength of the CRR model lies in its simplicity. It is a good model when dealing with many tree levels. The CRR model yields the correct expected value for each node of the tree and provides a good approximation for the corresponding local volatility. The approximation becomes better as the number of time steps represented in the tree is increased.

The EQP model is another discrete time model. It has the advantage of building a tree with the exact volatility in each tree node, even with small numbers of time steps. It also provides better results than CRR in some given trading environments, for example, when

stock volatility is low and interest rates are high. However, this additional precision causes increased complexity, which is reflected in the number of calculations required to build a tree.

The LR model is another discrete time model. It has the advantage of producing estimates close to the Black-Scholes model using only a few steps, while also minimizing the oscillation.

The ITT model is a CRR-style implied trinomial tree which takes advantage of prices quoted from liquid options in the market with varying strikes and maturities to build a tree that more accurately represents the market. An ITT model is commonly used to price exotic options in such a way that they are consistent with the market prices of standard options.

The STT model is another discrete time model. It is considered to produce more accurate results than the binomial model when fewer time steps are modeled. The STT model is sometimes more stable and accurate than the binomial model when pricing exotic options.

## Building Equity Binary Trees

The tree of stock prices is the fundamental unit representing the evolution of the price of a stock over a given period of time. The MATLAB functions `crree`, `eqptree`, and `lree` create CRR trees, EQP trees, and LR trees, respectively. These functions create an output tree structure along with information about the parameters used for creating the tree.

The functions `crree`, `eqptree`, and `lree` take three structures as input arguments:

- The stock parameter structure `StockSpec`
- The interest-rate term structure `RateSpec`
- The tree time layout structure `TimeSpec`

### Calling Sequence for Equity Binary Trees

The calling syntax for `crree` is:

```
CRRTree = crree (StockSpec, RateSpec, TimeSpec)
```

Similarly, the calling syntax for `eqptree` is:

```
EQPTree = eqptree (StockSpec, RateSpec, TimeSpec)
```

And, the calling syntax for `lmtree` is:

```
LRTree = lmtree(StockSpec, RateSpec, TimeSpec, Strike)
```

All three functions require the structures `StockSpec`, `RateSpec`, and `TimeSpec` as input arguments:

- `StockSpec` is a structure that specifies parameters of the stock whose price evolution is represented by the tree. This structure, created using the function `stockspec`, contains information such as the stock's original price, its volatility, and its dividend payment information.
- `RateSpec` is the interest-rate specification of the initial rate curve. Create this structure with the function `intenvset`.
- `TimeSpec` is the tree time layout specification. Create these structures with the functions `crrtimespec`, `eqptimespec`, and `lrtimespec`. The structures contain information regarding the mapping of relevant dates into the tree structure, plus the number of time steps used for building the tree.

#### **Specifying the Stock Structure for Equity Binary Trees**

The structure `StockSpec` encapsulates the stock-specific information required for building the binary tree of an individual stock's price movement.

You generate `StockSpec` with the function `stockspec`. This function requires two input arguments and accepts up to three additional input arguments that depend on the existence and type of dividend payments.

The syntax for calling `stockspec` is:

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...  
DividendAmounts, ExDividendDates)
```

where:

- `Sigma` is the decimal annual volatility of the underlying security.
- `AssetPrice` is the price of the stock at the valuation date.
- `DividendType` is a character vector specifying the type of dividend paid by the stock. Allowed values are `cash`, `constant`, or `continuous`.
- `DividendAmounts` has a value that depends on the specification of `DividendType`. For `DividendType cash`, `DividendAmounts` is a vector of cash dividends. For `DividendType constant`, it is a vector of constant annualized dividend yields. For

DividendType continuous, it is a scalar representing a continuously annualized dividend yield.

- ExDividendDates also has a value that depends on the nature of DividendType. For DividendType cash or constant, ExDividendDates is vector of dividend dates. For DividendType continuous, ExDividendDates is ignored.

### Stock Structure Example Using a Binary Tree

Consider a stock with a price of \$100 and an annual volatility of 15%. Assume that the stock pays three cash \$5.00 dividends on dates January 01, 2004, July 01, 2005, and January 01, 2006. You specify these parameters in MATLAB as:

```
Sigma = 0.15;
AssetPrice = 100;
DividendType = 'cash';
DividendAmounts = [5; 5; 5];
ExDividendDates = {'jan-01-2004', 'july-01-2005', 'jan-01-2006'};
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)
```

```
StockSpec =
```

```
      FinObj: 'StockSpec'
      Sigma: 0.1500
      AssetPrice: 100
      DividendType: 'cash'
      DividendAmounts: [3x1 double]
      ExDividendDates: [3x1 double]
```

### Specifying the Interest-Rate Term Structure for Equity Binary Trees

The RateSpec structure defines the interest rate environment used when building the stock price binary tree. “Modeling the Interest-Rate Term Structure” on page 2-67 explains how to create these structures using the function `intenvset`, given the interest rates, the starting and ending dates for each rate, and the compounding value.

### Specifying the Tree-Time Term Structure for Equity Binary Trees

The TimeSpec structure defines the tree layout of the binary tree:

- It maps the valuation and maturity dates to their corresponding times.
- It defines the time of the levels of the tree by dividing the time span between valuation and maturity into equally spaced intervals. By specifying the number of intervals, you define the granularity of the tree time structure.

The syntax for building a `TimeSpec` structure is:

```
TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec = eqptimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods)
```

where:

- `ValuationDate` is a scalar date marking the pricing date and first observation in the tree (location of the root node). You enter `ValuationDate` either as a serial date number (generated with `datenum`) or a date character vector.
- `Maturity` is a scalar date marking the maturity of the tree, entered as a serial date number or a date character vector.
- `NumPeriods` is a scalar defining the number of time steps in the tree; for example, `NumPeriods = 10` implies 10 time steps and 11 tree levels (0, 1, 2, ..., 9, 10).

#### TimeSpec Example Using a Binary Tree

Consider building a CRR tree, with a valuation date of January 1, 2003, a maturity date of January 1, 2008, and 20 time steps. You specify these parameters in MATLAB as:

```
ValuationDate = 'Jan-1-2003';
Maturity = 'Jan-1-2008';
NumPeriods = 20;
TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)

TimeSpec =
```

```
    FinObj: 'BinTimeSpec'
ValuationDate: 731582
    Maturity: 733408
    NumPeriods: 20
        Basis: 0
    EndMonthRule: 1
           tObs: [1x21 double]
           dObs: [1x21 double]
```

Two vector fields in the `TimeSpec` structure are of particular interest: `dObs` and `tObs`. These two fields represent the observation times and corresponding dates of all tree levels, with `dObs(1)` and `tObs(1)`, respectively, representing the root node (`ValuationDate`), and `dObs(end)` and `tObs(end)` representing the last tree level (`Maturity`).



---

**Note** There is no relationship between the dates specified for the tree and the implied tree level times, and the maturities specified in the interest-rate term structure. The rates in RateSpec are interpolated or extrapolated as required to meet the time distribution of the tree.

---

### Examples of Binary Tree Creation

You can now use the StockSpec and TimeSpec structures described previously to build an equal probability tree (EQPTree), a CRR tree (CRRTree), or an LR tree (LRTree). First, you must define the interest-rate term structure. For this example, assume that the interest rate is fixed at 10% annually between the valuation date of the tree (January 1, 2003) until its maturity.

```
ValuationDate = 'Jan-1-2003';
Maturity = 'Jan-1-2008';
Rate = 0.1;
RateSpec = intenvset('Rates', Rate, 'StartDates', ...
ValuationDate, 'EndDates', Maturity, 'Compounding', -1);
```

To build a CRRTree, enter:

```
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)
```

```
CRRTree =
```

```
    FinObj: 'BinStockTree'
    Method: 'CRR'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
             tObs: [1x21 double]
             dObs: [1x21 double]
             STree: {1x21 cell}
             UpProbs: [1x20 double]
```

To build an EQPTree, enter:

```
EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)
```

```
EQPTree =
```

```
    FinObj: 'BinStockTree'
    Method: 'EQP'
```

```
StockSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [1x21 double]
    dObs: [1x21 double]
    STree: {1x21 cell}
UpProbs: [1x20 double]
```

## Building Implied Trinomial Trees

The tree of stock prices is the fundamental unit representing the evolution of the price of a stock over a given period of time. The MATLAB function `itttree` creates an output tree structure along with the information about the parameters used to create the tree.

The function `itttree` takes four structures as input arguments:

- The stock parameter structure `StockSpec`
- The interest-rate term structure `RateSpec`
- The tree time layout structure `TimeSpec`
- The stock option specification structure `StockOptSpec`

## Calling Sequence for Implied Trinomial Trees

The calling syntax for `itttree` is:

```
ITTTree = itttree (StockSpec,RateSpec,TimeSpec,StockOptSpec)
```

- `StockSpec` is a structure that specifies parameters of the stock whose price evolution is represented by the tree. This structure, created using the function `stockspec`, contains information such as the stock's original price, its volatility, and its dividend payment information.
- `RateSpec` is the interest-rate specification of the initial rate curve. Create this structure with the function `intenvset`.
- `TimeSpec` is the tree time layout specification. Create these structures with the function `itttimespec`. This structure contains information regarding the mapping of relevant dates into the tree structure, plus the number of time steps used for building the tree.
- `StockOptSpec` is a structure containing parameters of European stock options instruments. Create this structure with the function `stockoptspec`.

## Specifying the Stock Structure for Implied Trinomial Trees

The structure `StockSpec` encapsulates the stock-specific information required for building the trinomial tree of an individual stock's price movement.

You generate `StockSpec` with the function `stockspec`. This function requires two input arguments and accepts up to three additional input arguments that depend on the existence and type of dividend payments.

The syntax for calling `stockspec` is:

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
    DividendAmounts, ExDividendDates)
```

where:

- `Sigma` is the decimal annual volatility of the underlying security.
- `AssetPrice` is the price of the stock at the valuation date.
- `DividendType` is a character vector specifying the type of dividend paid by the stock. Allowed values are `cash`, `constant`, or `continuous`.
- `DividendAmounts` has a value that depends on the specification of `DividendType`. For `DividendType` `cash`, `DividendAmounts` is a vector of cash dividends. For `DividendType` `constant`, it is a vector of constant annualized dividend yields. For `DividendType` `continuous`, it is a scalar representing a continuously annualized dividend yield.
- `ExDividendDates` also has a value that depends on the nature of `DividendType`. For `DividendType` `cash` or `constant`, `ExDividendDates` is vector of dividend dates. For `DividendType` `continuous`, `ExDividendDates` is ignored.

## Stock Structure Example Using an Implied Trinomial Tree

Consider a stock with a price of \$100 and an annual volatility of 12%. Assume that the stock is expected to pay a dividend yield of 6%. You specify these parameters in MATLAB as:

```
So=100;
DividendYield = 0.06;
Sigma=.12;

StockSpec = stockspec(Sigma, So, 'continuous', DividendYield)

StockSpec =
```

```
    FinObj: 'StockSpec'  
      Sigma: 0.1200  
    AssetPrice: 100  
    DividendType: 'continuous'  
    DividendAmounts: 0.0600  
    ExDividendDates: []
```

#### **Specifying the Interest-Rate Term Structure for Implied Trinomial Trees**

The structure `RateSpec` defines the interest rate environment used when building the stock price binary tree. “Modeling the Interest-Rate Term Structure” on page 2-67 explains how to create these structures using the function `intenvset`, given the interest rates, the starting and ending dates for each rate, and the compounding value.

#### **Specifying the Tree-Time Term Structure for Implied Trinomial Trees**

The `TimeSpec` structure defines the tree layout of the trinomial tree:

- It maps the valuation and maturity dates to their corresponding times.
- It defines the time of the levels of the tree by dividing the time span between valuation and maturity into equally spaced intervals. By specifying the number of intervals, you define the granularity of the tree time structure.

The syntax for building a `TimeSpec` structure is:

```
TimeSpec = itttimespec(ValuationDate, Maturity, NumPeriods)
```

where:

- `ValuationDate` is a scalar date marking the pricing date and first observation in the tree (location of the root node). You enter `ValuationDate` either as a serial date number (generated with `datenum`) or a date character vector.
- `Maturity` is a scalar date marking the maturity of the tree, entered as a serial date number or a date character vector.
- `NumPeriods` is a scalar defining the number of time steps in the tree; for example, `NumPeriods = 10` implies 10 time steps and 11 tree levels (0, 1, 2, ..., 9, 10).

#### **TimeSpec Example Using an Implied Trinomial Tree**

Consider building an ITT tree, with a valuation date of January 1, 2006, a maturity date of January 1, 2008, and four time steps. You specify these parameters in MATLAB as:

```

ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
NumPeriods = 4;

TimeSpec = itttimespec(ValuationDate, EndDate, NumPeriods)

TimeSpec =

    FinObj: 'ITTimeSpec'
ValuationDate: 732678
    Maturity: 733408
    NumPeriods: 4
    Basis: 0
    EndMonthRule: 1
    tObs: [0 0.5000 1 1.5000 2]
    dObs: [732678 732860 733043 733225 733408]

```

Two vector fields in the `TimeSpec` structure are of particular interest: `dObs` and `tObs`. These two fields represent the observation times and corresponding dates of all tree levels, with `dObs(1)` and `tObs(1)`, respectively, representing the root node (`ValuationDate`), and `dObs(end)` and `tObs(end)` representing the last tree level (`Maturity`).

### Specifying the Option Stock Structure for Implied Trinomial Trees

The `StockOptSpec` structure encapsulates the option-stock-specific information required for building the implied trinomial tree. You generate `StockOptSpec` with the function `stockoptspec`. This function requires five input arguments. An optional sixth argument `InterpMethod`, specifying the interpolation method, can be included. The syntax for calling `stockoptspec` is:

```
[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)
```

where:

- `Optprice` is a NINST-by-1 vector of European option prices.
- `Strike` is a NINST-by-1 vector of strike prices.
- `Settle` is a scalar date marking the settlement date.
- `Maturity` is a NINST-by-1 vector of maturity dates.
- `OptSpec` is a NINST-by-1 cell array of character vectors for the values 'call' or 'put'.

#### Option Stock Structure Example Using an Implied Trinomial Tree

Consider the following data quoted from liquid options in the market with varying strikes and maturity. You specify these parameters in MATLAB as:

```
Settle = '01/01/06';  
Maturity = ['07/01/06';  
           '07/01/06';  
           '07/01/06';  
           '01/01/07';  
           '01/01/07';  
           '01/01/07';  
           '01/01/07';  
           '07/01/07';  
           '07/01/07';  
           '07/01/07';  
           '07/01/07';  
           '01/01/08';  
           '01/01/08';  
           '01/01/08';  
           '01/01/08'];  
Strike = [113;  
         101;  
         100;  
         88;  
         128;  
         112;  
         100;  
         78;  
         144;  
         112;  
         100;  
         69;  
         162;  
         112;  
         100;  
         61];  
OptPrice = [ 0;  
            4.807905472659144;  
            1.306321897011867;  
            0.048039195057173;  
            0;  
            2.310953054191461;  
            1.421950392866235;  
            0.020414826276740;  
            0;  
            5.091986935627730;  
            1.346534812295291;  
            0.005101325584140;  
            0;  
            8.047628153217246;
```

```

1.219653432150932;
0.001041436654748];

OptSpec = { 'call';
            'call';
            'put';
            'put';
            'call';
            'call';
            'put';
            'put';
            'call';
            'call';
            'put';
            'put';
            'call';
            'call';
            'put';
            'put'};

StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)

StockOptSpec =

    FinObj: 'StockOptSpec'
    OptPrice: [16x1 double]
    Strike: [16x1 double]
    Settle: 732678
    Maturity: [16x1 double]
    OptSpec: {16x1 cell}
    InterpMethod: 'price'

```

**Note** The algorithm for building the ITT tree requires specifying option prices for all tree nodes. The maturities of those options correspond to those of the tree levels, and the strike to the prices on the tree nodes. The types of option are Calls for the nodes above the central nodes, and Puts for those below and including the central nodes.

Clearly, all these options will not be available in the market, hence making interpolation, and extrapolation necessary to obtain the node option prices. The degree to which the tree reflects the market will unavoidably be tied to the results of these interpolations and extrapolations. Keeping in mind that extrapolation is less accurate than interpolation, and more so the further away the extrapolated points are from the data points, the function `itttree` issues a warning with a list of the options for which extrapolation was necessary.

Sometimes, it may be desirable to view a list of ideal option prices to form an idea of the ranges needed. This can be achieved by calling the function `itttree` specifying only the first three input arguments. The second output argument is a structure array containing the list of ideal options needed.

---

#### Creating an Implied Trinomial Tree

You can now use the `StockSpec`, `TimeSpec`, and `StockOptSpec` structures described in “Stock Structure Example Using an Implied Trinomial Tree” on page 3-9, “TimeSpec Example Using an Implied Trinomial Tree” on page 3-10, and “Option Stock Structure Example Using an Implied Trinomial Tree” on page 3-12 to build an implied trinomial tree (ITT). First, you must define the interest rate term structure. For this example, assume that the interest rate is fixed at 8% annually between the valuation date of the tree (January 1, 2006) until its maturity.

```
Rate = 0.08;
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';

RateSpec = intenvset('StartDates', ValuationDate, 'EndDates', EndDate, ...
    'ValuationDate', ValuationDate, 'Rates', Rate, 'Compounding', -1);
```

To build an ITTtree, enter:

```
ITTtree = itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)

ITTtree =

    FinObj: 'ITStockTree'
    StockSpec: [1x1 struct]
    StockOptSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 0.5000000000000000 1 1.5000000000000000 2]
    dObs: [732678 732860 733043 733225 733408]
    STree: {1x5 cell}
    Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
```

#### Building Standard Trinomial Trees

The tree of stock prices is the fundamental unit representing the evolution of the price of a stock over a given period of time. The MATLAB function `stttree` creates an output tree structure along with the information about the parameters used to create the tree.

The function `stttree` takes three structures as input arguments:



- The stock parameter structure `StockSpec`
- The interest-rate term structure `RateSpec`
- The tree time layout structure `TimeSpec`

### Calling Sequence for Standard Trinomial Trees

The calling syntax for `stttree` is:

```
STTTree = stttree (StockSpec,RateSpec,TimeSpec)
```

- `StockSpec` is a structure that specifies parameters of the stock whose price evolution is represented by the tree. This structure, created using the function `stockspec`, contains information such as the stock's original price, its volatility, and its dividend payment information.
- `RateSpec` is the interest-rate specification of the initial rate curve. Create this structure with the function `intenvset`.
- `TimeSpec` is the tree time layout specification. Create these structures with the function `stttimespec`. This structure contains information regarding the mapping of relevant dates into the tree structure, plus the number of time steps used for building the tree.

### Specifying the Stock Structure for Standard Trinomial Trees

The structure `StockSpec` encapsulates the stock-specific information required for building the trinomial tree of an individual stock's price movement.

You generate `StockSpec` with the function `stockspec`. This function requires two input arguments and accepts up to three additional input arguments that depend on the existence and type of dividend payments.

The syntax for calling `stockspec` is:

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...  
DividendAmounts, ExDividendDates)
```

where:

- `Sigma` is the decimal annual volatility of the underlying security.
- `AssetPrice` is the price of the stock at the valuation date.
- `DividendType` is a character vector specifying the type of dividend paid by the stock. Allowed values are `cash`, `constant`, or `continuous`.

- `DividendAmounts` has a value that depends on the specification of `DividendType`. For `DividendType` `cash`, `DividendAmounts` is a vector of cash dividends. For `DividendType` `constant`, it is a vector of constant annualized dividend yields. For `DividendType` `continuous`, it is a scalar representing a continuously annualized dividend yield.
- `ExDividendDates` also has a value that depends on the nature of `DividendType`. For `DividendType` `cash` or `constant`, `ExDividendDates` is vector of dividend dates. For `DividendType` `continuous`, `ExDividendDates` is ignored.

#### Stock Structure Example Using a Standard Trinomial Tree

Consider a stock with a price of \$100 and an annual volatility of 12%. Assume that the stock is expected to pay a dividend yield of 6%. You specify these parameters in MATLAB as:

```
So=100;
DividendYield = 0.06;
Sigma=.12;

StockSpec = stockspec(Sigma, So, 'continuous', DividendYield)

StockSpec =

    FinObj: 'StockSpec'
    Sigma: 0.1200
    AssetPrice: 100
    DividendType: 'continuous'
    DividendAmounts: 0.0600
    ExDividendDates: []
```

#### Specifying the Interest-Rate Term Structure for Standard Trinomial Trees

The structure `RateSpec` defines the interest rate environment used when building the stock price binary tree. “Modeling the Interest-Rate Term Structure” on page 2-67 explains how to create these structures using the function `intenvset`, given the interest rates, the starting and ending dates for each rate, and the compounding value.

#### Specifying the Tree-Time Term Structure for Standard Trinomial Trees

The `TimeSpec` structure defines the tree layout of the trinomial tree:

- It maps the valuation and maturity dates to their corresponding times.

- It defines the time of the levels of the tree by dividing the time span between valuation and maturity into equally spaced intervals. By specifying the number of intervals, you define the granularity of the tree time structure.

The syntax for building a `TimeSpec` structure is:

```
TimeSpec = stttimespec(ValuationDate, Maturity, NumPeriods)
```

where:

- `ValuationDate` is a scalar date marking the pricing date and first observation in the tree (location of the root node). You enter `ValuationDate` either as a serial date number (generated with `datenum`) or a date character vector.
- `Maturity` is a scalar date marking the maturity of the tree, entered as a serial date number or a date character vector.
- `NumPeriods` is a scalar defining the number of time steps in the tree; for example, `NumPeriods = 10` implies 10 time steps and 11 tree levels (0, 1, 2, ..., 9, 10).

### TimeSpec Example Using a Standard Trinomial Tree

Consider building an STT tree, with a valuation date of January 1, 2006, a maturity date of January 1, 2008, and four time steps. You specify these parameters in MATLAB as:

```
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
NumPeriods = 4;
```

```
TimeSpec = stttimespec(ValuationDate, EndDate, NumPeriods)
```

```
TimeSpec =
```

```

    FinObj: 'STTTimeSpec'
ValuationDate: 732678
    Maturity: 733408
    NumPeriods: 4
        Basis: 0
    EndMonthRule: 1
        tObs: [0 0.5000 1 1.5000 2]
        dObs: [732678 732860 733043 733225 733408]
```

Two vector fields in the `TimeSpec` structure are of particular interest: `dObs` and `tObs`. These two fields represent the observation times and corresponding dates of all tree levels, with `dObs(1)` and `tObs(1)`, respectively, representing the root node

(ValuationDate), and dObs(end) and tObs(end) representing the last tree level (Maturity).

#### Creating a Standard Trinomial Tree

You can now use the StockSpec, TimeSpec structures described in “Stock Structure Example Using an Implied Trinomial Tree” on page 3-9 and “TimeSpec Example Using an Implied Trinomial Tree” on page 3-10, to build a standard trinomial tree (STT). First, you must define the interest rate term structure. For this example, assume that the interest rate is fixed at 8% annually between the valuation date of the tree (January 1, 2006) until its maturity.

```
Rate = 0.08;
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';

RateSpec = intenvset('StartDates', ValuationDate, 'EndDates', EndDate, ...
    'ValuationDate', ValuationDate, 'Rates', Rate, 'Compounding', -1);
```

To build an STTTree, enter:

```
STTTree = stttree(StockSpec, RateSpec, TimeSpec)

STTTree =

    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
           tObs: [0 0.5000 1 1.5000 2]
           dObs: [732678 732860 733043 733225 733408]
           STree: {1x5 cell}
           Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
```

#### Examining Equity Trees

Financial Instruments Toolbox uses equity binary and trinomial trees to represent prices of equity options and of underlying stocks. At the highest level, these trees have structures wrapped around them. The structures encapsulate information required to interpret information in the tree.

To examine an equity, binary, or trinomial tree, load the data in the MAT-file `deriv.mat` into the MATLAB workspace.

```
load deriv.mat
```

Display the list of variables loaded from the MAT-file with the `whos` command.

Name	Size	Bytes	Class	Attributes
BDTInstSet	1x1	27344	struct	
BDTTree	1x1	7322	struct	
BKInstSet	1x1	27334	struct	
BKTree	1x1	8532	struct	
CRRInstSet	1x1	21066	struct	
CRRTree	1x1	7086	struct	
EQPInstSet	1x1	21066	struct	
EQPTree	1x1	7086	struct	
HJMInstSet	1x1	27336	struct	
HJMTree	1x1	8334	struct	
HWInstSet	1x1	27334	struct	
HWTree	1x1	8532	struct	
ITTInstSet	1x1	21070	struct	
ITTTree	1x1	12660	struct	
STTInstSet	1x1	21070	struct	
STTTree	1x1	7782	struct	
ZeroInstSet	1x1	17458	struct	
ZeroRateSpec	1x1	2152	struct	

## Examining a CRRTree

You can examine in some detail the contents of the CRRTree structure contained in this file.

CRRTree

CRRTree =

```

    FinObj: 'BinStockTree'
    Method: 'CRR'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [731582 731947 732313 732678 733043]
    STree: {[100] [110.5171 90.4837] [122.1403 100 81.8731] [1x4 double] [1x5 double]}
    UpProbs: [0.7309 0.7309 0.7309 0.7309]

```

The Method field of the structure indicates that this is a CRR tree, not an EQP tree.

The fields StockSpec, TimeSpec, and RateSpec hold the original structures passed into the function crrtree. They contain all the context information required to interpret the tree data.

The fields tObs and dObs are vectors containing the observation times and dates, that is, the times and dates of the levels of the tree. In this particular case, tObs reveals that the tree has a maturity of four years (tObs(end) = 4) and that it has four time steps (the length of tObs is five).

The field dObs shows the specific dates for the tree levels, with a granularity of one day. This means that all values in tObs that correspond to a given day from 00:00 hours to

24:00 hours are mapped to the corresponding value in `dObs`. You can use the function `datestr` to convert these MATLAB serial dates into their character vector representations.

The field `UpProbs` is a vector representing the probabilities for up movements from any node in each level. This vector has one element per tree level. All nodes for a given level have the same probability of an up movement. In the specific case being examined, the probability of an up movement is 0.7309 for all levels, and the probability for a down movement is 0.2691 ( $1 - 0.7309$ ).

Finally, the field `STree` contains the actual stock tree. It is represented in MATLAB as a cell array with each cell array element containing a vector of prices corresponding to a tree level. The prices are in descending order, that is, `CRRTree.STree{3}(1)` represents the topmost element of the third level of the tree, and `CRRTree.STree{3}(end)` represents the bottom element of the same level of the tree.

#### Examining an ITTTree

You can examine in some detail the contents of the `ITTTree` structure contained in this file.

`ITTTree`

`ITTTree =`

```
    FinObj: 'ITStockTree'  
    StockSpec: [1x1 struct]  
    StockOptSpec: [1x1 struct]  
    TimeSpec: [1x1 struct]  
    RateSpec: [1x1 struct]  
    tObs: [0 1 2 3 4]  
    dObs: [732678 733043 733408 733773 734139]  
    STree: {1x5 cell}  
    Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
```

The fields `StockSpec`, `StockOptSpec`, `TimeSpec`, and `RateSpec` hold the original structures passed into the function `itttree`. They contain all the context information required to interpret the tree data.

The fields `tObs` and `dObs` are vectors containing the observation times and dates and the times and dates of the levels of the tree. In this particular case, `tObs` reveals that the tree has a maturity of four years (`tObs(end) = 4`) and that it has four time steps (the length of `tObs` is five).

The field `dObs` shows the specific dates for the tree levels, with a granularity of one day. This means that all values in `tObs` that correspond to a given day from 00:00 hours to

24:00 hours are mapped to the corresponding value in `dObs`. You can use the function `datestr` to convert these MATLAB serial dates into their character vector representations.

The field `Probs` is a vector representing the probabilities for movements from any node in each level. This vector has three elements per tree node. In the specific case being examined, at `tObs = 1`, the probability for an up movement is 0.4675, and the probability for a down movement is 0.1934.

Finally, the field `STree` contains the actual stock tree. It is represented in MATLAB as a cell array with each cell array element containing a vector of prices corresponding to a tree level. The prices are in descending order, that is, `ITTTree.STree{4}(1)` represents the top element of the fourth level of the tree, and `ITTTree.STree{4}(end)` represents the bottom element of the same level of the tree.

### Isolating a Specific Node for a CRRTree

The function `treepath` can isolate a specific set of nodes of a binary tree by specifying the path used to reach the final node. As an example, consider the nodes tapped by starting from the root node, then following a down movement, then an up movement, and finally a down movement. You use a vector to specify the path, with 1 corresponding to an up movement and 2 corresponding to a down movement. An up-down-up path is then represented as `[2 1 2]`. To obtain the values of all nodes tapped by this path, enter:

```
SVals = treepath(CRRTree.STree, [2 1 2])
```

```
SVals =
```

```
100.0000
 90.4837
100.0000
 90.4837
```

The first value in the vector `SVals` corresponds to the root node, and the last value corresponds to the final node reached by following the path indicated.

### Isolating a Specific Node for an ITTTree

The function `trintreepath` can isolate a specific set of nodes of a trinomial tree by specifying the path used to reach the final node. As an example, consider the nodes tapped by starting from the root node, then following an up movement, then a middle movement, and finally a down movement. You use a vector to specify the path, with 1 corresponding to an up movement, 2 corresponding to a middle movement, and 3

corresponding to a down movement. An up-down-middle-down path is then represented as [1 3 2 3]. To obtain the values of all nodes tapped by this path, enter:

```
pathSVals = trintreepath(ITTree, [1 3 2 3])
```

```
pathSVals =
```

```
50.0000  
66.3448  
50.0000  
50.0000  
37.6819
```

The first value in the vector `pathSVals` corresponds to the root node, and the last value corresponds to the final node reached by following the path indicated.

## Differences Between CRR and EQP Tree Structures

In essence, the structures representing CRR trees and EQP trees are similar. If you create a CRR or an EQP tree using identical input arguments, only a few of the tree structure fields differ:

- The `Method` field has a value of 'CRR' or 'EQP' indicating the method used to build the structure.
- The prices in the `STree` cell array have the same structure, but the prices within the cell array are different.
- For EQP, the structure field `UpProb` always holds a vector with all elements set to 0.5, while for CRR, these probabilities are calculated based on the input arguments passed when building the tree.

## See Also

```
crrtimespec | crrtree | eqptimespec | eqptree | intenvset | itttimespec |  
itttree | lrtimespec | lrtree | stockoptspec | stockspect | treepath |  
trintreepath
```

## Related Examples

- “Pricing Equity Derivatives Using Trees” on page 3-130
- “Creating Instruments or Properties” on page 1-21



- “Graphical Representation of Equity Derivative Trees” on page 3-142

### **More About**

- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44
- “Supported Interest-Rate Instruments” on page 2-2

## Supported Equity Derivatives

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### Asian Option

An Asian option is a path-dependent option with a payoff linked to the average value of the underlying asset during the life (or some part of the life) of the option. They are similar to lookback options in that there are two types of Asian options: fixed (average price option) and floating (average strike option). Fixed Asian options have a specified strike, while floating Asian options have a strike equal to the average value of the underlying asset over the life of the option.

There are four Asian option types, each with its own characteristic payoff formula:

- Fixed call (average price option):  $\max(0, S_{av} - X)$
- Fixed put (average price option):  $\max(0, X - S_{av})$
- Floating call (average strike option):  $\max(0, S - S_{av})$
- Floating put (average strike option):  $\max(0, S_{av} - S)$

where:

$S_{av}$  is the average price of underlying asset.

$S$  is the price of the underlying asset.

$X$  is the strike price (applicable only to fixed Asian options).

$S_{av}$  is defined using either a geometric or an arithmetic average.

The following functions support Asian options.

Function	Purpose
asianbycrr	Price Asian options from a CRR binomial tree.
asianbyeqp	Price Asian options from an EQP binomial tree.
asianbyitt	Price Asian options using an implied trinomial tree (ITT).
asianbystt	Price Asian options using a standard trinomial tree (STT).
instasian	Construct an Asian option.
asianbyls	Price European or American Asian options using the Longstaff-Schwartz model.
asiansensbyls	Calculate prices and sensitivities of European or American Asian options using the Longstaff-Schwartz model.
asianbykv	Price European geometric Asian options using the Kemna Vorst model.
asiansensbykv	Calculate prices and sensitivities of European geometric Asian options using the Kemna Vorst model.
asianbylevy	Price European arithmetic Asian options using the Levy model.
asiansensbylevy	Calculate prices and sensitivities of European arithmetic Asian options using the Levy model.
asianbyhgm	Calculate prices of European discrete arithmetic fixed Asian options using the Haug, Haug, Margrabe model.

<b>Function</b>	<b>Purpose</b>
asiansensbyhbm	Calculate prices and sensitivities of European discrete arithmetic fixed Asian options using the Haug, Haug, Margrabe model
asianbytw	Calculate prices of European arithmetic fixed Asian options using the Turnbull Wakeman model.
asiansensbytw	Calculate prices and sensitivities of European arithmetic fixed Asian options using the Turnbull Wakeman model.

## **Barrier Option**

A barrier option is similar to a vanilla put or call option, but its life either begins or ends when the price of the underlying stock passes a predetermined barrier value. There are four types of barrier options.

### **Up Knock-In**

This option becomes effective when the price of the underlying stock passes above a barrier that is above the initial stock price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves below the barrier again.

### **Up Knock-Out**

This option terminates when the price of the underlying stock passes above a barrier that is above the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves below the barrier again.

### **Down Knock-In**

This option becomes effective when the price of the underlying stock passes below a barrier that is below the initial stock price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves above the barrier again.

### **Down Knock-Out**

This option terminates when the price of the underlying stock passes below a barrier that is below the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves above the barrier again.

## Rebates

If a barrier option fails to exercise, the seller may pay a rebate to the buyer of the option. Knock-outs may pay a rebate when they are knocked out, and knock-ins may pay a rebate if they expire without ever knocking in.

The following functions support barrier options.

Function	Purpose
barrierbycrr	Price barrier options from a CRR binomial tree.
barrierbyeqp	Price barrier options from an EQP binomial tree.
barrierbyitt	Price barrier options using an implied trinomial tree (ITT).
barrierbystt	Price barrier options using a standard trinomial tree (STT).
barrierbyfd	Price barrier option using finite difference method.
barriersensbyfd	Calculate barrier option price and sensitivities using finite difference method.
barrierbybls	Price a European barrier option using Black-Scholes option pricing model.
barriersensbybls	Calculate price and sensitivities for a European barrier option using Black-Scholes option pricing model.
barrierbyls	Price a barrier option using Longstaff-Schwartz model.
barriersensbyls	Calculate price and sensitivities for a barrier option using Longstaff-Schwartz model.
instbarrier	Construct a barrier option.

## Double Barrier Option

A double barrier option is similar to the standard single barrier option except that they have two barrier levels: a lower barrier (LB) and an upper barrier (UB). The payoff for a double barrier option depends on whether the underlying asset remains between the barrier levels during the life of the option. Double barrier options are less expensive than single barrier options as the probability of being knocked out is higher. Because of this, double barrier options allow investors to achieve reduction in the option premiums as and match an investor's belief about the future movement of the underlying price process.

There are two types of double barrier options:

- Double Knock-in

This option becomes effective when the price of the underlying asset reaches one of the barriers. It gives the option holder, the right but not the obligation to buy or sell the underlying security at the strike price, if the underlying asset goes above or below the barrier levels during the life of the option.

- Double Knock-out

This option gives the option holder, the right but not the obligation to buy or sell the underlying security at the strike price, as long as the underlying asset remains between the barrier levels during the life of the option. This option terminates when the price of the underlying asset passes one of the barriers.

The following functions support double barrier options.

Function	Purpose
<code>dblbarrierbybls</code>	Price European double barrier options using the Black-Scholes option pricing model.
<code>dblbarriersensbybls</code>	Calculate the price and sensitivities for a European double barrier options using the Black-Scholes option pricing model.
<code>dblbarrierbyfd</code>	Price double barrier option prices using the finite difference method.
<code>dblbarriersensbyfd</code>	Calculate the price and sensitivities for a double barrier option using the finite difference method.

## Basket Option

A basket option is an option on a portfolio of several underlying equity assets. Payout for a basket option depends on the cumulative performance of the collection of the individual assets. A basket option tends to be cheaper than the corresponding portfolio of plain vanilla options for these reasons:

- If the basket components correlate negatively, movements in the value of one component neutralize opposite movements of another component. Unless all the components correlate perfectly, the basket option is cheaper than a series of individual options on each of the assets in the basket.
- A basket option minimizes transaction costs because an investor has to purchase only one option instead of several individual options.

The payoff for a basket option is as follows:

- For a call:  $\max(\sum W_i * S_i - K; 0)$
- For a put:  $\max(\sum K - W_i * S_i; 0)$

where:

$S_i$  is the price of asset  $i$  in the basket.

$W_i$  is the quantity of asset  $i$  in the basket.

$K$  is the strike price.

Financial Instruments Toolbox software supports Longstaff-Schwartz and Nengiu Ju models for pricing basket options. The Longstaff-Schwartz model supports both European, Bermuda, and American basket options. The Nengiu Ju model only supports European basket options. If you want to price either an American or Bermuda basket option, use the functions for the Longstaff-Schwartz model. To price a European basket option, use either the functions for the Longstaff-Schwartz model or the Nengiu Ju model.

Function	Purpose
basketbyls	Price basket options using the Longstaff-Schwartz model.
basketsensbyls	Calculate price and sensitivities for basket options using the Longstaff-Schwartz model.
basketbyju	Price European basket options using the Nengiu Ju approximation model.
basketsensbyju	Calculate European basket options price and sensitivity using the Nengiu Ju approximation model.
basketstockspec	Specify a basket stock structure.

## Chooser Option

A chooser option enables the holder to decide before the option expiration date whether the option is a call or put.

A chooser option has a specified decision time  $t_1$  where the holder has to make the decision whether the option is a call or put. At the expiration time  $t_2$  the option expires. If the holder chooses a call option, the payout is  $\max(S - K, 0)$ . For the choice of a put

option, the payout is  $\max ( K - S , 0 )$  where  $K$  is the strike price of the option and  $S$  is the equity price at expiry.

Function	Purpose
chooserbybls	Price a European simple chooser options using Black-Scholes model.

## Compound Option

A compound option is basically an option on an option; it gives the holder the right to buy or sell another option. With a compound option, a vanilla stock option serves as the underlying instrument. Compound options thus have two strike prices and two exercise dates.

There are four types of compound options:

- Call on a call
- Put on a put
- Call on a put
- Put on a call

---

**Note** The payoff formulas for compound options are too complex for this discussion. If you are interested in the details, consult the paper by Mark Rubinstein entitled “Double Trouble,” published in *Risk* 5 (1991).

---

Consider the third type, a call on a put. It gives the holder the right to buy a put option. In this case, on the first exercise date, the holder of the compound option pay the first strike price and receives a put option. The put option gives the holder the right to sell the underlying asset for the second strike price on the second exercise date.

The following functions support compound options.

Function	Purpose
compoundbycrr	Price compound options from a CRR binomial tree.
compoundbyeqp	Price compound options from an EQP binomial tree.
compoundbyitt	Price compound options using an implied trinomial tree (ITT).



Function	Purpose
compoundbystt	Price compound options using a standard trinomial tree (STT).
instcompound	Construct a compound option.

## Convertible Bond

A convertible bond is a financial instrument that combines equity and debt features. It is a bond with the embedded option to turn it into a fixed number of shares. The holder of a convertible bond has the right, but not the obligation, to exchange the convertible security for a predetermined number of equity shares at a preset price. The debt component is derived from the coupon payments and the principal. The equity component is provided by the conversion feature.

Convertible bonds have several defining features:

- **Coupon** — The coupon in convertible bonds are typically lower than coupons in vanilla bonds since investors are willing to take the lower coupon for the opportunity to participate in the company's stock via the conversion.
- **Maturity** — Most convertible bonds are issued with long-stated maturities. Short-term maturity convertible bonds usually do not have call or put provisions.
- **Conversion ratio** — Conversion ratio is the number of shares that the holder of the convertible bond will receive from exercising the call option of the convertible bond:  

$$\text{Conversion ratio} = \frac{\text{par value convertible bond}}{\text{conversion price of equity}}$$

For example, a conversion ratio of 25 means a bond can be exchanged for 25 shares of stock. This also implies a conversion price of \$40 (1000/25). This, \$40, would be the price at which the owner would buy the shares. This can be expressed as a ratio or as the conversion price and is specified in the contract along with other provisions.

- **Option type:**
  - **Callable Convertible:** a convertible bond that is callable by the issuer. The issuer of the bond forces conversion, removing the advantage that conversion is at the discretion of the bondholder. Upon call, the bondholder can either convert the bond or redeem at the call price. This option enables the issuer to control the price of the convertible bond and if necessary refinance the debt with a new cheaper one.
  - **Puttable Convertible:** a convertible bond with a put feature that allows the bondholder to sell back the bond at a premium on a specific date. This option protects the holder against rising interest rates by reducing the year to maturity.

Function	Purpose
cbondbycrr	Price convertible bonds using a CRR binomial tree with the Tsiveriotis and Fernandes model.
cbondbyeqp	Price convertible bonds using an EQP binomial tree with the Tsiveriotis and Fernandes model.
cbondbyitt	Price convertible bonds using an implied trinomial tree with the Tsiveriotis and Fernandes model.
cbondbystt	Price convertible bonds using a standard trinomial tree with the Tsiveriotis and Fernandes model.
instcbond	Construct a cbond instrument for a convertible bond.

## Lookback Option

A lookback option is a path-dependent option based on the maximum or minimum value the underlying asset achieves during the entire life of the option.

Financial Instruments Toolbox software supports two types of lookback options: fixed and floating. Fixed lookback options have a specified strike price, while floating lookback options have a strike price determined by the asset path. So, there are a total of four lookback option types, each with its own characteristic payoff formula:

- Fixed call:  $\max(0, S_{\max} - X)$
- Fixed put:  $\max(0, X - S_{\min})$
- Floating call:  $\max(0, S - S_{\min})$
- Floating put:  $\max(0, S_{\max} - S)$

where:

$S_{\max}$  is the maximum price of underlying stock found along the particular path followed to the node.

$S_{\min}$  is the minimum price of underlying stock found along the particular path followed to the node.

$S$  is the price of the underlying stock on the node.

$X$  is the strike price (applicable only to fixed lookback options).

The following functions support lookback options.

Function	Purpose
lookbackbycrr	Price lookback options from a CRR binomial tree.
lookbackbyeqp	Price lookback options from an EQP binomial tree.
lookbackbyitt	Price lookback options using an implied trinomial tree (ITT).
lookbackbystt	Price lookback options using standard trinomial tree.
instlookback	Construct a lookback option based on an equity tree model.
lookbackbycvgs	Calculate prices of European lookback fixed and floating strike options using the Conze-Viswanathan and Goldman-Sosin-Gatto models. For more information, see “Lookback Option” on page 3-51.
lookbacksensbycvgs	Calculate prices and sensitivities of European fixed and floating strike lookback options using the Conze-Viswanathan and Goldman-Sosin-Gatto models. For more information, see “Lookback Option” on page 3-51.
lookbackbyls	Calculate prices of lookback fixed and floating strike options using the Longstaff-Schwartz model. For more information, see “Lookback Option” on page 3-51.
lookbacksensbyls	Calculate prices and sensitivities of lookback fixed and floating strike options using the Longstaff-Schwartz model. For more information, see “Lookback Option” on page 3-51.

## Digital Option

A digital option is an option whose payoff is characterized as having only two potential values: a fixed payout, when the option is in the money or a zero payout otherwise. This is the case irrespective of how far the asset price at maturity is above (call) or below (put) the strike.

Digital options are attractive to sellers because they guarantee a known maximum loss when the option is exercised. This overcomes a fundamental problem with the vanilla options, where the potential loss is unlimited. Digital options are attractive to buyers because the option payoff is a known constant amount, and this amount can be adjusted to provide the exact quantity of protection required.

Financial Instruments Toolbox supports four types of digital options:

- Cash-or-nothing option — Pays some fixed amount of cash if the option expires in the money.
- Asset-or-nothing option — Pays the value of the underlying security if the option expires in the money.
- Gap option — One strike decides if the option is in or out of money; another strike decides the size of the payoff.
- Supershare — Pays out a proportion of the assets underlying a portfolio if the asset lies between a lower and an upper bound at the expiry of the option.

The following functions calculate pricing and sensitivity for digital options.

<b>Function</b>	<b>Purpose</b>
cashbybls	Calculate the price of cash-or-nothing digital options using the Black-Scholes model.
assetbybls	Calculate the price of asset-or-nothing digital options using the Black-Scholes model.
gapbybls	Calculate the price of gap digital options using the Black-Scholes model.
supersharebybls	Calculate the price of supershare digital options using the Black-Scholes model.
cashsensbybls	Calculate the price and sensitivities of cash-or-nothing digital options using the Black-Scholes model.
assetsensbybls	Calculate the price and sensitivities of asset-or-nothing digital options using the Black-Scholes model.
gapsensbybls	Calculate the price and sensitivities of gap digital options using the Black-Scholes model.
supersharesensbybls	Calculate the price and sensitivities of supershare digital options using the Black-Scholes model.

## Rainbow Option

A rainbow option payoff depends on the relative price performance of two or more assets. A rainbow option gives the holder the right to buy or sell the best or worst of two securities, or options that pay the best or worst of two assets.

Rainbow options are popular because of the lower premium cost of the structure relative to the purchase of two separate options. The lower cost reflects the fact that the payoff is generally lower than the payoff of the two separate options.

Financial Instruments Toolbox supports two types of rainbow options:

- Minimum of two assets — The option holder has the right to buy(sell) one of two risky assets, whichever one is worth less.
- Maximum of two assets — The option holder has the right to buy(sell) one of two risky assets, whichever one is worth more.

The following rainbow options speculate/hedge on two equity assets.

Function	Purpose
<code>minassetbystulz</code>	Calculate the European rainbow option price on minimum of two risky assets using the Stulz option pricing model.
<code>minassetsensbystulz</code>	Calculate the European rainbow option prices and sensitivities on minimum of two risky assets using the Stulz pricing model.
<code>maxassetbystulz</code>	Calculate the European rainbow option price on maximum of two risky assets using the Stulz option pricing model.
<code>maxassetsensbystulz</code>	Calculate the European rainbow option prices and sensitivities on maximum of two risky assets using the Stulz pricing model.

## Vanilla Option

A vanilla option is a category of options that includes only the most standard components. A vanilla option has an expiration date and straightforward strike price. American-style options and European-style options are both categorized as vanilla options.

The payoff for a vanilla option is as follows:

- For a call:  $\max(St - K, 0)$
- For a put:  $\max(K - St, 0)$

where:

$St$  is the price of the underlying asset at time  $t$ .

$K$  is the strike price.

The following functions support specifying or pricing a vanilla option.

Function	Purpose
<code>optstockbybaw</code>	Calculate the American options prices using the Barone-Adesi-Whaley option pricing model.
<code>optstocksensbybaw</code>	Calculate the American options prices and sensitivities using the Barone-Adesi-Whaley option pricing model.
<code>optstockbycrr</code>	Calculate the price of a European, Bermuda, or American stock option using a CRR tree.
<code>optstockbyeqp</code>	Calculate the price of a European, Bermuda, or American stock option using an EQP tree.
<code>optstockbyfd</code>	Calculate vanilla option prices using finite difference method.
<code>optstocksensbyfd</code>	Calculate vanilla option prices and sensitivities using finite difference method.
<code>optByLocalVolFD</code>	Calculate vanilla option price by local volatility model, using finite differences.
<code>optSensByLocalVolFD</code>	Calculate vanilla option price or sensitivities by local volatility model, using finite differences.
<code>optByHestonFD</code>	Calculate vanilla option price by Heston model using finite differences.
<code>optSensByHestonFD</code>	Calculate vanilla option price and sensitivities by Heston model using finite differences.
<code>optByBatesFD</code>	Calculates vanilla European option price by Bates model using finite differences.

<b>Function</b>	<b>Purpose</b>
optSensByBatesFD	Calculates vanilla European option price and sensitivities by Bates model using finite differences.
optByMertonFD	Calculates vanilla European option price by Merton76 model using finite differences.
optBySensMertonFD	Calculates vanilla European option price and sensitivities by Merton76 model using finite differences.
optstockbyitt	Calculate the price of a European, Bermuda, or American stock option using an ITT tree.
optstockbystt	Calculate the price of a European, Bermuda, or American stock option using an STT tree.
optstockbylr	Calculate the price of a European, Bermuda, or American stock option using the Leisen-Reimer (LR) binomial tree model.
optstocksensbylr	Calculate the price and sensitivities of a European, Bermuda, or American stock option using the Leisen-Reimer (LR) binomial tree model.
optstockbybls	Price options using the Black-Scholes option pricing model.
optstocksensbybls	Calculate option prices and sensitivities using the Black-Scholes option pricing model.
optstockbyrgw	Calculate American call option prices using the Roll-Geske-Whaley option pricing model.
optstocksensbyrgw	Calculate American call option prices and sensitivities using the Roll-Geske-Whaley option pricing model.
optstockbybjs	Price American options using the Bjerksund-Stensland 2002 option pricing model.
optstocksensbybjs	Calculate American option prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model.
optstockbyls	Price vanilla options using the Longstaff-Schwartz model.
optstocksensbyls	Calculate vanilla option prices and sensitivities using the Longstaff-Schwartz model.

<b>Function</b>	<b>Purpose</b>
optByHestonFFT	Calculate option price by Heston model using FFT and FRFT.
optSensByHestonFFT	Calculate option price and sensitivities by Heston model using FFT and FRFT.
optByHestonNI	Calculate option price by Heston model using numerical integration.
optSensByHestonNI	Calculate option price and sensitivities by Heston model using numerical integration.
optByBatesFFT	Calculate option price by Bates model using FFT and FRFT.
optSensByBatesFFT	Calculate option price and sensitivities by Bates model using FFT and FRFT.
optByBatesNI	Calculate option price by Bates model using numerical integration.
optSensByBatesNI	Calculate option price or sensitivities by Bates model using numerical integration.
optByMertonFFT	Calculate option price by Merton76 model using FFT and FRFT.
optSensByMertonFFT	Calculate option price and sensitivities by Merton76 model using FFT and FRFT.
optByMertonNI	Calculate option price by Merton76 model using numerical integration.
optSensByMertonNI	Calculate option price and sensitivities by Merton76 model using numerical integration.
instoptstock	Specify a European or Bermuda option.

### **Bermuda Put and Call Schedule**

A Bermuda option resembles a hybrid of American and European options. You exercise it on predetermined dates only, usually monthly. In Financial Instruments Toolbox software, you indicate the relevant information for a Bermuda option in two input matrices:

- **Strike** — Contains the strike price values for the option.
- **ExerciseDates** — Contains the schedule when you can exercise the option.



## Spread Option

A spread option is an option written on the difference of two underlying assets. For example, a European call on the difference of two assets  $X1$  and  $X2$  would have the following pay off at maturity:

$$\max(X1 - X2 - K, 0)$$

where:

$K$  is the strike price.

The following functions support spread options.

Function	Purpose
spreadbykirk	Price European spread options using the Kirk pricing model.
spreadsensbykirk	Calculate European spread option prices and sensitivities using the Kirk pricing model.
spreadbybjs	Price European spread options using the Bjerksund-Stensland pricing model.
spreadsensbybjs	Calculate European spread option prices and sensitivities using the Bjerksund-Stensland pricing model.
spreadbyfd	Price European or American spread options using the Alternate Direction Implicit (ADI) finite difference method.
spreadsensbyfd	Calculate price and sensitivities of European or American spread options using the Alternate Direction Implicit (ADI) finite difference method.
spreadbyls	Price European or American spread options using Monte Carlo simulations.
spreadsensbyls	Calculate price and sensitivities for European or American spread options using Monte Carlo simulations.

## Forwards Option

A forward option is a non-standardized contract between two parties to buy or to sell an asset at a specified future time at a price agreed upon today. The buyer of a forward option contract has the right to hold a particular forward position at a specific price any

time before the option expires. The forward option seller holds the opposite forward position when the buyer exercises the option. A call option is the right to enter into a long forward position and a put option is the right to enter into a short forward position. A closely related contract is a futures contract. A forward is like a futures in that it specifies the exchange of goods for a specified price at a specified future date. The table below displays some of the characteristics of forward and futures contracts.

Forwards	Futures
Customized contracts	Standardized contracts
Over the counter traded	Exchange traded
Exposed to default risk	Clearing house reduces default risk
Mostly used for hedging	Mostly used by hedgers and speculators
Settlement at the end of contract (no Margin required)	Daily changes are settled day by day (Margin required)
Delivery usually takes place	Delivery usually never happens

The payoff for a forward option, where the value of a forward position at maturity depends on the relationship between the delivery price ( $K$ ) and the underlying price ( $S_T$ ) at that time, is:

- For a long position:  $f_T = S_T - K$
- For a short position:  $f_T = K - S_T$

The following functions support pricing a forwards option.

Function	Purpose
optstockbyblk	Price options on forwards using the Black option pricing model.
optstocksensbyblk	Determine option prices and sensitivities on forwards using the Black pricing model.

## Futures Option

A future option is a standardized contract between two parties to buy or sell a specified asset of standardized quantity and quality for a price agreed upon today (the futures price) with delivery and payment occurring at a specified future date, the delivery date. The contracts are negotiated at a futures exchange, which acts as an intermediary

between the two parties. The party agreeing to buy the underlying asset in the future, the "buyer" of the contract, is said to be "long", and the party agreeing to sell the asset in the future, the "seller" of the contract, is said to be "short."

Forwards	Futures
Customized contracts	Standardized contracts
Over the counter traded	Exchange traded
Exposed to default risk	Clearing house reduces default risk
Mostly used for hedging	Mostly used by hedgers and speculators
Settlement at the end of contract (no Margin required)	Daily changes are settled day by day (Margin required)
Delivery usually takes place	Delivery usually never happens

A futures contract is the delivery of item  $J$  at time  $T$  and:

- There exists in the market a quoted price  $F(t, T)$ , which is known as the futures price at time  $t$  for delivery of  $J$  at time  $T$ .
- The price of entering a futures contract is equal to zero.
- During any time interval  $[t, s]$ , the holder receives the amount  $F(s, T) - F(t, T)$  (this reflects instantaneous marking to market).
- At time  $T$ , the holder pays  $F(T, T)$  and is entitled to receive  $J$ . Note that  $F(T, T)$  should be the spot price of  $J$  at time  $T$ .

The following functions support pricing a futures option.

Function	Purpose
optstockbyblk	Price options on futures using the Black option pricing model.
optstocksensbyblk	Determine option prices and sensitivities on futures using the Black pricing model.

## See Also

asianbycrr | asianbyeqp | asianbyitt | asianbykv | asianbylevy | asianbyls | asiansensbykv | asiansensbylevy | asiansensbyls | assetbybls | assetsensbybls | barrierbycrr | barrierbyeqp | barrierbyitt | basketbyju |

basketbyls | basketsensbyju | basketsensbyls | basketstockspec |  
basketstockspec | cashbyls | cashsensbyls | chooserbyls | compoundbycrr |  
compoundbyeqp | compoundbyitt | crprice | crsens | crtimespec | crtree |  
eqprice | eqpsens | eqptimespec | eqptree | gapbyls | gapsensbyls |  
impvbybjs | impvbyblk | impvbyls | impvbyrgw | instasian | instbarrier |  
instcompound | instlookback | instoptstock | ittprice | ittens |  
itttimespec | itttree | lookbackbycrr | lookbackbycvgs | lookbackbyeqp |  
lookbackbyitt | lookbackbyls | lookbackbyls | lookbacksensbycvgs |  
lookbacksensbyls | lookbacksensbyls | lrtimespec | lrtree |  
maxassetbystulz | maxassetsensbystulz | minassetbystulz |  
minassetsensbystulz | optpricebysim | optstockbybjs | optstockbyblk |  
optstockbyls | optstockbycrr | optstockbyeqp | optstockbyitt |  
optstockbylr | optstockbyls | optstockbyrgw | optstocksensbybjs |  
optstocksensbyblk | optstocksensbyls | optstocksensbylr |  
optstocksensbyls | optstocksensbyrgw | spreadbybjs | spreadbykirk |  
spreadbyls | spreadsensbybjs | spreadsensbykirk | spreadsensbyls |  
stockspec | supersharebyls | supersharesensbyls | treepath |  
trintreepath

### Related Examples

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- “Pricing Equity Derivatives Using Trees” on page 3-130
- “Creating Instruments or Properties” on page 1-21
- “Graphical Representation of Equity Derivative Trees” on page 3-142
- “Compute Option Prices on a Forward” on page 11-1814
- “Compute Forward Option Prices and Delta Sensitivities” on page 11-1883
- “Compute the Option Price on a Future” on page 11-1816
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- “Pricing Using the Bjerksund-Stensland Model” on page 3-158

### More About

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- “Bjerk Sund-Stensland 2002 Model” on page 3-153
- “Roll-Geske-Whaley Model” on page 3-152
- “Black Model” on page 3-151
- “Digital Option” on page 3-33
- “Supported Energy Derivatives” on page 3-44
- “Supported Interest-Rate Instruments” on page 2-2

## Supported Energy Derivatives

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"Spread Option" on page 3-50
"Lookback Option" on page 3-51
"Forwards Option" on page 3-53
"Futures Option" on page 3-54

### Asian Option

An Asian option is a path-dependent option with a payoff linked to the average value of the underlying asset during the life (or some part of the life) of the option. They are similar to lookback options in that there are two types of Asian options: fixed (average price option) and floating (average strike option). Fixed Asian options have a specified strike, while floating Asian options have a strike equal to the average value of the underlying asset over the life of the option.

There are four Asian option types, each with its own characteristic payoff formula:

- Fixed call (average price option):  $\max(0, S_{av} - X)$
- Fixed put (average price option):  $\max(0, X - S_{av})$
- Floating call (average strike option):  $\max(0, S - S_{av})$
- Floating put (average strike option):  $\max(0, S_{av} - S)$

where:

$S_{av}$  is the average price of underlying asset.

$S$  is the price of the underlying asset.

$X$  is the strike price (applicable only to fixed Asian options).

$S_{av}$  is defined using either a geometric or an arithmetic average.

The following functions support Asian options.

Function	Purpose
asianbyls	Price European or American Asian options using the Longstaff-Schwartz model.
asiansensbyls	Calculate prices and sensitivities of European or American Asian options using the Longstaff-Schwartz model.
asianbykv	Price European geometric Asian options using the Kemna Vorst model.
asiansensbykv	Calculate prices and sensitivities of European geometric Asian options using the Kemna Vorst model.
asianbylevy	Price European arithmetic Asian options using the Levy model.
asiansensbylevy	Calculate prices and sensitivities of European arithmetic Asian options using the Levy model.
asianbyhbm	Calculate prices of European discrete arithmetic fixed Asian options using the Haug, Haug, Margrabe model.
asiansensbyhbm	Calculate prices and sensitivities of European discrete arithmetic fixed Asian options using the Haug, Haug, Margrabe model.
asianbytw	Calculate prices of European arithmetic fixed Asian options using the Turnbull Wakeman model.
asiansensbytw	Calculate prices and sensitivities of European arithmetic fixed Asian options using the Turnbull Wakeman model.
asianbycrr	Price an Asian option from a Cox-Ross-Rubinstein binomial tree.
asianbyeqp	Price an Asian option from an Equal Probabilities binomial tree.
asianbyitt	Price an Asian option using an implied trinomial tree (ITT).
asianbystt	Price an Asian option using a standard trinomial tree.
instasian	Construct an Asian option.

## Barrier Option

A barrier option is similar to a vanilla put or call option, but its life either begins or ends when the price of the underlying asset passes a predetermined barrier value. There are four types of barrier options.

### Up Knock-In

This option becomes effective when the price of the underlying asset passes above a barrier that is above the initial asset price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves below the barrier again.

### Up Knock-Out

This option terminates when the price of the underlying asset passes above a barrier that is above the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves below the barrier again.

### Down Knock-In

This option becomes effective when the price of the underlying asset passes below a barrier that is below the initial stock price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves above the barrier again.

### Down Knock-Out

This option terminates when the price of the underlying asset passes below a barrier that is below the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves above the barrier again.

### Rebates

If a barrier option fails to exercise, the seller may pay a rebate to the buyer of the option. Knock-outs may pay a rebate when they are knocked out, and knock-ins may pay a rebate if they expire without ever knocking in.

The following functions support barrier options.

Function	Purpose
barrierbyfd	Price barrier option using finite difference method.



Function	Purpose
barriersensbyfd	Calculate barrier option price and sensitivities using finite difference method.
barrierbyls	Price European or American barrier options using Monte Carlo simulations.
barrierbybls	Price European barrier options using Black-Scholes option pricing model.
barrierbycrr	Price a barrier option from a Cox-Ross-Rubinstein binomial tree.
barrierbyeqp	Price a barrier option from an Equal Probabilities binomial tree.
barrierbyitt	Price a barrier option using an implied trinomial tree (ITT).
barrierbystt	Price a barrier options using a standard trinomial tree.

## Double Barrier Option

A double barrier option is similar to the standard single barrier option except that they have two barrier levels: a lower barrier (LB) and an upper barrier (UB). The payoff for a double barrier option depends on whether the underlying asset remains between the barrier levels during the life of the option. Double barrier options are less expensive than single barrier options as the probability of being knocked out is higher. Because of this, double barrier options allow investors to achieve reduction in the option premiums as and match an investor's belief about the future movement of the underlying price process.

There are two types of double barrier options:

- Double Knock-in

This option becomes effective when the price of the underlying asset reaches one of the barriers. It gives the option holder, the right but not the obligation to buy or sell the underlying security at the strike price, if the underlying asset goes above or below the barrier levels during the life of the option.

- Double Knock-out

This option gives the option holder, the right but not the obligation to buy or sell the underlying security at the strike price, as long as the underlying asset remains between the barrier levels during the life of the option. This option terminates when the price of the underlying asset passes one of the barriers.

The following functions support double barrier options.

Function	Purpose
<code>dblbarrierbybls</code>	Price European double barrier options using the Black-Scholes option pricing model.
<code>dblbarriersensbybls</code>	Calculate the price and sensitivities for a European double barrier options using the Black-Scholes option pricing model.
<code>dblbarrierbyfd</code>	Price double barrier option prices using the finite difference method.
<code>dblbarriersensbyfd</code>	Calculate the price and sensitivities for a double barrier option using the finite difference method.

## Vanilla Option

A vanilla option is a category of options that includes only the most standard components. A vanilla option has an expiration date and straightforward strike price. American-style options and European-style options are both categorized as vanilla options.

The payoff for a vanilla option is as follows:

- For a call:  $\max(St - K, 0)$
- For a put:  $\max(K - St, 0)$

where:

$St$  is the price of the underlying asset at time  $t$ .

$K$  is the strike price.

The following functions support specifying or pricing a vanilla option.

Function	Purpose
<code>optstockbyls</code>	Price European, Bermudan, or American vanilla options using the Longstaff-Schwartz model.
<code>optstocksensbyls</code>	Calculate European, Bermudan, or American vanilla option prices and sensitivities using the Longstaff-Schwartz model.

<b>Function</b>	<b>Purpose</b>
optstockbyfd	Calculate vanilla option prices using finite difference method.
optstocksensbyfd	Calculate vanilla option prices and sensitivities using finite difference method.
optstockbybaw	Calculate American options prices using Barone-Adesi and Whaley option pricing model.
optstocksensbybaw	Calculate American options prices and sensitivities using Barone-Adesi and Whaley option pricing model.
optstockbyrgw	Calculate American call option prices using Roll-Geske-Whaley option pricing model.
optstocksensbyrgw	Calculate American call option prices or sensitivities using Roll-Geske-Whaley option pricing model.
optByLocalVolFD	Calculate vanilla option price by local volatility model, using finite differences.
optstockbybjs	Price American options using Bjerksund-Stensland 2002 option pricing model.
optstocksensbybjs	Determine American option prices or sensitivities using Bjerksund-Stensland 2002 option pricing model.
optSensByLocalVolFD	Calculate vanilla option price or sensitivities by local volatility model, using finite differences.
optByHestonFD	Calculate vanilla option price by Heston model using finite differences.
optSensByHestonFD	Calculate vanilla option price and sensitivities by Heston model using finite differences.
optByBatesFD	Calculates vanilla European option price by Bates model using finite differences.
optSensByBatesFD	Calculates vanilla European option price and sensitivities by Bates model using finite differences.
optByMertonFD	Calculates vanilla European option price by Merton76 model using finite differences.
optBySensMertonFD	Calculates vanilla European option price and sensitivities by Merton76 model using finite differences.

Function	Purpose
optByBatesFFT	Calculate option price by Bates model using FFT and FRFT.
optByHestonFFT	Calculate option price by Heston model using FFT and FRFT.
optByMertonFFT	Calculate option price by Merton76 model using FFT and FRFT.
optstockbycrr	Price an option from a Cox-Ross-Rubinstein binomial tree.
optstockbyeqp	Price an option from an Equal Probabilities binomial tree.
optstockbyitt	Price an option using an implied trinomial tree (ITT).
optstockbystt	Price an option using a standard trinomial tree.

## Spread Option

A spread option is an option written on the difference of two underlying assets. For example, a European call on the difference of two assets  $X1$  and  $X2$  would have the following pay off at maturity:

$$\max(X1 - X2 - K, 0)$$

where:

$K$  is the strike price.

The following functions support spread options.

Function	Purpose
spreadbykirk	Price European spread options using the Kirk pricing model.
spreadsensbykirk	Calculate European spread option prices and sensitivities using the Kirk pricing model.
spreadbybjs	Price European spread options using the Bjerksund-Stensland pricing model.
spreadsensbybjs	Calculate European spread option prices and sensitivities using the Bjerksund-Stensland pricing model.

Function	Purpose
spreadbyfd	Price European or American spread options using the Alternate Direction Implicit (ADI) and Crank-Nicolson finite difference methods.
spreadsensbyfd	Calculate price and sensitivities of European or American spread options using the Alternate Direction Implicit (ADI) and Crank-Nicolson finite difference methods.
spreadbyls	Price European or American spread options using Monte Carlo simulations.
spreadsensbyls	Calculate price and sensitivities for European or American spread options using Monte Carlo simulations.

For more information on using spread options, see “Pricing European and American Spread Options” on page 3-57.

## Lookback Option

A lookback option is a path-dependent option based on the maximum or minimum value the underlying asset (e.g. electricity, stock) achieves during the entire life of the option. Basically the holder of the option can ‘look back’ over time to determine the payoff. This type of option provides price protection over a selected period, reduces uncertainties with the timing of market entry, moderates the need for the ongoing management, and therefore, is usually more expensive than vanilla options.

Lookback call options give the holder the right to buy the underlying asset at the lowest price. Lookback put options give the right to sell the underlying asset at the highest price.

Financial Instruments Toolbox software supports two types of lookback options: fixed and floating. The difference is related to how the strike price is set in the contract. Fixed lookback options have a specified strike price and the option pays out the maximum of the difference between the highest (lowest) observed price of the underlying during the life of the option and the strike. Floating lookback options have a strike price determined at maturity, and it is set at the lowest (highest) price of the underlying reached during the life of the option. This means that for a floating strike lookback call (put), the holder has the right to buy (sell) the underlying asset at its lowest (highest) price observed during the life of the option. So, there are a total of four lookback option types, each with its own characteristic payoff formula:

- Fixed call:  $\max(0, S_{\max} - X)$
- Fixed put:  $\max(0, X - S_{\min})$
- Floating call:  $\max(0, S - S_{\min})$
- Floating put:  $\max(0, S_{\max} - S)$

where:

$S_{\max}$  is the maximum price of underlying asset.

$S_{\min}$  is the minimum price of underlying asset.

$S$  is the price of the underlying asset at maturity.

$X$  is the strike price.

The following functions support lookback options.

Function	Purpose
lookbackbycvgs	Calculate prices of European lookback fixed and floating strike options using the Conze-Viswanathan and Goldman-Sosin-Gatto models.
lookbacksensbycvgs	Calculate prices and sensitivities of European fixed and floating strike lookback options using the Conze-Viswanathan and Goldman-Sosin-Gatto models.
lookbackbyls	Calculate prices of lookback fixed and floating strike options using the Longstaff-Schwartz model.
lookbacksensbyls	Calculate prices and sensitivities of lookback fixed and floating strike options using the Longstaff-Schwartz model.
lookbackbycrr	Price a lookback option from a Cox-Ross-Rubinstein binomial tree.
lookbackbyeqp	Price a lookback option from an Equal Probabilities binomial tree.
lookbackbyitt	Price a lookback option using an implied trinomial tree (ITT).
lookbackbystt	Price a lookback option using a standard trinomial tree.

Lookback options and Asian options are instruments used in the electricity market to manage purchase timing risk. Electricity purchasers cover part of their expected electricity consumption on the forward market to avoid the volatility and limited liquidity of the spot market. Using Asian options as a hedging tool is a passive approach to solving the purchase timing problem. An Asian option instrument diminishes the wrong timing risk but it also reduces any potential benefit to the buyer from falling prices. On the other hand, lookback options allow the purchasers to buy electricity at the lowest price, but as mentioned before, this instrument is more expensive than Asian and vanilla options.

## Forwards Option

A forward option is a non-standardized contract between two parties to buy or to sell an asset at a specified future time at a price agreed upon today. The buyer of a forward option contract has the right to hold a particular forward position at a specific price any time before the option expires. The forward option seller holds the opposite forward position when the buyer exercises the option. A call option is the right to enter into a long forward position and a put option is the right to enter into a short forward position. A closely related contract is a futures contract. A forward is like a futures in that it specifies the exchange of goods for a specified price at a specified future date. The following table displays some of the characteristics of forward and futures contracts.

Forwards	Futures
Customized contracts	Standardized contracts
Over the counter traded	Exchange traded
Exposed to default risk	Clearing house reduces default risk
Mostly used for hedging	Mostly used by hedgers and speculators
Settlement at the end of contract (no Margin required)	Daily changes are settled day by day (Margin required)
Delivery usually takes place	Delivery usually never happens

The payoff for a forward option, where the value of a forward position at maturity depends on the relationship between the delivery price ( $K$ ) and the underlying price ( $S_T$ ) at that time, is:

- For a long position:  $f_T = S_T - K$
- For a short position:  $f_T = K - S_T$

The following functions support pricing a forwards option.

Function	Purpose
optstockbyblk	Price options on forwards using the Black option pricing model.
optstocksensbyblk	Determine option prices and sensitivities on forwards using the Black pricing model.

## Futures Option

A future option is a standardized contract between two parties to buy or sell a specified asset of standardized quantity and quality for a price agreed upon today (the futures price) with delivery and payment occurring at a specified future date, the delivery date. The contracts are negotiated at a futures exchange, which acts as an intermediary between the two parties. The party agreeing to buy the underlying asset in the future, the "buyer" of the contract, is said to be "long", and the party agreeing to sell the asset in the future, the "seller" of the contract, is said to be "short."

Forwards	Futures
Customized contracts	Standardized contracts
Over the counter traded	Exchange traded
Exposed to default risk	Clearing house reduces default risk
Mostly used for hedging	Mostly used by hedgers and speculators
Settlement at the end of contract (no Margin required)	Daily changes are settled day by day (Margin required)
Delivery usually takes place	Delivery usually never happens

A futures contract is the delivery of item  $J$  at time  $T$  and:

- There exists in the market a quoted price  $F(t, T)$ , which is known as the futures price at time  $t$  for delivery of  $J$  at time  $T$ .
- The price of entering a futures contract is equal to zero.
- During any time interval  $[t, s]$ , the holder receives the amount  $F(s, T) - F(t, T)$  (this reflects instantaneous marking to market).
- At time  $T$ , the holder pays  $F(T, T)$  and is entitled to receive  $J$ . Note that  $F(T, T)$  should be the spot price of  $J$  at time  $T$ .

The following functions support pricing a futures option.



Function	Purpose
optstockbyblk	Price options on futures using the Black option pricing model.
optstocksensbyblk	Determine option prices and sensitivities on futures using the Black pricing model.

## See Also

asianbykv | asianbylevy | asianbyls | asiainsensbykv | asiainsensbylevy | asiainsensbyls | lookbackbycvsg | lookbackbyls | lookbacksensbycvsg | lookbacksensbyls | optpricebysim | optstockbyblk | optstockbyls | optstocksensbyblk | optstocksensbyls | spreadbybjs | spreadbyfd | spreadbykirk | spreadbyls | spreadsensbybjs | spreadsensbyfd | spreadsensbykirk | spreadsensbyls

## Related Examples

- “Pricing European and American Spread Options” on page 3-57
- “Hedging Strategies Using Spread Options” on page 3-76
- “Pricing Swing Options using the Longstaff-Schwartz Method” on page 3-84
- “Compute Option Prices on a Forward” on page 11-1814
- “Compute Forward Option Prices and Delta Sensitivities” on page 11-1883
- “Compute the Option Price on a Future” on page 11-1816
- “Simulating Electricity Prices with Mean-Reversion and Jump-Diffusion” on page 3-98
- “Pricing Asian Options” on page 3-113

## More About

- “Forwards Option” on page 3-53
- “Futures Option” on page 3-54
- “Spread Option” on page 3-50
- “Asian Option” on page 3-44
- “Vanilla Option” on page 3-48

- “Lookback Option” on page 3-51
- “Supported Equity Derivatives” on page 3-24
- “Supported Interest-Rate Instruments” on page 2-2

### **External Websites**

- Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## Pricing European and American Spread Options

This example shows how to price and calculate sensitivities for European and American spread options using various techniques. First, the price and sensitivities for a European spread option is calculated using closed form solutions. Then, price and sensitivities for an American spread option is calculated using finite difference and Monte Carlo simulations. Finally, further analysis is conducted on spread options with a different range of inputs.

*Spread options* are options on the difference of two underlying asset prices. For example, a call option on the spread between two assets has the following payoff at maturity:

$$\max(X_1 - X_2 - K, 0)$$

where  $X_1$  is the price of the first underlying asset,  $X_2$  is the price of the second underlying asset, and  $K$  is the strike price. At maturity, if the spread  $X_1 - X_2$  is greater than the strike price  $K$ , the option holder exercises the option and gains the difference between the spread and the strike price. If the spread is less than 0, the option holder does not exercise the option, and the payoff is 0. Spread options are frequently traded in the energy market. Two examples are:

- *Crack spreads*: Options on the spread between refined petroleum products and crude oil. The spread represents the refinement margin made by the oil refinery by "cracking" the crude oil into a refined petroleum product.
- *Spark spreads*: Options on the spread between electricity and some type of fuel. The spread represents the margin of the power plant, which takes fuel to run its generator to produce electricity.

### Overview of the Pricing Methods

There are several methods to price spread options, as discussed in [1]. This example uses the closed form, finite difference, and Monte Carlo simulations to price spread options. The advantages and disadvantages of each method are discussed below:

- Closed form solutions and approximations of partial differential equations (PDE) are advantageous because they are very fast, and extend well to computing sensitivities (Greeks). However, closed form solutions are not always available, for example for American spread options.
- The finite difference method is a numerical procedure to solve PDEs by discretizing the price and time variables into a grid. A detailed analysis of this method can be

found in [2]. It can handle cases where closed form solutions are not available. Also, finite difference extends well to calculating sensitivities because it outputs a grid of option prices for a range of underlying prices and times. However, it is slower than the closed form solutions.

- Monte Carlo simulation uses random sampling to simulate movements of the underlying asset prices. It handles cases where closed solutions do not exist. However, it usually takes a long time to run, especially if sensitivities are calculated.

#### Pricing a European Spread Option

The following example demonstrates the pricing of a crack spread option.

A refiner is concerned about its upcoming maintenance schedule and needs to protect against decreasing crude oil prices and increasing heating oil prices. During the maintenance the refiner needs to continue providing customers with heating oil to meet their demands. The refiner's strategy is to use spread options to manage its hedge.

On January 2013, the refiner buys a 1:1 crack spread option by purchasing heating oil futures and selling crude oil futures. CLF14 WTI crude oil futures is at \$100 per barrel and HOF14 heating oil futures contract is at \$2.6190 per gallon.

```
clear;
```

```
% Price, volatility, and dividend of heating oil
```

```
Price1gallon = 2.6190;      % $/gallon
```

```
Price1 = Price1gallon*42;   % $/barrel
```

```
Vol1 = 0.10;
```

```
Div1 = 0.03;
```

```
% Price, volatility, and dividend of WTI crude oil
```

```
Price2 = 100;              % $/barrel
```

```
Vol2 = 0.15;
```

```
Div2 = 0.02;
```

```
% Correlation of underlying prices
```

```
Corr = 0.3;
```

```
% Option type
```

```
OptSpec = 'call';
```

```
% Strike
```

```
Strike = 5;
```

```

% Settlement date
Settle = '01-Jan-2013';

% Maturity
Maturity = '01-Jan-2014';

% Risk free rate
RiskFreeRate = 0.05;

```

The pricing functions take an interest-rate term structure and stock structure as inputs. Also, you need to specify which outputs are of interest.

```

% Define RateSpec
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', RiskFreeRate, 'Compounding', ...
    Compounding, 'Basis', Basis);

% Define StockSpec for the two assets
StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);
StockSpec2 = stockspec(Vol2, Price2, 'Continuous', Div2);

% Specify price and sensitivity outputs
OutSpec = {'Price', 'Delta', 'Gamma'};

```

The Financial Instruments Toolbox™ contains two types of closed form approximations for calculating price and sensitivities of European spread options: the Kirk's approximation (`spreadbykirk`, `spreadsensbykirk`) and the Bjerksund and Stensland model (`spreadbybjs`, `spreadsensbybjs`) [3].

The function `spreadsensbykirk` calculates prices and sensitivities for a European spread option using the Kirk's approximation.

```

% Kirk's approximation
[PriceKirk, DeltaKirk, GammaKirk] = ...
    spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, ...
    Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)

PriceKirk = 8.3636

DeltaKirk = 1x2

    0.6108    -0.5590

```

```
GammaKirk = 1x2  
    0.0225    0.0249
```

The function `spreadsensbybjs` calculates the prices and sensitivities for a European spread option using the Bjerksund and Stensland model. In [3], Bjerksund and Stensland explains that the Kirk's approximation tends to underprice the spread option when the strike is close to zero, and overprice when the strike is further away from zero. In comparison, the model by Bjerksund and Stensland has higher precision.

```
% Bjerksund and Stensland model  
[PriceBJS, DeltaBJS, GammaBJS] = ...  
    spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...  
    Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)  
  
PriceBJS = 8.3662  
  
DeltaBJS = 1x2  
    0.6115    -0.5597  
  
GammaBJS = 1x2  
    0.0225    0.0248
```

A comparison of the calculated prices show that the two closed form models produce similar results for price and sensitivities. In addition to delta and gamma, the functions can also calculate theta, vega, lambda, and rho.

```
displayComparison('Kirk', 'BJS', PriceKirk, PriceBJS, DeltaKirk, DeltaBJS, GammaKirk, C  
  
Comparison of prices:  
  
Kirk:    8.363641  
BJS :    8.366158  
  
Comparison of delta:  
  
Kirk:    0.610790    -0.558959  
BJS :    0.611469    -0.559670  
  
Comparison of gamma:
```

```
Kirk:    0.022533      0.024850
BJS :    0.022495      0.024819
```

### Pricing an American Spread Option

Although the closed form approximations are fast and well suited for pricing European spread options, they cannot price American spread options. Using the finite difference method and the Monte Carlo method, an American spread option can be priced. In this example, an American spread option is priced with the same attributes as the above crack spread option.

The finite difference method numerically solves a PDE by discretizing the underlying price and time variables into a grid. The Financial Instrument Toolbox™ contains the functions `spreadbyfd` and `spreadsensbyfd`, which calculate prices and sensitivities for European and American spread options using the finite difference method. For the finite difference method, the composition of the grid has a large impact on the quality of the output and the execution time. Generally, a finely discretized grid will result in outputs that are closer to the theoretical value, but it comes at the cost of longer execution times. The composition of the grid is controlled using optional parameters `PriceGridSize`, `TimeGridSize`, `AssetPriceMin` and `AssetPriceMax`.

To indicate pricing an American option, add an optional input of `AmericanOpt` with a value of 1 to the argument of the function.

```
% Finite difference method for American spread option
[PriceFD, DeltaFD, GammaFD, PriceGrid, AssetPrice1, ...
 AssetPrice2] = ...
 spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
 Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec, ...
 'PriceGridSize', [500 500], 'TimeGridSize', 100, ...
 'AssetPriceMin', [0 0], 'AssetPriceMax', [2000 2000], ...
 'AmericanOpt', 1);

% Display price and sensitivities
PriceFD

PriceFD = 8.5463

DeltaFD

DeltaFD = 1x2
```

```
0.6306 -0.5777
```

```
GammaFD
```

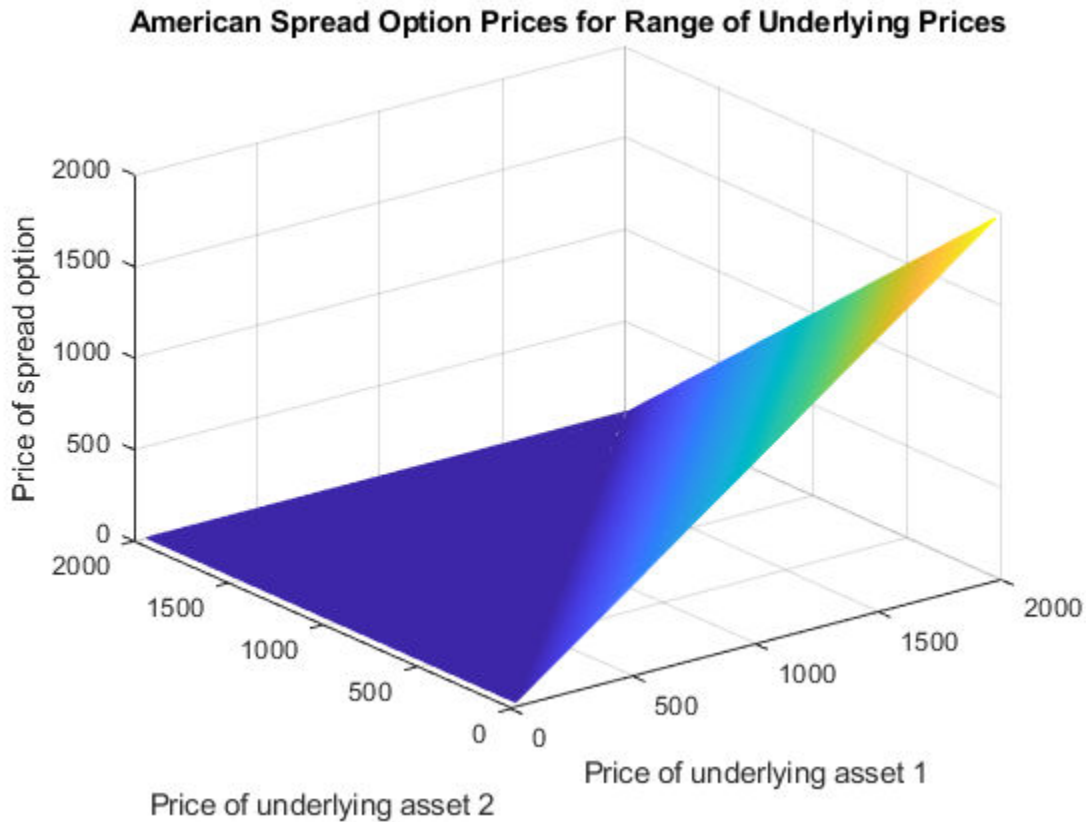
```
GammaFD = 1x2
```

```
0.0233 0.0259
```

The function `spreadsensbyfd` also returns a grid that contains the option prices for a range of underlying prices and times. The grid of option prices at time zero, which is the option prices at the settle date, can be plotted for a range of underlying prices.

```
% Plot option prices
figure;
mesh(AssetPrice1, AssetPrice2, PriceGrid(:, :, 1)');
title('American Spread Option Prices for Range of Underlying Prices');
xlabel('Price of underlying asset 1');
ylabel('Price of underlying asset 2');
zlabel('Price of spread option');
```





An American style option can be priced by Monte Carlo methods using the least square method of Longstaff and Schwartz [4]. The Financial Instruments Toolbox™ contains the functions `spreadbyls` and `spreadsensbyls`, that calculate prices and sensitivities of European and American options using simulations. The Monte Carlo simulation method in `spreadsensbyls` generates multiple paths of simulations according to a geometric Brownian motion (GBM) for the two underlying asset prices. Similar to the finite difference method where the granularity of the grid determined the quality of the output and the execution time, the quality of output and execution time of the Monte Carlo simulation depends on the number of paths (`NumTrials`) and the number of time periods per path (`NumPeriods`). Also, the results obtained by Monte Carlo simulations are not deterministic. Each run will have different results depending on the simulation outcomes.

```
% To indicate that we are pricing an American option using the Longstaff  
% and Schwartz method, add an optional input of |AmericanOpt| with a value  
% of |1| to the argument of the function.
```

```
% Monte Carlo method for American spread option  
[PriceMC, DeltaMC, GammaMC] = ...  
    spreadsensbyls(RateSpec, StockSpec1, StockSpec2, Settle, ...  
    Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec, ...  
    'NumTrials', 1000, 'Antithetic', true, 'AmericanOpt', 1)
```

```
PriceMC = 8.4999
```

```
DeltaMC = 1x2
```

```
    0.6325    -0.5931
```

```
GammaMC = 1x2
```

```
   -0.0873    0.0391
```

The results of the two models are compared. The prices and sensitivities calculated by the Longstaff and Schwartz method will vary at each run, depending on the outcome of the simulations. It is important to note that the quality of the results from the finite difference method and the Monte Carlo simulation depend on the optional input parameters. For example, increasing the number of paths (NumTrials) for the spreadsensbyls function will result in more precise results at the cost of longer execution times.

```
displayComparison('Finite Difference', 'Monte Carlo', PriceFD, PriceMC, DeltaFD, DeltaMC)
```

```
Comparison of prices:
```

```
Finite Difference:    8.546285  
Monte Carlo          :    8.499894
```

```
Comparison of delta:
```

```
Finite Difference:    0.630606    -0.577686  
Monte Carlo          :    0.632549    -0.593106
```

```
Comparison of gamma:
```

```
Finite Difference:    0.023273    0.025852  
Monte Carlo          :   -0.087340    0.039120
```

## Comparing Results for a Range of Strike Prices

As discussed earlier, the Kirk's approximation tends to overprice spread options when the strike is further away from zero. To confirm this, a spread option is priced with the same attributes as before, but for a range of strike prices.

```
% Specify outputs
OutSpec = {'Price', 'Delta'};
```

```
% Range of strike prices
Strike = [-25; -15; -5; 0; 5; 15; 25];
```

The results from the Kirk's approximation and the Bjerksund and Stensland model are compared against the numerical approximation from the finite difference method. Since `spreadsensbyfd` can only price one option at a time, it is called in a loop for each strike value. The Monte Carlo simulation (`spreadsensbyls`) with a large number of trial paths can also be used as a benchmark, but the finite difference is used for this example.

```
% Kirk's approximation
[PriceKirk, DeltaKirk] = ...
    spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, ...
        Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);
```

```
% Bjerksund and Stensland model
[PriceBJS, DeltaBJS] = ...
    spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
        Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);
```

```
% Finite difference
PriceFD = zeros(numel(Strike), 1);
DeltaFD = zeros(numel(Strike), 2);
for i = 1:numel(Strike)
    [PriceFD(i), DeltaFD(i,:)] = ...
        spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
            Maturity, OptSpec, Strike(i), Corr, 'OutSpec', OutSpec, ...
            'PriceGridSize', [500 500], 'TimeGridSize', 100, ...
            'AssetPriceMin', [0 0], 'AssetPriceMax', [2000 2000]);
end
```

```
displayComparisonPrices(PriceKirk, PriceBJS, PriceFD, Strike)
```

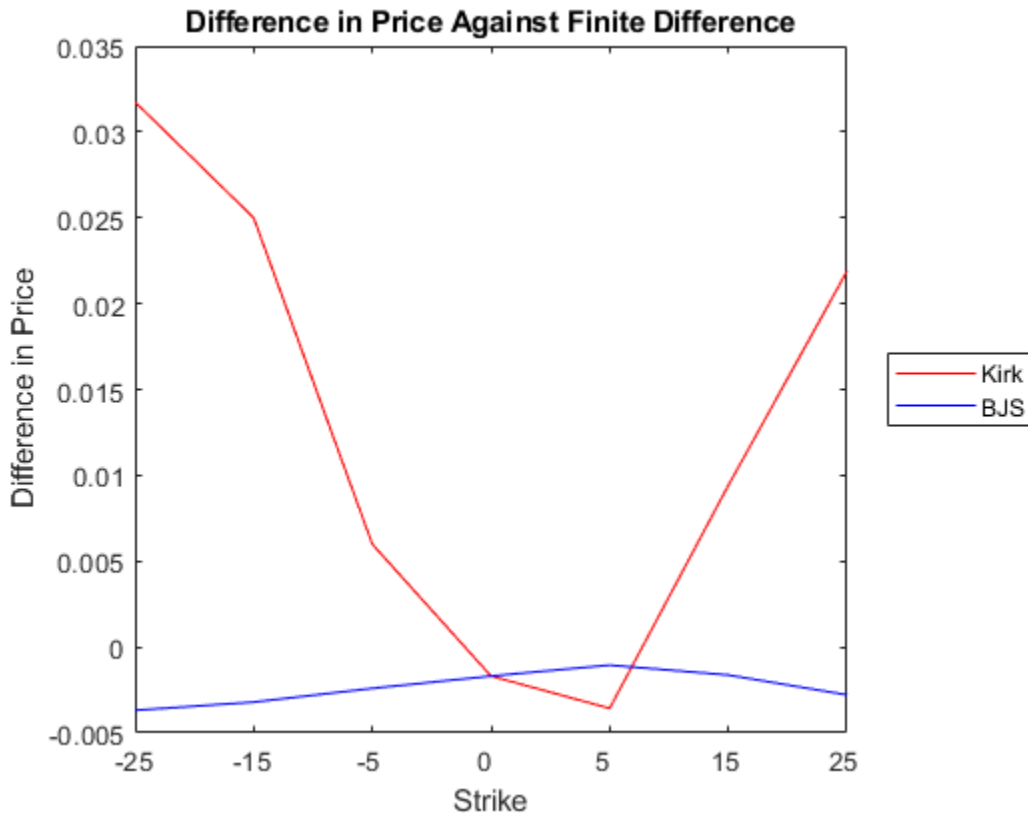
```
Prices for range of strikes:
```

```
Kirk          BJS          FD
```

32.707787	32.672353	32.676040
23.605307	23.577099	23.580307
15.236908	15.228510	15.230919
11.560332	11.560332	11.562023
8.363641	8.366158	8.367212
3.689909	3.678862	3.680493
1.243753	1.219079	1.221866

The difference in prices between the closed form and finite difference method is plotted below. It is clear that as the strike moves further away from 0, the difference between the Kirk's approximation and finite difference (red line) increases, while the difference between the Bjerksund and Stensland model and finite difference (blue line) stays at the same level. As stated in [3], the Kirk's approximation is overpricing the spread option when the strike is far away from 0.

```
% Plot of difference in price against the benchmark
figure;
plot(PriceKirk-PriceFD, 'Color', 'red');
hold on;
plot(PriceBJS-PriceFD, 'Color', 'blue');
hold off;
title('Difference in Price Against Finite Difference');
legend('Kirk', 'BJS', 'Location', 'EastOutside');
xlabel('Strike');
ax = gca;
ax.XTickLabel = Strike;
ylabel('Difference in Price');
```

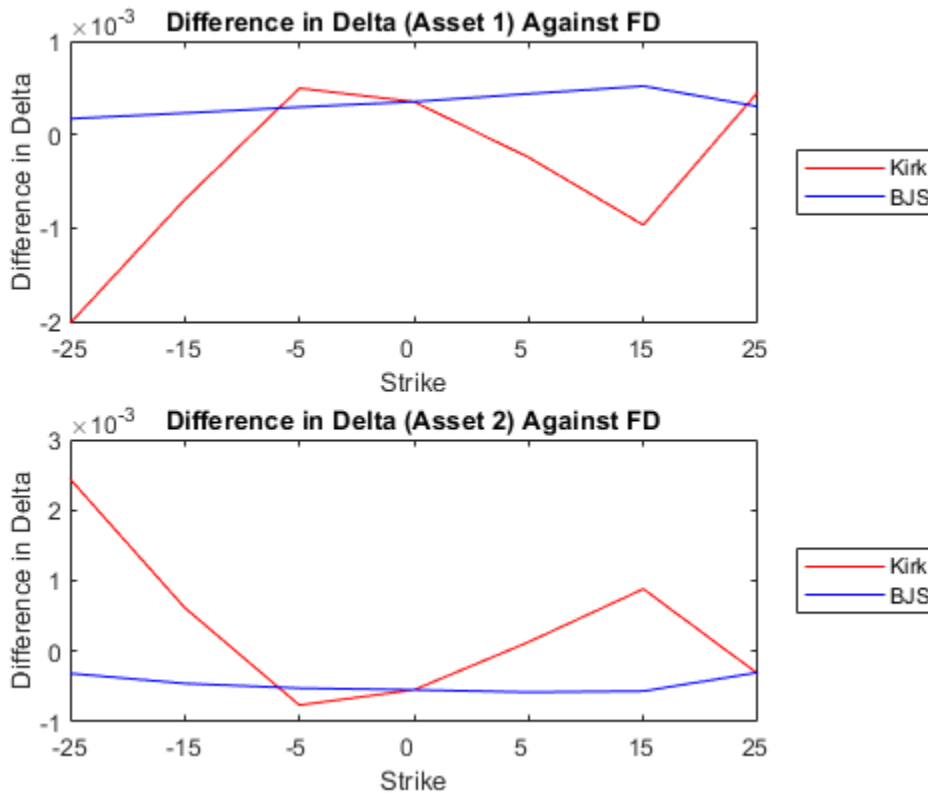


Next, the difference in delta between the closed form models and finite difference is plotted. The top plot shows the difference in delta for the first asset, and the bottom plot shows the difference in delta for the second asset. As seen from the small increments in the y-axis of the order  $10e-3$ , it can be seen that all three models (Kirk, BJS, finite difference) produce similar values for delta.

```
% Plot of difference in delta of first asset against the benchmark
figure;
subplot(2, 1, 1);
plot(DeltaKirk(:,1)-DeltaFD(:,1), 'Color', 'red');
hold on;
plot(DeltaBJS(:,1)-DeltaFD(:,1), 'Color', 'blue');
hold off;
title('Difference in Delta (Asset 1) Against FD');
```

```
legend('Kirk', 'BJS', 'Location', 'EastOutside');
xlabel('Strike');
ax = gca;
ax.XTickLabel = Strike;
ylabel('Difference in Delta');

% Plot of difference in delta of second asset against the benchmark
subplot(2, 1, 2);
plot(DeltaKirk(:,2)-DeltaFD(:,2), 'Color', 'red');
hold on;
plot(DeltaBJS(:,2)-DeltaFD(:,2), 'Color', 'blue');
hold off;
title('Difference in Delta (Asset 2) Against FD');
legend('Kirk', 'BJS', 'Location', 'EastOutside');
xlabel('Strike');
ax = gca;
ax.XTickLabel = Strike;
ylabel('Difference in Delta');
```



### Analyzing Prices and Vega at Different Levels of Volatility

To further show the type of analysis that can be conducted using these models, the above spread option is priced at different levels of volatility for the first asset. The price and vega are compared at three levels of volatility for the first asset: 0.1, 0.3, and 0.5. The Bjerksund and Stensland model is used for this analysis.

```
% Strike
Strike = 5;

% Specify output
OutSpec = {'Price', 'Vega'};

% Different levels of volatility for asset 1
```

```
Vol1 = [0.1, 0.3, 0.5];

StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);

% Bjerksund and Stensland model
[PriceBJS, VegaBJS] = ...
    spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
        Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);

displaySummary(Vol1, PriceBJS, VegaBJS)

Prices for different vol levels in asset 1:

8.366158
14.209112
21.795746

Asset 1 vega for different vol levels in asset 1:

15.534849
36.212192
38.794348

Asset 2 vega for different vol levels in asset 1:

29.437036
7.133657
-0.557852
```

The change in the price and vega with respect to the volatility of the first asset is plotted below. You can observe that as the volatility of the first asset increases, the price of the spread option also increases. Also, the changes in vega indicate that the price of the spread option becomes more sensitive to the volatility of the first asset and less sensitive to the volatility of the second asset.

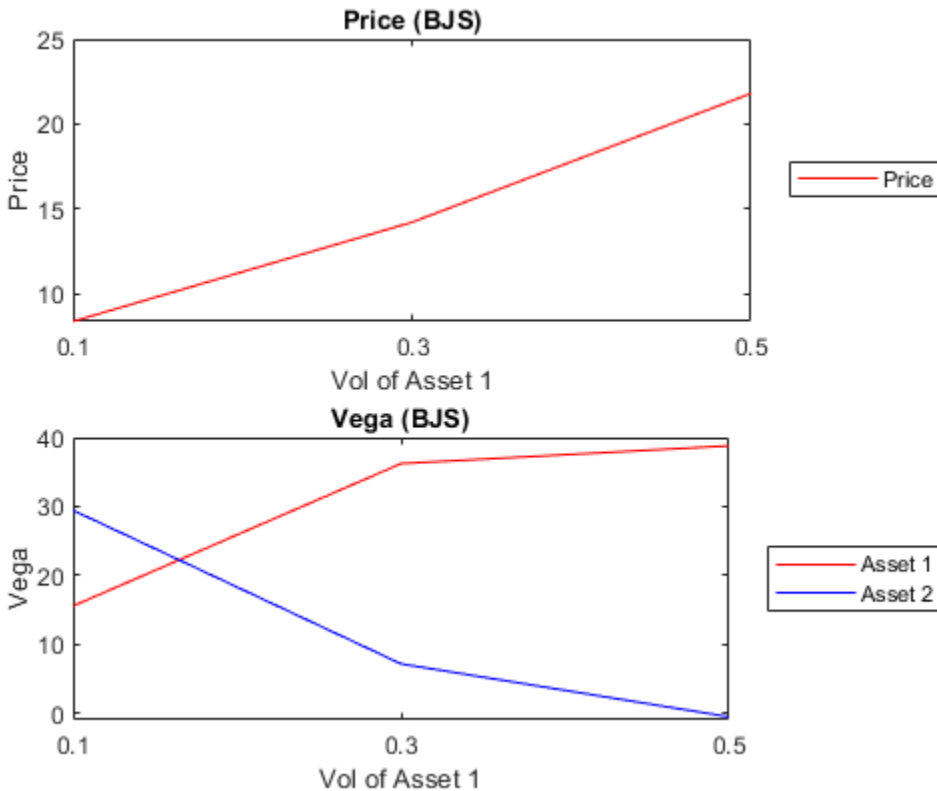
```
figure;

% Plot price for BJS model
subplot(2, 1, 1);
plot(PriceBJS, 'Color', 'red');
title('Price (BJS)');
legend('Price', 'Location', 'EastOutside');
xlabel('Vol of Asset 1');
ax = gca;
ax.XTick = 1:3;
```



```
ax.XTickLabel = Vol1;
ylabel('Price');

% Plot of vega for BJS model
subplot(2, 1, 2);
plot(VegaBJS(:,1), 'Color', 'red');
hold on;
plot(VegaBJS(:,2), 'Color', 'blue');
hold off;
title('Vega (BJS)');
legend('Asset 1', 'Asset 2', 'Location', 'EastOutside');
xlabel('Vol of Asset 1');
ax = gca;
ax.XTick = 1:3;
ax.XTickLabel = Vol1;
ax.YLim = [-1 40];
ylabel('Vega');
```



### Summary

In this example, European and American spread options are priced and analyzed using various techniques. The Financial Instruments Toolbox™ provides functions for two types of closed form solutions (Kirk, BJS), the finite difference method, and the Monte Carlo simulation method. The closed form solutions are well suited for pricing and sensitivity calculation of European spread options because they are fast. However, they cannot price American spread options. The finite difference method and Monte Carlo method can price both European and American options. However, they are not as fast in pricing European spread options as compared to closed form solutions.

## References

- [1] Carmona, Rene, Durrleman, Valdo. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, 2003, pp. 627-685.
- [2] Wilmott, Paul, Dewynne, Jeff, Howison, Sam. *Option Pricing*. Oxford Financial Press, 1993.
- [3] Bjerksund, Petter, Stensland, Gunnar. "Closed form spread option valuation." Department of Finance, NHH, 2006.
- [4] Longstaff, Francis A, Schwartz, Eduardo S. "Valuing American Options by Simulation: A Simple Least-Squares Approach." Anderson Graduate School of Management, UC Los Angeles, 2001.

## Utility Functions

```
function displayComparison(model1, model2, price1, price2, delta1, delta2, gamma1, gamma2)
% Pad the model name with additional spaces
additionalSpaces = numel(model1) - numel(model2);
if additionalSpaces > 0
    model2 = [model2 repmat(' ', 1, additionalSpaces)];
else
    model1 = [model1 repmat(' ', 1, abs(additionalSpaces))];
end

% Comparison of calculated prices
fprintf('Comparison of prices:\n');
fprintf('\n');
fprintf('%s: % f\n', model1, price1);
fprintf('%s: % f\n', model2, price2);
fprintf('\n');

% Comparison of Delta
fprintf('Comparison of delta:\n');
fprintf('\n');
fprintf('%s: % f % f\n', model1, delta1(1), delta1(2));
fprintf('%s: % f % f\n', model2, delta2(1), delta2(2));
fprintf('\n');

% Comparison of Gamma
fprintf('Comparison of gamma:\n');
fprintf('\n');
fprintf('%s: % f % f\n', model1, gamma1(1), gamma1(2));
```

```
fprintf('%s: % f % f\n', model2, gamma2(1), gamma2(2));
fprintf('\n');
end

function displayComparisonPrices(PriceKirk, PriceBJS, PriceFD, Strike)
% Comparison of calculated prices
fprintf('Prices for range of strikes:\n');
fprintf('\n');
fprintf('Kirk \tBJS \tFD \n');
for i = 1:numel(Strike)
    fprintf('%f\t%f\t%f\n', PriceKirk(i), PriceBJS(i), PriceFD(i));
end
end

function displaySummary(Vol1, PriceBJS, VegaBJS)
% Display price
fprintf('Prices for different vol levels in asset 1:\n');
fprintf('\n');
for i = 1:numel(Vol1)
    fprintf('%f\n', PriceBJS(i));
end
fprintf('\n');

% Display vega for first asset
fprintf('Asset 1 vega for different vol levels in asset 1:\n');
fprintf('\n');
for i = 1:numel(Vol1)
    fprintf('%f\n', VegaBJS(i,1));
end
fprintf('\n');

% Display vega for second asset
fprintf('Asset 2 vega for different vol levels in asset 1:\n');
fprintf('\n');
for i = 1:numel(Vol1)
    fprintf('%f\n', VegaBJS(i,2));
end
end
```

## See Also

asianbykv | asianbylevy | asianbyls | asiainsensbykv | asiainsensbylevy | asiainsensbyls | lookbackbycvgsg | lookbackbyls | lookbacksensbycvgsg |

---

lookbacksensbyls | optpricebysim | optstockbyblk | optstockbyls |  
optstocksensbyblk | optstocksensbyls | spreadbybjs | spreadbyfd |  
spreadbykirk | spreadbyls | spreadsensbybjs | spreadsensbyfd |  
spreadsensbykirk | spreadsensbyls

## Related Examples

- “Hedging Strategies Using Spread Options” on page 3-76
- “Pricing Swing Options using the Longstaff-Schwartz Method” on page 3-84
- “Compute Option Prices on a Forward” on page 11-1814
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- “Simulating Electricity Prices with Mean-Reversion and Jump-Diffusion” on page 3-98
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- “Asian Option” on page 3-44
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- “Supported Equity Derivatives” on page 3-24
- “Supported Interest-Rate Instruments” on page 2-2

## External Websites

- Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## Hedging Strategies Using Spread Options

This example shows different hedging strategies to minimize exposure in the Energy market using Crack Spread Options.

### Understanding Crack Spread Options

In the petroleum industry, refiners are concerned about the difference between their input costs (crude oil) and output prices (refined products - gasoline, heating oil, diesel fuel, and so on). The differential between these two underlying commodities is referred to as a *Crack Spread*. It represents the profit margin between crude oil and the refined products.

A *Spread option* is an option on the spread where the holder has the right, but not the obligation, to enter into a spot or forward spread contract. Crack Spread Options are often used to protect against declines in the crack spread or to monetize volatility or price expectations on the spread.

### Example 1: Protecting Margins using a 1:1 Crack Spread Option

A marketer is interested in protecting his gasoline margin since current prices are strong. A crack spread option strategy is used to maintain profits for the following season. In March the June WTI crude oil futures are at \$91.10 per barrel and RBOB gasoline futures contract are at \$2.72 per gallon. The marketer's strategy is a long crack call involving purchasing RBOB gasoline futures and selling crude oil futures.

```
OldFormat = get(0, 'format');
format bank

% Price and volatility of RBOB gasoline
Price1gallon = 2.72;           % $/gallon
Price1 = Price1gallon * 42;    % $/barrel
Vol1 = 0.39;

% Price and volatility of WTI crude oil
Price2 = 91.10;               % $/barrel
Vol2 = 0.34;

% Assume the following data
% Spread Option
Strike = 20;
OptSpec = 'call';
Settle = '01-March-2013';
```

```
Maturity = '01-June-2013';
Corr = 0.45;      % Correlation of underlying commodities
```

Define the RateSpec and StockSpec.

```
% Define RateSpec
Rate = 0.035;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rate, 'Compounding', ...
    Compounding, 'Basis', Basis);

% Define StockSpec for the two assets
StockSpec1 = stockspec(Vol1, Price1);
StockSpec2 = stockspec(Vol2, Price2);
```

Price the Crack Spread Option

Use the function `spreadbybjs` in the Financial Instruments Toolbox™ to price the spread option using the Bjerksund and Stensland model.

```
Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
    Maturity, OptSpec, Strike, Corr)
```

```
Price =
    9.91
```

The 1:1 implied current crack spread between these two underlyings is \$23.14 per barrel.

```
CrackSpread = Price1 - Price2      % $/barrel
```

```
CrackSpread =
    23.14
```

Suppose that by expiration day, June crude oil prices decrease to \$90.34 per barrel and gasoline prices rise to \$2.89 per gallon. The price changes cause the marketer's profit margin (the new implied crack spread) to increase from \$23.14/barrel to \$31.04/barrel:

```
NewCrackSpread = (2.89 * 42) - 90.34
```

```
NewCrackSpread =  
    31.04
```

Since the marketer purchased a long crack call on the \$20 call, the option is now in the money by \$11.04.

```
(NewCrackSpread - Strike)
```

```
ans =  
    11.04
```

The marketer paid \$9.91 from the long crack call, this protects the margin by \$1.13.

```
(NewCrackSpread - Strike - Price)
```

```
ans =  
    1.13
```

This strategy provides the marketer protection during spread increase scenarios.

#### **Example 2: Creating a Floor with Crack Spread Options**

A refiner is interested in covering its fixed and operating costs, but still profit from a favorable move in the market. In March the May WTI crude oil futures are at \$99.43 per barrel and RBOB gasoline futures contract are at \$3.04 per gallon. The refiner believes that the spread between those commodities of \$28.25 per barrel is favorable. Of this, \$11 corresponds to operating and fixed costs, and \$17.25 is the net refining margin. The refiner's strategy is to sell the crack spread by selling 10 RBOB gasoline futures and buying 10 crude oil futures.

```
% Price and volatility of RBOB gasoline  
Pricelgallon = 3.04;           % $/gallon  
Pricel = Pricelgallon * 42;   % $/barrel  
Vol1 = 0.35;  
Div1 = 0.0783;
```

```
% Price and volatility of WTI crude oil  
Price2 = 99.43;             % $/barrel  
Vol2 = 0.38;  
Div2 = 0.0571;
```

The refiner purchases 10 May RBOB gasoline crack spread puts with a strike price of \$25.



```

% Spread Option
Strike = 25;
OptSpec = 'put';
Settle = '01-March-2013';
Maturity = '01-May-2013';
Corr = 0.30;      % Correlation of underlying commodities

```

Define the RateSpec and StockSpec.

```

% Define RateSpec
Rate = 0.035;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rate, 'Compounding', ...
    Compounding, 'Basis', Basis);

```

% Define StockSpec for the two assets

```

StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);
StockSpec2 = stockspec(Vol2, Price2, 'Continuous', Div2);

```

Price the Crack Spread Option

Use the function `spreadbyfd` in the Financial Instruments Toolbox™ to price the American spread option using the finite difference method.

```

Price = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
    Maturity, OptSpec, Strike, Corr, 'AmericanOpt', 1)

```

```

Price =
    6.61

```

By expiration, if the option is exercised, the refiner would have hedged the cost of purchasing 10000 barrels of crude oil with the revenue of selling 10000 barrels of RBOB gasoline. The futures contract represents 1000 barrels of crude oil and 42000 gallons of gasoline.

```

CostOfHedge = Price * 10000 % Option premium

```

```

CostOfHedge =
    66122.24

```

The hedge cost is \$66386 to implement and guarantee that neither a fall in RBOB gasoline prices or an increase in WTI crude oil prices will diminish the refining margin below \$25.

```
ProfitMargin = 14 * 10000    %$
```

```
ProfitMargin =  
    140000.00
```

```
CrackingMargin = ProfitMargin - CostOfHedge
```

```
CrackingMargin =  
    73877.76
```

This strategy allows a cracking margin of \$73613.

Another strategy for the refiner could be to buy the \$22 puts at a price of \$5.38.

```
StrikeNew = 22;
```

```
PriceNew = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...  
                    Maturity, OptSpec, StrikeNew, Corr, 'AmericanOpt', 1)
```

```
PriceNew =  
    5.36
```

This time the hedge would have cost \$53823, but it also guarantees a \$11 per barrel or a \$56176 cracking margin.

```
NewCostOfHedge = PriceNew * 10000    % Option premium
```

```
NewCostOfHedge =  
    53570.97
```

```
NewProfitMargin = 11 * 10000
```

```
NewProfitMargin =  
    110000.00
```

```
CrackingMargin = NewProfitMargin - NewCostOfHedge
```

```
CrackingMargin =  
56429.03
```

### Example 3: Using Collars to Reduce the Cost of Hedging

A refiner is concerned about its cost of hedging and decides to use a collar strategy. In April the crack spread is trading at \$4.23 per barrel. The refiner is not convinced to lock in this margin, but also wants to protect against price changes causing the refinery margin to decrease less than \$4 per barrel.

```
% Price and volatility of heating oil  
Price1gallon = 2.52;           % $/gallon  
Price1 = Price1gallon * 42;    % $/barrel  
Vol1 = 0.38;  
Div1 = 0.0762;
```

```
% Price and volatility of WTI crude oil  
Price2 = 101.61;             % $/barrel  
Vol2 = 0.34;  
Div2 = 0.1169;
```

To accomplish the collar strategy the refiner sells a call spread option with a strike of \$4.50 and uses the premium income to offset the cost of purchasing a put spread option with a strike of \$4. This allows the refiner to benefit if market prices move up, and protects it if market prices move down.

```
% Assume the following data  
Strike = [4.50;4];  
OptSpec = {'call';'put'};  
Settle = '01-April-2013';  
Maturity = '01-June-2013';  
Corr = 0.35;                % Correlation of underlying commodities
```

Define the RateSpec and StockSpec.

```
% Define RateSpec  
Rate = 0.035;  
Compounding = -1;  
Basis = 1;  
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...  
    'EndDates', Maturity, 'Rates', Rate, 'Compounding', ...  
    Compounding, 'Basis', Basis);
```

```
% Define StockSpec for the two assets
StockSpec1 = stockspec(Vol1, Price1, 'Continuous', Div1);
StockSpec2 = stockspec(Vol2, Price2, 'Continuous', Div2);
```

Price the Crack Spread Options

Use the function `spreadbybjs` in the Financial Instruments Toolbox™ to price the spread options using the Bjerksund and Stensland model.

```
Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
                  Maturity, OptSpec, Strike, Corr)
```

```
Price = 2×1
        7.06
        6.43
```

The collar strategy allows the refiner to reduce the cost of the hedge to \$0.63.

```
% CostOfHedge = Premium of Call - Premium of Put
CostOfHedge = Price(1) - Price(2)
```

```
CostOfHedge =
        0.63
```

The refiner is protected if the crack spread narrows to less than \$4. If the crack spread widens to more than \$4.50, the refiner will not benefit over this amount if he has hedged 100% of all its market exposure.

```
set(0, 'format', OldFormat);
```

## See Also

`asianbykv` | `asianbylevy` | `asianbyls` | `asiansensbykv` | `asiansensbylevy` | `asiansensbyls` | `lookbackbycvgsg` | `lookbackbyls` | `lookbacksensbycvgsg` | `lookbacksensbyls` | `optpricebysim` | `optstockbyblk` | `optstockbyls` | `optstocksensbyblk` | `optstocksensbyls` | `spreadbybjs` | `spreadbyfd` | `spreadbykirk` | `spreadbyls` | `spreadsensbybjs` | `spreadsensbyfd` | `spreadsensbykirk` | `spreadsensbyls`

## Related Examples

- “Pricing European and American Spread Options” on page 3-57
- “Pricing Swing Options using the Longstaff-Schwartz Method” on page 3-84
- “Compute Option Prices on a Forward” on page 11-1814
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## External Websites

- Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## Pricing Swing Options using the Longstaff-Schwartz Method

This example shows how to price a swing option using a Monte Carlo simulation and the Longstaff-Schwartz method. A risk-neutral simulation of the underlying natural gas price is conducted using a mean-reverting model. The simulation results are used to price a swing option based on the Longstaff-Schwartz method [6]. This approach uses a regression technique to approximate the continuation value of the option. A comparison is made between a polynomial and spline basis to fit the regression. Finally, the resulting prices are analyzed against lower and upper price boundaries derived from standard European and American options.

### Overview of Swing Options

Swing options are popular financial instruments in the energy market, which provide flexibility in the volume of the delivered asset. In order for energy consumers to protect themselves against fluctuations in energy prices, they want to lock in a price by purchasing a forward contract, called the *baseload forward contract*. However, consumers do not know exactly how much energy will be used in the future, and energy commodities such as electricity and gas cannot easily be stored. Therefore, the consumer wants the flexibility to change the amount of energy that is delivered at each delivery date. Swing options provide this flexibility. Thus, the full contract is composed of two parts: the baseload forward contract, and the swing option component.

Swing options are generally over-the-counter (OTC) contracts that can be highly customized. Therefore, there are many different types of constraints and penalties (see [5] for more details). In this example, a swing option is priced where the only constraint is the daily volume, which is known as the Daily Contract Quantity (DCQ). When a swing right is exercised, the volume cannot go below the minimum DCQ ( $\text{minDCQ}$ ), or go above the maximum DCQ ( $\text{maxDCQ}$ ).

There are several methods to price swing options, such as finite differences, simulation, and dynamic programming based on trees [5]. This example uses the simulation-based approach with the Longstaff-Schwartz method. The benefit of the simulation-based approach is that the dynamics used to simulate the underlying asset price are separated from the pricing algorithm. In the finite difference and tree based methods, the pricing algorithm must be changed in order to consider pricing with a different underlying price dynamic.

### Risk-Neutral Simulation of Natural Gas Price

In this example, natural gas is used as the underlying asset with the following mean-reverting dynamic [8]:

$$dS_t = \kappa(\mu - \log(S_t))S_t dt + \sigma S_t dW_t$$

where  $W_t$  is a standard Brownian motion. Applying Ito's Lemma to the logarithm of the price leads to an Orstein-Uhlenbeck process:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t$$

where  $X_t = \log(S_t)$ ,  $\kappa > 0$ , and  $\theta$  is defined as:

$$\theta = \mu - \frac{\sigma^2}{2\kappa}$$

$\theta$  is the mean-reversion level that determines the value at which the simulated values will revert to in the long run.  $\kappa$  is the mean-reversion speed that determines how fast this reversion occurs.  $\sigma$  is the volatility of  $X$ . We first proceed by simulating the logarithm of the price. Afterwards, the exponential of the simulated values are taken to obtain the prices.

The length of the simulation is for a one year period, with the initial price of 3.9 dollars per MMBtu. The Monte Carlo simulation is conducted for 1,000 trials, with daily periods. In practice, these parameters are calibrated against market data. In this example,  $\kappa = 1.2$ ,  $\theta = 1.7$ , and  $\sigma = 59\%$ . The HWV object from the Financial Toolbox™ is used to simulate the mean-reverting dynamics of the natural gas price.

```
% Settlement date
Settle = '01-Jun-2014';

% Maturity Date
Maturity = '01-Jun-2015';

% Actual/Actual basis
Basis = 0;

% Initial log(price in $/MMBtu)
X0 = log(3.9);

% Volatility of log(price)
```

```
Sigma = 0.59;

% Number of trials in the Monte Carlo simulation
NumTrials = 1000;

% Number of periods (daily)
NumPeriods = daysdif(Settle, Maturity, Basis);

% Daily time step
dt = 1/NumPeriods;

% Mean reversion speed of log(price)
Kappa = 1.2;

% Mean reversion level of log(price)
Theta = 1.7;

% Create HWV object
hwvobj = hwv(Kappa, Theta, Sigma, 'StartState', X0);

The simulation is run and plotted below.

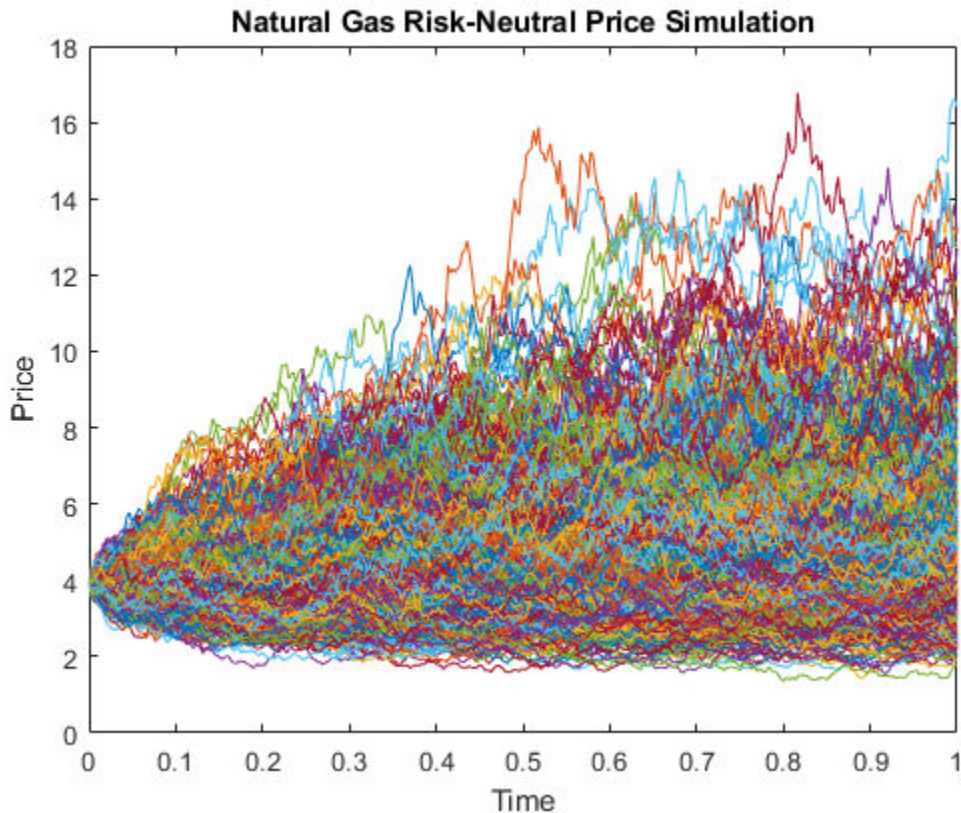
% Set random number generator seed
savedState = rng(0, 'twister');

% Simulate gas prices
[Paths, Times] = hwvobj.simBySolution(NumPeriods, 'NTRIALS', NumTrials, ...
    'DeltaTime', dt);
Paths = squeeze(exp(Paths));

% Restore random number generator state
rng(savedState);

% Plot paths
figure;
plot(Times, Paths);
title('Natural Gas Risk-Neutral Price Simulation');
xlabel('Time');
ylabel('Price');
```





In this example, natural gas is used as the underlying asset with a mean-reverting dynamic. However, the Longstaff-Schwartz algorithm can be used for other underlying assets, such as electricity, with any underlying price dynamic.

### Pricing the Swing Option

We consider a swing option with five swing rights at the strike of \$4.69/MMBtu, which can be exercised daily between the day after the settlement date and the maturity date. The Daily Contract Quantity (DCQ) is 10,000 MMBtu, which is the average amount of natural gas that the consumer expects to purchase on a given day. The consumer has the flexibility to reduce the purchase amount (downswing) in one day to the minimum DCQ of 2,500 MMBtu, or increase the purchase (upswing) to 15,000 MMBtu. The continuously compounded annual risk-free rate is 1%.

RateSpec is used to represent the interest-rate term structure. For the sake of simplicity, we consider a flat interest-rate term structure in this example. The values of RateSpec can be modified to accommodate any interest-rate curve. The function `hswingbyls` in this example assumes a daily exercise if the `ExerciseDates` input is empty.

```
% Define RateSpec
rfrate = 0.01;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
                    'EndDates', Maturity, 'Rates', rfrate, ...
                    'Compounding', Compounding, 'Basis', Basis);

% Daily exercise
% hswingbyls assumes daily exercise for empty ExerciseDates
ExerciseDates = [];

% Number of swings
NumSwings = 5;

% Daily Contract Quantity in MMBtu
DCQ = 10000;

% Minimum DCQ constraint in MMBtu
minDCQ = 2500;

% Maximum DCQ constraint in MMBtu
maxDCQ = 15000;

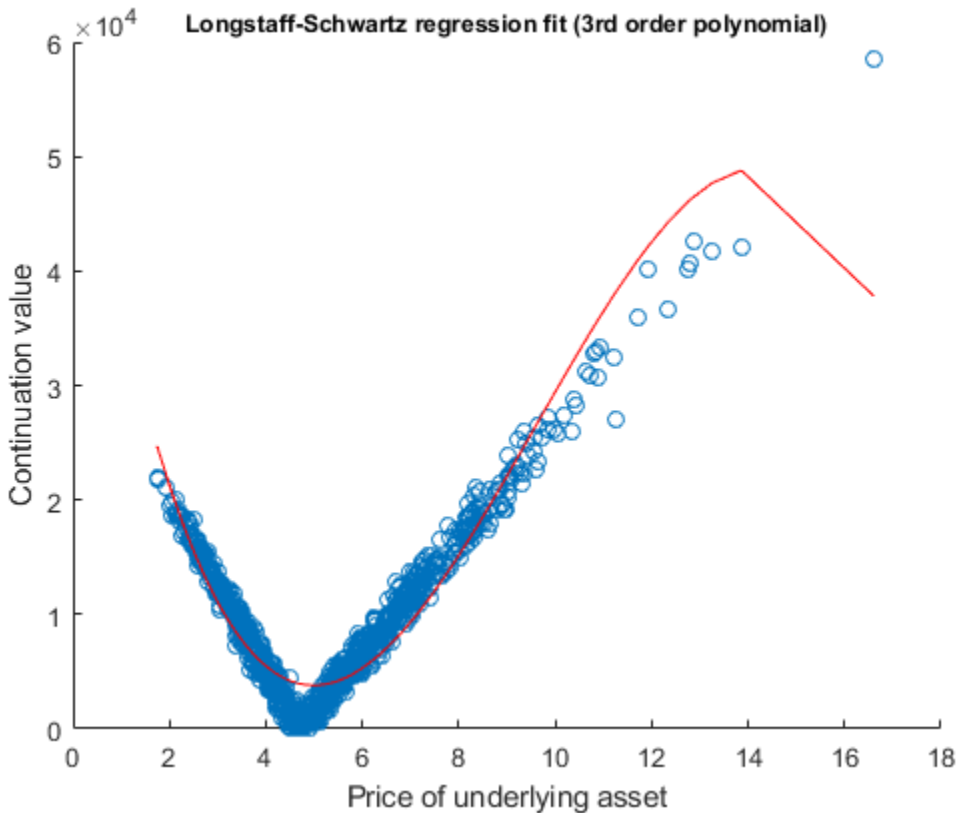
% Strike
Strike = 4.69;
```

The Longstaff-Schwartz method is a backward iteration algorithm, which steps backward in time from the maturity date. At each exercise date, the algorithm approximates the continuation value, which is the value of the option if it is not exercised. This is done by fitting a regression against the values of the simulated prices and the discounted future value of the option at the next exercise date. The future value of the option is known as the algorithm moves backward in time. The continuation value is compared to the sum of the payoff from immediate exercise (a downswing or upswing) and the continuation value of a swing option with one less swing right. If this sum is smaller, the option holder's optimal strategy is to not exercise on that date. The function `hswingbyls` in this example uses this method to determine the optimal exercise strategy and the price for swing options [1,2,7].

As discussed earlier, the only constraint considered in this example is the minimum and maximum DCQ. In this case, the optimal early exercise strategy is of a "bang-bang" type. This means that when it is optimal to upswing or downswing at a certain exercise date, the option holder should always exercise at the maximum or minimum DCQ to maximize profit. The "bang-bang" exercise would not be the optimal strategy if, for example, there is a terminal penalty based on volume. The pricing algorithm would then need to additionally keep track of all possible volume levels, which significantly adds to the runtime performance cost.

First, the swing option is priced using a 3rd order polynomial to fit the regression of the Longstaff-Schwartz method. The function `hswingbyls` also generates a plot of the regression between the underlying price and the continuation value at the exercise date before maturity.

```
% Price swing option using 3rd order polynomial to fit Longstaff-Schwartz
% regression
tic;
useSpline = false;
SwingPrice = hswingbyls(Paths, Times, RateSpec, Settle, Maturity, ...
    Strike, ExerciseDates, NumSwings, DCQ, minDCQ, maxDCQ, useSpline, ...
    [], true)
```



```
SwingPrice = 5.6943e+04
```

```
lsPolyTime = toc;
```

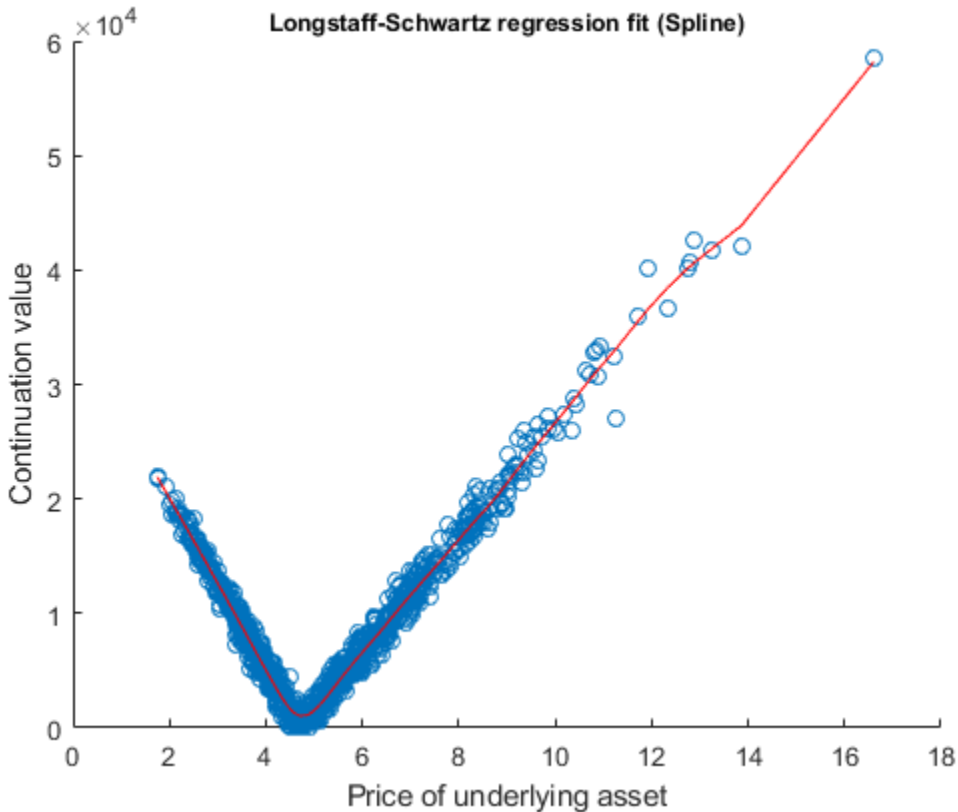
The above plot of the regression fit shows that the 3rd order polynomial does not fit the continuation value perfectly, especially near the hinge and at the extreme points. We now use the `csaps` function to fit the regression using a cubic smoothing spline with a smoothing parameter of 0.7. The Curve Fitting Toolbox™ is required to run the remainder of the example.

```
% Price swing option using smoothed splines to fit Longstaff-Schwartz
% regression
tic;
useSpline = true;
```

```

smoothingParam = 0.7;
SwingPriceSpline = hswingbyls(Paths, Times, RateSpec, Settle, Maturity, ...
    Strike, ExerciseDates, NumSwings, DCQ, minDCQ, maxDCQ, useSpline, ...
    smoothingParam, true)

```



```
SwingPriceSpline = 6.0729e+04
```

```
lsSplineTime = toc;
```

The plot of the regression shows that the cubic smoothing spline has a better fit against the data, thus obtaining a more accurate value for the continuation values. However, the comparison below shows that using a cubic smoothing spline takes longer than a 3rd order polynomial.

```
% Print comparison of running times
displayRunningTimes(lsPolyTime, lsSplineTime)
```

Comparison of running times:

```
3rd order polynomial: 4.57 sec
Spline                : 11.94 sec
```

Also, it is important to note that the price represents solely the optionality component. Hence, the price of the baseload forward contract is not included in the above calculated price. Because we used a fixed strike price, the baseload contract has a non-zero value, which can be calculated by:

$$BaseLoadPrice = \sum_{i=1}^N e^{-rt_i} E(S_{t_i} - K)$$

where  $t_i, i = 1, \dots, N$ , are the exercise dates (see [3] for more details). The full price of the contract, including the baseload and the swing option, is calculated below using the swing option price from the smoothed cubic spline.

```
% Obtain discount factors
RS2 = intenvset(RateSpec, 'StartTimes', 0, 'EndTimes', Times(2:end));
D = intenvget(RS2, 'Disc');

% Calculate baseload price
BaseLoadPrice = DCQ.*mean(Paths(2:end,:)-Strike,2)*D;

% Calculate full contract price, based on results from cubic spline LS
FullContractPrice = BaseLoadPrice + SwingPriceSpline

FullContractPrice = 1.2479e+05
```

#### Price Bounds

A lower bound for the swing option is a strip of European options, and the upper bound is a strip of American options [4]. Compared to European options, swing options have an early exercise premium at each exercise date, thus the price should be higher. The price is lower than the American option strips, because only a single swing right can be exercised at each exercise date. More than one strip can be exercised in a single day using American options.

The prices for the strips of the lower and upper bounds are calculated below to check that the swing option prices are within these bounds. The European strip prices are calculated against the last five exercise dates.

```
% Obtain discount factor for the last NumSwings exercise dates
D = D(end-NumSwings+1:end);

% European lower bound
idx = size(Paths, 1):-1:(size(Paths, 1) - NumSwings + 1);
putEuro = D'*mean(max(Strike - Paths(idx,:), 0),2);
callEuro = D'*mean(max(Paths(idx,:) - Strike, 0),2);
lowerBound = ((DCQ-minDCQ).*putEuro+(maxDCQ-DCQ).*callEuro);

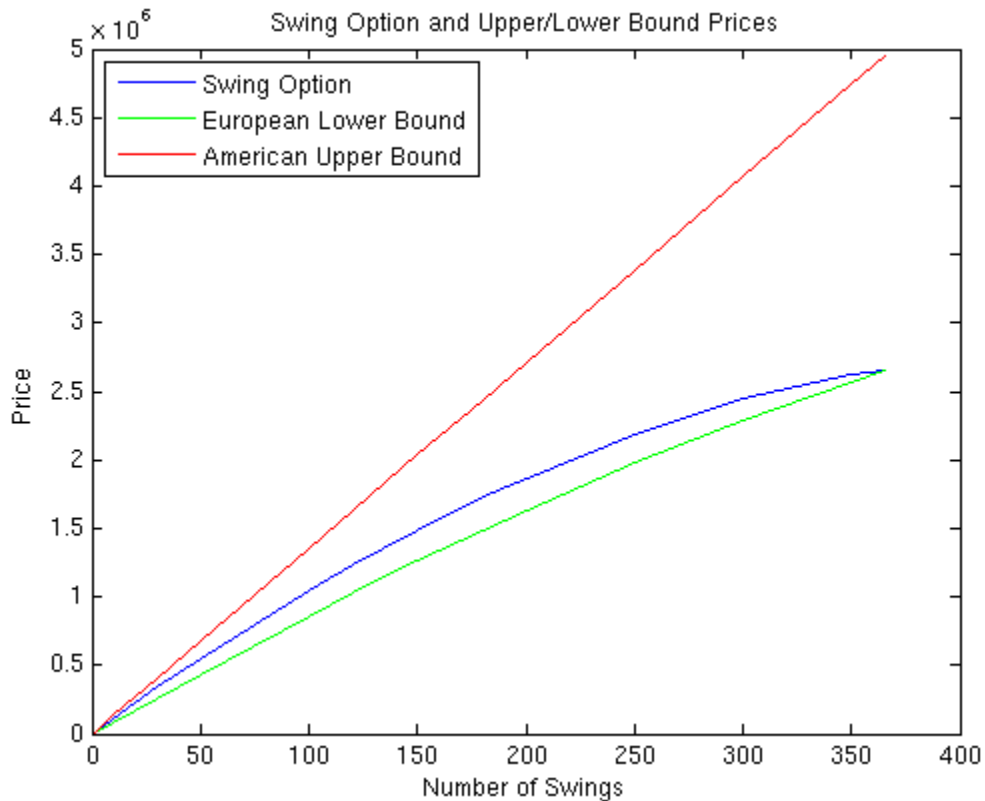
% American upper bound
[putAmer, callAmer] = hamericanPrice(Paths, Times, RateSpec, Strike);
upperBound = NumSwings.*((DCQ-minDCQ).*putAmer+(maxDCQ-DCQ).*callAmer);

% Print price and lower/upper bounds
displaySummary(SwingPriceSpline, lowerBound, upperBound);

Comparison to lower and upper bounds:

Lower bound (European) : 44412.14
Swing Option Price      : 60729.00
Upper bound (American) : 68181.42
```

The prices calculated using the Longstaff-Schwartz algorithm are within the lower and upper bounds. The plot below shows a comparison between the swing option and the upper and lower bounds as the number of swings increases. When the number of swings is 1, the swing option is equivalent to an American option. In the case of daily exercise opportunity (`NumSwings = 365`), the swing option is equivalent to the strip of European options with daily maturity.



### Conclusion

The example shows the use of the Longstaff-Schwartz method to price a swing option where the underlying asset follows a mean-reverting dynamic. A 3rd order polynomial and a smoothed cubic spline are used to fit the regression in the Longstaff-Schwartz algorithm to approximate the continuation value. It was shown that the smoothed cubic spline fits the data better at the cost of slower performance. Finally, the resulting swing option prices were checked against the lower bound of a strip of European options and an upper bound of a strip of American options.



## References

- [1] Boogert, A., de Jong, C., Gas storage valuation using a Monte Carlo method. *Journal of Derivatives*, 15(3):81-98, 2008.
- [2] Dorr, Uwe, Valuation of Swing Options and Estimation of Exercise strategies by Monte Carlo Techniques, Oxford, 2002.
- [3] Hull, John C., Options, Futures, and Other Derivatives, Sixth Edition, Pearson Education, Inc., 2006.
- [4] Jaillet, P., Ronn, E. I., Tompaidis, S., Valuation of commodity-based swing options, *Management Science*, 50(7):909-921, 2004.
- [5] Loland, Ambers, Lindqvist, Ola, Valuation of Commodity-Based Swing Options: A Survey, Norsk, Regnesentral, 2008.
- [6] Longstaff, Francis A., Schwartz, Eduardo S, Valuing American Options by Simulation: A Simple Least-Squares Approach, *The Review of Financial Studies*, 14(1):113-147, 2001.
- [7] Meinshausen, N., Hambly, B.M., Monte Carlo methods for the valuation of multiple exercise options, *Mathematical Science*, 14:557-583, 2004.
- [8] Schwartz, Eduardo S, The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging, *The Journal of Finance*, 52(3):923-973, 1997.

## Utility Functions

```
function displaySummary(SwingPriceSpline, lowerBound, upperBound)
fprintf('Comparison to lower and upper bounds:\n');
fprintf('\n')
fprintf('Lower bound (European) : %.2f\n', lowerBound);
fprintf('Swing Option Price      : %.2f\n', SwingPriceSpline);
fprintf('Upper bound (American) : %.2f\n\n', upperBound);
end

function displayRunningTimes(lsPolyTime, lsSplineTime)
fprintf('Comparison of running times:\n');
fprintf('\n')
fprintf('3rd order polynomial: %.2f sec\n', lsPolyTime);
```

```
fprintf('Spline           : %.2f sec\n\n', lsSplineTime);  
end
```

### See Also

asianbykv | asianbylevy | asianbyls | asiainsensbykv | asiainsensbylevy |  
asiainsensbyls | lookbackbycvgs | lookbackbyls | lookbacksensbycvgs |  
lookbacksensbyls | optpricebysim | optstockbyblk | optstockbyls |  
optstocksensbyblk | optstocksensbyls | spreadbybjs | spreadbyfd |  
spreadbykirk | spreadbyls | spreadsensbybjs | spreadsensbyfd |  
spreadsensbykirk | spreadsensbyls

### Related Examples

- “Pricing European and American Spread Options” on page 3-57
- “Hedging Strategies Using Spread Options” on page 3-76
- “Compute Option Prices on a Forward” on page 11-1814
- “Compute Forward Option Prices and Delta Sensitivities” on page 11-1883
- “Compute the Option Price on a Future” on page 11-1816
- “Simulating Electricity Prices with Mean-Reversion and Jump-Diffusion” on page 3-98
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### More About

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- “Spread Option” on page 3-50
- “Asian Option” on page 3-44
- “Vanilla Option” on page 3-48
- “Lookback Option” on page 3-51
- “Supported Equity Derivatives” on page 3-24
- “Supported Interest-Rate Instruments” on page 2-2

## **External Websites**

- Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## Simulating Electricity Prices with Mean-Reversion and Jump-Diffusion

This example shows how to simulate electricity prices using a mean-reverting model with seasonality and a jump component. The model is calibrated under the real-world probability using historical electricity prices. The market price of risk is obtained from futures prices. A risk-neutral Monte Carlo simulation is conducted using the calibrated model and the market price of risk. The simulation results are used to price a Bermudan option with electricity prices as the underlying.

### Overview of the Model

Electricity prices exhibit jumps in prices at periods of high demand when additional, less efficient electricity generation methods, are brought on-line to provide a sufficient supply of electricity. In addition, they have a prominent seasonal component, along with reversion to mean levels. Therefore, these characteristics should be incorporated into a model of electricity prices [2].

In this example, electricity price is modeled as:

$$\log(P_t) = f(t) + X_t$$

where  $P_t$  is the spot price of electricity. The logarithm of electricity price is modeled with two components:  $f(t)$  and  $X_t$ . The component  $f(t)$  is the deterministic seasonal part of the model, and  $X_t$  is the stochastic part of the model. Trigonometric functions are used to model  $f(t)$  as follows [3]:

$$f(t) = s_1 \sin(2\pi t) + s_2 \cos(2\pi t) + s_3 \sin(4\pi t) + s_4 \cos(4\pi t) + s_5$$

where  $s_i, i = 1, \dots, 5$  are constant parameters, and  $t$  is the annualized time factors. The stochastic component  $X_t$  is modeled as an Ornstein-Uhlenbeck process (mean-reverting) with jumps:

$$dX_t = (\alpha - \kappa X_t)dt + \sigma dW_t + J(\mu_J, \sigma_J)d\Pi(\lambda)$$

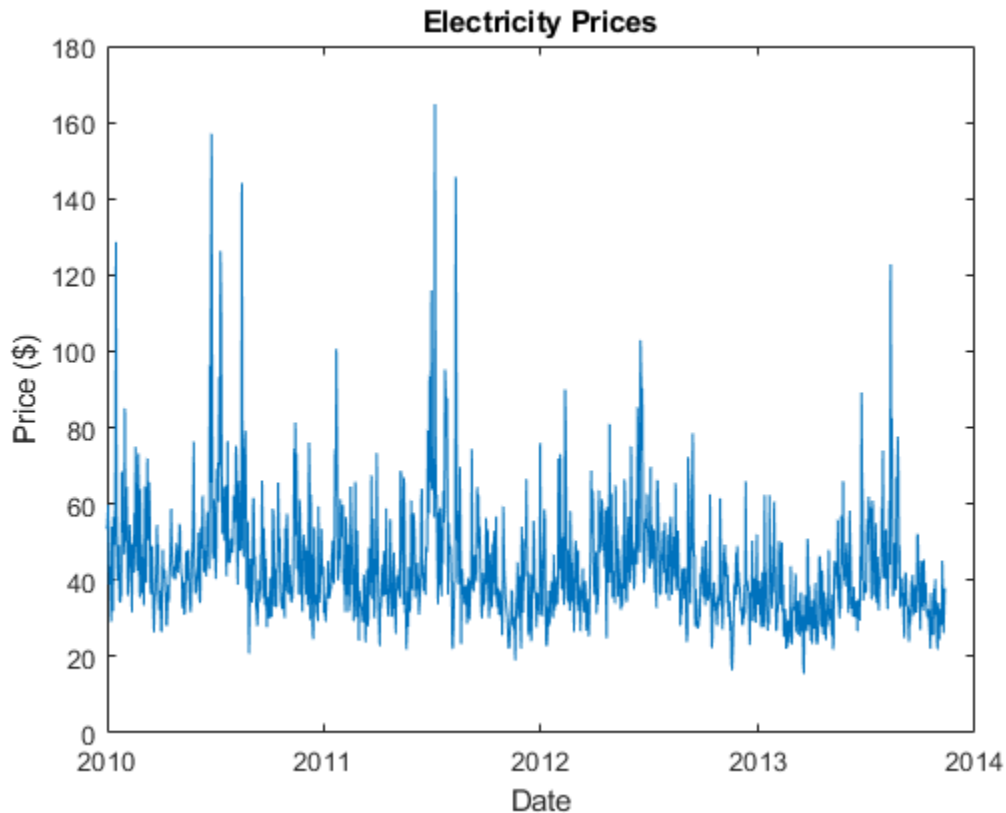
The parameters  $\alpha$  and  $\kappa$  are the mean-reversion parameters. Parameter  $\sigma$  is the volatility, and  $W_t$  is a standard Brownian motion. The jump size is  $J(\mu_J, \sigma_J)$ , with a normally distributed mean  $\mu_J$ , and a standard deviation  $\sigma_J$ . The Poisson process  $\Pi(\lambda)$  has a jump intensity of  $\lambda$ .

## Electricity Prices

Sample electricity prices from January 1, 2010 to November 11, 2013 are loaded and plotted below. `Prices` contain the electricity prices, and `PriceDates` contain the dates associated with the prices. The logarithm of the prices and annual time factors are calculated.

```
% Load electricity prices and futures prices
load('electricity_prices.mat');

% Plot electricity prices
figure;
plot(PriceDates, Prices);
datetick();
title('Electricity Prices');
xlabel('Date');
ylabel('Price ($)');
```



```
% Obtain log of prices  
logPrices = log(Prices);
```

```
% Obtain annual time factors from dates  
PriceTimes = yearfrac(PriceDates(1), PriceDates);
```

### Calibration

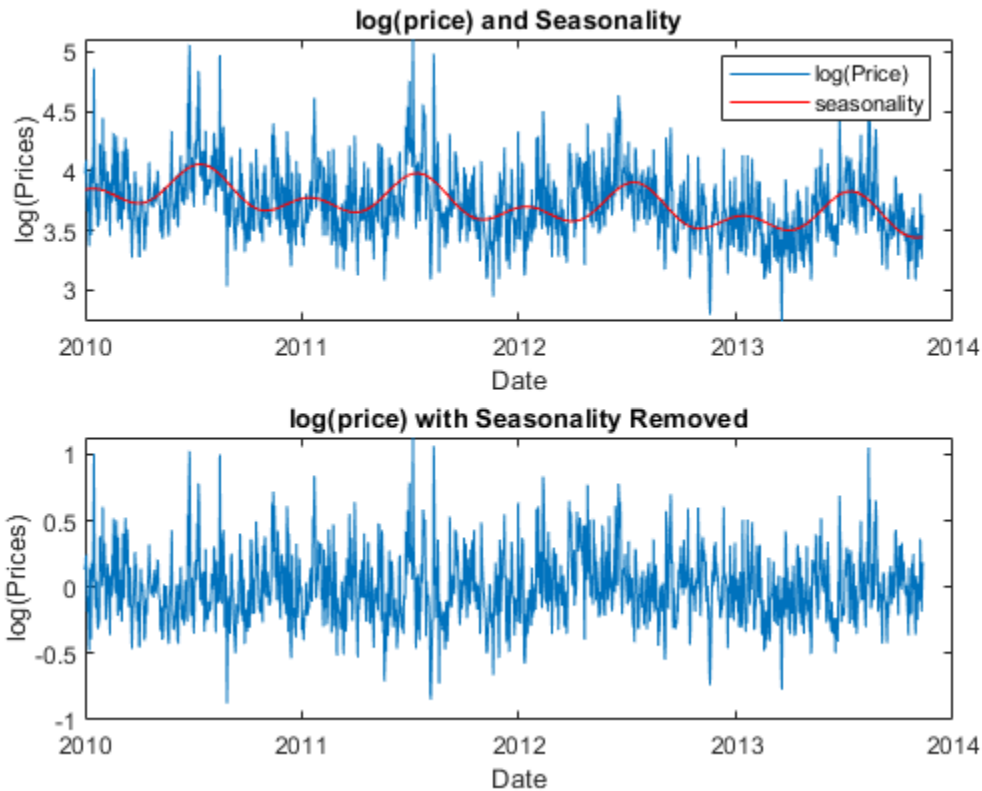
First, the deterministic seasonality part is calibrated using the least squares method. Since the seasonality function is linear with respect to the parameters  $s_i$ , the backslash operator (`mldivide`) is used. After the calibration, the seasonality is removed from the logarithm of price. The logarithm of price and seasonality trends are plotted below. Also, the de-seasonalized logarithm of price is plotted.

```

% Calibrate parameters for the seasonality model
seasonMatrix = @(t) [sin(2.*pi.*t) cos(2.*pi.*t) sin(4.*pi.*t) ...
    cos(4.*pi.*t) t ones(size(t, 1), 1)];
C = seasonMatrix(PriceTimes);
seasonParam = C\logPrices;

% Plot log price and seasonality line
figure;
subplot(2, 1, 1);
plot(PriceDates, logPrices);
datetick();
title('log(price) and Seasonality');
xlabel('Date');
ylabel('log(Prices)');
hold on;
plot(PriceDates, C*seasonParam, 'r');
hold off;
legend('log(Price)', 'seasonality');

% Plot de-seasonalized log price
X = logPrices-C*seasonParam;
subplot(2, 1, 2);
plot(PriceDates, X);
datetick();
title('log(price) with Seasonality Removed');
xlabel('Date');
ylabel('log(Prices)');
    
```



The second stage is to calibrate the stochastic part. The model for  $X_t$  needs to be discretized to conduct the calibration. To discretize, assume that there is a Bernoulli process for the jump events. That is, there is at most one jump per day since this example is calibrating against daily electricity prices. The discretized equation is:

$$X_t = \alpha\Delta t + \phi X_{t-1} + \sigma\xi$$

with probability  $(1 - \lambda\Delta t)$  and,

$$X_t = \alpha\Delta t + \phi X_{t-1} + \sigma\xi + \mu_J + \sigma_J\xi_J$$

with probability  $\lambda\Delta t$ , where  $\xi$  and  $\xi_J$  are independent standard normal random variables, and  $\phi = 1 - \kappa\Delta t$ . The density function of  $X_t$  given  $X_{t-1}$  is [1,4]:



$$f(X_t|X_{t-1}) = (\lambda\Delta t)N_1(X_t|X_{t-1}) + (1 - \lambda\Delta t)N_2(X_t|X_{t-1})$$

$$N_1(X_t|X_{t-1}) = (2\pi(\sigma^2 + \sigma_J^2))^{-\frac{1}{2}} \exp\left(\frac{-(X_t - \alpha\Delta t - \phi X_{t-1} - \mu_J)^2}{2(\sigma^2 + \sigma_J^2)}\right)$$

$$N_2(X_t|X_{t-1}) = (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left(\frac{-(X_t - \alpha\Delta t - \phi X_{t-1})^2}{2\sigma^2}\right)$$

The parameters  $\theta = \{\alpha, \phi, \mu_J, \sigma^2, \sigma_J^2, \lambda\}$  can be calibrated by minimizing the negative log likelihood function:

$$\min_{\theta} - \sum_{t=1}^T \log(f(X_t|X_{t-1}))$$

$$\text{subject to } \phi < 1, \sigma^2 > 0, \sigma_J^2 > 0, 0 \leq \lambda\Delta t \leq 1$$

The first inequality constraint,  $\phi < 1$ , is equivalent to  $\kappa > 0$ . The volatilities  $\sigma$  and  $\sigma_J$  must be positive. In the last inequality,  $\lambda\Delta t$  is between 0 and 1, because it represents the probability of a jump occurring in  $\Delta t$  time. In this example, assume that  $\Delta t$  is one day. Therefore, there is at most 365 jumps in one year. The `mle` function from the Statistics and Machine Learning Toolbox™ is well suited to solve the above maximum likelihood problem.

```
% Prices at t, X(t)
Pt = X(2:end);
```

```
% Prices at t-1, X(t-1)
Pt_1 = X(1:end-1);
```

```
% Discretization for daily prices
dt = 1/365;
```

```
% PDF for discretized model
mrjpdf = @(Pt, a, phi, mu_J, sigmaSq, sigmaSq_J, lambda) ...
    lambda.*exp((- (Pt-a-phi.*Pt_1-mu_J).^2)./ ...
    (2.*(sigmaSq+sigmaSq_J))).* (1/sqrt(2.*pi.*(sigmaSq+sigmaSq_J))) + ...
    (1-lambda).*exp((- (Pt-a-phi.*Pt_1).^2)/(2.*sigmaSq)).* ...
    (1/sqrt(2.*pi.*sigmaSq));
```

```
% Constraints:
```

```
% phi < 1 (k > 0)
% sigmaSq > 0
% sigmaSq_J > 0
% 0 <= lambda <= 1
lb = [-Inf -Inf -Inf 0 0 0];
ub = [Inf 1 Inf Inf Inf 1];

% Initial values
x0 = [0 0 0 var(X) var(X) 0.5];

% Solve maximum likelihood
params = mle(Pt, 'pdf', mrjpdf, 'start', x0, 'lowerbound', lb, 'upperbound', ub, ...
            'optimfun', 'fmincon');

% Obtain calibrated parameters
alpha = params(1)/dt

alpha = -20.1060

kappa = (1-params(2))/dt

kappa = 188.2535

mu_J = params(3)

mu_J = 0.2044

sigma = sqrt(params(4)/dt);
sigma_J = sqrt(params(5))

sigma_J = 0.2659

lambda = params(6)/dt

lambda = 98.3357
```

#### Monte Carlo Simulation

The calibrated parameters and the discretized model allow us to simulate electricity prices under the real-world probability. The simulation is conducted for approximately 2 years with 10,000 trials. It exceeds 2 years to include all the dates in the last month of simulation. This is because the expected simulation prices for the futures contract expiry date is required in the next section to calculate the market price of risk. The seasonality is added back on the simulated paths. A plot for a single simulation path is plotted below.

```
rng default;
```

```

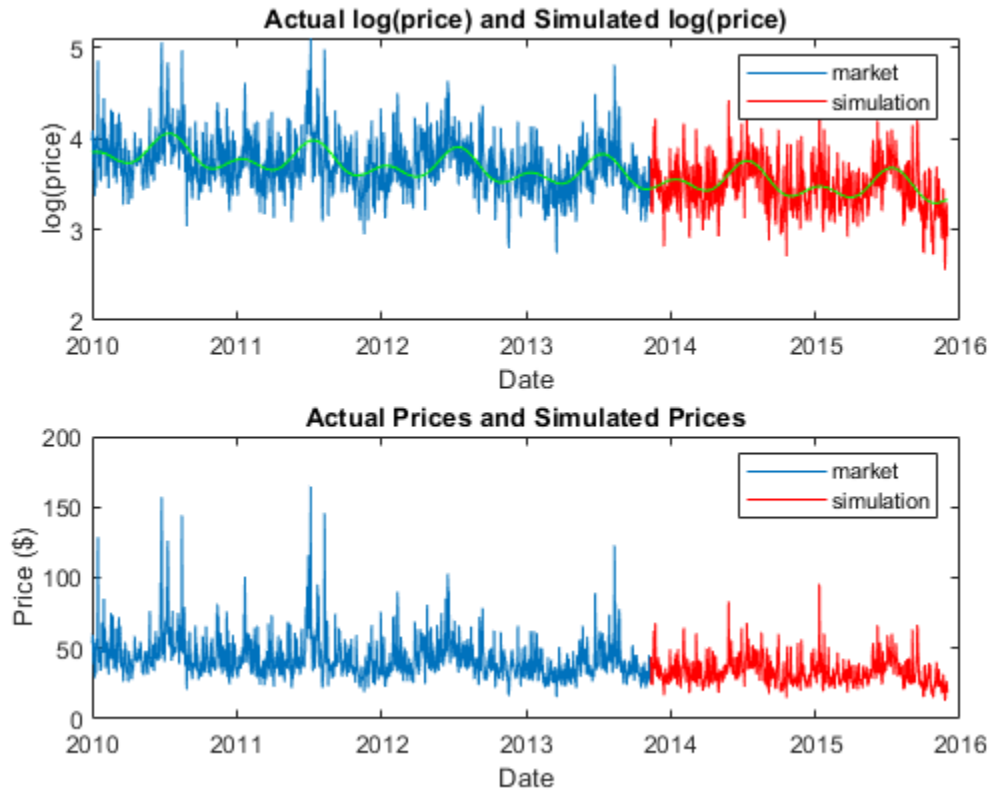
% Simulate for about 2 years
nPeriods = 365*2+20;
nTrials = 10000;
n1 = randn(nPeriods,nTrials);
n2 = randn(nPeriods, nTrials);
j = binornd(1, lambda*dt, nPeriods, nTrials);
SimPrices = zeros(nPeriods, nTrials);
SimPrices(1,:) = X(end);
for i=2:nPeriods
    SimPrices(i,:) = alpha*dt + (1-kappa*dt)*SimPrices(i-1,:) + ...
        sigma*sqrt(dt)*n1(i,:) + j(i,:).*(mu_J+sigma_J*n2(i,:));
end

% Add back seasonality
SimPriceDates = daysadd(PriceDates(end),0:nPeriods-1);
SimPriceTimes = yearfrac(PriceDates(1), SimPriceDates);
CSim = seasonMatrix(SimPriceTimes);
logSimPrices = SimPrices + repmat(CSim*seasonParam,1,nTrials);

% Plot logarithm of Prices and simulated logarithm of Prices
figure;
subplot(2, 1, 1);
plot(PriceDates, logPrices);
hold on;
plot(SimPriceDates(2:end), logSimPrices(2:end,1), 'red');
seasonLine = seasonMatrix([PriceTimes; SimPriceTimes(2:end)])*seasonParam;
plot([PriceDates; SimPriceDates(2:end)], seasonLine, 'green');
hold off;
datetick();
title('Actual log(price) and Simulated log(price)');
xlabel('Date');
ylabel('log(price)');
legend('market', 'simulation');

% Plot prices and simulated prices
PricesSim = exp(logSimPrices);
subplot(2, 1, 2);
plot(PriceDates, Prices);
hold on;
plot(SimPriceDates, PricesSim(:,1), 'red');
hold off;
datetick();
title('Actual Prices and Simulated Prices');
xlabel('Date');
    
```

```
ylabel('Price ($)');
legend('market', 'simulation');
```



### Calibration of the Market Price of Risk

Up to this point, the parameters were calibrated under the real-world probability. However, to price options, you need the simulation under the risk-neutral probability. To obtain this, calculate the market price of risk from futures prices to derive the risk-neutral parameters. Suppose that there are monthly futures contracts available on the market, which are settled daily during the contract month. For example, such futures for the PJM electricity market are listed on the Chicago Mercantile Exchange [5].

The futures are settled daily during the contract month. Therefore, you can obtain daily futures values by assuming the futures value is constant for the contract month. The

expected futures prices from the real-world measure are also needed to calculate the market price of risk. This can be obtained from the simulation conducted in the previous section.

```

% Obtain daily futures prices
FutPricesDaily = zeros(size(SimPriceDates));
for i=1:nPeriods
    idx = find(year(SimPriceDates(i)) == year(FutExpiry) & ...
              month(SimPriceDates(i)) == month(FutExpiry));
    FutPricesDaily(i) = FutPrices(idx);
end

% Calculate expected futures price under real-world measure
SimPricesExp = mean(PricesSim, 2);
    
```

To calibrate the market price of risk against market futures values, use the following equation:

$$\log\left(\frac{F_t}{E_t}\right) = -\sigma e^{-kt} \int_0^t e^{ks} m_s ds$$

where  $F_t$  is the observed futures value at time  $t$ , and  $E_t$  is the expected value under the real-world measure at time  $t$ . The equation was obtained using the same methodology as described in [3]. This example assumes that the market price of risk is fully driven by the Brownian motion. The market price of risk,  $m_t$ , can be solved by discretizing the above equation and solving a system of linear equations.

```

% Setup system of equations
t0 = yearfrac(PriceDates(1), FutValuationDate);
tz = SimPriceTimes-t0;
b = -log(FutPricesDaily(2:end) ./ SimPricesExp(2:end)) ./ ...
    (sigma.*exp(-kappa.*tz(2:end)));
A = (1/kappa).*(exp(kappa.*tz(2:end)) - exp(kappa.*tz(1:end-1)));
A = tril(repmat(A', size(A,1), 1));

% Precondition to stabilize numerical inversion
P = diag(1./diag(A));
b = P*b;
A = P*A;

% Solve for market price of risk
riskPremium = A\b;
    
```

### Simulation of Risk-Neutral Prices

Once  $m_t$  is obtained, risk-neutral simulation can be conducted using the following dynamics:

$$X_t = \alpha\Delta t + \phi X_{t-1} - \sigma m_{t-1}\Delta t + \sigma\xi$$

with probability  $(1 - \lambda\Delta t)$  and

$$X_t = \alpha\Delta t + \phi X_{t-1} - \sigma m_{t-1}\Delta t + \sigma\xi + \mu_J + \sigma_J\xi_J$$

with probability  $\lambda\Delta t$ .

```
nTrials = 10000;
n1 = randn(nPeriods, nTrials);
n2 = randn(nPeriods, nTrials);
j = binornd(1, lambda*dt, nPeriods, nTrials);

SimPrices = zeros(nPeriods, nTrials);
SimPrices(1,:) = X(end);
for i=2:nPeriods
    SimPrices(i,:) = alpha*dt + (1-kappa*dt)*SimPrices(i-1,:) + ...
        sigma*sqrt(dt)*n1(i,:) - sigma*dt*riskPremium(i-1) + ...
        j(i,:).*(mu_J+sigma_J*n2(i,:));
end

% Add back seasonality
CSim = seasonMatrix(SimPriceTimes);
logSimPrices = SimPrices + repmat(CSim*seasonParam,1,nTrials);

% Convert log(Price) to Price
PricesSim = exp(logSimPrices);
```

The expected values from the risk-neutral simulation are plotted against the market futures values. This confirms that the risk-neutral simulation closely reproduces the market futures values.

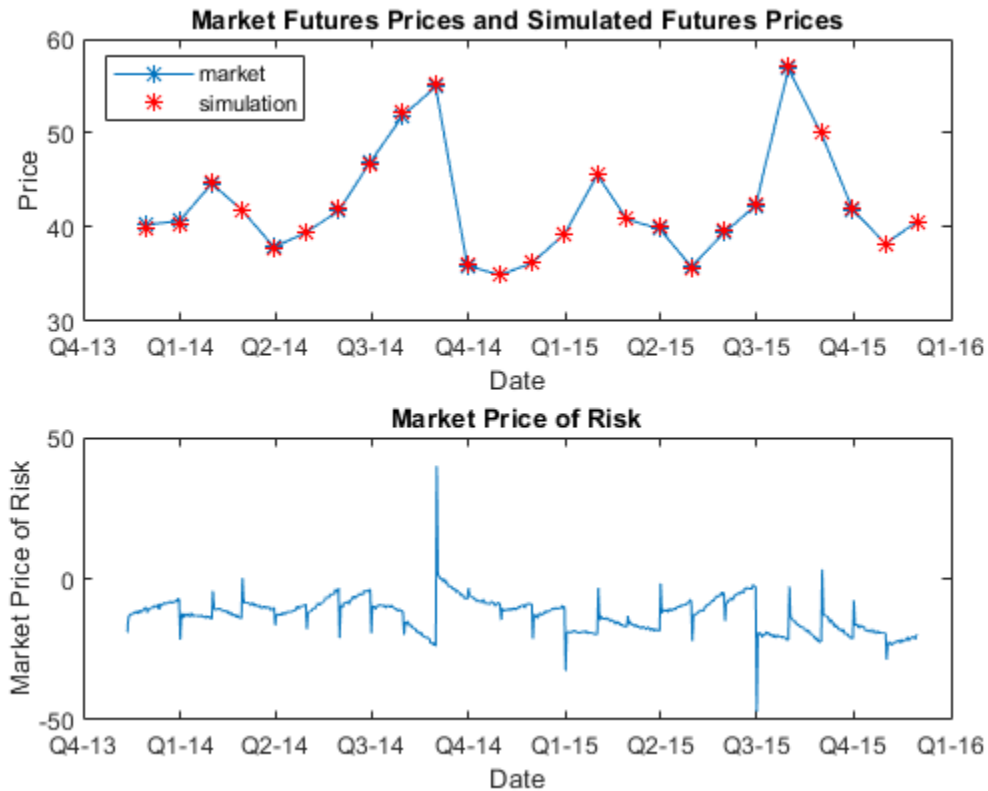
```
% Obtain expected values from the risk-neutral simulation
SimPricesExp = mean(PricesSim,2);
fexp = zeros(size(FutExpiry));
for i = 1:size(FutExpiry,1)
    idx = SimPriceDates == FutExpiry(i);
    if sum(idx)==1
        fexp(i) = SimPricesExp(idx);
```

```

        end
    end

% Plot expected values from the simulation against market futures prices
figure;
subplot(2,1,1);
plot(FutExpiry, FutPrices(1:size(FutExpiry,1)), '-*');
hold on;
plot(FutExpiry, fexp, '*r');
datetick();
hold off;
title('Market Futures Prices and Simulated Futures Prices');
xlabel('Date');
ylabel('Price');
legend('market', 'simulation', 'Location', 'NorthWest');
subplot(2,1,2);
plot(SimPriceDates(2:end), riskPremium);
datetick();
title('Market Price of Risk');
xlabel('Date');
ylabel('Market Price of Risk');

```



#### Pricing a Bermudan Option

The risk-neutral simulated values are used as input into the function `optpricebysim` in the Financial Instruments Toolbox™ to price a European, Bermudan, or American option on electricity prices. Below, the price is calculated for a two-year Bermudan call option with two exercise opportunities. The first exercise is after one year, and the second is at the maturity of the option.

```
% Settle, maturity of option
Settle = FutValuationDate;
Maturity = addtodate(FutValuationDate, 2, 'year');

% Create interest rate term structure
riskFreeRate = 0.01;
```



```

Basis = 0;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rate', riskFreeRate, 'Compounding', ...
    Compounding, 'Basis', Basis);

% Cutoff simulation at maturity
endIdx = find(SimPriceDates == Maturity);
SimPrices = PricesSim(1:endIdx,:);
Times = SimPriceTimes(1:endIdx) - SimPriceTimes(1);

% Bermudan call option with strike 60, two exercise opportunities, after
% one year and at maturity.
OptSpec = 'call';
Strike = 60;
ExerciseTimes = [Times(366) Times(end)];
Price = optpricebysim(RateSpec, SimPrices, Times, OptSpec, Strike, ...
    ExerciseTimes)

Price = 1.1085

```

## References

- [1] Escribano, Alvaro, Pena, Juan Ignacio, Villaplana, Pablo. "Modeling Electricity Prices: International Evidence." Universidad Carlos III de Madrid, Working Paper 02-27, 2002.
- [2] Lucia, Julio J., Schwartz, Eduardo. "Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange." *Review of Derivatives Research*. Vol. 5, Issue 1, pp 5-50, 2002.
- [3] Seifert, Jan, Uhrig-Homburg, Marliese. "Modelling Jumps in Electricity Prices: Theory and Empirical Evidence." *Review of Derivatives Research*. Vol. 10, pp 59-85, 2007.
- [4] Villaplana, Pablo. "Pricing Power Derivatives: A Two-Factor Jump-Diffusion Approach." Universidad Carlos III de Madrid, Working Paper 03-18, 2003.
- [5] <https://www.cmegroup.com>

## See Also

asianbykv | asianbylevy | asianbyls | asiandsensbykv | asiandsensbylevy | asiandsensbyls | lookbackbycvgsg | lookbackbyls | lookbacksensbycvgsg | lookbacksensbyls | optpricebysim | optstockbyblk | optstockbyls |

optstocksensbyblk | optstocksensbyls | spreadbybjs | spreadbyfd |  
spreadbykirk | spreadbyls | spreadsensbybjs | spreadsensbyfd |  
spreadsensbykirk | spreadsensbyls

### Related Examples

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- “Hedging Strategies Using Spread Options” on page 3-76
- “Pricing Swing Options using the Longstaff-Schwartz Method” on page 3-84
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### More About

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### External Websites

- Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## Pricing Asian Options

This example shows how to price a European Asian option using six methods in the Financial Instruments Toolbox™. This example demonstrates four closed form approximations (Kemna-Vorst, Levy, Turnbull-Wakeman, and Haug-Haug-Margrabe), a lattice model (Cox-Ross-Rubinstein), and Monte Carlo simulation. All these methods involve some tradeoffs between numerical accuracy and computational efficiency. This example also demonstrates how variations in spot prices, volatility, and strike prices affect option prices on European Vanilla and Asian options.

### Overview of Asian Options

Asian options are securities with payoffs that depend on the average value of an underlying asset over a specific period of time. Underlying assets can be stocks, commodities, or financial indices.

Two types of Asian options are found in the market: average price options and average strike options. Average price options have a fixed strike value and the average used is the asset price. Average strike options have a strike equal to the average value of the underlying asset.

The payoff at maturity of an average price European Asian option is:

$\max(0, Savg - K)$  for a call

$\max(0, K - Savg)$  for a put

The payoff at maturity of an average strike European Asian option is:

$\max(0, St - Savg)$  for a call

$\max(0, Savg - St)$  for a put

where  $Savg$  is the average price of underlying asset,  $St$  is the price at maturity of underlying asset, and  $K$  is the strike price.

The average can be arithmetic or geometric.

### Pricing Asian Options Using Closed Form Approximations

The Financial Instruments Toolbox™ supports four closed form approximations for European Average Price options. The Kemna-Vorst method is based on the geometric

mean of the price of the underlying during the life of the option [1]. The Levy and Turnbull-Wakeman models provide a closed form pricing solution to continuous arithmetic averaging options [2,3]. The Haug-Haug-Margrabe approximation is used for pricing discrete arithmetic averaging options [4].

All the pricing functions `asianbykv`, `asianbylevy`, `asianbytw`, and `asianbyhmm` take an interest-rate term structure and stock structure as inputs.

Consider the following example:

```
% Create RateSpec from the interest rate term structure
StartDates = '12-March-2014';
EndDates = '12-March-2020';
Rates = 0.035;
Compounding = -1;
Basis = 1;

RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', ...
    Compounding, 'Basis', Basis);

% Define StockSpec with the underlying asset information
Sigma = 0.20;
AssetPrice = 100;

StockSpec = stockspec(Sigma, AssetPrice);

% Define the Asian option
Settle = '12-March-2014';
ExerciseDates = '12-March-2015';
Strike = 90;
OptSpec = 'call';

% Kemna-Vorst closed form model
PriceKV = asianbykv(RateSpec, StockSpec, OptSpec, Strike, Settle,...
    ExerciseDates);

% Levy model approximation
PriceLevy = asianbylevy(RateSpec, StockSpec, OptSpec, Strike, Settle,...
    ExerciseDates);

% Turnbull-Wakeman approximation
PriceTW = asianbytw(RateSpec, StockSpec, OptSpec, Strike, Settle,...
    ExerciseDates);
```

```

% Haug-Haug-Margrabe approximation
PriceHHM = asianbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle,...
    ExerciseDates);

% Comparison of calculated prices for the geometric and arithmetic options
% using different closed form algorithms.
displayPricesClosedForm(PriceKV, PriceLevy, PriceTW, PriceHHM)

Comparison of Asian Arithmetic and Geometric Prices:

Kemna-Vorst:      11.862580
Levy:             12.164734
Turnbull-Wakeman: 12.164734
Haug-Haug-Margrabe: 12.108746

```

### Computing Asian Options Prices Using the Cox-Ross-Rubinstein Model

In addition to closed form approximations, the Financial Instruments Toolbox™ supports pricing European Average Price options using CRR trees via the function `asianbycrr`.

The lattice pricing function `asianbycrr` takes an interest-rate tree ( `CRRTree` ) and stock structure as inputs. You can price the previous options by building a `CRRTree` using the interest-rate term structure and stock specification from the example above.

```

% Create the time specification of the tree
NPeriods = 20;
TreeValuationDate = '12-March-2014';
TreeMaturity = '12-March-2024';
TimeSpec = crrtimespec(TreeValuationDate, TreeMaturity, NPeriods);

% Build the tree
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec);

% Price the European Asian option using the CRR lattice model.
% The function 'asianbycrr' computes prices of arithmetic and geometric
% Asian options.
AvgType = {'arithmetic'; 'geometric'};
AmericanOpt = 0;
PriceCRR20 = asianbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates,...
    AmericanOpt, AvgType);

% Increase the numbers of periods in the tree and compare results
NPeriods = 40;
TimeSpec = crrtimespec(TreeValuationDate, TreeMaturity, NPeriods);

```

```
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec);  
PriceCRR40 = asianbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates,...  
    AmericanOpt, AvgType);
```

```
% Display prices  
displayPricesCRR(PriceCRR20, PriceCRR40)
```

Asian Prices using the CRR lattice model:

```
PriceArithmetic(CRR20): 11.934380  
PriceArithmetic(CRR40): 12.047243  
PriceGeometric (CRR20): 11.620899  
PriceGeometric (CRR40): 11.732037
```

The results above compare the findings from calculating both geometric and arithmetic Asian options, using CRR trees with 20 and 40 levels. As the number of levels increases, the results approach the closed form solutions.

#### Calculating Prices of Asian Options Using Monte Carlo Simulation

Another method to price European Average Price options with the Financial Instruments Toolbox™ is via Monte Carlo simulations.

The pricing function `asianbyls` takes an interest-rate term structure and stock structure as inputs. The output and execution time of the Monte Carlo simulation depends on the number of paths ( `NumTrials` ) and the number of time periods per path ( `NumPeriods` ).

You can price the same options of previous examples using Monte Carlo.

```
% Simulation Parameters  
NumTrials = 500;  
NumPeriods = 200;  
  
% Price the arithmetic option  
PriceAMC = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,...  
    ExerciseDates, 'NumTrials', NumTrials, ...  
    'NumPeriods', NumPeriods);  
  
% Price the geometric option  
PriceGMC = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,...  
    ExerciseDates, 'NumTrials', NumTrials, ...  
    'NumPeriods', NumPeriods, 'AvgType', AvgType(2));
```

```

% Use the antithetic variates method to value the options
Antithetic = true;
PriceAMCAntithetic = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,...
    ExerciseDates,'NumTrials', NumTrials, 'NumPeriods',...
    NumPeriods, 'Antithetic', Antithetic);

PriceGMCantithetic = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,...
    ExerciseDates,'NumTrials', NumTrials, 'NumPeriods',...
    NumPeriods, 'Antithetic', Antithetic,'AvgType', AvgType(2));

% Display prices
displayPricesMonteCarlo(PriceAMC, PriceAMCAntithetic, PriceGMC, PriceGMCantithetic)

```

Asian Prices using Monte Carlo Method:

```

Arithmetic Asian
Standard Monte Carlo:          12.304046
Variate Antithetic Monte Carlo: 12.304046

```

```

Geometric Asian
Standard Monte Carlo:          12.048434
Variate Antithetic Monte Carlo: 12.048434

```

The use of variate antithetic accelerates the conversion process by reducing the variance.

You can create a plot to display the difference between the geometric Asian price using the Kemna-Vorst model, standard Monte Carlo, and antithetic Monte Carlo.

```

nTrials = [50:5:100 110:10:250 300:50:500 600:100:2500]';
PriceKVVector = PriceKV * ones(size(nTrials));
PriceGMCVector = nan(size(nTrials));
PriceGMCantitheticVector = nan(size(nTrials));
TimeGMCantitheticVector = nan(length(nTrials),1);
TimeGMCVector = nan(length(nTrials),1);
idx = 1;
for iNumTrials = nTrials'
    PriceGMCVector(idx) = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,...
        ExerciseDates,'NumTrials', iNumTrials, 'NumPeriods',...
        NumPeriods, 'AvgType', AvgType(2));

    PriceGMCantitheticVector(idx) = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,...
        ExerciseDates,'NumTrials', iNumTrials, 'NumPeriods',...
        NumPeriods, 'Antithetic', Antithetic,'AvgType', AvgType(2));

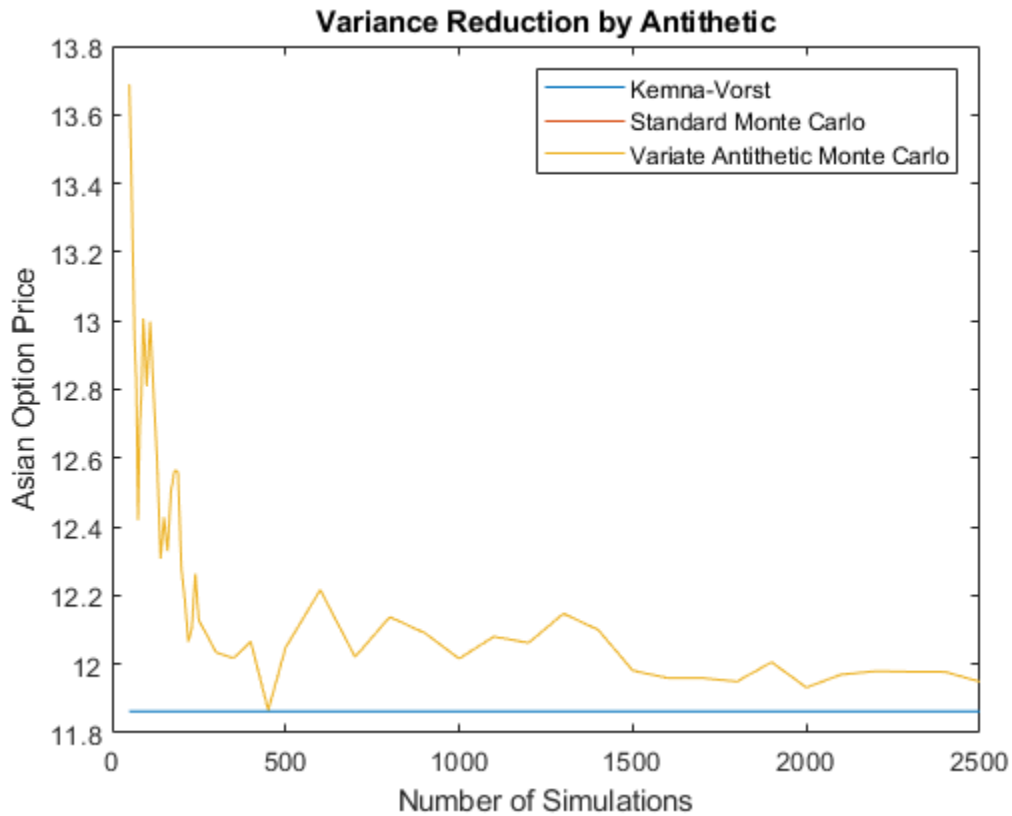
    idx = idx+1;
end

```

```

figure('menubar', 'none', 'numbertitle', 'off')
plot(nTrials, [PriceKVVector PriceGMCVector PriceGMCantitheticVector]);
title 'Variance Reduction by Antithetic'
xlabel 'Number of Simulations'
ylabel 'Asian Option Price'
legend('Kemna-Vorst', 'Standard Monte Carlo', 'Variate Antithetic Monte Carlo ', 'locat

```



The graph above shows how oscillation in simulated price is reduced by using variate antithetic.



## Compare Pricing Model Results

Prices calculated by the Monte Carlo method varies depending on the outcome of the simulations. Increase NumTrials and analyze the results.

```
NumTrials = 2000;
```

```
PriceAMCAntithetic2000 = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Exerc...
    'NumTrials', NumTrials, 'NumPeriods', NumPeriods, 'Antithetic', Antithetic);
```

```
PriceGMCantithetic2000 = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,...
    ExerciseDates, 'NumTrials', NumTrials, 'NumPeriods', ...
    NumPeriods, 'Antithetic', Antithetic, 'AvgType', AvgType(2));
```

```
% Comparison of calculated Asian call prices
```

```
displayComparisonAsianCallPrices(PriceLevy, PriceTW, PriceHHM, PriceCRR40, PriceAMCAnt...
```

```
Comparison of Asian call prices:
```

```
Arithmetic Asian
```

```
Levy: 12.164734
Turnbull-Wakeman: 12.164734
Haug-Haug-Margrabe: 12.108746
Cox-Ross-Rubinstein: 12.047243
Monte Carlo(500 trials): 12.304046
Monte Carlo(2000 trials): 12.196848
```

```
Geometric Asian
```

```
Kemna-Vorst: 11.862580
Cox-Ross-Rubinstein: 11.732037
Monte Carlo(500 trials): 12.048434
Monte Carlo(2000 trials): 11.932017
```

The table above contrasts the results from closed approximation models against price simulations implemented via CRR trees and Monte Carlo.

## Asian and Vanilla Call Options

Asian options are popular instruments since they tend to be less expensive than comparable Vanilla calls and puts. This is because the volatility in the average value of an underlier tends to be lower than the volatility of the value of the underlier itself.

The Financial Instruments Toolbox™ supports several algorithms for pricing vanilla options. Let us compare the price of Asian options against their Vanilla counterpart.

First, compute the price of a European Vanilla Option using the Black Scholes model.

```
PriceBLS = optstockbybls(RateSpec, StockSpec, Settle, ExerciseDates,...  
                        OptSpec, Strike);
```

```
% Comparison of calculated call prices.
```

```
displayComparisonVanillaAsian('Prices', PriceBLS, PriceKV, PriceLevy, PriceTW, PriceHMM)
```

```
Comparison of Vanilla and Asian Prices:
```

```
Vanilla BLS:           15.743809  
Asian Kemna-Vorst:    11.862580  
Asian Levy:           12.164734  
Asian Turnbull-Wakeman: 12.164734  
Asian Haug-Haug-Margrabe: 12.108746
```

Both geometric and arithmetic Asians price lower than their Vanilla counterpart.

You can analyze options prices at different levels of the underlying asset. Using the Financial Instruments Toolbox™, it is possible to observe the effect of different parameters on the price of the options. Consider for example, the effect of variations in the price of the underlying asset.

```
StockPrices = (50:5:150)';  
PriceBLS = nan(size(StockPrices));  
PriceKV = nan(size(StockPrices));  
PriceLevy = nan(size(StockPrices));  
PriceTW = nan(size(StockPrices));  
PriceHMM = nan(size(StockPrices));  
idx = 1;  
for So = StockPrices'  
    SP = stockspec(Sigma, So);  
    PriceBLS(idx) = optstockbybls(RateSpec, SP, Settle, ExerciseDates,...  
                                OptSpec, Strike);  
  
    PriceKV(idx) = asianbykv(RateSpec, SP, OptSpec, Strike, Settle,...  
                            ExerciseDates);  
  
    PriceLevy(idx) = asianbylevy(RateSpec, SP, OptSpec, Strike, Settle,...  
                                ExerciseDates);  
  
    PriceKV(idx) = asianbykv(RateSpec, SP, OptSpec, Strike, Settle,...  
                            ExerciseDates);  
  
    PriceKV(idx) = asianbykv(RateSpec, SP, OptSpec, Strike, Settle,...
```

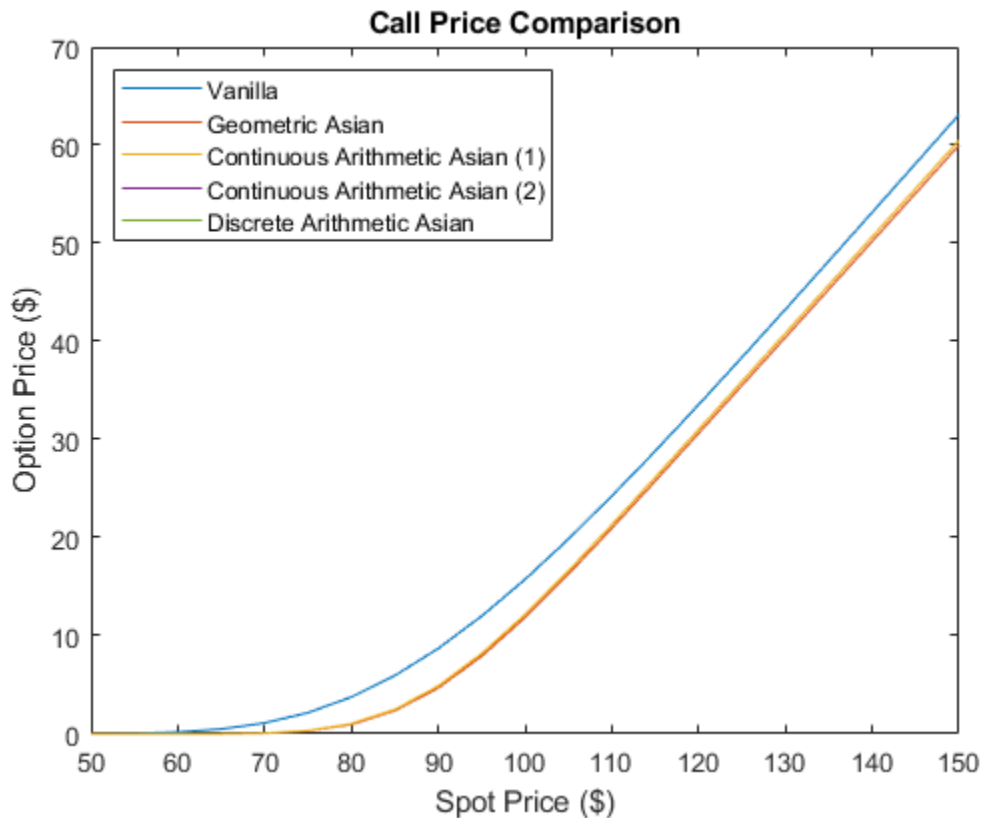
```

ExerciseDates);

    idx = idx+1;
end

figure('menubar', 'none', 'numbertitle', 'off')
plot(StockPrices, [PriceBLS PriceKV PriceLevy PriceTW PriceHHM]);
xlabel 'Spot Price ($)'
ylabel 'Option Price ($)'
title 'Call Price Comparison'
legend('Vanilla', 'Geometric Asian', 'Continuous Arithmetic Asian (1)', 'Continuous Ar

```



It can be observed that the price of the Asian option is cheaper than the price of the Vanilla option.

Also, it is possible to observe the effect of changes in the volatility of the underlying asset. The table below shows what happens to Asian and Vanilla option prices when the constant volatility changes.

Call Option (ITM)

Strike = 90 AssetPrice = 100

-----  
 -----

Volatility Haug-Haug-Margrabe Turnbull-Wakeman Levy Kemna-Vorst BLS

10% 11.3946 11.3987 11.3987 11.3121 13.4343

20% 12.1087 12.1647 12.1647 11.8626 15.7438

30% 13.5374 13.6512 13.6512 13.0338 18.8770

40% 15.2823 15.4464 15.4464 14.4086 22.2507

A comparison of the calculated prices show that Asian options are less sensitive to volatility changes, since averaging reduces the volatility of the value of the underlying asset. Also, Asian options that use arithmetic average are more expensive than those that use geometric average.

Now, examine the effect of strike on option prices.

```

Strikes = (90:5:120)';
NStrike = length(Strikes);
PriceBLS = nan(size(Strikes));
PriceKV = nan(size(Strikes));
PriceLevy = nan(size(Strikes));
PriceTW = nan(size(Strikes));
PriceHHM = nan(size(Strikes));
idx = 1;
for ST = Strikes'
    SP = stockspec(Sigma, AssetPrice);
    PriceBLS(idx) = optstockbyblys(RateSpec, SP, Settle, ExerciseDates,...
        OptSpec, ST);

    PriceKV(idx) = asianbykv(RateSpec, SP, OptSpec, ST, Settle,...
        ExerciseDates);

```

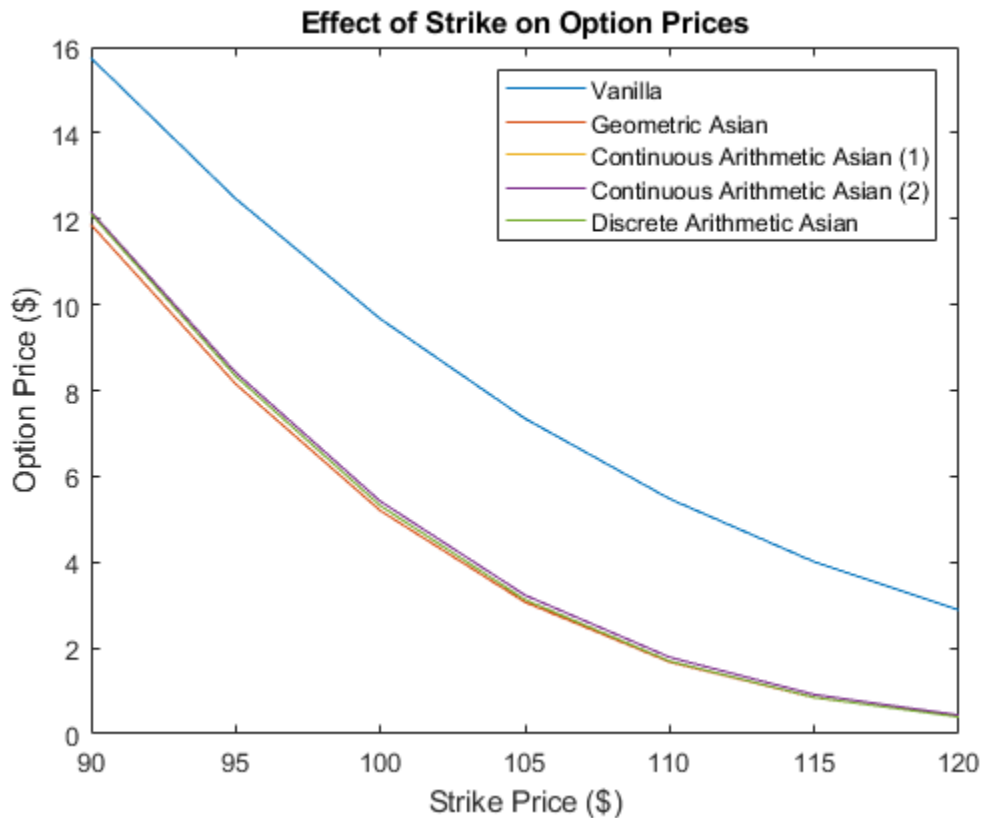
```
PriceLevy(idx) = asianbylevy(RateSpec, SP, OptSpec, ST, Settle,...
                             ExerciseDates);

PriceTW(idx) = asianbytw(RateSpec, SP, OptSpec, ST, Settle,...
                         ExerciseDates);

PriceHHM(idx) = asianbyhbm(RateSpec, SP, OptSpec, ST, Settle,...
                           ExerciseDates);

    idx = idx+1;
end

figure('menubar', 'none', 'numbertitle', 'off')
plot(Strike, [PriceBLS PriceKV PriceLevy PriceTW PriceHHM]);
xlabel 'Strike Price ($)'
ylabel 'Option Price ($)'
title 'Effect of Strike on Option Prices'
legend('Vanilla', 'Geometric Asian', 'Continuous Arithmetic Asian (1)', 'Continuous Ar
```



The figure above displays the option price with respect to strike price. Since call option value decreases as strike price increases, the Asian call curve is under the Vanilla call curve. It can be observed that the Asian call option is less expensive than the Vanilla call.

### Hedging

Hedging is an insurance to minimize exposure to market movements on the value of a position or portfolio. As the underlying changes, the proportions of the instruments forming the portfolio may need to be adjusted to keep the sensitivities within the desired range. Delta measures the option price sensitivity to changes in the price of the underlying.

Assume that you have a portfolio of two options with the same strike and maturity. You can use the Financial Instruments Toolbox™ to compute Delta for the Vanilla and Average Price options.

```
OutSpec = 'Delta';
```

```
% Vanilla option using Black Scholes
```

```
DeltaBLS = optstocksensbybls(RateSpec, StockSpec, Settle, ExerciseDates,...
                             OptSpec, Strike, 'OutSpec', OutSpec);
```

```
% Asian option using Kemna-Vorst method
```

```
DeltaKV = asiansensbykv(RateSpec, StockSpec, OptSpec, Strike, Settle,...
                        ExerciseDates, 'OutSpec', OutSpec);
```

```
% Asian option using Levy model
```

```
DeltaLevy = asiansensbylevy(RateSpec, StockSpec, OptSpec, Strike, Settle,...
                             ExerciseDates, 'OutSpec', OutSpec);
```

```
% Asian option using Turnbull-Wakeman model
```

```
DeltaTW = asiansensbytw(RateSpec, StockSpec, OptSpec, Strike, Settle,...
                         ExerciseDates, 'OutSpec', OutSpec);
```

```
% Asian option using Haug-Haug-Margrabe model
```

```
DeltaHHM = asiansensbyhmm(RateSpec, StockSpec, OptSpec, Strike, Settle,...
                            ExerciseDates, 'OutSpec', OutSpec);
```

```
% Delta Comparison
```

```
displayComparisonVanillaAsian('Delta', DeltaBLS, DeltaKV, DeltaLevy, DeltaTW, DeltaHHM)
```

```
Comparison of Vanilla and Asian Delta:
```

```
Vanilla BLS:          0.788666
Asian Kemna-Vorst:   0.844986
Asian Levy:          0.852806
Asian Turnbull-Wakeman: 0.852806
Asian Haug-Haug-Margrabe: 0.857864
```

The following graph demonstrates the behavior of Delta for the Vanilla and Asian options as a function of the underlying price.

```
StockPrices = (40:5:120)';
NStockPrices = length(StockPrices);
DeltaBLS = nan(size(StockPrices));
DeltaKV = nan(size(StockPrices));
DeltaLevy = nan(size(StockPrices));
```

```
DeltaTW = nan(size(StockPrices));
DeltaHHM = nan(size(StockPrices));

idx = 1;
for SPrices = StockPrices'
    SP = stockspec(Sigma, SPrices);
    DeltaBLS(idx) = optstocksensbybls(RateSpec, SP, Settle, ...
        ExerciseDates, OptSpec, Strike, 'OutSpec', OutSpec);

    DeltaKV(idx) = asiandsensbykv(RateSpec, SP, OptSpec, Strike, ...
        Settle, ExerciseDates, 'OutSpec', OutSpec);

    DeltaLevy(idx) = asiandsensbylevy(RateSpec, SP, OptSpec, Strike, ...
        Settle, ExerciseDates, 'OutSpec', OutSpec);

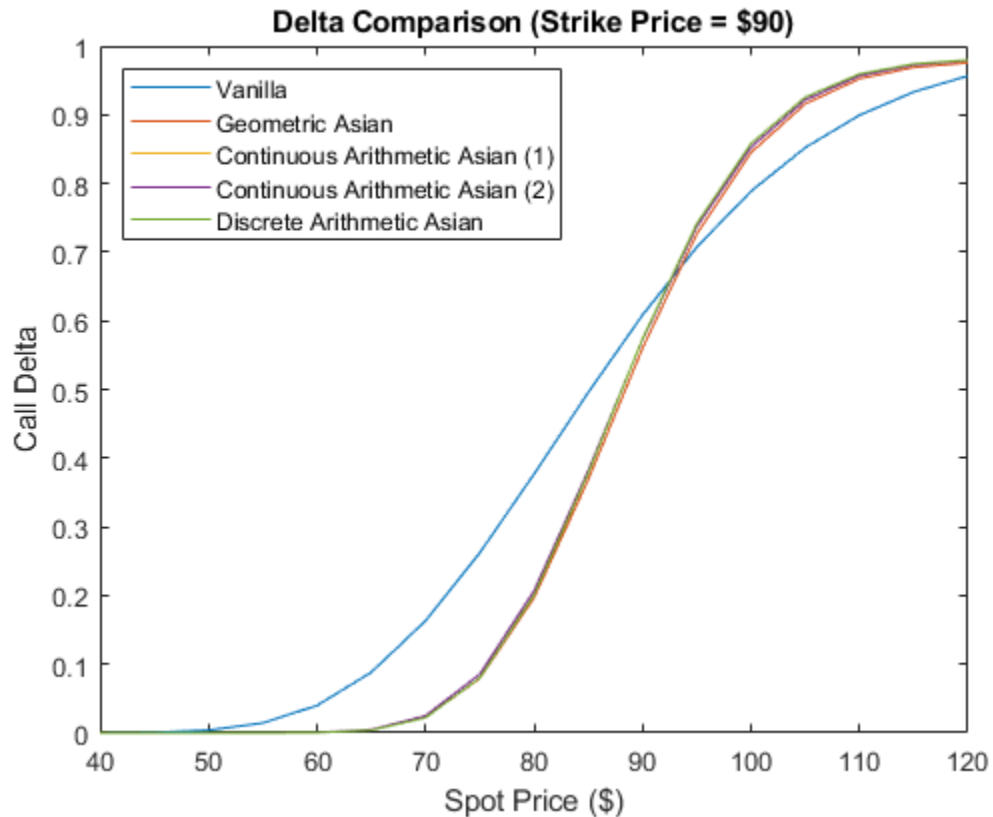
    DeltaTW(idx) = asiandsensbytw(RateSpec, SP, OptSpec, Strike, ...
        Settle, ExerciseDates, 'OutSpec', OutSpec);

    DeltaHHM(idx) = asiandsensbyhhm(RateSpec, SP, OptSpec, Strike, ...
        Settle, ExerciseDates, 'OutSpec', OutSpec);

    idx = idx+1;
end

figure('menubar', 'none', 'numbertitle', 'off')
plot(StockPrices, [DeltaBLS DeltaKV DeltaLevy DeltaTW DeltaHHM]);
xlabel 'Spot Price ($)'
ylabel 'Call Delta'
title 'Delta Comparison (Strike Price = $90)'
legend('Vanilla', 'Geometric Asian', 'Continuous Arithmetic Asian (1)', 'Continuous Ar
```





A Vanilla, or Asian, in the money (ITM) call option is more sensitive to price movements than an out of the money (OTM) option. If the asset price is deep in the money, then it is more likely to be exercised. The opposite occurs for an out of the money option. Asian delta is lower for out of the money options and is higher for in the money options than its Vanilla European counterpart. The geometric Asian delta is lower than the arithmetic Asian delta.

### References

[1] Kemna, A. and Vorst, A. "A Pricing Method for Options Based on Average Asset Values." *Journal of Banking and Finance*. Vol. 14, 1990, pp. 113-129.

[2] Levy, E. "Pricing European Average Rate Currency Options." *Journal of International Money and Finance*. Vol. 11,1992, pp. 474-491.

[3] Turnbull, S. M., and Wakeman, L.M. "A Quick Algorithm for Pricing European Average Options." *Journal of Financial and Quantitative Analysis*. Vol. 26(3), 1991, pp. 377-389.

[4] Haug, E. G. *The Complete Guide to Option Pricing Formulas*. McGraw-Hill, New York, 2007.

#### Utility Functions

```
function displayPricesClosedForm(PriceKV, PriceLevy, PriceTW, PriceHHM)
fprintf('Comparison of Asian Arithmetic and Geometric Prices:\n');
fprintf('\n');
fprintf('Kemna-Vorst:           %f\n', PriceKV);
fprintf('Levy:                   %f\n', PriceLevy);
fprintf('Turnbull-Wakeman:      %f\n', PriceTW);
fprintf('Haug-Haug-Margrabe:    %f\n', PriceHHM);
end
```

```
function displayPricesCRR(PriceCRR20, PriceCRR40)
fprintf('Asian Prices using the CRR lattice model:\n');
fprintf('\n');
fprintf('PriceArithmetic(CRR20): %f\n', PriceCRR20(1));
fprintf('PriceArithmetic(CRR40): %f\n', PriceCRR40(1));
fprintf('PriceGeometric (CRR20): %f\n', PriceCRR20(2));
fprintf('PriceGeometric (CRR40): %f\n', PriceCRR40(2));
end
```

```
function displayPricesMonteCarlo(PriceAMC, PriceAMCAntithetic, PriceGMC, PriceGMCAntithetic)
fprintf('Asian Prices using Monte Carlo Method:\n');
fprintf('\n');
fprintf('Arithmetic Asian\n');
fprintf('Standard Monte Carlo:           %f\n', PriceAMC);
fprintf('Variate Antithetic Monte Carlo: %f\n\n', PriceAMCAntithetic);
fprintf('Geometric Asian\n');
fprintf('Standard Monte Carlo:           %f\n', PriceGMC);
fprintf('Variate Antithetic Monte Carlo: %f\n', PriceGMCAntithetic);
end
```

```
function displayComparisonAsianCallPrices(PriceLevy, PriceTW, PriceHHM, PriceCRR40, PriceCRR20)
fprintf('Comparison of Asian call prices:\n');
fprintf('\n');
fprintf('Arithmetic Asian\n');
```

```
fprintf('Levy: %f\n', PriceLevy);
fprintf('Turnbull-Wakeman: %f\n', PriceTW);
fprintf('Haug-Haug-Margrabe: %f\n', PriceHHM);
fprintf('Cox-Ross-Rubinstein: %f\n', PriceCRR40(1));
fprintf('Monte Carlo(500 trials): %f\n', PriceAMCAntithetic);
fprintf('Monte Carlo(2000 trials): %f\n', PriceAMCAntithetic2000);
fprintf('\n');
fprintf('Geometric Asian\n');
fprintf('Kemna-Vorst: %f\n', PriceKV);
fprintf('Cox-Ross-Rubinstein: %f\n', PriceCRR40(2));
fprintf('Monte Carlo(500 trials): %f\n', PriceGMCantithetic);
fprintf('Monte Carlo(2000 trials): %f\n', PriceGMCantithetic2000);
end

function displayComparisonVanillaAsian(type, BLS, KV, Levy, TW, HHM)
fprintf('Comparison of Vanilla and Asian %s:\n', type);
fprintf('\n');
fprintf('Vanilla BLS: %f\n', BLS);
fprintf('Asian Kemna-Vorst: %f\n', KV);
fprintf('Asian Levy: %f\n', Levy);
fprintf('Asian Turnbull-Wakeman: %f\n', TW);
fprintf('Asian Haug-Haug-Margrabe: %f\n', HHM);
end
```

## Pricing Equity Derivatives Using Trees

In this section...
"Computing Instrument Prices" on page 3-130
"Computing Prices Using CRR" on page 3-131
"Computing Prices Using EQP" on page 3-133
"Computing Prices Using ITT" on page 3-135
"Computing Prices Using STT" on page 3-137
"Examining Output from the Pricing Functions" on page 3-139
"Graphical Representation of Equity Derivative Trees" on page 3-142

### Computing Instrument Prices

The portfolio pricing functions `crrprice`, `eqpprice`, and `ittprice` calculate the price of any set of supported instruments based on a binary equity price tree, an implied trinomial price tree, or a standard trinomial tree. These functions are capable of pricing the following instrument types:

- Vanilla stock options
  - American and European puts and calls
- Exotic options
  - Asian
  - Barrier
  - Compound
  - Lookback
  - Stock options (Bermuda put and call schedules)

The syntax for calling the function `crrprice` is:

```
[Price, PriceTree] = crrprice(CRRTree, InstSet, Options)
```

The syntax for `eqpprice` is:

```
[Price, PriceTree] = eqpprice(EQPtree, InstSet, Options)
```

The syntax for `ittprice` is:

```
Price = ittprice(ITTree, ITInstSet, Options)
```

The syntax for `sttprice` is:

```
[Price, PriceTree] = sttprice(STTree, InstSet, Name, Value)
```

These functions require two input arguments: the equity price tree and the set of instruments, `InstSet`, and allow a third optional argument.

### Required Arguments

`CRRTree` is a CRR equity price tree created using `crrtree`. `EQPTree` is an equal probability equity price tree created using `eqptree`. `ITTree` is an ITT equity price tree created using `itttree`. `STTree` is a standard trinomial equity price tree created using `stttree`. See “Building Equity Binary Trees” on page 3-3 and “Building Implied Trinomial Trees” on page 3-8 to learn how to create these structures.

`InstSet` is a structure that represents the set of instruments to be priced independently using the model.

### Optional Argument

You can enter a third optional argument, `Options`, used when pricing barrier options. For more specific information, see “Pricing Options Structure” on page B-2.

These pricing functions internally classify the instruments and call the appropriate individual instrument pricing function for each of the instrument types. The CRR pricing functions are `asianbycrr`, `barrierbycrr`, `compoundbycrr`, `lookbackbycrr`, and `optstockbycrr`. A similar set of functions exists for EQP, ITT, and STT pricing. You can also use these functions directly to calculate the price of sets of instruments of the same type. See the reference pages for these individual functions for further information.

## Computing Prices Using CRR

Consider the following example, which uses the portfolio and stock price data in the MATLAB file `deriv.mat` included in the toolbox. Load the data into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

Name	Size	Bytes	Class	Attributes
BDTInstSet	1x1	27344	struct	
BDTree	1x1	7322	struct	
BKInstSet	1x1	27334	struct	
BKTree	1x1	8532	struct	
CRRInstSet	1x1	21066	struct	
CRRTree	1x1	7086	struct	
EQPInstSet	1x1	21066	struct	
EQPTree	1x1	7086	struct	
HJMInstSet	1x1	27336	struct	
HJMTree	1x1	8334	struct	
HWInstSet	1x1	27334	struct	
HWTree	1x1	8532	struct	
ITTIInstSet	1x1	21070	struct	
ITTree	1x1	12660	struct	
STTIInstSet	1x1	21070	struct	
STTree	1x1	7782	struct	
ZeroInstSet	1x1	17458	struct	
ZeroRateSpec	1x1	2152	struct	

`CRRTree` and `CRRInstSet` are the required input arguments to call the function `crrprice`.

Use `instdisp` to examine the set of instruments contained in the variable `CRRInstSet`.

`instdisp(CRRInstSet)`

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity			
1	OptStock	call	105	01-Jan-2003	01-Jan-2005	1	Call1	10			
2	OptStock	put	105	01-Jan-2003	01-Jan-2006	0	Put1	5			
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Barrier	Rebate	Name	Quantity
3	Barrier	call	105	01-Jan-2003	01-Jan-2006	1	ui	102	0	Barrier1	1
Index	Type	UOptSpec	UStrike	USettle	UExerciseDates	UAmericanOpt	COptSpec	CStrike	CSettle	CExerciseDates	
4	Compound	call	130	01-Jan-2003	01-Jan-2006	1	put	5	01-Jan-2003	01-Jan-2005	
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity			
5	Lookback	call	115	01-Jan-2003	01-Jan-2006	0	Lookback1	7			
6	Lookback	call	115	01-Jan-2003	01-Jan-2007	0	Lookback2	9			
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPrice	AvgDate	Name	Quantity
7	Asian	put	110	01-Jan-2003	01-Jan-2006	0	arithmetic	NaN	NaN	Asian1	4
8	Asian	put	110	01-Jan-2003	01-Jan-2007	0	arithmetic	NaN	NaN	Asian2	6

---

**Note** Because of space considerations, the compound option above (Index 4) has been condensed to fit the page. The `instdisp` command displays all compound option fields on your computer screen.

---

The instrument set contains eight instruments:

- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `crrprice`.

Now use `crrprice` to calculate the price of each instrument in the instrument set.

```
Price = crrprice(CRRTree, CRRInstSet)
```

```
Price =
    8.2863
    2.5016
   12.1272
    3.3241
    7.6015
   11.7772
    4.1797
    3.4219
```

## Computing Prices Using EQP

Load the data into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

Name	Size	Bytes	Class	Attributes
BDTInstSet	1x1	27344	struct	
BDTTree	1x1	7322	struct	
BKInstSet	1x1	27334	struct	
BKTree	1x1	8532	struct	
CRRInstSet	1x1	21066	struct	
CRRTree	1x1	7086	struct	
EQPInstSet	1x1	21066	struct	
EQPTree	1x1	7086	struct	

```

HJMInstSet      1x1      27336 struct
HJMTree         1x1      8334  struct
HWInstSet       1x1      27334 struct
HWTTree         1x1      8532  struct
ITTInstSet      1x1      21070 struct
ITTTTree        1x1      12660 struct
STTInstSet      1x1      21070 struct
STTTTree        1x1      7782  struct
ZeroInstSet     1x1      17458 struct
ZeroRateSpec    1x1      2152  struct

```

**EQPtree** and **EQPInstSet** are the input arguments required to call the function **eqpprice**.

Use the command **instdisp** to examine the set of instruments contained in the variable **EQPInstSet**.

**instdisp(EQPInstSet)**

```

Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt Name      Quantity
1      OptStock call   105   01-Jan-2003 01-Jan-2005 1          Call1 10
2      OptStock put   105   01-Jan-2003 01-Jan-2006 0          Put1  5

Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt BarrierSpec Barrier Rebate Name      Quantity
3      Barrier call   105   01-Jan-2003 01-Jan-2006 1          ui      102    0    Barrier1 1

Index Type      UOptSpec UStrike USettle      UExerciseDates UAmericanOpt COptSpec CStrike CSettle      CExerciseDates
4      Compound call   130   01-Jan-2003 01-Jan-2006 1          put    5      01-Jan-2003 01-Jan-2005

Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt Name      Quantity
5      Lookback call   115   01-Jan-2003 01-Jan-2006 0          Lookback1 7
6      Lookback call   115   01-Jan-2003 01-Jan-2007 0          Lookback2 9

Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt AvgType      AvgPrice AvgDate Name      Quantity
7      Asian put     110   01-Jan-2003 01-Jan-2006 0          arithmetic NaN    NaN    Asian1 4
8      Asian put     110   01-Jan-2003 01-Jan-2007 0          arithmetic NaN    NaN    Asian2 6

>> instdisp(EQPInstSet)
Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt Name      Quantity
1      OptStock call   105   01-Jan-2003 01-Jan-2005 1          Call1 10
2      OptStock put   105   01-Jan-2003 01-Jan-2006 0          Put1  5

Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt BarrierSpec Barrier Rebate Name      Quantity
3      Barrier call   105   01-Jan-2003 01-Jan-2006 1          ui      102    0    Barrier1 1

Index Type      UOptSpec UStrike USettle      UExerciseDates UAmericanOpt COptSpec CStrike CSettle      CExerciseDates
4      Compound call   130   01-Jan-2003 01-Jan-2006 1          put    5      01-Jan-2003 01-Jan-2005

Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt Name      Quantity
5      Lookback call   115   01-Jan-2003 01-Jan-2006 0          Lookback1 7
6      Lookback call   115   01-Jan-2003 01-Jan-2007 0          Lookback2 9

Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt AvgType      AvgPrice AvgDate Name      Quantity
7      Asian put     110   01-Jan-2003 01-Jan-2006 0          arithmetic NaN    NaN    Asian1 4
8      Asian put     110   01-Jan-2003 01-Jan-2007 0          arithmetic NaN    NaN    Asian2 6

```



---

**Note** Because of space considerations, the compound option above (Index 4) has been condensed to fit the page. The `instdisp` command displays all compound option fields on your computer screen.

---

The instrument set contains eight instruments:

- Two vanilla options (`Call1`, `Put1`)
- One barrier option (`Barrier1`)
- One compound option (`Compound1`)
- Two lookback options (`Lookback1`, `Lookback2`)
- Two Asian options (`Asian1`, `Asian2`)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `eqpprice`.

Now use `eqpprice` to calculate the price of each instrument in the instrument set.

```
Price = eqpprice(EQPTree, EQPInstSet)
```

```
Price =
    8.4791
    2.6375
   12.2632
    3.5091
    8.7941
   12.9577
    4.7444
    3.9178
```

## Computing Prices Using ITT

Consider the following example, which uses the portfolio and stock price data in the MAT-file `deriv.mat` included in the toolbox. Load the data into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

Name	Size	Bytes	Class	Attributes
------	------	-------	-------	------------

BDTInstSet	1x1	27344	struct
BDTTree	1x1	7322	struct
BKInstSet	1x1	27334	struct
BKTree	1x1	8532	struct
CRRInstSet	1x1	21066	struct
CRRTree	1x1	7086	struct
EQPInstSet	1x1	21066	struct
EQPTree	1x1	7086	struct
HJMInstSet	1x1	27336	struct
HJMTree	1x1	8334	struct
HWInstSet	1x1	27334	struct
HWTree	1x1	8532	struct
ITTIInstSet	1x1	21070	struct
ITTTree	1x1	12660	struct
STTIInstSet	1x1	21070	struct
STTTree	1x1	7782	struct
ZeroInstSet	1x1	17458	struct
ZeroRateSpec	1x1	2152	struct

ITTTree and ITTIInstSet are the input arguments required to call the function `ittpprice`. Use the command `instdisp` to examine the set of instruments contained in the variable `ITTIInstSet`.

`instdisp(ITTIInstSet)`

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
1	OptStock	call	95	01-Jan-2006	31-Dec-2008	1	Call1	10
2	OptStock	put	80	01-Jan-2006	01-Jan-2010	0	Put1	4

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Barrier	Rebate	Name	Quantity
3	Barrier	call	85	01-Jan-2006	31-Dec-2008	1	ui	115	0	Barrier1	1

Index	Type	UOptSpec	UStrike	USettle	UExerciseDates	UAmericanOpt	COptSpec	CStrike	CSettle	CExerciseDates
4	Compound	call	99	01-Jan-2006	01-Jan-2010	1	put	5	01-Jan-2006	01-Jan-2010

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
5	Lookback	call	85	01-Jan-2006	01-Jan-2008	0	Lookback1	7
6	Lookback	call	85	01-Jan-2006	01-Jan-2010	0	Lookback2	9

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPrice	AvgDate	Name	Quantity
7	Asian	call	55	01-Jan-2006	01-Jan-2008	0	arithmetic	NaN	NaN	Asian1	5
8	Asian	call	55	01-Jan-2006	01-Jan-2010	0	arithmetic	NaN	NaN	Asian2	7

The instrument set contains eight instruments:

- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `ittprice`.

Now use `ittprice` to calculate the price of each instrument in the instrument set.

```
Price = ittprice(ITTree, ITTInstSet)
```

```
Price =
    1.6506
   10.6832
    2.4074
    3.2294
    0.5426
    6.1845
    3.2052
    6.6074
```

## Computing Prices Using STT

Consider the following example, which uses the portfolio and stock price data in the MATLAB file `deriv.mat` included in the toolbox. Load the data into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MATLAB file.

Name	Size	Bytes	Class	Attributes
BDTInstSet	1x1	27344	struct	
BDTTree	1x1	7322	struct	
BKInstSet	1x1	27334	struct	
BKTree	1x1	8532	struct	
CRRInstSet	1x1	21066	struct	
CRRTree	1x1	7086	struct	
EQPInstSet	1x1	21066	struct	
EQPTree	1x1	7086	struct	
HJMInstSet	1x1	27336	struct	
HJMTree	1x1	8334	struct	
HWInstSet	1x1	27334	struct	
HWTTree	1x1	8532	struct	
ITTInstSet	1x1	21070	struct	
ITTree	1x1	12660	struct	
STTInstSet	1x1	21070	struct	
STTree	1x1	7782	struct	
ZeroInstSet	1x1	17458	struct	
ZeroRateSpec	1x1	2152	struct	

STTtree and STTInstSet are the input arguments required to call the function sttprice. Use the command instdisp to examine the set of instruments contained in the variable STTInstSet.

instdisp(STTInstSet)

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
1	OptStock	call	100	01-Jan-2009	01-Jan-2011	1	Call1	10
2	OptStock	put	80	01-Jan-2009	01-Jan-2012	0	Put1	5

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Barrier	Rebate	Name	Quantity
3	Barrier	call	105	01-Jan-2009	01-Jan-2012	1	ui	115	0	Barrier1	1

Index	Type	UOptSpec	UStrike	USettle	UExerciseDates	UAmericanOpt	COptSpec	CStrike	CSettle	CExerciseDates
4	Compound	call	95	01-Jan-2009	01-Jan-2012	1	put	5	01-Jan-2009	01-Jan-2011

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
5	Lookback	call	90	01-Jan-2009	01-Jan-2012	0	Lookback1	7
6	Lookback	call	95	01-Jan-2009	01-Jan-2013	0	Lookback2	9

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPrice	AvgDate	Name	Quantity
7	Asian	call	100	01-Jan-2009	01-Jan-2012	0	arithmetic	NaN	NaN	Asian1	4
8	Asian	call	100	01-Jan-2009	01-Jan-2013	0	arithmetic	NaN	NaN	Asian2	6

The instrument set contains eight instruments:

- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by sttprice.

Now use sttprice to calculate the price of each instrument in the instrument set.

Price = sttprice(STTtree, STTInstSet)

Price =

```

4.5025
3.0603
3.7977
1.7090
11.7296
12.9120

```

```
1.6905
2.6203
```

## Examining Output from the Pricing Functions

The prices in the output vector `Price` correspond to the prices at observation time zero (`tObs = 0`), which is defined as the valuation date of the equity tree. The instrument indexing within `Price` is the same as the indexing within `InstSet`.

In the CRR example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(CRRInstSet, 'FieldName', 'Name')
```

```
InstNames =
```

```
Call1
Put1
Barrier1
Compound1
Lookback1
Lookback2
Asian1
Asian2
```

So, in the `Price` vector, the fourth element, 3.3241, represents the price of the fourth instrument (`Compound1`), and the sixth element, 11.7772, represents the price of the sixth instrument (`Lookback2`).

In the ITT example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(ITTInstSet, 'FieldName', 'Name')
```

```
InstNames =
```

```
Call1
Put1
Barrier1
Compound1
Lookback1
Lookback2
Asian1
Asian2
```

So, in the `Price` vector, the first element, 1.650, represents the price of the first instrument (`Call1`), and the eighth elements, 6.607, represents the price of the eighth instrument (`Asian2`).

#### Price Tree Output for CRR

If you call a pricing function with two output arguments, for example:

```
[Price, PriceTree] = crrprice(CRRTree, CRRInstSet)
```

you generate a price tree structure along with the price information.

This price tree structure `PriceTree` holds all pricing information.

```
PriceTree =  
FinObj: 'BinPriceTree'  
PTree: {[8x1 double] [8x2 double] [8x3 double] [8x4 double] [8x5 double]}  
tObs: [0 1 2 3 4]  
dObs: [731582 731947 732313 732678 733043]
```

The first field of this structure, `FinObj`, indicates that this structure represents a price tree. The second field, `PTree`, is the tree holding the prices of the instruments in each node of the tree. Finally, the third and fourth fields, `tObs` and `dObs`, represent the observation time and date of each level of `PTree`, with `tObs` using units in terms of compounding periods.

Using the command-line interface, you can directly examine `PriceTree.PTree`, the field within the `PriceTree` structure that contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
PriceTree.PTree{1}
```

```
ans =  
8.2863  
2.5016  
12.1272  
3.3241  
7.6015  
11.7772  
4.1797  
3.4219
```

With this interface, you can observe the prices for all instruments in the portfolio at a specific time.

The function `eqpprice` also returns a price tree that you can examine in the same way.

### Price Tree Output for ITT

If you call a pricing function with two output arguments, for example:

```
[Price, PriceTree] = ittprice(ITTree, ITInstSet)
```

you generate a price tree structure along with the price information.

This price tree structure `PriceTree` holds all pricing information.

```
PriceTree =
```

```
FinObj: 'TrinPriceTree'
PTree: {[8x1 double] [8x3 double] [8x5 double] [8x7 double] [8x9 double]}
tObs: [0 1 2 3 4]
dObs: [732678 733043 733408 733773 734139]
```

The first field of this structure, `FinObj`, indicates that this structure represents a trinomial price tree. The second field, `PTree` is the tree holding the prices of the instruments in each node of the tree. Finally, the third and fourth fields, `tObs` and `dObs`, represent the observation time and date of each level of `PTree`, with `tObs` using units in terms of compounding periods.

Using the command-line interface, you can directly examine `PriceTree.PTree`, the field within the `PriceTree` structure that contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
PriceTree.PTree{1}
```

```
ans =
```

```
1.6506
10.6832
2.4074
3.2294
0.5426
6.1845
3.2052
6.6074
```

With this interface, you can observe the prices for all instruments in the portfolio at a specific time.

### Prices for Lookback and Asian Options for Equity Trees

Lookback options and Asian options are path-dependent, and, as such, there are no unique prices for any node except the root node. So, the corresponding values for

lookback and Asian options in the price tree are set to NaN, the only exception being the root node. This becomes apparent if you examine the prices in the second node (`tobs = 1`) of the CRR price tree:

```
PriceTree.PTree{2}
```

```
ans =
```

```
11.9176      0
 0.9508      7.1914
16.4600      2.6672
 2.5896      5.0000
   NaN      NaN
   NaN      NaN
   NaN      NaN
   NaN      NaN
```

Examining the prices in the second node (`tobs = 1`) of the ITT price tree displays:

```
PriceTree.PTree{2}
```

```
ans =
```

```
3.9022      0      0
6.3736     13.3743    22.1915
5.6914      0      0
2.7663      3.8594    5.0000
   NaN      NaN      NaN
   NaN      NaN      NaN
   NaN      NaN      NaN
   NaN      NaN      NaN
```

## Graphical Representation of Equity Derivative Trees

You can use the function `treeviewer` to display a graphical representation of a tree, allowing you to examine interactively the prices and rates on the nodes of the tree until maturity. The graphical representations of CRR, EQP, and LR trees are equivalent to Black-Derman-Toy (BDT) trees, given that they are all binary recombining trees. The graphical representations of ITT and STT trees are equivalent to Hull-White (HW) trees, given that they are all trinomial recombining trees. See “Graphical Representation of Trees” on page 2-157 for an overview on the use of `treeviewer` with CRR trees, EQP trees, LR trees, ITT trees, and STT trees and their corresponding option price trees. Follow the instructions for BDT trees.



## See Also

asianbycrr | asianbyeqp | asianbyitt | asianbystt | barrierbycrr | barrierbyeqp | barrierbyitt | barrierbystt | compoundbycrr | compoundbyeqp | compoundbyitt | compoundbystt | crrprice | crrsens | crrtimespec | crrtree | eqpprice | eqpsens | eqptimespec | eqptree | instasian | instbarrier | instcompound | instlookback | instoptstock | ittprice | ittens | itttimespec | itttree | lookbackbycrr | lookbackbyeqp | lookbackbyitt | lookbackbystt | lrtimespec | lrtree | optstockbycrr | optstockbyeqp | optstockbyitt | optstockbylr | optstockbystt | optstocksensbylr | stockspec | sttprice | sttsens | treepath | trintreepath

## Related Examples

- “Understanding Equity Trees” on page 3-2
- “Computing Equity Instrument Sensitivities” on page 3-144
- “Creating Instruments or Properties” on page 1-21
- “Graphical Representation of Equity Derivative Trees” on page 3-142
- “Pricing European Call Options Using Different Equity Models” on page 3-163
- “Pricing Asian Options” on page 3-113

## More About

- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44
- “Supported Interest-Rate Instruments” on page 2-2

## Computing Equity Instrument Sensitivities

Sensitivities can be reported either as dollar price changes or percentage price changes. The delta, gamma, and vega sensitivities that the toolbox computes are dollar sensitivities.

The functions `crrsens`, `eqpsens`, `ittsens`, and `sttsens` compute the delta, gamma, and vega sensitivities of instruments using a stock tree. They also optionally return the calculated price for each instrument. The sensitivity functions require the same two input arguments used by the pricing functions (`CRRTree` and `CRRInstSet` for CRR, `EQPTree` and `EQPInstSet` for EQP, `ITTTree` and `ITTInstSet` for ITT, and `STTTree` and `STTInstSet` for STT).

As with the instrument pricing functions, the optional input argument `Options` is also allowed. You would include this argument if you want a sensitivity function to generate a price for a barrier option as one of its outputs and want to control the method that the toolbox uses to perform the pricing operation. See “Pricing Options Structure” on page B-2 or the `derivset` function for more information.

For path-dependent options (lookback and Asian), delta and gamma are computed by finite differences in calls to `crrprice`, `eqpprice`, `ittprice`, and `sttprice`. For the other options (stock option, barrier, and compound), delta and gamma are computed from the CRR, EQP, ITT, and STT trees and the corresponding option price tree. (See Chriss, Neil, *Black-Scholes and Beyond*, pp. 308-312.)

### CRR Sensitivities Example

The calling syntax for the sensitivity function is:

```
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, InstSet, Options)
```

Using the example data in `deriv.mat`, calculate the sensitivity of the instruments.

```
load deriv.mat  
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, CRRInstSet);
```

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

```
format bank  
All = [Delta, Gamma, Vega, Price]
```

```
All =
```

0.59	0.04	53.45	8.29
-0.31	0.03	67.00	2.50
0.69	0.03	67.00	12.13
-0.12	-0.01	-98.08	3.32
-0.40	-45926.32	88.18	7.60
-0.42	-112143.15	119.19	11.78
0.60	45926.32	49.21	4.18
0.82	112143.15	41.71	3.42

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in `CRRInstSet`. To view the per-dollar sensitivities, divide each dollar sensitivity by the corresponding instrument price.

```
All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]
```

```
All =
```

0.07	0.00	6.45	8.29
-0.12	0.01	26.78	2.50
0.06	0.00	5.53	12.13
-0.04	-0.00	-29.51	3.32
-0.05	-6041.77	11.60	7.60
-0.04	-9522.02	10.12	11.78
0.14	10987.98	11.77	4.18
0.24	32771.92	12.19	3.42

## ITT Sensitivities Example

The calling syntax for the sensitivity function is:

```
[Delta, Gamma, Vega, Price] = ittens(ITTree, ITInstSet, Options)
```

Using the example data in `deriv.mat`, calculate the sensitivity of the instruments.

```
load deriv.mat
```

```
warning('off', 'fininst:itttree:Extrapolation');
[Delta, Gamma, Vega, Price] = ittens(ITTree, ITInstSet);
```

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

```
format bank
All = [Delta, Gamma, Vega, Price]

All =
```

0.24	0.03	19.35	1.65
-0.43	0.02	49.69	10.68
0.35	0.04	12.29	2.41
-0.07	0.00	6.73	3.23
0.63	142945.66	38.90	0.54
0.60	22703.21	68.92	6.18
0.32	-142945.66	18.48	3.21
0.67	-22703.21	17.75	6.61

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in ITTInstSet.

---

**Note** In this example, the extrapolation warnings are turned off before calculating the sensitivities to avoid displaying many warnings on the Command Window as the sensitivities are calculated.

---

If the extrapolation warnings are turned on

```
warning('on', 'fininst:itttree:Extrapolation');
```

and ittsens is rerun, the extrapolation warnings scroll as the command executes:

```
[Delta, Gamma, Vega, Price] = ittsens(ITTree, ITTInstSet)
```

```
Warning: The option set specified in StockOptSpec was too narrow for the
generated tree.
This made extrapolation necessary. Below is a list of the options that were
outside of the
range of those specified in StockOptSpec.
```

```
Option Type: 'call'   Maturity: 01-Jan-2007   Strike=67.2897
Option Type: 'put'   Maturity: 01-Jan-2007   Strike=37.1528
Option Type: 'put'   Maturity: 01-Jan-2008   Strike=27.6066
Option Type: 'put'   Maturity: 31-Dec-2008   Strike=20.5132
Option Type: 'call'  Maturity: 01-Jan-2010   Strike=164.0157
Option Type: 'put'   Maturity: 01-Jan-2010   Strike=15.2424
```

```
> In itttree>InterpOptPrices (line 680)
   In itttree (line 285)
   In stocktreesens>stocktreevega (line 193)
   In stocktreesens (line 94)
   In ittsens (line 79)
```

```
Delta =
```

```

0.24
-0.43
0.35
-0.07
0.63
0.60
0.32
0.67

Gamma =

0.03
0.02
0.04
0.00
142945.66
22703.21
-142945.66
-22703.21

Vega =

19.35
49.69
12.29
6.73
38.90
68.92
18.48
17.75

Price =

1.65
10.68
2.41
3.23
0.54
6.18
3.21
6.61

```

These warnings are a consequence of having to extrapolate to find the option price of the tree nodes. In this example, the set of inputs options was too narrow for the shift in the tree nodes introduced by the disturbance used to calculate the sensitivities. As a consequence extrapolation for some of the nodes was needed. Since the input data is quite close the extrapolated data, the error introduced by extrapolation is fairly low.

### STT Sensitivities Example

The calling syntax for the sensitivity function is:

```
[Delta, Gamma, Vega, Price] = sttsens(STTtree, InstSet, Name, Value)
```

Using the example data in `deriv.mat`, calculate the sensitivity of the instruments.

```
load deriv.mat
[Delta, Gamma, Vega, Price] = sttsens(STTTree, STTInstSet);
```

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

```
format bank
All = [Delta, Gamma, Vega, Price]
```

```
All =
```

0.53	0.02	52.90	4.50
-0.09	0.00	42.44	3.06
0.47	0.03	25.98	3.80
-0.06	0.00	-9.53	1.71
0.23	-186495.25	70.38	11.73
0.33	-191186.43	92.92	12.91
0.57	186495.25	25.81	1.69
0.66	191186.43	37.88	2.62

## See Also

asianbycrr | asianbyeqp | asianbyitt | asianbystt | barrierbycrr | barrierbyeqp | barrierbyitt | barrierbystt | compoundbycrr | compoundbyeqp | compoundbyitt | compoundbystt | crrprice | crrsens | crrtimespec | crrtree | eqpprice | eqpsens | eqptimespec | eqptree | instasian | instbarrier | instcompound | instlookback | instoptstock | ittprice | ittens | itttimespec | itttree | lookbackbycrr | lookbackbyeqp | lookbackbyitt | lookbackbystt | lrtimespec | lrtree | optstockbycrr | optstockbyeqp | optstockbyitt | optstockbylr | optstockbystt | optstocksensbylr | stockspect | sttprice | sttsens | treepath | trintreepath

## Related Examples

- “Understanding Equity Trees” on page 3-2
- “Pricing Equity Derivatives Using Trees” on page 3-130
- “Graphical Representation of Equity Derivative Trees” on page 3-142
- “Creating Instruments or Properties” on page 1-21

- “Graphical Representation of Equity Derivative Trees” on page 3-142
- “Pricing European Call Options Using Different Equity Models” on page 3-163
- “Pricing Asian Options” on page 3-113

### **More About**

- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44
- “Supported Interest-Rate Instruments” on page 2-2

## Equity Derivatives Using Closed-Form Solutions

### In this section...

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“Black-Scholes Model” on page 3-150

“Black Model” on page 3-151

“Roll-Geske-Whaley Model” on page 3-152

“Bjerk Sund-Stensland 2002 Model” on page 3-153

“Barone-Adesi-Whaley Model” on page 3-153

“Pricing Using the Black-Scholes Model” on page 3-154

“Pricing Using the Black Model” on page 3-156

“Pricing Using the Roll-Geske-Whaley Model” on page 3-157

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“Compute American Option Prices Using the Barone-Adesi and Whaley Option Pricing Model” on page 3-160

### Introduction

Financial Instruments Toolbox supports four types of closed-form solutions and analytical approximations to calculate price and sensitivities (greeks) of vanilla options:

- Black-Scholes model
- Black model
- Roll-Geske-Whaley model
- Bjerk Sund-Stensland 2002 model

### Black-Scholes Model

The Black-Scholes model is one of the most commonly used models to price European calls and puts. It serves as a basis for many closed-form solutions used for pricing options. The standard Black-Scholes model is based on the following assumptions:

- There are no dividends paid during the life of the option.



- The option can only be exercised at maturity.
- The markets operate under a Markov process in continuous time.
- No commissions are paid.
- The risk-free interest rate is known and constant.
- Returns on the underlying stocks are log-normally distributed.

---

**Note** The Black-Scholes model implemented in Financial Instruments Toolbox software allows dividends. The following three dividend methods are supported:

- Cash dividend
- Continuous dividend yield
- Constant dividend yield

However, not all Black-Scholes closed-form pricing functions support all three dividend methods. For more information on specifying the dividend methods, see `stockspec`.

---

Closed-form solutions based on a Black-Scholes model support the following tasks.

<b>Task</b>	<b>Function</b>
Price European options with different dividends using the Black-Scholes option pricing model.	<code>optstockbybls</code>
Calculate European option prices and sensitivities using the Black-Scholes option pricing model.	<code>optstocksensbybls</code>
Calculate implied volatility on European options using the Black-Scholes option pricing model.	<code>impvbybls</code>
Price European simple chooser options using Black-Scholes model.	<code>chooserbybls</code>

For an example using the Black-Scholes model, see “Pricing Using the Black-Scholes Model” on page 3-154.

## Black Model

Use the Black model for pricing European options on physical commodities, forwards or futures. The Black model supported by Financial Instruments Toolbox software is a

special case of the Black-Scholes model. The Black model uses a forward price as an underlier in place of a spot price. The assumption is that the forward price at maturity of the option is log-normally distributed.

Closed-form solutions for a Black model support the following tasks.

<b>Task</b>	<b>Function</b>
Price European options on futures using the Black option pricing model.	optstockbyblk
Calculate European option prices and sensitivities on futures using the Black option pricing model.	optstocksensbyblk
Calculate implied volatility for European options using the Black option pricing model.	impvbyblk

For an example using the Black model, see “Pricing Using the Black Model” on page 3-156.

## **Roll-Geske-Whaley Model**

Use the Roll-Geske-Whaley approximation method to price American call options paying a single cash dividend. This model is based on the modification of the observed stock price for the present value of the dividend and also supports a compound option to account for the possibility of early exercise. The Roll-Geske-Whaley model has drawbacks due to an escrowed dividend price approach which may lead to arbitrage. For further explanation, see *Options, Futures, and Other Derivatives* by John Hull.

Closed-form solutions for a Roll-Geske-Whaley model support the following tasks.

<b>Task</b>	<b>Function</b>
Price American call options with a single cash dividend using the Roll-Geske-Whaley option pricing model.	optstockbyrgw
Calculate American call prices and sensitivities using the Roll-Geske-Whaley option pricing model.	optstocksensbyrgw
Calculate implied volatility for American call options using the Roll-Geske-Whaley option pricing model.	impvbyrgw

For an example using the Roll-Geske-Whaley model, see “Pricing Using the Roll-Geske-Whaley Model” on page 3-157.

## Bjerksund-Stensland 2002 Model

Use the Bjerksund-Stensland 2002 model for pricing American puts and calls with continuous dividend yield. This model works by dividing the time to maturity of the option in two separate parts, each with its own flat exercise boundary (trigger price). The Bjerksund-Stensland 2002 method is a generalization of the Bjerksund and Stensland 1993 method and is considered to be computationally efficient. For further explanation, see *Closed Form Valuation of American Options* by Bjerksund and Stensland.

Closed-form solutions for a Bjerksund-Stensland 2002 model support the following tasks.

Task	Function
Price American options with continuous dividend yield using the Bjerksund-Stensland 2002 option pricing model.	optstockbybjs
Calculate American options prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model.	optstocksensbybjs
Calculate implied volatility for American options using the Bjerksund-Stensland 2002 option pricing model.	impvbybjs

For an example using the Bjerksund-Stensland 2002 model, see “Pricing Using the Bjerksund-Stensland Model” on page 3-158.

## Barone-Adesi-Whaley Model

The Barone-Adesi-Whaley model is used for pricing American vanilla options. Closed-form solutions for a Barone-Adesi-Whaley model support the following tasks.

Task	Function
Calculate the prices of an American call and put options using the Barone-Adesi-Whaley approximation model.	optstockbybaw

Task	Function
Calculate the prices and sensitivities of an American call and put options using the Barone-Adesi-Whaley approximation model.	optstocksensbybaw
Calculate the implied volatility for American options using the Barone-Adesi-Whaley model.	impvbybaw

For an example using the Barone-Adesi-Whaley model, see “Compute American Option Prices Using the Barone-Adesi and Whaley Option Pricing Model” on page 3-160.

## Pricing Using the Black-Scholes Model

Consider a European stock option with an exercise price of \$40 on January 1, 2008 that expires on July 1, 2008. Assume that the underlying stock pays dividends of \$0.50 on March 1 and June 1. The stock is trading at \$40 and has a volatility of 30% per annum. The risk-free rate is 4% per annum. Using this data, calculate the price of a call and a put option on the stock using the Black-Scholes option pricing model:

```
Strike = 40;
AssetPrice = 40;
Sigma = .3;
Rates = 0.04;
Settle = 'Jan-01-08';
Maturity = 'Jul-01-08';
```

```
Div1 = 'March-01-2008';
Div2 = 'Jun-01-2008';
```

Create RateSpec and StockSpec:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...
Maturity, 'Rates', Rates, 'Compounding', -1);
```

```
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, 0.50, {Div1, Div2});
```

Define two options, one call and one put:

```
OptSpec = {'call'; 'put'};
```

Calculate the price of the European options:

```
Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
```

```
Price =
```

```

3.2063
3.4027

```

The first element of the `Price` vector represents the price of the call (\$3.21); the second is the price of the put (\$3.40). Use the function `optstocksensbybls` to compute six sensitivities for the Black-Scholes model: `delta`, `gamma`, `vega`, `lambda`, `rho`, and `theta` and the price of the option.

The selection of output parameters and their order is determined by the optional input parameter `OutSpec`. This parameter is a cell array of character vectors, each one specifying a desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the character vectors contained in `OutSpec`.

As an example, consider the same options as the previous example. To calculate their `Delta`, `Rho`, `Price`, and `Gamma`, build the cell array `OutSpec` as follows:

```

OutSpec = {'delta', 'rho', 'price', 'gamma'};

[Delta, Rho, Price, Gamma] =optstocksensbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)

```

Delta =

```

0.5328
-0.4672

```

Rho =

```

8.7902
-10.8138

```

Price =

```

3.2063
3.4027

```

Gamma =

```

0.0480
0.0480

```

## Pricing Using the Black Model

Consider two European call options on a futures contract with exercise prices of \$20 and \$25 that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at \$20 and has a volatility of 35% per annum. The risk-free rate is 4% per annum. Using this data, calculate the price of the call futures options using the Black model:

```
Strike = [20; 25];
AssetPrice = 20;
Sigma = .35;
Rates = 0.04;
Settle = 'May-01-08';
Maturity = 'Sep-01-08';
```

Create RateSpec and StockSpec:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);
```

```
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the call option:

```
OptSpec = {'call'};
```

Calculate price and all sensitivities of the European futures options:

```
OutSpec = {'All'}
```

```
[Delta, Gamma, Vega, Lambda, Rho, Theta, Price] = optstocksensbyblk(RateSpec, ...
StockSpec, Settle, Maturity, OptSpec, Strike, 'OutSpec', OutSpec);
```

```
Price =
```

```
1.5903
```

```
0.3037
```

The first element of the Price vector represents the price of the call with an exercise price of \$20 (\$1.59); the second is the price of the call with an exercise price of \$25 (\$2.89).

The function `impvbyblk` is used to compute the implied volatility using the Black option pricing model. Assuming that the previous European call futures are trading at \$1.5903 and \$0.3037, you can calculate their implied volatility:

```
Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, ...
OptSpec, Strike, Price);
```

As expected, you get volatilities of 35%. If the call futures were trading at \$1.50 and \$0.50 in the market, the implied volatility would be 33% and 42%:

```
Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, Strike, [1.50;0.5])
```

```
Volatility =
```

```
    0.3301
    0.4148
```

## Pricing Using the Roll-Geske-Whaley Model

Consider two American call options, with exercise prices of \$110 and \$100 on June 1, 2008, that expire on June 1, 2009. Assume that the underlying stock pays dividends of \$0.001 on December 1, 2008. The stock is trading at \$80 and has a volatility of 20% per annum. The risk-free rate is 6% per annum. Using this data, calculate the price of the American calls using the Roll-Geske-Whaley option pricing model:

```
AssetPrice = 80;
Settle = 'Jun-01-2008';
Maturity = 'Jun-01-2009';
Strike = [110; 100];
```

```
Rate = 0.06;
Sigma = 0.2;
```

```
DivAmount = 0.001;
DivDate = 'Dec-01-2008';
```

Create RateSpec and StockSpec:

```
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);
```

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

Calculate the call prices:

```
Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike)
```

```
Price =
```

```
    0.8398
    2.0236
```

The first element of the `Price` vector represents the price of the call with an exercise price of \$110 (\$0.84); the second is the price of the call with an exercise price of \$100 (\$2.02).

## Pricing Using the Bjerk Sund-Stensland Model

Consider four American stock options (two calls and two puts) with an exercise price of \$100 that expire on July 1, 2008. Assume that the underlying stock pays a continuous dividend yield of 4% as of January 1, 2008. The stock has a volatility of 20% per annum and the risk-free rate is 8% per annum. Using this data, calculate the price of the American calls and puts assuming the following current prices of the stock: \$80, \$90 (for the calls) and \$100 and \$110 (for the puts):

```
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Strike = 100;
AssetPrice = [80; 90; 100; 110];
DivYield = 0.04;
```

```
Rate = 0.08;
Sigma = 0.20;
```

Create `RateSpec` and `StockSpec`:

```
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

Define the option type:

```
OptSpec = {'call'; 'call'; 'put'; 'put'};
```

Compute the option prices:

```
Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
```

```
Price =
    0.4144
    2.1804
    4.7253
    1.7164
```

The first two elements of the `Price` vector represent the price of the calls (\$0.41 and \$2.18), the last two elements represent the price of the put options (\$4.72 and \$1.72).



Use the function `optstocksensbybjs` to compute six sensitivities for the Bjerksund-Stensland model: `delta`, `gamma`, `vega`, `lambda`, `rho`, and `theta` and the price of the option. The selection of output parameters and their order is determined by the optional input parameter `OutSpec`. This parameter is a cell array of character vectors, each one specifying a desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the character vectors contained in `OutSpec`. As an example, consider the same options as the previous example. To calculate their `delta`, `gamma`, and `price`, build the cell array `OutSpec` as follows:

```
OutSpec = {'delta', 'gamma', 'price'};
```

The outputs of `optstocksensbybjs` are in the same order as in `OutSpec`.

```
[Delta, Gamma, Price]= optstocksensbybjs(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)
```

```
Delta =
```

```
    0.0843
    0.2912
    0.4803
    0.2261
```

```
Gamma =
```

```
    0.0136
    0.0267
    0.0304
    0.0217
```

```
Price =
```

```
    0.4144
    2.1804
    4.7253
    1.7164
```

For more information on the Bjerksund-Stensland model, see “Closed-Form Solutions Modeling” on page C-4.

## Compute American Option Prices Using the Barone-Adesi and Whaley Option Pricing Model

Consider an American call option with an exercise price of \$120. The option expires on Jan 1, 2018. The stock has a volatility of 14% per annum, and the annualized continuously compounded risk-free rate is 4% per annum as of Jan 1, 2016. Using this data, calculate the price of the American call, assuming the price of the stock is \$125 and pays a dividend of 2%.

```
StartDate = 'Jan-1-2016';
EndDate = 'jan-1-2018';
Basis = 1;
Compounding = -1;
Rates = 0.04;
```

Define the RateSpec.

```
RateSpec = intenvset('ValuationDate',StartDate,'StartDate',StartDate,'EndDate',EndDate,
'Rates',Rates,'Basis',Basis,'Compounding',Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9231
    Rates: 0.0400
    EndTimes: 2
    StartTimes: 0
    EndDates: 737061
    StartDates: 736330
    ValuationDate: 736330
    Basis: 1
    EndMonthRule: 1
```

Define the StockSpec.

```
Dividend = 0.02;
AssetPrice = 125;
Volatility = 0.14;
```

```
StockSpec = stockspec(Volatility,AssetPrice,'Continuous',Dividend)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
```

```

        Sigma: 0.1400
    AssetPrice: 125
    DividendType: {'continuous'}
    DividendAmounts: 0.0200
    ExDividendDates: []

```

Define the American option.

```

OptSpec = 'call';
Strike = 120;
Settle = 'Jan-1-2016';
Maturity = 'jan-1-2018';

```

Compute the price for the American option.

```

Price = optstockbybaw(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike)

Price = 14.5180

```

## See Also

asianbykv | asianbylevy | asianbyls | asiainsensbykv | asiainsensbylevy | asiainsensbyls | assetbybls | assetsensbybls | basketbyju | basketbyls | basketsensbyju | basketsensbyls | basketstockspec | basketstockspec | cashbybls | cashsensbybls | chooserbybls | gapbybls | gapsensbybls | impvbybjs | impvbyblk | impvbybls | impvbyrgw | lookbackbycvgs | lookbackbyls | lookbacksensbycvgs | lookbacksensbyls | maxassetbystulz | maxassetsensbystulz | minassetbystulz | minassetsensbystulz | optpricebysim | optstockbybaw | optstockbybjs | optstockbyblk | optstockbybls | optstockbyls | optstockbyrgw | optstocksensbybaw | optstocksensbybjs | optstocksensbyblk | optstocksensbybls | optstocksensbyls | optstocksensbyrgw | spreadbybjs | spreadbykirk | spreadbyls | spreadsensbybjs | spreadsensbykirk | spreadsensbyls | supersharebybls | supersharesensbybls

## Related Examples

- “Pricing European Call Options Using Different Equity Models” on page 3-163
- “Compute the Option Price on a Future” on page 3-172

- “Pricing European Call Options Using Different Equity Models”
- “Pricing Asian Options” on page 3-113

### **More About**

- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Pricing European Call Options Using Different Equity Models

This example illustrates how the Financial Instruments Toolbox™ is used to price European vanilla call options using different equity models.

The example compares call option prices using the Cox-Ross-Rubinstein model, the Leisen-Reimer model and the Black-Scholes closed formula.

### Define the Call Instrument

Consider a European call option, with an exercise price of \$30 on January 1, 2010. The option expires on Sep 1, 2010. Assume that the underlying stock provides no dividends. The stock is trading at \$25 and has a volatility of 35% per annum. The annualized continuously compounded risk-free rate is 1.11% per annum.

```
% Option
Settle = 'Jan-01-2010';
Maturity = 'Sep-01-2010';
Strike = 30;
OptSpec = 'call';
```

```
% Stock
AssetPrice = 25;
Sigma = .35;
```

### Create the Interest Rate Term Structure

```
StartDates = '01 Jan 2010';
EndDates = '01 Jan 2013';
Rates = 0.0111;
ValuationDate = '01 Jan 2010';
Compounding = -1;
```

```
RateSpec = intenvset('Compounding',Compounding,'StartDates', StartDates,...
                    'EndDates', EndDates, 'Rates', Rates, 'ValuationDate', ValuationDate);
```

### Create the Stock Structure

Suppose we want to create two scenarios. The first one assumes that `AssetPrice` is currently \$25, the option is out of the money (OTM). The second scenario assumes that the option is at the money (ATM), and therefore `AssetPriceATM = 30`.

```
AssetPriceATM = 30;
```

```
StockSpec = stockspec(Sigma, AssetPrice);  
StockSpecATM = stockspec(Sigma, AssetPriceATM);
```

#### **Price the Options Using the Black-Scholes Closed Formula**

Use the function `optstockbybls` in the Financial Instruments Toolbox to compute the price of the European call options.

```
% Price the option with AssetPrice = 25  
PriceBLS = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike);
```

```
% Price the option with AssetPrice = 30  
PriceBLSATM = optstockbybls(RateSpec, StockSpecATM, Settle, Maturity, OptSpec, Strike);
```

#### **Build the Cox-Ross-Rubinstein Tree**

```
% Create the time specification of the tree  
NumPeriods = 15;
```

```
CRRTIMEspec = crrtimespec(ValuationDate, Maturity, NumPeriods);
```

```
% Build the tree  
CRRTree = crrtree(StockSpec, RateSpec, CRRTIMEspec);  
CRRTreeATM = crrtree(StockSpecATM, RateSpec, CRRTIMEspec);
```

#### **Build the Leisen-Reimer Tree**

```
% Create the time specification of the tree  
LRTIMEspec = lrtimespec(ValuationDate, Maturity, NumPeriods);
```

```
% Use the default method 'PP1' (Peizer-Pratt method 1 inversion) to build  
% the tree  
LRTree = lrtree(StockSpec, RateSpec, LRTIMEspec, Strike);  
LRTreeATM = lrtree(StockSpecATM, RateSpec, LRTIMEspec, Strike);
```

#### **Price the Options Using the Cox-Ross-Rubinstein (CRR) Model**

```
PriceCRR = optstockbycrr(CRRTree, OptSpec, Strike, Settle, Maturity);  
PriceCRRATM = optstockbycrr(CRRTreeATM, OptSpec, Strike, Settle, Maturity);
```

#### **Price the Options Using the Leisen-Reimer (LR) Model**

```
PriceLR = optstockbylr(LRTree, OptSpec, Strike, Settle, Maturity);  
PriceLRATM = optstockbylr(LRTreeATM, OptSpec, Strike, Settle, Maturity);
```

### Compare BLS, CRR and LR Results

```
printf('PriceBLS: \t%f\nPriceCRR: \t%f\nPriceLR:\t%f\n', PriceBLS, ...
      PriceCRR, PriceLR)
```

```
ans =
  'PriceBLS:      1.275075
  PriceCRR:      1.294979
  PriceLR:       1.275838
  '
```

```
printf('\t== ATM ==\nPriceBLS ATM: \t%f\nPriceCRR ATM: \t%f\nPriceLR ATM:\t%f\n', PriceBLS,
      PriceCRRATM, PriceLRATM)
```

```
ans =
  '      == ATM ==
  PriceBLS ATM:      3.497891
  PriceCRR ATM:      3.553938
  PriceLR ATM:       3.498571
  '
```

### Convergence of CRR and LR Models to a BLS Solution

The following tables compare call option prices using the CRR and LR models against the results obtained with the Black-Scholes formula.

While the CRR binomial model and the Black-Scholes model converge as the number of time steps gets large and the length of each step gets small, this convergence, except for at the money options, is anything but smooth or uniform.

The tables below show that the Leisen-Reimer model reduces the size of the error with even as few steps of 45.

Strike = 30, Asset Price = 30

-----

#Steps LR CRR

15 3.4986 3.5539

25 3.4981 3.5314

45 3.4980 3.5165

65 3.4979 3.5108

85 3.4979 3.5077

105 3.4979 3.5058

201 3.4979 3.5020

501 3.4979 3.4996

999 3.4979 3.4987

Strike = 30, Asset Price = 25

-----

#Steps LR CRR

15 1.2758 1.2950

25 1.2754 1.2627

45 1.2751 1.2851

65 1.2751 1.2692

85 1.2751 1.2812

105 1.2751 1.2766

201 1.2751 1.2723

501 1.2751 1.2759

999 1.2751 1.2756

#### **Analyze the Effect of the Number of Periods on the Price of the Options**

The following graphs show how convergence changes as the number of steps in the binomial calculation increases, as well as, the impact on convergence to changes to the stock price. Observe that the Leisen-Reimer model removes the oscillation and produces estimates close to the Black-Scholes model using only a small number of steps.



```

NPoints = 300;

% Cox-Ross-Rubinstein
NumPeriodCRR = 5 : 1 : NPoints;
NbStepCRR    = length(NumPeriodCRR);
PriceCRR     = nan(NbStepCRR, 1);
PriceCRRATM  = PriceCRR;

for i = 1 : NbStepCRR
    CRRTTimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriodCRR(i));
    CRRT = crrtree(StockSpec, RateSpec, CRRTTimeSpec);
    PriceCRR(i) = optstockbycrr(CRRT, OptSpec, Strike, ValuationDate, Maturity) ;

    CRRTATM = crrtree(StockSpecATM, RateSpec, CRRTTimeSpec);
    PriceCRRATM(i) = optstockbycrr(CRRTATM, OptSpec, Strike, ValuationDate, Maturity) ;
end

% Now with Leisen-Reimer
NumPeriodLR = 5 : 2 : NPoints;
NbStepLR    = length(NumPeriodLR);
PriceLR     = nan(NbStepLR, 1);
PriceLRATM  = PriceLR;

for i = 1 : NbStepLR
    LRTimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriodLR(i));
    LRT = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike);
    PriceLR(i) = optstockbylr(LRT, OptSpec, Strike, ValuationDate, Maturity) ;

    LRTATM = lrtree(StockSpecATM, RateSpec, LRTimeSpec, Strike);
    PriceLRATM(i) = optstockbylr(LRTATM, OptSpec, Strike, ValuationDate, Maturity) ;
end

First scenario: Out of the Money call option

% For Cox-Ross-Rubinstein
plot(NumPeriodCRR, PriceCRR);
hold on;
plot(NumPeriodCRR, PriceBLS*ones(NbStepCRR,1), 'Color', [0 0.9 0], 'linewidth', 1.5);

% For Leisen-Reimer
plot(NumPeriodLR, PriceLR, 'Color', [0.9 0 0], 'linewidth', 1.5);

% Concentrate in the area of interest by clipping on the Y axis at 5x the
% LR Price:

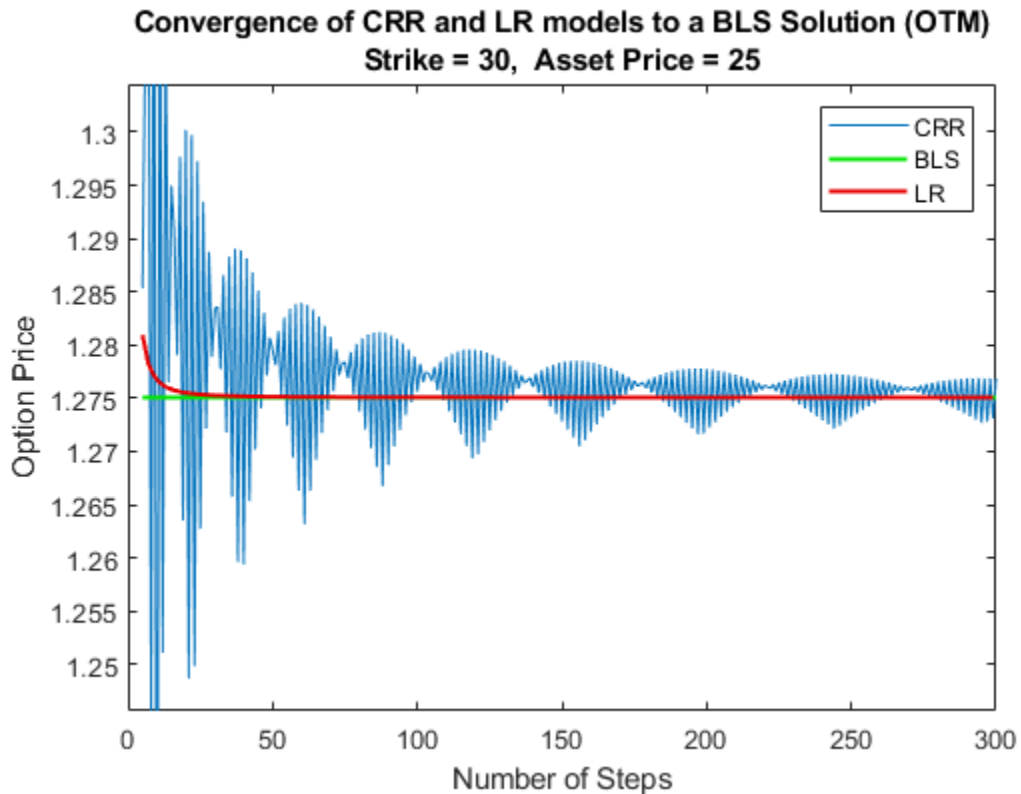
```

```

YLimDelta = 5*abs(PriceLR(1) - PriceBLS);
ax = gca;
ax.YLim = [PriceBLS-YLimDelta PriceBLS+YLimDelta];

% Annotate Plot
titleString = sprintf('\nConvergence of CRR and LR models to a BLS Solution (OTM)\nStrike = 30, Asset Price = 25');
title(titleString)
ylabel('Option Price')
xlabel('Number of Steps')
legend('CRR', 'BLS', 'LR', 'Location', 'NorthEast')

```

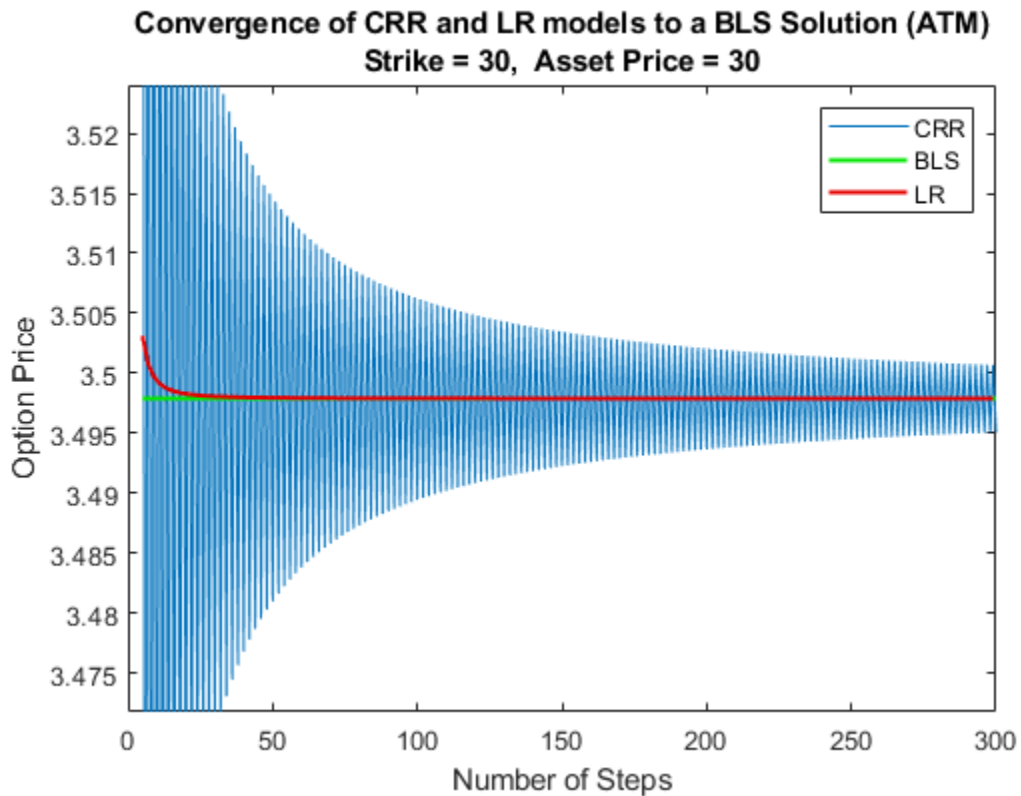


Second scenario: At the Money call option

```
% For Cox-Ross-Rubinstein
figure;
plot(NumPeriodCRR, PriceCRRATM);
hold on;
plot(NumPeriodCRR, PriceBLSATM*ones(NbStepCRR,1), 'Color',[0 0.9 0], 'linewidth', 1.5);

% For Leisen-Reimer
plot(NumPeriodLR, PriceLRATM, 'Color',[0.9 0 0], 'linewidth', 1.5);

% Concentrate in the area of interest by clipping on the Y axis at 5x the
% LR Price:
YLimDelta = 5*abs(PriceLRATM(1) - PriceBLSATM);
ax = gca;
ax.YLim = [PriceBLSATM-YLimDelta PriceBLSATM+YLimDelta];
% Annotate Plot
titleString = sprintf('\nConvergence of CRR and LR models to a BLS Solution (ATM)\nStr:
title(titleString)
ylabel('Option Price')
xlabel('Number of Steps')
legend('CRR', 'BLS', 'LR', 'Location', 'NorthEast')
```



## See Also

asianbykv | asianbylevy | asianbyls | asiandsensbykv | asiandsensbylevy | asiandsensbyls | assetbybls | assetsensbybls | basketbyju | basketbyls | basketsensbyju | basketsensbyls | basketstockspec | basketstockspec | cashbybls | cashsensbybls | chooserbybls | gapbybls | gapsensbybls | impvbybjs | impvbyblk | impvbybls | impvbyrgw | lookbackbycvgs | lookbackbyls | lookbacksensbycvgs | lookbacksensbyls | maxassetbystulz | maxassetsensbystulz | minassetbystulz | minassetsensbystulz | optpricebysim | optstockbybjs | optstockbyblk | optstockbybls | optstockbyls | optstockbyrgw | optstocksensbybaw | optstocksensbybjs | optstocksensbyblk | optstocksensbybls | optstocksensbyls |

optstocksensbyrgw | spreadbybjs | spreadbykirk | spreadbyls |  
spreadsensbybjs | spreadsensbykirk | spreadsensbyls | supersharebybjs |  
supersharesensbyls

### **Related Examples**

- “Equity Derivatives Using Closed-Form Solutions” on page 3-150
- “Compute the Option Price on a Future” on page 3-172
- “Pricing Asian Options” on page 3-113

### **More About**

- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44

## Compute the Option Price on a Future

Consider a call European option on the Crude Oil Brent futures. The option expires on December 1, 2014 with an exercise price of \$120. Assume that on April 1, 2014 futures price is at \$105, the annualized continuously compounded risk-free rate is 3.5% per annum and volatility is 22% per annum. Using this data, compute the price of the option.

Define the RateSpec.

```
ValuationDate = 'January-1-2014';
EndDates = 'January-1-2015';
Rates = 0.035;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate,...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9656
    Rates: 0.0350
    EndTimes: 1
    StartTimes: 0
    EndDates: 735965
    StartDates: 735600
    ValuationDate: 735600
    Basis: 1
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 105;
Sigma = 0.22;
StockSpec = stockspect(Sigma, AssetPrice)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2200
    AssetPrice: 105
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the option.

```
Settle = 'April-1-2014';
Maturity = 'Dec-1-2014';
Strike = 120;
OptSpec = {'call'};
```

Price the futures call option.

```
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
```

```
Price = 2.5847
```

## See Also

asianbykv | asianbylevy | asianbyls | asiainsensbykv | asiainsensbylevy | asiainsensbyls | assetbybls | assetsensbybls | basketbyju | basketbyls | basketsensbyju | basketsensbyls | basketstockspec | basketstockspec | cashbybls | cashsensbybls | chooserbybls | gapbybls | gapsensbybls | impvbybjs | impvbyblk | impvbybls | impvbyrgw | lookbackbycvgs | lookbackbyls | lookbacksensbycvgs | lookbacksensbyls | maxassetbystulz | maxassetsensbystulz | minassetbystulz | minassetsensbystulz | optpricebysim | optstockbybaw | optstockbybjs | optstockbyblk | optstockbybls | optstockbyls | optstockbyrgw | optstocksensbybaw | optstocksensbybjs | optstocksensbyblk | optstocksensbybls | optstocksensbyls | optstocksensbyrgw | spreadbybjs | spreadbykirk | spreadbyls | spreadsensbybjs | spreadsensbykirk | spreadsensbyls | supersharebybls | supersharesensbybls

## Related Examples

- “Equity Derivatives Using Closed-Form Solutions” on page 3-150
- “Pricing European Call Options Using Different Equity Models”
- “Pricing Asian Options” on page 3-113

## More About

- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44





# Hedging Portfolios

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- “Hedging” on page 4-2
- “Hedging Functions” on page 4-4
- “Pricing and Hedging a Portfolio Using the Black-Karasinski Model” on page 4-17
- “Specifying Constraints with ConSet” on page 4-32
- “Hedging with Constrained Portfolios” on page 4-37

# Hedging

Hedging is an important consideration in modern finance. Whether or not to hedge, how much portfolio insurance is adequate, and how often to rebalance a portfolio are important considerations for traders, portfolio managers, and financial institutions alike.

If there were no transaction costs, financial professionals would prefer to rebalance portfolios continually, thereby minimizing exposure to market movements. However, in practice, the transaction costs associated with frequent portfolio rebalancing may be expensive. Therefore, traders and portfolio managers must carefully assess the cost required to achieve a particular portfolio sensitivity (for example, maintaining delta, gamma, and vega neutrality). Thus, the hedging problem involves the fundamental tradeoff between portfolio insurance and the cost of such insurance coverage.

## See Also

`hedgeopt` | `hedgeslf`

## Related Examples

- “Portfolio Creation” on page 1-8
- “Adding Instruments to an Existing Portfolio” on page 1-11
- “Instrument Constructors” on page 1-20
- “Creating Instruments or Properties” on page 1-21
- “Searching or Subsetting a Portfolio” on page 1-23
- “Hedging Functions” on page 4-4
- “Hedging with `hedgeopt`” on page 4-5
- “Self-Financing Hedges with `hedgeslf`” on page 4-12
- “Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13
- “Pricing and Hedging a Portfolio Using the Black-Karasinski Model” on page 4-17
- “Specifying Constraints with `ConSet`” on page 4-32
- “Portfolio Rebalancing” on page 4-34
- “Hedging with Constrained Portfolios” on page 4-37

## **More About**

- “Instrument Constructors” on page 1-20

## Hedging Functions

In this section...
“Introduction” on page 4-4
“Hedging with hedgeopt” on page 4-5
“Self-Financing Hedges with hedgeslf” on page 4-12

### Introduction

Hedging is an investment to reduce the risk of adverse price movements in an asset. Financial Instruments Toolbox offers two functions for assessing the fundamental hedging tradeoff, `hedgeopt` and `hedgeslf`.

The first function, `hedgeopt`, addresses the most general hedging problem. It allocates an optimal hedge to satisfy either of two goals:

- Minimize the cost of hedging a portfolio given a set of target sensitivities.
- Minimize portfolio sensitivities for a given set of maximum target costs.

`hedgeopt` allows investors to modify portfolio allocations among instruments according to either of the goals. The problem is cast as a constrained linear least-squares problem. For additional information about `hedgeopt`, see “Hedging with `hedgeopt`” on page 4-5.

The second function, `hedgeslf`, attempts to allocate a self-financing hedge among a portfolio of instruments. In particular, `hedgeslf` attempts to maintain a constant portfolio value consistent with reduced portfolio sensitivities (that is, the rebalanced portfolio is hedged against market moves and is closest to being self-financing). If `hedgeslf` cannot find a self-financing hedge, it rebalances the portfolio to minimize overall portfolio sensitivities. For additional information on `hedgeslf`, see “Self-Financing Hedges with `hedgeslf`” on page 4-12.

The examples in this section consider the *delta*, *gamma*, and *vega* sensitivity measures. In this toolbox, when you work with *interest-rate derivatives*, delta is the price sensitivity measure of shifts in the forward yield curve, gamma is the delta sensitivity measure of shifts in the forward yield curve, and vega is the price sensitivity measure of shifts in the volatility process. See `bdtsens` or `hjmsens` for details on the computation of sensitivities for interest-rate derivatives.

For *equity exotic options*, the underlying instrument is the stock price instead of the forward yield curve. So, delta now represents the price sensitivity measure of shifts in the stock price, gamma is the delta sensitivity measure of shifts in the stock price, and vega is the price sensitivity measure of shifts in the volatility of the stock. See `crsens`, `eqpsens`, `ittsens`, or `sttsens` for details on the computation of sensitivities for equity derivatives.

For examples showing the computation of sensitivities for interest-rate based derivatives, see “Computing Instrument Sensitivities” on page 2-74. Likewise, for examples showing the computation of sensitivities for equity exotic options, see “Computing Equity Instrument Sensitivities” on page 3-144.

---

**Note** The delta, gamma, and vega sensitivities that the toolbox calculates are dollar sensitivities.

---

## Hedging with `hedgeopt`

---

**Note** The numerical results in this section are displayed in the MATLAB bank format. Although the calculations are performed in floating-point double precision, only two decimal places are displayed.

---

To illustrate the hedging facility, consider the portfolio `HJMInstSet` obtained from the example file `deriv.mat`. The portfolio consists of eight instruments: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap.

Both hedging functions require some common inputs, including the current portfolio holdings (allocations), and a matrix of instrument sensitivities. To create these inputs, load the example portfolio into memory

```
load deriv.mat;
```

```
compute price and sensitivities
```

```
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet);
```

```
Warning: Not all cash flows are aligned with the tree. Result will  
be approximated.
```

```
and extract the current portfolio holdings.
```

```
Holdings = instget(HJMIInstSet, 'FieldName', 'Quantity');
```

For convenience place the delta, gamma, and vega sensitivity measures into a matrix of sensitivities.

```
Sensitivities = [Delta Gamma Vega];
```

Each row of the `Sensitivities` matrix is associated with a different instrument in the portfolio, and each column with a different sensitivity measure.

To summarize the portfolio information

```
disp([Price Holdings Sensitivities])
```

98.72	100.00	-272.65	1029.90	0.00
97.53	50.00	-347.43	1622.69	-0.04
0.05	-50.00	-8.08	643.40	34.07
98.72	80.00	-272.65	1029.90	0.00
100.55	8.00	-1.04	3.31	0
6.28	30.00	294.97	6852.56	93.69
0.05	40.00	-47.16	8459.99	93.69
3.69	10.00	-282.05	1059.68	0.00

The first column above is the dollar unit price of each instrument, the second is the holdings of each instrument (the quantity held or the number of contracts), and the third, fourth, and fifth columns are the dollar delta, gamma, and vega sensitivities, respectively.

The current portfolio sensitivities are a weighted average of the instruments in the portfolio.

```
TargetSens = Holdings' * Sensitivities
```

```
TargetSens =
```

```
    -61910.22    788946.21    4852.91
```

### Maintaining Existing Allocations

To illustrate using `hedgeopt`, suppose that you want to maintain your existing portfolio. The first form of `hedgeopt` minimizes the cost of hedging a portfolio given a set of target sensitivities. If you want to maintain your existing portfolio composition and exposure, you should be able to do so without spending any money. To verify this, set the target sensitivities to the current sensitivities.

```
FixedInd = [1 2 3 4 5 6 7 8];
```

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, Holdings, FixedInd, [], [], TargetSens);
```

Holdings =

100.00  
 50.00  
 -50.00  
 80.00  
 8.00  
 30.00  
 40.00  
 10.00

Sens =

-61910.22      788946.21      4852.91

Cost =

0

Quantity =

Columns 1 through 6

100.00      50.00      -50.00      80.00      8.00      30.00

Columns 7 through 8

40.00      10.00

Portfolio composition and sensitivities are unchanged, and the cost associated with doing nothing is zero. The cost is defined as the change in portfolio value. This number cannot be less than zero because the rebalancing cost is defined as a nonnegative number.

If  $Value_0$  and  $Value_1$  represent the portfolio value before and after rebalancing, respectively, the zero cost can also be verified by comparing the portfolio values.

$Value_0 = Holdings' * Price$

$Value_0 =$

23674.62

```
Value1 = Quantity * Price
```

```
Value1 =
```

```
23674.62
```

### Partially Hedged Portfolio

Building on the example in “Maintaining Existing Allocations” on page 4-6, suppose you want to know the cost to achieve an overall portfolio dollar sensitivity of [ -23000 -3300 3000], while allowing trading only in instruments 2, 3, and 6 (holding the positions of instruments 1, 4, 5, 7, and 8 fixed). To find the cost, first set the target portfolio dollar sensitivity.

```
TargetSens = [-23000 -3300 3000];
```

Then, specify the instruments to be fixed.

```
FixedInd = [1 4 5 7 8];
```

Finally, call `hedgeopt`

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...  
Holdings, FixedInd, [], [], TargetSens);
```

and again examine the results.

```
Sens =
```

```
-23000.00    -3300.00    3000.00
```

```
Cost =
```

```
19174.02
```

```
Quantity' =
```

```
100.00  
-141.03  
137.26  
80.00  
8.00  
-57.96  
40.00  
10.00
```



Recompute `Value1`, the portfolio value after rebalancing.

```
Value1 = Quantity * Price
```

```
Value1 =
```

```
    4500.60
```

As expected, the cost, \$19174.02, is the difference between `Value0` and `Value1`, \$23674.62 — \$4500.60. Only the positions in instruments 2, 3, and 6 have been changed.

### Fully Hedged Portfolio

The example in “Partially Hedged Portfolio” on page 4-8 illustrates a partial hedge, but perhaps the most interesting case involves the cost associated with a fully hedged portfolio (simultaneous delta, gamma, and vega neutrality). In this case, set the target sensitivity to a row vector of 0s and call `hedgeopt` again. The following example uses data from “Hedging with `hedgeopt`” on page 4-5.

```
TargetSens = [0 0 0];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
Holdings, FixedInd, [], [], TargetSens);
```

Examining the outputs reveals that you have obtained a fully hedged portfolio

```
Sens =
```

```
    -0.00    -0.00    -0.00
```

but at an expense of over \$20,000.

```
Cost =
```

```
    23055.90
```

The positions required to achieve a fully hedged portfolio

```
Quantity' =
```

```
    100.00
   -182.36
    -19.55
     80.00
      8.00
   -32.97
```

```
40.00  
10.00
```

result in the new portfolio value

```
Value1 = Quantity * Price
```

```
Value1 =
```

```
618.72
```

### Minimizing Portfolio Sensitivities

The examples in “Fully Hedged Portfolio” on page 4-9 illustrate how to use `hedgeopt` to determine the minimum cost of hedging a portfolio given a set of target sensitivities. In these examples, portfolio target sensitivities are treated as equality constraints during the optimization process. You tell `hedgeopt` what sensitivities you want, and it tells you what it will cost to get those sensitivities.

A related problem involves minimizing portfolio sensitivities for a given set of maximum target costs. For this goal, the target costs are treated as inequality constraints during the optimization process. You tell `hedgeopt` the most you are willing spend to insulate your portfolio, and it tells you the smallest portfolio sensitivities you can get for your money.

To illustrate this use of `hedgeopt`, compute the portfolio dollar sensitivities along the entire cost frontier. From the previous examples, you know that spending nothing replicates the existing portfolio, while spending \$23,055.90 completely hedges the portfolio.

Assume, for example, you are willing to spend as much as \$50,000, and want to see what portfolio sensitivities will result along the cost frontier. Assume that the same instruments are held fixed, and that the cost frontier is evaluated from \$0 to \$50,000 at increments of \$1000.

```
MaxCost = [0:1000:50000];
```

Now, call `hedgeopt`.

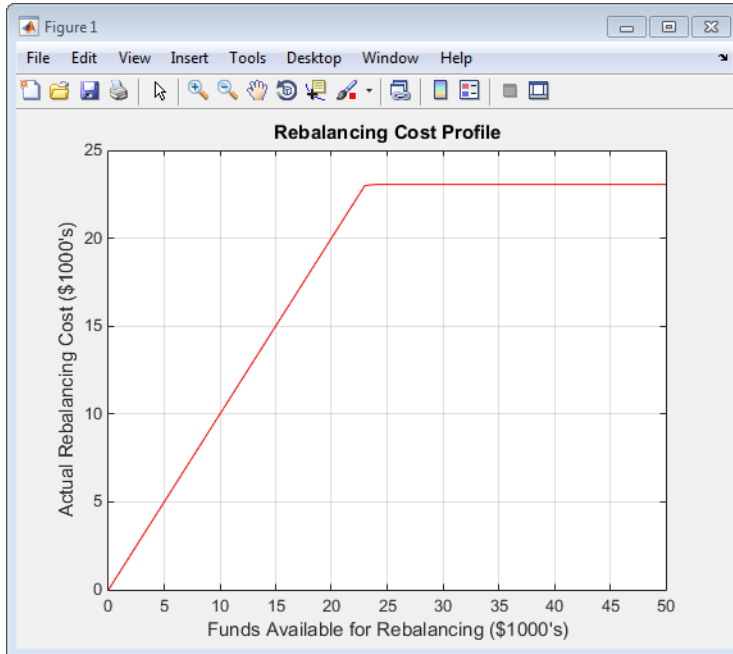
```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...  
Holdings, FixedInd, [], MaxCost);
```

With this data, you can plot the required hedging cost versus the funds available (the amount you are willing to spend)

```

plot(MaxCost/1000, Cost/1000, 'red'), grid
xlabel('Funds Available for Rebalancing ($1000's)')
ylabel('Actual Rebalancing Cost ($1000's)')
title ('Rebalancing Cost Profile')

```



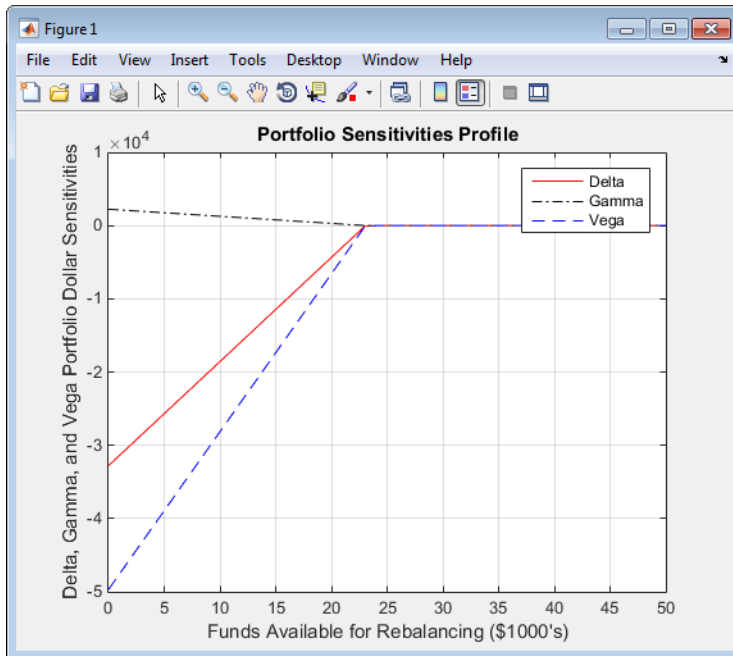
### Rebalancing Cost Profile

and the portfolio dollar sensitivities versus the funds available.

```

figure
plot(MaxCost/1000, Sens(:,1), '-red')
hold('on')
plot(MaxCost/1000, Sens(:,2), '-.black')
plot(MaxCost/1000, Sens(:,3), '--blue')
grid
xlabel('Funds Available for Rebalancing ($1000's)')
ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
title ('Portfolio Sensitivities Profile')
legend('Delta', 'Gamma', 'Vega', 0)

```



### Funds Available for Rebalancing

### Self-Financing Hedges with hedgeslf

The figures “Rebalancing Cost Profile” on page 4-11 and “Funds Available for Rebalancing” on page 4-12 indicate that there is no benefit because the funds available for hedging exceed \$23,055.90, the point of maximum expense required to obtain simultaneous delta, gamma, and vega neutrality. You can also find this point of delta, gamma, and vega neutrality using `hedgeslf`.

```
[Sens, Value1, Quantity] = hedgeslf(Sensitivities, Price,...
Holdings, FixedInd);
```

Sens =

```
-0.00
-0.00
-0.00
```

```

Value1 =
    618.72
Quantity =
    100.00
   -182.36
    -19.55
     80.00
     8.00
   -32.97
    40.00
    10.00

```

Similar to `hedgeopt`, `hedgelf` returns the portfolio dollar sensitivities and instrument quantities (the rebalanced holdings). However, in contrast, the second output parameter of `hedgelf` is the value of the rebalanced portfolio, from which you can calculate the rebalancing cost by subtraction.

```

Value0 - Value1
ans =
    23055.90

```

In this example, the portfolio is clearly not self-financing, so `hedgelf` finds the best possible solution required to obtain zero sensitivities.

There is, in fact, a third calling syntax available for `hedgeopt` directly related to the results shown above for `hedgelf`. Suppose, instead of directly specifying the funds available for rebalancing (the most money you are willing to spend), you want to simply specify the number of points along the cost frontier. This call to `hedgeopt` samples the cost frontier at 10 equally spaced points between the point of minimum cost (and potentially maximum exposure) and the point of minimum exposure (and maximum cost).

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
Holdings, FixedInd, 10)
```

```

Sens =
   -32784.46    2231.83   -49694.33
   -29141.74    1983.85   -44172.74
   -25499.02    1735.87   -38651.14
   -21856.30    1487.89   -33129.55
   -18213.59    1239.91   -27607.96
   -14570.87     991.93   -22086.37

```

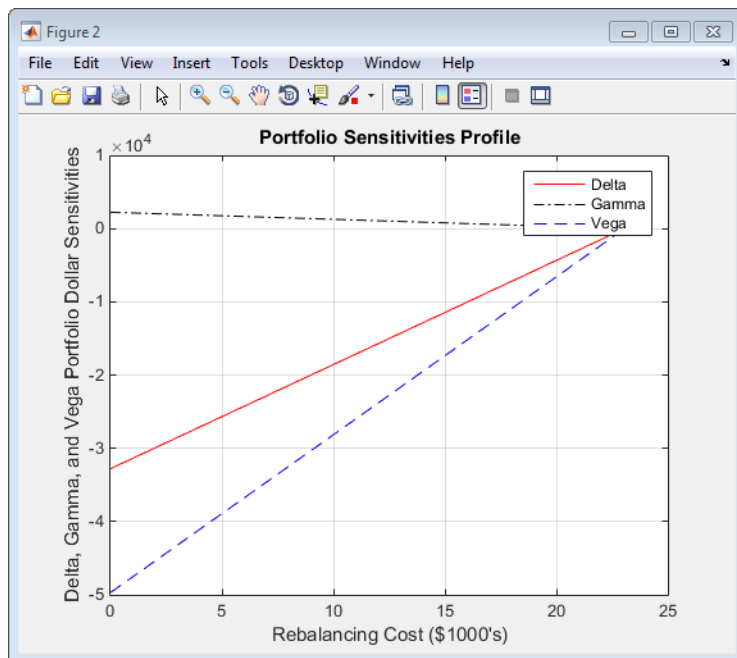
-10928.15	743.94	-16564.78
-7285.43	495.96	-11043.18
-3642.72	247.98	-5521.59
0.00	-0.00	0.00

Cost =

0.00
2561.77
5123.53
7685.30
10247.07
12808.83
15370.60
17932.37
20494.14
23055.90

Now plot this data.

```
figure
plot(Cost/1000, Sens(:,1), '-red')
hold('on')
plot(Cost/1000, Sens(:,2), '-.black')
plot(Cost/1000, Sens(:,3), '--blue')
grid
xlabel('Rebalancing Cost ($1000's)')
ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
title('Portfolio Sensitivities Profile')
legend('Delta', 'Gamma', 'Vega', 0)
```



## Rebalancing Cost

In this calling form, `hedgeopt` calls `hedgeslf` internally to determine the maximum cost needed to minimize the portfolio sensitivities (\$23,055.90), and evenly samples the cost frontier between \$0 and \$23,055.90.

Both `hedgeopt` and `hedgeslf` cast the optimization problem as a constrained linear least squares problem. Depending on the instruments and constraints, neither function is guaranteed to converge to a solution. In some cases, the problem space may be unbounded, and additional instrument equality constraints, or user-specified constraints, may be necessary for convergence. See “Hedging with Constrained Portfolios” on page 4-37 for additional information.

## See Also

`hedgeopt` | `hedgeslf`

### **Related Examples**

- “Portfolio Creation” on page 1-8
- “Adding Instruments to an Existing Portfolio” on page 1-11
- “Instrument Constructors” on page 1-20
- “Creating Instruments or Properties” on page 1-21
- “Searching or Subsetting a Portfolio” on page 1-23
- “Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13
- “Pricing and Hedging a Portfolio Using the Black-Karasinski Model” on page 4-17
- “Specifying Constraints with ConSet” on page 4-32
- “Portfolio Rebalancing” on page 4-34
- “Hedging with Constrained Portfolios” on page 4-37

### **More About**

- “Instrument Constructors” on page 1-20
- “Hedging” on page 4-2



## Pricing and Hedging a Portfolio Using the Black-Karasinski Model

This example illustrates how MATLAB® can be used to create a portfolio of interest-rate derivatives securities, and price it using the Black-Karasinski interest-rate model. The example also shows some hedging strategies to minimize exposure to market movements.

### Create the Interest-Rate Term Structure Based on Reported Data

The structure `RateSpec` is an interest-rate term structure that defines the initial rate specification from which the tree rates are derived. Use the information of annualized zero coupon rates in the table below to populate the `RateSpec` structure.

From	To	Rate
27 Feb 2007	27 Feb 2008	0.0493
27 Feb 2007	27 Feb 2009	0.0459
27 Feb 2007	27 Feb 2010	0.0450
27 Feb 2007	27 Feb 2012	0.0446
27 Feb 2007	27 Feb 2014	0.0445
27 Feb 2007	27 Feb 2017	0.0450
27 Feb 2007	27 Feb 2027	0.0473

This data could be retrieved from the Federal Reserve Statistical Release page by using the Datafeed Toolbox™. In this case, the Datafeed Toolbox™ will connect to FRED® and pull back the rates of the following treasury notes.

Terms	Symbol
=====	=====
1	= DGS1
2	= DGS2
3	= DGS3
5	= DGS5
7	= DGS7
10	= DGS10
20	= DGS20

Create the connection object:

```
c = fred;
```

Create the symbol fetch list:

```
FredNames = { ...
    'DGS1'; ... % 1 Year
```

```
'DGS2'; ... % 2 Year
'DGS3'; ... % 3 Year
'DGS5'; ... % 5 Year
'DGS7'; ... % 7 Year
'DGS10'; ... % 10 Year
'DGS20'}; % 20 Year
```

Define the Terms:

```
Terms = [ 1; ... % 1 Year
          2; ... % 2 Year
          3; ... % 3 Year
          5; ... % 5 Year
          7; ... % 7 Year
          10; ... % 10 Year
          20]; % 20 Year
```

Set the StartDate to Feb 27, 2007:

```
StartDate = datenum('Feb-27-2007');
FredRet = fetch(c,FredNames,StartDate);
```

Set the ValuationDate based on the StartDate:

```
ValuationDate = StartDate;
EndDates = [];
Rates =[];
```

Create the EndDates:

```
for idx = 1:length(FredRet)

    %Pull the rates associated with Feb 27, 2007. All the Fred Rates come
    %back as percents
    Rates = [Rates; ...
             FredRet(idx).Data(1,2) / 100];

    %Determine the EndDates by adding the Term to the year of the
    %StartDate
    EndDates = [EndDates; ...
               round(datenum(...
                    year(StartDate)+ Terms(idx,1), ...
                    month(StartDate),...
                    day(StartDate)))];
```

```
end
```

Use the function `intenvset` to create the `RateSpec` with the following data:

```
Compounding = 1;
StartDate = '27-Feb-2007';
Rates = [0.0493; 0.0459; 0.0450; 0.0446; 0.0446; 0.0450; 0.0473];
EndDates = {'27-Feb-2008'; '27-Feb-2009'; '27-Feb-2010'; '27-Feb-2012'; ...
            '27-Feb-2014'; '27-Feb-2017'; '27-Feb-2027'};
ValuationDate = StartDate;

RateSpec = intenvset('Compounding',Compounding,'StartDates', StartDate,...
                    'EndDates', EndDates, 'Rates', Rates, 'ValuationDate', ValuationDate);
```

```
RateSpec =
```

```
struct with fields:
```

```
    FinObj: 'RateSpec'
  Compounding: 1
        Disc: [7x1 double]
        Rates: [7x1 double]
    EndTimes: [7x1 double]
  StartTimes: [7x1 double]
    EndDates: [7x1 double]
  StartDates: 733100
ValuationDate: 733100
        Basis: 0
  EndMonthRule: 1
```

### Specify the Volatility Model

Create the structure `VolSpec` that specifies the volatility process with the following data.

```
Volatility = [0.011892; 0.01563; 0.02021; 0.02125; 0.02165; 0.02065; 0.01803];
Alpha = [0.0001];
VolSpec = bkvolspec(ValuationDate, EndDates, Volatility, EndDates(end), Alpha)
```

```
VolSpec =
```

```
struct with fields:
```

```
        FinObj: 'BKVolSpec'  
ValuationDate: 733100  
    VolDates: [7x1 double]  
    VolCurve: [7x1 double]  
    AlphaCurve: 1.0000e-04  
    AlphaDates: 740405  
VolInterpMethod: 'linear'
```

### Specify the Time Structure of the Tree

The structure `TimeSpec` specifies the time structure for an interest-rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

```
TimeSpec = bktimespec(ValuationDate, EndDates)
```

```
TimeSpec =
```

```
    struct with fields:
```

```
        FinObj: 'BKTimeSpec'  
ValuationDate: 733100  
    Maturity: [7x1 double]  
    Compounding: -1  
        Basis: 0  
    EndMonthRule: 1
```

### Create the BK Tree

Use the previously computed values for `RateSpec`, `VolSpec`, and `TimeSpec` to create the BK tree.

```
BKTree = bktree(VolSpec, RateSpec, TimeSpec)
```

```
BKTree =
```

```
    struct with fields:
```

```
        FinObj: 'BKFwdTree'  
    VolSpec: [1x1 struct]  
    TimeSpec: [1x1 struct]
```

```

RateSpec: [1x1 struct]
  tObs: [0 1 2 3 5 7 10]
  dObs: [733100 733465 733831 734196 734926 735657 736753]
  CFlowT: {1x7 cell}
  Probs: {1x6 cell}
  Connect: {1x6 cell}
  FwdTree: {1x7 cell}

```

Observe the Interest-Rate Tree.

Visualize the interest rate evolution along the tree by looking at the output structure `BKTree`. The function `bktree` returns an inverse discount tree, which you can convert into an interest rate tree with the `cvtree` function.

```
BKTreeR = cvtree(BKTree)
```

```
BKTreeR =
```

```
struct with fields:
```

```

  FinObj: 'BKRateTree'
  VolSpec: [1x1 struct]
  TimeSpec: [1x1 struct]
  RateSpec: [1x1 struct]
  tObs: [0 1 2 3 5 7 10]
  dObs: [733100 733465 733831 734196 734926 735657 736753]
  CFlowT: {1x7 cell}
  Probs: {1x6 cell}
  Connect: {1x6 cell}
  RateTree: {1x7 cell}

```

Look at the upper, middle and lower branch paths of the tree:

```
OldFormat = get(0, 'format');
format short
```

```
%Rate at root node:
```

```
RateRoot = trintreepath(BKTreeR, 0)
```

```
%Rates along upper branch:
```

```
RatePathUp = trintreepath(BKTreeR, [1 1 1 1 1 1])
```

```
%Rates along middle branch:  
RatePathMiddle = trintreepath(BKTreeR, [2 2 2 2 2 2])
```

```
%Rates along lower branch:  
RatePathDown = trintreepath(BKTreeR, [3 3 3 3 3 3])
```

```
RateRoot =
```

```
    0.0481
```

```
RatePathUp =
```

```
    0.0481  
    0.0425  
    0.0446  
    0.0478  
    0.0510  
    0.0555  
    0.0620
```

```
RatePathMiddle =
```

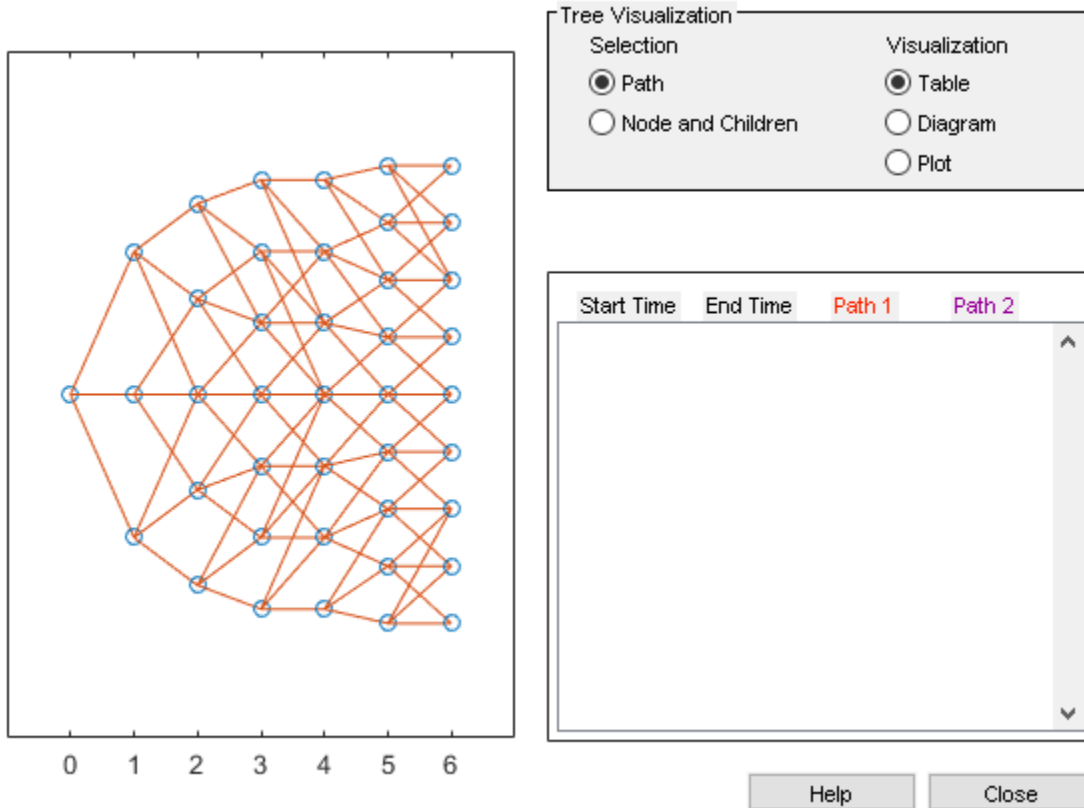
```
    0.0481  
    0.0416  
    0.0423  
    0.0430  
    0.0436  
    0.0449  
    0.0484
```

```
RatePathDown =
```

```
    0.0481  
    0.0408  
    0.0401  
    0.0388  
    0.0373  
    0.0363  
    0.0378
```

You can also display a graphical representation of the tree to examine interactively the rates on the nodes of the tree until maturity. The function `treeviewer` displays the structure of the rate tree in the left window. The tree visualization in the right window is blank, but by selecting Table/Diagram and clicking on the nodes you can examine the rates along the paths.

```
treeviewer(BKTreeR);
```



### Create an Instrument Portfolio

Create a portfolio consisting of two bonds instruments and an option on the 5% bond.

```
% Two Bonds  
CouponRate = [0.04;0.05];
```

```

Settle = '27 Feb 2007';
Maturity = {'27 Feb 2009'; '27 Feb 2010'};
Period = 1;

% American Option on the 5% Bond
OptSpec = {'call'};
Strike = 98;
ExerciseDates = '27 Feb 2010';
AmericanOpt = 1;

InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period);
InstSet = instadd(InstSet, 'OptBond', 2, OptSpec, Strike, ExerciseDates, AmericanOpt);

% Assign Names and Holdings
Holdings = [10; 15; 3];
Names = {'4% Bond'; '5% Bond'; 'Option 98'};

InstSet = instsetfield(InstSet, 'Index', 1:3, 'FieldName', {'Quantity'}, 'Data', Holdings);
InstSet = instsetfield(InstSet, 'Index', 1:3, 'FieldName', {'Name'}, 'Data', Names );

```

Examine the set of instruments contained in the variable `InstSet`.

```
instdisp(InstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.04	27-Feb-2007	27-Feb-2009	1	0	1	NaN
2	Bond	0.05	27-Feb-2007	27-Feb-2010	1	0	1	NaN

Index	Type	UnderInd	OptSpec	Strike	ExerciseDates	AmericanOpt	Quantity	Name
3	OptBond	2	call	98	27-Feb-2010	1	3	Option 98

### Price the Portfolio Using the BK Model

Calculate the price of each instrument in the portfolio.

```
[Price, PTree] = bkprice(BKTree, InstSet)
```

```
Price =
```

```

    98.8841
   101.3470
    3.3470

```



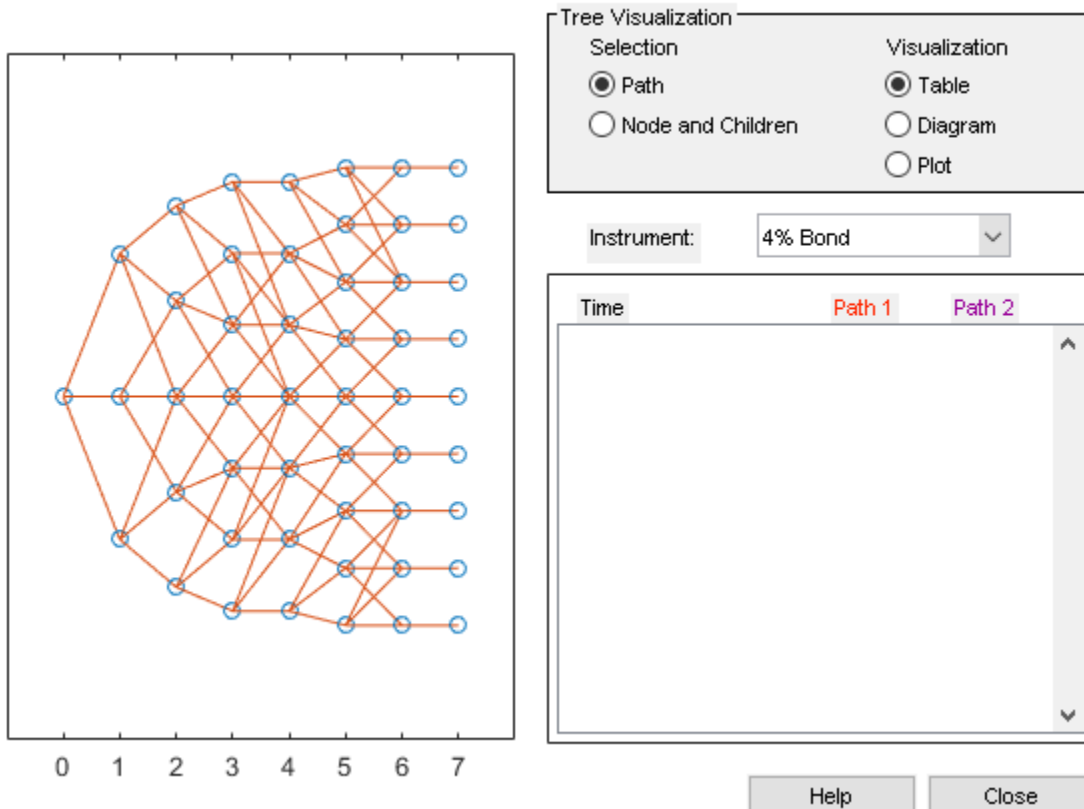
```
PTree =  
  
    struct with fields:  
  
        FinObj: 'BKPriceTree'  
        PTree: {1x8 cell}  
        AITree: {1x8 cell}  
        tObs: [0 1 2 3 5 7 10 20]  
        Connect: {1x6 cell}  
        Probs: {1x6 cell}
```

The prices in the output vector `Price` correspond to the prices at observation time zero (`tObs = 0`), which is defined as the Valuation Date of the interest-rate tree.

In the `Price` vector, the first element, 98.884, represents the price of the first instrument (4% Bond); the second element, 101.347, represents the price of the second instrument (5% Bond), and 3.347 represents the price of the American call option.

You can also display a graphical representation of the price tree to examine the prices on the nodes of the tree until maturity.

```
treeviewer(PTree,InstSet);
```



### Add More Instruments to the Existing Portfolio

Add instruments to the existing portfolio: cap, floor, floating rate note, vanilla swap and a puttable and callable bond.

```
% Cap
StrikeC =0.035;
InstSet = instadd(InstSet,'Cap', StrikeC, Settle, '27 Feb 2010');

% Floor
StrikeF =0.05;
InstSet = instadd(InstSet,'Floor', StrikeF, Settle, '27 Feb 2009');

% Floating Rate Note
```

```

InstSet = instadd(InstSet, 'Float', 30, Settle, '27 Feb 2009');

% Vanilla Swap
LegRate = [0.04 5];
InstSet = instadd(InstSet, 'Swap', LegRate, Settle, '27 Feb 2010');

% Puttable and Callable Bonds
InstSet = instadd(InstSet, 'OptEmBond', CouponRate, Settle, '27 Feb 2010', {'put'; 'call'};
    Strike, '27 Feb 2010', 'AmericanOpt', 1, 'Period', 1);

% Process Names and Holdings
Holdings = [15 ;5 ;8; 7; 9; 4];
Names = {'3.5% Cap'; '5% Floor'; '30BP Float'; '4%/5BP Swap'; 'PuttBond'; 'CallBond' };

InstSet = instsetfield(InstSet, 'Index',4:9, 'FieldName', {'Quantity'}, 'Data', Holdings);
InstSet = instsetfield(InstSet, 'Index',4:9, 'FieldName', {'Name'}, 'Data', Names );
    
```

Examine the set of instruments contained in the variable InstSet.

```
instdisp(InstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.04	27-Feb-2007	27-Feb-2009	1	0	1	NaN
2	Bond	0.05	27-Feb-2007	27-Feb-2010	1	0	1	NaN

Index	Type	UnderInd	OptSpec	Strike	ExerciseDates	AmericanOpt	Quantity	Name
3	OptBond	2	call	98	27-Feb-2010	1	3	Option 98

Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal	Quantity	Name
4	Cap	0.035	27-Feb-2007	27-Feb-2010	1	0	100	15	3.5%

Index	Type	Strike	Settle	Maturity	FloorReset	Basis	Principal	Quantity	Name
5	Floor	0.05	27-Feb-2007	27-Feb-2009	1	0	100	5	5%

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRule
6	Float	30	27-Feb-2007	27-Feb-2009	1	0	100	1

Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	End
7	Swap	[0.04 5]	27-Feb-2007	27-Feb-2010	[NaN]	0	100	[NaN]	1

Index	Type	CouponRate	Settle	Maturity	OptSpec	Strike	ExerciseDates
8	OptEmBond	0.04	27-Feb-2007	27-Feb-2010	put	98	27-Feb-2007
9	OptEmBond	0.05	27-Feb-2007	27-Feb-2010	call	98	27-Feb-2007

### Hedging

The idea behind hedging is to minimize exposure to market movements. As the underlying changes, the proportions of the instruments forming the portfolio may need to be adjusted to keep the sensitivities within the desired range.

Calculate sensitivities using the BK model.

```
[Delta, Gamma, Vega, Price] = bksens(BKTree, InstSet);
```

Get the current portfolio holdings.

```
Holdings = instget(InstSet, 'FieldName', 'Quantity');
```

Create a matrix of sensitivities.

```
Sensitivities = [Delta Gamma Vega];
```

Each row of the Sensitivities matrix is associated with a different instrument in the portfolio, and each column with a different sensitivity measure.

Display the information.

```
format bank
disp([Price Holdings Sensitivities])
```

98.88	10.00	-185.47	528.47	0
101.35	15.00	-277.51	1045.05	-0.00
3.35	3.00	-223.52	11843.32	-0.00
2.77	15.00	250.04	2921.11	0
0.75	5.00	-132.97	11566.69	0
100.56	8.00	-0.80	2.02	0
-1.53	7.00	-272.08	1027.85	0
98.60	9.00	-168.92	21712.82	-0.00
98.00	4.00	-53.99	-10798.27	0

The first column above is the dollar unit price of each instrument, the second column is the number of contracts of each instrument, and the third, fourth, and fifth columns are the dollar delta, gamma, and vega sensitivities.

The current portfolio sensitivities are a weighted average of the instruments in the portfolio.

```
TargetSens = Holdings' * Sensitivities
```

```
TargetSens =
      -7249.21      317573.92      -0.00
```

### Obtain a Neutral Sensitivity Portfolio

Suppose you want to obtain a delta, gamma and vega neutral portfolio. The function `hedgeslf` finds the reallocation in a portfolio of financial instruments closest to being self-financing (maintaining constant portfolio value).

```
[Sens, Value1, Quantity]= hedgeslf(Sensitivities, Price,Holdings)
```

```
Sens =
      -0.00
       0.00
      -0.00
```

```
Value1 =
      4637.54
```

```
Quantity =
      10.00
       5.26
      -5.11
       7.06
      -3.05
      12.45
      -7.36
       8.47
      10.37
```

The function `hedgeslf` returns the portfolio dollar sensitivities (`Sens`), the value of the rebalanced portfolio (`Value1`) and the new allocation for each instrument (`Quantity`). If `Value0` and `Value1` represent the portfolio value before and after rebalancing, you can verify the cost by comparing the portfolio values.

```
Value0 = Holdings' * Price
```

```
Value0 =  
    4637.54
```

In this example, the portfolio is fully hedged (simultaneous delta, gamma, and vega neutrality) and self-financing (the values of the portfolio before and after balancing (`Value0` and `Value1`) are the same).

### Adding Constraints to Hedge a Portfolio

Suppose that you want to place upper and lower bounds on the individual instruments in the portfolio. Let's say that you want to bound the position of all instruments to within +/- 11 contracts.

Applying these constraints disallows the current positions in the fifth and eighth instruments. All other instruments are currently within the upper/lower bounds.

```
% Specify the lower and upper bounds
```

```
LowerBounds = [-11 -11 -11 -11 -11 -11 -11 -11];  
UpperBounds = [ 11  11  11  11  11  11  11  11];
```

```
% Use the function portcons to build the constraints
```

```
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

```
% Apply the constraints to the portfolio
```

```
[Sens, Value, Quantity1] = hedgeslf(Sensitivities, Price, Holdings, [], ConSet)
```

```
Sens =  
  
    0  
    0  
    0
```

```
Value =  
  
    0
```

```
Quantity1 =
```

```
0  
0  
0  
0  
0  
0  
0  
0  
0
```

Observe that the `hedgelsf` function enforces the bounds on the fifth and eighth instruments, and the portfolio continues to be fully hedged and self-financing.

```
set(0, 'format', OldFormat);
```

## See Also

`hedgeopt` | `hedgelsf`

## Related Examples

- “Portfolio Creation” on page 1-8
- “Adding Instruments to an Existing Portfolio” on page 1-11
- “Instrument Constructors” on page 1-20
- “Creating Instruments or Properties” on page 1-21
- “Searching or Subsetting a Portfolio” on page 1-23
- “Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13
- “Specifying Constraints with ConSet” on page 4-32
- “Portfolio Rebalancing” on page 4-34
- “Hedging with Constrained Portfolios” on page 4-37

## More About

- “Instrument Constructors” on page 1-20
- “Hedging” on page 4-2

## Specifying Constraints with ConSet

### In this section...

“Introduction” on page 4-32  
 “Setting Constraints” on page 4-32  
 “Portfolio Rebalancing” on page 4-34

### Introduction

Both `hedgeopt` and `hedgeslf` accept an optional input argument, `ConSet`, that allows you to specify a set of linear inequality constraints for instruments in your portfolio. The examples in this section are brief. For additional information regarding portfolio constraint specifications, refer to “Analyzing Portfolios” (Financial Toolbox).

### Setting Constraints

For the first example of setting constraints, return to the fully hedged portfolio example that used `hedgeopt` to determine the minimum cost of obtaining simultaneous delta, gamma, and vega neutrality (target sensitivities all 0). Recall that when `hedgeopt` computes the cost of rebalancing a portfolio, the input target sensitivities you specify are treated as equality constraints during the optimization process. The situation is reproduced next for convenience.

```
TargetSens = [0 0 0];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens);
```

The outputs provide a fully hedged portfolio

```
Sens =
      -0.00      -0.00      -0.00
```

at an expense of over \$23,000.

```
Cost =
 23055.90
```

The positions required to achieve this fully hedged portfolio are

```
Quantity' =
```



```

100.00
-182.36
-19.55
80.00
8.00
-32.97
40.00
10.00

```

Suppose now that you want to place some upper and lower bounds on the individual instruments in your portfolio. You can specify these constraints, along with a variety of general linear inequality constraints, with Financial Toolbox™ function `portcons`.

As an example, assume that, in addition to holding instruments 1, 4, 5, 7, and 8 fixed as before, you want to bound the position of all instruments to within +/- 180 contracts (for each instrument, you cannot short or long more than 180 contracts). Applying these constraints disallows the current position in the second instrument (short 182.36). All other instruments are currently within the upper/lower bounds.

You can generate these constraints by first specifying the lower and upper bounds vectors and then calling `portcons`.

```

LowerBounds = [-180 -180 -180 -180 -180 -180 -180 -180];
UpperBounds = [ 180  180  180 180  180  180  180  180];
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);

```

To impose these constraints, call `hedgeopt` with `ConSet` as the last input.

```

[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens, ConSet);

```

Examine the outputs and see that they are all set to `NaN`, indicating that the problem, given the constraints, is not solvable. Intuitively, the results mean that you cannot obtain simultaneous delta, gamma, and vega neutrality with these constraints at any price.

To see how close you can get to portfolio neutrality with these constraints, call `hedgeslf`.

```

[Sens, Value1, Quantity] = hedgeslf(Sensitivities, Price,...
Holdings, FixedInd, ConSet);

```

```

Sens =
-352.43
21.99

```

```

-498.77
Value1 =
855.10
Quantity =
100.00
-180.00
-37.22
80.00
8.00
-31.86
40.00
10.00

```

hedgeslf enforces the lower bound for the second instrument, but the sensitivity is far from neutral. The cost to obtain this portfolio is

```

Value0 - Value1
ans =
22819.52

```

## Portfolio Rebalancing

As a final example of user-specified constraints, rebalance the portfolio using the second hedging goal of hedgeopt. Assume that you are willing to spend as much as \$20,000 to rebalance your portfolio, and you want to know what minimum portfolio sensitivities you can get for your money. In this form, recall that the target cost (\$20,000) is treated as an inequality constraint during the optimization process.

For reference, start up hedgeopt without any user-specified linear inequality constraints.

```

[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], 20000);
Sens =
-4345.36      295.81     -6586.64
Cost =

```

```

20000.00
Quantity' =
    100.00
   -151.86
   -253.47
    80.00
     8.00
   -18.18
    40.00
    10.00

```

This result corresponds to the \$20,000 point along the Portfolio Sensitivities Profile shown in the figure “Rebalancing Cost” on page 4-15.

Assume that, in addition to holding instruments 1, 4, 5, 7, and 8 fixed as before, you want to bound the position of all instruments to within +/- 150 contracts (for each instrument, you cannot short more than 150 contracts and you cannot long more than 150 contracts). These bounds disallow the current position in the second and third instruments (-151.86 and -253.47). All other instruments are currently within the upper/lower bounds.

As before, you can generate these constraints by first specifying the lower and upper bounds vectors and then calling `portcons`.

```

LowerBounds = [-150 -150 -150 -150 -150 -150 -150 -150];
UpperBounds = [ 150  150  150  150  150  150  150  150];
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);

```

To impose these constraints, again call `hedgeopt` with `ConSet` as the last input.

```

[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
Holdings, FixedInd, [], 20000, [], ConSet);

Sens =
   -8818.47      434.43   -4010.79

Cost =
   19876.89

Quantity' =

```

```
100.00  
-150.00  
-150.00  
80.00  
8.00  
-28.32  
40.00  
10.00
```

With these constraints, `hedgeopt` enforces the lower bound for the second and third instruments. The cost incurred is \$19,876.89.

## See Also

`hedgeopt` | `hedgeslf`

## Related Examples

- “Portfolio Creation” on page 1-8
- “Adding Instruments to an Existing Portfolio” on page 1-11
- “Instrument Constructors” on page 1-20
- “Creating Instruments or Properties” on page 1-21
- “Searching or Subsetting a Portfolio” on page 1-23
- “Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13
- “Pricing and Hedging a Portfolio Using the Black-Karasinski Model” on page 4-17
- “Hedging with Constrained Portfolios” on page 4-37

## More About

- “Instrument Constructors” on page 1-20
- “Hedging” on page 4-2

## Hedging with Constrained Portfolios

### In this section...

“Overview” on page 4-37

“Example: Fully Hedged Portfolio” on page 4-37

“Example: Minimize Portfolio Sensitivities” on page 4-39

“Example: Under-Determined System” on page 4-40

“Example: Portfolio Constraints with hedgeslf” on page 4-42

### Overview

Both hedging functions cast the optimization as a constrained linear least-squares problem. (See the function `lsqlin` for details.) In particular, `lsqlin` attempts to minimize the constrained linear least squares problem

$$\min_x \frac{1}{2} \|Cx - d\|_2^2 \text{ such that } \begin{aligned} A \cdot x &\leq b \\ Aeq \cdot x &= beq \\ lb &\leq x \leq ub \end{aligned}$$

where  $C$ ,  $A$ , and  $Aeq$  are matrices, and  $d$ ,  $b$ ,  $beq$ ,  $lb$ , and  $ub$  are vectors. For Financial Instruments Toolbox software,  $x$  is a vector of asset holdings (contracts).

Depending on the constraint and the number of assets in the portfolio, a solution to a particular problem may or may not exist. Furthermore, if a solution is found, it may not be unique. For a unique solution to exist, the least squares problem must be sufficiently and appropriately constrained.

### Example: Fully Hedged Portfolio

Recall that `hedgeopt` allows you to allocate an optimal hedge by one of two goals:

- Minimize the cost of hedging a portfolio given a set of target sensitivities.
- Minimize portfolio sensitivities for a given set of maximum target costs.

As an example, reproduce the results for the fully hedged portfolio example.

```
TargetSens = [0 0 0];
FixedInd    = [1 4 5 7 8];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
    Holdings, FixedInd, [], [], TargetSens);
```

Sens =

```
        -0.00          -0.00          -0.00
```

Cost =

```
23055.90
```

Quantity' =

```
    98.72
   -182.36
    -19.55
    80.00
     8.00
   -32.97
    40.00
    10.00
```

This example finds a unique solution at a cost of just over \$23,000. The matrix C (formed internally by `hedgeopt` and passed to `lsqlin`) is the asset Price vector expressed as a row vector.

```
C = Price' = [98.72 97.53 0.05 98.72 100.55 6.28 0.05 3.69]
```

The vector `d` is the current portfolio value `Value0 = 23674.62`. The example maintains, as closely as possible, a constant portfolio value subject to the specified constraints.

### Additional Constraints

In the absence of any additional constraints, the least squares objective involves a single equation with eight unknowns. This is an under-determined system of equations. Because such systems generally have an infinite number of solutions, you need to specify additional constraints to achieve a solution with practical significance.

The additional constraints can come from two sources:

- User-specified equality constraints

- Target sensitivity equality constraints imposed by `hedgeopt`

The example in “Fully Hedged Portfolio” on page 4-9 specifies five equality constraints associated with holding assets 1, 4, 5, 7, and 8 fixed. This reduces the number of unknowns from eight to three, which is still an under-determined system. However, when combined with the first goal of `hedgeopt`, the equality constraints associated with the target sensitivities in `TargetSens` produce an additional system of three equations with three unknowns. This additional system guarantees that the weighted average of the delta, gamma, and vega of assets 2, 3, and 6, together with the remaining assets held fixed, satisfy the overall portfolio target sensitivity needs in `TargetSens`.

Combining the least-squares objective equation with the three portfolio sensitivity equations provides an overall system of four equations with three unknown asset holdings. This is no longer an under-determined system, and the solution is as shown.

If the assets held fixed are reduced, for example, `FixedInd = [1 4 5 7]`, `hedgeopt` returns a no cost, fully hedged portfolio (`Sens = [0 0 0]` and `Cost = 0`).

If you further reduce `FixedInd` (for example, `[1 4 5]`, `[1 4]`, or even `[]`), `hedgeopt` always returns a no cost, fully hedged portfolio. In these cases, insufficient constraints result in an under-determined system. Although `hedgeopt` identifies no cost, fully hedged portfolios, there is nothing unique about them. These portfolios have little practical significance.

Constraints must be *sufficient* and *appropriately defined*. Additional constraints having no effect on the optimization are called *dependent constraints*. As a simple example, assume that parameter  $Z$  is constrained such that  $Z \leq 1$ . Furthermore, assume that you somehow add another constraint that effectively restricts  $Z \leq 0$ . The constraint  $Z \leq 1$  now has no effect on the optimization.

## Example: Minimize Portfolio Sensitivities

To illustrate using `hedgeopt` to minimize portfolio sensitivities for a given maximum target cost, specify a target cost of \$20,000 and determine the new portfolio sensitivities, holdings, and cost of the rebalanced portfolio.

```
MaxCost = 20000;
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
Holdings, [1 4 5 7 8], [], MaxCost);
```

```
Sens =
```

```
          -4345.36      295.81      -6586.64
Cost =
          20000.00
Quantity' =
          100.00
          -151.86
          -253.47
           80.00
           8.00
          -18.18
           40.00
           10.00
```

This example corresponds to the \$20,000 point along the cost axis in the figures “Rebalancing Cost Profile” on page 4-11, “Funds Available for Rebalancing” on page 4-12, and “Rebalancing Cost” on page 4-15.

When minimizing sensitivities, the maximum target cost is treated as an inequality constraint; in this case, `MaxCost` is the most you are willing to spend to hedge a portfolio. The least-squares objective matrix `C` is the matrix transpose of the input asset sensitivities

```
C = Sensitivities'
```

a 3-by-8 matrix in this example, and `d` is a 3-by-1 column vector of zeros, `[0 0 0]'`.

Without any additional constraints, the least-squares objective results in an under-determined system of three equations with eight unknowns. By holding assets 1, 4, 5, 7, and 8 fixed, you reduce the number of unknowns from eight to three. Now, with a system of three equations with three unknowns, `hedgeopt` finds the solution shown.

### Example: Under-Determined System

Reducing the number of assets held fixed creates an under-determined system with meaningless solutions. For example, see what happens with only four assets constrained.

```
FixedInd = [1 4 5 7];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], MaxCost);
```



```

Sens =
      -0.00      -0.00      -0.00

Cost =
      20000.00

Quantity' =
      100.00
      -149.31
      -14.91
      80.00
      8.00
      -34.64
      40.00
      -32.60

```

You have spent \$20,000 (all the funds available for rebalancing) to achieve a fully hedged portfolio.

With an increase in available funds to \$50,000, you still spend all available funds to get another fully hedged portfolio.

```

MaxCost = 50000;
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
Holdings, FixedInd, [],MaxCost);

```

```

Sens =
      -0.00      0.00      0.00

Cost =
      50000.00

Quantity' =
      100.00
      -473.78
      -60.51
      80.00
      8.00
      -18.20

```

```
40.00  
385.60
```

All solutions to an under-determined system are meaningless. You buy and sell various assets to obtain zero sensitivities, spending all available funds every time. If you reduce the number of fixed assets any further, this problem is insufficiently constrained, and you find no solution (the outputs are all NaN).

Note also that no solution exists whenever constraints are *inconsistent*. Inconsistent constraints create an infeasible solution space; the outputs are all NaN.

### Example: Portfolio Constraints with `hedgeslf`

The other hedging function, `hedgeslf`, attempts to minimize portfolio sensitivities such that the rebalanced portfolio maintains a constant value (the rebalanced portfolio is hedged against market moves and is closest to being self-financing). If a self-financing hedge is not found, `hedgeslf` tries to rebalance a portfolio to minimize sensitivities.

From a least-squares systems approach, `hedgeslf` first attempts to minimize cost in the same way that `hedgeopt` does. If it cannot solve this problem (a no cost, self-financing hedge is not possible), `hedgeslf` proceeds to minimize sensitivities like `hedgeopt`. Thus, the discussion of constraints for `hedgeopt` is directly applicable to `hedgeslf` as well.

To illustrate this hedging facility using equity exotic options, consider the portfolio `CRRInstSet` obtained from the example MAT-file `deriv.mat`. The portfolio consists of eight option instruments: two stock options, one barrier, one compound, two lookback, and two Asian.

The hedging functions require inputs that include the current portfolio holdings (allocations) and a matrix of instrument sensitivities. To create these inputs, start by loading the example portfolio into memory

```
load deriv.mat;
```

Next, compute the prices and sensitivities of the instruments in this portfolio.

```
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, CRRInstSet);
```

Extract the current portfolio holdings (the quantity held or the number of contracts).

```
Holdings = instget(CRRInstSet, 'FieldName', 'Quantity');
```

For convenience place the delta, gamma, and vega sensitivity measures into a matrix of sensitivities.

```
Sensitivities = [Delta Gamma Vega];
```

Each row of the `Sensitivities` matrix is associated with a different instrument in the portfolio and each column with a different sensitivity measure.

```
disp([Price Holdings Sensitivities])
```

8.29	10.00	0.59	0.04	53.45
2.50	5.00	-0.31	0.03	67.00
12.13	1.00	0.69	0.03	67.00
3.32	3.00	-0.12	-0.01	-98.08
7.60	7.00	-0.40	-45926.32	88.18
11.78	9.00	-0.42	-112143.15	119.19
4.18	4.00	0.60	45926.32	49.21
3.42	6.00	0.82	112143.15	41.71

The first column contains the dollar unit price of each instrument, the second contains the holdings of each instrument, and the third, fourth, and fifth columns contain the delta, gamma, and vega dollar sensitivities, respectively.

Suppose that you want to obtain a delta, gamma, and vega neutral portfolio using `hedgeslf`.

```
[Sens, Value1, Quantity]= hedgeslf(Sensitivities, Price, ...
Holdings)
```

```
Sens =
```

```
0.00
-0.00
0.00
```

```
Value1 =
```

```
313.93
```

```
Quantity =
```

```
10.00
7.64
-1.56
26.13
9.94
3.73
```

```
-0.75  
8.11
```

`hedgeslf` returns the portfolio dollar sensitivities (`Sens`), the value of the rebalanced portfolio (`Value1`) and the new allocation for each instrument (`Quantity`).

If `Value0` and `Value1` represent the portfolio value before and after rebalancing, respectively, you can verify the cost by comparing the portfolio values.

```
Value0= Holdings' * Price
```

```
Value0 =
```

```
313.93
```

In this example, the portfolio is fully hedged (simultaneous delta, gamma, and vega neutrality) and self-financing (the values of the portfolio before and after balancing (`Value0` and `Value1`) are the same).

Suppose now that you want to place some upper and lower bounds on the individual instruments in your portfolio. By using Financial Toolbox function `portcons`, you can specify these constraints, along with various general linear inequality constraints.

As an example, assume that, in addition to holding instrument 1 fixed as before, you want to bound the position of all instruments to within +/- 20 contracts (for each instrument, you cannot short or long more than 20 contracts). Applying these constraints disallows the current position in the fourth instrument (long 26.13). All other instruments are currently within the upper/lower bounds.

You can generate these constraints by first specifying the lower and upper bounds vectors and then calling `portcons`.

```
LowerBounds = [-20 -20 -20 -20 -20 -20 -20 -20];  
UpperBounds = [20 20 20 20 20 20 20 20];  
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

To impose these constraints, call `hedgeslf` with `ConSet` as the last input.

```
[Sens, Cost, Quantity1] = hedgeslf(Sensitivities, Price, ...  
Holdings, 1, ConSet)
```

```
Sens =
```

```
-0.00
```

```
0.00
0.00

Cost =

313.93

Quantity1 =

10.00
5.28
10.98
20.00
20.00
-6.99
-20.00
9.39
```

Observe that `hedgeslf` enforces the upper bound on the fourth instrument, and the portfolio continues to be fully hedged and self-financing.

## See Also

`hedgeopt` | `hedgeslf`

## Related Examples

- “Portfolio Creation” on page 1-8
- “Adding Instruments to an Existing Portfolio” on page 1-11
- “Instrument Constructors” on page 1-20
- “Creating Instruments or Properties” on page 1-21
- “Searching or Subsetting a Portfolio” on page 1-23
- “Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13
- “Pricing and Hedging a Portfolio Using the Black-Karasinski Model” on page 4-17
- “Specifying Constraints with `ConSet`” on page 4-32
- “Portfolio Rebalancing” on page 4-34

**More About**

- “Instrument Constructors” on page 1-20
- “Hedging” on page 4-2

# Mortgage-Backed Securities

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- “What Are Mortgage-Backed Securities?” on page 5-2
- “Fixed-Rate Mortgage Pool” on page 5-3
- “Computing Option-Adjusted Spread” on page 5-11
- “Prepayments with Fewer Than 360 Months Remaining” on page 5-14
- “Pools with Different Numbers of Coupons Remaining” on page 5-17
- “Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model” on page 5-19
- “Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model” on page 5-42
- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51
- “Prepayment Risk” on page 5-52
- “CMO Workflow” on page 5-60
- “Create PAC and Sequential CMO” on page 5-63

## What Are Mortgage-Backed Securities?

Mortgage-backed securities (MBSs) are a type of investment that represents ownership in a group of mortgages. Principal and interest from the individual mortgages are used to pay principal and interest on the MBS.

Ownership in a group of mortgages is typically represented by a *pass-through certificate* (PC). Most pass-through certificates are issued by the Government National Mortgage Agency, a branch of the United States government, or by one of two private corporations: Fannie Mae or Freddie Mac. With these certificates, homeowners' payments pass from the originating bank through the issuing agency to holders of the certificates. These agencies also frequently guarantee that the certificate holder receives timely payment of principal and interest from the PCs.

### See Also

mbscfamounts | mbsconvp | mbsconvy | mbsdurp | mbsdury | mbsnoprepay |  
mboas2price | mboas2yield | mbspassthrough | mbsprice | mbsprice2oas |  
mbsprice2speed | mbswal | mbsyield | mbsyield2oas | mbsyield2speed |  
psaspeed2default | psaspeed2rate

### Related Examples

- “Fixed-Rate Mortgage Pool” on page 5-3



# Fixed-Rate Mortgage Pool

## In this section...

“Introduction” on page 5-3

“Inputs to Functions” on page 5-3

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Risk Measurement” on page 5-8

“Mortgage Pool Valuation” on page 5-9

## Introduction

Financial Instruments Toolbox software supports calculations involved with generic fixed-rate mortgage pools and balloon mortgages. Generic fixed-rate mortgage pools and balloon mortgages have pass-through certificates (PC) that typically have embedded call options in the form of prepayment. Prepayment is an excess payment applied to the principal of a PC. These accelerated payments reduce the effective life of a PC.

The toolbox comes with a standard Bond Market Association (PSA) prepayment model and can generate multiples of standard prepayment speeds. The Public Securities Association provides a set of uniform practices for calculating the characteristics of mortgage-backed securities when there is an assumed prepayment function.

Alternatively, aside from the standard PSA implementation in this toolbox, you can supply your own projected prepayment vectors. Currently, however, custom prepayment functionality that incorporates pool-specific information and interest rate forecasts are not available in this toolbox. If you plan to use custom prepayment vectors in your calculations, you presumably already own such a suite in MATLAB.

## Inputs to Functions

Because of the generic, all-purpose nature of the toolbox pass-through functions, you can fine-tune them to conform to a particular mortgage. Most functions require at least this set of inputs:

- Gross coupon rate

- Settlement date
- Issue (effective) date
- Maturity date

Typical optional inputs include standard prepayment speed (or customized vector), net coupon rate (if different from gross coupon rate), and payment delay in number of days.

All calculations are based on expected payment dates and actual cash flow to the investor. For example, when `GrossRate` and `CouponRate` differ as inputs to `mbsdurp`, the function returns a modified duration based on `CouponRate`. (A notable exception is `mbspassthrough`, which returns interest quantities based on the `GrossRate`.)

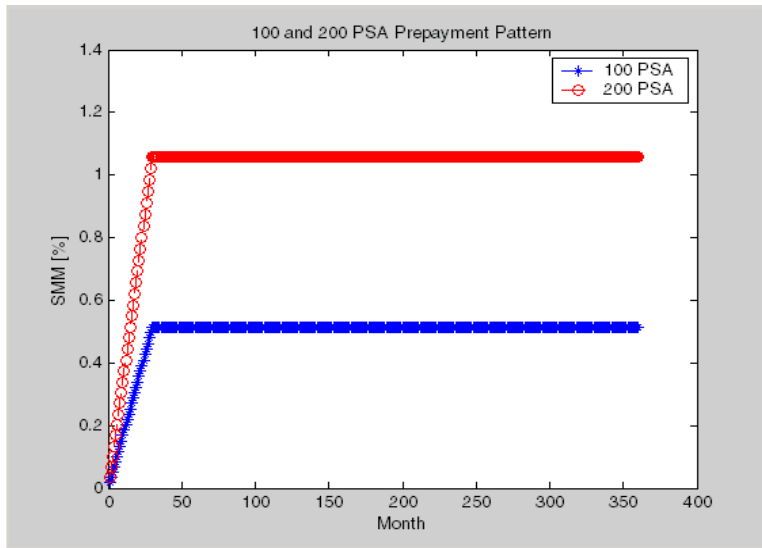
### Generating Prepayment Vectors

You can generate PSA multiple prepayment vectors quickly. To generate prepayment vectors of 100 and 200 PSA, type

```
PSASpeed = [100, 200];  
[CPR, SMM] = psaspeed2rate(PSASpeed);
```

This function computes two prepayment values: conditional prepayment rate (CPR) and single monthly mortality (SMM) rate. CPR is the percentage of outstanding principal prepaid in one year. SMM is the percentage of outstanding principal prepaid in one month. In other words, CPR is an annual version of SMM.

Since the entire 360-by-2 array is too long to show in this document, observe the SMM (100 and 200 PSA) plots, spaced one month apart, instead.



Prepayment assumptions form the basis upon which far more comprehensive MBS calculations are based. As an illustration, observe the following example, which shows the use of the function `mbscfamounts` to generate cash flows and timings based on a set of standard prepayments.

Consider three mortgage pools that were sold on the issue date (which starts unamortized). The first two pools "balloon out" in 60 months, and the third is regularly amortized to the end. The prepayment speeds are assumed to be 100, 200, and 200 PSA, respectively.

```
Settle      = [datenum('1-Feb-2000');
               datenum('1-Feb-2000');
               datenum('1-Feb-2000')];

Maturity    = [datenum('1-Feb-2030')];

IssueDate   = datenum('1-Feb-2000');
GrossRate   = 0.08125;
CouponRate  = 0.075;
Delay       = 14;

PSASpeed    = [100, 200];
[CPR, SMM]  = psaspeed2rate(PSASpeed);
```

```

PrepayMatrix = ones(360,3);
PrepayMatrix(1:60,1:2) = SMM(1:60,1:2);
PrepayMatrix(:,3) = SMM(:,2);

[CFlowAmounts, CFlowDates, TFactors, Factors] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, [], PrepayMatrix);

```

The fourth output argument, `Factors`, indicates the fraction of the balance still outstanding at the beginning of each month. A snapshot of this argument in the MATLAB Variables editor illustrates the 60-month life of the first two of the mortgages with balloon payments and the continuation of the third mortgage until the end (360 months).

	59	60	61	62	63
1	0.7627	0.7580	0.7533	0	0
2	0.6021	0.5951	0.5882	0	0
3	0.6021	0.5951	0.5882	0.5813	0.5746

You can readily see that `mbscfamounts` is the building block of most fixed rate and balloon pool cash flows.

## Mortgage Prepayments

Prepayment is beneficial to the pass-through owner when a mortgage pool has been purchased at discount. The next example compares mortgage yields (compounded monthly) versus the purchase clean price with constant prepayment speed. The example illustrates that when you have purchased a pool at a discount, prepayment generates a higher yield with decreasing purchase price.

```

Price = [85; 90; 95];
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;

```

Compute the mortgage and bond-equivalent yields.

```
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, ...  
IssueDate, GrossRate, CouponRate, Delay, Speed)
```

```
MYield =
```

```
0.1018  
0.0918  
0.0828
```

```
BEMBSYield =
```

```
0.1040  
0.0936  
0.0842
```

If for this same pool of mortgages, there was no prepayment ( $\text{Speed} = 0$ ), the yields would decline to

```
MYield =
```

```
0.0926  
0.0861  
0.0802
```

```
BEMBSYield =
```

```
0.0944  
0.0877  
0.0815
```

Likewise, if the rate of prepayment doubled ( $\text{Speed} = 200$ ), the yields would increase to

```
MYield =
```

```
0.1124  
0.0984  
0.0858
```

```
BEMBSYield =
```

```
0.1151  
0.1004  
0.0873
```

For the same prepayment vector, deeper discount pools earn higher yields. For more information, see `mbsprice` and `mbsyield`.

## Risk Measurement

Financial Instruments Toolbox provides the most basic risk measures of a pool portfolio:

- Modified duration
- Convexity
- Average life of pool

Consider the following example, which calculates the Macaulay and modified durations given the price of a mortgage pool.

```
Price = [95; 100; 105];  
Settle = datenum('15-Apr-2002');  
Maturity = datenum('1-Jan-2030');  
IssueDate = datenum('1-Jan-2000');  
GrossRate = 0.08125;  
CouponRate = 0.075;  
Delay = 14;  
Speed = 100;
```

```
[YearDuration, ModDuration] = mbsdurp(Price, Settle, ...  
Maturity, IssueDate, GrossRate, CouponRate, Delay, Speed)
```

```
YearDuration =
```

```
6.1341  
6.3882  
6.6339
```

```
ModDuration =
```

```
5.8863  
6.1552  
6.4159
```

Using Financial Instruments Toolbox functions, you can obtain modified duration and convexity from either price or yield, as long as you specify a prepayment vector or an assumed prepayment speed. The toolbox risk-measurement functions (`mbsdurp`, `mbsdury`, `mbsconvp`, `mbsconvy`, and `mbswal`) adhere to the guidelines listed in the *PSA Uniform Practices* manual.

## Mortgage Pool Valuation

For accurate valuation of a mortgage pool, you must generate interest-rate paths and use them with mortgage pool characteristics to properly value the pool. A widely used methodology is the option-adjusted spread (OAS). OAS measures the yield spread that is not directly attributable to the characteristics of a fixed-income investment.

### Calculating OAS

Prepayment alters the cash flows of an otherwise regularly amortizing mortgage pool. A comprehensive option-adjusted spread calculation typically begins with the generation of a set of paths of spot rates to predict prepayment. A path is collection of  $i$  spot-rate paths, with corresponding  $j$  cash flows on each of those paths.

The effect of the OAS on pool pricing is shown mathematically in the following equation, where  $K$  is the option-adjusted spread.

$$PoolPrice = \frac{1}{NumberofPaths} \times \sum_i^{NumberofPaths} \sum_j^{CF_{ij}} \frac{CF_{ij}}{(1 + zerorates_{ij} + K)^{T_{ij}}}$$

### Calculating Effective Duration

Alternatively, if you are more interested in the sensitivity of a mortgage pool to interest rate changes, use effective duration, which is a more appropriate measure. Effective duration is defined mathematically with the following equation.

$$Effective\ Duration = \frac{P(y + \Delta y) - P(y - \Delta y)}{2P(y)\Delta y}$$

### Calculating Market Price

The toolbox has all the components required to calculate OAS and effective duration if you supply prepayment vectors or assumptions. For OAS, given a prepayment vector, you can generate a set of cash flows with `mbscfamounts`. Discounting these cash flows with the reference curve and then adding OAS produces the market price. See “Computing Option-Adjusted Spread” on page 5-11 for a discussion on the computation of option-adjusted spread.

Effective duration is a more difficult issue. While modified duration changes the discounting process (by changing the yield used to discount cash flows), effective duration must account for the change in cash flow because of the change in yield. A

possible solution is to recompute prices using `mbsprice` for a small change in yield, in both the upwards and downwards directions. In this case, you must recompute the prepayment input. Internally, this alters the cash flows of the mortgage pool. Assuming that the OAS stays constant in all yield environments, you can apply a set of discounting factors to the cash flows in up and down yield environments to find the effective duration.

### See Also

`mbscfamounts` | `mbsconvp` | `mbsconvy` | `mbsdurp` | `mbsdury` | `mbsnoprepay` | `mboas2price` | `mboas2yield` | `mbspassthrough` | `mbsprice` | `mbsprice2oas` | `mbsprice2speed` | `mbswal` | `mbsyield` | `mbsyield2oas` | `mbsyield2speed` | `psaspeed2default` | `psaspeed2rate`

### Related Examples

- “Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model” on page 5-19
- “Computing Option-Adjusted Spread” on page 5-11
- “Prepayments with Fewer Than 360 Months Remaining” on page 5-14
- “Pools with Different Numbers of Coupons Remaining” on page 5-17
- “Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model” on page 5-42
- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51

### More About

- “What Are Mortgage-Backed Securities?” on page 5-2



## Computing Option-Adjusted Spread

The option-adjusted spread (OAS) is an amount of extra interest added above (or below if negative) the reference zero curve. To compute the OAS, you must provide the zero curve as an extra input. You can specify the zero curve in any intervals and with any compounding method. (To minimize any error due to interpolation, keep the intervals as regular and frequent as possible.) You must supply a prepayment vector or specify a speed corresponding to a standard PSA prepayment vector.

One way to compute the appropriate zero curve for an agency is to look at its bond yields and bootstrap them from the shortest maturity onwards. You can do this with Financial Toolbox functions `zbtprice` and `zbtyield`.

The following example shows how to calculate an appropriate zero curve followed by computation of the pool's OAS. This example calculates the OAS of a 30-year fixed rate mortgage with about a 28-year weighted average maturity left, given an assumption of 0, 50, and 100 PSA prepayment speeds.

Create curve for zerorates.

```
Bonds = [datenum('11/21/2002') 0 100 0 2 1;
         datenum('02/20/2003') 0 100 0 2 1;
         datenum('07/31/2004') 0.03 100 2 3 1;
         datenum('08/15/2007') 0.035 100 2 3 1;
         datenum('08/15/2012') 0.04875 100 2 3 1;
         datenum('02/15/2031') 0.05375 100 2 3 1];

Yields = [0.0162;
          0.0163;
          0.0211;
          0.0328;
          0.0420;
          0.0501];
```

Since the above is Treasury data and not selected agency data, a term structure of spread is assumed. In this example, the spread declines proportionally from a maximum of 250 basis points at the shortest maturity.

```
Yields = Yields + 0.025 * (1./[1:6]');
```

Get parameters from Bonds matrix.

```
Settle = datenum('20-Aug-2002');
Maturity = Bonds(:,1);
CouponRate = Bonds(:,2);
Face = Bonds(:,3);
Period = Bonds(:,4);
```

```
Basis = Bonds(:,5);
EndMonthRule = Bonds(:,6);

[Prices, AccruedInterest] = bndprice(Yields, CouponRate, ...
Settle, Maturity, Period, Basis, EndMonthRule, [], [], [], [], ...
Face);
```

Use `zbtprice` to solve for zero rates.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCompounding = 2*ones(size(ZeroRatesP));
ZeroMatrix = [CurveDatesP, ZeroRatesP, ZeroCompounding];
```

Use output from `zbtprice` to calculate the OAS.

```
Price = 95;
Settle = datenum('20-Aug-2002');
Maturity = datenum('2-Jan-2030');
IssueDate = datenum('2-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0; 50; 100];

OAS = mbsprice2oas(ZeroMatrix, Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Interpolation, ...
PrepaySpeed)
```

```
OAS =

    26.0502
    28.6348
    31.2222
```

This example shows that one cash flow set is being discounted and solved for its OAS, as contrasted with the `NumberOfPaths` set of cash flows as shown in “Mortgage Pool Valuation” on page 5-9. Averaging the sets of cash flows resulting from all simulations into one average cash flow vector and solving for the OAS, discounts the averaged cash flows to have a present value of today's (average) price.

While this example uses the mortgage pool price (`mbsprice2oas`) to determine the OAS, you can also use yield to resolve it (`mbsyield2oas`). Also, there are reverse OAS functions that return prices and yields given OAS (`mbsoas2price` and `mbsoas2yield`).

The example also restates earlier examples that show discount securities benefit from higher level of prepayment, keeping everything else unchanged. The relation is reversed for premium securities.

## See Also

mbscfamounts | mbsconvp | mbsconvy | mbsdurp | mbsdury | mbsnoprepay | mboas2price | mboas2yield | mbspassthrough | mbsprice | mbsprice2oas | mbsprice2speed | mbswal | mbsyield | mbsyield2oas | mbsyield2speed | psaspeed2default | psaspeed2rate

## Related Examples

- “Fixed-Rate Mortgage Pool” on page 5-3
- “Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model” on page 5-19
- “Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model” on page 5-42
- “Prepayments with Fewer Than 360 Months Remaining” on page 5-14
- “Pools with Different Numbers of Coupons Remaining” on page 5-17
- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51

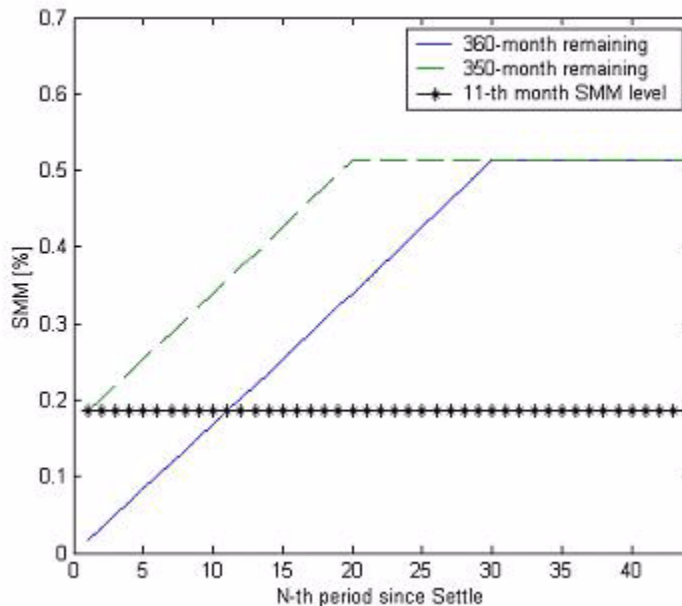
## More About

- “What Are Mortgage-Backed Securities?” on page 5-2

## Prepayments with Fewer Than 360 Months Remaining

When fewer than 360 months remain in the pool, the applicable PSA prepayment vector is "seasoned" by the pool's age. (Elements in the 360-element prepayment vector that represent past payments are skipped. For example, on a 30-year mortgage that is 10 months old, only the final 350 prepayments are applied.)

Assume, for example, that you have two 30-year loans, one new and another 10 months old. Both have the same PSA speed of 100 and prepay using the vectors plotted below.



Still within the scope of relative valuation, you could also solve for the percentage of the standard PSA prepayment vector given the pool's arbitrary, user-supplied prepayment vector, such that the PSA speed gives the same Macaulay duration as the user-supplied prepayment vector.

If you supply a custom prepayment vector, you must account for the number of months remaining.

```
Price = 101;
Settle = datenum('1-Jan-2001');
```

```

Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
PrepayMatrix = 0.005*ones(348,1);
CouponRate = 0.075;
Delay = 14;

ImpliedSpeed = mbsprice2speed(Price, Settle, Maturity, ...
IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay)

ImpliedSpeed =

    104.2543

```

Examine the prepayment input. The remaining 29 years require 348 monthly elements in the prepayment vector. Suppose then, keeping everything the same, you change `Settle` to February 14, 2003.

```
Settle = datenum('14-Feb-2003');
```

You can use `cpncount` to count all incoming coupons received after `Settle` by invoking

```

NumCouponsRemaining = cpncount(Settle, Maturity, 12, 1, [], ...
IssueDate)

NumCouponsRemaining =

    323

```

The input 12 defines the monthly payment frequency, 1 defines the 30/360 basis, and `IssueDate` defines aging and determination-of-holder date. Thus, you must supply a 323-element vector to account for a prepayment corresponding to each monthly payment.

## See Also

`mbscfamounts` | `mbsconvp` | `mbsconvy` | `mbsdurp` | `mbsdury` | `mbsnoprepay` |  
`mbsoas2price` | `mbsoas2yield` | `mbspassthrough` | `mbsprice` | `mbsprice2oas` |  
`mbsprice2speed` | `mbswal` | `mbsyield` | `mbsyield2oas` | `mbsyield2speed` |  
`psaspeed2default` | `psaspeed2rate`

## Related Examples

- “Fixed-Rate Mortgage Pool” on page 5-3

- “Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model” on page 5-19
- “Computing Option-Adjusted Spread” on page 5-11
- “Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model” on page 5-42
- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51

### **More About**

- “What Are Mortgage-Backed Securities?” on page 5-2

## Pools with Different Numbers of Coupons Remaining

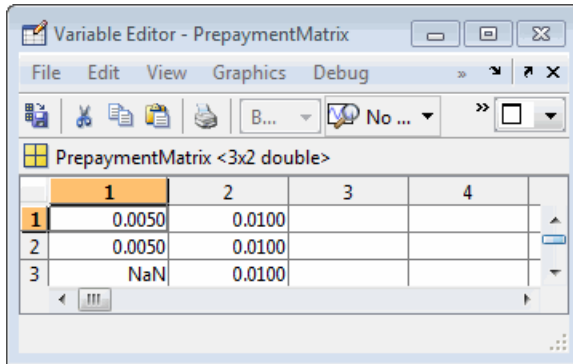
Suppose that one pool has two remaining coupons, and the other has three. MATLAB expects the prepayment matrix to be in the following format:

```
V11      V21
V12      V22
NaN      V23
```

$V_{ij}$  denotes the single monthly mortality (SMM) rate for pool  $i$  during the  $j$ th coupon period since `Settle`.

The use of `NaN` to pad the prepayment matrix is necessary because MATLAB cannot concatenate vectors of different lengths into a matrix. Also, it can serve as an error check against any unintended operation (any MATLAB operation that would return `NaN`).

For example, assume that the 2-month pool has a constant SMM of 0.5% and the 3-month pool has a constant SMM of 1% in every period. The prepayment matrix you would create is depicted below.



	1	2	3	4
1	0.0050	0.0100		
2	0.0050	0.0100		
3	NaN	0.0100		

Create this input in whatever manner is best for you.

### Summary of Prepayment Data Vector Representation

- When you specify a PSA prepayment speed, MATLAB "seasons" the pool according to its age.

- When you specify your own prepayment matrix, identify the maximum number of coupons remaining using `cpncount`. Then supply the matrix elements up to the point when cash flow ceases to exist.
- When different length pools must exist in the same matrix, pad the shorter one(s) with `NaN`. Each column of the prepayment matrix corresponds to a specific pool.

### See Also

`mbscfamounts` | `mbsconvp` | `mbsconvy` | `mbsdurp` | `mbsdury` | `mbsnoprepay` | `mbsoas2price` | `mbsoas2yield` | `mbspassthrough` | `mbsprice` | `mbsprice2oas` | `mbsprice2speed` | `mbswal` | `mbsyield` | `mbsyield2oas` | `mbsyield2speed` | `psaspeed2default` | `psaspeed2rate`

### Related Examples

- “Fixed-Rate Mortgage Pool” on page 5-3
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### More About

- “What Are Mortgage-Backed Securities?” on page 5-2



# Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model

This example shows how to model prepayment in MATLAB® using functionality from the Financial Instruments Toolbox™. Specifically, a variation of the Richard and Roll prepayment model is implemented using a two factor Hull-White interest-rate model and a LIBOR Market Model to simulate future interest-rate paths. A mortgage-backed security is priced with both the custom and default prepayment models.

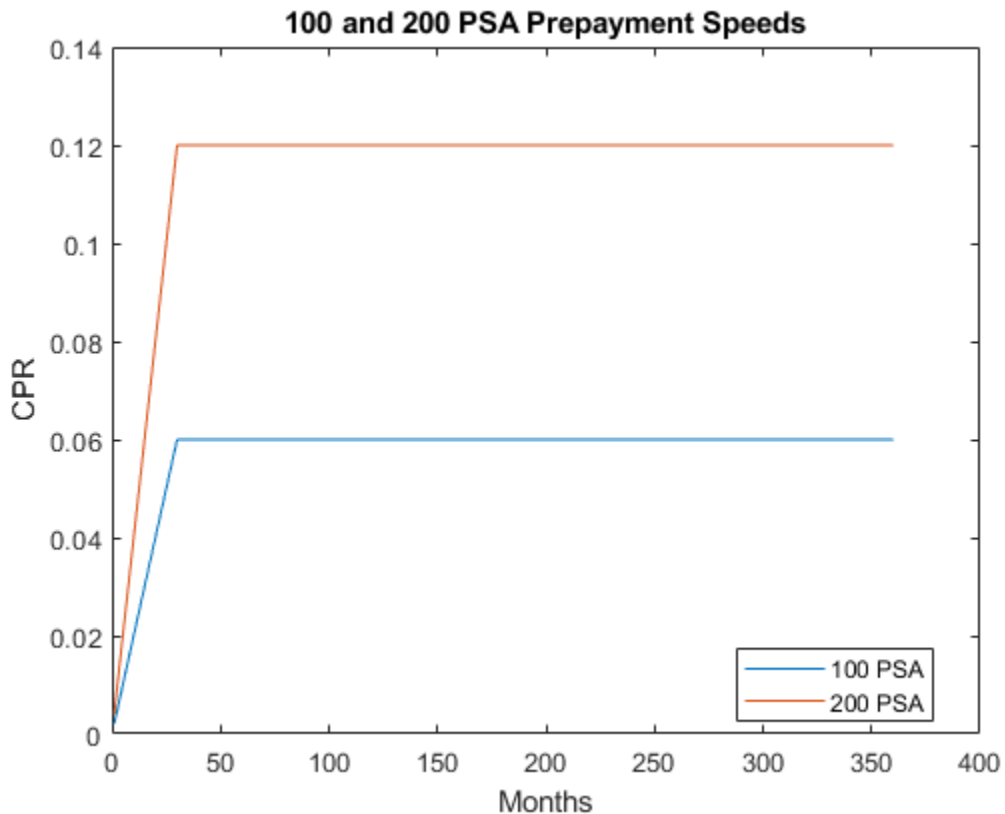
## Introduction

Prepayment modeling is crucial to the analysis of mortgage-backed securities (MBS). Prepayments by individual mortgage holders affect both the amount and timing of cash flows -- and for collateralized mortgage obligations (for example, interest-only securities), prepayment can greatly affect the value of the securities.

## PSA Model

The most basic prepayment model is the Public Securities Association (PSA) model, which assumes a ramp-up phase and then a constant conditional prepayment rate (CPR). The PSA model can be generated in MATLAB using the Financial Instruments Toolbox function `psaspeed2rate`.

```
G2PP_CPR = psaspeed2rate([100 200]);  
figure  
plot(G2PP_CPR)  
title('100 and 200 PSA Prepayment Speeds')  
xlabel('Months')  
ylabel('CPR')  
ylim([0 .14])  
legend({'100 PSA', '200 PSA'}, 'Location', 'Best')
```



### Mortgage-Backed Security

The MBS analyzed in this example matures in 2020 and has the properties outlined in this section. Cash flows are generated for PSA prepayment speeds simply by entering the PSA speed as an input argument.

```
% Parameters for MBS passthrough to be priced
Settle = datenum('15-Dec-2007');
Maturity = datenum('15-Dec-2020');
IssueDate = datenum('15-Dec-2000');
GrossRate = .0475;
CouponRate = .045;
Delay = 14;
Period = 12;
```

```

Basis = 4;

% Generate cash flows and dates for baseline case using 100 PSA
[CFlowAmounts, CFlowDates] = mbscfamounts(Settle,Maturity, IssueDate,...
    GrossRate, CouponRate, Delay,100);
CFlowTimes = yearfrac(Settle,CFlowDates);
NumCouponsRemaining = cpncount(Settle, Maturity, Period,Basis, 1, IssueDate);
    
```

### Richard and Roll Model

While prepayment modeling often involves complex and sophisticated modeling, often at the loan level, this example uses a slightly modified approach based on the model proposed by Richard and Roll in [6].

The Richard and Roll prepayment model involves the following factors:

- Refinancing incentive
- Seasonality (month of the year)
- Seasoning or age of the mortgage
- Burnout

Richard and Roll propose a multiplicative model of the following:

$$CPR = RefiIncentive * SeasoningMultiplier * SeasonalityMultiplier * BurnoutMultiplier$$

For the custom model in this example, the *Burnout Multiplier*, which describes the tendency of prepayment to slow when a significant number of homeowners have already refinanced, is ignored and the first three terms are used.

The refinancing incentive is a function of the ratio of the coupon-rate of the mortgage to the available mortgage rate at that particular point in time. For example, the Office of Thrift Supervision (OTS) proposes the following model:

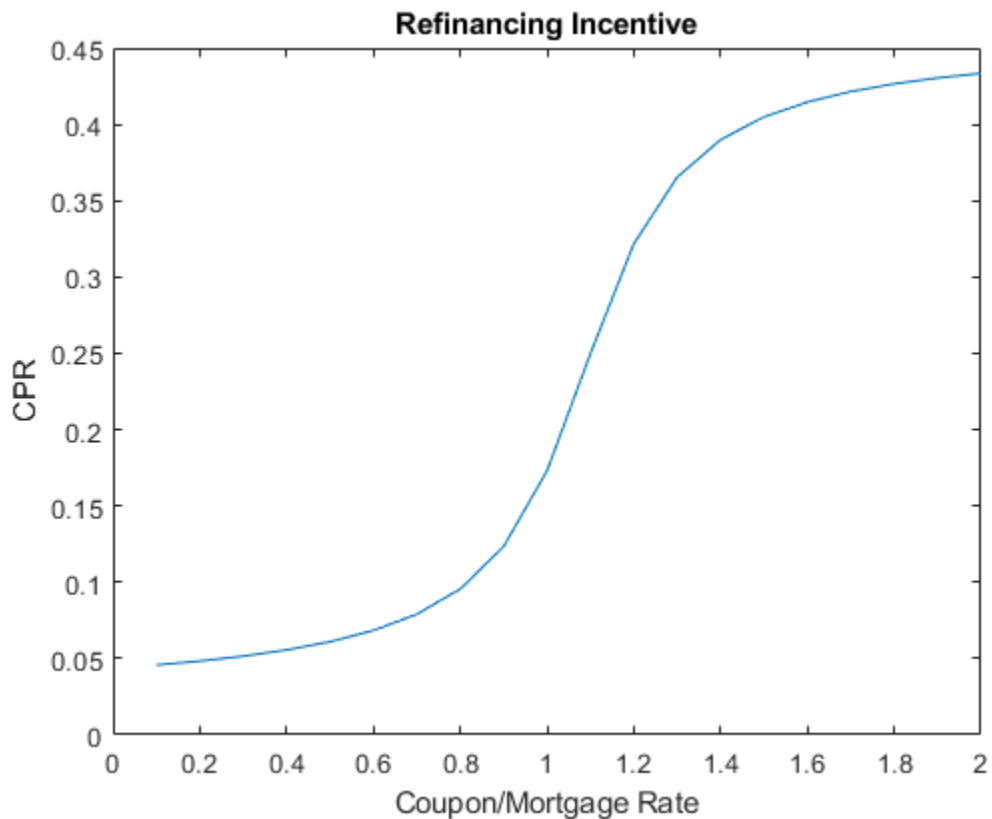
$$Refi = .2406 - .1389 * \arctan\left(5.952 * \left(1.089 - \frac{CouponRate}{MortgageRate}\right)\right)$$

The refinancing incentive requires a simulation of future interest rates. This will be discussed later in this example.

```

C_M = .1:.1:2;
G2PP_Refi = .2406 - .1389 * atan(5.952*(1.089 - C_M));
figure
    
```

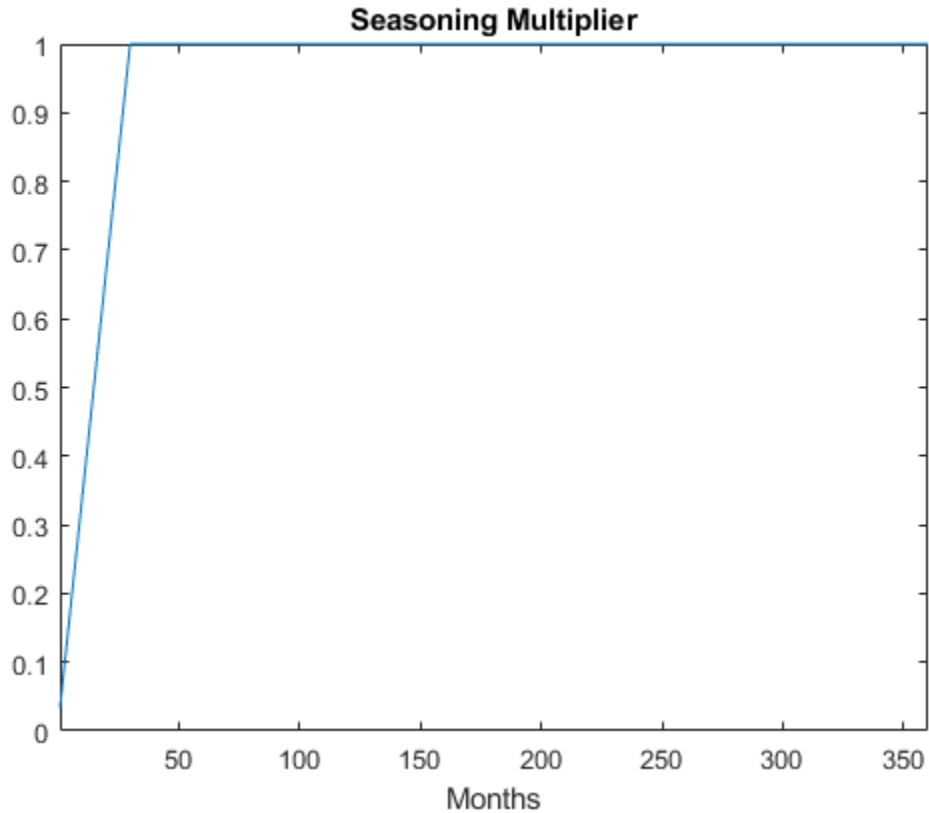
```
plot(C_M,G2PP_Refi)
xlabel('Coupon/Mortgage Rate')
ylabel('CPR')
title('Refinancing Incentive')
```



Seasoning captures the tendency of prepayment to ramp up at the beginning of a mortgage before leveling off. The OTS models the seasoning multiplier as follows:

```
Seasoning = ones(360,1);
Seasoning(1:29) = (1:29)/30;
figure
plot(Seasoning)
xlim([1 360])
```

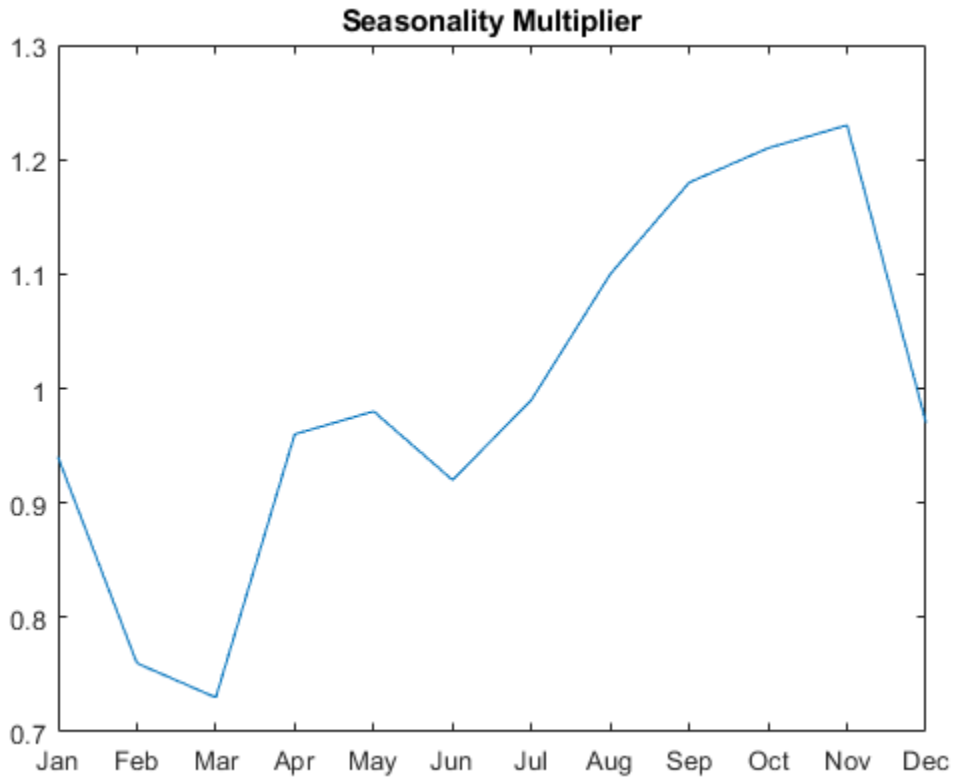
```
title('Seasoning Multiplier')  
xlabel('Months')
```



The seasonality multiplier simply models the seasonal behavior of prepayments -- this data is based on Figure 3 of [6], which applies to the behavior of Ginnie Mae 30-year, single-family MBSs.

```
Seasonality = [.94 .76 .73 .96 .98 .92 .99 1.1 1.18 1.21 1.23 .97];  
figure  
plot(Seasonality)  
xlim([1 12])  
ax = gca;  
ax.XTick = 1:12;  
ax.XTickLabel = {'Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', ...
```

```
'Sep','Oct','Nov','Dec'};  
title('Seasonality Multiplier')
```



### G2++ Interest-Rate Model

Since the refinancing incentive requires a simulation of future interest rates, an interest-rate model must be used. One choice is a two-factor additive Gaussian model, referred to as G2++ by Brigo and Mercurio [2].

The G2++ Interest Rate Model is:

$$r(t) = x(t) + y(t) + \varphi(t)$$

$$dx(t) = -ax(t)dt + \sigma dW_1(t)$$

$$dy(t) = -by(t)dt + \eta dW_2(t)$$

where  $dW_1(t)dW_2(t)$  is a two-dimensional Brownian motion with correlation  $\rho$

$$dW_1(t)dW_2(t) = \rho dt$$

$$\varphi(T) = f^M(0, T) + \frac{\sigma^2}{2a^2}(1 - e^{-aT})^2 + \frac{\eta^2}{2b^2}(1 - e^{-bT})^2 + \rho \frac{\sigma\eta}{ab}(1 - e^{-aT})(1 - e^{-bT})$$

and  $r(t)$  is the short rate,  $a$  and  $b$  are mean reversion constants and  $\sigma$  and  $\eta$  are volatility constants, and  $f^M(0, T)$  is the market forward rate, or the forward rate observed on the Settle date.

### LIBOR Market Model

The LIBOR Market Model (LMM) differs from short-rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate:

$$\frac{dF_i(t)}{F_i} = -\mu_i dt + \sigma_i(t) dW_i$$

where

$dW$  is an  $N$  dimensional geometric Brownian motion with:

$$dW_i(t)dW_j(t) = \rho_{ij} dt$$

The LMM relates the drifts of the forward rates based on no-arbitrage arguments. Specifically, under the Spot LIBOR measure, the drifts are expressed as the following:

$$\mu_i(t) = -\sigma_i(t) \sum_{j=q(t)}^i \frac{\tau_j \rho_{i,j} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)}$$

where

$\tau_i$  is the time fraction associated with the  $i$ th forward rate

$q(t)$  is an index function defined by the relation  $T_{q(t)-1} < t < T_{q(t)}$

and the Spot LIBOR numeraire is defined as the following:

$$B(t) = P(t, T_{q(t)}) \prod_{n=0}^{q(t)-1} (1 + \tau_n F_n(T_n))$$

Given the above, the choice with the LMM is how to model volatility and correlation.

The volatility of the rates can be modeled with a stochastic volatility, but for this example a deterministic volatility is used, and so a functional form needs to be specified. One of the most popular functional forms in the literature is the following:

$$\sigma_i(t) = \phi_i(a(T_i - t) + b)e^{c(T_i - t)} + d$$

where  $\phi$  adjusts the curve to match the volatility for the  $i^{\text{th}}$  forward rate.

Similarly, the correlation between the forward rates needs to be specified. This can be estimated from historical data or fitted to option prices. For this example, the following functional form will be used:

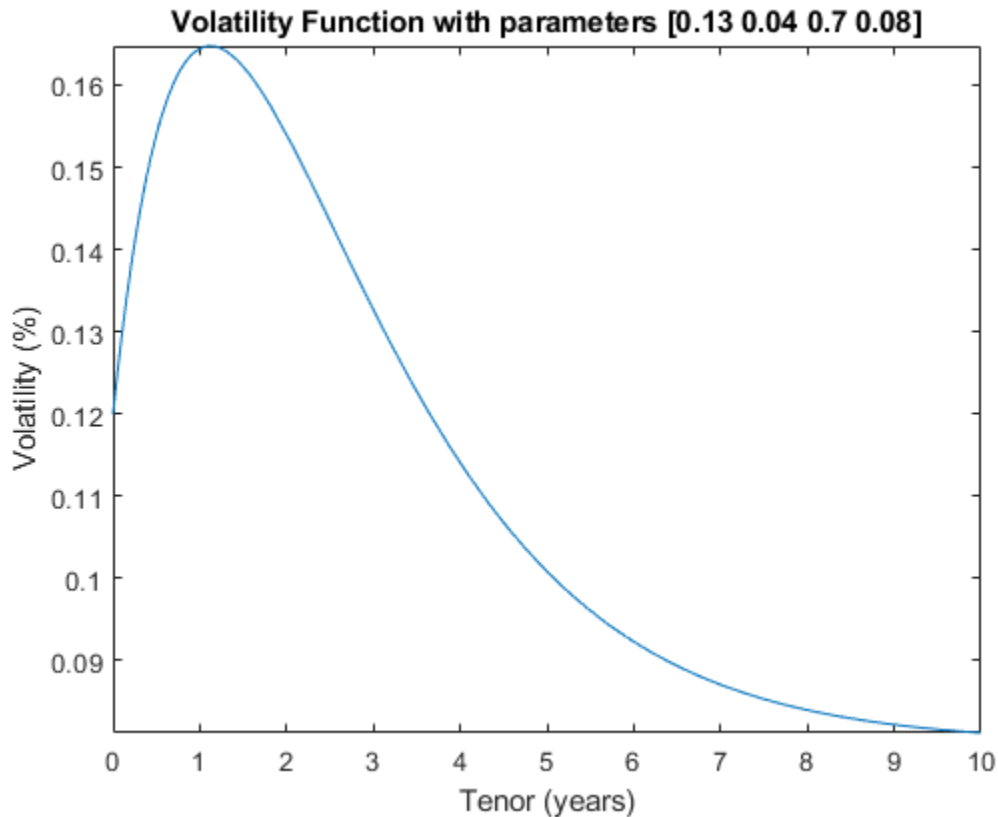
$$\rho_{i,j} = e^{-\beta|i-j|}$$

Once the volatility and correlation are specified, the parameters need to be calibrated -- this can be done with historical or market data, typically swaptions or caps/floors. For this example, we simply use reasonable estimates for the correlation and volatility parameters.

```
% The volatility function to be used -- and one choice for the parameters
LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.13 .04 .7 .08];
```

```
% Volatility specification
fplot(@(t) LMMVolFunc(LMMVolParams,t),[0 10])
title(['Volatility Function with parameters ' mat2str(LMMVolParams)])
ylabel('Volatility (%)')
xlabel('Tenor (years)')
```





### Calibration to Market Data

The parameters in the G2++ model can be calibrated to market data. Typically, the parameters are calibrated to observed interest-rate cap, floor and/or swaption data. For now, market cap data is used for calibration.

This data is hardcoded but could be imported into MATLAB with the Database Toolbox™ or Datafeed Toolbox™.

```
% Zero Curve -- this data is hardcoded for now, but could be bootstrapped
% using the |bootstrap| method of |IRDataCurve|.
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
ZeroDates = daysadd(Settle,360*ZeroTimes,1);
```

```
DiscountRates = zero2disc(ZeroRates,ZeroDates,Settle);
irdc = IRDataCurve('Zero',Settle,ZeroDates,ZeroRates);

figure
plot(ZeroDates,ZeroRates)
datetick
title(['US Zero Curve for ' datestr(Settle)])

% Cap Data
Reset = 2;
Notional = 100;
CapMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);
CapVolatility = [.28 .30 .32 .31 .30 .27 .23 .2 .18 .17 .165]';

% ATM strikes could be computed with swapbyzero
Strike = [0.0353 0.0366 0.0378 0.0390 0.0402 0.0421 0.0439 ...
          0.0456 0.0471 0.0471 0.0471]';

% This could be computed with capbyblk
BlackCapPrices = [0.1532 0.6416 1.3366 2.0290 2.7366 4.2960 6.5992 ...
                  9.6787 12.2580 14.0969 15.7873]';

figure
scatter(CapMaturity,CapVolatility)
datetick
title(['ATM Volatility for Caps on ' datestr(Settle)])

%
% To calibrate the model parameters, a parameter set will be found that
% minimizes the sum of the squared differences between the G2++ predicted
% Cap values and the observed Black Cap values. The Optimization Toolbox(TM)
% function <docid:optim_ug#buuhch7 lsqnonlin> is used in this example, although other
% (for example, Global Optimization) may also be applicable. The function
% <docid:fininst_ug#btzewvg-313 capbylg2f>
% computes the analytic values for the caps given parameter values.
%
% Upper and lower bounds for the model parameters are set to be
% relatively constrained. As Brigo and Mercurio discuss, the correlation
% parameter, $$ rho $$, can often be close to |-1| when fitting a G2++ model
% to interest-rate cap prices. Therefore, $$ rho $$ is constrained
% to be between |-.7| and |.7| to ensure that the parameters represent a truly
% two-factor model. The remaining mean reversion and volatility
% parameters are constrained to be between |0| and |.5|. Calibration remains a
% complex task, and while the plot below indicates that the best fit
```

```
% parameters seem to do a reasonably good job of reproducing the Cap
% prices, it should be noted that the procedure outlined here simply
% represents one approach.

% Call to lsqnonlin to calibrate parameters
objfun = @(x) BlackCapPrices - capbylg2f(irdc,x(1),x(2),x(3),x(4),x(5),Strike,CapMatur
x0 = [.5 .05 .1 .01 -.1];
lb = [0 0 0 0 -.7];
ub = [.5 .5 .5 .5 .7];

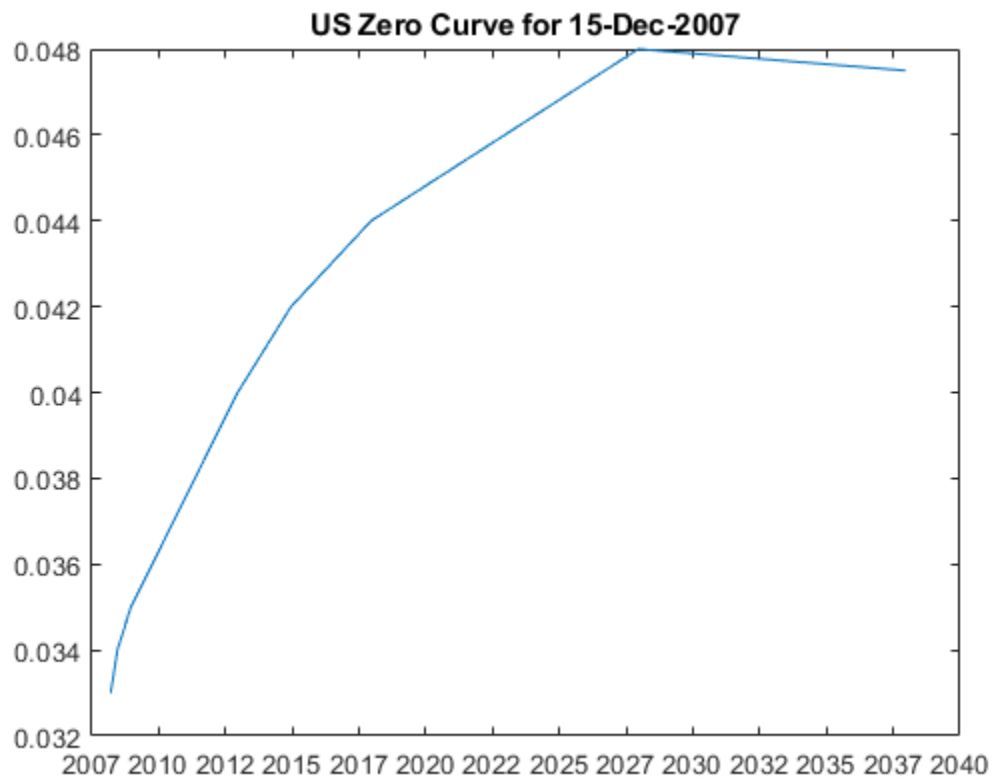
G2PP_Params = lsqnonlin(objfun,x0,lb,ub);

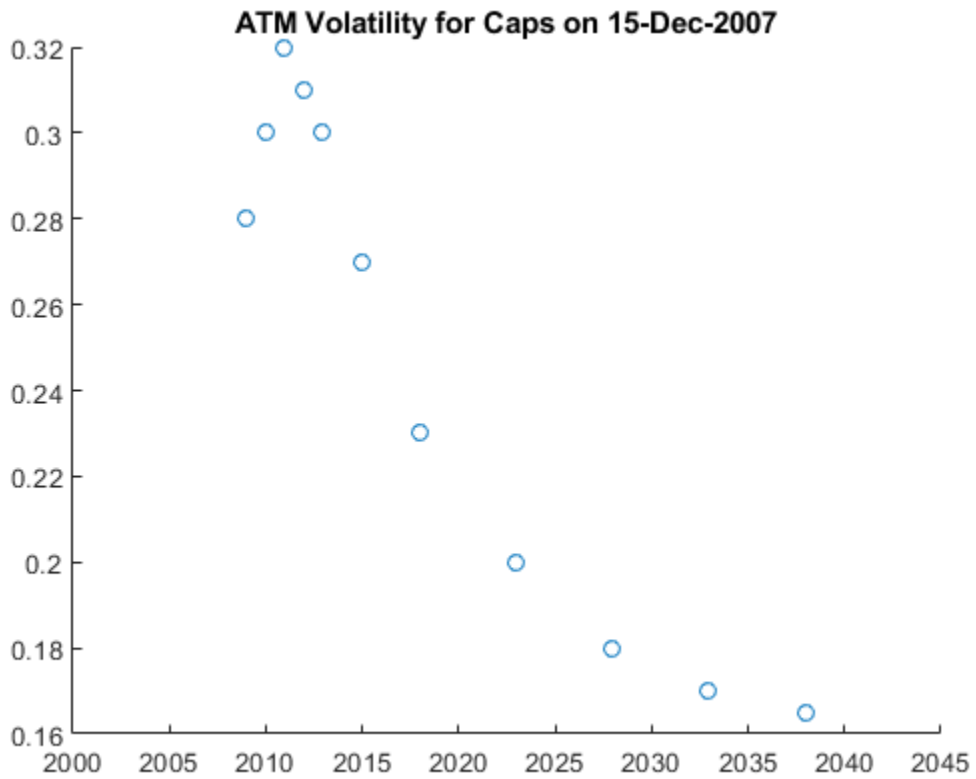
a = G2PP_Params(1);
b = G2PP_Params(2);
sigma = G2PP_Params(3);
eta = G2PP_Params(4);
rho = G2PP_Params(5);

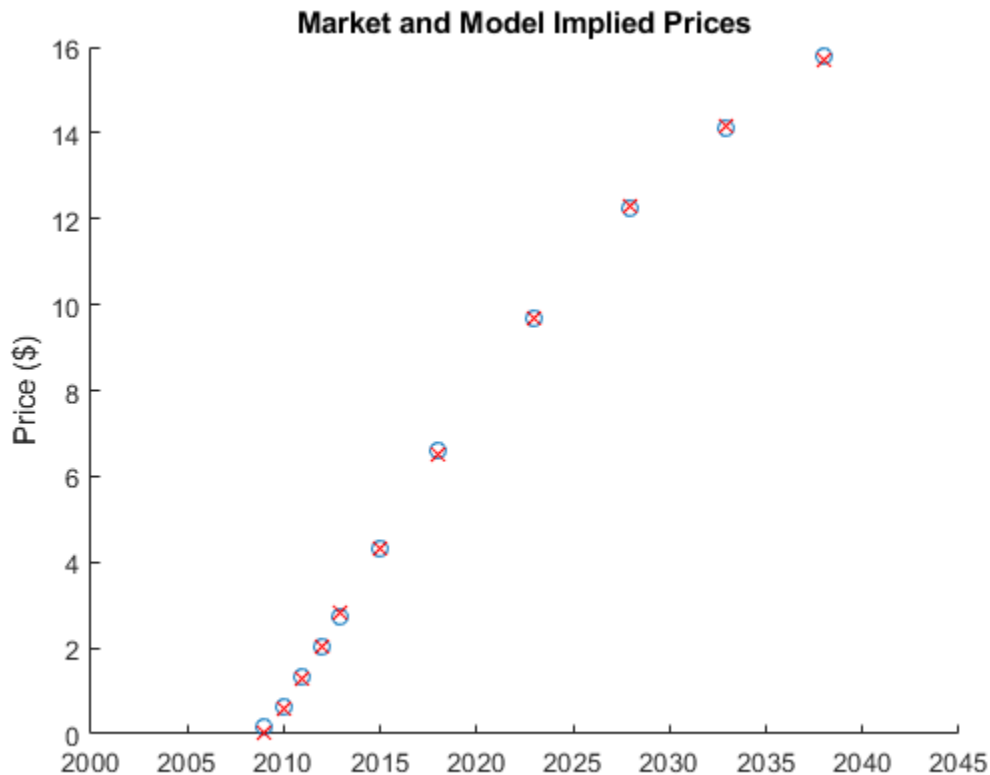
% Compare the results
figure
scatter(CapMaturity,BlackCapPrices)
hold on
scatter(CapMaturity,capbylg2f(irdc,a,b,sigma,eta,rho,Strike,CapMaturity),'rx')
datetick
title('Market and Model Implied Prices')
ylabel('Price ($)')
```

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the value of the function tolerance.







### G2++ Model Implementation

The `LinearGaussian2F` model can be used to specify the G2++ model and simulate future paths interest rates.

```
% G2++ model from Brigo and Mercurio with time homogeneous volatility
% parameters
G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho);
```

### LIBOR Market Model Implementation

After the volatility and correlation have been calibrated, Monte Carlo simulation is used to evolve the rates forward in time. The `LiborMarketModel` object is used to simulate the forward rates.

While factor reduction is often used with the LMM to reduce computational complexity, there is no factor reduction in this example.

6M LIBOR rates are chosen to be evolved in this simulation. Since a monthly prepayment vector must be computed, interpolation is used to generate the intermediate rates. Simple linear interpolation is used.

```
numForwardRates = 46;

% Instead of being fit, VolPhi is simply hard-coded --
% representative of a declining volatility over time.
VolPhi = linspace(1.2, .8, numForwardRates-1)';

Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
CorrMat = CorrFunc(meshgrid(1:numForwardRates-1), meshgrid(1:numForwardRates-1), Beta);

VolFunc = cell(length(VolPhi), 1);
for jdx = 1:length(VolPhi)
    VolFunc(jdx) = {@(t) VolPhi(jdx)*ones(size(t)).*(LMMVolParams(1)*t + ...
        LMMVolParams(2)).*exp(-LMMVolParams(3)*t) + LMMVolParams(4)};
end

LMM = LiborMarketModel(irdc, VolFunc, CorrMat);
```

### G2++ Monte Carlo Simulation

The various interest-rate paths can be simulated by calling the `simTermStructs` method.

One limitation to two-factor Gaussian models like this one is that it does not permit negative interest rates. This is a concern, particularly in low interest-rate environments. To handle this possibility, any interest-rate paths with negative rates are simply rejected.

```
nPeriods = NumCouponsRemaining;
nTrials = 100;
DeltaTime = 1/12;

% Generate factors and short rates
Tenor = [1/12 1 2 3 4 5 7 10 15 20 30];
G2PP_SimZeroRates = G2PP.simTermStructs(nPeriods, 'NTRIALS', nTrials, ...
    'Tenor', Tenor, 'DeltaTime', DeltaTime);

SimDates = daysadd(Settle, 360*DeltaTime*(0:nPeriods), 1);
```

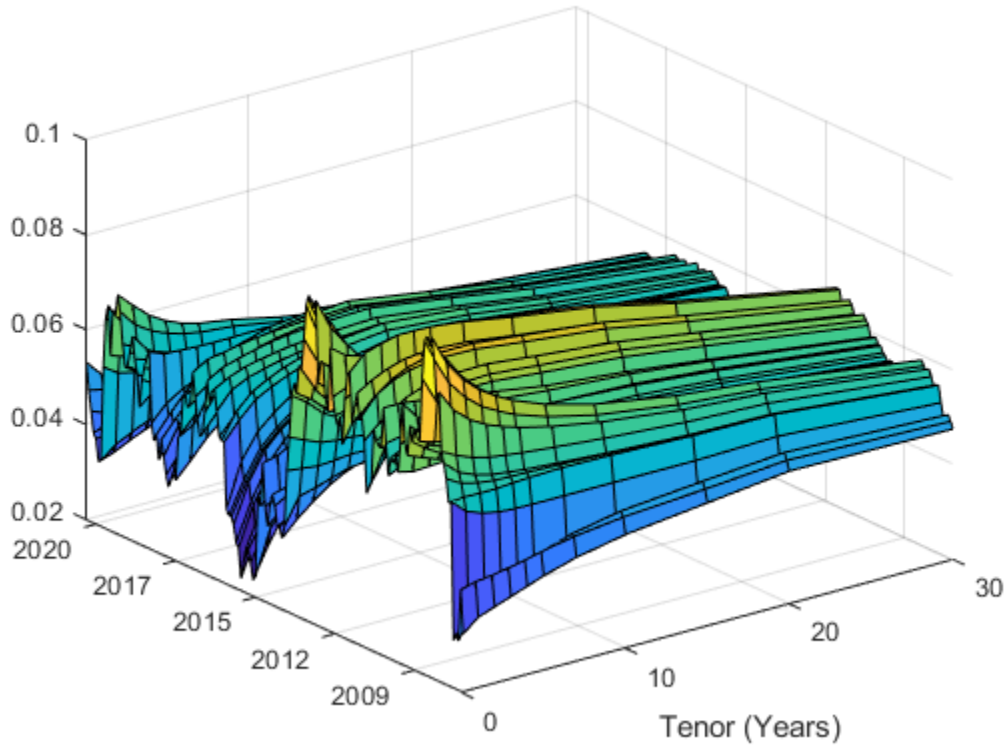
```
% Tenors that will be recovered for each simulation date. The stepsize is
% included here to facilitate computing a discount factor for each
% simulation path.

% Remove any paths that go negative
NegIdx = squeeze(any(any(G2PP_SimZeroRates < 0,1),2));
G2PP_SimZeroRates(:, :, NegIdx) = [];
nTrials = size(G2PP_SimZeroRates,3);

% Plot evolution of one sample path
trialIdx = 1;
figure
surf(Tenor, SimDates, G2PP_SimZeroRates(:, :, trialIdx))
datetick y keep ticks keep limits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of G2++ Model'])
xlabel('Tenor (Years)')
```



Evolution of the Zero Curve for Trial:1 of G2++ Model



### LIBOR Market Model Simulation

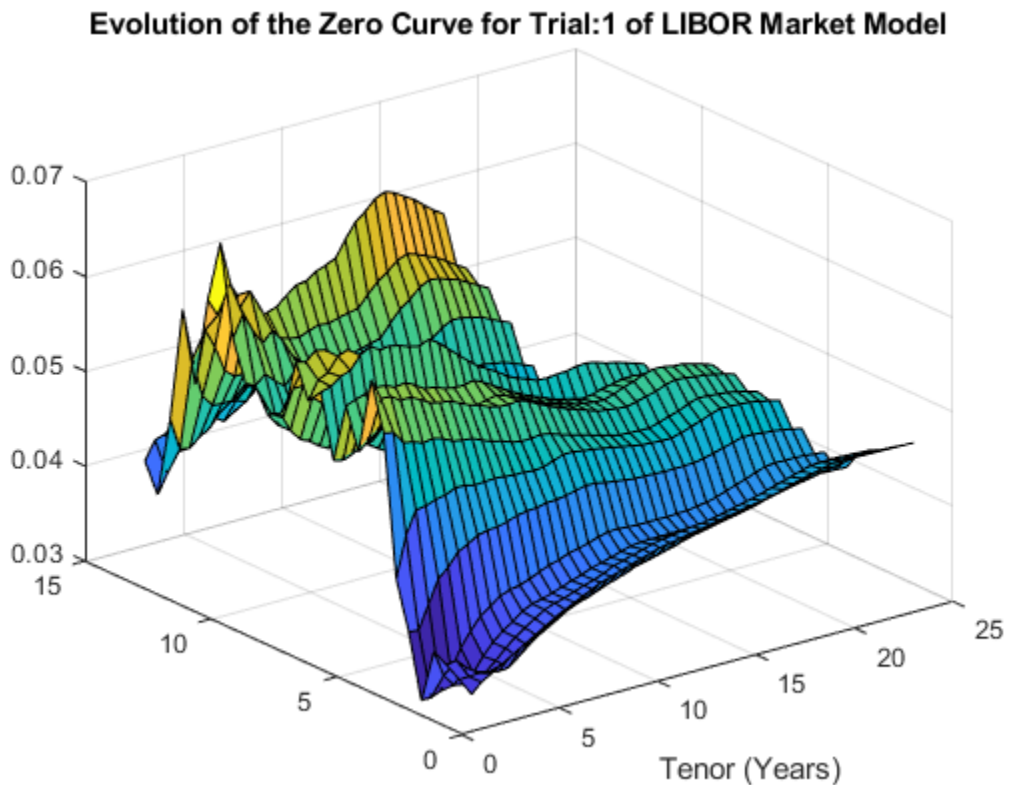
The various interest-rate paths can be simulated by calling the `simTermStructs` method of the `LiborMarketModel` object.

```
LMMPeriod = 2; % Semi-annual rates
LMMNumPeriods = NumCouponsRemaining/12*LMMPeriod; % Number of semi-annual periods
LMMDeltaTime = 1/LMMPeriod;
LMMNTRIALS = 100;
```

```
% Simulate
```

```
[LMMZeroRates, LMMForwardRates] = LMM.simTermStructs(LMMNumPeriods, 'nTrials', LMMNTRIALS);
ForwardTimes = 1/2:1/2:numForwardRates/2;
LMMSimTimes = 0:1/LMMPeriod:LMMNumPeriods/LMMPeriod;
```

```
% Plot evolution of one sample path
trialIdx = 1;
figure
tmpPlotData = LMMZeroRates(:, :, trialIdx);
tmpPlotData(tmpPlotData == 0) = NaN;
surf(ForwardTimes, LMMSimTimes, tmpPlotData)
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of LIBOR Market Model'])
xlabel('Tenor (Years)')
```



## Compute Mortgage Rates from Simulation

Once the interest-rate paths have been simulated, the mortgage rate needs to be computed -- one approach, discussed by [7], is to compute the mortgage rate from a combination of the 2-year and 10-year rates.

For this example, the following is used:

$$\text{MortgageRate} = .024 + .2 * \text{TwoYearRate} + .6 * \text{TenYearRate}$$

```
% Compute mortgage rates from interest rate paths
```

```
TwoYearRates = squeeze(G2PP_SimZeroRates(:,Tenor == 2,:));
TenYearRates = squeeze(G2PP_SimZeroRates(:,Tenor == 7,:));
G2PP_MortgageRates = .024 + .2*TwoYearRates + .6*TenYearRates;
```

```
LMMMortgageRates = squeeze(.024 + .2*LMMZeroRates(:,4,:) + .6*LMMZeroRates(:,20,:));
LMMDiscountFactors = squeeze(cumprod(1./(1 + LMMZeroRates(:,1,:)*.5)));
```

```
% Interpolate to get monthly mortgage rates
```

```
MonthlySimTimes = 0:1/12:LMMNumPeriods/LMMPeriod;
LMMMonthlyMortgageRates = zeros(nPeriods+1,LMMNTRIALS);
LMMMonthlyDF = zeros(nPeriods+1,LMMNTRIALS);
for trialidx = 1:LMMNTRIALS
    LMMMonthlyMortgageRates(:,trialidx) = interp1(LMMSimTimes,LMMMortgageRates(:,trialidx),MonthlySimTimes);
    LMMMonthlyDF(:,trialidx) = interp1(LMMSimTimes,LMMDiscountFactors(:,trialidx),MonthlySimTimes);
end
```

## Computing CPR and Generating and Valuing Cash Flows

Once the Mortgage Rates have been simulated, the CPR can be computed from the multiplicative model for each interest-rate path.

```
% Compute Seasoning and Refinancing Multipliers
```

```
Seasoning = ones(nPeriods+1,1);
Seasoning(1:30) = 1/30*(1:30);
G2PP_Refi = .2406 - .1389 * atan(5.952*(1.089 - CouponRate./G2PP_MortgageRates));
LMM_Refi = .2406 - .1389 * atan(5.952*(1.089 - CouponRate./LMMMonthlyMortgageRates));
```

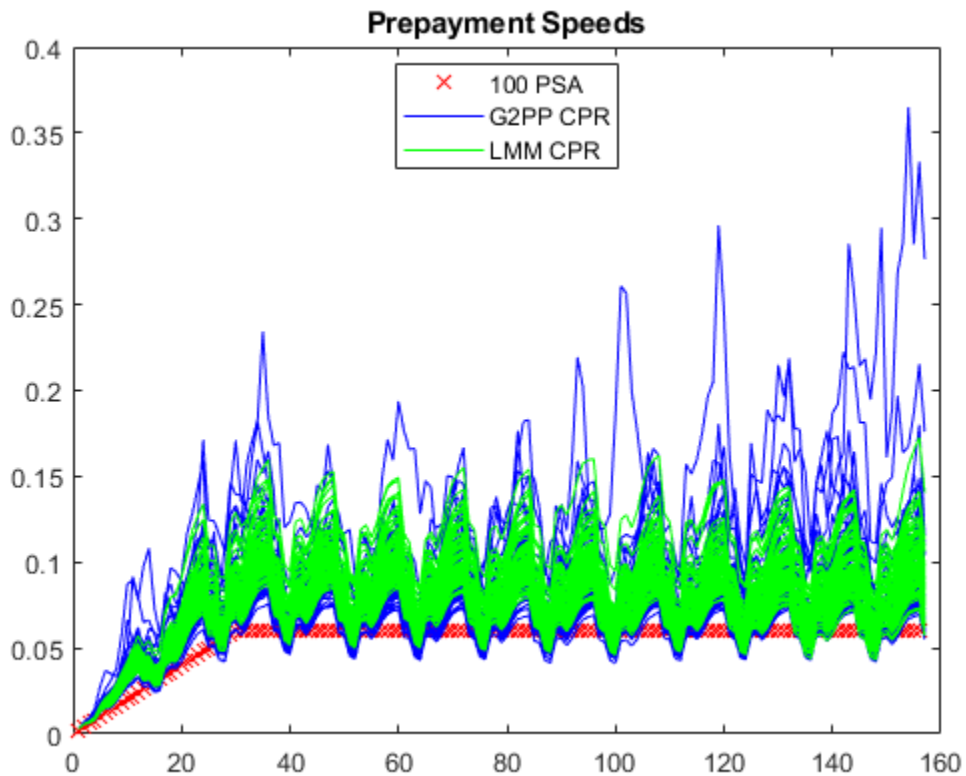
```
% CPR is simply computed by evaluating the multiplicative model
```

```
G2PP_CPR = bsxfun(@times,G2PP_Refi,Seasoning.*(Seasonality(month(CFlowDates))'));
LMM_CPR = bsxfun(@times,LMM_Refi,Seasoning.*(Seasonality(month(CFlowDates))'));
```

```
% Compute single monthly mortality (SMM) from CPR
```

```
G2PP_SMM = 1 - (1 - G2PP_CPR).^(1/12);
```

```
LMM_SMM = 1 - (1 - LMM_CPR).^(1/12);  
  
% Plot CPR's against 100 PSA  
CPR_PSA100 = psaspeed2rate(100);  
figure  
PSA_handle = plot(CPR_PSA100(1:nPeriods),'rx');  
hold on  
G2PP_handle = plot(G2PP_CPR,'b');  
LMM_handle = plot(LMM_CPR,'g');  
title('Prepayment Speeds')  
legend([PSA_handle(1) G2PP_handle(1) LMM_handle(1)],{'100 PSA', 'G2PP CPR', 'LMM CPR'}, 'l')
```



## Generate Cash Flows and Compute Present Value

With a vector of single monthly mortalities (SMM) computed for each interest-rate path, cash flows for the MBS can be computed and discounted.

```
% Compute the baseline zero rate at each cash flow time
CFlowZero = interp1(ZeroTimes,ZeroRates,CFlowTimes,'linear','extrap');

% Compute DF for each cash flow time
CFlowDF_Zero = zero2disc(CFlowZero,CFlowDates,Settle);

% Compute the price of the MBS using the zero curve
Price_Zero = CFlowAmounts*CFlowDF_Zero';

% Generate the cash flows for each IR Path
G2PP_CFlowAmounts = mbscfamounts(Settle, ...
    repmat(Maturity,1,nTrials), IssueDate, GrossRate, CouponRate, Delay, [], G2PP_SMM(2));

% Compute the DF for each IR path
G2PP_CFlowDFSim = cumprod(exp(squeeze(-G2PP_SimZeroRates(:,1,:).*DeltaTime)));

% Present value the cash flows for each MBS
G2PP_Price_Ind = sum(G2PP_CFlowAmounts.*G2PP_CFlowDFSim',2);
G2PP_Price = mean(G2PP_Price_Ind);

% Repeat for LMM
LMM_CFlowAmounts = mbscfamounts(Settle, ...
    repmat(Maturity,1,LMMNTRIALS), IssueDate, GrossRate, CouponRate, Delay, [], LMM_SMM(2));

% Present value the cash flows for each MBS
LMM_Price_Ind = sum(LMM_CFlowAmounts.*LMMMonthlyDF',2);
LMM_Price = mean(LMM_Price_Ind);
```

The results from the different approaches can be compared. The number of trials for the G2++ model will typically be less than 100 due to the filtering out of any paths that produce negative interest rates.

Additionally, while the number of trials for the G2++ model in this example is set to be 100, it is often the case that a larger number of simulations need to be run to produce an accurate valuation.

```
fprintf('          # of Monte Carlo Trials: %8d\n' , nTrials)
fprintf('          # of Time Periods/Trial: %8d\n\n' , nPeriods)
fprintf('          MBS Price with PSA 100: %8.4f\n' , Price_Zero)
```

```
fprintf(' MBS Price with Custom G2PP Prepayment Model: %8.4f\n\n', G2PP_Price)
fprintf(' MBS Price with Custom LMM Prepayment Model: %8.4f\n\n', LMM_Price)

# of Monte Carlo Trials:      73
# of Time Periods/Trial:     156

MBS Price with PSA 100:      1.0187
MBS Price with Custom G2PP Prepayment Model:  0.9871

MBS Price with Custom LMM Prepayment Model:  0.9993
```

## **Conclusion**

This example shows how to calibrate and simulate a G2++ interest-rate model and how to use the generated interest-rate paths in a prepayment model loosely based on the Richard and Roll model. This example also provides a useful starting point to using the G2++ and LMM interest-rate models in other financial applications.

## **Bibliography**

This example is based on the following books, papers and journal articles:

- 1** Andersen, L. and V. Piterbarg (2010). Interest Rate Modeling, Atlantic Financial Press.
- 2** Brigo, D. and F. Mercurio (2001). Interest Rate Models - Theory and Practice with Smile, Inflation and Credit (2nd ed. 2006 ed.). Springer Verlag. ISBN 978-3-540-22149-4.
- 3** Hayre, L, ed., Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities. New York: John Wiley & Sons, 2001b
- 4** Karpishpan, Y., O. Turel, and A. Hasha, Introducing the Citi LMM Term Structure Model for Mortgages, The Journal of Fixed Income, Volume 20 (2010) 44-58.
- 5** Rebonato, R., K. McKay, and R. White (2010). The Sabr/Libor Market Model: Pricing, Calibration and Hedging for Complex Interest-Rate Derivatives. John Wiley & Sons.
- 6** Richard, S. F., and R. Roll, 1989, "Prepayments on Fixed Rate Mortgage-Backed Securities" ,Journal of Portfolio Management.
- 7** Office of Thrift Supervision, "Net Portfolio Value Model Manual", March 2000.
- 8** Stein, H. J., Belikoff, A. L., Levin, K. and Tian, X., Analysis of Mortgage Backed Securities: Before and after the Credit Crisis (January 5, 2007). Credit Risk Frontiers: Subprime Crisis, Pricing and Hedging, CVA, MBS, Ratings, and Liquidity; Bielecki,

Tomasz,; Damiano Brigo and Frederic Patras, eds., February 2011. Available at SSRN:  
[https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=955358](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=955358)

## See Also

mbscfamounts | mbsconvp | mbsconvy | mbsdurp | mbsdury | mbsnoprepay |  
mboas2price | mboas2yield | mbspassthrough | mbsprice | mbsprice2oas |  
mbsprice2speed | mbswal | mbsyield | mbsyield2oas | mbsyield2speed |  
psaspeed2default | psaspeed2rate

## Related Examples

- “Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model” on page 5-42
- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51

## More About

- “What Are Mortgage-Backed Securities?” on page 5-2

## Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model

This example illustrates how the Financial Toolbox™ and Financial Instruments Toolbox™ are used to price a level mortgage backed security using the BDT model.

### Load the BDT Tree Stored in the Data File

```
load mbsexample.mat
```

### Observe the Interest-Rate Tree

Visualize the interest rate evolution along the tree by looking at the output structure BDTTree. BDTTree returns an inverse discount tree, which you can convert into an interest-rate tree with the `cvtree` function.

```
BDTTreeR = cvtree(BDTTree);
```

Look at the upper branch and lower branch paths of the tree:

```
OldFormat = get(0, 'format');  
format short
```

```
%Rate at root node:
```

```
RateRoot = treepath(BDTTreeR.RateTree, 0)
```

```
RateRoot = 0.0399
```

```
%Rates along upper branch:
```

```
RatePathUp = treepath(BDTTreeR.RateTree, [1 1 1 1 1])
```

```
RatePathUp = 6×1
```

```
0.0399  
0.0397  
0.0391  
0.0383  
0.0373  
0.0360
```



```

%Rates along lower branch:
RatePathDown = treepath(BDTreeR.RateTree, [2 2 2 2 2])

RatePathDown = 6x1

    0.0399
    0.0470
    0.0550
    0.0638
    0.0734
    0.0841

```

### Compute the Price Tree for the Non-Prepayable Mortgage

Let's say that we have a 3 year \$10000 level prepayable loan, with a mortgage interest rate of 4.64% semi-annually compounded.

```

MortgageAmount = 10000;
CouponRate = 0.0464;
Period = 2;
Settle='01-Jan-2007';
Maturity='01-Jan-2010';
Compounding = BDTree.TimeSpec.Compounding;

```

```
format bank
```

Use the function `amortize` in the Financial Toolbox to calculate the mortgage payment of the loan (MP), the interest and principal components, and the outstanding principal balance.

```

NumPeriods = date2time(Settle,Maturity, Compounding)';

[Principal, InterestPayment, OutstandingBalance, MP] = amortize(CouponRate/Period, Num
% Display Principal, Interest and Outstanding balances
PrincipalAmount = Principal'

```

```

PrincipalAmount = 6x1

    1572.59
    1609.07
    1646.40
    1684.60

```

1723.68  
1763.67

InterestPaymentAmount = InterestPayment'

InterestPaymentAmount = 6×1

232.00  
195.52  
158.19  
119.99  
80.91  
40.92

OutstandingBalanceAmount = OutstandingBalance'

OutstandingBalanceAmount = 6×1

8427.41  
6818.34  
5171.94  
3487.35  
1763.67  
0.00

CFlowAmounts = MP\*ones(1,NumPeriods);

% The CFlowDates are the same as the tree level dates

CFlowDates= {'01-Jul-2007' , '01-Jan-2008' , '01-Jul-2008' , '01-Jan-2009' , '01-Jul-2009'}

% Calculate the price of the non-prepayable mortgage

[PriceNonPrepayableMortgage, PriceTreeNonPrepayableMortgage] = cfbybdt(BDTree, CFlowAmounts, CFlowDates)

for iLevel = 2:length(PriceTreeNonPrepayableMortgage.PTree)

PriceTreeNonPrepayableMortgage.PTree{iLevel}(:, :) = PriceTreeNonPrepayableMortgage.PTree{iLevel-1}(:, :);

end

% Look at the price of the mortgage today t0bs = 0

PriceNonPrepayableMortgage

PriceNonPrepayableMortgage =

10017.47

```
% The value of the non-prepayable mortgage is $10017.47. This value exceeds  
% the $10000 amount borrowed since the homeowner received not only $10000, but  
% also a prepayment option.
```

```
% Look at the value of the mortgage on the last date, right after the last  
% mortgage payment, is zero:
```

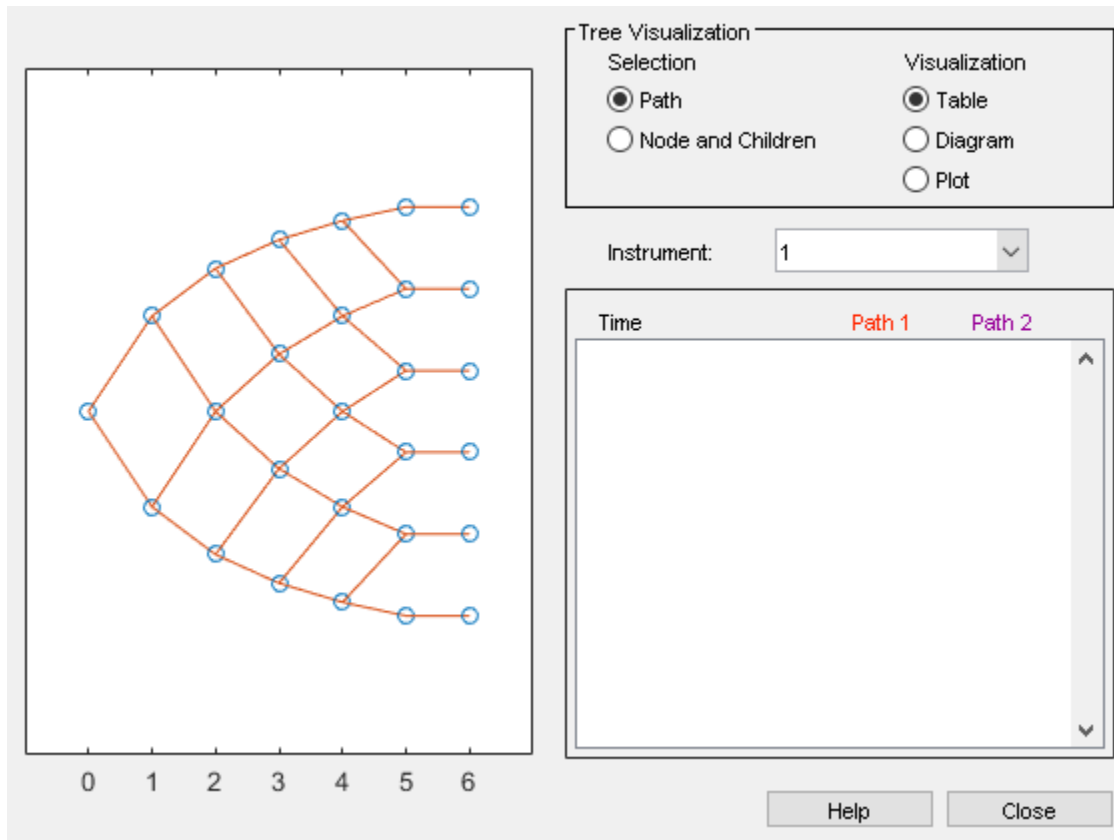
```
PriceTreeNonPrepayableMortgage.PTree{end}
```

```
ans = 1x6
```

```
0 0 0 0 0 0
```

```
% Visualize the price tree for the non-prepayable mortgage.
```

```
treeviewer(PriceTreeNonPrepayableMortgage)
```



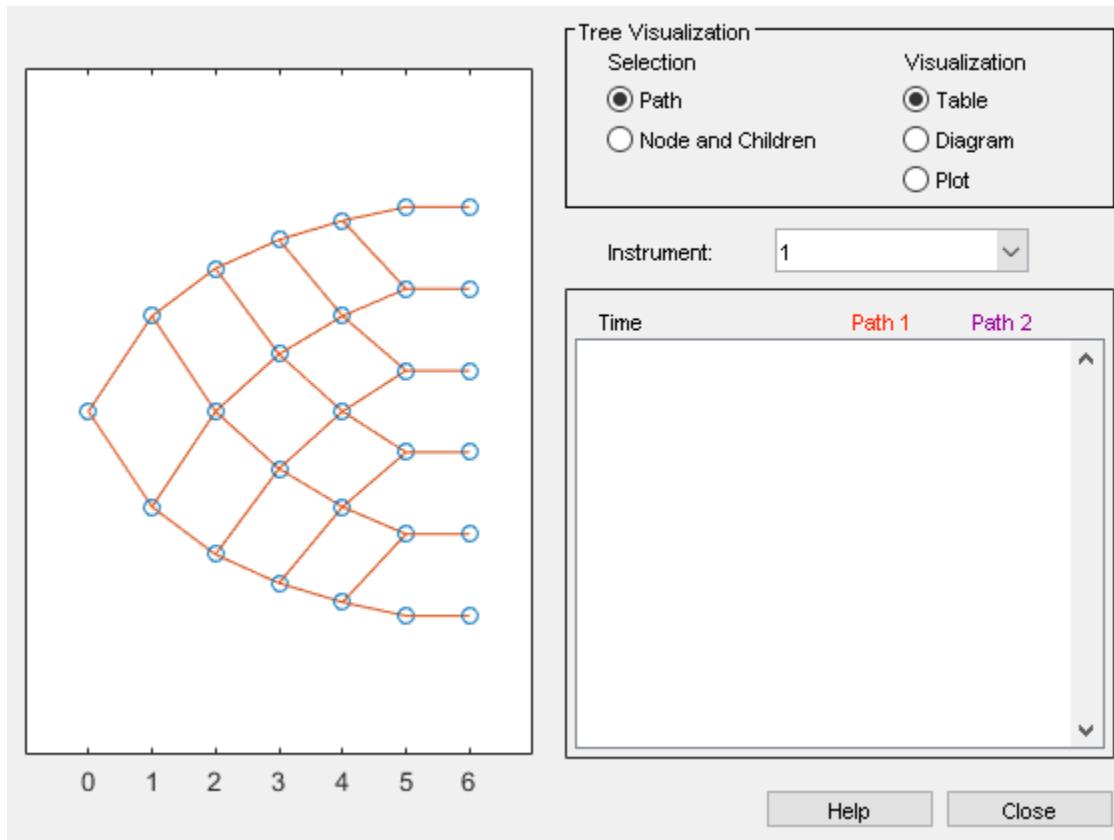
### Compute the Price Tree of the Prepayment Option

```
% The Prepayment option is like a call option on a bond.
%
% The exercise price or strike will be equal to the outstanding principal amount
% which has been calculated using the function |amortize|.
```

```
OptSpec = 'call';
Strike = [MortgageAmount OutstandingBalance];
ExerciseDates =[Settle CFlowDates];
AmericanOpt = 0;
Maturity = CFlowDates(end);
```

```
% Compute the price of the prepayment option:
```

```
[PricePrepaymentOption, PriceTreePrepaymentOption] = prepaymentbybdt(BDTree, OptSpec,  
    0, Settle, Maturity,[], [], [], ...  
    [], [], [], [], 0, [], CFlowAmounts);  
  
% Look at the price of the prepayment option today (t0bs = 0)  
PricePrepaymentOption  
  
PricePrepaymentOption =  
    17.47  
  
% The value of the prepayment option is $17.47 as expected.  
  
% Visualize the price tree for the prepayment option  
treeviewer(PriceTreePrepaymentOption)
```



### Calculate the Price Tree of the Prepayable Mortgage.

`% Compute the price of the prepayable mortgage.`

```
PricePrepayableMortgage = PriceNonPrepayableMortgage - PricePrepaymentOption;
```

```
PriceTreePrepayableMortgage = PriceTreeNonPrepayableMortgage;
```

```
for iLevel = 1:length(PriceTreeNonPrepayableMortgage.PTree)
```

```
    PriceTreePrepayableMortgage.PTree{iLevel}(:, :) = PriceTreeNonPrepayableMortgage.PTree{iLevel}(:, :) - PriceTreePrepaymentOption.PTree{iLevel}(:, :);
```

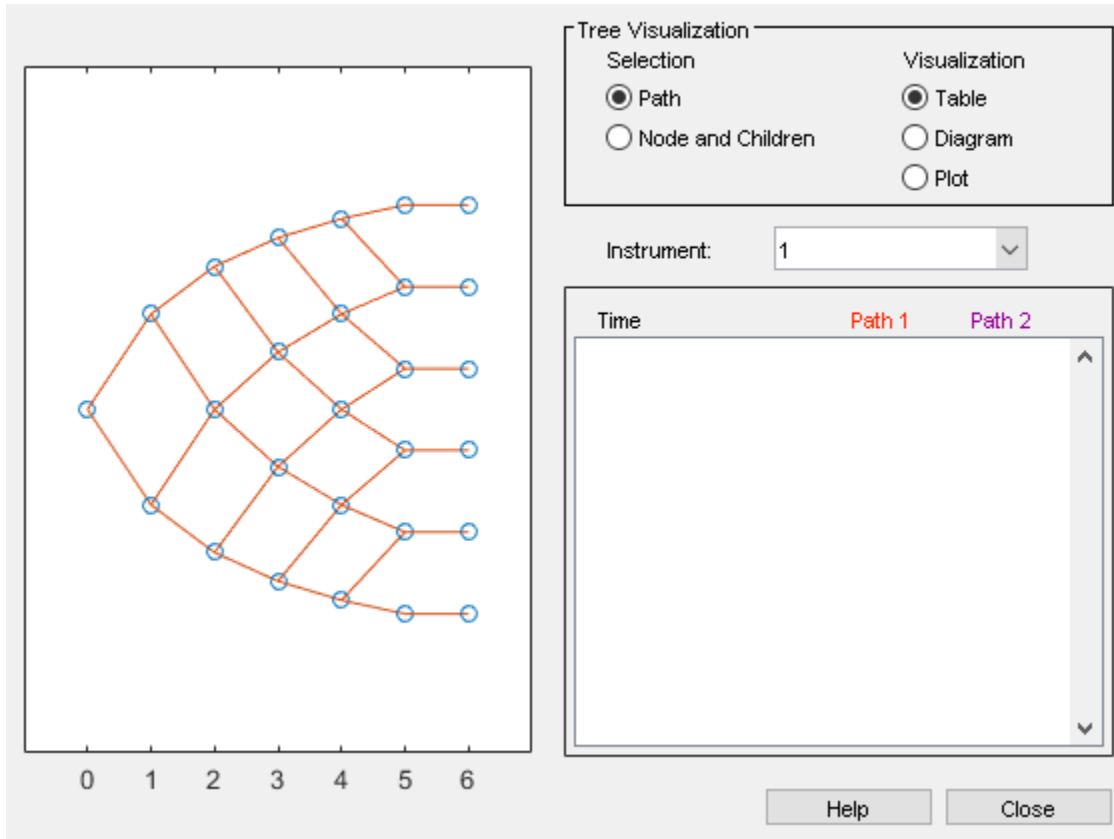
```
end
```

```
% Look at the price of the prepayable mortgage today (t0bs = 0)
PricePrepayableMortgage
```

```
PricePrepayableMortgage =
    10000.00
```

```
% The value of the prepayable mortgage is $10000 as expected.
```

```
% Visualize the price and price tree for the prepayable mortgage
treeviewer(PriceTreePrepayableMortgage)
```



```
set(0, 'format', OldFormat);
```

### See Also

[mbscfamounts](#) | [mbsconvp](#) | [mbsconvy](#) | [mbsdurp](#) | [mbsdury](#) | [mbsnoprepay](#) | [mbsoas2price](#) | [mbsoas2yield](#) | [mbspassthrough](#) | [mbsprice](#) | [mbsprice2oas](#) | [mbsprice2speed](#) | [mbswal](#) | [mbsyield](#) | [mbsyield2oas](#) | [mbsyield2speed](#) | [psaspeed2default](#) | [psaspeed2rate](#)

### Related Examples

- “Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model” on page 5-19
- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51

### More About

- “What Are Mortgage-Backed Securities?” on page 5-2



# Using Collateralized Mortgage Obligations (CMOs)

## What Are CMOs?

Financial Instruments Toolbox supports collateralized mortgage obligations (CMOs) to provide investors with a greater range of risk and return characteristics than mortgage-backed securities (MBS). In contrast to an MBS, which simply redirects principal and interest cash flows to investors on a pro rata basis, a CMO structures cash flows to different tranches, or slices, to create securities that are better tailored to specific investors.

For example, banks might be primarily concerned with extension risk, or the risk that their investment lengthens in time due to increasing interest rates, given that they typically have short-term deposits as liabilities. Insurance companies and pension funds might be concerned primarily with contraction risk, or the risk that their investment will pay off too soon, with liabilities that have much longer lives. A CMO structure addresses the interest-rate risk of extension or contraction with a blend of short-term and long-term CMO securities, called tranches.

## See Also

[cmosched](#) | [cmoschedcf](#) | [cmoseqcf](#) | [mbscfamounts](#) | [mbspassthrough](#)

## Related Examples

- “CMO Workflow” on page 5-60
- “Prepayment Risk” on page 5-52
- “Create PAC and Sequential CMO” on page 5-63
- “Fixed-Rate Mortgage Pool” on page 5-3

## More About

- “What Are Mortgage-Backed Securities?” on page 5-2

## Prepayment Risk

Prepayment risk is the risk that the term of the security varies according to differing rates of repayment of principal by borrowers (repayments from refinancings, sales, curtailments, or foreclosures). In a CMO, you can structure the principal (and associated coupon) stream from the underlying mortgage pool collateral to allocate prepayment risk. If principal is prepaid faster than expected (for example, if mortgage rates fall and borrowers refinance), then the overall term of the mortgage pool collateral shortens.

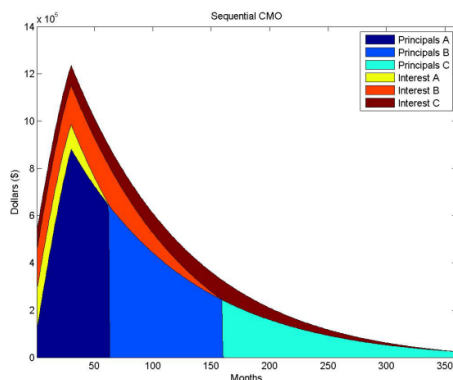
You cannot remove prepayment risk, but you can reallocate it among CMO tranches so that some tranches have some protection against this risk, and other tranches will absorb more of this risk. To facilitate this allocation of prepayment risk, CMOs are structured such that prepayments are allocated among tranches using a fixed set of rules. The most common schemes for prepayment tranching are:

- Sequential tranching, with or without, Z-bond tranching
- Schedule bond tranching
  - Planned amortization class (PAC) bonds
  - Target amortization class (TAC) bonds

Financial Instruments Toolbox supports these schemes for prepayment tranching for CMOs and tools for pricing and scheduling cash flows between the tranches, as well as analyzing the price and yield for CMOs. Financial Instruments Toolbox functionality for CMOs does not model credit risk. Therefore, this functionality is most appropriate for CMOs where credit risk is not an issue (for example, agency CMOs where the underlying mortgage pool collateral is insured for default by the agency Government-Sponsored Enterprises (GSEs), such as Fannie Mae and Freddie Mac).

### Sequential Tranches Without a Z-Bond

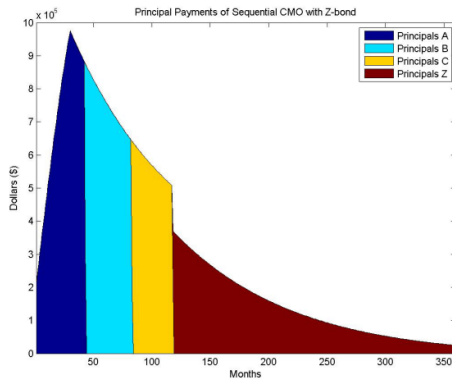
All available principal and interest payments go to the first sequential tranche, until its balance decrements to zero, then to the second, and so on. For example, consider the following example where all principal and interest from the underlying mortgage pool is repaid on tranche A first, then tranche B, then tranche C. Interest is paid on each tranche as long as the principal for the tranche has not been retired.



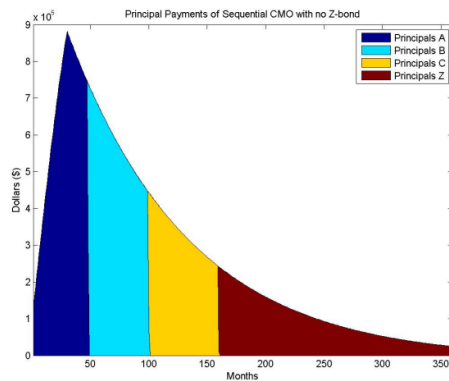
## Sequential Tranches with a Z-Bond

The Z-bond, also called an accrual bond, is a type of interest and principal pay rule. The Z-bond tranche supports other sequential pay tranches by not receiving an interest payment. The interest payment that would have accrued to the Z-bond tranche pays off the principal of other bonds, and the principal of the Z-bond tranche increases. The Z-bond tranche starts receiving interest and principal payments only after the other tranches in the CMO have been fully paid. The Z-bond tranche is used in a sequential-pay structure to accelerate the principal repayments of the sequential-pay bonds.

A Z-bond differs from other CMO instruments because it is not tranching principal but interest. The Z-bond receives no cash flows until all other securities have been paid off. In the interim, the interest that is owed to the Z-bond is accrued to its principal. The following chart demonstrates the difference between a Z-bond and a normal sequential pay tranche. The C tranche pays off sooner with the Z-bond, because the interest cash flows to the Z-bond are being used to pay down the principal of the C tranche.



For comparison, the following graphic is the same sequential CMO with no Z-bond.



## PAC Tranches

Planned amortization class (PAC) bonds help reduce the effects of prepayment risk. They are designed to produce more stable cash flows by redirecting prepayments from the underlying mortgage collateral to other classes (tranches) called companion or support classes. PAC bonds have a principal payment rate over a predetermined period of time. The PAC bond payment schedule is determined by two different prepayment rates, which together form a band (also called a collar). Early in the life of the CMO, the prepayment at the lower PSA yields a lower prepayment. Later in its life, the principal in the higher PSA declines enough that it yields a lower prepayment. The PAC tranche receives whichever

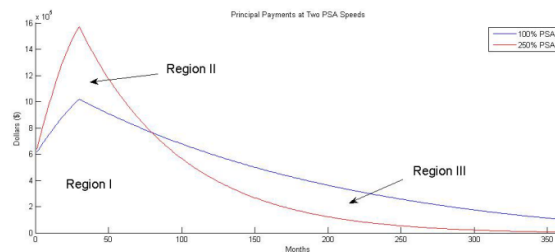
rate is lower, so it will change prepayment at one PSA for the first part of its life, then switch to the other rate. The ability to stay on this schedule is maintained by a support bond, which absorbs excess prepayments, and receives fewer prepayments to prevent extension of average life.

However, the PAC is only protected from extension to the amount that prepayments are made on the underlying MBSs. If there is a sustained period of fast prepayments, then that might completely eliminate a PAC bond's outstanding support class. When the principal of the associated PAC bond is exhausted, the CMO is called a "busted PAC", or "busted collar". Alternatively, in times of slow prepayments, amortization of the support bonds is delayed if there is not enough principal for the currently paying PAC bond. This extends the average life of the class.

A PAC bond protects against both extension and contraction risk by:

- Specifying a schedule of principal payments for the PAC bond
- Including support tranches that are allocated prepayments inside a specified prepayment band

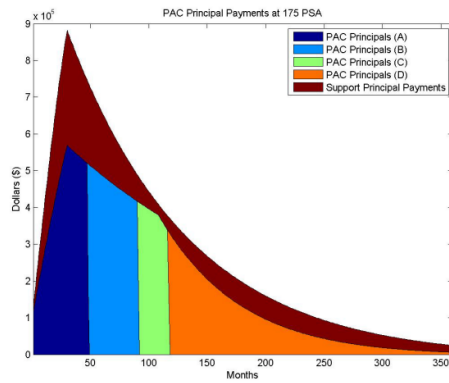
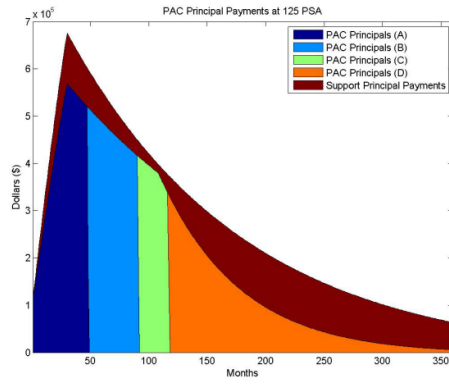
PAC bonds typically specify a band expressed using the PSA model. A PAC bond with a range of 100-250% has this principal schedule.

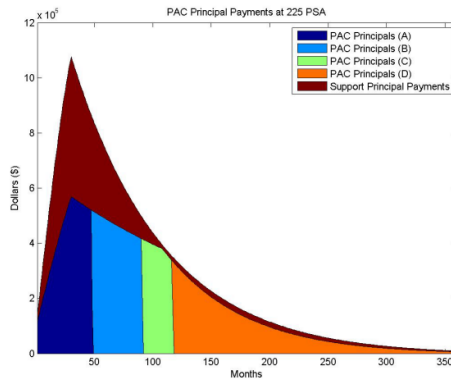


The principal repayment schedule is the minimum principal payment as Region 1 shows. This is the principal payment schedule as long as the actual prepayment stays within the prepayment band of 100-250% PSA.

For example, for different prepayment speeds of 125%, 175%, and 225% PSA, the actual principal payments are shown in the following graphs. At higher prepayment speeds, the support tranche is allocated principal earlier while the principal timing for the other tranches remains constant.

# 5 Mortgage-Backed Securities



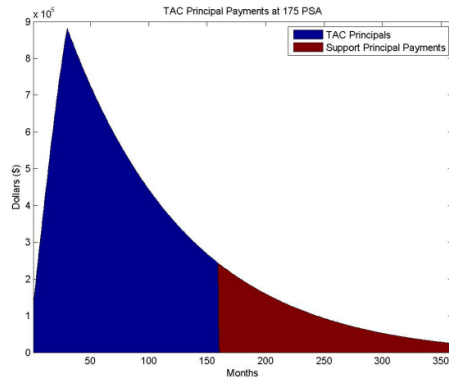
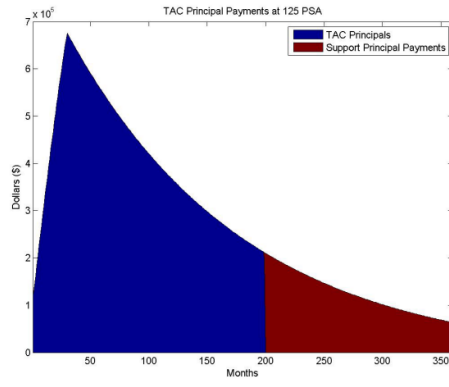


## TAC Tranches

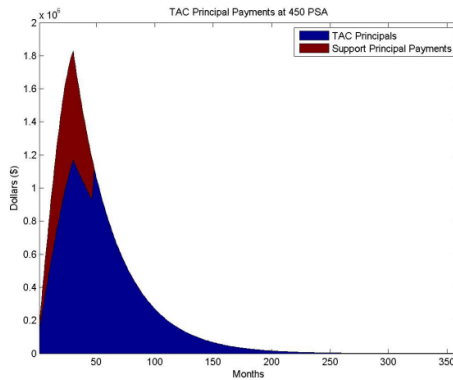
Target amortization class (TAC) bonds are similar to PAC bonds, but they do not provide protection against extension of average life. Create the schedule of principal payments by using just a single PSA. TAC bonds pay a “targeted” principal payment schedule at a single, constant prepayment speed. As long as the underlying mortgage collateral does not prepay at a rate slower than this speed, the TAC bond payment schedule is met. TAC bonds can protect against increasing prepayments and early retirement of the TAC bond investment. If the principal cash flow from the mortgage collateral exceeds the TAC schedule, the excess is allocated to TAC companion (support) classes. Alternatively, if prepayments fall below the speed necessary to maintain the TAC schedule, the weighted average life of the TAC is extended. The TAC bond does not protect against low prepayment rates.

For example, here is a TAC structure rated for 125%, 175%, and 450% PSA.

# 5 Mortgage-Backed Securities







For prepayments below 175% PSA, the TAC bond extends like a normal sequential pay CMO. TAC bonds are appealing because they offer higher yields than comparable PAC bonds. The unaddressed risk from low prepayment rates generally does not concern investors as much as risk from high prepayment rates.

## See Also

[cmosched](#) | [cmoschedcf](#) | [cmoseqcf](#) | [mbscfamounts](#) | [mbspassthrough](#)

## Related Examples

- “CMO Workflow” on page 5-60
- “Prepayment Risk” on page 5-52
- “Create PAC and Sequential CMO” on page 5-63
- “Fixed-Rate Mortgage Pool” on page 5-3

## More About

- “What Are CMOs?” on page 5-51
- “What Are Mortgage-Backed Securities?” on page 5-2

## CMO Workflow

In general, the CMO workflow is:

- 1 Calculate underlying mortgage cash flows.
- 2 Define CMO tranches
- 3 If using a PAC or TAC CMO, calculate the principal schedule.
- 4 Calculate cash flows for each tranche.
- 5 Analyze the CMO by computing price, yield, spread of CMO cash flows.

### Calculate Underlying Mortgage Cash Flows

Underlying mortgage pool pass-through cash flows are calculated by the existing function `mbspassthrough`. The CMO cash flow functions require the principal payments (including prepayments) calculated from existing functions `mbspassthrough` or `mbscfamounts`.

```
principal = 10000000;  
coupon = 0.06;  
terms = 360;  
psa = 150;  
  
[principal_balance, monthly_payments, sched_principal_payments,...  
interest_payments, prepayments] = mbspassthrough(principal,...  
coupon, terms, terms, psa, []);  
  
principal_payments = sched_principal_payments.' + prepayments.';
```

After determining principal payments for the underlying mortgage collateral, you can generate cash flows for a sequential CMO, with or without a Z-bond, by using `cmoseqcf`. For a PAC or TAC CMO, the cash flows are generated using `cmoschedcf`

### Define CMO Tranches

Define CMO tranche; for example, define a CMO with two tranches:

```
TranchePrincipals = [500000; 500000];  
TrancheCoupons = [0.06; 0.06];
```

## If Using a PAC or TAC CMO, Calculate Principal Schedule

Calculate the PAC/TAC principal balance schedule based on a band of PSA speeds. For scheduled CMOs (PAC/TAC), the CMO cash flow functions additionally take in the principal balance schedule calculated by the CMO schedule function `cmosched`.

```
speed = [100 300];
[balanceSchedule, initialBalance] = cmosched(principal, coupon,...
terms, terms, speed, TranchePrincipals(1));
```

## Calculate Cash Flows for Each Tranche

You can reuse the output from the cash flow generation functions to further divide the cash flows into tranches. For example, the output from `cmoschedcf` for a PAC tranche can be divided into sequential tranches by passing the principal cash flows of the PAC tranche into the `cmoschedcf` function. The outputs of the CMO cash flow functions are the principal and interest cash flows, and the principal balance.

```
[principal_balances, principal_cashflows, interest_cashflows] = cmoschedcf(principal_payments,...
TranchePrincipals, TrancheCoupons, balanceSchedule);
```

## Analyze CMO by Computing Price, Yield, and Spread of CMO Cash Flows

The outputs from the CMO functions (`cmoseqcf` and `cmoschedcf`) are cash flows. The functions used to analyze a CMO are based on these cash flows. To that end, you can use `cfbyzero`, `cfs spread`, `cfyield`, and `cfprice` to compute prices, yield, and spreads for the CMO cash flows. In addition, using the following, you can calculate a weighted average life (WAL) for each tranche in the CMO:

$$WAL = \sum_{i=1}^n \frac{P_i}{\bar{P}} t_i$$

where:

$P$  is the total principal.

$P_i$  is the principal repayment of the coupon  $i$ .

$\frac{P_i}{\bar{P}}$  is the fraction of the principal repaid in coupon  $i$ .

$t_i$  is the time in years from the start to coupon  $i$ .

### See Also

[cmosched](#) | [cmoschedcf](#) | [cmoseqcf](#) | [mbscfamounts](#) | [mbspassthrough](#)

### Related Examples

- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51
- “Create PAC and Sequential CMO” on page 5-63
- “Fixed-Rate Mortgage Pool” on page 5-3

### More About

- “What Are Mortgage-Backed Securities?” on page 5-2

## Create PAC and Sequential CMO

This example shows how to use an underlying mortgage-backed security (MBS) pool for a 30-year fixed-rate mortgage of 6% to define a PAC bond, and then define a sequential CMO from the PAC bond. Analyze the CMO by comparing the CMO spread to a zero-rate curve for a 30-year Treasury bond and then calculate the weighted-average life (WAL) for the PAC bond.

### Step 1. Define the underlying mortgage pool.

```
principal = 100000000;
grossrate = 0.06;
coupon = 0.05;
originalTerm = 360;
termRemaining = 360;
speed = 100;
delay = 14;

Settle      = datenum('1-Jan-2011');
IssueDate   = datenum('1-Jan-2011');
Maturity    = addtodate(IssueDate, 360, 'month');
```

### Step 2. Calculate underlying pool cash flow.

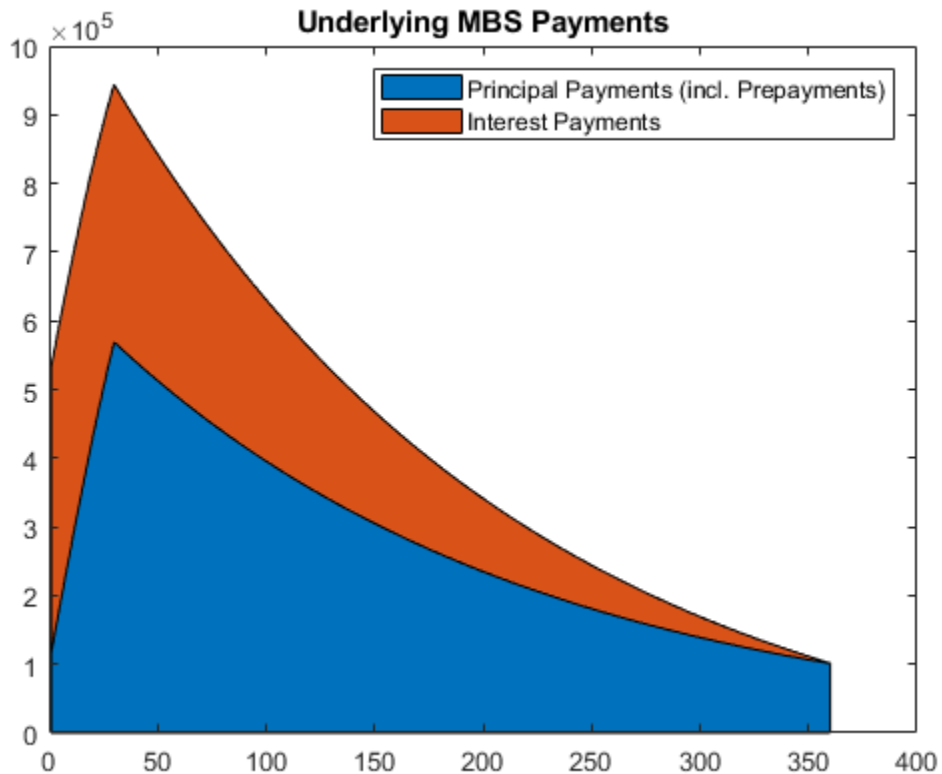
```
[CFlowAmounts, CFlowDates, ~, ~, ~, UnitPrincipal, UnitInterest, ...
UnitPrepayment] = mbscfamounts(Settle, Maturity, IssueDate, grossrate, ...
coupon, delay, speed, []);
```

### Step 3. Calculate prepayments.

```
principalPayments = UnitPrincipal * principal;
netInterest = UnitInterest * principal;
prepayments = UnitPrepayment * principal;
dates = CFlowDates' + delay;
```

### Step 4. Generate a plot for underlying MBS payments.

```
area([principalPayments'+prepayments', netInterest'])
title('Underlying MBS Payments');
legend('Principal Payments (incl. Prepayments)', 'Interest Payments')
```

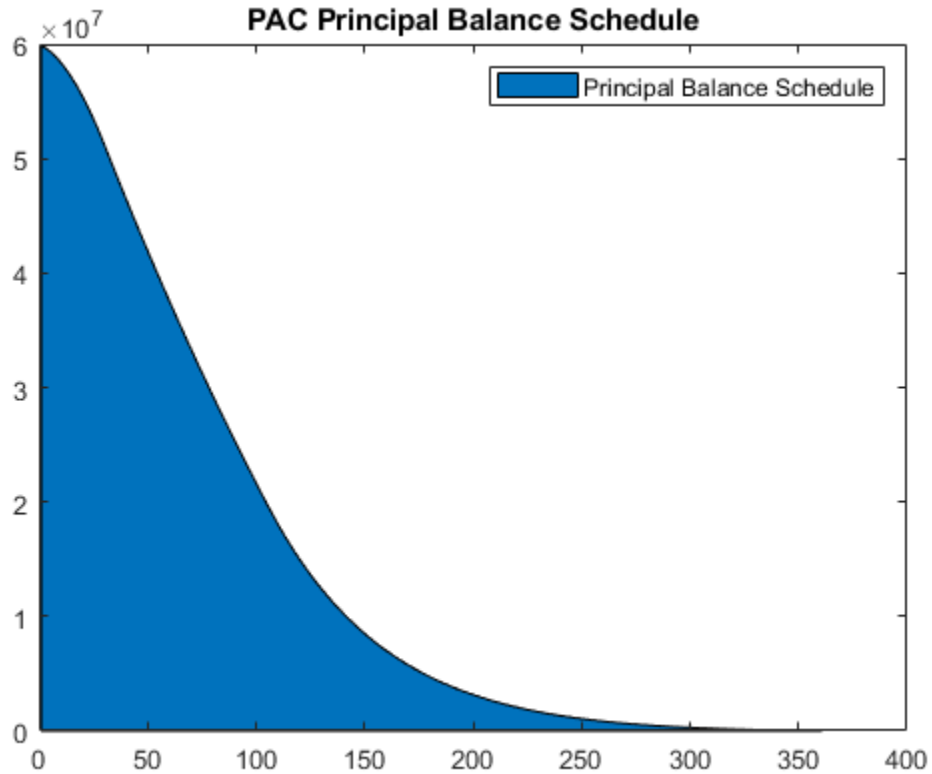


### Step 5. Calculate the PAC schedule.

```
pacSpeed = [80 300];
[balanceSchedule, pacInitBalance] = ...
cmosched(principal, grossrate, originalTerm, termRemaining, ...
pacSpeed, []);
```

### Step 6. Generate a plot for the PAC principal balance schedule.

```
figure;
area([pacInitBalance'; balanceSchedule'])
title('PAC Principal Balance Schedule');
legend('Principal Balance Schedule');
```



### Step 7. Calculate PAC cash flow.

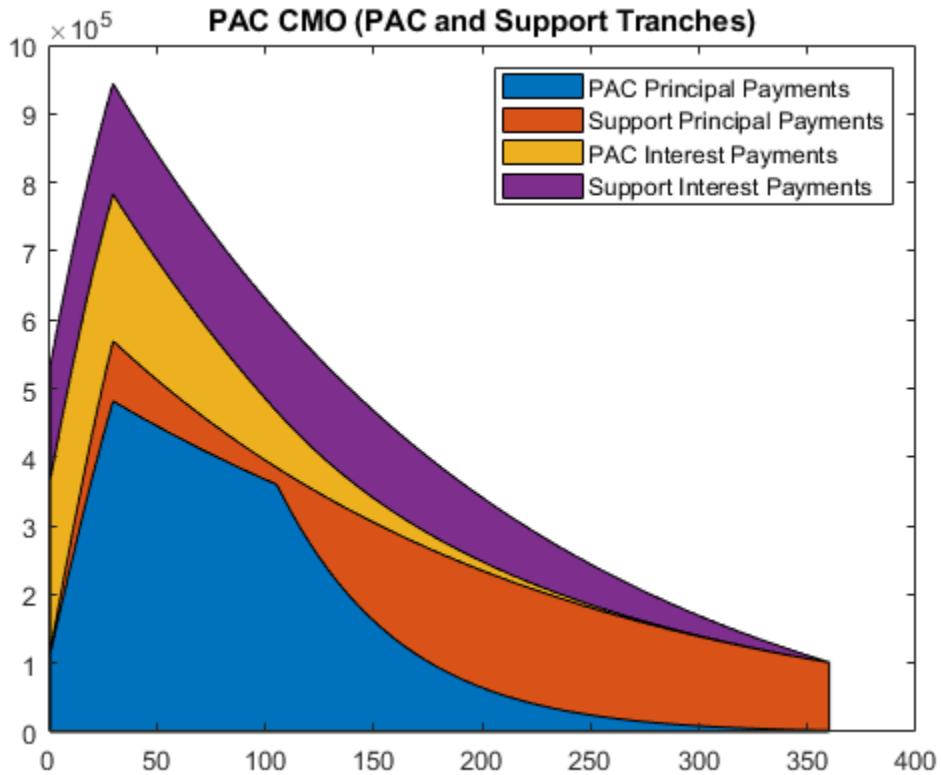
```
pacTranchePrincipals = [pacInitBalance; principal-pacInitBalance];
pacTrancheCoupons = [0.05; 0.05];
[pacBalances, pacPrincipals, pacInterests] = ...
cmoschedcf(principalPayments+prepayments, ...
pacTranchePrincipals, pacTrancheCoupons, balanceSchedule);
```

### Step 8. Generate a plot for the PAC CMO tranches.

Generate a plot for the PAC CMO tranches:

```
figure;
area([pacPrincipals' pacInterests']);
title('PAC CMO (PAC and Support Tranches)');
```

```
legend('PAC Principal Payments', 'Support Principal Payments', ...
      'PAC Interest Payments', 'Support Interest Payments');
```



**Step 9. Create sequential CMO from the PAC bond.**

CMO tranches, A, B, C, and D

```
seqTranchePrincipals = ...
[20000000; 20000000; 10000000; pacInitBalance-50000000];
seqTrancheCoupons = [0.05; 0.05; 0.05; 0.05];
```



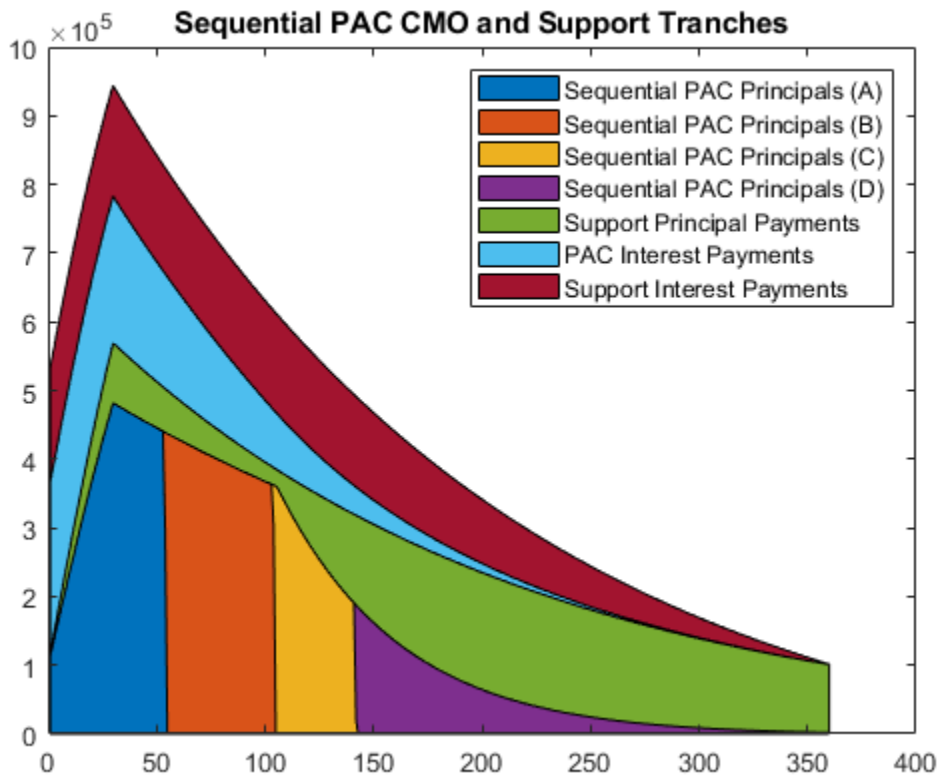
**Step 10. Calculate cash flows for each tranche.**

```
[seqBalances, seqPrincipals, seqInterests] = ...  
cmoseqcf(pacPrincipals(1, :), seqTranchePrincipals, ...  
seqTrancheCoupons, false);
```

**Step 11. Generate a plot for the sequential PAC CMO.**

Generate a plot for the sequential PAC CMO:

```
figure  
area([seqPrincipals' pacPrincipals(2, :)' pacInterests']);  
title('Sequential PAC CMO and Support Tranches');  
legend('Sequential PAC Principals (A)', 'Sequential PAC Principals (B)', ...  
'Sequential PAC Principals (C)', 'Sequential PAC Principals (D)', ...  
'Support Principal Payments', 'PAC Interest Payments', ...  
'Support Interest Payments');
```



**Step 12. Create the discount curve.**

```
CurveSettle = datenum('1-Jan-2011');
ZeroRates = [0.01 0.03 0.10 0.19 0.45 0.81 1.76 2.50 3.18 4.09 4.38]'/100;
CurveTimes = [1/12 3/12 6/12 1 2 3 5 7 10 20 30]';
CurveDates = daysadd(CurveSettle, 360 * CurveTimes, 1);
zeroCurve = intenvset('Rates', ZeroRates, 'StartDates', CurveSettle, ...
    'EndDates', CurveDates);
```

**Step 13. Price the CMO cash flows.**

The cash flow for the sequential PAC principal A tranche is calculated using the cash flow functions `cfbyzero`, `cfyield`, `cfprice`, and `cfsread`.

```
cflows = seqPrincipals(1, :)+seqInterests(1, :);
cfdates = dates(2:end)';
price1 = cfbyzero(zeroCurve, cflows, cfdates, Settle, 4)

price1 = 2.2109e+07

yield = cfyield(cflows, cfdates, price1, Settle, 'Basis', 4)

yield = 0.0090

price2 = cfprice(cflows, cfdates, yield, Settle, 'Basis', 4)

price2 = 2.2109e+07

spread = cfsread(zeroCurve, price2, cflows, cfdates, Settle, 'Basis', 4)

spread = 8.5711e-13

WAL = sum(cflows .* yearfrac(Settle, cfdates, 4)) / sum(cflows)

WAL = 2.5408
```

The weighted average life (WAL) for the sequential PAC principal A tranche is 2.54 years.

## See Also

cfbyzero | cfbyzero | cfprice | cfsread | cfyield | cmosched | cmoschedcf |  
cmoseqcf | mbscfamounts

## Related Examples

- “Fixed-Rate Mortgage Pool” on page 5-3

## More About

- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-51



# Debt Instruments

---

- “Agency Option-Adjusted Spreads” on page 6-2
- “Using Zero-Coupon Bonds” on page 6-6
- “Stepped-Coupon Bonds” on page 6-11
- “Term Structure Calculations” on page 6-14

## Agency Option-Adjusted Spreads

Often bonds are issued with embedded options, which then makes standard price/yield or spread measures irrelevant. For example, a municipality concerned about the chance that interest rates may fall in the future might issue bonds with a provision that allows the bond to be repaid before the bond's maturity. This is a call option on the bond and must be incorporated into the valuation of the bond. Option-adjusted spread (OAS), which adjusts a bond spread for the value of the option, is the standard measure for valuing bonds with embedded options. Financial Instruments Toolbox software supports computing option-adjusted spreads for bonds with single embedded options using the agency model.

The Securities Industry and Financial Markets Association (SIFMA) has a simplified approach to compute OAS for agency issues (Government Sponsored Entities like Fannie Mae and Freddie Mac) termed "Agency OAS". In this approach, the bond has only one call date (European call) and uses Black's model (a variation on Black Scholes, [http://en.wikipedia.org/wiki/Black\\_model](http://en.wikipedia.org/wiki/Black_model)) to value the bond option. The price of the bond is computed as follows:

$$\text{Price}_{\text{Callable}} = \text{Price}_{\text{NonCallable}} - \text{Price}_{\text{Option}}$$

where

$\text{Price}_{\text{Callable}}$  is the price of the callable bond.

$\text{Price}_{\text{NonCallable}}$  is the price of the noncallable bond, that is, price of the bond using `bndspread`.

$\text{Price}_{\text{Option}}$  is the price of the option, that is, price of the option using Black's model.

The Agency OAS is the spread, when used in the previous formula, yields the market price. Financial Instruments Toolbox software supports these functions:

### Agency OAS

Agency OAS Functions	Purpose
<code>agencyoas</code>	Compute the OAS of the callable bond using the Agency OAS model.
<code>agencyprice</code>	Price the callable bond OAS using Agency using the OAS model.

## Computing the Agency OAS for Bonds

To compute the Agency OAS using `agencyoas`, you must provide the zero curve as the input `ZeroData`. You can specify the zero curve in any intervals and with any compounding method. You can do this using Financial Toolbox™ functions `zbtprice` and `zbtyield`. Or, you can use `IRDataCurve` to construct an `IRDataCurve` object, and then use the `getZeroRates` to convert to dates and data for use in the `ZeroData` input.

After creating the `ZeroData` input for `agencyoas`, you can then:

- 1 Assign parameters for `CouponRate`, `Settle`, `Maturity`, `Vol`, `CallDate`, and `Price`.
- 2 Compute the option-adjusted spread using `agencyoas` to derive the OAS output.

If you have the Agency OAS for the callable bond, you can use the OAS value as an input to `agencyprice` to determine the price for a callable bond.

In the following example, the Agency OAS is computed using `agencyoas` for a range of bond prices and the spread of an identically priced noncallable bond is calculated using `bndspread`.

```
%% Data
% Bond data -- note that there is only 1 call date
Settle = datenum('20-Jan-2010');
Maturity = datenum('30-Dec-2013');
Coupon = .022;
Vol = .5117;
CallDate = datenum('30-Dec-2010');
Period = 2;
Basis = 1;
Face = 100;

% Zero Curve data
ZeroTime = [.25 .5 1 2 3 4 5 7 10 20 30]';
ZeroDates = daysadd(Settle,360*ZeroTime,1);
ZeroRates = [.0008 .0017 .0045 .0102 .0169 .0224 .0274 .0347 .0414 .0530 .0740]';
ZeroData = [ZeroDates ZeroRates];
CurveCompounding = 2;
CurveBasis = 1;

Price = 94:104;
OAS = agencyoas(ZeroData, Price', Coupon, Settle,Maturity, Vol, CallDate,'Basis',Basis)
Spread = bndspread(ZeroData, Price', Coupon, Settle, Maturity)
plot(OAS,Price)
hold on
plot(Spread,Price,'r')
xlabel('Spread (bp)')
ylabel('Price')
title('AOAS and Spread for an Agency and Equivalent Noncallable Bond')
legend({'Callable Issue','Noncallable Issue'})
```

OAS =

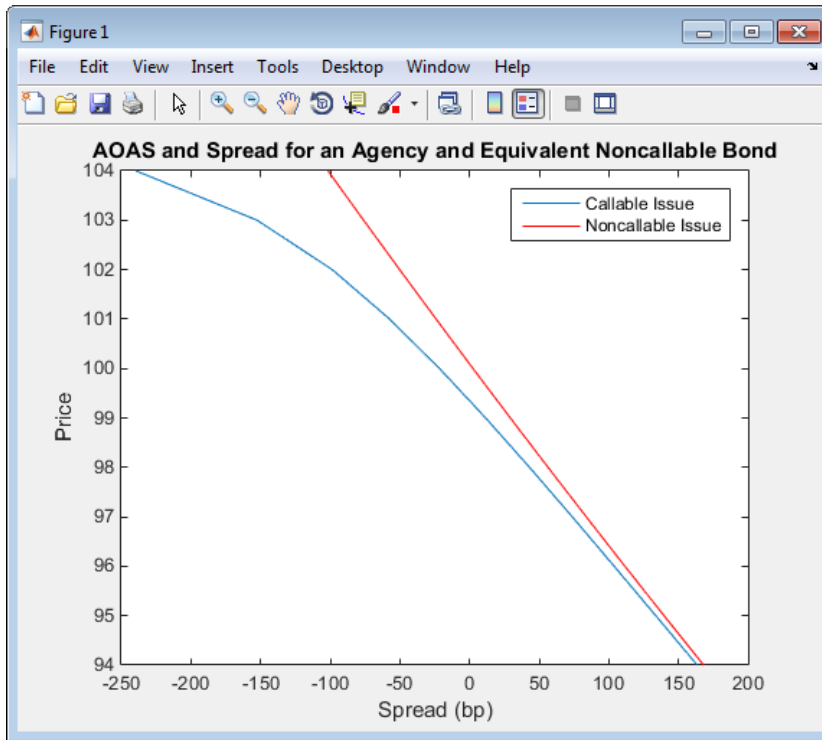
163.4942  
133.7306  
103.8735  
73.7505  
43.1094  
11.5608  
-21.5412  
-57.3869  
-98.5675  
-152.5226  
-239.6462

Spread =

168.1412  
139.7047  
111.6123  
83.8561  
56.4286  
29.3227  
2.5314  
-23.9523  
-50.1348  
-76.0226  
-101.6218

The following plot demonstrates as the price increases, the value of the embedded option in the Agency issue increases, and the value of the issue itself does not increase as much as it would for a noncallable bond, illustrating the negative convexity of this issue:





## See Also

[agencyoas](#) | [agencyprice](#)

## Related Examples

- “Using Zero-Coupon Bonds” on page 6-6
- “Stepped-Coupon Bonds” on page 6-11
- “Term Structure Calculations” on page 6-14

## More About

- “Supported Interest-Rate Instruments” on page 2-2

## Using Zero-Coupon Bonds

In this section...
“Introduction” on page 6-6
“Measuring Zero-Coupon Bond Function Quality” on page 6-6
“Pricing Treasury Notes” on page 6-7
“Pricing Corporate Bonds” on page 6-8

### Introduction

A zero-coupon bond is a corporate, Treasury, or municipal debt instrument that pays no periodic interest. Typically, the bond is redeemed at maturity for its full face value. It is a security issued at a discount from its face value, or it may be a coupon bond stripped of its coupons and repackaged as a zero-coupon bond.

Financial Instruments Toolbox software provides functions for valuing zero-coupon debt instruments. These functions supplement existing coupon bond functions such as `bndprice` and `bndyield` that are available in Financial Toolbox software.

### Measuring Zero-Coupon Bond Function Quality

Zero-coupon function quality is measured by how consistent the results are with coupon-bearing bonds. Because the zero coupon's yield is bond-equivalent, comparisons with coupon-bearing bonds are possible.

In the textbook case, where time ( $t$ ) is measured continuously and the rate ( $r$ ) is continuously compounded, the value of a zero bond is the principal multiplied by  $e^{-rt}$ . In reality, the rate quoted is continuous and the basis can be variable, requiring a more consistent approach to meet the stricter demands of accurate pricing.

The following two examples

- “Pricing Treasury Notes” on page 6-7
- “Pricing Corporate Bonds” on page 6-8

show how the zero functions are consistent with supported coupon bond functions.

## Pricing Treasury Notes

A Treasury note can be considered to be a package of zeros. The toolbox functions that price zeros require a coupon bond equivalent yield. That yield can originate from any type of coupon paying bond, with any periodic payment, or any accrual basis. The next example shows the use of the toolbox to price a Treasury note and compares the calculated price with the actual price quotation for that day.

```
Settle = datenum('02-03-2003');
MaturityCpn = datenum('05-15-2009');
Period = 2;
Basis = 0;
```

```
% Quoted yield.
QYield = 0.03342;
```

```
% Quoted price.
QPriceACT = 112.127;
```

```
CouponRate = 0.055;
```

Extract the cash flow and compute price from the sum of zeros discounted.

```
[CFlows, CDates] = cfamounts(CouponRate, Settle, MaturityCpn, ...
Period, Basis);
MaturityofZeros = CDates;
```

Compute the price of the coupon bond identically as a collection of zeros by multiplying the discount factors to the corresponding cash flows.

```
PriceofZeros = CFlows * zeroprice(QYield, Settle, ...
MaturityofZeros, Period, Basis)/100;
```

The following table shows the intermediate calculations.

Cash Flows	Discount Factors	Discounted Cash Flows
-1.2155	1.0000	-1.2155
2.7500	0.9908	2.7246
2.7500	0.9745	2.6799
2.7500	0.9585	2.6359
2.7500	0.9427	2.5925

Cash Flows	Discount Factors	Discounted Cash Flows
2.7500	0.9272	2.5499
2.7500	0.9120	2.5080
2.7500	0.8970	2.4668
2.7500	0.8823	2.4263
2.7500	0.8678	2.3864
2.7500	0.8535	2.3472
2.7500	0.8395	2.3086
2.7500	0.8257	2.2706
102.7500	0.8121	83.4451
		Tota 112.1263
	1	

Compare the quoted price and the calculated price based on zeros.

```
[QPriceACT PriceofZeros]
```

```
ans =
```

```
112.1270    112.1263
```

This example shows that `zeroprice` can satisfactorily price a Treasury note, a semiannual actual/actual basis bond, as if it were a composed of a series of zero-coupon bonds.

## Pricing Corporate Bonds

You can similarly price a corporate bond, for which there is no corresponding zero-coupon bond, as opposed to a Treasury note, for which corresponding zeros exist. You can create a synthetic zero-coupon bond and arrive at the quoted coupon-bond price when you later sum the zeros.

```
Settle = datenum('02-05-2003');
MaturityCpn = datenum('01-14-2009');
Period = 2;
Basis = 1;
% Quoted yield.
QYield = 0.05974;
```

```
% Quoted price.
QPrice30 = 99.382;
CouponRate = 0.05850;
```

Extract cash flow and compute price from the sum of zeros.

```
[CFlows, CDates] = cfamounts(CouponRate, Settle, MaturityCpn, ...
Period, Basis);

Maturity = CDates;
```

Compute the price of the coupon bond identically as a collection of zeros by multiplying the discount factors to the corresponding cash flows.

```
Price30 = CFlows * zeroprice(QYield, Settle, Maturity, Period, ...
Basis)/100;
```

Compare quoted price and calculated price based on zeros.

```
[QPrice30 Price30]
```

```
ans =
```

```
99.3820    99.3828
```

As a test of fidelity, intentionally giving the wrong basis, say actual/actual (`Basis = 0`) instead of 30/360, gives a price of 99.3972. Such a systematic error, if recurring in a more complex pricing routine, quickly adds up to large inaccuracies.

In summary, the zero functions in MATLAB software facilitate extraction of present value from virtually any fixed-coupon instrument, up to any period in time.

## See Also

`bndprice` | `bndyield`

## Related Examples

- “Agency Option-Adjusted Spreads” on page 6-2
- “Stepped-Coupon Bonds” on page 6-11
- “Term Structure Calculations” on page 6-14

**More About**

- “Supported Interest-Rate Instruments” on page 2-2

## Stepped-Coupon Bonds

### In this section...

“Introduction” on page 6-11

“Cash Flows from Stepped-Coupon Bonds” on page 6-11

“Price and Yield of Stepped-Coupon Bonds” on page 6-12

### Introduction

A stepped-coupon bond has a fixed schedule of changing coupon amounts. Like fixed coupon bonds, stepped-coupon bonds could have different periodic payments and accrual bases.

The functions `stepcpnprice` and `stepcpnyield` compute prices and yields of such bonds. An accompanying function `stepcpncfamounts` produces the cash flow schedules pertaining to these bonds.

### Cash Flows from Stepped-Coupon Bonds

Consider a bond that has a schedule of two coupons. Suppose that the bond starts out with a 2% coupon that steps up to 4% in 2 years and onward to maturity. Assume that the issue and settlement dates are both March 15, 2003. The bond has a 5-year maturity. Use `stepcpncfamounts` to generate the cash flow schedule and times.

```
Settle      = datenum('15-Mar-2003');
Maturity    = datenum('15-Mar-2008');
ConvDates   = [datenum('15-Mar-2005')];
CouponRates = [0.02, 0.04];

[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ...
ConvDates, CouponRates)
```

Notably, `ConvDates` has one less element than `CouponRates` because MATLAB software assumes that the first element of `CouponRates` indicates the coupon schedule between `Settle` (March 15, 2003) and the first element of `ConvDates` (March 15, 2005), shown diagrammatically below.

	Pay 2% from March 15, 2003		Pay 4% from March 15, 2003
--	----------------------------	--	----------------------------

Effective 2% on March 15, 2003		Effective 4% on March 15, 2005	
--------------------------------	--	--------------------------------	--

Coupon Dates	Semiannual Coupon Payment
15-Mar-03	0
15-Sep-03	1
15-Mar-04	1
15-Sep-04	1
15-Mar-05	1
15-Sep-05	2
15-Mar-06	2
15-Sep-06	2
15-Mar-07	2
15-Sep-07	2
15-Mar-08	102

The payment on March 15, 2005 is still a 2% coupon. Payment of the 4% coupon starts with the next payment, September 15, 2005. March 15, 2005 is the end of first coupon schedule, not to be confused with the beginning of the second.

In summary, MATLAB takes user input as the end dates of coupon schedules and computes the next coupon dates automatically.

The payment due on settlement (zero in this case) represents the accrued interest due on that day. It is negative if such amount is nonzero. Comparison with `cfamounts` in Financial Toolbox shows that the two functions operate identically.

## Price and Yield of Stepped-Coupon Bonds

The toolbox provides two basic analytical functions to compute price and yield for stepped-coupon bonds. Using the above bond as an example, you can compute the price when the yield is known.

You can estimate the yield to maturity as a number-of-year weighted average of coupon rates. For this bond, the estimated yield is:



$$\frac{(2 \times 2) + (4 \times 3)}{5}$$

or 3.33%. While definitely not exact (due to nonlinear relation of price and yield), this estimate suggests close to par valuation and serves as a quick first check on the function.

```
Yield = 0.0333;
```

```
[Price, AccruedInterest] = stepcpnprice(Yield, Settle, ...
Maturity, ConvDates, CouponRates)
```

The price returned is 99.2237 (per \$100 notional), and the accrued interest is zero, consistent with our earlier assertions.

To validate that there is consistency among the stepped-coupon functions, you can use the above price and see if indeed it implies a 3.33% yield by using `stepcpnyield`.

```
YTM = stepcpnyield(Price, Settle, Maturity, ConvDates, ...
CouponRates)
```

```
YTM =
```

```
0.0333
```

## See Also

`stepcpncfamounts` | `stepcpnprice` | `stepcpnyield`

## Related Examples

- “Agency Option-Adjusted Spreads” on page 6-2
- “Using Zero-Coupon Bonds” on page 6-6
- “Term Structure Calculations” on page 6-14

## More About

- “Supported Interest-Rate Instruments” on page 2-2

## Term Structure Calculations

### In this section...

"Introduction" on page 6-14

"Computing Spot and Forward Curves" on page 6-14

"Computing Spreads" on page 6-16

### Introduction

So far, a more formal definition of "yield" and its application has not been developed. In many situations when cash flow is available, discounting factors to the cash flows may not be immediately apparent. In other cases, what is relevant is often a *spread*, the difference between curves (also known as the term structure of spread).

All these calculations require one main ingredient, the Treasury spot, par-yield, or forward curve. Typically, the generation of these curves starts with a series of on-the-run and selected off-the-run issues as inputs.

MATLAB software uses these bonds to find spot rates one at a time, from the shortest maturity onwards, using bootstrap techniques. All cash flows are used to construct the spot curve, and rates between maturities (for these coupons) are interpolated linearly.

### Computing Spot and Forward Curves

For an illustration of how this works, observe the use of `zbtyield` (or equivalently `zbtprice`) on a portfolio of six Treasury bills and bonds.

Bills	Maturity Date	Current Yield
3 month	4/17/03	1.15
6 month	7/17/03	1.18

Notes/Bonds	Coupon	Maturity Date	Current Yield
2 year	1.750	12/31/04	1.68
5 year	3.000	11/15/07	2.97
10 year	4.000	11/15/12	4.01

Notes/Bonds	Coupon	Maturity Date	Current Yield
30 year	5.375	2/15/31	4.92

You can specify prices or yields to the bonds above to infer the spot curve. The function `zbyield` accepts yields (bond-equivalent yield, to be exact).

To proceed, first assemble the above table into a variable called `Bonds`. The first column contains maturities, the second contains coupons, and the third contains notionals or face values of the bonds. (Note that bills have zero coupons.)

```
Bonds = [datenum('04/17/2003')    0      100;
          datenum('07/17/2003')    0      100;
          datenum('12/31/2004')  0.0175  100;
          datenum('11/15/2007')    0.03    100;
          datenum('11/15/2012')    0.04    100;
          datenum('02/15/2031')    0.05375 100];
```

Then specify the corresponding yields.

```
Yields = [0.0115;
          0.0118;
          0.0168;
          0.0297;
          0.0401;
          0.0492];
```

You are now ready to compute the spot curve for each of these six maturities. The spot curve is based on a settlement date of January 17, 2003.

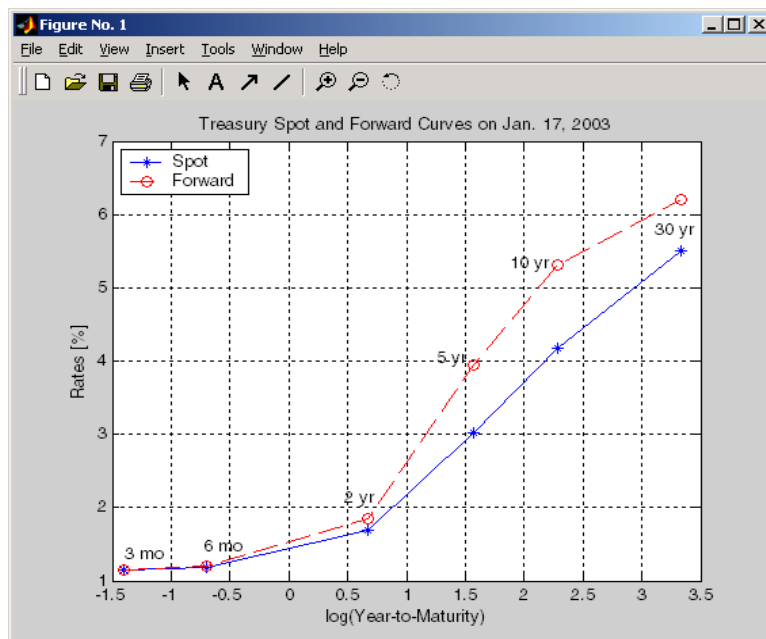
```
Settle = datenum('17-Jan-2003');
[ZeroRates, CurveDates] = zbyield(Bonds, Yields, Settle)
```

This gets you the Treasury spot curve for the day.

You can compute the forward curve from this spot curve with `zero2fwd`.

```
[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, ...
Settle)
```

Here the notion of forward rates refers to rates between the maturity dates shown above, not to a certain period (forward 3-month rates, for example).



## Computing Spreads

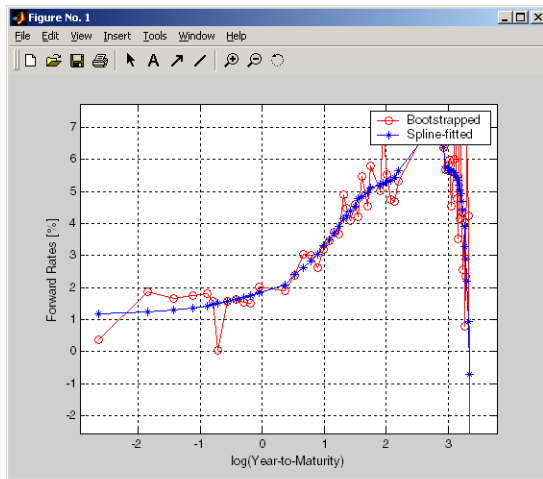
Calculating the spread between specific, fixed forward periods (such as the Treasury-Eurodollar spread) requires an extra step. Interpolate the zero rates (or zero prices, instead) for the corresponding maturities on the interval dates. Then use the interpolated zero rates to deduce the forward rates, and thus the spread of Eurodollar forward curve segments versus the relevant forward segments from Treasury bills.

Additionally, the variety of curve functions (including `zero2fwd`) helps to standardize such calculations. For instance, by making both rates quoted with quarterly compounding and on an actual/360 basis, the resulting spread structure is fully comparable. This avoids the small inconsistency that occurs when directly comparing the bond-equivalent yield of a Treasury bill to the quarterly forward rates implied by Eurodollar futures.

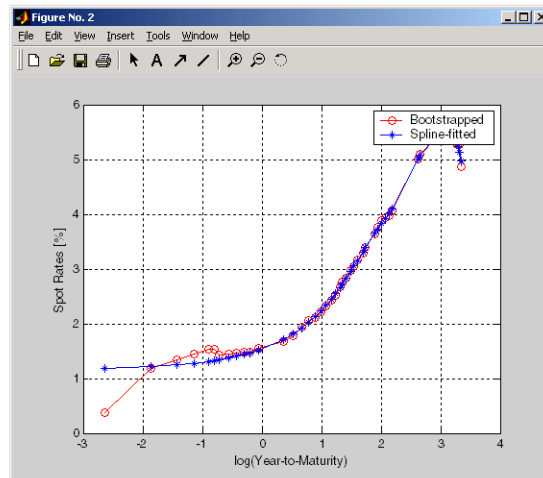
## Noise in Curve Computations

When introducing more bonds in constructing curves, noise may become a factor and may need some “smoothing” (with splines, for example); this helps obtain a smoother forward curve.

The following spot and forward curves are constructed from 67 Treasury bonds. The fitted and bootstrapped spot curve (bottom right figure) displays comparable stability. The forward curve (upper-left figure) contains significant noise and shows an improbable forward rate structure. The noise is not necessarily bad; it could uncover trading opportunities for a relative-value approach. Yet, a more balanced approach is desired when the bootstrapped forward curve oscillates this much and contains a negative rate as large as -10% (not shown in the plot because it is outside the limits).



**Implied Forward Curves.**  
 The jagged curve comes from direct bootstrapping.  
 The smooth curve shows the effect of smoothing with splines.



**Implied Spot Rate Curves.**  
 These curves correspond to the forward curve above.

This example uses `termfit`, an example function from Financial Toolbox software that also requires the use of Curve Fitting Toolbox™ software.

## See Also

`zbtprice` | `zbtyield`

## **Related Examples**

- “Agency Option-Adjusted Spreads” on page 6-2
- “Using Zero-Coupon Bonds” on page 6-6
- “Stepped-Coupon Bonds” on page 6-11

## **More About**

- “Supported Interest-Rate Instruments” on page 2-2





# Derivative Securities

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- “Interest Rate Swaps” on page 7-2
- “Bond Futures” on page 7-10
- “Analysis of Bond Futures” on page 7-13
- “Managing Present Value with Bond Futures” on page 7-16
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17
- “Fitting the Diebold Li Model” on page 7-25

## Interest Rate Swaps

In this section...
“Swap Pricing Assumptions” on page 7-2
“Swap Pricing Example” on page 7-3
“Portfolio Hedging” on page 7-8

### Swap Pricing Assumptions

Financial Instruments Toolbox contains the function `liborfloat2fixed`, which computes a fixed-rate par yield that equates the floating-rate side of a swap to the fixed-rate side. The solver sets the present value of the fixed side to the present value of the floating side without having to line up and compare fixed and floating periods.

#### Assumptions on Floating-Rate Input

- Rates are quarterly, for example, that of Eurodollar futures.
- Effective date is the first third Wednesday after the settlement date.
- All delivery dates are spaced 3 months apart.
- All periods start on the third Wednesday of delivery months.
- All periods end on the same dates of delivery months, 3 months after the start dates.
- Accrual basis of floating rates is actual/360.
- Applicable forward rates are estimated by interpolation in months when forward-rate data is not available.

#### Assumptions on Fixed-Rate Output

- Design allows you to create a bond of any coupon, basis, or frequency, based on the floating-rate input.
- The start date is a valuation date, that is, a date when an agreement to enter into a contract by the settlement date is made.
- Settlement can be on or after the start date. If it is after, a forward fixed-rate contract results.
- Effective date is assumed to be the first third Wednesday after settlement, the same date as that of the floating rate.

- The end date of the bond is a designated number of years away, on the same day and month as the effective date.
- Coupon payments occur on anniversary dates. The frequency is determined by the period of the bond.
- Fixed rates are not interpolated. A fixed-rate bond of the same present value as that of the floating-rate payments is created.

## Swap Pricing Example

This example shows the use of the functions in computing the fixed rate applicable to a series of 2-, 5-, and 10-year swaps based on Eurodollar market data. According to the Chicago Mercantile Exchange (<https://www.cmegroup.com>), Eurodollar data on Friday, October 11, 2002, was as shown in the following table.

---

**Note** This example illustrates swap calculations in MATLAB software. Timing of the data set used was not rigorously examined and was assumed to be the proxy for the swap rate reported on October 11, 2002.

---

**Eurodollar Data on Friday, October 11, 2002**

<b>Month</b>	<b>Year</b>	<b>Settle</b>
10	2002	98.21
11	2002	98.26
12	2002	98.3
1	2003	98.3
2	2003	98.31
3	2003	98.275
6	2003	98.12
9	2003	97.87
12	2003	97.575
3	2004	97.26
6	2004	96.98
9	2004	96.745
12	2004	96.515
3	2005	96.33
6	2005	96.135
9	2005	95.955
12	2005	95.78
3	2006	95.63
6	2006	95.465
9	2006	95.315
12	2006	95.16
3	2007	95.025
6	2007	94.88
9	2007	94.74
12	2007	94.595
3	2008	94.48
6	2008	94.375
9	2008	94.28

Month	Year	Settle
12	2008	94.185
3	2009	94.1
6	2009	94.005
9	2009	93.925
12	2009	93.865
3	2010	93.82
6	2010	93.755
9	2010	93.7
12	2010	93.645
3	2011	93.61
6	2011	93.56
9	2011	93.515
12	2011	93.47
3	2012	93.445
6	2012	93.41
9	2012	93.39

Using this data, you can compute 1-, 2-, 3-, 4-, 5-, 7-, and 10-year swap rates with the toolbox function `liborfloat2fixed`. The function requires you to input only Eurodollar data, the settlement date, and tenor of the swap. MATLAB software then performs the required computations.

To illustrate how this function works, first load the data contained in the supplied Excel® worksheet `EDdata.xls`.

```
[EDRawData, textdata] = xlsread('EDdata.xls');
```

Extract the month from the first column and the year from the second column. The rate used as proxy is the arithmetic average of rates on opening and closing.

```
Month = EDRawData(:,1);
Year = EDRawData(:,2);
IMMData = (EDRawData(:,4)+EDRawData(:,6))/2;
EDFutData = [Month, Year, IMMData];
```

Next, input the current date.

```
Settle = datenum('11-Oct-2002');
```

To compute for the 2-year swap rate, set the tenor to 2.

```
Tenor = 2;
```

Finally, compute the swap rate with `liborfloat2fixed`.

```
[FixedSpec, ForwardDates, ForwardRates] = ...  
liborfloat2fixed(EDFutData, Settle, Tenor)
```

MATLAB returns a par-swap rate of 2.23% using the default setting (quarterly compounding and 30/360 accrual), and forward dates and rates data (quarterly compounded).

```
FixedSpec =
```

```
    Coupon: 0.0223  
    Settle: '16-Oct-2002'  
    Maturity: '16-Oct-2004'  
    Period: 4  
    Basis: 1
```

```
ForwardDates =
```

```
731505  
731596  
731687  
731778  
731869  
731967  
732058  
732149
```

```
ForwardRates =
```

```
0.0178  
0.0168  
0.0171  
0.0189  
0.0216  
0.0250  
0.0280  
0.0306
```

In the `FixedSpec` output, note that the swap rate actually goes forward from the third Wednesday of October 2002 (October 16, 2002), 5 days after the original `Settle` input (October 11, 2002). This, however, is still the best proxy for the swap rate on `Settle`, as the assumption merely starts the swap's effective period and does not affect its valuation method or its length.

The correction suggested by Hull and White improves the result by turning on convexity adjustment as part of the input to `liborfloat2fixed`. (See Hull, J., *Options, Futures, and Other Derivatives*, 4th Edition, Prentice-Hall, 2000.) For a long swap, for example, five years or more, this correction could prove to be large.

The adjustment requires additional parameters:

- `StartDate`, which you make the same as `Settle` (the default) by providing an empty matrix `[]` as input.
- `ConvexAdj` to tell `liborfloat2fixed` to perform the adjustment.
- `RateParam`, which provides the parameters  $a$  and  $S$  as input to the Hull-White short rate process.
- Optional parameters `InArrears` and `Sigma`, for which you can use empty matrices `[]` to accept the MATLAB defaults.
- `FixedCompound`, with which you can facilitate comparison with values cited in Table H15 of *Federal Reserve Statistical Release* by turning the default quarterly compounding into semiannual compounding, with the (default) basis of 30/360.

```
StartDate = [];
Interpolation = [];
ConvexAdj = 1;
RateParam = [0.03; 0.017];
FixedCompound = 2;
[FixedSpec, ForwardDates, ForwardRates] = ...
liborfloat2fixed(EDFutData, Settle, Tenor, StartDate, ...
Interpolation, ConvexAdj, RateParam, [], [], FixedCompound)
```

This returns 2.21% as the 2-year swap rate, quite close to the reported swap rate for that date.

Analogously, the following table summarizes the solutions for 1-, 3-, 5-, 7-, and 10-year swap rates (convexity-adjusted and unadjusted).

### Calculated and Market Average Data of Swap Rates on Friday, October 11, 2002

Swap Length (Years)	Unadjusted	Adjusted	Table H15	Adjusted Error (Basis Points)
1	1.80%	1.79%	1.80%	-1
2	2.24%	2.21%	2.22%	-1
3	2.70%	2.66%	2.66%	0
4	3.12%	3.03%	3.04%	-1
5	3.50%	3.37%	3.36%	+1
7	4.16%	3.92%	3.89%	+3
10	4.87%	4.42%	4.39%	+3

### Portfolio Hedging

You can use these results further, such as for hedging a portfolio. The `liborduration` function provides a duration-hedging capability. You can isolate assets (or liabilities) from interest-rate risk exposure with a swap arrangement.

Suppose that you own a bond with these characteristics:

- \$100 million face value
- 7% coupon paid semiannually
- 5% yield to maturity
- Settlement on October 11, 2002
- Maturity on January 15, 2010
- Interest accruing on an actual/365 basis

Use of the `bnddury` function from Financial Toolbox software shows a modified duration of 5.6806 years.

To immunize this asset, you can enter into a pay-fixed swap, specifically a swap in the amount of notional principal ( $N_s$ ) such that  $N_s * \text{SwapDuration} + \$100\text{M} * 5.6806 = 0$  (or  $N_s = -100 * 5.6806 / \text{SwapDuration}$ ).

Suppose again, you choose to use a 5-, 7-, or 10-year swap (3.37%, 3.92%, and 4.42% from the previous table) as your hedging tool.



```
SwapFixRate = [0.0337; 0.0392; 0.0442];
Tenor = [5; 7; 10];
Settle = '11-Oct-2002';
PayFixDuration = liborduration(SwapFixRate, Tenor, Settle)
```

This gives a duration of -3.6835, -4.7307, and -6.0661 years for 5-, 7-, and 10-year swaps. The corresponding notional amount is computed by

```
Ns = -100*5.6806./PayFixDuration
```

```
Ns =
```

```
154.2163
120.0786
93.6443
```

The notional amount entered in pay-fixed side of the swap instantaneously immunizes the portfolio.

## See Also

[liborduration](#) | [liborfloat2fixed](#) | [liborprice](#)

## Related Examples

- “Analysis of Bond Futures” on page 7-13
- “Fitting the Diebold Li Model” on page 7-25
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17

## More About

- “Supported Interest-Rate Instruments” on page 2-2

## Bond Futures

Bond futures are futures contracts where the commodity for delivery is a government bond. There are established global markets for government bond futures. Bond futures provide a liquid alternative for managing interest-rate risk.

In the U.S. market, the Chicago Mercantile Exchange (CME) offers futures on Treasury bonds and notes with maturities of 2, 5, 10, and 30 years. Typically, the following bond future contracts from the CME have maturities of 3, 6, 9, and 12 months:

- 30-year U.S. Treasury bond
- 10-year U.S. Treasury bond
- 5-year U.S. Treasury bond
- 2-year U.S. Treasury bond

The short position in a Treasury bond or note future contract must deliver to the long position in one of many possible existing Treasury bonds. For example, in a 30-year Treasury bond future, the short position must deliver a Treasury bond with at least 15 years to maturity. Because these bonds have different values, the bond future contract is standardized by computing a conversion factor. The conversion factor normalizes the price of a bond to a theoretical bond with a coupon of 6%. The price of a bond future contract is represented as:

$$\text{InvoicePrice} = \text{FutPrice} \times \text{CF} + \text{AI}$$

where:

*FutPrice* is the price of the bond future.

*CF* is the conversion factor for a bond to deliver in a futures contract.

*AI* is the accrued interest.

The short position in a futures contract has the option of which bond to deliver and, in the U.S. bond market, when in the delivery month to deliver the bond. The short position typically chooses to deliver the bond known as the Cheapest to Deliver (CTD). The CTD bond most often delivers on the last delivery day of the month.

Financial Instruments Toolbox software supports the following bond futures:

- U.S. Treasury bonds and notes
- German Bobl, Bund, Buxl, and Schatz
- UK gilts
- Japanese government bonds (JGBs)

The functions supporting all bond futures are:

Function	Purpose
convfactor	Calculates bond conversion factors for U.S. Treasury bonds, German Bobl, Bund, Buxl, and Schatz, U.K. gilts, and JGBs.
bndfutprice	Prices bond future given repo rates.
bndfutimrepo	Calculates implied repo rates for a bond future given price.

The functions supporting U.S. Treasury bond futures are:

Function	Purpose
tfutbyprice	Calculates future prices of Treasury bonds given the spot price.
tfutbyyield	Calculates future prices of Treasury bonds given current yield.
tfutimrepo	Calculates implied repo rates for the Treasury bond future given price.
tfutpricebyrepo	Calculates Treasury bond futures price given the implied repo rates.
tfutyieldbyrepo	Calculates Treasury bond futures yield given the implied repo rates.

## See Also

bnddurp | bnddury | bndfutimrepo | bndfutprice | convfactor | tfutbyprice | tfutbyyield | tfutimrepo | tfutpricebyrepo | tfutyieldbyrepo

## Related Examples

- “Analysis of Bond Futures” on page 7-13
- “Fitting the Diebold Li Model” on page 7-25
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17

**More About**

- “Supported Interest-Rate Instruments” on page 2-2

## Analysis of Bond Futures

The following example demonstrates analyzing German Euro-Bund futures traded on Eurex. However, `convfactor`, `bndfutprice`, and `bndfutimprepo` apply to bond futures in the U.S., U.K., Germany, and Japan. The workflow for this analysis is:

- 1 Calculate bond conversion factors.
- 2 Calculate implied repo rates to find the CTD bond.
- 3 Price the bond future using the term implied repo rate.

### Calculating Bond Conversion Factors

Use conversion factors to normalize the price of a particular bond for delivery in a futures contract. When using conversion factors, the assumption is that a bond for delivery has a 6% coupon. Use `convfactor` to calculate conversion factors for all bond futures from the U.S., Germany, Japan, and U.K.

For example, conversion factors for Euro-Bund futures on Eurex are listed at [www.eurexchange.com](http://www.eurexchange.com). The delivery date for Euro-Bund futures is the 10th day of the month, as opposed to bond futures in the U.S., where the short position has the option of choosing when to deliver the bond.

For the 4% bond, compute the conversion factor with:

```
CF1 = convfactor('10-Sep-2009', '04-Jul-2018', .04, .06, 3)
```

```
CF1 =
```

```
0.8659
```

This syntax for `convfactor` works fine for bonds with standard coupon periods. However, some deliverable bonds have long or short first coupon periods. Compute the conversion factors for such bonds using the optional input parameters `StartDate` and `FirstCouponDate`. Specify all optional input arguments for `convfactor` as parameter/value pairs:

```
CF2 = convfactor('10-Sep-2009', '04-Jan-2019', .0375, 'Convention', 3, 'startdate', ...
datenum('14-Nov-2008'))
```

```
CF2 =
```

```
0.8426
```

## Calculating Implied Repo Rates to Find the CTD Bond

To determine the availability of the cheapest bond for deliverable bonds against a futures contract, compute the implied repo rate for each bond. The bond with the highest repo rate is the cheapest because it has the lowest initial value, thus yielding a higher return, provided you deliver it with the stated futures price. Use `bndfutimprepo` to calculate repo rates:

```
% Bond Properties
CouponRate = [.0425;.0375;.035];
Maturity = [datenum('04-Jul-2018');datenum('04-Jan-2019');datenum('04-Jul-2019')];
CF = [0.882668;0.842556;0.818193];
Price = [105.00;100.89;98.69];

% Futures Properties
FutSettle = '09-Jun-2009';
FutPrice = 118.54;
Delivery = '10-Sep-2009';

% Note that the default for BDNFUTIMPREPO is for the bonds to be
% semi-annual with a day count basis of 0. Since these are German
% bonds, we need to have a Basis of 8 and a Period of 1
ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, CF, ...
    CouponRate, Maturity,'Basis',8,'Period',1)

ImpRepo =

    0.0261
   -0.0022
   -0.0315
```

## Pricing Bond Futures Using the Term Implied Repo Rate

Use `bndfutprice` to perform price calculations for all bond futures from the U.S., Germany, Japan, and U.K. To price the bond, given a term repo rate:

```
% Assume a term repo rate of .0091;
RepoRate = .0091;
[FutPrice,AccrInt] = bndfutprice(RepoRate, Price(1), FutSettle,...
    Delivery, CF(1), CouponRate(1), Maturity(1),...
    'Basis',8,'Period',1)

FutPrice =

    118.0126

AccrInt =

    0.7918
```

## See Also

bnddurp | bnddury | bndfutimprepo | bndfutprice | convfactor | tfutbyprice |  
tfutbyyield | tfutimprepo | tfutpricebyrepo | tfutyieldbyrepo

## Related Examples

- “Managing Present Value with Bond Futures” on page 7-16
- “Fitting the Diebold Li Model” on page 7-25
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17

## More About

- “Bond Futures” on page 7-10
- “Supported Interest-Rate Instruments” on page 2-2

## Managing Present Value with Bond Futures

The Present Value of a Basis Point (PVBP) is used to manage interest-rate risk. PVBP is a measure that quantifies the change in price of a bond given a one-basis point shift in interest rates. The PVBP of a bond is computed with the following:

$$PVBP_{Bond} = \frac{Duration \times MarketValue}{100}$$

The PVBP of a bond futures contract can be computed with the following:

$$PVBP_{Futures} = \frac{PVBP_{CTDBond}}{CTDConversionFactor}$$

Use `bnddurp` and `bnddury` from Financial Toolbox software to compute the modified durations of CTD bonds. For more information, see “Managing Interest-Rate Risk with Bond Futures” on page 7-17 and “Fitting the Diebold Li Model” on page 7-25.

### See Also

`bnddurp` | `bnddury` | `bndfutimprepo` | `bndfutprice` | `convfactor` | `tfutbyprice` | `tfutbyyield` | `tfutimprepo` | `tfutpricebyrepo` | `tfutyieldbyrepo`

### Related Examples

- “Analysis of Bond Futures” on page 7-13
- “Fitting the Diebold Li Model” on page 7-25
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17

### More About

- “Bond Futures” on page 7-10
- “Supported Interest-Rate Instruments” on page 2-2



## Managing Interest-Rate Risk with Bond Futures

This example shows how to hedge the interest-rate risk of a portfolio using bond futures.

### **Modifying the Duration of a Portfolio with Bond Futures**

In managing a bond portfolio, you can use a benchmark portfolio to evaluate performance. Sometimes a manager is constrained to keep the portfolio's duration within a particular band of the duration of the benchmark. One way to modify the duration of the portfolio is to buy and sell bonds, however, there may be reasons why a portfolio manager wishes to maintain the existing composition of the portfolio (for example, the current holdings reflect fundamental research/views about future returns). Therefore, another option for modifying the duration is to buy and sell bond futures.

Bond futures are futures contracts where the commodity to be delivered is a government bond that meets the standard outlined in the futures contract (for example, the bond has a specified remaining time to maturity).

Since often many bonds are available, and each bond may have a different coupon, you can use a conversion factor to normalize the payment by the long to the short.

There exist well developed markets for government bond futures. Specifically, the Chicago Board of Trade offers futures on the following:

- 2 Year Note
- 3 Year Note
- 5 Year Note
- 10 Year Note
- 30 Year Bond

<https://www.cmegroup.com/trading/interest-rates/>

Eurex offers futures on the following:

- Euro-Schatz Futures 1.75 to 2.25
- Euro-Bobl Futures 4.5 to 5.5
- Euro-Bund Futures 8.5 to 10.5
- Euro-Buxl Futures 24.0 to 35

<https://www.eurexchange.com/exchange-en/>

Bond futures can be used to modify the duration of a portfolio. Since bond futures derive their value from the underlying instrument, the duration of a bond futures contract is related to the duration of the underlying bond.

There are two challenges in computing this duration:

- Since there are many available bonds for delivery, the short in the contract has a choice in which bond to deliver.
- Some contracts allow the short flexibility in choosing the delivery date.

Typically, the bond used for analysis is the bond that is cheapest for the short to deliver (CTD).

One approach is to compute duration measures using the CTD's duration and the conversion factor.

For example, the Present Value of a Basis Point (PVBP) can be computed from the following:

$$PVBP_{Futures} = \frac{PVBP_{CTD}}{ConversionFactor_{CTD}}$$

$$PVBP_{CTD} = \frac{Duration_{CTD} * Price_{CTD}}{100}$$

Note that these definitions of duration for the futures contract are approximate, and do not account for the value of the delivery options for the short.

If the goal is to modify the duration of a portfolio, use the following:

$$NumContracts = \frac{(Dur_{Target} - Dur_{Initial}) * Value_{Portfolio}}{Dur_{CTD} * Price_{CTD} * ContractSize} * ConvFactor_{CTD}$$

Note that the contract size is typically for 100,000 face value of a bond -- so the contract size is typically 1000, as the bond face value is 100.

The following example assumes an initial duration, portfolio value, and target duration for a portfolio with exposure to the Euro interest rate. The June Euro-Bund Futures contract is used to modify the duration of the portfolio.

Note that typically futures contracts are offered for March, June, September and December.

```

% Assume the following for the portfolio and target
PortfolioDuration = 6.4;
PortfolioValue = 100000000;
BenchmarkDuration = 4.8;

% Deliverable Bunds -- note that these conversion factors may also be
% computed with the MATLAB(R) function convfactor
BondPrice = [106.46;108.67;104.30];
BondMaturity = datenum({'04-Jan-2018','04-Jul-2018','04-Jan-2019'});
BondCoupon = [.04;.0425;.0375];
ConversionFactor = [.868688;.880218;.839275];

% Futures data -- found from http://www.eurexchange.com
FuturesPrice = 122.17;
FuturesSettle = '23-Apr-2009';
FuturesDelivery = '10-Jun-2009';

% To find the CTD bond we can compute the implied repo rate
ImpliedRepo = bndfutimprepo(BondPrice,FuturesPrice,FuturesSettle,...
    FuturesDelivery,ConversionFactor,BondCoupon,BondMaturity);

% Note that the bond with the highest implied repo rate is the CTD
[CTDImpRepo,CTDIndex] = max(ImpliedRepo);

% Compute the CTD's Duration -- note the period and basis for German Bunds
Duration = bnddurp(BondPrice,BondCoupon,FuturesSettle,BondMaturity,1,8);

ContractSize = 1000;

% Use the formula above to compute the number of contracts to sell
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...
    (BondPrice(CTDIndex)*ContractSize*Duration(CTDIndex))*ConversionFactor(CTDIndex);

disp(['To achieve the target duration, ' num2str(abs(round(NumContracts))) ...
    ' Euro-Bund Futures must be sold.'])

To achieve the target duration, 180 Euro-Bund Futures must be sold.

```

### Modifying the Key Rate Durations of a Portfolio with Bond Futures

One of the shortcomings of using duration as a risk measure is that it assumes parallel shifts in the yield curve. While many studies have shown that this explains roughly 85% of the movement in the yield curve, changes in the slope or shape of the yield curve are not

captured by duration, and therefore, hedging strategies are not successful at addressing these dynamics.

One approach is to use key rate duration -- this is particularly relevant when using bond futures with multiple maturities, like Treasury futures.

The following example uses 2, 5, 10 and 30 year Treasury Bond futures to hedge the key rate duration of a portfolio.

Computing key rate durations requires a zero curve. This example uses the zero curve published by the Treasury and found at the following location:

<https://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml>

Note that this zero curve could also be derived using the Interest-Rate Curve functionality found in `IRDataCurve` and `IRFunctionCurve`.

```
% Assume the following for the portfolio and target, where the duration
% vectors are key rate durations at 2, 5, 10, and 30 years.
PortfolioDuration = [.5 1 2 6];
PortfolioValue = 100000000;
BenchmarkDuration = [.4 .8 1.6 5];

% The following are the CTD Bonds for the 30, 10, 5 and 2 year futures
% contracts -- these were determined using the procedure outlined in the
% previous section.
CTDCoupon = [4.75 3.125 5.125 7.5]'/100;
CTDMaturity = datenum({'3/31/2011', '08/31/2013', '05/15/2016', '11/15/2024'});
CTDConversion = [0.9794 0.8953 0.9519 1.1484]';
CTDPrice = [107.34 105.91 117.00 144.18]';

ZeroRates = [0.07 0.10 0.31 0.50 0.99 1.38 1.96 2.56 3.03 3.99 3.89]'/100;
ZeroDates = daysadd(FuturesSettle,[30 360 360*2 360*3 360*5 ...
    360*7 360*10 360*15 360*20 360*25 360*30],1);

% Compute the key rate durations for each of the CTD bonds.
CTDKRD = bndkrdr([ZeroDates ZeroRates], CTDCoupon,FuturesSettle,...
    CTDMaturity,'KeyRates',[2 5 10 30]);

% Note that the contract size for the 2 Year Note Future is $200,000
ContractSize = [2000;1000;1000;1000];

NumContracts = (bsxfun(@times,CTDPrice.*ContractSize./CTDConversion,CTDKRD))\...
```

```

(BenchmarkDuration - PortfolioDuration)*PortfolioValue;

sprintf(['To achieve the target duration, \n' ...
num2str(-round(NumContracts(1))) ' 2 Year Treasury Note Futures must be sold, \n'
num2str(-round(NumContracts(2))) ' 5 Year Treasury Note Futures must be sold, \n'
num2str(-round(NumContracts(3))) ' 10 Year Treasury Note Futures must be sold, \n'
num2str(-round(NumContracts(4))) ' Treasury Bond Futures must be sold, \n'])

ans =
'To achieve the target duration,
 24 2 Year Treasury Note Futures must be sold,
 47 5 Year Treasury Note Futures must be sold,
 68 10 Year Treasury Note Futures must be sold,
120 Treasury Bond Futures must be sold,
'
```

### Improving the Performance of a Hedge with Regression

An additional component to consider in hedging interest-rate risk with bond futures, again related to movements in the yield curve, is that typically the yield curve moves more at the short end than at the long end.

Therefore, if a position is hedged with a future where the CTD bond has a maturity that is different than the portfolio this could lead to a situation where the hedge under- or over-compensates for the actual interest-rate risk of the portfolio.

One approach is to perform a regression on historical yields at different maturities to determine a Yield Beta, which is a value that represents how much more the yield changes for different maturities.

This example shows how to use this approach with UK Long Gilt futures and historical data on Gilt Yields.

Market data on Gilt futures is found at the following:

<https://www.euronext.com>

Historical data on gilts is found at the following;

<https://www.dmo.gov.uk>

Note that while this approach does offer the possibility of improving the performance of a hedge, any analysis using historical data depends on historical relationships remaining consistent.

Also note that an additional enhancement takes into consideration the correlation between different maturities. While this approach is outside the scope of this example, you can use this to implement a minimum variance hedge.

```
% Assume the following for the portfolio and target
```

```
PortfolioDuration = 6.4;  
PortfolioValue = 100000000;  
BenchmarkDuration = 4.8;
```

```
% This is the CTD Bond for the Long Gilt Futures contract
```

```
CTDBondPrice = 113.40;  
CTDBondMaturity = datenum('7-Mar-2018');  
CTDBondCoupon = .05;  
CTDConversionFactor = 0.9325024;
```

```
% Market data for the Long Gilt Futures contract
```

```
FuturesPrice = 120.80;  
FuturesSettle = '23-Apr-2009';  
FuturesDelivery = '10-Jun-2009';
```

```
CTDDuration = bnddurp(CTDBondPrice,CTDBondCoupon,FuturesSettle,CTDBondMaturity);
```

```
ContractSize = 1000;
```

```
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...  
    (CTDBondPrice*ContractSize*CTDDuration)*CTDConversionFactor;
```

```
disp(['To achieve the target duration with a conventional hedge ' ...  
    num2str(-round(NumContracts)) ...  
    ' Long Gilt Futures must be sold.'])
```

```
To achieve the target duration with a conventional hedge 182 Long Gilt Futures must be
```

To improve the accuracy of this hedge, historical data is used to determine a relationship between the standard deviation of the yields. Specifically, standard deviation of yields is plotted and regressed vs bond duration. This relationship is then used to compute a Yield Beta for the hedge.

```
% Load data from XLS spreadsheet
```

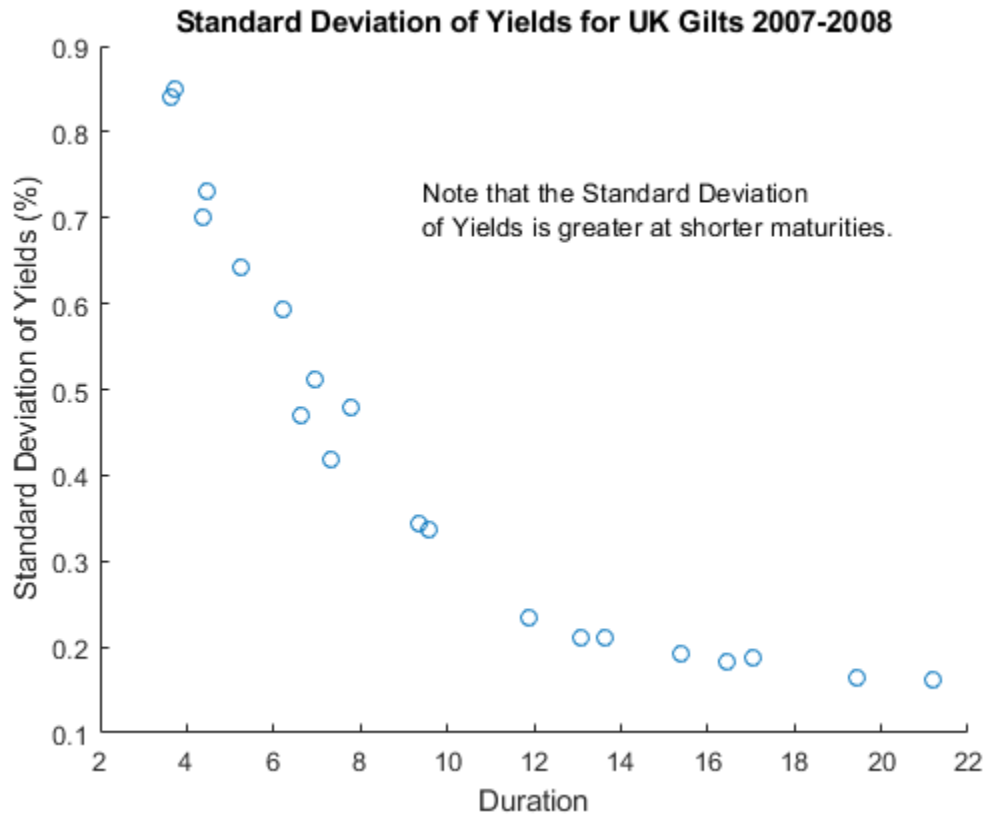
```
load ukbonddata_20072008
```

```

Duration = bnddury(Yield(1,:)',Coupon,Dates(1,:),Maturity);

scatter(Duration,100*std(Yield))
title('Standard Deviation of Yields for UK Gilts 2007-2008')
ylabel('Standard Deviation of Yields (%)')
xlabel('Duration')
annotation(gcf,'textbox',[0.4067 0.685 0.4801 0.0989],...
    'String',{'Note that the Standard Deviation',...
    'of Yields is greater at shorter maturities.'},...
    'FitBoxToText','off',...
    'EdgeColor','none');

```



```
stats = regstats(std(Yield),Duration);  
YieldBeta = (stats.beta'*[1 PortfolioDuration]')./(stats.beta'*[1 CTDDuration]');
```

Now the Yield Beta is used to compute a new value for the number of contracts to be sold. Note that since the duration of the portfolio was less than the duration of the CTD Gilt, the number of futures to sell is actually greater than in the first case.

```
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...  
    (CTDBondPrice*ContractSize*CTDDuration)*CTDConversionFactor*YieldBeta;  
  
disp(['To achieve the target duration using a Yield Beta-modified hedge, ' ...  
    num2str(abs(round(NumContracts))) ...  
    ' Long Gilt Futures must be sold.'])
```

To achieve the target duration using a Yield Beta-modified hedge, 193 Long Gilt Futures

### Bibliography

This example is based on the following books and papers:

- [1] Burghardt, G., T. Belton, M. Lane and J. Papa. The Treasury Bond Basis. New York, NY: McGraw-Hill, 2005.
- [2] Krgin, D. Handbook of Global Fixed Income Calculations. New York, NY: John Wiley & Sons, 2002.
- [3] CFA Program Curriculum, Level III, Volume 4, Reading 31. CFA Institute, 2009.

### See Also

[bnddurp](#) | [bnddury](#) | [bndfutimprepo](#) | [bndfutprice](#) | [convfactor](#) | [tfutbyprice](#) | [tfutbyyield](#) | [tfutimprepo](#) | [tfutpricebyrepo](#) | [tfutyieldbyrepo](#)

### Related Examples

- “Analysis of Bond Futures” on page 7-13
- “Fitting the Diebold Li Model” on page 7-25

### More About

- “Supported Interest-Rate Instruments” on page 2-2



## Fitting the Diebold Li Model

This example shows how to construct a Diebold Li model of the US yield curve for each month from 1990 to 2010. This example also demonstrates how to forecast future yield curves by fitting an autoregressive model to the time series of each parameter.

The paper can be found here:

<https://www.sas.upenn.edu/~fdiebold/papers/paper49/Diebold-Li.pdf>

### Load the Data

The data used are monthly Treasury yields from 1990 through 2010 for tenors of 1 Mo, 3 Mo, 6 Mo, 1 Yr, 2 Yr, 3 Yr, 5 Yr, 7 Yr, 10 Yr, 20 Yr, 30 Yr.

Daily data can be found here:

<https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldAll>

Data is stored in a MATLAB® data file as a MATLAB dataset object.

```
load Data_USYieldCurve
```

```
% Extract data for the last day of each month
MonthYearMat = repmat((1990:2010)',1,12)';
EOMDates = lbusdate(MonthYearMat(:), repmat((1:12)',21,1));
MonthlyIndex = find(ismember(Dataset.Properties.ObsNames,datestr(EOMDates)));
Estimationdataset = Dataset(MonthlyIndex,:);
EstimationData = double(Estimationdataset);
```

### Diebold Li Model

Diebold and Li start with the Nelson Siegel model

$$y = \beta_0 + (\beta_1 + \beta_2) \frac{\tau}{m} (1 - e^{-\frac{m}{\tau}}) - \beta_2 e^{-\frac{m}{\tau}}$$

and rewrite it to be the following:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right)$$

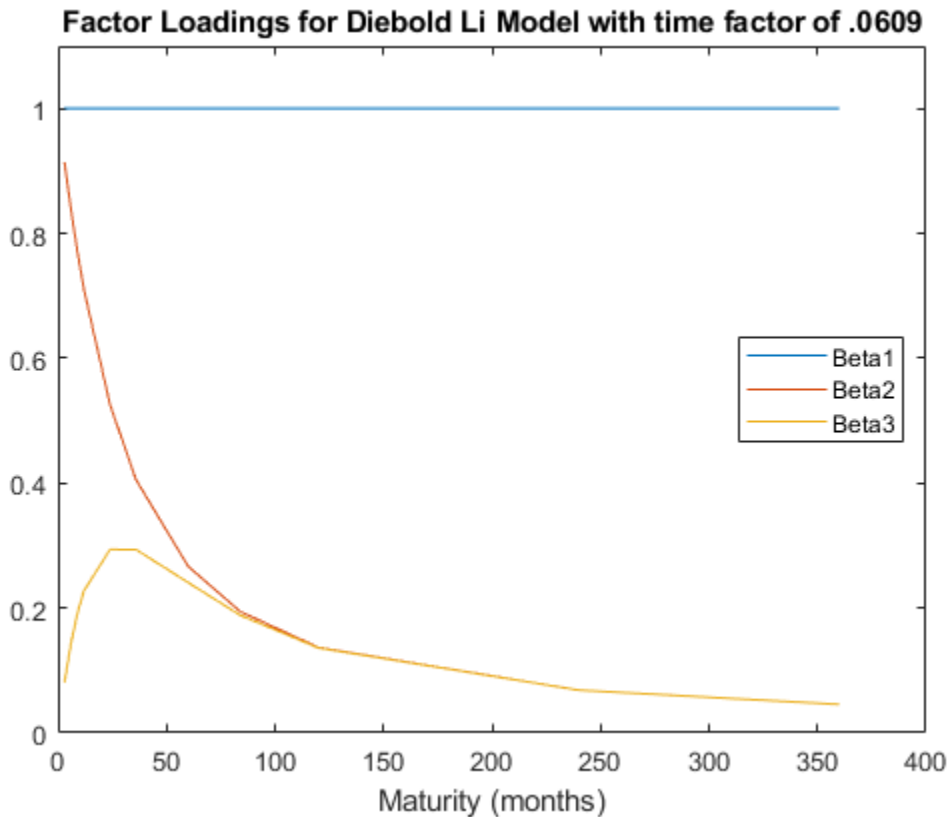
The above model allows the factors to be interpreted in the following way: Beta1 corresponds to the long term/level of the yield curve, Beta2 corresponds to the short term/slope, and Beta3 corresponds to the medium term/curvature.  $\lambda$  determines the maturity at which the loading on the curvature is maximized, and governs the exponential decay rate of the model.

Diebold and Li advocate setting  $\lambda$  to maximize the loading on the medium term factor, Beta3, at 30 months. This also transforms the problem from a nonlinear fitting to a simple linear regression.

```
% Explicitly set the time factor lambda
lambda_t = .0609;

% Construct a matrix of the factor loadings
% Tenors associated with data
TimeToMat = [3 6 9 12 24 36 60 84 120 240 360]';
X = [ones(size(TimeToMat)) (1 - exp(-lambda_t*TimeToMat))./(lambda_t*TimeToMat) ...
     ((1 - exp(-lambda_t*TimeToMat))./(lambda_t*TimeToMat) - exp(-lambda_t*TimeToMat))]

% Plot the factor loadings
plot(TimeToMat,X)
title('Factor Loadings for Diebold Li Model with time factor of .0609')
xlabel('Maturity (months)')
ylim([0 1.1])
legend({'Beta1', 'Beta2', 'Beta3'}, 'location', 'east')
```



### Fit the Model

A `DieboldLi` object is developed to facilitate fitting the model from yield data. The `DieboldLi` object inherits from the `IRCurve` object, so the `getZeroRates`, `getDiscountFactors`, `getParYields`, `getForwardRates`, and `toRateSpec` methods are all implemented. Additionally, the method `fitYieldsFromBetas` is implemented to estimate the Beta parameters given a lambda parameter for observed market yields.

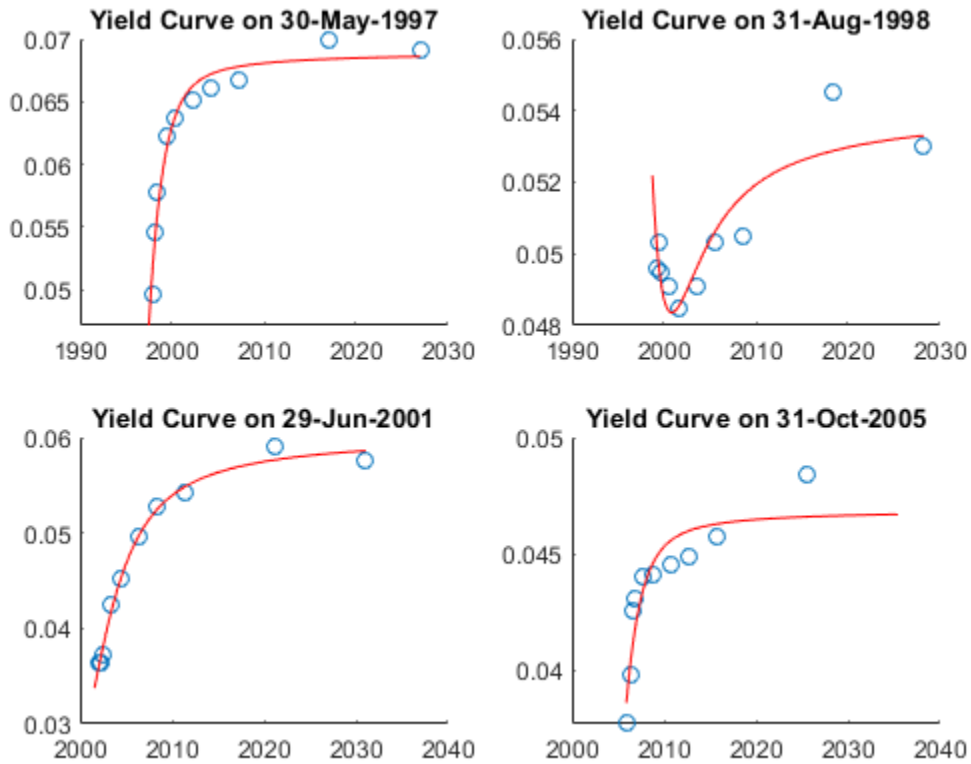
The `DieboldLi` object is used to fit a Diebold Li model for each month from 1990 through 2010.

```
% Preallocate the Betas
Beta = zeros(size(EstimationData,1),3);
```

```
% Loop through and fit each end of month yield curve
for jdx = 1:size(EstimationData,1)
    tmpCurveModel = DieboldLi.fitBetasFromYields(EOMDates(jdx),lambda_t*12,daysadd(EOMD
    Beta(jdx,:) = [tmpCurveModel.Beta1 tmpCurveModel.Beta2 tmpCurveModel.Beta3];
end
```

The Diebold Li fits on selected dates are included here

```
PlotSettles = datenum({'30-May-1997','31-Aug-1998','29-Jun-2001','31-Oct-2005'});
figure
for jdx = 1:length(PlotSettles)
    subplot(2,2,jdx)
    tmpIdx = find(strcmpi(Estimationdataset.Properties.ObsNames,datestr(PlotSettles(jdx)
    tmpCurveModel = DieboldLi.fitBetasFromYields(PlotSettles(jdx),lambda_t*12,...
        daysadd(PlotSettles(jdx),30*TimeToMat),EstimationData(tmpIdx,:)');
    scatter(daysadd(PlotSettles(jdx),30*TimeToMat),EstimationData(tmpIdx,:))
    hold on
    PlottingDates = (PlotSettles(jdx)+30:30:PlotSettles(jdx)+30*360)';
    plot(PlottingDates,tmpCurveModel.getParYields(PlottingDates),'r-')
    title(['Yield Curve on ' datestr(PlotSettles(jdx))])
    datetick
end
```



## Forecasting

The Diebold Li model can be used to forecast future yield curves. Diebold and Li propose fitting an AR(1) model to the time series of each Beta parameter. This fitted model can then be used to forecast future values of each parameter, and by extension, future yield curves.

For this example the MATLAB function `regress` is used to estimate the parameters for an AR(1) model for each Beta.

The confidence intervals for the regression fit are also used to generate two additional yield curve forecasts that serve as additional possible scenarios for the yield curve.

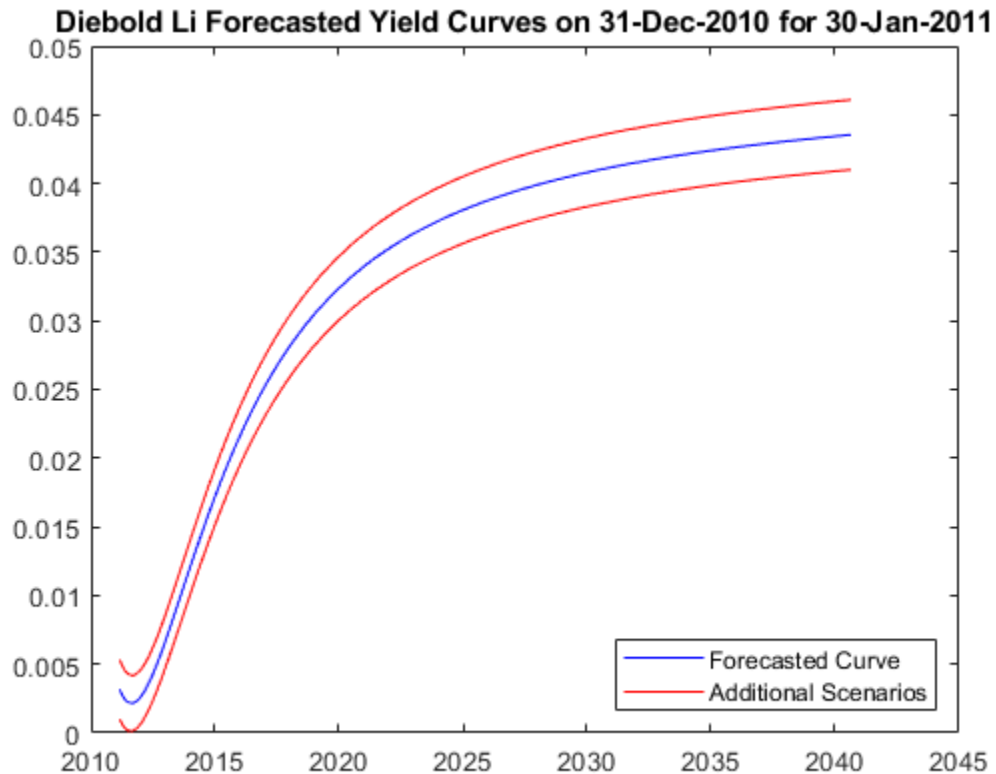
The MonthsLag variable can be adjusted to make different period ahead forecasts. For example, changing the value from 1 to 6 would change the forecast from a 1 month ahead to 6 month ahead forecast.

```
MonthsLag = 1;
```

```
[tmpBeta,bint] = regress(Beta(MonthsLag+1:end,1),[ones(size(Beta(MonthsLag+1:end,1)))
ForecastBeta(1,1) = [1 Beta(end,1)]*tmpBeta;
ForecastBeta_Down(1,1) = [1 Beta(end,1)]*bint(:,1);
ForecastBeta_Up(1,1) = [1 Beta(end,1)]*bint(:,2);
[tmpBeta,bint] = regress(Beta(MonthsLag+1:end,2),[ones(size(Beta(MonthsLag+1:end,2)))
ForecastBeta(1,2) = [1 Beta(end,2)]*tmpBeta;
ForecastBeta_Down(1,2) = [1 Beta(end,2)]*bint(:,1);
ForecastBeta_Up(1,2) = [1 Beta(end,2)]*bint(:,2);
[tmpBeta,bint] = regress(Beta(MonthsLag+1:end,3),[ones(size(Beta(MonthsLag+1:end,3)))
ForecastBeta(1,3) = [1 Beta(end,3)]*tmpBeta;
ForecastBeta_Down(1,3) = [1 Beta(end,3)]*bint(:,1);
ForecastBeta_Up(1,3) = [1 Beta(end,3)]*bint(:,2);
```

```
% Forecasted yield curve
```

```
figure
Settle = daysadd(EOMDates(end),30*MonthsLag);
DieboldLi_Forecast = DieboldLi('ParYield',Settle,[ForecastBeta lambda_t*12]);
DieboldLi_Forecast_Up = DieboldLi('ParYield',Settle,[ForecastBeta_Up lambda_t*12]);
DieboldLi_Forecast_Down = DieboldLi('ParYield',Settle,[ForecastBeta_Down lambda_t*12]);
PlottingDates = (Settle+30:30:Settle+30*360)';
plot(PlottingDates,DieboldLi_Forecast.getParYields(PlottingDates),'b-')
hold on
plot(PlottingDates,DieboldLi_Forecast_Up.getParYields(PlottingDates),'r-')
plot(PlottingDates,DieboldLi_Forecast_Down.getParYields(PlottingDates),'r-')
title(['Diebold Li Forecasted Yield Curves on ' datestr(EOMDates(end)) ' for ' datestr(EOMDates(end))])
legend({'Forecasted Curve','Additional Scenarios'},'location','southeast')
datetick
```



## Bibliography

This example is based on the following paper:

[1] Francis X. Diebold, Canlin Li, Forecasting the term structure of government bond yields, *Journal of Econometrics*, Volume 130, Issue 2, February 2006, Pages 337-364

## See Also

bnddurp | bnddury | bndfutimprepo | bndfutprice | convfactor | tfutbyprice | tfutbyyield | tfutimprepo | tfutpricebyrepo | tfutyieldbyrepo

### **Related Examples**

- “Analysis of Bond Futures” on page 7-13
- “Managing Interest-Rate Risk with Bond Futures” on page 7-17

### **More About**

- “Supported Interest-Rate Instruments” on page 2-2



# Credit Derivatives

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- “Counterparty Credit Risk and CVA” on page 8-2
- “First-to-Default Swaps” on page 8-25
- “Credit Default Swap Option” on page 8-37
- “Pricing a Single-Name CDS Option” on page 8-38
- “Pricing a CDS Index Option” on page 8-40
- “Wrong Way Risk with Copulas” on page 8-44

## Counterparty Credit Risk and CVA

This example shows how to compute the unilateral credit value (valuation) adjustment (CVA) for a bank holding a portfolio of vanilla interest-rate swaps with several counterparties. CVA is the expected loss on an over-the-counter contract or portfolio of contracts due to counterparty default. The CVA for a particular counterparty is defined as the sum over all points in time of the discounted expected exposure at each moment multiplied by the probability that the counterparty defaults at that moment, all multiplied by 1 minus the recovery rate. The CVA formula is:

$$CVA = (1 - R) \int_0^T discEE(t) dPD(t)$$

Where  $R$  is the recovery,  $discEE$  the discounted expected exposure at time  $t$ , and  $PD$  the default probability distribution.

The expected exposure is computed by first simulating many future scenarios of risk factors for the given contract or portfolio. Risk factors can be interest rates, as in this example, but will differ based on the portfolio and can include FX rates, equity or commodity prices, or anything that will affect the market value of the contracts. Once a sufficient set of scenarios has been simulated, the contract or portfolio can be priced on a series of future dates for each scenario. The result is a matrix, or "cube", of contract values.

These prices are converted into exposures after taking into account collateral agreements that the bank might have in place as well as netting agreements, as in this example, where the values of several contracts may offset each other, lowering their total exposure.

The contract values for each scenario are discounted to compute the discounted exposures. The discounted expected exposures can then be computed by a simple average of the discounted exposures at each simulation date.

Finally, counterparty default probabilities are typically derived from credit default swap (CDS) market quotes and the CVA for the counterparty can be computed according to the above formula. Assume that a counterparty default is independent of its exposure (no wrong-way risk).

This example demonstrates a portfolio of vanilla interest-rate swaps with the goal of computing the CVA for a particular counterparty.

## Read Swap Portfolio

The portfolio of swaps is close to zero value at time  $t = 0$ . Each swap is associated with a counterparty and may or may not be included in a netting agreement.

```
% Read swaps from spreadsheet
swapFile = 'cva-swap-portfolio.xls';
swaps = readtable(swapFile, 'Sheet', 'Swap Portfolio');
swaps.LegType = [swaps.LegType ~swaps.LegType];
swaps.LegRate = [swaps.LegRateReceiving swaps.LegRatePaying];
swaps.LegReset = ones(size(swaps,1),1);

numSwaps = size(swaps,1);
```

For more information on the swap parameters for CounterpartyID and NettingID, see `creditexposures`. For more information on the swap parameters for Principal, Maturity, LegType, LegRate, LatestFloatingRate, Period, and LegReset, see `swapbyzero`.

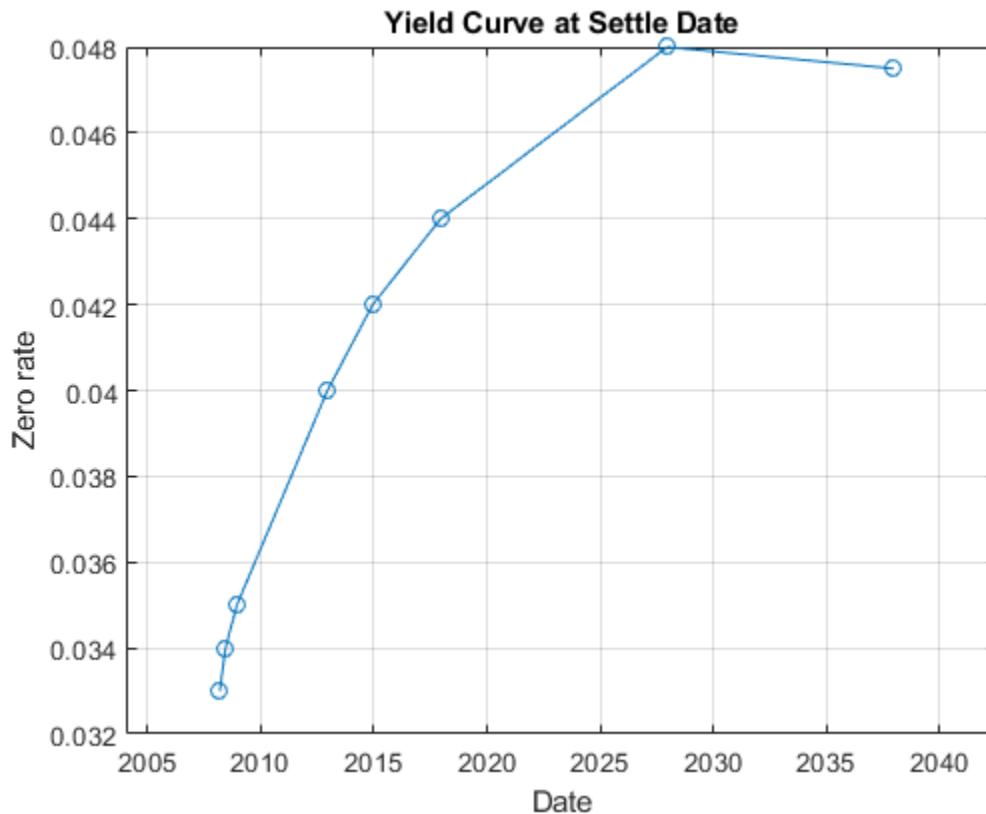
## Create RateSpec from the Interest-Rate Curve

```
Settle = datenum('14-Dec-2007');

Tenor = [3 6 12 5*12 7*12 10*12 20*12 30*12]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';

ZeroDates = datemnth(Settle,Tenor);
Compounding = 2;
Basis = 0;
RateSpec = intenvset('StartDates', Settle, 'EndDates', ZeroDates, ...
    'Rates', ZeroRates, 'Compounding', Compounding, 'Basis', Basis);

figure;
plot(ZeroDates, ZeroRates, 'o-');
xlabel('Date');
datetick('keeplimits');
ylabel('Zero rate');
grid on;
title('Yield Curve at Settle Date');
```



### Set Changeable Simulation Parameters

You can vary the number of simulated interest-rate scenarios that you generate. This example sets the simulation dates to be more frequent at first, then turning less frequent further in the future.

```
% Number of Monte Carlo simulations
numScenarios = 1000;

% Compute monthly simulation dates, then quarterly dates later.
simulationDates = datemnth(Settle,0:12);
simulationDates = [simulationDates datemnth(simulationDates(end),3:3:74)]';
numDates = numel(simulationDates);
```

### Compute Floating Reset Dates

For each simulation date, compute previous floating reset date for each swap.

```
floatDates = cfdates(Settle-360,swaps.Maturity,swaps.Period);
swaps.FloatingResetDates = zeros(numSwaps,numDates);
for i = numDates:-1:1
    thisDate = simulationDates(i);
    floatDates(floatDates > thisDate) = 0;
    swaps.FloatingResetDates(:,i) = max(floatDates,[],2);
end
```

### Setup Hull-White Single Factor Model

The risk factor that is simulated to value the contracts is the zero curve. For this example, you model the interest-rate term structure using the one-factor Hull-White model. This is a model of the short rate and is defined as:

$$dr = [\theta(t) - ar]dt + \sigma dz$$

where

- $dr$ : Change in the short rate after a small change in time,  $dt$
- $a$ : Mean reversion rate
- $\sigma$ : Volatility of the short rate
- $dz$ : A Weiner process (a standard normal process)
- $\theta(t)$ : Drift function defined as:

$$\theta(t) = F_t(0, t) + aF(0, t) + \frac{\sigma^2}{2a}(1 - e^{-2at})$$

$F(0, t)$ : Instantaneous forward rate at time  $t$

$F_t(0, t)$ : Partial derivative of  $F$  with respect to time

Once you have simulated a path of the short rate, generate a full yield curve at each simulation date using the formula:

$$R(t, T) = -\frac{1}{(T-t)}\ln A(t, T) + \frac{1}{(T-t)}B(t, T)r(t)$$

$$\ln A(t, T) = \ln \frac{P(0, T)}{P(0, t)} + B(t, T)F(0, t) - \frac{1}{4a^3}\sigma^2(e^{-aT} - e^{-at})^2(e^{2at} - 1)$$

$$B(t, T) = \frac{1 - e^{-a(T-t)}}{a}$$

$R(t, T)$ : Zero rate at time  $t$  for a period of  $T - t$

$P(t, T)$ : Price of a zero coupon bond at time  $t$  that pays one dollar at time  $T$

Each scenario contains the full term structure moving forward through time, modeled at each of our selected simulation dates.

Refer to the “Calibrating Hull-White Model Using Market Data” on page 2-111 example in the Financial Instruments Toolbox™ Users' Guide for more details on Hull-White one-factor model calibration.

```
Alpha = 0.2;
Sigma = 0.015;
```

```
hw1 = HullWhite1F(RateSpec, Alpha, Sigma);
```

### Simulate Scenarios

For each scenario, simulate the future interest-rate curve at each valuation date using the Hull-White one-factor interest-rate model.

```
% Use reproducible random number generator (vary the seed to produce
% different random scenarios).
```

```
prevRNG = rng(0, 'twister');
```

```
dt = diff(yearfrac(Settle, simulationDates, 1));
```

```
nPeriods = numel(dt);
```

```
scenarios = hw1.simTermStructs(nPeriods, ...
```

```
    'nTrials', numScenarios, ...
```

```
    'deltaTime', dt);
```

```
% Restore random number generator state
```

```
rng(prevRNG);
```

```
% Compute the discount factors through each realized interest rate
```

```
% scenario.
```

```
dfactors = ones(numDates, numScenarios);
```

```
for i = 2:numDates
```

```
    tenorDates = datemnth(simulationDates(i-1), Tenor);
```

```
    rateAtNextSimDate = interp1(tenorDates, squeeze(scenarios(i-1, :, :)), ...
        simulationDates(i), 'linear', 'extrap');
```

```

% Compute D(t1,t2)
dfactors(i,:) = zero2disc(rateAtNextSimDate, ...
    repmat(simulationDates(i),1,numScenarios),simulationDates(i-1),-1,3);
end
dfactors = cumprod(dfactors,1);

```

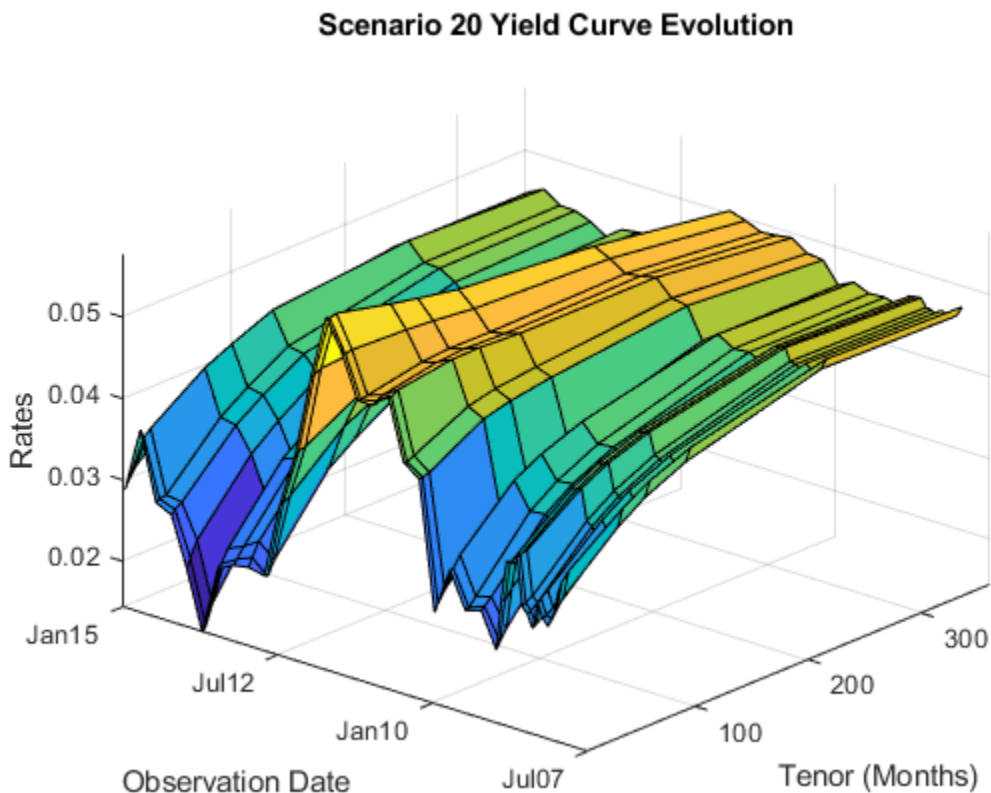
### Inspect a Scenario

Create a surface plot of the yield curve evolution for a particular scenario.

```

i = 20;
figure;
surf(Tenor, simulationDates, scenarios(:, :, i))
axis tight
datetick('y', 'mmyy');
xlabel('Tenor (Months)');
ylabel('Observation Date');
zlabel('Rates');
ax = gca;
ax.View = [-49 32];
title(sprintf('Scenario %d Yield Curve Evolution\n', i));

```



### Compute Mark to Market Swap Prices

For each scenario the swap portfolio is priced at each future simulation date. Prices are computed using a price approximation function, `hswapapprox`. It is common in CVA applications to use simplified approximation functions when pricing contracts due to the performance requirements of these Monte Carlo simulations.

Since the simulation dates do not correspond to the swaps cash flow dates (where the floating rates are reset) estimate the latest floating rate with the 1-year rate (all swaps have period 1 year) interpolated between the nearest simulated rate curves.

The swap prices are then aggregated into a "cube" of values which contains all future contract values at each simulation date for each scenario. The resulting cube of contract



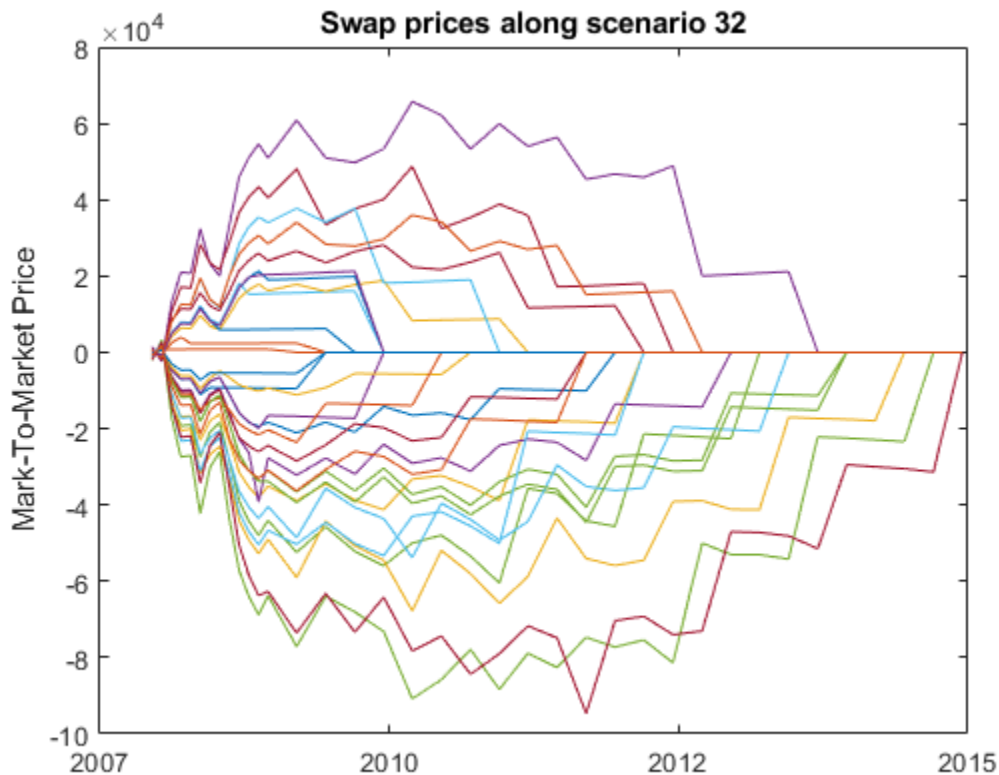
prices is a 3-dimensional matrix where each row represents a simulation date, each column a contract, and each "page" a different simulated scenario.

```
% Compute all mark-to-market values for this scenario. Use an  
% approximation function here to improve performance.  
values = hcomputeMTMValues(swaps,simulationDates,scenarios,Tenor);
```

### **Inspect Scenario Prices**

Create a plot of the evolution of all swap prices for a particular scenario.

```
i = 32;  
figure;  
plot(simulationDates, values(:, :, i));  
datetick;  
ylabel('Mark-To-Market Price');  
title(sprintf('Swap prices along scenario %d', i));
```

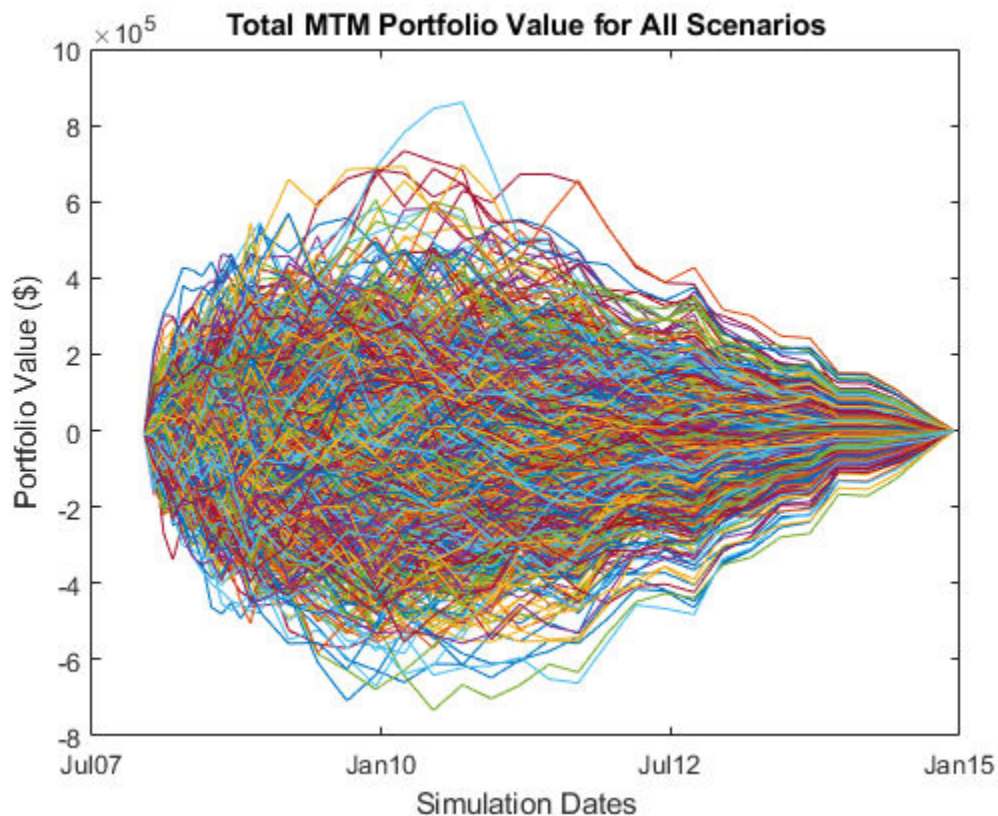


### Visualize Simulated Portfolio Values

Plot the total portfolio value for each scenario of the simulation. As each scenario moves forward in time, the values of the contracts move up or down depending on how the modeled interest-rate term structure changes. As the swaps get closer to maturity, their values will begin to approach zero since the aggregate value of all remaining cash flows will decrease after each cash flow date.

```
% View portfolio value over time
figure;
totalPortValues = squeeze(sum(values, 2));
plot(simulationDates, totalPortValues);
title('Total MTM Portfolio Value for All Scenarios');
datetick('x', 'mmmyy')
```

```
ylabel('Portfolio Value ($)')
xlabel('Simulation Dates')
```



### Compute Exposure by Counterparty

The exposure of a particular contract ( $i$ ) at time  $t$  is the maximum of the contract value ( $V_i$ ) and  $\theta$ :

$$E_i(t) = \max\{V_i(t), 0\}$$

And the exposure for a particular counterparty is simply a sum of the individual contract exposures:

$$E_{cp}(t) = \sum E_i(t) = \sum \max\{V_i(t), 0\}$$

In the presence of netting agreements, however, contracts are aggregated together and can offset each other. Therefore the total exposure of all contracts in a netting agreement is:

$$E_{na}(t) = \max\{\sum V_i(t), 0\}$$

Compute these exposures for the entire portfolio as well as each counterparty at each simulation date using the `creditexposures` function.

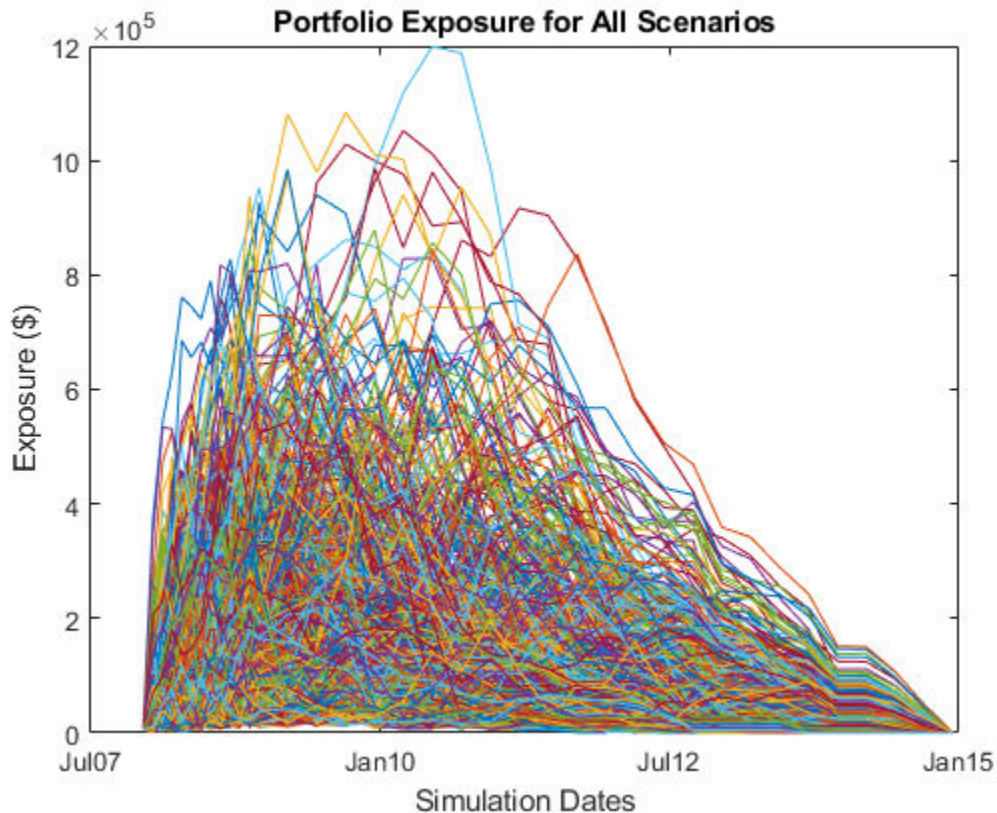
Unnetted contracts are indicated using a NaN in the `NettingID` vector. Exposure of an unnetted contract is equal to the market value of the contract if it has positive value, otherwise it is zero.

Contracts included in a netting agreement have their values aggregated together and can offset each other. See the references for more details on computing exposure from mark-to-market contract values.

```
[exposures, expcpty] = creditexposures(values,swaps.CounterpartyID, ...  
    'NettingID',swaps.NettingID);
```

Plot the total portfolio exposure for each scenario in our simulation. Similar to the plot of contract values, the exposures for each scenario will approach zero as the swaps mature.

```
% View portfolio exposure over time  
figure;  
totalPortExposure = squeeze(sum(exposures,2));  
plot(simulationDates,totalPortExposure);  
title('Portfolio Exposure for All Scenarios');  
datetick('x','mmyy')  
ylabel('Exposure ($)')  
xlabel('Simulation Dates')
```



### Exposure Profiles

Several exposure profiles are useful when analyzing the potential future exposure of a bank to a counterparty. Here you can compute several (non-discounted) exposure profiles per counterparty, as well as, for the entire portfolio.

- PFE (Potential Future Exposure): A high percentile (95%) of the distribution of exposures at any particular future date (also called Peak Exposure (PE))
- MPFE (Maximum Potential Future Exposure): The maximum PFE across all dates
- EE : (Expected Exposure): The mean (average) of the distribution of exposures at each date

- EPE (Expected Positive Exposure): Weighted average over time of the expected exposure
- EffEE (Effective Expected Exposure): The maximum expected exposure at any time,  $t$ , or previous time
- EffEPE (Effective Expected Positive Exposure): The weighted average of the effective expected exposure

For further definitions, see for example the Basel II document in references.

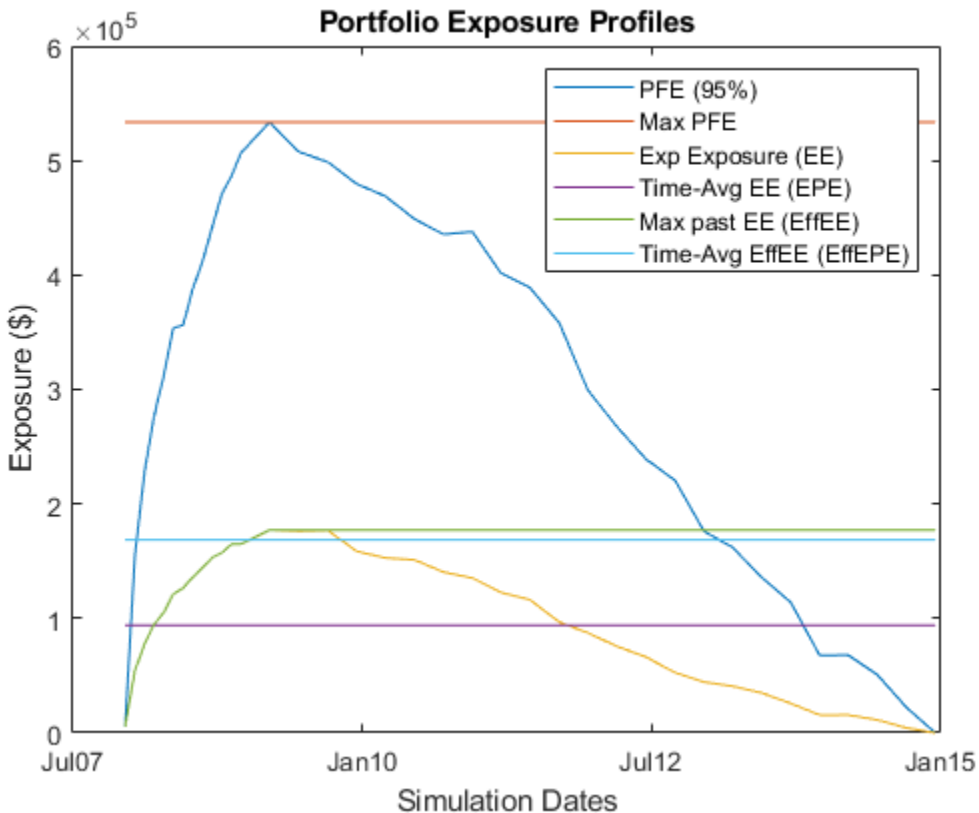
```
% Compute entire portfolio exposure
portExposures = sum(exposures,2);
```

```
% Compute exposure profiles for each counterparty and entire portfolio
cpProfiles = exposureprofiles(simulationDates,exposures);
portProfiles = exposureprofiles(simulationDates,portExposures);
```

Visualize the exposure profiles, first for the entire portfolio, then for a particular counterparty.

```
% Visualize portfolio exposure profiles
figure;
plot(simulationDates,portProfiles.PFE, ...
      simulationDates,portProfiles.MPFE * ones(numDates,1), ...
      simulationDates,portProfiles.EE, ...
      simulationDates,portProfiles.EPE * ones(numDates,1), ...
      simulationDates,portProfiles.EffEE, ...
      simulationDates,portProfiles.EffEPE * ones(numDates,1));
legend({'PFE (95%)', 'Max PFE', 'Exp Exposure (EE)', 'Time-Avg EE (EPE)', ...
       'Max past EE (EffEE)', 'Time-Avg EffEE (EffEPE)'});

datetick('x','mmyy')
title('Portfolio Exposure Profiles');
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```



Visualize exposure profiles for a particular counterparty.

```

cpIdx = find(expcpty == 5);
figure;
plot(simulationDates,cpProfiles(cpIdx).PFE, ...
     simulationDates,cpProfiles(cpIdx).MPFE * ones(numDates,1), ...
     simulationDates,cpProfiles(cpIdx).EE, ...
     simulationDates,cpProfiles(cpIdx).EPE * ones(numDates,1), ...
     simulationDates,cpProfiles(cpIdx).EffEE, ...
     simulationDates,cpProfiles(cpIdx).EffEPE * ones(numDates,1));
legend({'PFE (95%)','Max PFE','Exp Exposure (EE)','Time-Avg EE (EPE)', ...
       'Max past EE (EffEE)','Time-Avg EffEE (EffEPE)'});

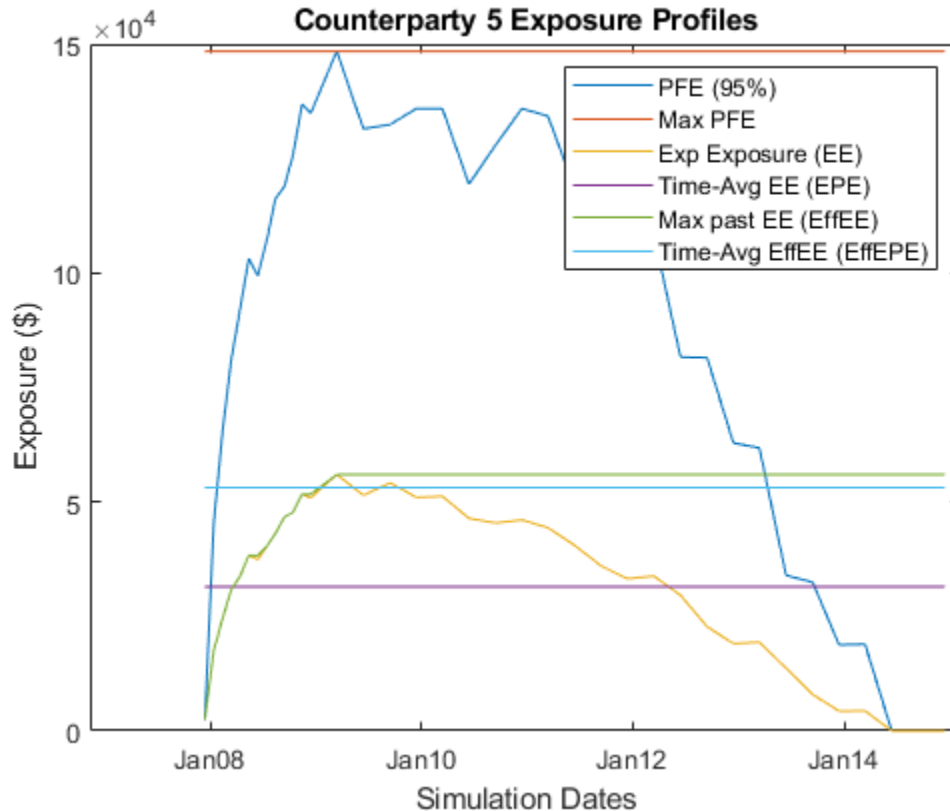
datetick('x','mmyy','keeplimits')

```

```

title(sprintf('Counterparty %d Exposure Profiles', cpIdx));
ylabel('Exposure ($)')
xlabel('Simulation Dates')

```



### Discounted Exposures

Compute the discounted expected exposures using the discount factors from each simulated interest-rate scenario. The discount factor for a given valuation date in a given scenario is the product of the incremental discount factors from one simulation date to the next, along with the interest-rate path of that scenario.

```

% Get discounted exposures per counterparty, for each scenario
discExp = zeros(size(exposures));
for i = 1:numScenarios

```



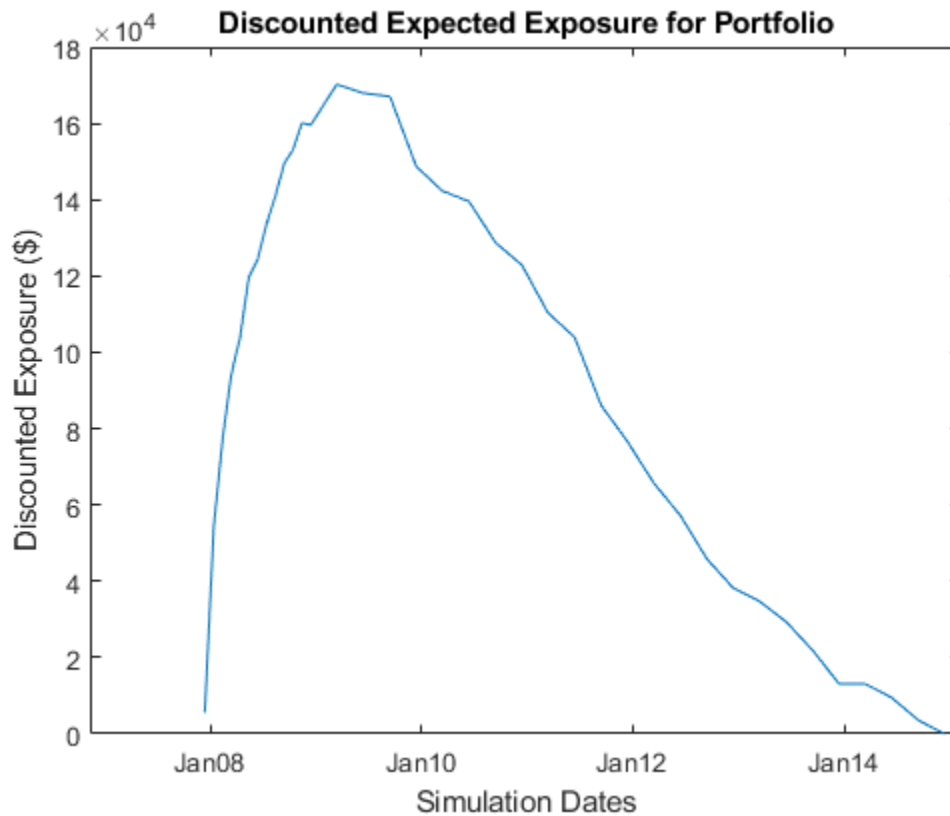
```
    discExp(:, :, i) = bsxfun(@times, dfactors(:, i), exposures(:, :, i));  
end
```

```
% Discounted expected exposure  
discProfiles = exposureprofiles(simulationDates, discExp, ...  
    'ProfileSpec', 'EE');
```

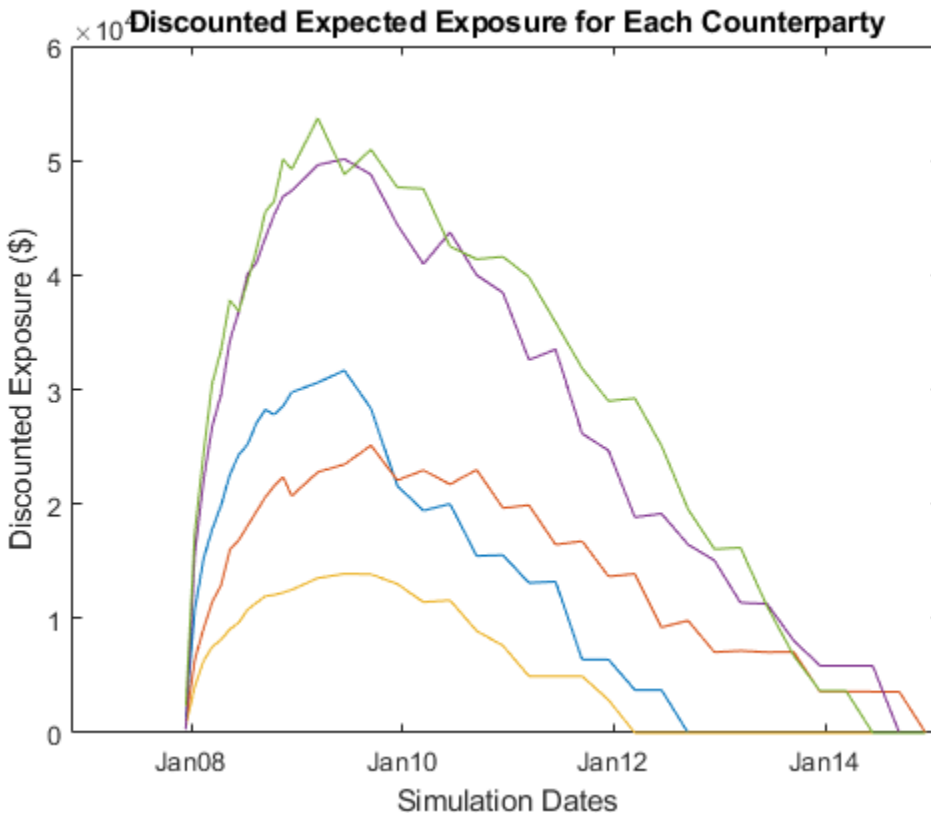
Plot the discounted expected exposures for the aggregate portfolio as well as for each counterparty.

```
% Aggregate the discounted EE for each counterparty into a matrix  
discEE = [discProfiles.EE];
```

```
% Portfolio discounted EE  
figure;  
plot(simulationDates, sum(discEE, 2))  
datetick('x', 'mmmy', 'keeplimits')  
title('Discounted Expected Exposure for Portfolio');  
ylabel('Discounted Exposure ($)')  
xlabel('Simulation Dates')
```



```
% Counterparty discounted EE
figure;
plot(simulationDates,discEE)
datetick('x','mmyy','keeplimits')
title('Discounted Expected Exposure for Each Counterparty');
ylabel('Discounted Exposure ($)')
xlabel('Simulation Dates')
```



### Calibrating Probability of Default Curve for Each Counterparty

The default probability for a given counterparty is implied by the current market spreads of the counterparty's CDS. Use the function `cdsbootstrap` to generate the cumulative probability of default at each simulation date.

```
% Import CDS market information for each counterparty
CDS = readtable(swapFile, 'Sheet', 'CDS Spreads');
disp(CDS);
```

Date	cp1	cp2	cp3	cp4	cp5
'3/20/2008'	140	85	115	170	140

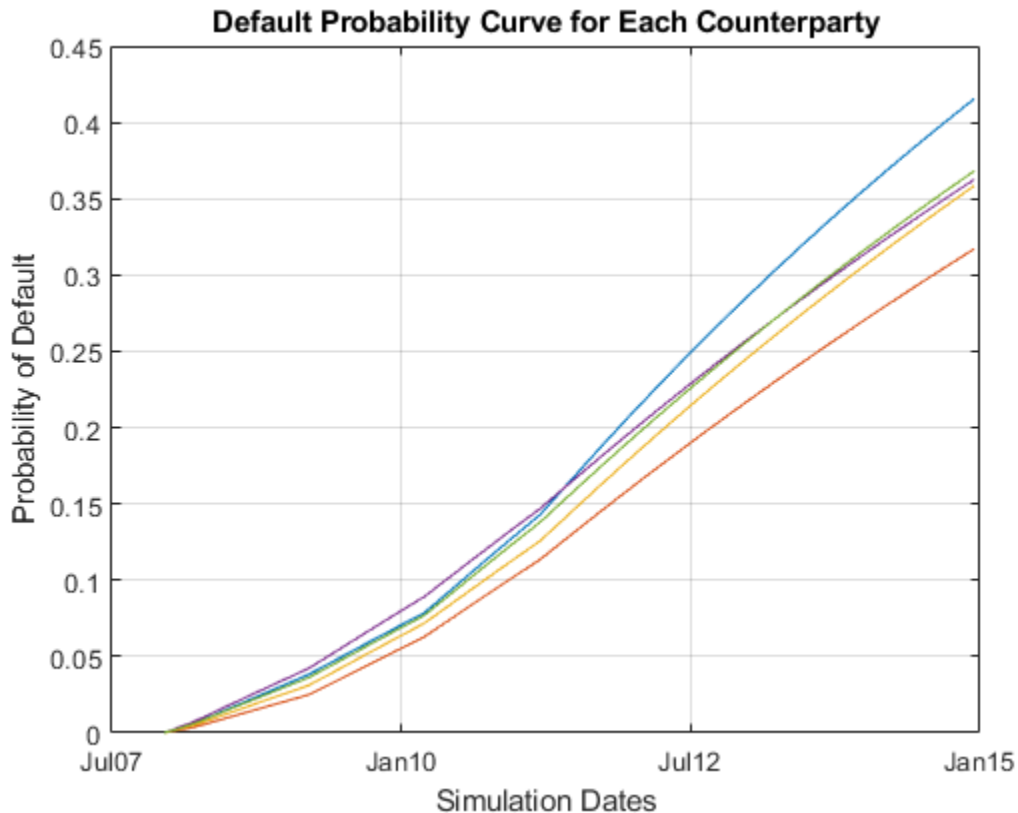
'3/20/2009'	185	120	150	205	175
'3/20/2010'	215	170	195	245	210
'3/20/2011'	275	215	240	285	265
'3/20/2012'	340	255	290	320	310

```
CSDSDates = datenum(CDS.Date);
CDSspreads = table2array(CDS(:,2:end));

ZeroData = [RateSpec.EndDates RateSpec.Rates];

% Calibrate default probabilities for each counterparty
DefProb = zeros(length(simulationDates), size(CDSSpreads,2));
for i = 1:size(DefProb,2)
    probData = cdsbootstrap(ZeroData, [CSDSDates CDSSpreads(:,i)], ...
        Settle, 'probDates', simulationDates);
    DefProb(:,i) = probData(:,2);
end

% Plot of the cumulative probability of default for each counterparty.
figure;
plot(simulationDates,DefProb)
title('Default Probability Curve for Each Counterparty');
xlabel('Date');
grid on;
ylabel('Cumulative Probability')
datetick('x','mmyy')
ylabel('Probability of Default')
xlabel('Simulation Dates')
```



### CVA Computation

The Credit Value (Valuation) Adjustment (CVA) formula is:

$$CVA = (1 - R) \int_0^T \text{discEE}(t) dPD(t)$$

Where  $R$  is the recovery,  $\text{discEE}$  the discounted expected exposure at time  $t$ , and  $PD$  the default probability distribution. This assumes the exposure is independent of default (no wrong-way risk), and it also assumes that the exposures were obtained using risk-neutral probabilities.

Approximate the integral with a finite sum over the valuation dates as:

$$CVA(\text{approx}) = (1 - R) \sum_{i=2}^n \text{discEE}(t_i)(PD(t_i) - PD(t_{i-1}))$$

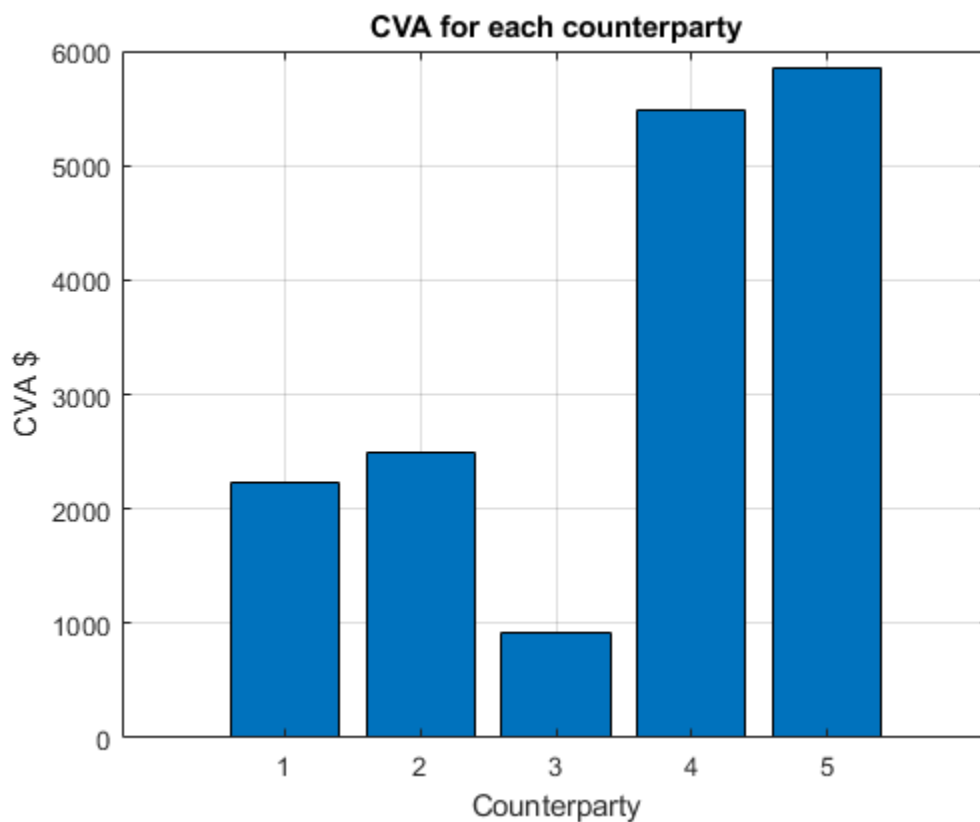
where  $t_1$  is today's date,  $t_2, \dots, t_n$  the future valuation dates.

Assume that the CDS information corresponds to the counterparty with index  $cpIdx$ . The computed CVA is the present market value of our credit exposure to counterparty  $cpIdx$ . For this example, set the recovery rate at 40%.

```
Recovery = 0.4;
CVA = (1-Recovery) * sum(discEE(2:end,:) .* diff(DefProb));
for i = 1:numel(CVA)
    fprintf('CVA for counterparty %d = $%.2f\n',i,CVA(i));
end
```

```
CVA for counterparty 1 = $2228.36
CVA for counterparty 2 = $2487.60
CVA for counterparty 3 = $920.39
CVA for counterparty 4 = $5478.50
CVA for counterparty 5 = $5859.30
```

```
figure;
bar(CVA);
title('CVA for each counterparty');
xlabel('Counterparty');
ylabel('CVA $');
grid on;
```



### References

- 1 Pykhtin, Michael, and Steven Zhu, *A Guide to Modeling Counterparty Credit Risk*, GARP, July/August 2007, issue 37, pp. 16-22.
- 2 Pykhtin, Michael, and Dan Rosen, *Pricing Counterparty Risk at the Trade Level and CVA*, 2010.
- 3 Basel II: <https://www.bis.org/publ/bcbs128.pdf> page 256.

### See Also

cdsbootstrap | cdsprice | cdsrpv01 | cdsspread

### **Related Examples**

- “First-to-Default Swaps” on page 8-25
- “Credit Default Swap Option” on page 8-37

### **External Websites**

- Pricing and Valuation of Credit Default Swaps (4 min 22 sec)



## First-to-Default Swaps

This example shows how to price first-to-default (FTD) swaps under the homogeneous loss assumption.

A first-to-default swap is an instrument that pays a predetermined amount when (and if) the first of a basket of credit instruments defaults. The credit instruments in the basket are usually bonds. If we assume that the loss amount following a credit event is the same for all credits in the basket, we are under the *homogeneous loss* assumption. This assumption makes models simpler, because any default in the basket triggers the same payment amount. This example is an implementation of the pricing methodology for these instruments, as described in O'Kane [2]. There are two steps in the methodology: a) Compute the survival probability for the basket numerically; b) Use this survival curve and standard single-name credit-default swap (CDS) functionality to find FTD spreads and to price existing FTD swaps.

### Fit Probability Curves to Market Data

Given CDS market quotes for each issuer in the basket, use `cdsbootstrap` to calibrate individual default probability curves for each issuer.

```
% Interest rate curve
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];

% CDS spreads
% Each row in MarketSpreads corresponds to a different issuer; each
% column to a different maturity date (corresponding to MarketDates)
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
'20-Sep-16'});
MarketSpreads = [
    160 195 230 285 330;
    130 165 205 260 305;
    150 180 210 260 300;
    165 200 225 275 295];
% Number of issuers equals number of rows in MarketSpreads
nIssuers = size(MarketSpreads,1);

% Settlement date
Settle = datenum('17-Jul-2009');
```

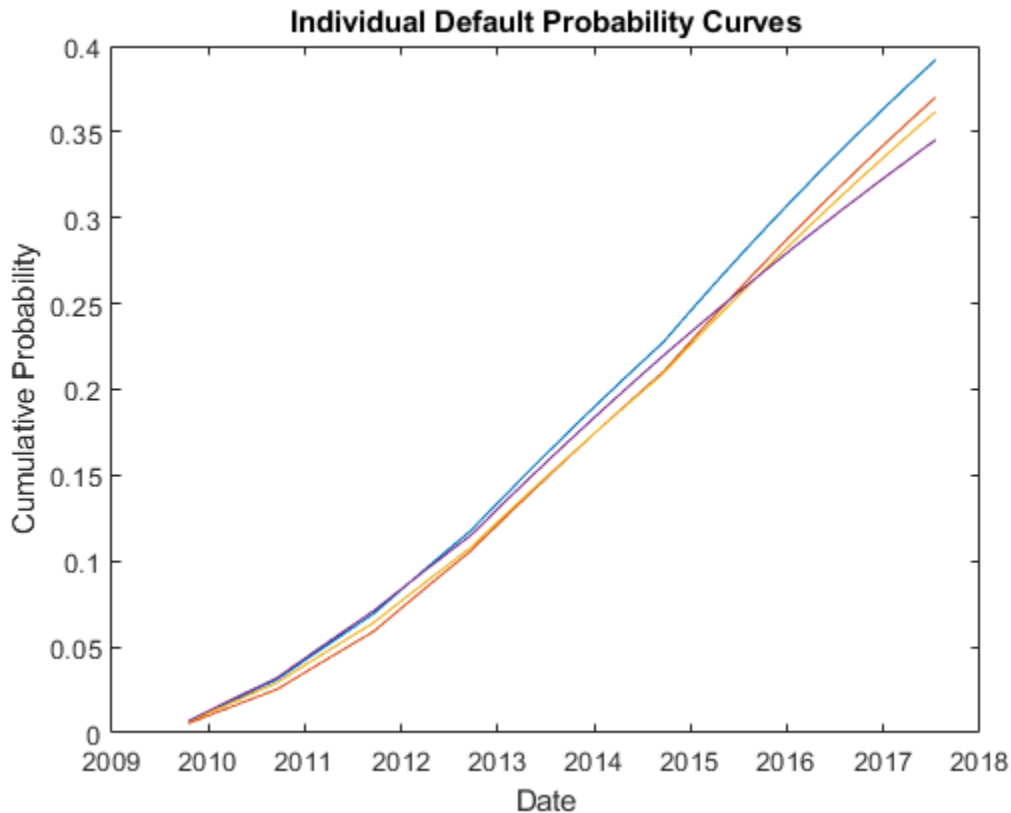
In practice, the time axis is discretized and the FTD survival curve is only evaluated at grid points. We use one point every three months. To request that `cdsbootstrap` returns default probability values over the specific grid points we want, use the optional argument `'ProbDates'`. We add the original standard CDS market dates to the grid, otherwise the default probability information on those dates would be interpolated using the two closest dates on the grid, and the prices on market dates would be inconsistent with the original market data.

```
ProbDates = union(MarketDates,daysadd(Settle,360*(0.25:0.25:8),1));
nProbDates = length(ProbDates);
DefProb = zeros(nIssuers,nProbDates);

for ii = 1:nIssuers
    MarketData = [MarketDates MarketSpreads(ii,:)'];
    ProbData = cdsbootstrap(ZeroData,MarketData,Settle,...
        'ProbDates',ProbDates);
    DefProb(ii,:) = ProbData(:,2)';
end
```

These are the calibrated default probability curves for each credit in the basket.

```
figure
plot(ProbDates',DefProb)
datetick
title('Individual Default Probability Curves')
ylabel('Cumulative Probability')
xlabel('Date')
```



### Determine Latent Variable Thresholds

Latent variables are used in different credit risk contexts, with different interpretations. In some contexts, a latent variable is a proxy for a *change in the value of assets*, and the domain of this variable is binned, with each bin corresponding to a credit rating. The bins limits, or thresholds, are determined from credit migration matrices. In our context, the latent variable is associated to a *time to default*, and the thresholds determine bins in a discretized time grid where defaults may occur.

Formally, if the time to default of a particular issuer is denoted by  $\tau$ , and we know its default probability function  $P(t)$ , a latent variable  $A$  and corresponding thresholds  $C(t)$  satisfy

$$Pr(\tau \leq t) = P(t) = Pr(A \leq C(t))$$

or

$$Pr(s \leq \tau \leq t) = P(t) - P(s) = Pr(C(s) \leq A \leq C(t))$$

These relationships make latent variable approaches convenient for both simulations and analytical derivations. Both  $P(t)$  and  $C(t)$  are functions of time.

The choice of a distribution for the variable  $A$  determines the thresholds  $C(t)$ . In the standard latent variable model, the variable  $A$  is chosen to follow a standard normal distribution, from which

$$C(t) = \Phi^{-1}(P(t))$$

where  $\Phi$  is the cumulative standard normal distribution.

Use the previous formula to determine the *default-time thresholds*, or simply *default thresholds*, corresponding to the default probabilities previously obtained for the credits in the basket.

```
DefThresh = norminv(DefProb);
```

### Derive Survival Curve for the Basket

Following O'Kane [2], we use a one-factor latent variable model to derive expressions for the survival probability function of the basket.

Given parameters  $\beta_i$  for each issuer  $i$ , and given independent standard normal variables  $Z$  and  $\epsilon_i$ , the one-factor latent variable model assumes that the latent variable  $A_i$  associated to issuer  $i$  satisfies

$$A_i = \beta_i * Z + \sqrt{1 - \beta_i^2} * \epsilon_i$$

This induces a correlation between issuers  $i$  and  $j$  of  $\beta_i \beta_j$ . All latent variables  $A_i$  share the common factor  $Z$  as a source of uncertainty, but each latent variable also has an idiosyncratic source of uncertainty  $\epsilon_i$ . The larger the coefficient  $\beta_i$ , the more the latent variable resembles the common factor  $Z$ .

Using the latent variable model, we derive an analytic formula for the survival probability of the basket.

The probability that issuer  $i$  survives past time  $t_j$ , in other words, that its default time  $\tau_i$  is greater than  $t_j$  is

$$Pr(\tau_i > t_j) = 1 - Pr(A_i \leq C_i(t_j))$$

where  $C_i(t_j)$  is the default threshold computed above for issuer  $i$ , for the  $j$ -th date in the discretization grid.

Conditional on the value of the one-factor  $Z$ , the probability that all issuers survive past time  $t_j$  is

$$Pr(\text{No defaults by time } t_j | Z) = Pr(\tau_i > t_j \text{ for all } i | Z) = \prod_i [1 - Pr(A_i \leq C_i(t_j) | Z)]$$

where the product is justified because all the  $\epsilon_i$ 's are independent. Therefore, conditional on  $Z$ , the  $A_i$ 's are independent.

The unconditional probability of no defaults by time  $t_j$  is the integral over all values of  $Z$  of the previous conditional probability

$$Pr(\text{No defaults by time } t_j) = \int_Z \prod_i [1 - Pr(A_i \leq C_i(t_j) | Z)] \phi(Z) dZ$$

with  $\phi(Z)$  the standard normal density.

By evaluating this one-dimensional integral for each point  $t_j$  in the grid, we get a discretization of the survival curve for the whole basket, which is the FTD survival curve.

The latent variable model can also be used to simulate default times, which is the back engine of many pricing methodologies for credit instruments. Loeffler and Posch [1], for example, estimate the survival probability of a basket via simulation. In each simulated scenario a time to default is determined for each issuer. With some bookkeeping, the probability of having the first default on each bucket of the grid can be estimated from the simulation. The simulation approach is also discussed in O'Kane [2]. Simulation is very flexible and applicable to many credit instruments. However, analytic approaches are

preferred, when available, because they are much faster and more accurate than simulation.

To compute the FTD survival probabilities in our example, we set all betas to the square root of a target correlation. Then we loop over all dates in the time grid to compute the one dimensional integral that gives the survival probability of the basket.

Regarding implementation, the conditional survival probability as a function of a scalar  $Z$  would be

```
condProb=@(Z)prod(normcdf((-DefThresh(:,jj)+beta*Z)./sqrt(1-beta.^2)));
```

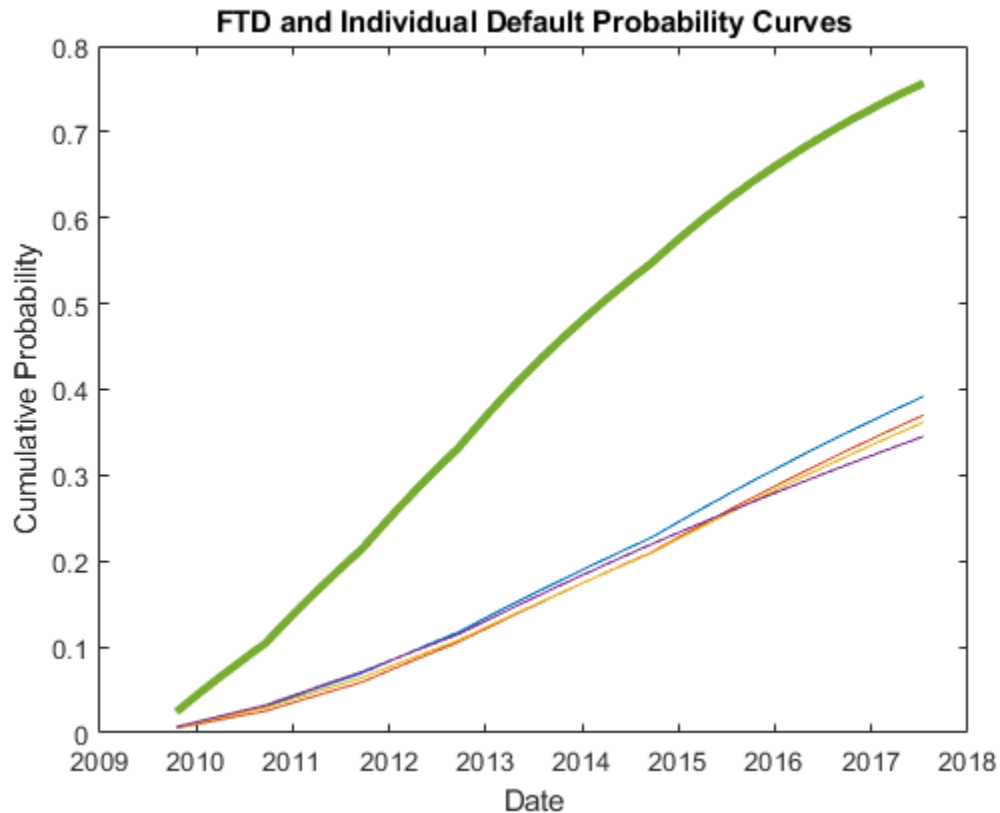
However, the integration function we use requires that the function handle of the integrand accepts vectors. Although a loop around the scalar version of the conditional probability would work, it is far more efficient to vectorize the conditional probability using `bsxfun`.

```
beta = sqrt(0.25)*ones(nIssuers,1);

FTDSurvProb = zeros(size(ProbDates));
for jj = 1:nProbDates
    % vectorized conditional probability as a function of Z
    vecCondProb = @(Z)prod(normcdf(bsxfun(@rdivide,...
        - repmat(DefThresh(:,jj),1,length(Z))+bsxfun(@times,beta,Z),...
        sqrt(1-beta.^2))));
    % truncate domain of normal distribution to [-5,5] interval
    FTDSurvProb(jj) = integral(@(Z)vecCondProb(Z).*normpdf(Z),-5,5);
end
FTDDefProb = 1-FTDSurvProb;
```

Compare the FTD probability to the default probabilities of the individual issuers.

```
figure
plot(ProbDates',DefProb)
datetick
hold on
plot(ProbDates,FTDDefProb,'LineWidth',3)
datetick
hold off
title('FTD and Individual Default Probability Curves')
ylabel('Cumulative Probability')
xlabel('Date')
```



### Find FTD Spreads and Price Existing FTD Swaps

Under the assumption that all instruments in the basket have the same recovery rate, or homogeneous loss assumption (see O'Kane in References), we get the spread for the FTD swap using the `cdspread` function, but passing the FTD probability data just computed.

```
Maturity = MarketDates;
ProbDataFTD = [ProbDates, FTDDefProb];
FTDSpread = cdspread(ZeroData, ProbDataFTD, Settle, Maturity);
```

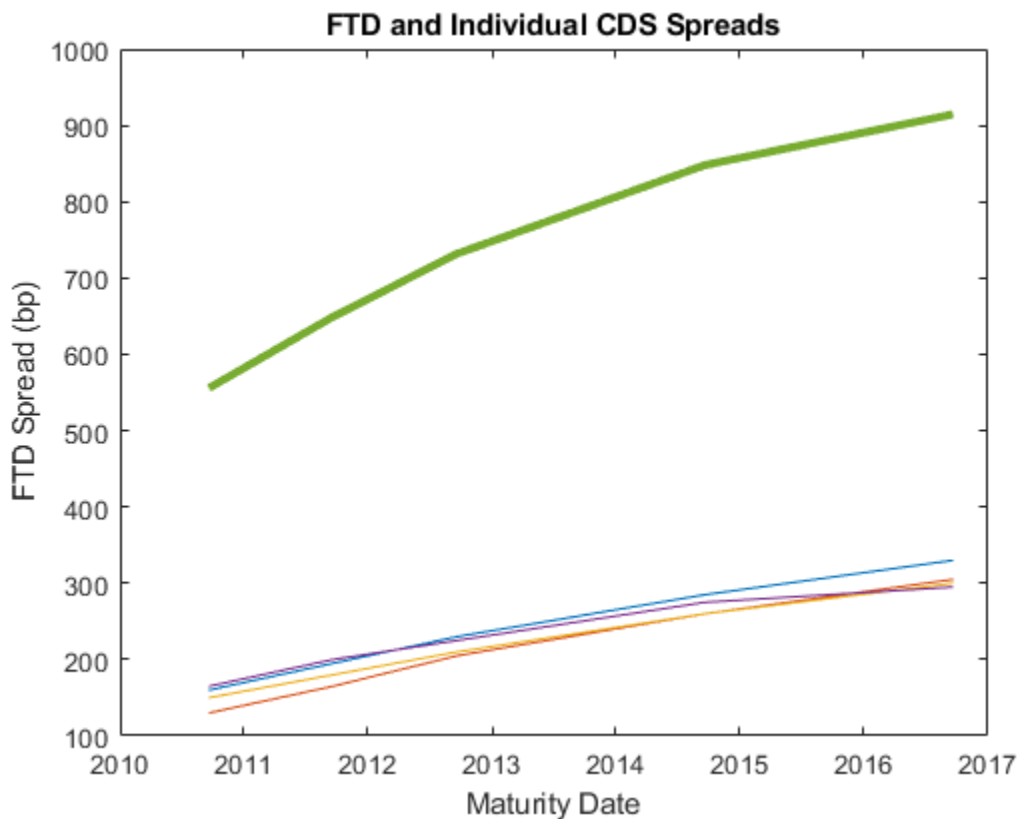
Compare the FTD spreads with the individual spreads.

```
figure
plot(MarketDates, MarketSpreads')
```

```

datetick
hold on
plot(MarketDates,FTDSpread,'LineWidth',3)
hold off
title('FTD and Individual CDS Spreads')
ylabel('FTD Spread (bp)')
xlabel('Maturity Date')

```



An existing FTD swap can be priced with `cdsprice`, using the same FTD probability.

```

Maturity0 = MarketDates(1); % Assume maturity on nearest market date
Spread0 = 540; % Spread of existing FTD contract
% Assume default values of recovery and notional

```



```
FTDPrice = cdsprice(ZeroData,ProbDataFTD,Settle,Maturity0,Spread0);
fprintf('Price of existing FTD contract: %g\n',FTDPrice)
```

```
Price of existing FTD contract: 17644.7
```

### Analyze Sensitivity to Correlation

To illustrate the sensitivity of the FTD spreads to model parameters, we calculate the market spreads for a range of correlation values.

```
corr = [0 0.01 0.10 0.25 0.5 0.75 0.90 0.99 1];
FTDSpreadByCorr = zeros(length(Maturity),length(corr));
FTDSpreadByCorr(:,1) = sum(MarketSpreads)';
FTDSpreadByCorr(:,end) = max(MarketSpreads)';

for ii = 2:length(corr)-1
    beta = sqrt(corr(ii))*ones(nIssuers,1);
    FTDSurvProb = zeros(length(ProbDates));
    for jj = 1:nProbDates
        % vectorized conditional probability as a function of Z
        condProb = @(Z)prod(normcdf(bsxfun(@divide,...
            - repmat(DefThresh(:,jj),1,length(Z))+bsxfun(@times,beta,Z),...
            sqrt(1-beta.^2)))));
        % truncate domain of normal distribution to [-5,5] interval
        FTDSurvProb(jj) = integral(@(Z)condProb(Z).*normpdf(Z),-5,5);
    end
    FTDDefProb = 1-FTDSurvProb;
    ProbDataFTD = [ProbDates, FTDDefProb];
    FTDSpreadByCorr(:,ii) = cdsspread(ZeroData,ProbDataFTD,Settle,Maturity);
end
```

The FTD spreads lie in a band between the sum and the maximum of individual spreads. As the correlation increases to one, the FTD spreads decrease towards the maximum of the individual spreads in the basket (all credits default together). As the correlation decreases to zero, the FTD spreads approach the sum of the individual spreads (independent credits).

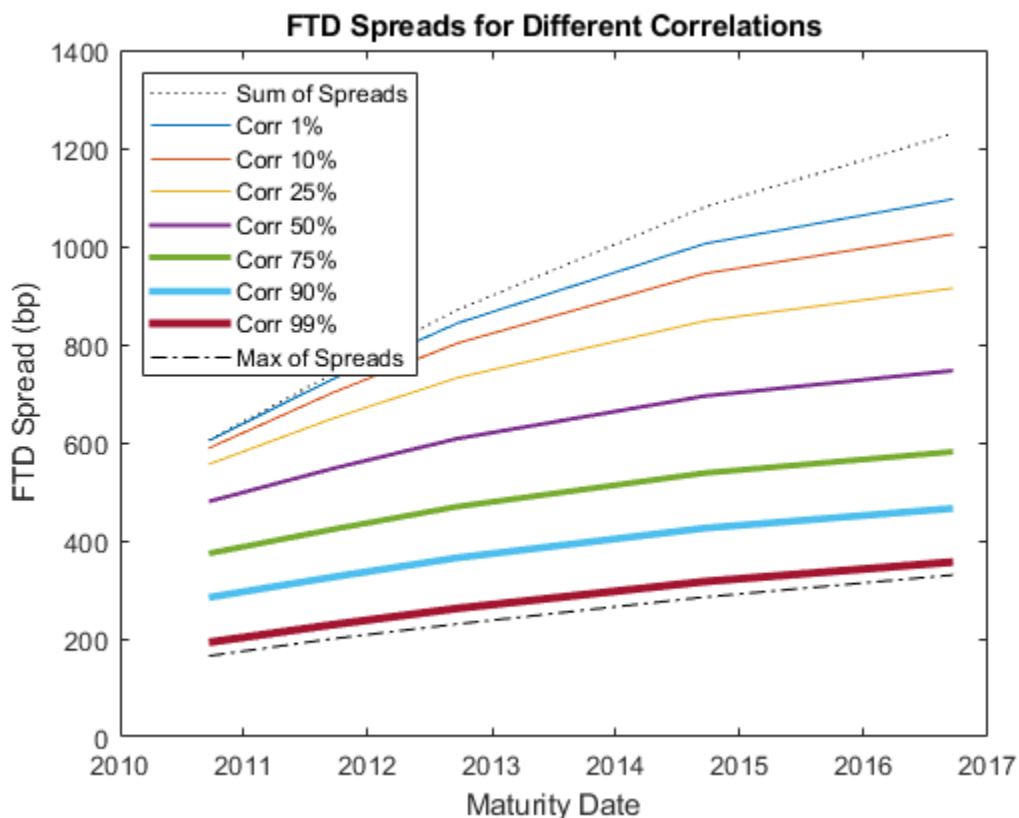
```
figure
legends = cell(1,length(corr));
plot(MarketDates,FTDSpreadByCorr(:,1),'k:');
legends{1} = 'Sum of Spreads';
datetick
hold on
for ii = 2:length(corr)-1
    plot(MarketDates,FTDSpreadByCorr(:,ii),'LineWidth',3*corr(ii))
```

```

legends{ii} = ['Corr ' num2str(corr(ii)*100) '%'];
end
plot(MarketDates,FTDSpreadByCorr(:,end),'k-.')
legends{end} = 'Max of Spreads';

hold off
title('FTD Spreads for Different Correlations')
ylabel('FTD Spread (bp)')
xlabel('Maturity Date')
legend(legends,'Location','NW')

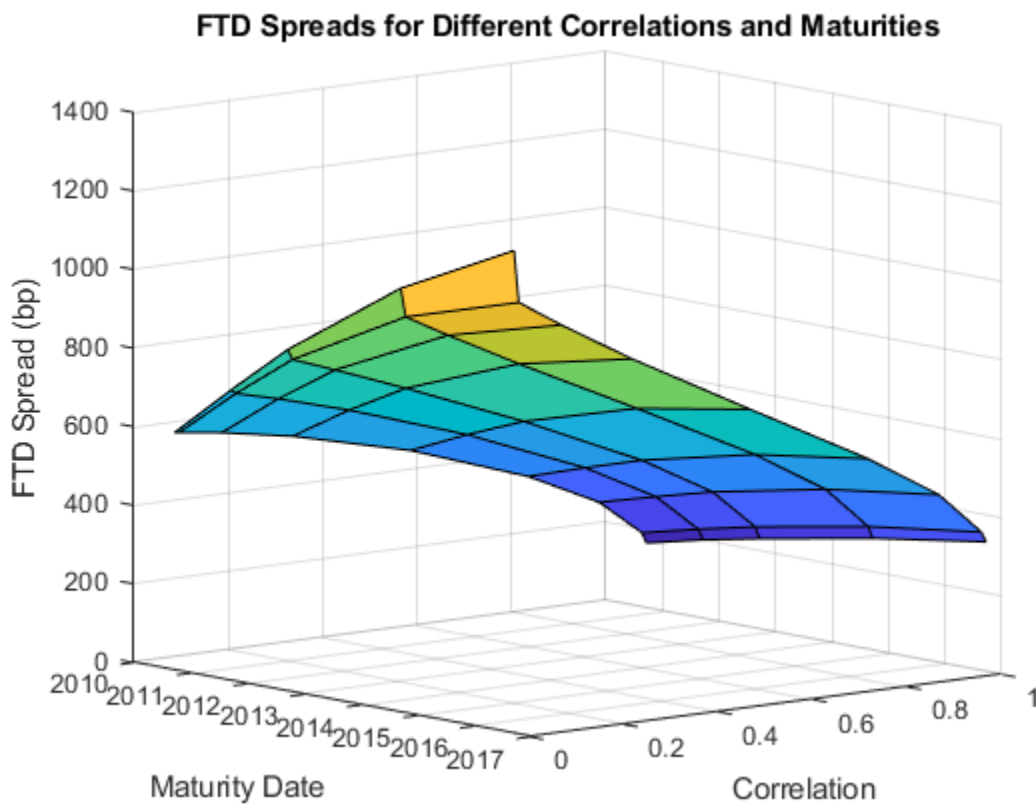
```



For short maturities and small correlations, the basket is effectively independent (the FTD spread is very close to the sum of individual spreads). The correlation effect becomes more significant for longer maturities.

Here is an alternative visualization of the dependency of FTD spreads on correlation.

```
figure
surf(corr,MarketDates,FTDSpreadByCorr)
datetick('y')
ax = gca;
ax.YDir = 'reverse';
view(-40,10)
title('FTD Spreads for Different Correlations and Maturities')
xlabel('Correlation')
ylabel('Maturity Date')
zlabel('FTD Spread (bp)')
```



### References

[1] Loeffler, Gunter and Peter Posch, *Credit risk modeling using Excel and VBA*, Wiley Finance, 2007.

[2] O'Kane, Dominic, *Modelling single-name and multi-name Credit Derivatives*, Wiley Finance, 2008.

### See Also

`cdsbootstrap` | `cdsprice` | `cdsrpv01` | `cdsspread`

### Related Examples

- “Credit Default Swap Option” on page 8-37

### External Websites

- Pricing and Valuation of Credit Default Swaps (4 min 22 sec)

## Credit Default Swap Option

A credit default swap (CDS) option, or credit default swaption, is a contract that provides the holder with the right, but not the obligation, to enter into a credit default swap in the future. CDS options can either be payer swaptions or receiver swaptions. If a payer swaption, the option holder has the right to enter into a CDS where they pay premiums; and, if a receiver swaption, the option holder receives premiums. Financial Instruments Toolbox software provides `cdsoptprice` for pricing payer and receiver credit default swaptions. Also, with some additional steps, `cdsoptprice` can be used for pricing multi-name CDS index options.

### References

O'Kane, D., *Modelling Single-name and Multi-name Credit Derivatives*, Wiley, 2008.

### See Also

`cdsoptprice` | `cdsrpv01` | `cdsspread`

### Related Examples

- “Pricing a Single-Name CDS Option” on page 8-38
- “Pricing a CDS Index Option” on page 8-40
- “Credit Default Swap (CDS)” (Financial Toolbox)

## Pricing a Single-Name CDS Option

This example shows how to price a single-name CDS option using `cdsoptprice`. The function `cdsoptprice` is based on the Black's model as described in O'Kane (2008). The optional `knockout` argument for `cdsoptprice` supports two variations of the mechanics of a CDS option. CDS options can be knockout or non-knockout options.

- A knockout option cancels with no payments if there is a credit event before the option expiry date.
- A non-knockout option does not cancel if there is a credit event before the option expiry date. In this case, the option holder of a non-knockout payer swaption can take delivery of the underlying long protection CDS on the option expiry date and exercise the protection, delivering a defaulted obligation in return for par. This portion of protection from option initiation to option expiry is known as the front-end protection (FEP). While this distinction does not affect the receiver swaption, the price of a non-knockout payer swaption is obtained by adding the value of the FEP to the knockout payer swaption price.

Define the CDS instrument.

```
Settle = datenum('12-Jun-2012');
OptionMaturity = datenum('20-Sep-2012');
CDSMaturity = datenum('20-Sep-2017');
OptionStrike = 200;
SpreadVolatility = .4;
```

Define the zero rate.

```
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [.5 .75 1.5 1.7 1.9 2.2]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate]
```

```
ZeroData = 6×2
105 ×
```

```
7.3521    0.0000
7.3540    0.0000
7.3576    0.0000
7.3613    0.0000
7.3649    0.0000
7.3686    0.0000
```

Define the market data.

```
Market_Time = [1 2 3 5 7 10]';
Market_Rate = [100 120 145 220 245 270]';
Market_Dates = daysadd(Settle,360*Market_Time,1);
MarketData = [Market_Dates Market_Rate];

ProbData = cdsbootstrap(ZeroData, MarketData, Settle)
```

```
ProbData = 6×2
105 ×

    7.3540    0.0000
    7.3576    0.0000
    7.3613    0.0000
    7.3686    0.0000
    7.3759    0.0000
    7.3868    0.0000
```

Define the CDS option.

```
[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, ...
    CDSMaturity, OptionStrike, SpreadVolatility, 'Knockout', true);
fprintf('    Payer: %.0f    Receiver: %.0f (Knockout)\n',Payer,Receiver);

    Payer: 196    Receiver: 23 (Knockout)

[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, ...
    CDSMaturity, OptionStrike, SpreadVolatility, 'Knockout', false);
fprintf('    Payer: %.0f    Receiver: %.0f (Non-Knockout)\n',Payer,Receiver);

    Payer: 224    Receiver: 23 (Non-Knockout)
```

## See Also

[cdsoptprice](#) | [cdsrpv01](#) | [cdsspread](#)

## Related Examples

- “Pricing a CDS Index Option” on page 8-40
- “Credit Default Swap (CDS)” (Financial Toolbox)

## Pricing a CDS Index Option

This example shows how to price CDS index options by using `cdsoptprice` with the forward spread adjustment. Unlike a single-name CDS, a CDS portfolio index contains multiple credits. When one or more of the credits default, the corresponding contingent payments are made to the protection buyer but the contract still continues with reduced coupon payments. Considering the fact that the CDS index option does not cancel when some of the underlying credits default before expiry, one might attempt to price CDS index options using the Black's model for non-knockout single-name CDS option. However, Black's model in this form is not appropriate for pricing CDS index options because it does not capture the exercise decision correctly when the strike spread ( $K$ ) is very high, nor does it ensure put-call parity when ( $K$ ) is not equal to the contractual spread (O'Kane, 2008).

However, with the appropriate modifications, Black's model for single-name CDS options used in `cdsoptprice` can provide a good approximation for CDS index options. While there are some variations in the way the Black's model is modified for CDS index options, they usually involve adjusting the forward spread  $F$ , the strike spread  $K$ , or both. Here we describe the approach of adjusting the forward spread only. In the Black's model for single-name CDS options, the forward spread  $F$  is defined as:

$$F = S(t, t_E, T) = \frac{S(t, T)RPV01(t, T) - S(t, t_E)RPV01(t, t_E)}{RPV01(t, t_E, T)}$$

where

$S$  is the spread.

$RPV01$  is the risky present value of a basis point (see `cdsrpv01`).

$t$  is the valuation date.

$t_E$  is the option expiry date.

$T$  is the CDS maturity date.

To capture the exercise decision correctly for CDS index options, we use the knockout form of the Black's model and adjust the forward spread to incorporate the  $FEP$  as follows:

$$F_{Adj} = F + \frac{FEP}{RPV01(t, t_E, T)}$$



with  $FEP$  defined as

$$FEP = (1 - R)Z(t, t_E)(1 - Q(t, t_E))$$

where

$R$  is the recovery rate.

$Z$  is the discount factor.

$Q$  is the survival probability.

In `cdsoptprice`, forward spread adjustment can be made with the `AdjustedForwardSpread` parameter. When computing the adjusted forward spread, we can compute the spreads using `cdsspread` and the RPV01s using `cdsrpv01`.

Set up the data for the CDS index, its option, and zero curve. The underlying is a 5-year CDS index maturing on 20-Jun-2017 and the option expires on 20-Jun-2012. A flat index spread is assumed when bootstrapping the default probability curve.

```
% CDS index and option data
Recovery = .4;
Basis = 2;
Period = 4;
CDSMaturity = datenum('20-Jun-2017');
ContractSpread = 100;
IndexSpread = 140;
BusDayConvention = 'follow';
Settle = datenum('13-Apr-2012');
OptionMaturity = datenum('20-Jun-2012');
OptionStrike = 140;
SpreadVolatility = .69;

% Zero curve data
MM_Time = [1 2 3 6]';
MM_Rate = [0.004111 0.00563 0.00757 0.01053]';
MM_Dates = daysadd(Settle,30*MM_Time,1);
Swap_Time = [1 2 3 4 5 6 7 8 9 10 12 15 20 30]';
Swap_Rate = [0.01387 0.01035 0.01145 0.01318 0.01508 0.01700 0.01868 ...
             0.02012 0.02132 0.02237 0.02408 0.02564 0.02612 0.02524]';
Swap_Dates = daysadd(Settle,360*Swap_Time,1);

InstTypes = [repmat({'deposit'},size(MM_Time));repmat({'swap'},size(Swap_Time))];
Instruments = [repmat(Settle,size(InstTypes)) [MM_Dates;Swap_Dates] [MM_Rate;Swap_Rate]];

ZeroCurve = IRDataCurve.bootstrap('zero',Settle,InstTypes,Instruments);

% Bootstrap the default probability curve assuming a flat index spread.
MarketData = [CDSMaturity IndexSpread];
ProbDates = datemnth(OptionMaturity,(0:5*12)');
ProbData = cdsbootstrap(ZeroCurve, MarketData, Settle, 'ProbDates', ProbDates);
```

Compute the spot and forward RPV01s, which will be used later in the computation of the adjusted forward spread. For this purpose, we can use `cdsrpv01`.

```
% RPV01(t,T)
RPV01_CDSMaturity = cdsrpv01(ZeroCurve,ProbData,Settle,CDSMaturity)

% RPV01(t,t_E,T)
RPV01_OptionExpiryForward = cdsrpv01(ZeroCurve,ProbData,Settle,CDSMaturity,...
    'StartDate',OptionMaturity)

% RPV01(t,t_E) = RPV01(t,T) - RPV01(t,t_E,T)
RPV01_OptionExpiry = RPV01_CDSMaturity - RPV01_OptionExpiryForward
```

```
RPV01_CDSMaturity =
```

```
4.7853
```

```
RPV01_OptionExpiryForward =
```

```
4.5971
```

```
RPV01_OptionExpiry =
```

```
0.1882
```

Compute the spot spreads using `cdsspread`.

```
% S(t,t_E)
Spread_OptionExpiry = cdsspread(ZeroCurve,ProbData,Settle,OptionMaturity,...
    'Period',Period,'Basis',Basis,'BusDayConvention',BusDayConvention,...
    'PayAccruedPremium',true,'recoveryrate',Recovery)

% S(t,T)
Spread_CDSMaturity = cdsspread(ZeroCurve,ProbData,Settle,CDSMaturity,...
    'Period',Period,'Basis',Basis,'BusDayConvention',BusDayConvention,...
    'PayAccruedPremium',true,'recoveryrate',Recovery)
```

```
Spread_OptionExpiry =
```

```
139.9006
```

```
Spread_CDSMaturity =
```

```
140.0000
```

The spot spreads and RPV01s are then used to compute the forward spread.

```
% F = S(t,t_E,T)
ForwardSpread = (Spread_CDSMaturity.*RPV01_CDSMaturity - ...
    Spread_OptionExpiry.*RPV01_OptionExpiry)./RPV01_OptionExpiryForward
```

```
ForwardSpread =
```

```
140.0040
```

Compute the front-end protection (FEP).

```
FEP = 10000*(1-Recovery)*ZeroCurve.getDiscountFactors(OptionMaturity)*ProbData(1,2)
```

```
FEP =
```

```
26.3108
```

Compute the adjusted forward spread.

```
AdjustedForwardSpread = ForwardSpread + FEP./RPV01_OptionExpiryForward
```

```
AdjustedForwardSpread =
```

```
145.7273
```

Compute the option prices using `cdsoptprice` with the adjusted forward spread. Note again that the `Knockout` parameter should be set to be `true` because the FEP was already incorporated into the adjusted forward spread.

```
[Payer,Receiver] = cdsoptprice(ZeroCurve, ProbData, Settle, OptionMaturity, ...
    CDSMaturity, OptionStrike, SpreadVolatility,'Knockout',true,...
    'AdjustedForwardSpread', AdjustedForwardSpread,'PayAccruedPremium',true);
fprintf(' Payer: %.0f Receiver: %.0f \n',Payer,Receiver);
```

```
Payer: 92 Receiver: 66
```

## See Also

`cdsoptprice` | `cdsrpv01` | `cdsspread`

## Related Examples

- “Pricing a Single-Name CDS Option” on page 8-38
- “Credit Default Swap (CDS)” (Financial Toolbox)

## Wrong Way Risk with Copulas

This example shows an approach to modeling wrong-way risk for Counterparty Credit Risk using a Gaussian copula.

A basic approach to Counterparty Credit Risk (CCR) (see “Counterparty Credit Risk and CVA” example) assumes that market and credit risk factors are independent of each other. A simulation of market risk factors drives the exposures for all contracts in the portfolio. In a separate step, Credit-Default Swap (CDS) market quotes determine the default probabilities for each counterparty. Exposures, default probabilities, and a given recovery rate are used to compute the Credit-Value Adjustment (CVA) for each counterparty, which is a measure of expected loss. The simulation of risk factors and the default probabilities are treated as independent of each other.

In practice, default probabilities and market factors are correlated. The relationship may be negligible for some types of instruments, but for others, the relationship between market and credit risk factors may be too important to be ignored when computing risk measures.

When the probability of default of a counterparty and the exposure resulting from particular contract tend to increase together we say that the contract has wrong-way risk (WWR).

This example demonstrates an implementation of the wrong-way risk methodology described in Garcia Cespedes et al. (see References).

### Exposures Simulation

Many financial institutions have systems that simulate market risk factors and value all the instruments in their portfolios at given simulation dates. These simulations are used to compute exposures and other risk measures. Because the simulations are computationally intensive, reusing them for subsequent risk analyses is important.

This example uses the data and the simulation results from the “Counterparty Credit Risk and CVA” example, previously saved in the `ccr.mat` file. The `ccr.mat` file contains:

- `RateSpec`: The rate spec when contract values were calculated
- `Settle`: The settle date when contract values were calculated
- `simulationDates`: A vector of simulation dates
- `swaps`: A struct containing the swap parameters

- `values`: The `NUMDATES` x `NUMCONTRACT` x `NUMSCENARIOS` cube of simulated contract values over each date/scenario

This example looks at expected losses over a one-year time horizon only, so the data is cropped after one year of simulation. Simulation dates over the first year are at a monthly frequency, so the 13th simulation date is our one-year time horizon (the first simulation date is the settle date).

```
load ccr.mat

oneYearIdx = 13;
values = values(1:oneYearIdx, :, :);
dates = simulationDates(1:oneYearIdx);

numScenarios = size(values, 3);
```

The credit exposures are computed from the simulated contract values. These exposures are monthly credit exposures per counterparty from the settle date to our one-year time horizon.

Since defaults can happen at any time during the one-year time period, it is common to model the exposure at default (EAD) based on the idea of expected positive exposure (EPE). The time-averaged exposure for each scenario is computed, which is called PE (positive exposure). The average of the PE's, including all scenarios, is the EPE, which can also be obtained from the `exposureprofiles` function.

The positive exposure matrix PE contains one row per simulated scenario and one column per counterparty. This is used as the EAD in our analysis.

```
% Compute counterparty exposures
[exposures, counterparties] = creditexposures(values, swaps.Counterparty, ...
    'NettingID', swaps.NettingID);
numCP = numel(counterparties);

% Compute PE (time-averaged exposures) per scenario
intervalWeights = diff(dates) / (dates(end) - dates(1));
exposureMidpoints = 0.5 * (exposures(1:end-1, :, :) + exposures(2:end, :, :));
weightedContributions = bsxfun(@times, intervalWeights, exposureMidpoints);
PE = squeeze(sum(weightedContributions));

% Compute total portfolio exposure per scenario
totalExp = sum(PE, 2);
```

```
% Display size of PE and totalExp
whos PE totalExp
```

Name	Size	Bytes	Class	Attributes
PE	1000x5	40000	double	
totalExp	1000x1	8000	double	

### Credit Simulation

A common approach for simulating credit defaults is based on a "one-factor model", sometimes called the "asset-value approach" (see Gupton et al., 1997). This is an efficient way to simulate correlated defaults.

Each company  $i$  is associated with a random variable  $Y_i$ , such that

$$Y_i = \beta_i Z + \sqrt{1 - \beta_i^2} \epsilon_i$$

where  $Z$  is the "one-factor", a standard normal random variable that represents a systematic credit risk factor whose values affect all companies. The correlation between company  $i$  and the common factor is given by  $\beta_i$ , the correlation between companies  $i$  and  $j$  is  $\beta_i \beta_j$ . The idiosyncratic shock  $\epsilon_i$  is another standard normal variable that may reduce or increase the effect of the systematic factor, independently of what happens with any other company.

If the default probability for company  $i$  is  $PD_i$ , a default occurs when

$$\Phi(Y_i) < PD_i$$

where  $\Phi$  is the cumulative standard normal distribution.

The  $Y_i$  variable is sometimes interpreted as asset returns, or sometimes referred to as a latent variable.

This model is a Gaussian copula that introduces a correlation between credit defaults. Copulas offer a particular way to introduce correlation, or more generally, co-dependence between two random variables whose co-dependence is unknown.

Use CDS spreads to bootstrap the one-year default probabilities for each counterparty. The CDS quotes come from the swap-portfolio spreadsheet used in the "Counterparty Credit Risk and CVA" example.

```
% Import CDS market information for each counterparty
swapFile = 'cva-swap-portfolio.xls';
```

```

cds = readtable(swapFile, 'Sheet', 'CDS Spreads');
cdsDates = datenum(cds.Date);
cdsSpreads = table2array(cds(:,2:end));

% Bootstrap default probabilities for each counterparty
zeroData = [RateSpec.EndDates RateSpec.Rates];
defProb = zeros(1, size(cdsSpreads,2));
for i = 1:numel(defProb)
    probData = cdsbootstrap(zeroData, [cdsDates cdsSpreads(:,i)], ...
        Settle, 'probDates', dates(end));
    defProb(i) = probData(2);
end

```

Now simulate the credit scenarios. Because defaults are rare, it is common to simulate a large number of credit scenarios.

The sensitivity parameter *beta* is set to 0.3 for all counterparties. This value can be calibrated or tuned to explore model sensitivities. See the references for more information.

```

numCreditScen = 100000;
rng('default');

% Z is the single credit factor
Z = randn(numCreditScen,1);

% epsilon is the idiosyncratic factor
epsilon = randn(numCreditScen,numCP);

% beta is the counterparty sensitivity to the credit factor
beta = 0.3 * ones(1,numCP);

% Counterparty latent variables
Y = bsxfun(@times,beta,Z) + bsxfun(@times,sqrt(1 - beta.^2),epsilon);

% Default indicator
isDefault = bsxfun(@lt,normcdf(Y),defProb);

```

### Correlating Exposure and Credit Scenarios

Now that there is a set of sorted portfolio exposure scenarios and a set of default scenarios, follow the approach in Garcia Cespedes et al. and use a Gaussian copula to generate correlated exposure-default scenario pairs.

Define a latent variable  $Y_e$  that maps into the distribution of simulated exposures.  $Y_e$  is defined as

$$Y_e = \rho Z + \sqrt{1 - \rho^2} \epsilon_e$$

where  $Z$  is the systemic factor computed in the credit simulation,  $\epsilon_e$  is an independent standard normal variable and  $\rho$  is interpreted as a market-credit correlation parameter. By construction,  $Y_e$  is a standard normal variable correlated with  $Z$  with correlation parameter  $\rho$ .

The mapping between  $Y_e$  and the simulated exposures requires us to order the exposure scenarios in a meaningful way, based on some sortable criterion. The criterion can be any meaningful quantity, for example, it could be an underlying risk factor for the contract values (such as an interest rate), the total portfolio exposure, and so on.

In this example, use the total portfolio exposure (`totalExp`) as the exposure scenario criterion to correlate the credit factor with the total exposure. If  $\rho$  is negative, low values of the credit factor  $Z$  tend to get linked to high values of  $Y_e$ , hence high exposures. This means negative values of  $\rho$  introduce WWR.

To implement the mapping between  $Y_e$  and the exposure scenarios, sort the exposure scenarios by the `totalExp` values. Suppose that the number of exposure scenarios is  $S$  (`numScenarios`). Given  $Y_e$ , find the value  $j$  such that

$$\frac{j-1}{S} \leq \Phi(Y_e) < \frac{j}{S}$$

and select the scenario  $j$  from the sorted exposure scenarios.

$Y_e$  is correlated to the simulated exposures and  $Z$  is correlated to the simulated defaults. The correlation  $\rho$  between  $Y_e$  and  $Z$  is, therefore, the correlation link between the exposures and the credit simulations.

```
% Sort the total exposure
[~,totalExpIdx] = sort(totalExp);

% Scenario cut points
cutPoints = 0:1/numScenarios:1;

% epsilonExp is the idiosyncratic factor for the latent variable
epsilonExp = randn(numCreditScen,1);
```



```

% Set a market-credit correlation value
rho = -0.75;

% Latent variable
Ye = rho * Z + sqrt(1 - rho^2) * epsilonExp;

% Find corresponding exposure scenario
binIdx = discretize(normcdf(Ye),cutPoints);
scenIdx = totalExpIdx(binIdx);
totalExpCorr = totalExp(scenIdx);
PECorr = PE(scenIdx,:);

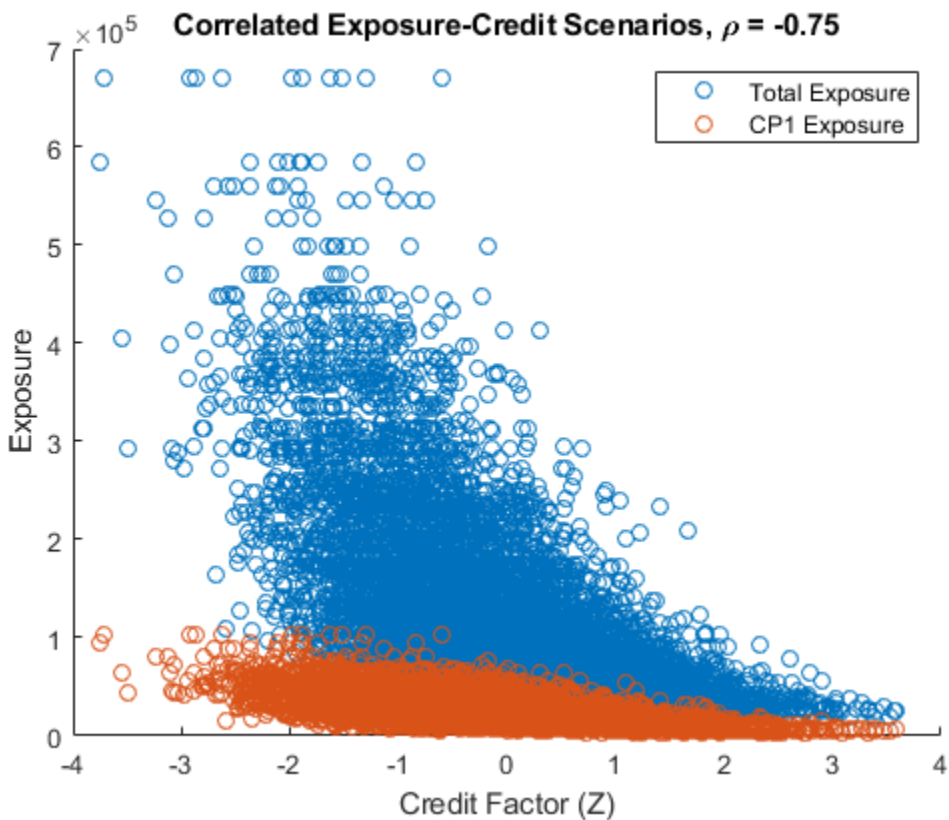
```

The following plot shows the correlated exposure-credit scenarios for the total portfolio exposure as well as for the first counterparty. Because of the negative correlation, negative values of the credit factor  $Z$  correspond to high exposure levels (wrong-way risk).

```

% We only plot up to 10000 scenarios
numScenPlot = min(10000,numCreditScen);
figure;
scatter(Z(1:numScenPlot),totalExpCorr(1:numScenPlot))
hold on
scatter(Z(1:numScenPlot),PECorr(1:numScenPlot,1))
xlabel('Credit Factor (Z)')
ylabel('Exposure')
title(['Correlated Exposure-Credit Scenarios, \rho = ' num2str(rho)])
legend('Total Exposure','CP1 Exposure')
hold off

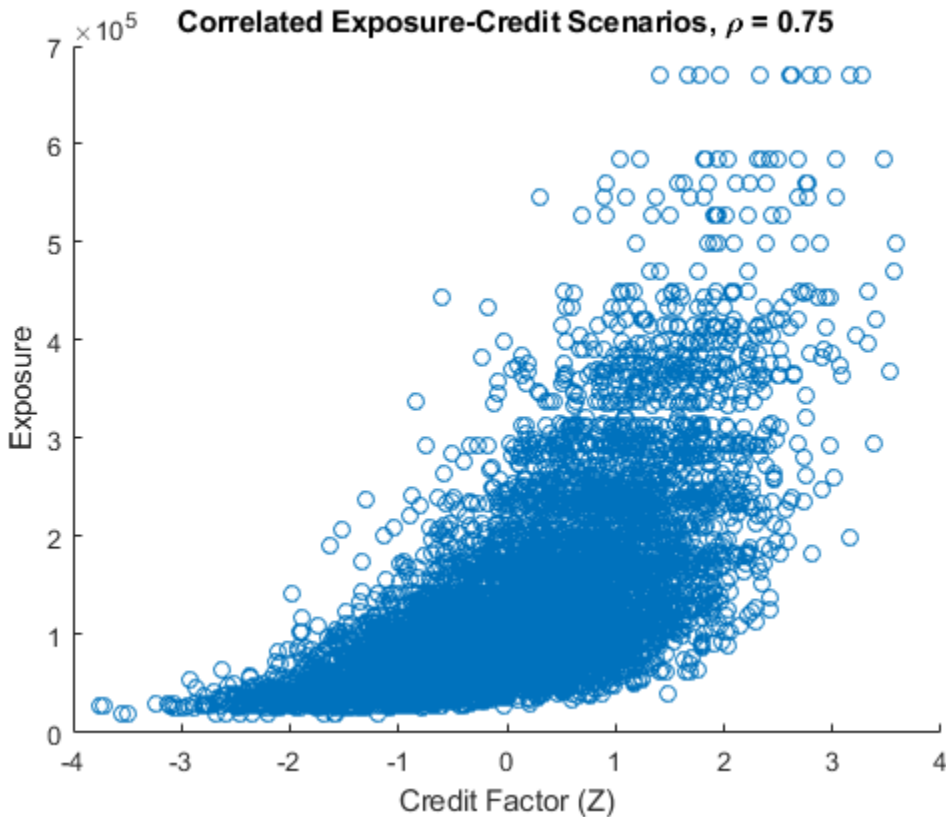
```



For positive values of  $\rho$ , the relationship between the credit factor and the exposures is reversed (right-way risk).

```
rho = 0.75;
Ye = rho * Z + sqrt(1 - rho^2) * epsilonExp;
binidx = discretize(normcdf(Ye),cutPoints);
scenIdx = totalExpIdx(binidx);
totalExpCorr = totalExp(scenIdx);

figure;
scatter(Z(1:numScenPlot),totalExpCorr(1:numScenPlot))
xlabel('Credit Factor (Z)')
ylabel('Exposure')
title(['Correlated Exposure-Credit Scenarios, \rho = ' num2str(rho)])
```



### Sensitivity to Correlation

You can explore the sensitivity of the exposures or other risk measures to a range of values for  $\rho$ .

For each value of  $\rho$ , compute the total losses per credit scenario as well as the expected losses per counterparty. This example assumes a 40% recovery rate.

```
Recovery = 0.4;
rhoValues = -1:0.1:1;
```

```
totalLosses = zeros(numCreditScen,numel(rhoValues));
expectedLosses = zeros(numCP, numel(rhoValues));
```

```

for i = 1:numel(rhoValues)
    rho = rhoValues(i);

    % Latent variable
    Ye = rho * Z + sqrt(1 - rho^2) * epsilonExp;

    % Find corresponding exposure scenario
    binIdx = discretize(normcdf(Ye),cutPoints);
    scenIdx = totalExpIdx(binIdx);
    simulatedExposures = PE(scenIdx,:);

    % Compute actual losses based on exposures and default events
    losses = isDefault .* simulatedExposures * (1-Recovery);
    totalLosses(:,i) = sum(losses,2);

    % We compute the expected losses per counterparty
    expectedLosses(:,i) = mean(losses)';
end
displayExpectedLosses(rhoValues, expectedLosses)

```

Rho	Expected Losses				
	CP1	CP2	CP3	CP4	CP5
-1.0	604.10	260.44	194.70	1234.17	925.95
-0.9	583.67	250.45	189.02	1158	

You can visualize the sensitivity of the Economic Capital (EC) to the market-credit correlation parameter. Define EC as the difference between a percentile  $q$  of the distribution of losses, minus the expected loss.

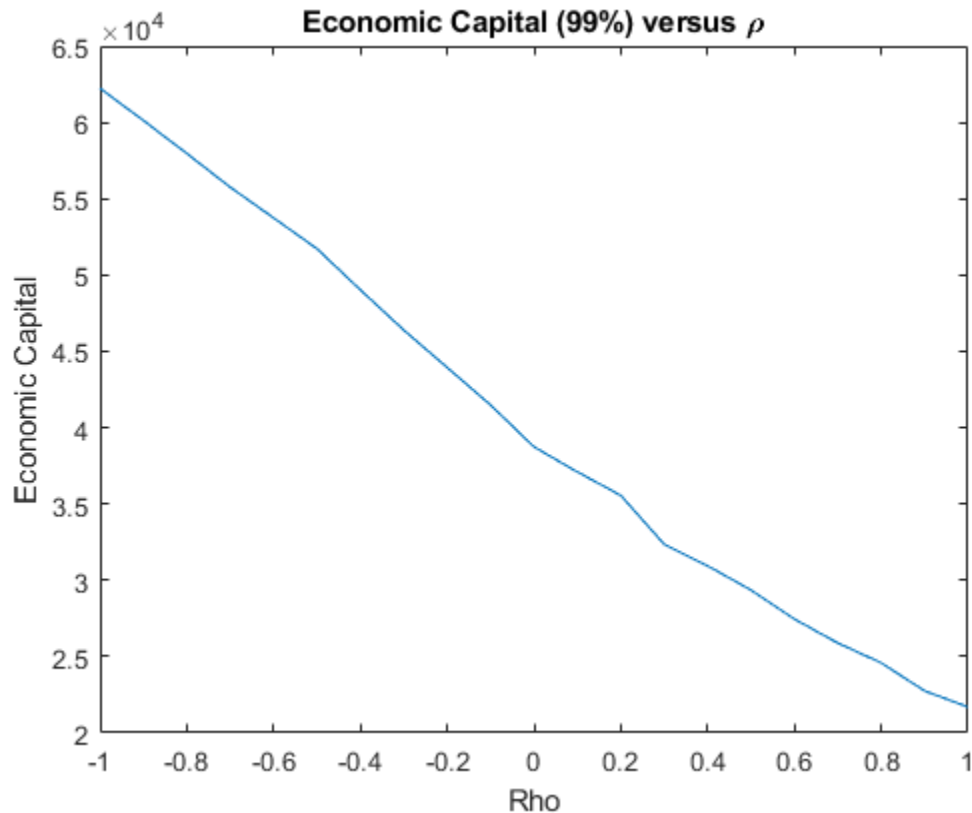
Negative values of  $\rho$  result in higher capital requirements because of WWR.

```

pct = 99;
ec = prctile(totalLosses,pct) - mean(totalLosses);

figure;
plot(rhoValues,ec)
title('Economic Capital (99%) versus \rho')
xlabel('Rho');
ylabel('Economic Capital');

```



### Final Remarks

This example implements a copula-based approach to WWR, following Garcia Cespedes et al. The methodology can efficiently reuse existing exposures and credit simulations, and the sensitivity to the market-credit correlation parameter can be efficiently computed and conveniently visualized for all correlation values.

The single-parameter copula approach presented here can be extended for a more thorough exploration of the WWR of a portfolio. For example, different types of copulas can be applied, and different criteria can be used to sort the exposure scenarios. Other extensions include simulating multiple systemic credit risk variables (a multi-factor model), or switching from a one-year to a multi-period framework to calculate measures such as credit value adjustment (CVA), as in Rosen and Saunders (see References).

## References

- 1 Garcia Cespedes, J. C., "Effective Modeling of Wrong-Way Risk, Counterparty Credit Risk Capital, and Alpha in Basel II," *The Journal of Risk Model Validation*, Volume 4 / Number 1, pp. 71-98, Spring 2010.
- 2 Gupton, G., C. Finger, and M. Bathia, *CreditMetrics™ - Technical Document*, J.P. Morgan, New York, 1997.
- 3 Rosen, D., and D. Saunders, "CVA the Wrong Way," *Journal of Risk Management in Financial Institutions*, Vol. 5, No. 3, pp. 252-272, 2012.

## Local Functions

```
function displayExpectedLosses(rhoValues, expectedLosses)
fprintf('          Expected Losses\n');
fprintf(' Rho    CP1    CP2    CP3    CP4    CP5\n');
fprintf('-----\n');
for i = 1:numel(rhoValues)
    % Display expected loss
    fprintf('% .1f%9.2f%9.2f%9.2f%9.2f%9.2f', rhoValues(i), expectedLosses(:,i));
end
end
```

## See Also

[cdsbootstrap](#) | [cdsprice](#) | [cdsrpv01](#) | [cdssspread](#)

## Related Examples

- "First-to-Default Swaps" on page 8-25
- "Credit Default Swap Option" on page 8-37

## External Websites

- Pricing and Valuation of Credit Default Swaps (4 min 22 sec)

# Interest-Rate Curve Objects

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- “Interest-Rate Curve Objects and Workflow” on page 9-2
- “Creating Interest-Rate Curve Objects” on page 9-4
- “Creating an IRDataCurve Object” on page 9-6
- “Bootstrapping a Swap Curve” on page 9-13
- “Dual Curve Bootstrapping” on page 9-17
- “Creating an IRFunctionCurve Object” on page 9-22
- “Fitting Interest Rate Curve Functions” on page 9-33
- “Converting an IRDataCurve or IRFunctionCurve Object” on page 9-41

## Interest-Rate Curve Objects and Workflow

### In this section...

“Class Structure” on page 9-2

“Workflow Using Interest-Rate Curve Objects” on page 9-3

### Class Structure

Financial Instruments Toolbox class structure supports interest-rate curve objects. The class structure supports five classes.

#### Class Structure

Class Name	Description
“@IRCurve” on page A-4	Base abstract class for interest-rate curves. <code>IRCurve</code> is an abstract class; you cannot create instances of it directly. You can create <code>IRFunctionCurve</code> and <code>IRDataCurve</code> objects that are derived from this class.
“@IRDataCurve” on page A-7	Creates a representation of an interest-rate curve with dates and data. <code>IRDataCurve</code> is constructed directly by specifying dates and corresponding interest rates or discount factors, or you can bootstrap an <code>IRDataCurve</code> object from market data.
“@IRFunctionCurve” on page A-13	Creates a representation of an interest-rate curve with a function. <code>IRFunctionCurve</code> is constructed directly by specifying a function handle, or you can fit a function to market data using methods of the <code>IRFunctionCurve</code> object.
“@IRBootstrapOptions” on page A-2	The <code>IRBootstrapOptions</code> object lets you specify options relating to the bootstrapping of an <code>IRDataCurve</code> object.
“@IRFitOptions” on page A-11	The <code>IRFitOptions</code> object lets you specify options relating to the fitting process for an <code>IRFunctionCurve</code> object.



## Workflow Using Interest-Rate Curve Objects

The supported workflow model for using interest-rate curve objects is:

- 1 Create an interest-rate curve based on an `IRDataCurve` object or an `IRFunctionCurve` object.
  - To create an `IRDataCurve` object:
    - Use vectors of dates and data with interpolation methods.
    - Use bootstrapping based on market instruments.

For more information on creating an `IRDataCurve` object, see “Creating an `IRDataCurve` Object” on page 9-6.

  - To create an `IRFunctionCurve` object:
    - Specify a function handle.
    - Fit a function using the Nelson-Siegel model, Svensson model, or smoothing spline model.
    - Fit a custom function.
- 2 Use methods of the `IRDataCurve` or `IRFunctionCurve` objects to extract forward, zero, discount factor, or par yield curves for the interest-rate curve object.
- 3 Convert an interest-rate curve from an `IRDataCurve` or `IRFunctionCurve` object to a `RateSpec` structure. This `RateSpec` structure is identical to the `RateSpec` produced by the Financial Instruments Toolbox function `intenvset`. Using the `RateSpec` for an interest-rate curve object, you can then use Financial Instruments Toolbox functions to model an interest-rate structure and price.

## See Also

`IRBootstrapOptions` | `IRDataCurve` | `IRFitOptions` | `IRFunctionCurve`

## Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4

## Creating Interest-Rate Curve Objects

Depending on your data and purpose for analysis, you can create an interest-rate curve object by using an `IRDataCurve` or `IRFunctionCurve` object.

To create an `IRDataCurve` object, you can:

- Use the `IRDataCurve` constructor using vector of dates and data with interpolation methods.
- Use the `IRDataCurve` method `bootstrap` using market instruments.

For more information on creating an `IRDataCurve` object, see “Creating an `IRDataCurve` Object” on page 9-6.

Using an `IRDataCurve` object, you can use the following methods to determine:

- Forward rate curve — `getForwardRates`
- Zero rate curve — `getZeroRates`
- Discount rate curve — `getDiscountFactors`
- Par yield curve — `getParYields`

Alternatively, to create an `IRFunctionCurve` object, you can:

- Use the `IRFunctionCurve` constructor and directly specify a function handle.
- Use `IRFunctionCurve` methods:
  - `fitNelsonSiegel` fits a “Fitting `IRFunctionCurve` Object Using Nelson-Siegel Method” on page 9-22 to market data for bonds.
  - `fitSvensson` fits a “Fitting `IRFunctionCurve` Object Using Svensson Method” on page 9-24 to market data for bonds.
  - `fitSmoothingSpline` fits a “Fitting `IRFunctionCurve` Object Using Smoothing Spline Method” on page 9-26 function to market data for bonds.
  - `fitFunction` custom fits an interest-rate curve object to market data for bonds.

Using an `IRFunctionCurve` object, you can use the following method to determine:

- Forward rate curve — `getForwardRates`
- Zero rate curve — `getZeroRates`

- Discount rate curve — `getDiscountFactors`
- Par yield curve — `getParYields`

In addition, you can convert an `IRDataCurve` or `IRFunctionCurve` to a `RateSpec` structure. For more information, see “Converting an `IRDataCurve` or `IRFunctionCurve` Object” on page 9-41.

## See Also

`IRBootstrapOptions` | `IRDataCurve` | `IRFitOptions` | `IRFunctionCurve`

## Related Examples

- “Creating an `IRDataCurve` Object” on page 9-6

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2

## Creating an IRDataCurve Object

To create an IRDataCurve object, see the following options:

### In this section...

“IRDataCurve Constructor with Dates and Data” on page 9-6

“IRDataCurve Bootstrapping Based on Market Instruments” on page 9-7

### IRDataCurve Constructor with Dates and Data

Use the IRDataCurve constructor with vectors of dates and data to create an interest-rate curve object. When constructing the IRDataCurve object, you can also use optional inputs to define how the interest-rate curve is constructed from the dates and data.

#### Example

In this example, you create the vectors for Dates and Data for an interest-rate curve:

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
```

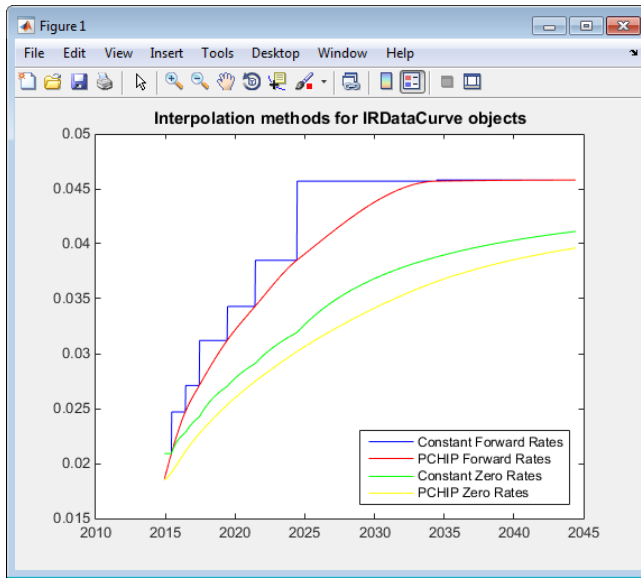
Use the IRDataCurve constructor to build interest-rate objects based on the constant and pchip interpolation methods:

```
irdc_const = IRDataCurve('Forward',today,Dates,Data,'InterpMethod','constant');
irdc_pchip = IRDataCurve('Forward',today,Dates,Data,'InterpMethod','pchip');
```

Plot the forward and zero rate curves for the two IRDataCurve objects based on constant and pchip interpolation methods:

```
PlottingDates = daysadd(today,180:10:360*30,1);
plot(PlottingDates, getForwardRates(irdc_const, PlottingDates),'b')
hold on
plot(PlottingDates, getForwardRates(irdc_pchip, PlottingDates),'r')
plot(PlottingDates, getZeroRates(irdc_const, PlottingDates),'g')
plot(PlottingDates, getZeroRates(irdc_pchip, PlottingDates),'yellow')
legend({'Constant Forward Rates','PCHIP Forward Rates','Constant Zero Rates',...
'PCHIP Zero Rates'},'location','SouthEast')
title('Interpolation methods for IRDataCurve objects')
datetick
```

The plot demonstrates the relationship of the forward and zero rate curves.



## IRDataCurve Bootstrapping Based on Market Instruments

Use the bootstrapping method, based on market instruments, to create an interest-rate curve object. When bootstrapping, you also have the option to define a range of interpolation methods (linear, spline, constant, and pchip).

### Example 1

In this example, you bootstrap a swap curve from deposits, Eurodollar Futures and swaps. The input market data for this example is hard-coded and specified as two cell arrays of data; one cell array indicates the type of instrument and the other contains the `Settle`, `Maturity` values and a market quote for the instrument. For deposits and swaps, the quote is a rate; for the EuroDollar futures, the quote is a price. Although bonds are not used in this example, a bond would also be quoted with a price.

```
InstrumentTypes = {'Deposit'; 'Deposit'; 'Deposit'; 'Deposit'; 'Deposit'; ...
    'Futures'; 'Futures'; ...
    'Futures'; 'Futures'; 'Futures'; ...
    'Futures'; 'Futures'; 'Futures'; ...
    'Futures'; 'Futures'; 'Futures'; ...
    'Futures'; 'Futures'; 'Futures'; ...
    'Swap'; 'Swap'; 'Swap'; 'Swap'; 'Swap'; 'Swap'; 'Swap'};
```

```

Instruments = [datenum('08/10/2007'),datenum('08/17/2007'),.0532063; ...
    datenum('08/10/2007'),datenum('08/24/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('10/17/2007'),.0534000; ...
    datenum('08/10/2007'),datenum('11/17/2007'),.0535866; ...
    datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
    datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...
    datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
    datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...
    datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...
    datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
    datenum('08/08/2007'),datenum('17-Jun-2009'),9494.5; ...
    datenum('08/08/2007'),datenum('16-Sep-2009'),9489; ...
    datenum('08/08/2007'),datenum('16-Dec-2009'),9481.5; ...
    datenum('08/08/2007'),datenum('17-Mar-2010'),9478; ...
    datenum('08/08/2007'),datenum('16-Jun-2010'),9474; ...
    datenum('08/08/2007'),datenum('15-Sep-2010'),9469.5; ...
    datenum('08/08/2007'),datenum('15-Dec-2010'),9464.5; ...
    datenum('08/08/2007'),datenum('16-Mar-2011'),9462.5; ...
    datenum('08/08/2007'),datenum('15-Jun-2011'),9456.5; ...
    datenum('08/08/2007'),datenum('21-Sep-2011'),9454; ...
    datenum('08/08/2007'),datenum('21-Dec-2011'),9449.5; ...
    datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
    datenum('08/08/2007'),datenum('08/08/2017'),.0545; ...
    datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
    datenum('08/08/2007'),datenum('08/08/2022'),.0559; ...
    datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
    datenum('08/08/2007'),datenum('08/08/2032'),.0566; ...
    datenum('08/08/2007'),datenum('08/08/2037'),.0566];

```

The `bootstrap` method is called as a static method of the “@IRDataCurve” on page A-7 class. Inputs to this method include the curve Type (zero or forward), Settle date, InstrumentTypes, and Instrument data. The `bootstrap` method also supports optional arguments, including an interpolation method, compounding, basis, and an options structure for bootstrapping. For example, you are passing in an “@IRBootstrapOptions” on page A-2 object which includes information for the ConvexityAdjustment to forward rates.

```

IRsigma = .01;
CurveSettle = datenum('08/10/2007');
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...
InstrumentTypes, Instruments,'InterpMethod','pchip',...
'Compounding',-1,'IRBootstrapOptions',...
IRBootstrapOptions('ConvexityAdjustment',@(t) .5*IRsigma^2.*t.^2))

```

```
bootModel =
```

```
IRDataCurve
```

```

    Type: Forward
    Settle: 733264 (10-Aug-2007)
    Compounding: -1

```

```

Basis: 0 (actual/actual)
InterpMethod: pchip
Dates: [29x1 double]
Data: [29x1 double]

```

The `bootstrap` method uses an Optimization Toolbox function to solve for any bootstrapped rates.

Plot the forward and zero curves:

```

PlottingDates = (CurveSettle+20:30:CurveSettle+365*25)';
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
BootstrappedForwardRates = getForwardRates(bootModel, PlottingDates);
BootstrappedZeroRates = getZeroRates(bootModel, PlottingDates);

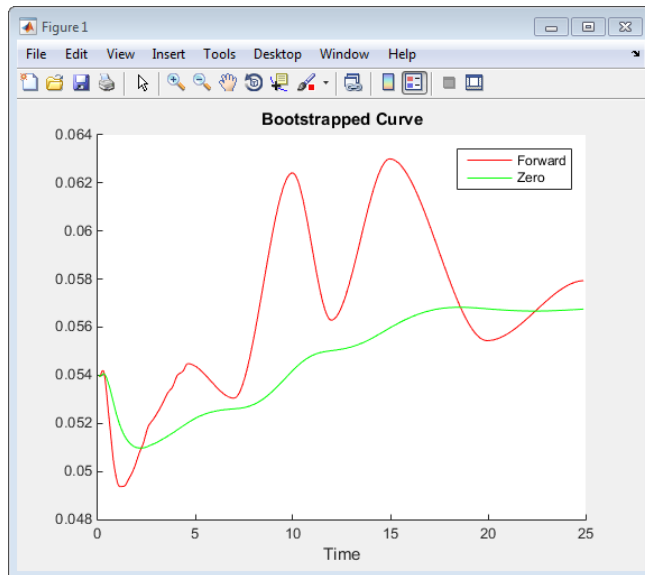
```

```

figure
hold on
plot(TimeToMaturity,BootstrappedForwardRates,'r')
plot(TimeToMaturity,BootstrappedZeroRates,'g')
title('Bootstrapped Curve')
xlabel('Time')
legend({'Forward','Zero'})

```

The plot demonstrates the forward and zero rate curves for the market data.



## Example 2

In this example, you bootstrap a swap curve from deposits, Eurodollar futures, and swaps. The input market data for this example is hard-coded and specified as two cell arrays of data; one cell array indicates the type of instrument and the other cell array contains the `Settle`, `Maturity` values and a market quote for the instrument. This example of bootstrapping also demonstrates the use of an `InstrumentBasis` for each `Instrument` type:

```
InstrumentTypes = {'Deposit';'Deposit';...
'Futures';'Futures';'Futures';'Futures';'Futures';'Futures';...
'Swap';'Swap';'Swap';'Swap';};

Instruments = [datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
datenum('08/10/2007'),datenum('11/17/2007'),.0535866; ...
datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...
datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...
datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...
datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
datenum('08/08/2007'),datenum('08/08/2037'),.0566];

CurveSettle = datenum('08/10/2007');
```

The `bootstrap` method is called as a static method of the “@IRDataCurve” on page A-7 class. Inputs to this method include the curve `Type` (zero or forward), `Settle` date, `InstrumentTypes`, and `Instrument` data. The `bootstrap` method also supports optional arguments, including an interpolation method, compounding, basis, and an options structure for bootstrapping. In this example, you are passing an additional `Basis` value for each instrument type:

```
bootModel=IRDataCurve.bootstrap('Forward',CurveSettle,InstrumentTypes, ...
Instruments,'InterpMethod','pchip','InstrumentBasis',[repmat(2,8,1);repmat(0,4,1)])
```

```
bootModel =
```

```
IRDataCurve
    Type: Forward
    Settle: 733264 (10-Aug-2007)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: pchip
    Dates: [12x1 double]
    Data: [12x1 double]
```

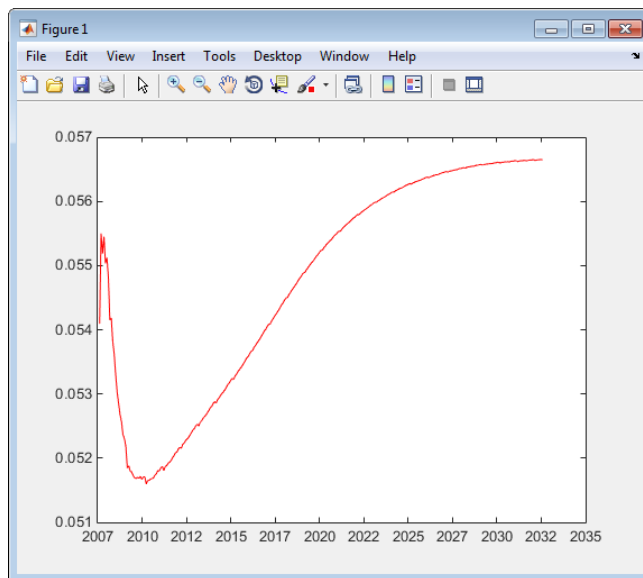


The `bootstrap` method uses an Optimization Toolbox function to solve for any bootstrapped rates.

Plot the par yields curve using the `getParYields` method:

```
PlottingDates = (datenum('08/11/2007'):30:CurveSettle+365*25)';  
plot(PlottingDates, getParYields(bootModel, PlottingDates), 'r')  
datetick
```

The plot demonstrates the par yields curve for the market data.



## See Also

[IRBootstrapOptions](#) | [IRDataCurve](#) | [IRFitOptions](#) | [IRFunctionCurve](#)

## Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4
- “Bootstrapping a Swap Curve” on page 9-13
- “Dual Curve Bootstrapping” on page 9-17

- “Creating an IRFunctionCurve Object” on page 9-22

### **More About**

- “Interest-Rate Curve Objects and Workflow” on page 9-2

### **External Websites**

- Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)
- Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk Applications (30 min 00 sec)

## Bootstrapping a Swap Curve

This example shows how to bootstrap an interest-rate curve, often referred to as a swap curve, using the `IRDataCurve` object. The static bootstrap method takes as inputs a cell array of market instruments (which can be deposits, interest-rate futures, swaps, and bonds) and bootstraps an interest-rate curve of either the forward or the zero curve. It is also possible to specify multiple interpolation methods, including piecewise constant, linear, and Piecewise Cubic Hermite Interpolating Polynomial (PCHIP).

### Obtain Data

A curve is bootstrapped from market data. In this example, we will bootstrap a swap curve from deposits, Eurodollar Futures, and swaps.

For this example, we have hard-coded the input market data, which is simply specified as 2 cell arrays of data, one which indicates the type of instrument and a second cell array containing the `Settle`, `Maturity`, and `Market Quote` for the instrument. For deposits and swaps, the quote is a rate, and for the EuroDollar Futures, the quote is a price. Although bonds are not used in this example, a bond would be quoted with a price.

```
InstrumentTypes = {'Deposit';'Deposit';'Deposit';'Deposit';'Deposit'; ...
    'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Swap';'Swap';'Swap';'Swap';'Swap';'Swap';'Swap'};

Instruments = [datenum('08/10/2007'),datenum('08/17/2007'),.0532063; ...
    datenum('08/10/2007'),datenum('08/24/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('10/17/2007'),.0534000; ...
    datenum('08/10/2007'),datenum('11/17/2007'),.0535866; ...
    datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
    datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...
    datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
    datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...
    datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...
    datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
    datenum('08/08/2007'),datenum('17-Jun-2009'),9494.5; ...
    datenum('08/08/2007'),datenum('16-Sep-2009'),9489; ...
    datenum('08/08/2007'),datenum('16-Dec-2009'),9481.5; ...
```

```

datenum('08/08/2007'),datenum('17-Mar-2010'),9478; ...
datenum('08/08/2007'),datenum('16-Jun-2010'),9474; ...
datenum('08/08/2007'),datenum('15-Sep-2010'),9469.5; ...
datenum('08/08/2007'),datenum('15-Dec-2010'),9464.5; ...
datenum('08/08/2007'),datenum('16-Mar-2011'),9462.5; ...
datenum('08/08/2007'),datenum('15-Jun-2011'),9456.5; ...
datenum('08/08/2007'),datenum('21-Sep-2011'),9454; ...
datenum('08/08/2007'),datenum('21-Dec-2011'),9449.5; ...
datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'),datenum('08/08/2017'),.0545; ...
datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'),datenum('08/08/2022'),.0559; ...
datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
datenum('08/08/2007'),datenum('08/08/2032'),.0566; ...
datenum('08/08/2007'),datenum('08/08/2037'),.0566];

```

### Construct the Curve via Bootstrapping

The `bootstrap` method is called as a static method of the `IRDataCurve` class. Inputs to this method include the curve type (Zero or Forward), settle date, instrument types, instrument data, and optional arguments including an interpolation method, compounding, and an options structure for bootstrapping. Note that in this example, we are passing in an `IRBootstrapOptions` object which includes information for the convexity adjustment to forward rates.

```

IRsigma = .01;
CurveSettle = datenum('08/10/2007');
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...
    InstrumentTypes, Instruments, 'InterpMethod', 'pchip', ...
    'Compounding', -1, 'IRBootstrapOptions', ...
    IRBootstrapOptions('ConvexityAdjustment', @(t) .5*IRsigma^2.*t.^2));

```

### Plot

We can now plot both the forward and zero curves.

```

PlottingDates = (CurveSettle+20:30:CurveSettle+365*25)';
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
BootstrappedForwardRates = bootModel.getForwardRates(PlottingDates);
BootstrappedZeroRates = bootModel.getZeroRates(PlottingDates);

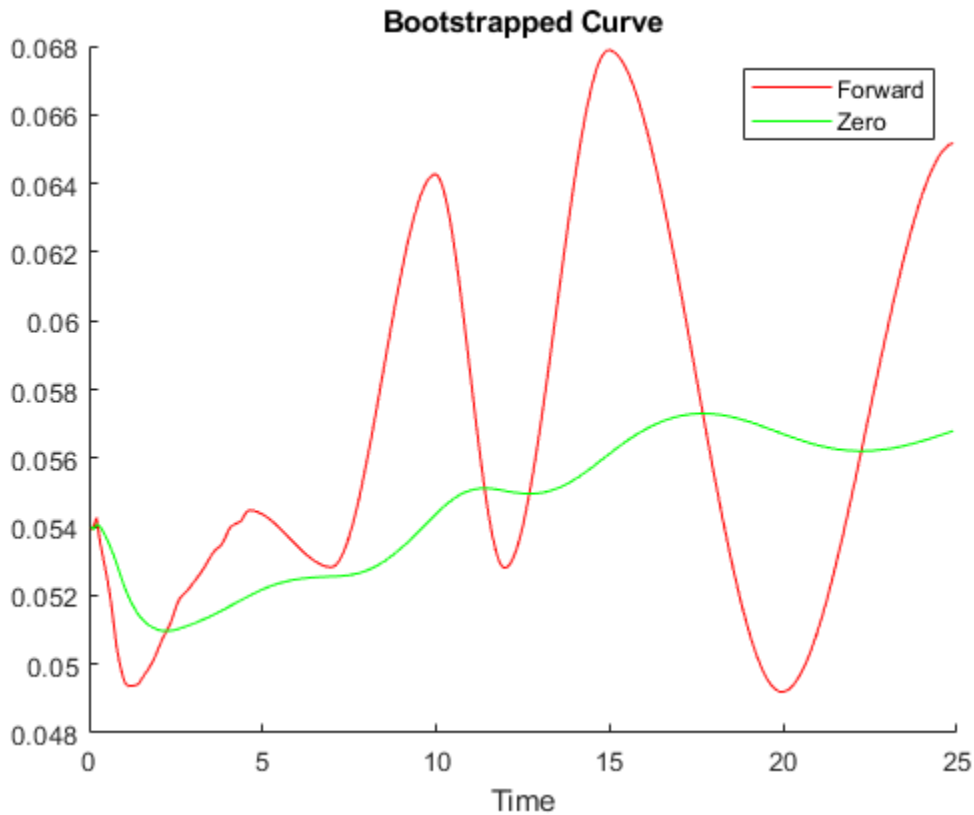
figure
hold on
plot(TimeToMaturity,BootstrappedForwardRates,'r')

```

```

plot(TimeToMaturity, BootstrappedZeroRates, 'g')
title('Bootstrapped Curve')
xlabel('Time')
legend({'Forward', 'Zero'})

```



### Bibliography

This example draws from the following papers and journal articles:

- [1] Hagan, P, West, G. (2006), "Interpolation Methods for Curve Construction", Applied Mathematical Finance, Vol 13, No. 2

[2] Ron, Uri(2000), "A Practical Guide to Swap Curve Construction", Working Papers 00-17, Bank of Canada.

## Dual Curve Bootstrapping

This example shows how to bootstrap a forward curve using a different curve for discounting.

### Define the Data

Data is needed for both the forward and discount curve. For this particular example, it is assumed that the data is provided for EONIA (the discount curve) and EURIBOR (the forward curve). However, this approach can be used in any case where the curve to be built is different than the curve used for discounting cash flows. While the data in this example is hardcoded, it could also be imported into MATLAB with Datafeed Toolbox™ or Database Toolbox™.

```
Settle = datenum('20-Aug-2013');

% Deposit data
EONIADepositRates = [.0007 .00067]';
EONIADepositMat = datenum({'3-Sep-2013', '20-Sep-2013'});
EONIADepositBasis = 2; % act/360
EONIADepositPeriod = 0;

% FRA
EONIAFRARates = [.00025 .0003 .00043 .00054]';
EONIAFRASStartDate = datenum({'11-Sep-2013', '9-Oct-2013', '13-Nov-2013', '11-Dec-2013'});
EONIAFRAEndDate = datenum({'9-Oct-2013', '13-Nov-2013', '11-Dec-2013', '11-Jan-2014'});
EONIAFRABasis = 2; % act/360
EONIAFRAPeriod = 0;

% Swap data
EONIASwapRates = [.0003 .001 .002 .004 .008 .012 .0155 .018 .0193 .02]';
EONIASwapMat = datemnth(Settle, 12*[2:5 7 10 15 20 25 30]');
EONIASwapBasis = 5; % 30/360 ISDA
EONIASwapPeriod = 1;

% EURIBOR Deposit data
EURIBORDepositRates = [.0022 .0021 .002 .0019]';
EURIBORDepositMat = datenum({'3-Sep-2013', '20-Sep-2013', '21-Oct-2013', '20-Nov-2013'});
EURIBORDepositBasis = 2; % act/360
EURIBORDepositPeriod = 0;

% EURIBOR Futures
EURIBORFRARates = [9982 9978 9976 9975]';
EURIBORFRASStartDate = datenum({'18-Dec-2013', '19-Mar-2014', '18-Jun-2014', '17-Sep-2014'});
EURIBORFRAEndDate = datenum({'18-Mar-2014', '19-Jun-2014', '18-Sep-2014', '17-Dec-2014'});
EURIBORFRABasis = 2; % act/360
EURIBORFRAPeriod = 4;

% EURIBOR Swap data
EURIBORSwapRates = [.0026 .0044 .0062 .0082 .012 .015 .018 .02 .021 .0215]';
EURIBORSwapMat = datemnth(Settle, 12*[2:5 7 10 15 20 25 30]');
EURIBORSwapBasis = 5; % 30/360 ISDA
EURIBORSwapPeriod = 1;
```

## Construct a EONIA Discount Curve

Build the EONIA curve. This is essentially the same as the single curve case.

```
CurveType = 'zero';
CurveCompounding = 1;
CurveBasis = 3; % act/365

nEONIADeposits = length(EONIADepositMat);
nEONIAFRA = length(EONIAFRAEndDate);
nEONIASwaps = length(EONIASwapMat);

EONIAInstrumentTypes = [ repmat({'deposit'},nEONIADeposits,1);
    repmat({'fra'},nEONIAFRA,1); repmat({'swap'},nEONIASwaps,1)];

EONIAPeriod = [ repmat(EONIADepositPeriod,nEONIADeposits,1);
    repmat(EONIAFRAPeriod,nEONIAFRA,1); repmat(EONIASwapPeriod,nEONIASwaps,1)];

EONIABasis = [ repmat(EONIADepositBasis,nEONIADeposits,1);
    repmat(EONIAFRABasis,nEONIAFRA,1); repmat(EONIASwapBasis,nEONIASwaps,1)];

EONIAInstrumentData = [[ repmat(Settle,[nEONIADeposits 1]);EONIAFRASStartDate; repmat(Settle,[nEONIASwaps 1])] ...
    [EONIADepositMat;EONIAFRAEndDate;EONIASwapMat] ...
    [EONIADepositRates;EONIAFRARates;EONIASwapRates]];

EONIACurve = IRDataCurve.bootstrap(CurveType,Settle,EONIAInstrumentTypes,...
    EONIAInstrumentData,'Compounding',CurveCompounding,'Basis',CurveBasis,...
    'InstrumentPeriod',EONIAPeriod,'InstrumentBasis',EONIABasis)

EONIACurve =

    Type: zero
    Settle: 735466 (20-Aug-2013)
    Compounding: 1
    Basis: 3 (actual/365)
    InterpMethod: linear
    Dates: [16x1 double]
    Data: [16x1 double]
```

## Construct a EURIBOR Forward Curve

The EURIBOR forward curve is built first using a single curve approach.

```
nEURIBORDeposits = length(EURIBORDepositMat);
nEURIBORFRA = length(EURIBORFRAEndDate);
nEURIBORSwaps = length(EURIBORSwapMat);

EURIBORInstrumentTypes = [ repmat({'deposit'},nEURIBORDeposits,1);
    repmat({'futures'},nEURIBORFRA,1); repmat({'swap'},nEURIBORSwaps,1)];

EURIBORPeriod = [ repmat(EURIBORDepositPeriod,nEURIBORDeposits,1);
    repmat(EURIBORFRAPeriod,nEURIBORFRA,1); repmat(EURIBORSwapPeriod,nEURIBORSwaps,1)];

EURIBORBasis = [ repmat(EURIBORDepositBasis,nEURIBORDeposits,1);
    repmat(EURIBORFRABasis,nEURIBORFRA,1); repmat(EURIBORSwapBasis,nEURIBORSwaps,1)];
```



```

EURIBORInstrumentData = [ repmat(Settle, size(EURIBORInstrumentTypes)) ...
    [EURIBORDepositMat; EURIBORFRAEndDate; EURIBORSwapMat] ...
    [EURIBORDepositRates; EURIBORFRARates; EURIBORSwapRates]];

EURIBORCurve_Single = IRDataCurve.bootstrap(CurveType, Settle, EURIBORInstrumentTypes, ...
    EURIBORInstrumentData, 'Compounding', CurveCompounding, 'Basis', CurveBasis, ...
    'InstrumentPeriod', EURIBORPeriod, 'InstrumentBasis', EURIBORBasis)

EURIBORCurve_Single =

    Type: zero
    Settle: 735466 (20-Aug-2013)
    Compounding: 1
    Basis: 3 (actual/365)
    InterpMethod: linear
    Dates: [18x1 double]
    Data: [18x1 double]

```

## Build the EURIBOR Curve with the EONIA Curve

Next, build a curve using the EONIA curve as a discounting curve. To do this, specify the EONIA curve as an optional input argument.

```

EURIBORCurve = IRDataCurve.bootstrap(CurveType, Settle, EURIBORInstrumentTypes, ...
    EURIBORInstrumentData, 'DiscountCurve', EONIACurve, 'Compounding', ...
    CurveCompounding, 'Basis', CurveBasis, 'InstrumentPeriod', EURIBORPeriod, ...
    'InstrumentBasis', EURIBORBasis)

EURIBORCurve =

    Type: zero
    Settle: 735466 (20-Aug-2013)
    Compounding: 1
    Basis: 3 (actual/365)
    InterpMethod: linear
    Dates: [18x1 double]
    Data: [18x1 double]

```

## Plot the Results

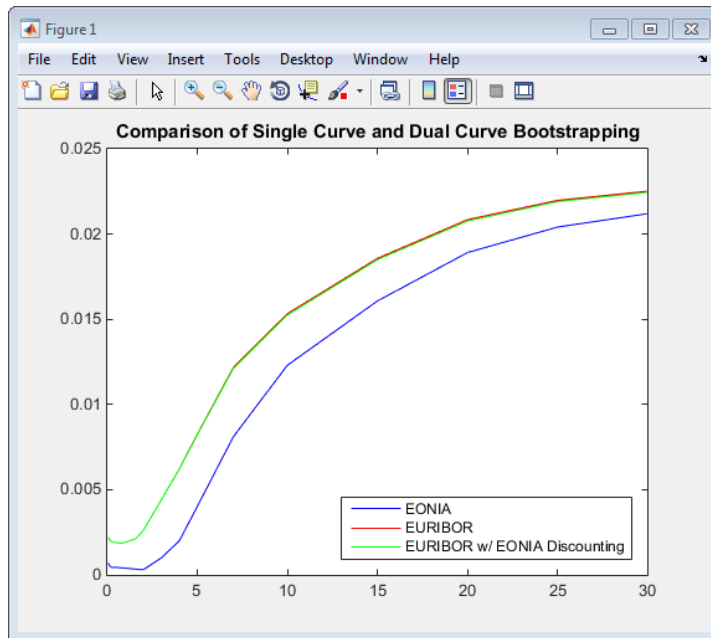
Plot the results to compare the curves.

```

PlottingDates = (Settle+20:30:Settle+365*30)';
TimeToMaturity = yearfrac(Settle, PlottingDates);

figure
plot(TimeToMaturity, getZeroRates(EONIACurve, PlottingDates), 'b')
hold on
plot(TimeToMaturity, getZeroRates(EURIBORCurve_Single, PlottingDates), 'r')
plot(TimeToMaturity, getZeroRates(EURIBORCurve, PlottingDates), 'g')
title('Comparison of Single Curve and Dual Curve Bootstrapping')
legend({'EONIA', 'EURIBOR', 'EURIBOR w/ EONIA Discounting'}, 'location', 'southeast')

```



As expected, the difference between the two different EURIBOR curves is small but nontrivial.

## Bibliography

This example draws from the following papers and journal articles:

- [1] Ametrano, F, and Bianchetti, M., *Everything You Always Wanted to Know About Multiple Interest Rate Curve Bootstrapping but Were Afraid to Ask* (April 2, 2013), available at: [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=2219548](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2219548).
- [2] Bianchetti, M., *Two Curves, One Price*, Risk Magazine, pages 74–80, August 2010.
- [3] Fujii, M, Shimada, Y, Takahashi, A., *A Note on Construction of Multiple Swap Curves with and without Collateral* (January 2, 2010), CARF Working Paper Series No. CARF-F-154, available at: [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1440633](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1440633).
- [4] Mercurio, Fabio, *Interest Rates and The Credit Crunch: New Formulas and Market Models* (February 5, 2009), Bloomberg Portfolio Research Paper No. 2010-01-FRONTIERS.

[5] Nashikkar, A., *Understanding OIS Discounting*, Barclays Capital Interest Rate Strategy, February 24, 2011.

## See Also

`IRDataCurve` | `bootstrap` | `floatbyzero` | `getZeroRates` | `swapbyzero`

## Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4
- “Bootstrapping a Swap Curve” on page 9-13

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2

## External Websites

- Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)
- Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk Applications (30 min 00 sec)

## Creating an IRFunctionCurve Object

To create an IRFunctionCurve object, see the following options:

### In this section...

“Fitting IRFunctionCurve Object Using a Function Handle” on page 9-22

“Fitting IRFunctionCurve Object Using Nelson-Siegel Method” on page 9-22

“Fitting IRFunctionCurve Object Using Svensson Method” on page 9-24

“Fitting IRFunctionCurve Object Using Smoothing Spline Method” on page 9-26

“Using fitFunction to Create Custom Fitting Function” on page 9-29

### Fitting IRFunctionCurve Object Using a Function Handle

You can use the constructor IRFunctionCurve with a MATLAB function handle to define an interest-rate curve. For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.

#### Example

This example uses a FunctionHandle argument with a value  $@(t) t.^2$  to construct an interest-rate curve:

```
rr = IRFunctionCurve('Zero',today,@(t) t.^2)
```

```
rr =
```

```
Properties:
  FunctionHandle: @(t)t.^2
             Type: 'Zero'
             Settle: 733600
  Compounding: 2
             Basis: 0
```

### Fitting IRFunctionCurve Object Using Nelson-Siegel Method

Use the method, fitNelsonSiegel, for the Nelson-Siegel model that fits the empirical form of the yield curve with a prespecified functional form of the spot rates which is a function of the time to maturity of the bonds.

The Nelson-Siegel model represents a dynamic three-factor model: level, slope, and curvature. However, the Nelson-Siegel factors are unobserved, or latent, which allows for measurement error, and the associated loadings have economic restrictions (forward rates are always positive, and the discount factor approaches zero as maturity increases). For more information, see “Zero-coupon yield curves: technical documentation,” *BIS Papers*, Bank for International Settlements, Number 25, October 2005.

### Example

This example uses `IRFunctionCurve` to model the default-free term structure of interest rates in the United Kingdom.

Load the data:

```
load ukdata20080430
```

Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

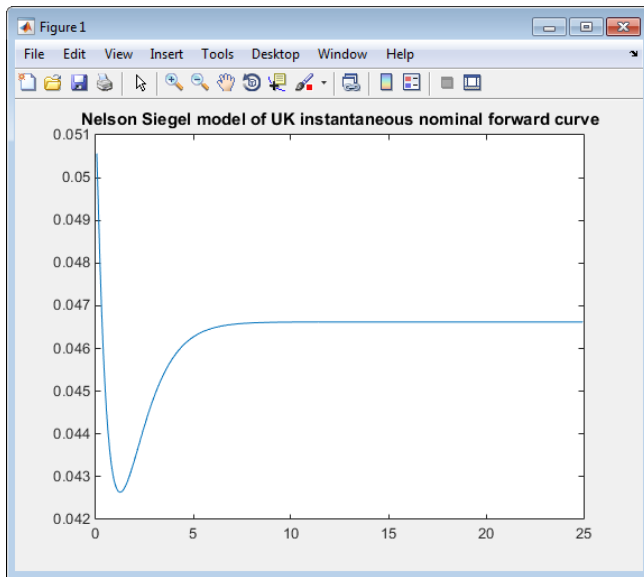
```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

The `IRFunctionCurve` object provides the capability to fit a Nelson-Siegel curve to observed market data with the `fitNelsonSiegel` method. The fitting is done by calling the Optimization Toolbox function `lsqnonlin`. This method has required inputs of `Type`, `Settle`, and a matrix of instrument data.

```
NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',CurveSettle,...
Instruments,'Compounding',-1,'InstrumentPeriod',InstrumentPeriod);
```

Plot the Nelson-Siegel interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
NSForwardRates = getForwardRates(NSModel, PlottingDates);
plot(TimeToMaturity,NSForwardRates)
title('Nelson Siegel model of UK instantaneous nominal forward curve')
```



## Fitting IRFunctionCurve Object Using Svensson Method

Use the method, `fitSvensson`, for the Svensson model to improve the flexibility of the curves and the fit for a Nelson-Siegel model. In 1994, Svensson extended Nelson and Siegel's function by adding a further term that allows for a second "hump." The extra precision is achieved at the cost of adding two more parameters,  $\beta_3$  and  $\tau_2$ , which have to be estimated.

### Example

In this example of using the `fitSvensson` method, an `IRFitOptions` structure, previously defined using the `IRFitOptions` constructor, is used. Thus, you must specify `FitType`, `InitialGuess`, `UpperBound`, `LowerBound`, and the `OptOptions` optimization parameters for `lsqnonlin`.

Load the data:

```
load ukdata20080430
```

Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0, size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

Define OptOptions for the IRFitOptions constructor:

```
OptOptions = optimoptions('lsqnonlin','MaxFunEvals',1000);
fIRFitOptions = IRFitOptions([5.82 -2.55 -.87 0.45 3.9 0.44],...
'FitType','durationweightedprice','OptOptions',OptOptions,...
'LowerBound',[0 -Inf -Inf -Inf 0 0],'UpperBound',[Inf Inf Inf Inf Inf]);
```

Fit the interest-rate curve using a Svensson model:

```
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',CurveSettle,...
Instruments,'IRFitOptions', fIRFitOptions, 'Compounding',-1,...
'InstrumentPeriod',InstrumentPeriod)
```

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the default value of the function tolerance.

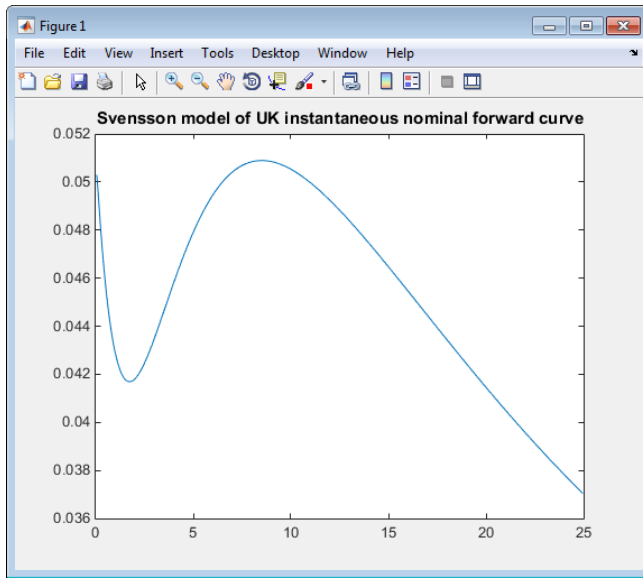
SvenssonModel =

```
    Type: Zero
    Settle: 733528 (30-Apr-2008)
    Compounding: -1
    Basis: 0 (actual/actual)
```

The status message, output from `lsqnonlin`, indicates that the optimization to find parameters for the Svensson equation terminated successfully.

Plot the Svensson interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
SvenssonForwardRates = getForwardRates(SvenssonModel, PlottingDates);
plot(TimeToMaturity,SvenssonForwardRates)
title('Svensson model of UK instantaneous nominal forward curve')
```



### Fitting IRFunctionCurve Object Using Smoothing Spline Method

Use the method, `fitSmoothingSpline`, to model the term structure with a spline, specifically, the term structure represents the forward curve with a cubic spline.

---

**Note** You must have a license for Curve Fitting Toolbox software to use the `fitSmoothingSpline` method.

---

#### Example

The `IRFunctionCurve` object is used to fit a smoothing spline representation of the forward curve with a penalty function. Required inputs are `Type`, `Settle`, the matrix of Instruments, and `LambdaFun`, a function handle containing the penalty function

Load the data:

```
load ukdata20080430
```

Convert repo rates to be equivalent zero coupon bonds:



```
RepoCouponRate = repmat(0,size(RepoRates));  
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

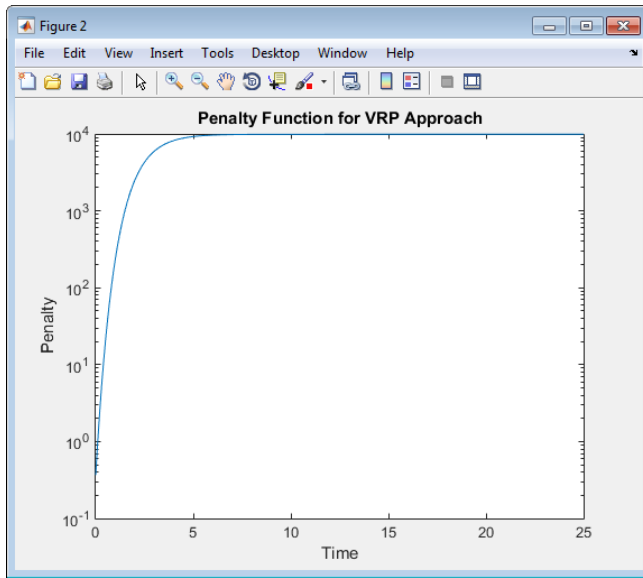
```
Settle = [RepoSettle;BondSettle];  
Maturity = [RepoMaturity;BondMaturity];  
CleanPrice = [RepoPrice;BondCleanPrice];  
CouponRate = [RepoCouponRate;BondCouponRate];  
Instruments = [Settle Maturity CleanPrice CouponRate];  
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];  
CurveSettle = datenum('30-Apr-2008');
```

Choose parameters for Lambdafun:

```
L = 9.2;  
S = -1;  
mu = 1;
```

Define the Lambdafun penalty function:

```
lambdafun = @(t) exp(L - (L-S)*exp(-t/mu));  
t = 0:.1:25;  
y = lambdafun(t);  
figure  
semilogy(t,y);  
title('Penalty Function for VRP Approach')  
ylabel('Penalty')  
xlabel('Time')
```

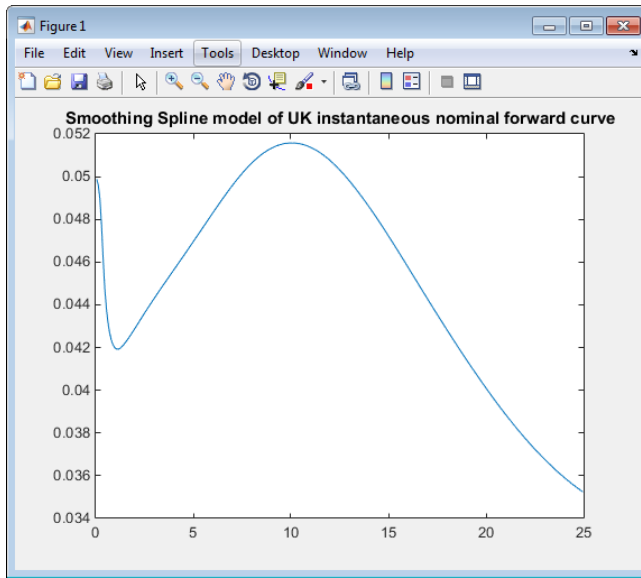


Use the `fitSmoothingSpline` method to fit the interest-rate curve and model the `Lambdafun` penalty function:

```
VRPModel = IRFunctionCurve.fitSmoothingSpline('Forward',CurveSettle,...
Instruments,lambdafun,'Compounding',-1,'InstrumentPeriod',InstrumentPeriod);
```

Plot the smoothing spline interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
VRPForwardRates = getForwardRates(VRPModel, PlottingDates);
plot(TimeToMaturity,VRPForwardRates)
title('Smoothing Spline model of UK instantaneous nominal forward curve')
```



## Using fitFunction to Create Custom Fitting Function

When using an `IRFunctionCurve` object, you can create a custom fitting function with the `fitFunction` method. To use `fitFunction`, you must define a `FunctionHandle`. In addition, you must also use the constructor `IRFitOptions` to define `IRFitOptionsObj` to support an `InitialGuess` for the parameters of the curve function.

### Example

The following example demonstrates the use of `fitFunction` with a `FunctionHandle` and an `IRFitOptionsObj`:

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];

CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
CurveSettle = datenum('30-Apr-2008');
```

Define the `FunctionHandle`:

```
functionHandle = @(t,theta) polyval(theta,t);
```

Define the OptOptions for IRFitOptions:

```
OptOptions = optimoptions('lsqnonlin','display','iter');
```

Define fitFunction:

```
CustomModel = IRFunctionCurve.fitFunction('Zero', CurveSettle, ...
functionHandle, Instruments, IRFitOptions([.05 .05 .05], 'FitType', 'price', ...
'OptOptions', OptOptions));
```

Iteration	Func-count	f(x)	Norm of step	First-order optimality	CG-iterations
0	4	38036.7		4.92e+04	
1	8	38036.7	10	4.92e+04	0
2	12	38036.7	2.5	4.92e+04	0
3	16	38036.7	0.625	4.92e+04	0
4	20	38036.7	0.15625	4.92e+04	0
5	24	30741.5	0.0390625	1.72e+05	0
6	28	30741.5	0.078125	1.72e+05	0
7	32	30741.5	0.0195312	1.72e+05	0
8	36	28713.6	0.00488281	2.33e+05	0
9	40	20323.3	0.00976562	9.47e+05	0
10	44	20323.3	0.0195312	9.47e+05	0
11	48	20323.3	0.00488281	9.47e+05	0
12	52	20323.3	0.0012207	9.47e+05	0
13	56	19698.8	0.000305176	1.08e+06	0
14	60	17493	0.000610352	7e+06	0
15	64	17493	0.0012207	7e+06	0
16	68	17493	0.000305176	7e+06	0
17	72	15455.1	7.62939e-05	2.25e+07	0
18	76	15455.1	0.000177499	2.25e+07	0
19	80	13317.1	3.8147e-05	3.18e+07	0
20	84	12865.3	7.62939e-05	7.83e+07	0
21	88	11779.8	7.62939e-05	7.58e+06	0
22	92	11747.6	0.000152588	1.45e+05	0
23	96	11720.9	0.000305176	2.33e+05	0
24	100	11667.2	0.000610352	1.48e+05	0
25	104	11558.6	0.0012207	3.55e+05	0
26	108	11335.5	0.00244141	1.57e+05	0
27	112	10863.8	0.00488281	6.36e+05	0
28	116	9797.14	0.00976562	2.53e+05	0
29	120	6882.83	0.0195312	9.18e+05	0
30	124	6882.83	0.0373993	9.18e+05	0
31	128	3218.45	0.00934981	1.96e+06	0
32	132	612.703	0.0186996	3.01e+06	0
33	136	13.0998	0.0253882	3.05e+06	0
34	140	0.0762922	0.00154002	5.05e+04	0
35	144	0.0731652	3.61102e-06	29.9	0
36	148	0.0731652	6.32335e-08	0.063	0

Local minimum possible.

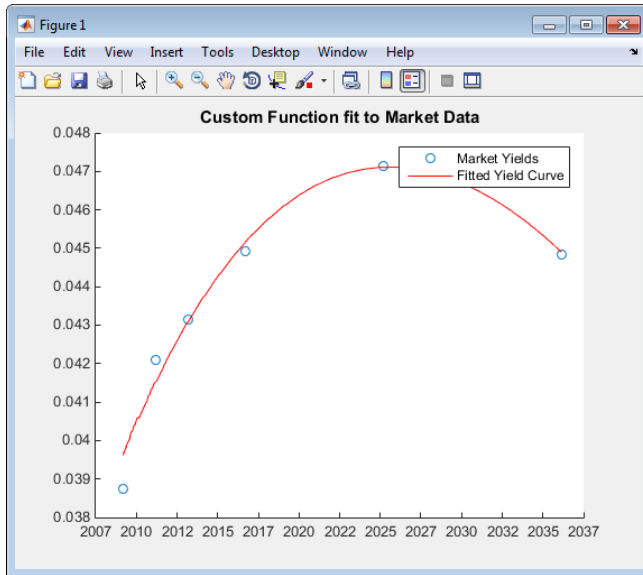
lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the default value of the function tolerance.

Plot the custom function that is defined using fitFunction:

```

Yields = bndyield(CleanPrice,CouponRate,Settle(1),Maturity);
scatter(Maturity,Yields);
PlottingPoints = min(Maturity):30:max(Maturity);
hold on;
plot(PlottingPoints, getParYields(CustomModel, PlottingPoints),'r');
datetick
legend('Market Yields','Fitted Yield Curve')
title('Custom Function fit to Market Data')

```



## See Also

[IRBootstrapOptions](#) | [IRDataCurve](#) | [IRFitOptions](#) | [IRFunctionCurve](#)

## Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4
- “Bootstrapping a Swap Curve” on page 9-13
- “Dual Curve Bootstrapping” on page 9-17
- “Creating an IRDataCurve Object” on page 9-6
- “Converting an IRDataCurve or IRFunctionCurve Object” on page 9-41
- “Analysis of Inflation Indexed Instruments”

- “Fitting Interest Rate Curve Functions” on page 9-33

### **More About**

- “Interest-Rate Curve Objects and Workflow” on page 9-2

### **External Websites**

- Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)
- Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk Applications (30 min 00 sec)

## Fitting Interest Rate Curve Functions

This example shows how to use `IRFunctionCurve` objects to model the term structure of interest rates (also referred to as the yield curve). This can be contrasted with modeling the term structure with vectors of dates and data and interpolating between the points (which can currently be done with the function `prbyzero`).

The term structure can refer to at least three different curves: the discount curve, zero curve, or forward curve.

The `IRFunctionCurve` object allows you to model an interest-rate curve as a function.

This example explores using `IRFunctionCurve` to model the default-free term structure of interest rates in the United Kingdom. Three different forms for the term structure are implemented and are discussed in more detail later:

- Nelson-Siegel
- Svensson
- Smoothing Cubic Spline with a so-called Variable Roughness Penalty (VRP)

### Choosing the Data

The first question in modeling the yield curve is what data should be used. To model a default-free yield curve, default-free, option-free market instruments must be used. The most significant component of the data is UK Government Bonds (known as Gilts). Historical data is retrieved from the following site:

<https://www.dmo.gov.uk>

Repo data is used to construct the short end of the yield curve. Repo data is retrieved from the following site:

<https://www.ukfinance.org.uk/>

Note also that the data must be specified as a matrix where the columns are `Settle`, `Maturity`, `CleanPrice`, and `CouponRate` and that instruments must be bonds or synthetically converted to bonds.

Market data for a close date of April 30, 2008, has been downloaded and saved to the following data file (`ukdata20080430`), which is loaded into MATLAB® with the following command:

```
% Load the data
load ukdata20080430

% Convert repo rates to be equivalent zero coupon bonds
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);

% Aggregate the data
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];

CurveSettle = datenum('30-Apr-2008');
```

### Fit Nelson-Siegel Model to Market Data

The Nelson-Siegel model proposes that the instantaneous forward curve can be modeled with the following:

$$f = \beta_0 + \beta_1 e^{-\frac{m}{\tau}} + \beta_2 e^{-\frac{m}{\tau}} \frac{m}{\tau}$$

This can be integrated to derive an equation for the zero curve (see [6] for more information on the equations and the derivation):

$$s = \beta_0 + (\beta_1 + \beta_2) \frac{\tau}{m} (1 - e^{-\frac{m}{\tau}}) - \beta_2 e^{-\frac{m}{\tau}}$$

See [1] for more information.

The `IRFunctionCurve` object provides the capability to fit a Nelson Siegel curve to observed market data with the `fitNelsonSiegel` method. The fitting is done by calling the Optimization Toolbox™ function `lsqnonlin`.

The `fitNelsonSiegel` method has required inputs for Curve Type, Curve Settle, and a matrix of instrument data.

Optional input arguments, specified in name-value pair argument, are:



- `IRFitOptions` structure: Provides the capability to choose which quantity to be minimized (price, yield, or duration weighted price) and other optimization parameters (for example, upper and lower bounds for parameters).
- Curve Compounding and Basis (day-count convention)
- Additional instrument parameters, `Period`, `Basis`, `FirstCouponDate`, and so on.

```
NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',CurveSettle,...
    Instruments,'InstrumentPeriod',InstrumentPeriod);
```

### Fit Svensson Model

A very similar model to the Nelson-Siegel model is the Svensson model, which adds two additional parameters to account for greater flexibility in the term structure. This model proposes that the forward rate can be modeled with the following form:

$$f = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 e^{-\frac{m}{\tau_1}} \frac{m}{\tau_1} + \beta_3 e^{-\frac{m}{\tau_2}} \frac{m}{\tau_2}$$

As above, this can be integrated to derive an equation for the zero curve:

$$s = \beta_0 + \beta_1 (1 - e^{-\frac{m}{\tau_1}}) \left(-\frac{\tau_1}{m}\right) + \beta_2 \left((1 - e^{-\frac{m}{\tau_1}}) \frac{\tau_1}{m} - e^{-\frac{m}{\tau_1}}\right) + \beta_3 \left((1 - e^{-\frac{m}{\tau_2}}) \frac{\tau_2}{m} - e^{-\frac{m}{\tau_2}}\right)$$

See [2] for more information.

Fitting the parameters to this model proceeds in a similar fashion to the Nelson-Siegel model using the `fitSvensson` method.

```
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',CurveSettle,...
    Instruments,'InstrumentPeriod',InstrumentPeriod);
```

### Fit Smoothing Spline

The term structure can also be modeled with a spline, specifically, one way to model the term structure is by representing the forward curve with a cubic spline. To ensure that the spline is sufficiently smooth, a penalty is imposed relating to the curvature (second derivative) of the spline:

$$\sum_{i=1}^N \left[ \frac{P_i - \hat{P}_i(f)}{D_i} \right]^2 + \int_0^M \lambda_t(m) [f''(m)]^2 dm$$

where the first term is the difference between the observed price  $P$  and the predicted price,  $P_{hat}$ , (weighted by the bond's duration,  $D$ ) summed over all bonds in the data set, and the second term is the penalty term (where  $lambda$  is a penalty function and  $f$  is the spline).

See [3], [4], [5] below.

There have been different proposals for the specification of the penalty function  $lambda$ . One approach, advocated by [4], and currently used by the UK Debt Management Office, is a penalty function of the following form:

$$\log(\lambda(m)) = L - (L - S)e^{-\frac{m}{\mu}}$$

The parameters  $L$ ,  $S$ , and  $mu$  are typically estimated from historical data.

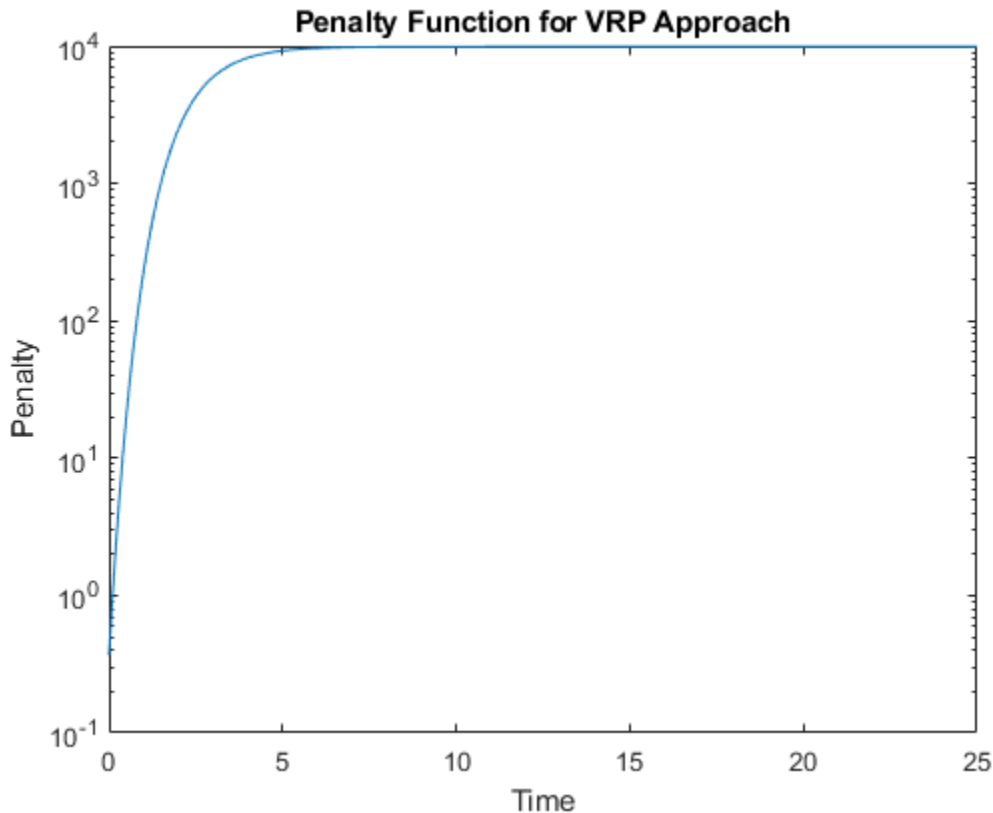
The `IRFunctionCurve` object can be used to fit a smoothing spline representation of the forward curve with a penalty function using the method `fitSmoothingSpline`.

Required inputs, like for the methods above, are a `CurveType`, `CurveSettle`, `Instruments` matrix, and a function handle (`Lambdafun`) containing the penalty function.

The optional parameters are similar to `fitNelsonSiegel` and `fitSvensson`.

```
% Parameters chosen to be roughly similar to [4] below.
L = 9.2;
S = -1;
mu = 1;

lambdafun = @(t) exp(L - (L-S)*exp(-t/mu)); % Construct penalty function
t = 0:.1:25; % Construct data to plot penalty function
y = lambdafun(t);
figure
semilogy(t,y);
title('Penalty Function for VRP Approach')
ylabel('Penalty')
xlabel('Time')
```



```
VRPModel = IRFunctionCurve.fitSmoothingSpline('Forward',CurveSettle,...
    Instruments,lambdafun,'Compounding',-1,...
    'InstrumentPeriod',InstrumentPeriod);
```

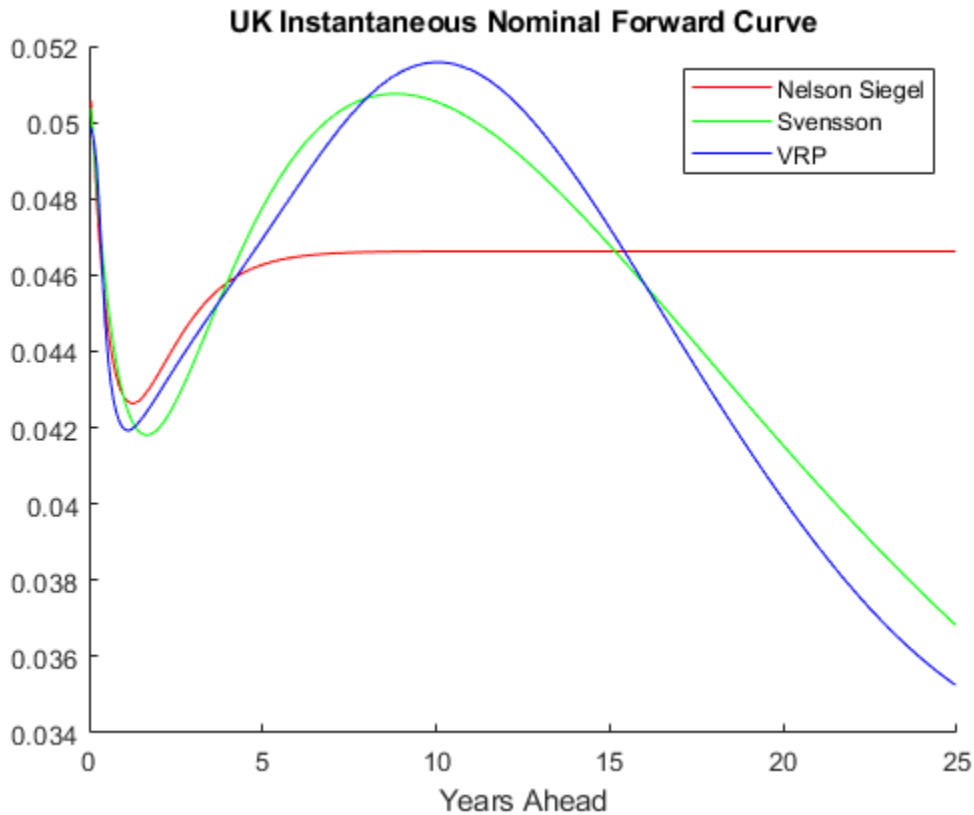
### Use Fitted Curves and Plot Results

Once a curve is constructed, methods are used to extract the Forward and Zero Rates and the Discount Factors. This curve can also be converted into a `RateSpec` structure using the `toRateSpec` method. The `RateSpec` can then be used with many other functions in the Financial Instruments Toolbox™

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
```

```
NSForwardRates = NSModel.getForwardRates(PlottingDates);  
SvenssonForwardRates = SvenssonModel.getForwardRates(PlottingDates);  
VRPForwardRates = VRPModel.getForwardRates(PlottingDates);
```

```
figure  
hold on  
plot(TimeToMaturity,NSForwardRates,'r')  
plot(TimeToMaturity,SvenssonForwardRates,'g')  
plot(TimeToMaturity,VRPForwardRates,'b')  
title('UK Instantaneous Nominal Forward Curve')  
xlabel('Years Ahead')  
legend({'Nelson Siegel', 'Svensson', 'VRP'})
```



## Compare with this Link

This link provides a live look at the derived yield curve published by the UK

<https://www.bankofengland.co.uk>

## Bibliography

This example is based on the following papers and journal articles:

[1] Nelson, C.R., Siegel, A.F., (1987), "Parsimonious Modelling of Yield Curves." *Journal of Business*. 60, pp 473-89.

[2] Svensson, L.E.O. (1994), "Estimating and Interpreting Forward Interest Rates: Sweden 1992-4." International Monetary Fund, IMF Working Paper, 1994/114.

[3] Fisher, M., Nychka, D., Zervos, D. (1995), "Fitting the Term Structure of Interest Rates with Smoothing Splines." Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper, 95-1.

[4] Anderson, N., Sleath, J. (1999), "New Estimates of the UK Real and Nominal Yield Curves." *Bank of England Quarterly Bulletin*. November, pp 384-92.

[5] Waggoner, D. (1997), "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices." Federal Reserve Board Working Paper, 97-10.

[6] "Zero-Coupon Yield Curves: Technical Documentation." BIS Papers No. 25, October 2005.

[7] Bolder, D.J., Gusba, S (2002), "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada." Working Papers 02-29, Bank of Canada.

[8] Bolder, D.J., Streliski, D (1999), "Yield Curve Modelling at the Bank of Canada." Technical Reports 84, Bank of Canada.

## See Also

[IRBootstrapOptions](#) | [IRDataCurve](#) | [IRFitOptions](#) | [IRFunctionCurve](#)

### Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4
- “Bootstrapping a Swap Curve” on page 9-13
- “Dual Curve Bootstrapping” on page 9-17
- “Creating an IRDataCurve Object” on page 9-6
- “Converting an IRDataCurve or IRFunctionCurve Object” on page 9-41
- “Analysis of Inflation Indexed Instruments”

### More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2

### External Websites

- Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)
- Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk Applications (30 min 00 sec)

## Converting an IRDataCurve or IRFunctionCurve Object

### In this section...

“Introduction” on page 9-41

“Using the toRateSpec Method” on page 9-41

“Using Vector of Dates and Data Methods” on page 9-43

### Introduction

The IRDataCurve and IRFunctionCurve objects for interest-rate curves support conversion to:

- A RateSpec structure. The RateSpec generated from an IRDataCurve or IRFunctionCurve object, using the toRateSpec method, is identical to the RateSpec structure created with intenvset using Financial Instruments Toolbox software.
- A vector of dates and data from an IRDataCurve object acceptable to prbyzero, bkcall, bkput, tfutbyprice, and tfutbyyield or any function that requires a term structure of interest rates.

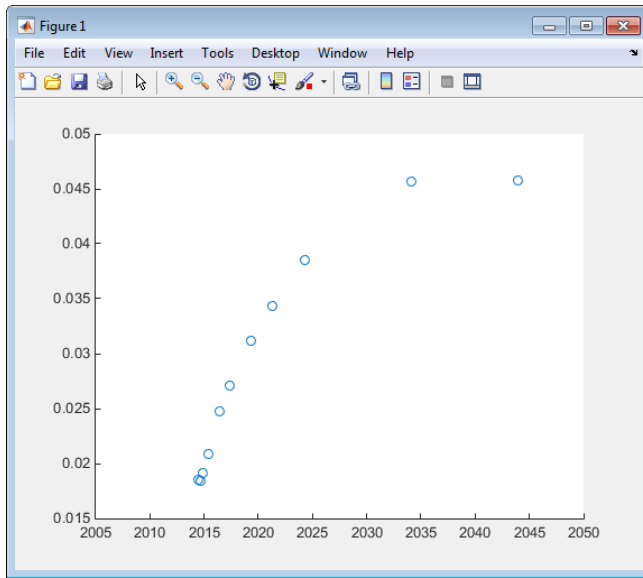
### Using the toRateSpec Method

To convert an IRDataCurve or IRFunctionCurve object to a RateSpec structure, you must first create an interest-rate curve object. Then, use the toRateSpec method for an IRDataCurve object or the toRateSpec method for an IRFunctionCurve object.

### Example

Create a data vector from the following data: <https://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml>:

```
Data = [1.85 1.84 1.91 2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[30 90 180 360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],2);
scatter(Dates,Data)
datetick
```



Create an IRDataCurve interest-rate curve object:

```
rr = IRDataCurve('Zero', today, Dates, Data);
```

Convert to a RateSpec:

```
toRateSpec(rr, today+30:30:today+365)
```

ans =

```

    FinObj: 'RateSpec'
  Compounding: 2
        Disc: [12x1 double]
        Rates: [12x1 double]
    EndTimes: [12x1 double]
  StartTimes: [12x1 double]
    EndDates: [12x1 double]
  StartDates: 733569
ValuationDate: 733569
        Basis: 0
  EndMonthRule: 1

```



## Using Vector of Dates and Data Methods

You can use the `getZeroRates` method for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `prbyzero` in Financial Toolbox software and `bkcall`, `bkput`, `tfutbyprice`, and `tfutbyyield` in Financial Instruments Toolbox software.

### Example

This is an example of using the `IRDataCurve` method `getZeroRates` with `prbyzero`:

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates>Data,'InterpMethod','pchip');
Maturity = daysadd(today,8*360,1);
CouponRate = .055;
ZeroDates = daysadd(today,180:180:8*360,1);
ZeroRates = getZeroRates(irdc, ZeroDates);
BondPrice = prbyzero([Maturity CouponRate], today, ZeroRates, ZeroDates)

BondPrice =
    113.9250
```

## See Also

[IRBootstrapOptions](#) | [IRDataCurve](#) | [IRFitOptions](#) | [IRFunctionCurve](#)

## Related Examples

- “Creating an `IRFunctionCurve` Object” on page 9-22
- “Dual Curve Bootstrapping” on page 9-17
- “Analysis of Inflation Indexed Instruments”
- “Fitting Interest Rate Curve Functions” on page 9-33

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2



# Numerix Workflows

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- “Working with Simple Numerix Trades” on page 10-2
- “Working with Advanced Numerix Trades” on page 10-5
- “Use Numerix to Price Cash Deposits” on page 10-10
- “Use Numerix for Interest-Rate Risk Assessment” on page 10-13
- “Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects” on page 10-16

## Working with Simple Numerix Trades

This example shows how to price a callable reverse floater using Numerix CROSSASSET.

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;

n = numerix('i:\NumeriX_java_10_3_0\data')

n =
    Path: 'i:\NumeriX_java_10_3_0\data'
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
    Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
    Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
    LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
    MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
    FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings'
    TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Create a market.

```
quotes = java.util.HashMap;
quotes.put('IR.USD-LIBOR-3M.SWAP-1Y.MID', 0.0066056);
quotes.put('IR.USD-LIBOR-3M.SWAP-10Y.MID', 0.022465005);
quotes.put('IR.USD-LIBOR-3M.SWAP-20Y.MID', 0.027544995);
market = Market('EOD_14-NOV-2011', DateExtensions.date('14-Nov-2011'), quotes.entrySet());
```

Define a trade instance for a callable reverse floater based on instrument template located in the Repository.

```
tradeDescriptor = 'TRADE.IR.CALLBLEREVERSEFLOATER';
tradeParameters = java.util.HashMap;
tradeParameters.put('Trade ID', '1001');
tradeParameters.put('Quote Type', 'MID');
tradeParameters.put('Currency', 'USD');
tradeParameters.put('Notional', 1000000.0);
tradeParameters.put('Effective Date', DateExtensions.date('1-Dec-2011'));
tradeParameters.put('Termination Date', DateExtensions.date('1-Dec-2021'));
tradeParameters.put('IR Index', 'LIBOR');
tradeParameters.put('IR Index Tenor', '3M');
tradeParameters.put('Structured Freq', '3M');
tradeParameters.put('Structured Side', 'Receive');
tradeParameters.put('Structured Coupon Floor', 0.0);
tradeParameters.put('Structured Coupon UpBd', 0.08);
tradeParameters.put('StructuredCoupon Multiplier', 1.4);
tradeParameters.put('Structured Coupon Cap', 0.05);
tradeParameters.put('Structured Basis', 'ACT/360');
tradeParameters.put('Funding Freq', '3M');
tradeParameters.put('Funding Side', 'Pay');
```

```

tradeParameters.put('Funding Spread', 0.003);
tradeParameters.put('Funding Basis', 'ACT/360');
tradeParameters.put('Call Start Date', DateExtensions.date('1-Dec-2013'));
tradeParameters.put('Call End Date', DateExtensions.date('1-Dec-2020'));
tradeParameters.put('Option Side', 'Short');
tradeParameters.put('Option Type', 'Right to Terminate');
tradeParameters.put('Call Frequency', '3M');
tradeParameters.put('Model', 'IR.USD-LIBOR-3M.MID.DET');
tradeParameters.put('Method', 'BackwardAnalytic');

```

Create the trade instance.

```
trade = RepositoryExtensions.createTradeInstance(n.Repository, tradeDescriptor, tradeParameters)
```

Price the trade.

```
results = CalculationContextExtensions.calculate(n.Context, trade, market, Request.getAll());
```

Parse the results for MATLAB and display.

```

r = n.parseResults(results)
disp([r.Name r.Category r.Currency r.Data])

```

r =

```

Category: {13x1 cell}
Currency: {13x1 cell}
Name: {13x1 cell}
Data: {13x1 cell}

'Reporting Currency'      'Price'      ''      'USD'
'Structured Cashflow Log' 'Cashflow'   ''      {41x20 cell}
'Structured Leg PV Accrued' 'Price'      'USD'   [ 0]
'PV'                     'Price'      'USD'   [ 6.4133e+04]
'Structured Leg PV Clean' 'Price'      'USD'   [ 4.2637e+05]
'Option PV'              'Price'      'USD'   [-1.3220e+05]
'Funding Cashflow Log'   'Cashflow'   ''      {41x20 cell}
'Structured Leg PV'      'Price'      'USD'   [ 4.2637e+05]
'Funding Leg PV'        'Price'      'USD'   [-2.3004e+05]
'Funding Leg PV Accrued' 'Price'      'USD'   [ 0]
'Funding Leg PV Clean'   'Price'      'USD'   [-2.3004e+05]
'Yield Risk Report'     ''           ''      { 4x30 cell}
'Messages'              ''           ''      { 4x1 cell}

```

## See Also

numerix | numerixCrossAsset | parseResults

## Related Examples

- “Working with Advanced Numerix Trades” on page 10-5
- “Use Numerix to Price Cash Deposits” on page 10-10

- “Use Numerix for Interest-Rate Risk Assessment” on page 10-13

### **External Websites**

- <https://www.numerix.com/CrossAsset>

## Working with Advanced Numerix Trades

This example shows how to price multiple trades from MATLAB using Numerix CROSSASSET.

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;

n = numerix('i:\NumeriX_java_10_3_0\data')

n =
    Path: 'i:\NumeriX_java_10_3_0\data'
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
    Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
    Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
    LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
    MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
    FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings'
    TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Specify the hybrid model for multiple trades.

```
hySpec = HybridModelSpecification;
hySpec.addHW1F('IR-USD', 'USD', 'LIBOR', '3M', 'MeanReversion(0.5),DiagonalSwaption(ATM, 10Y)');
hySpec.addHW1F('IR-EUR', 'EUR', 'EURIBOR', '6M', 'MeanReversion(0.5),DiagonalSwaption(ATM, 10Y)');
hySpec.addFXBlack('FX-USDEUR', 'USD', 'EUR', 'LIBOR', '3M', 'EURIBOR', '6M', 'StrikeFXEuropean(ATM, 10Y)');
% 5 Specify the factor correlations.
hyCorrelations = HybridModelCorrelationMatrix(hySpec);
hyCorrelations.add('IR-USD', 'IR-EUR', 0.5);
hyCorrelations.add('IR-USD', 'FX-USDEUR', 0.25);
hyCorrelations.add('IR-EUR', 'FX-USDEUR', 0.25);

% Specify the model parameters.
hybridModelParameters = java.util.HashMap;
hybridModelParameters.put('Quote Type', 'MID');
hybridModelParameters.put('Payout Currency', 'USD');
hybridModelParameters.put('Specification', hySpec);
hybridModelParameters.put('Correlations', hyCorrelations);
```

Specify exposure calculation parameters.

```
observationDates = CustomObservationSchedule;
observationDates.add(DateExtensions.date(2011, 12, 1));
for y = 2012:2013
    for m = 1:12
        observationDates.add(DateExtensions.date(y, m, 1));
    end
end

exposureParameters = java.util.HashMap;
exposureParameters.put('Model ID', 'HYBRID');
exposureParameters.put('Observation Dates', observationDates);
```

Define the first trade instance.

```
tradeParameters1 = java.util.HashMap;  
tradeParameters1.put('Trade ID', 'RVFL1001');  
tradeParameters1.put('Quote Type', 'MID');  
tradeParameters1.put('Currency', 'USD');  
tradeParameters1.put('Notional', 1000000.0);  
tradeParameters1.put('Effective Date', DateExtensions.date('1-Dec-2011'));  
tradeParameters1.put('Termination Date', DateExtensions.date('1-Dec-2021'));  
tradeParameters1.put('IR Index', 'LIBOR');  
tradeParameters1.put('IR Index Tenor', '3M');  
tradeParameters1.put('Structured Freq', '3M');  
tradeParameters1.put('Structured Side', 'Receive');  
tradeParameters1.put('Structured Coupon Floor', 0.0);  
tradeParameters1.put('Structured Coupon UpBd', 0.08);  
tradeParameters1.put('StructuredCoupon Multiplier', 1.4);  
tradeParameters1.put('Structured Coupon Cap', 0.05);  
tradeParameters1.put('Structured Basis', 'ACT/360');  
tradeParameters1.put('Funding Freq', '3M');  
tradeParameters1.put('Funding Side', 'Pay');  
tradeParameters1.put('Funding Spread', 0.003);  
tradeParameters1.put('Funding Basis', 'ACT/360');  
tradeParameters1.put('Call Start Date', DateExtensions.date('1-Dec-2013'));  
tradeParameters1.put('Call End Date', DateExtensions.date('1-Dec-2020'));  
tradeParameters1.put('Option Side', 'Short');  
tradeParameters1.put('Option Type', 'Right to Terminate');  
tradeParameters1.put('Call Frequency', '3M');  
tradeParameters1.put('Model', 'HYBRID');  
tradeParameters1.put('Method', 'BackwardMC');  
tradeInstance1 = RepositoryExtensions.createTradeInstance(n.Repository, 'TRADE.IR.CALLABLEREVERSEFLOATER', tradeParameters1);
```

Define the second trade instance.

```
tradeParameters2 = java.util.HashMap;  
tradeParameters2.put('Trade ID', 'CASHDEP1001');  
tradeParameters2.put('Quote Type', 'MID');  
tradeParameters2.put('Currency', 'USD');  
tradeParameters2.put('Coupon Rate', 0.05);  
tradeParameters2.put('Yield', 0.044);  
tradeParameters2.put('Notional', 100.0);  
tradeParameters2.put('Effective Date', DateExtensions.date('1-Apr-2012'));  
tradeParameters2.put('Maturity', DateExtensions.date('1-Apr-2013'));  
tradeParameters2.put('IR Index', 'LIBOR');  
tradeParameters2.put('IR Index Tenor', '3M');  
tradeParameters2.put('Model', 'HYBRID');  
tradeParameters2.put('Method', 'BACKWARDMC');  
tradeInstance2 = RepositoryExtensions.createTradeInstance(n.Repository, 'IR.CASHDEPOSIT', tradeParameters2);
```

Create the third trade instance.

```
tradeParameters3 = java.util.HashMap;  
tradeParameters3.put('Trade ID', 'FXFWD1001');  
tradeParameters3.put('Quote Type', 'MID');  
tradeParameters3.put('Base Currency', 'USD');  
tradeParameters3.put('Term Currency', 'EUR');  
tradeParameters3.put('Delivery Date', DateExtensions.date('1-Jun-2012'));  
tradeParameters3.put('Contract FX Forward Rate', 80.5);  
tradeParameters3.put('Base Notional', 10000000.0);  
tradeParameters3.put('Base IR Index', 'LIBOR');  
tradeParameters3.put('Term IR Index', 'EURIBOR');  
tradeParameters3.put('Base IR Index Tenor', '3m');
```



```

tradeParameters3.put('Term IR Index Tenor', '6m');
tradeParameters3.put('Calendar', 'NewYork Target');
tradeParameters3.put('Spot Lag', '2bd');
tradeParameters3.put('Model', 'HYBRID');
tradeParameters3.put('Method', 'BACKWARDMC');
tradeInstance3 = RepositoryExtensions.createTradeInstance(n.Repository, 'FX.FXFORWARD', tradeParameters3);

```

Set `tradeInstances` for all three trade instances.

```

tradeInstances = java.util.ArrayList;
tradeInstances.add(tradeInstance1);
tradeInstances.add(tradeInstance2);
tradeInstances.add(tradeInstance3);
n.Parameters.setInstances(tradeInstances);

```

Add a custom lookup so these trade instances reference the hybrid model.

```
n.Parameters.getLookups.add(0, ExactLookupRule('HYBRID', 'MODEL.HYBRID', hybridModelParameters.entrySet));
```

Add another custom lookup so that exposure report has parameters defined.

```
n.Parameters.getLookups.add(1, ExactLookupRule('RISK.REPORT.EXPOSURE', 'REPORT.EXPOSURE', exposureParameters.entrySet));
```

Perform the calculation.

```
results = n.Context.calculate(n.Parameters, Request.getExposure);
```

Parse the results for MATLAB and display.

```

r = n.parseResults(results)

disp([r.Trade(2) r.Market(2)])
disp([r.Results{2}.Name r.Results{2}.Category r.Results{2}.Currency r.Results{2}.Data])
disp([r.Results{2}.Name{1}])
disp([r.Results{2}.Data{1}])

```

r =

```

Trade: {3x1 cell}
Market: {3x1 cell}
Results: {3x1 cell}

```

```
'CASHDEP1001' 'EOD'
```

```

'Exposure' '' '' {21x501 cell}
'Exposure.Discount Factors' '' '' {21x501 cell}
'Messages' '' '' {12x1 cell}

```

Exposure

Columns 1 through 3

'DATE'	'VALUE 1'	'VALUE 2'
'Tue May 01 13:00:00 EDT 2012'	[104.198166609924]	[103.386222783828]
'Fri Jun 01 13:00:00 EDT 2012'	[ 104.09953599675]	[102.117465067435]
'Sun Jul 01 13:00:00 EDT 2012'	[105.524567506006]	[100.055731577867]
'Wed Aug 01 13:00:00 EDT 2012'	[105.787455961524]	[100.318762976796]
'Sat Sep 01 13:00:00 EDT 2012'	[104.417483614373]	[100.764337265155]
'Mon Oct 01 13:00:00 EDT 2012'	[104.692275556824]	[100.980213613911]
'Thu Nov 01 13:00:00 EDT 2012'	[104.443818312902]	[101.478508725115]

```
'Sat Dec 01 12:00:00 EST 2012' [104.736646932343] [101.679769557039]
'Tue Jan 01 12:00:00 EST 2013' [104.577562970494] [102.423339265735]
'Fri Feb 01 12:00:00 EST 2013' [ 104.28994278039] [103.117326879887]
'Fri Mar 01 12:00:00 EST 2013' [ 104.70469459715] [104.232180198939]
'Mon Apr 01 13:00:00 EDT 2013' [ 105.07334321718] [ 105.05089338769]
'Wed May 01 13:00:00 EDT 2013' [ 0] [ 0]
'Sat Jun 01 13:00:00 EDT 2013' [ 0] [ 0]
'Mon Jul 01 13:00:00 EDT 2013' [ 0] [ 0]
'Thu Aug 01 13:00:00 EDT 2013' [ 0] [ 0]
'Sun Sep 01 13:00:00 EDT 2013' [ 0] [ 0]
'Tue Oct 01 13:00:00 EDT 2013' [ 0] [ 0]
'Fri Nov 01 13:00:00 EDT 2013' [ 0] [ 0]
'Sun Dec 01 12:00:00 EST 2013' [ 0] [ 0]
```

Columns 499 through 501

```
'VALUE 498' 'VALUE 499' 'VALUE 500'
[ 105.36273206453] [104.335982034187] [104.141595030057]
[105.904822463264] [104.238089023172] [104.276676080686]
[103.893060436208] [103.613968079212] [106.188617261199]
[103.183889382889] [105.499763150412] [105.440275818983]
[103.310404527817] [105.233622768447] [105.267337892552]
[103.274239052394] [104.716952177783] [ 104.33099834332]
[103.583983117053] [104.710250522521] [105.501004542869]
[103.379982561438] [105.146939039653] [104.681616459661]
[103.821169954095] [105.567274949306] [104.835971977691]
[104.016530403399] [105.254054161819] [104.842156238753]
[104.481475787501] [105.197179985119] [104.962752610848]
[105.061984636083] [105.077227736476] [105.077766765965]
[ 0] [ 0] [ 0]
[ 0] [ 0] [ 0]
[ 0] [ 0] [ 0]
[ 0] [ 0] [ 0]
[ 0] [ 0] [ 0]
[ 0] [ 0] [ 0]
[ 0] [ 0] [ 0]
[ 0] [ 0] [ 0]
```

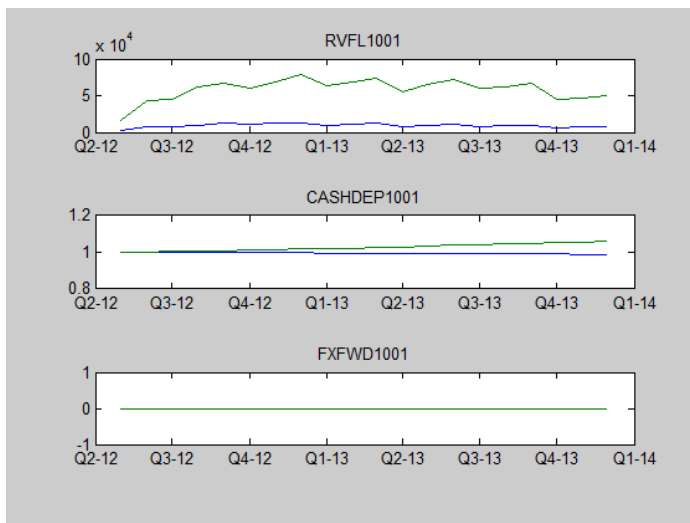
Plot the results for the second trade instance, CASHDEP1001, with the corresponding Exposure Discount Factors.

```
figure('Tag', 'NumerixAdvancedRiskExample');
for ii=1:3
    % Get dates
    dates = cell2mat(r.Results{ii}.Data{expIndex}(2:end,1));
    dates = dates(:,4:end);
    dates = floor(datenum(dates));
    % Get exposures
    mtm = cell2mat(r.Results{ii}.Data{expIndex}(2:end,2:end));
    exposures = max(0,mtm); % Exposure at contract level, no netting
    EE = mean(exposures); % Expected Exposure
    PFE = prctile(exposures,95); % Potential Future Exposure
    subplot(3,1,ii)
    plot(dates,EE,dates,PFE)
```

```

title(r.Trade{ii})
datetick
end

```



## See Also

`numerix` | `numerixCrossAsset` | `parseResults`

## Related Examples

- “Working with Simple Numerix Trades” on page 10-2
- “Use Numerix to Price Cash Deposits” on page 10-10
- “Use Numerix for Interest-Rate Risk Assessment” on page 10-13

## External Websites

- <https://www.numerix.com/CrossAsset>

## Use Numerix to Price Cash Deposits

This example shows how to use the Numerix CROSSASSET API to price a cash deposit from MATLAB. The trade parameters are read from the `Cashdeposit1.csv` in the Numerix Data Trades folder.

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;

n = numerix('i:\NumeriX_java_10_3_0\data')

n =
    Path: 'i:\NumeriX_java_10_3_0\data'
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
    Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
    Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
    LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
    MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
    FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings'
    TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Create a market.

```
market = Market('EOD_16-APR-2012', DateExtensions.date('16-APR-2012'), []);
```

Read the `Cashdeposit1.csv` file from the Numerix Trades folder.

```
[~,~,tradeInfo] = xlsread([n.TradesPath '\Cashdeposit1.csv'])
```

```
tradeInfo =
    'Template'      'String'      'TRADE_IR.CASHDEPOSIT'
    'Trade ID'     'ID'          'CASHDEP1001'
    'Quote Type'   'String'      'MID'
    'Effective Date' 'Date'        '4/1/2012'
    'Maturity'     'Date'        '4/1/2013'
    'Notional'     'Double'      [          100]
    'Currency'     'Currency'    'USD'
    'Coupon Rate'  'Double'      [          0.0500]
    'Yield'        'Double'      [          0.0440]
    'IR Index'     'String'      'Libor'
    'IR Index Tenor' 'Tenor'      '3m'
```

Define a trade instance from the imported CASHDEP1001 instrument.

```
tradeDescriptor = tradeInfo{1,3};
tradeParameters = java.util.HashMap;
numTradeInfoFields = size(tradeInfo,1);
for i = 2:numTradeInfoFields
    switch tradeInfo{i,2}
```

```

case {'DATE', 'Date'}
    tradeParameters.put(tradeInfo{i,1},DateExtensions.date(datestr(tradeInfo{i,3}, 'dd-mmm-yyyy')));
otherwise
    tradeParameters.put(tradeInfo{i,1},tradeInfo{i,3});
end
end

```

Create the trade instance.

```
trade = RepositoryExtensions.createTradeInstance(n.Repository, tradeDescriptor, tradeParameters);
```

Price the trade.

```
results = CalculationContextExtensions.calculate(n.Context, trade, market, Request.getAll);
```

Parse the results for MATLAB and display.

```
r = n.parseResults(results)
disp([r.Name r.Category r.Currency r.Data])
```

```

r =
Category: {9x1 cell}
Currency: {9x1 cell}
Name: {9x1 cell}
Data: {9x1 cell}

'Modified Duration'    'Price'    ''    [ 0.9349]
'Accrued Interest'    'Price'    'USD'  [ 0.2083]
'Reporting Currency'  'Price'    ''    'USD'
'PV'                  'Price'    'USD'  [ 100.7607]
'Instrument'          'Price'    ''    [1x85 char]
'Clean Price'         'Price'    'USD'  [ 100.5524]
'Convexity'          'Price'    ''    [ 1.7481]
'YTM'                 'Price'    ''    []
'Messages'            ''    ''    []

```

## See Also

[numerix](#) | [numerixCrossAsset](#) | [parseResults](#)

## Related Examples

- “Working with Simple Numerix Trades” on page 10-2
- “Working with Advanced Numerix Trades” on page 10-5
- “Use Numerix for Interest-Rate Risk Assessment” on page 10-13

## **External Websites**

- <https://www.numerix.com/CrossAsset>

## Use Numerix for Interest-Rate Risk Assessment

This example shows how to use the Numerix CROSSASSET API for interest-rate curve stripping for risk assessment.

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;

n = numerix('i:\NumeriX_java_10_3_0\data')

n =
    Path: 'i:\NumeriX_java_10_3_0\data'
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
    Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
    Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
    LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
    MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
    FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings'
    TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Specify the current market associated with the Numerix CROSSASSET environment.

```
markets = get(n.Parameters, 'Markets');
currentMarket = markets.get(0);
outInstance = RefObject(currentMarket);
```

Define the interest-rate curve key IR.USD-LIBOR-3M.MID.

```
n.Context.tryResolveId('IR.USD-LIBOR-3M.MID', outInstance);
currentInstance = outInstance.argvalue;
```

Set the instance and market.

```
n.Parameters.setMarkets(java.util.Arrays.asList(currentMarket));
n.Parameters.setInstances(java.util.Arrays.asList(currentInstance));
```

Calculate the interest-rate curve stripping.

```
results = n.Context.calculate(n.Parameters, Request.getAll());
```

The calculation returns the results from stripping the interest-rate curve for IR.USD-LIBOR-3M.MID. Parse the results for MATLAB and display.

```
% IR.USD-LIBOR-3M.MID
r = n.parseResults(results)

disp([r.Instance r.Market])
disp([r.Results{1}.Name r.Results{1}.Category r.Results{1}.Currency r.Results{1}.Data])
```

```
disp([r.Results{1}.Name{1}])
disp([r.Results{1}.Data{1}])

r =

    Instance: {'IR.USD-LIBOR-3M.MID'}
      Market: {'EOD'}
      Results: {[1x1 struct]}

'IR.USD-LIBOR-3M.MID'    'EOD'

'Curve Info'    ''    ''    {30x3 cell}
'Messages'     ''    ''    { 7x1 cell}
```

Curve Info

'KEY'	'DATE'	'DISCOUNTFACTOR'
'CASH RATE 16-APR-2012 17-APR-2012'	'Tue Apr 17 13:00:00 EDT 2012'	[ 1.0000]
'CASH RATE 16-APR-2012 18-APR-2012'	'Wed Apr 18 13:00:00 EDT 2012'	[ 1.0000]
'CASH RATE 16-APR-2012 23-APR-2012'	'Mon Apr 23 13:00:00 EDT 2012'	[ 1.0000]
'CASH RATE 16-APR-2012 30-APR-2012'	'Mon Apr 30 13:00:00 EDT 2012'	[ 0.9999]
'CASH RATE 16-APR-2012 16-MAY-2012'	'Wed May 16 13:00:00 EDT 2012'	[ 0.9998]
'CASH RATE 16-APR-2012 18-JUN-2012'	'Mon Jun 18 13:00:00 EDT 2012'	[ 0.9994]
'CASH RATE 16-APR-2012 16-JUL-2012'	'Mon Jul 16 13:00:00 EDT 2012'	[ 0.9988]
'CASH RATE 18-MAY-2012 12-AUG-2012'	'Sun Aug 12 13:00:00 EDT 2012'	[ 0.9987]
'CASH RATE 20-JUN-2012 20-SEP-2012'	'Thu Sep 20 13:00:00 EDT 2012'	[ 0.9981]
'CASH RATE 18-JUL-2012 18-OCT-2012'	'Thu Oct 18 13:00:00 EDT 2012'	[ 0.9975]
'CASH RATE 15-AUG-2012 15-NOV-2012'	'Thu Nov 15 12:00:00 EST 2012'	[ 0.9973]
'CASH RATE 19-SEP-2012 19-DEC-2012'	'Wed Dec 19 12:00:00 EST 2012'	[ 0.9968]
'CASH RATE 17-OCT-2012 17-JAN-2013'	'Thu Jan 17 12:00:00 EST 2013'	[ 0.9962]
'CASH RATE 19-DEC-2012 19-MAR-2013'	'Tue Mar 19 13:00:00 EDT 2013'	[ 0.9955]
'SWAP RATE 18-APR-2012 19-APR-2016'	'Tue Apr 19 13:00:00 EDT 2016'	[ 0.9645]
'SWAP RATE 18-APR-2012 18-APR-2017'	'Tue Apr 18 13:00:00 EDT 2017'	[ 0.9445]
'SWAP RATE 18-APR-2012 18-APR-2018'	'Wed Apr 18 13:00:00 EDT 2018'	[ 0.9199]
'SWAP RATE 18-APR-2012 18-APR-2019'	'Thu Apr 18 13:00:00 EDT 2019'	[ 0.8925]
'SWAP RATE 18-APR-2012 21-APR-2020'	'Tue Apr 21 13:00:00 EDT 2020'	[ 0.8639]
'SWAP RATE 18-APR-2012 19-APR-2021'	'Mon Apr 19 13:00:00 EDT 2021'	[ 0.8356]
'SWAP RATE 18-APR-2012 19-APR-2022'	'Tue Apr 19 13:00:00 EDT 2022'	[ 0.8069]
'SWAP RATE 18-APR-2012 18-APR-2023'	'Tue Apr 18 13:00:00 EDT 2023'	[ 0.7784]
'SWAP RATE 18-APR-2012 18-APR-2024'	'Thu Apr 18 13:00:00 EDT 2024'	[ 0.7506]
'SWAP RATE 18-APR-2012 19-APR-2027'	'Mon Apr 19 13:00:00 EDT 2027'	[ 0.6733]
'SWAP RATE 18-APR-2012 20-APR-2032'	'Tue Apr 20 13:00:00 EDT 2032'	[ 0.5682]
'SWAP RATE 18-APR-2012 20-APR-2037'	'Mon Apr 20 13:00:00 EDT 2037'	[ 0.4828]
'SWAP RATE 18-APR-2012 21-APR-2042'	'Mon Apr 21 13:00:00 EDT 2042'	[ 0.4112]
'SWAP RATE 18-APR-2012 18-APR-2052'	'Thu Apr 18 13:00:00 EDT 2052'	[ 0.3087]
'SWAP RATE 18-APR-2012 18-APR-2062'	'Tue Apr 18 13:00:00 EDT 2062'	[ 0.2414]

## See Also

numerix | numerixCrossAsset | parseResults

## Related Examples

- “Working with Simple Numerix Trades” on page 10-2
- “Working with Advanced Numerix Trades” on page 10-5



- “Use Numerix to Price Cash Deposits” on page 10-10

### **External Websites**

- <https://www.numerix.com/CrossAsset>

## Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects

This example shows how to use the Numerix CROSSASSET API to create and price a vanilla European option.

Construct a `numerixCrossAsset` object.

```
c = numerixCrossAsset
c =
numerixCrossAsset with properties:
Application: [1x1 com.numerix.pro.Application]
ApplicationWarning: [1x1 com.numerix.pro.ApplicationWarning]
```

Create and register data as a Matrix with the Numerix Cross Asset Integration Layer Application using the `applicationMatrix` method.

```
rowData = [41992, 42020, 42449, 42905, 43115];
colData = [390, 395, 400, 405];
volData = [0.35778, 0.35132, 0.34394, 0.33582;...
           0.33405, 0.32819, 0.32669, 0.31904;...
           0.31576, 0.31235, 0.30371, 0.30261;...
           0.29391, 0.29366, 0.28962, 0.28932;...
           0.28787, NaN, 0.28347, NaN ];
applicationMatrix(c, 'BYSTRIKEVOLDATA', rowData, colData, volData);
```

Create and register the yield curve data with the Application object. Use a table for optimal display purposes. Dates must be relative to '01/01/1900' and the Numerix Cross Asset Integration Layer API supports date number representation only. MATLAB `datetime`'s get converted automatically, otherwise date numbers must be input and based relative to '01/01/1900'.

```
dates = datetime({'18-Feb-2014'; '20-May-2014'; '18-Jun-2014'; '16-Jul-2014';
                 '20-Aug-2014'; '17-Sep-2014'; '15-Oct-2014'; '19-Nov-2014';
                 '17-Dec-2014'; '18-Mar-2015'; '17-Jun-2015'; '16-Sep-2015';
                 '16-Dec-2015'; '16-Mar-2016'; '15-Jun-2016'; '21-Sep-2016';
                 '21-Dec-2016'; '15-Mar-2017'; '20-Feb-2018'; '20-Feb-2019';
                 '20-Feb-2020'; '22-Feb-2021'; '22-Feb-2022'; '21-Feb-2023';
                 '20-Feb-2024'; '20-Feb-2025'; '20-Feb-2026'; '20-Feb-2029';
                 '21-Feb-2034'; '22-Feb-2039'; '22-Feb-2044'; '20-Feb-2054';
                 '20-Feb-2064'}, 'locale', 'en_US');
```

Define the corresponding discount factors.

```
discountFactors = [1;0.99942;0.999231;0.999037;0.998797;0.998616;0.998385;...
                  0.998122;0.997941;0.997159;0.996157;0.994825;0.993065;...
                  0.99078;0.987889;0.984092;0.979913;0.975459;0.952707;...
                  0.922223;0.888128;0.852291;0.816462;0.781228;0.746677;...
                  0.712892;0.680462;0.592285;0.474003;0.383493;0.312617;...
                  0.213809;0.152345];
```

Supported Numerix Cross Asset Integration Layer API names are DATE and DISCOUNTFACTOR for the creation of the data.

```
curveData = table(dates,discountFactors,'VariableNames',{'DATE','DISCOUNTFACTOR'});
applicationData(c,'USD_3MLIBOR_CURVE',curveData);
```

Define the headers for registering the RATESPEC and DIVSPEC call objects.

```
headers = {'ID','LOCAL ID','TIMER','TIMER CPU','UPDATED'};
```

Data is required to create dividend curve. Create and register the DIVSPEC call object using name-value pairs in this example.

```
applicationCall(c,headers,'ID','DIVSPEC','OBJECT','MARKET DATA','TYPE','DIVIDEND',...
    'COMMENT','Comments here','SKIP',false,'NOWDATE',41688,...
    'CURRENCY','USD','RATE/DIVIDEND',0,'BASIS','ACT/360');
```

Create the EQUITYVOLSPEC call object. BYSTRIKEVOLDATA denotes the volatility matrix object created previously, using an array of names and an array of values in this example.

```
applicationCall(c,headers,{'ID','OBJECT','TYPE','COMMENT','SKIP','NOWDATE','CURRENCY','VOLATILITYBASIS',...
    'DATA','INTERPMETHOD','INTERPVARIBLE','EXTRAPOLATION'},...
    {'EQVOLSPEC','MARKET DATA','EQ VOL','Comments here',...
    false,41688,'USD','ACT/360','BYSTRIKEVOLDATA',...
    'LINEAR','VOL','FLAT EXTRAPOLATION'});
```

Create the RATESPEC call object. USD\_3MLIBOR\_CURVE denotes yield curve data object created previously using name-value pairs.

```
applicationCall(c,headers,'ID','RATESPEC','OBJECT','MARKET DATA','TYPE','YIELD','COMMENT','Comments here',...
    'SKIP',false,'INTERPMETHOD','LogLinear','INTERPVARIBLE','DF',...
    'CURRENCY','USD','DATA','USD_3MLIBOR_CURVE','BASIS','ACT/360');
```

Create the EuropeanOptionEQ instrument. Create the STOCKSPEC call object using the applicationCall method.

```
applicationCall(c,headers,'ID','STOCKSPEC','OBJECT','INSTRUMENT','TYPE','EQ EUROPEAN',...
    'COMMENT','Comments here','SKIP',false,'FLAVOR','PUT',...
    'CURRENCY','USD','ENDDATE',43976,'SETTLEMENTDATE',43976,...
    'STRIKE',112,'SIGMA1',0.2,'NOTIONAL',100);
```

Price the portfolio by creating and registering call object to run pricing analytics. Create the OPTIONSPEC\_CLOSEFORM call object headers for registering the OPTIONSPEC\_CLOSEFORM call object.

```
headers = {'ATM','DELTA','DELTA TRADER','FORWARD DELTA','FORWARD DELTA TRADER',...
    'FUTURES DELTA','FUTURES DELTA TRADER','GAMMA','GAMMA TRADER',...
    'ID','LOCAL ID','NOTIONAL','PRICE','PV','RHO','RHO TRADER',...
    'SIGMA1','STRIKE','THETA','TIMER','TIMER CPU','UPDATED','VANNA',...
    'VANNA TRADER','VEGA','VEGA TRADER','VOLGA','VOLGA TRADER'};

applicationCall(c,headers,'ID','OPTIONSPEC_CLOSEFORM','OBJECT','ANALYTIC',...
    'TYPE','EUROPEAN OPTION','COMMENT','Comments here',...
    'SKIP',false,'NOWDATE',41688,'OPTION','STOCKSPEC',...);
```

```
'DIVIDENDCURVE', 'DIVSPEC', 'DOMESTICYIELDCURVE', 'RATESPEC', ...
'SPOTPRICE', 112, 'SPOTDATE', 41688, 'MODEL', 'BLACK');
```

Create an output structure in MATLAB from the Application object using the `getdata` method.

```
appData = getdata(c);
```

Display the results.

```
[appData.OPTIONSPEC_CLOSEFORM.OUTPUT_HEADERS
appData.OPTIONSPEC_CLOSEFORM.OUTPUT_VALUES]

ans =

28x2 cell array

'PRICE'          [          1467.24]
'PV'             [          1467.24]
'DELTA'          [           -30.54]
'FORWARD DELTA' [           -30.54]
'FUTURES DELTA' [           -26.83]
'GAMMA'          [             0.62]
'VEGA'           [          9827.91]
'VOLGA'          [          205.45]
'VANNA'          [            -1.44]
'DELTA TRADER'  [           -34.20]
'FORWARD DELTA TRADER' [           -34.20]
'FUTURES DELTA TRADER' [           -30.05]
'GAMMA TRADER'  [             0.78]
'VEGA TRADER'   [            98.28]
'VOLGA TRADER'  [             0.02]
'VANNA TRADER'  [            -0.02]
'SIGMA1'        [             0.20]
'STRIKE'         [            112.00]
'NOTIONAL'      [            100.00]
'RHO'           [          -30638.08]
'THETA'         [            -0.15]
'RHO TRADER'    [            -3.06]
'ATM'           [            127.48]
'UPDATED'       '12 @ 01:37:24 PM'
'ID'            'OPTIONSPEC_CLOSEFORM'
'TIMER'         [             0.17]
'TIMER CPU'     [             0.06]
'LOCAL ID'     'OPTIONSPEC_CLOSEFORM'
```

Close the `numerixCrossAsset` object.

```
close(c)
```

## See Also

`applicationCall` | `applicationData` | `applicationMatrix` | `close` | `getdata` | `numerixCrossAsset`

## **External Websites**

- <https://www.numerix.com/CrossAsset>



# Class Reference

## @IRBootstrapOptions

Create specific options for bootstrapping an interest-rate curve object

In this section...
“Hierarchy” on page A-2
“Constructor” on page A-2
“Public Read-Only Properties” on page A-2
“Methods” on page A-3

### Hierarchy

**Superclasses:** None

**Subclasses:** None

### Constructor

IRBootstrapOptions

### Public Read-Only Properties

Name	Description
ConvexityAdjustment	Controls the convexity adjustment to interest rate futures. This can be specified as a function handle that takes time to maturity as an input and returns a value which is ConvexityAdjustment. Alternatively, you can define ConvexityAdjustment as an N-by-1 vector of values, where N is the number of interest rate futures. In either case, the ConvexityAdjustment is subtracted from the futures rate.  For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.



## Methods

There are no methods.

## See Also

`IRDataCurve | bootstrap`

## Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4
- “Creating an IRDataCurve Object” on page 9-6
- “IRDataCurve Constructor with Dates and Data” on page 9-6
- “IRDataCurve Bootstrapping Based on Market Instruments” on page 9-7
- “Bootstrapping a Swap Curve” on page 9-13
- “Dual Curve Bootstrapping” on page 9-17

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2

## @IRCurve

Base abstract class for interest-rate curve objects

In this section...
“Hierarchy” on page A-4
“Description” on page A-4
“Constructor” on page A-4
“Public Read-Only Properties” on page A-4
“Methods” on page A-6

### Hierarchy

**Superclasses:** None

**Subclasses:** @IRDataCurve on page A-7, @IRFunctionCurve on page A-13

### Description

IRCurve is an abstract class; you cannot create instances of it directly. You can create IRDataCurve and IRFunctionCurve objects that are derived from this class.

### Constructor

@IRCurve is an abstract class. To construct an IRCurve object, use one of the subclass constructors, IRDataCurve or IRFunctionCurve.

### Public Read-Only Properties

Name	Description
Type	Type of interest-rate curve: zero, forward, or discount.
Settle	Scalar for the Settle date of the curve.

Name	Description
Compounding	<p data-bbox="498 300 1236 361">Scalar that sets the compounding frequency per year for the IRCurve object:</p> <ul data-bbox="498 390 1016 673" style="list-style-type: none"> <li data-bbox="498 390 912 420">• -1 = Continuous compounding</li> <li data-bbox="498 430 847 460">• 1 = Annual compounding</li> <li data-bbox="498 470 1016 499">• 2 = Semiannual compounding (default)</li> <li data-bbox="498 510 1016 539">• 3 = Compounding three times per year</li> <li data-bbox="498 550 877 579">• 4 = Quarterly compounding</li> <li data-bbox="498 590 884 619">• 6 = Bimonthly compounding</li> <li data-bbox="498 630 877 659">• 12 = Monthly compounding</li> </ul>
Basis	<p data-bbox="498 690 1265 720">Day-count basis of the interest-rate curve. A vector of integers.</p> <ul data-bbox="498 749 853 1333" style="list-style-type: none"> <li data-bbox="498 749 853 779">• 0 = actual/actual (default)</li> <li data-bbox="498 789 743 819">• 1 = 30/360 (SIA)</li> <li data-bbox="498 829 713 859">• 2 = actual/360</li> <li data-bbox="498 869 713 899">• 3 = actual/365</li> <li data-bbox="498 909 758 939">• 4 = 30/360 (BMA)</li> <li data-bbox="498 949 758 979">• 5 = 30/360 (ISDA)</li> <li data-bbox="498 989 817 1019">• 6 = 30/360 (European)</li> <li data-bbox="498 1029 853 1058">• 7 = actual/365 (Japanese)</li> <li data-bbox="498 1069 839 1098">• 8 = actual/actual (ICMA)</li> <li data-bbox="498 1109 813 1138">• 9 = actual/360 (ICMA)</li> <li data-bbox="498 1149 828 1178">• 10 = actual/365 (ICMA)</li> <li data-bbox="498 1189 802 1218">• 11 = 30/360E (ICMA)</li> <li data-bbox="498 1229 847 1258">• 12 = actual/actual (ISDA)</li> <li data-bbox="498 1269 709 1298">• 13 = BUS/252</li> </ul> <p data-bbox="498 1359 1169 1388">For more information, see <b>basis</b> on page Glossary-0 .</p>

## Methods

Classes that inherit from the `IRCurve` abstract class must implement the following methods.

Method	Description
<code>getForwardRates</code>	Returns forward rates for input dates.
<code>getZeroRates</code>	Returns zero rates for input dates.
<code>getDiscountFactors</code>	Returns discount factors for input dates.
<code>getParYields</code>	Returns par yields for input dates.
<code>toRateSpec</code>	Converts to be a <code>RateSpec</code> object. This is identical to the <code>RateSpec</code> structure produced by the Financial Instruments Toolbox function <code>intenvset</code> .

## See Also

`IRDataCurve` | `IRFunctionCurve`

## Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4
- “Creating an `IRDataCurve` Object” on page 9-6
- “Creating an `IRFunctionCurve` Object” on page 9-22

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2

## @IRDataCurve

Represent interest-rate curve object based on vector of dates and data

In this section...
"Hierarchy" on page A-7
"Description" on page A-7
"Constructor" on page A-7
"Public Read-Only Properties" on page A-8
"Methods" on page A-9

### Hierarchy

**Superclasses:** @IRCurve on page A-4

**Subclasses:** None

### Description

IRDataCurve is a representation of an interest-rate curve object with dates and data. You can construct this object directly by specifying dates and corresponding interest rates or discount factors; alternatively, you can bootstrap the object from market data. After an interest-rate curve object is constructed, you can:

- Calculate forward and zero rates and determine par yields.
- Extract the discount factors.
- Convert to a RateSpec structure that is identical to the RateSpec structure produced by the Financial Instruments Toolbox function `intenvset`.

### Constructor

IRDataCurve

## Public Read-Only Properties

Name	Description
Type	Type of interest-rate curve: zero, forward, or discount.
Settle	Scalar for the <code>Settle</code> date of the curve.
Compounding	Scalar that sets the compounding frequency per year for the <code>IRCurve</code> object: <ul style="list-style-type: none"><li>• -1 = Continuous compounding</li><li>• 0 = Simple interest (no compounding)</li><li>• 1 = Annual compounding</li><li>• 2 = Semiannual compounding (default)</li><li>• 3 = Compounding three times per year</li><li>• 4 = Quarterly compounding</li><li>• 6 = Bimonthly compounding</li><li>• 12 = Monthly compounding</li></ul>

Name	Description
Basis	<p>Day-count basis of the financial curve. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/actual (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
Dates	Dates corresponding to rate data.
Data	Interest-rate data or discount factors for the curve object.
InterpMethod	<p>Values are:</p> <ul style="list-style-type: none"> <li>• 'linear' — Linear interpolation (default).</li> <li>• 'constant' — Piecewise constant interpolation.</li> <li>• 'pchip' — Piecewise cubic Hermite interpolation.</li> <li>• 'spline' — Cubic spline interpolation.</li> </ul>

## Methods

The following table contains links to methods with supporting reference pages, including examples.

<b>Method</b>	<b>Description</b>
<code>getForwardRates</code>	Returns forward rates for input dates.
<code>getZeroRates</code>	Returns zero rates for input dates.
<code>getDiscountFactors</code>	Returns discount factors for input dates.
<code>getParYields</code>	Returns par yields for input dates.
<code>toRateSpec</code>	Converts to be a <code>RateSpec</code> object. This structure is identical to the <code>RateSpec</code> produced by the Financial Instruments Toolbox function <code>intenvset</code> .
<code>bootstrap</code>	Bootstraps an interest rate curve from market data.

## See Also

`IRBootstrapOptions` | `IRFunctionCurve` | `bootstrap` | `getDiscountFactors` | `getForwardRates` | `getParYields` | `getZeroRates` | `toRateSpec`

## Related Examples

- “Creating Interest-Rate Curve Objects” on page 9-4
- “Creating an `IRDataCurve` Object” on page 9-6
- “`IRDataCurve` Constructor with Dates and Data” on page 9-6
- “`IRDataCurve` Bootstrapping Based on Market Instruments” on page 9-7
- “Bootstrapping a Swap Curve” on page 9-13
- “Dual Curve Bootstrapping” on page 9-17
- “Converting an `IRDataCurve` or `IRFunctionCurve` Object” on page 9-41
- “Using the `toRateSpec` Method” on page 9-41
- “Using Vector of Dates and Data Methods” on page 9-43
- “Analysis of Inflation Indexed Instruments”

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2



## @IRFitOptions

Object to specify fitting options for an IRFunctionCurve interest-rate curve object

### In this section...

“Hierarchy” on page A-11

“Description” on page A-11

“Constructor” on page A-11

“Public Read-Only Properties” on page A-11

“Methods” on page A-12

### Hierarchy

**Superclasses:** None

**Subclasses:** None

### Description

The IRFitOptions object allows you to specify options relating to the fitting process for an IRFunctionCurve object. Input arguments are specified in parameter/value pairs. The IRFitOptions structure provides the capability to choose which quantity to be minimized and other optimization parameters.

### Constructor

IRFitOptions

### Public Read-Only Properties

Name	Description
FitType	Price, Yield, or DurationWeightedPrice determines which is minimized in the curve fitting process. DurationWeightedPrice is the default.
InitialGuess	Initial guess for the parameters of the curve function.

<b>Name</b>	<b>Description</b>
UpperBound	Upper bound for the parameters of the curve function.
LowerBound	Lower bound for the parameters of the curve function.
OptOptions	Optimization structure based on the output from the Optimization Toolbox function <code>optimset</code> or <code>optimoptions</code> . This optimization structure is evaluated by <code>lsqnonlin</code> .
A	Inequality constraint for parameters, ignored if <code>OptimFunction</code> is set to <code>lsqnonlin</code> .
b	Inequality constraint for parameters, ignored if <code>OptimFunction</code> is set to <code>lsqnonlin</code> .
OptimFunction	Optimization function used to fit function, either <code>lsqnonlin</code> or <code>fmincon</code> .

## Methods

There are no methods.

## See Also

`IRFunctionCurve`

## Related Examples

- “Fitting Interest Rate Curve Functions” on page 9-33
- “Using `fitFunction` to Create Custom Fitting Function” on page 9-29

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2

## @IRFunctionCurve

Represent an interest-rate curve object using a function

### In this section...

“Hierarchy” on page A-13

“Description” on page A-13

“Constructor” on page A-13

“Public Read-Only Properties” on page A-14

“Methods” on page A-15

### Hierarchy

**Superclasses:** @IRCurve on page A-4

**Subclasses:** None

### Description

IRFunctionCurve is a representation of an interest-rate curve object. You can construct this object directly by specifying a function handle or a function can be fit to market data using methods of the object. After an interest-rate curve object is constructed; you can:

- Calculate forward and zero rates and determine par yields.
- Extract the discount factors.
- Convert to a RateSpec structure; this is identical to the RateSpec structure produced by the Financial Instruments Toolbox function `intenvset`.

### Constructor

IRFunctionCurve

## Public Read-Only Properties

<b>Name</b>	<b>Description</b>
Type	Type of interest-rate curve: zero, forward, or discount.
Settle	Scalar for the <code>Settle</code> date of the curve.
Compounding	Scalar that sets the compounding frequency per year for the <code>IRCurve</code> object: <ul data-bbox="541 546 1061 829" style="list-style-type: none"><li>• -1 = Continuous compounding</li><li>• 1 = Annual compounding</li><li>• 2 = Semiannual compounding (default)</li><li>• 3 = Compounding three times per year</li><li>• 4 = Quarterly compounding</li><li>• 6 = Bimonthly compounding</li><li>• 12 = Monthly compounding</li></ul>

Name	Description
Basis	Day-count basis of the interest-rate curve. A vector of integers. <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/actual (ISDA)</li> <li>• 13 = BUS/252</li> </ul> For more information, see <b>basis</b> on page Glossary-0 .
FunctionHandle	Function handle that defines the interest-rate curve. For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.
Parameters	Fitted parameters for function.

## Methods

The following table contains links to methods with supporting reference pages, including examples.

Method	Description
getForwardRates	Returns forward rates for input dates.
getZeroRates	Returns zero rates for input dates.
getDiscountFactors	Returns discount factors for input dates.

<b>Method</b>	<b>Description</b>
<code>getParYields</code>	Returns par yields for input dates.
<code>toRateSpec</code>	Converts to be a <code>RateSpec</code> object. This is identical to the <code>RateSpec</code> structure produced by the Financial Instruments Toolbox function <code>intenvset</code> .
<code>fitSvensson</code>	Fits a Svensson function to market data.
<code>fitNelsonSiegel</code>	Fits a Nelson-Siegel function to market data.
<code>fitSmoothingSpline</code>	Fits a smoothing spline function to market data.
<code>fitFunction</code>	Fits a custom function to market data.

## See Also

`IRDataCurve` | `IRFitOptions` | `fitFunction` | `fitNelsonSiegel` | `fitSmoothingSpline` | `fitSvensson` | `getDiscountFactors` | `getForwardRates` | `getParYields` | `getZeroRates` | `toRateSpec`

## Related Examples

- “Creating an `IRFunctionCurve` Object” on page 9-22
- “Fitting Interest Rate Curve Functions” on page 9-33
- “Fitting `IRFunctionCurve` Object Using a Function Handle” on page 9-22
- “Fitting `IRFunctionCurve` Object Using Nelson-Siegel Method” on page 9-22
- “Fitting `IRFunctionCurve` Object Using Svensson Method” on page 9-24
- “Fitting `IRFunctionCurve` Object Using Smoothing Spline Method” on page 9-26
- “Using `fitFunction` to Create Custom Fitting Function” on page 9-29
- “Converting an `IRDataCurve` or `IRFunctionCurve` Object” on page 9-41
- “Analysis of Inflation Indexed Instruments”

## More About

- “Interest-Rate Curve Objects and Workflow” on page 9-2
- “Creating Interest-Rate Curve Objects” on page 9-4

## **External Websites**

- Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)
- Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk Applications (30 min 00 sec)





# Functions — Alphabetical List

---

## asianbycrr

Price Asian option from Cox-Ross-Rubinstein binomial tree

### Syntax

```
Price = asianbycrr(CRRTree,OptSpec,Strike,Settle,ExerciseDates)
Price = asianbycrr( ____,AmericanOpt,AvgType,AvgPrice,AvgDate)
```

### Description

Price = asianbycrr(CRRTree,OptSpec,Strike,Settle,ExerciseDates) prices Asian options using a Cox-Ross-Rubinstein binomial tree.

Price = asianbycrr( \_\_\_\_,AmericanOpt,AvgType,AvgPrice,AvgDate) adds optional arguments for AmericanOpt, AvgType, AvgPrice, and AvgDate.

### Examples

#### Price a Floating-Strike Asian Option Using a CRR Binomial Tree

This example shows how to price a floating-strike Asian option using a CRR binomial tree using the file deriv.mat, which provides CRRTree. The CRRTree structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;

OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';

Price = asianbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price = 1.2177
```

## Input Arguments

### **CRRTree** — Stock tree structure

structure

Stock tree structure, specified by using `crrtree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a cell array of character vectors.

Data Types: `char` | `cell`

### **Strike** — Option strike price value

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values.

To compute the value of a floating-strike Asian option, `Strike` must be specified as NaN. Floating-strike Asian options are also known as average strike options.

Data Types: `double`

### **Settle** — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the Asian option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every Asian option is set to the `ValuationDate` of the stock tree. The Asian argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a NINST-by-1 matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one ExerciseDates on the option expiry date.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

Data Types: double | char

### **AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

(Optional) Option type, specified as NINST-by-1 positive integer flags with values:

- 0 — European
- 1 — American

Data Types: double

### **AvgType — Average types**

arithmetic (default) | character vector with values of arithmetic or geometric

Average types, specified as arithmetic for arithmetic average, or geometric for geometric average.

Data Types: char

### **AvgPrice — Average price of underlying asset at Settle**

scalar

Average price of underlying asset at Settle, specified as a scalar.

---

**Note** Use this argument when AvgDate < Settle.

---

Data Types: double

### **AvgDate — Date averaging period begins**

scalar

Date averaging period begins, specified as a scalar.

Data Types: char | double

## **Output Arguments**

### **Price — Expected prices for Asian options at time 0**

vector

Expected prices for Asian options at time 0, returned as a NINST-by-1 vector. Pricing of Asian options is done using Hull-White (1993). Therefore, for these options there are no unique prices on the tree nodes except for the root node.

## **References**

- [1] Hull, J., and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Vol. 1, pp. 21-31.

## **See Also**

crmtree | instasian

## **Topics**

"Pricing Asian Options"

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Asian Option" on page 3-44

"Supported Equity Derivatives" on page 3-24

## **External Websites**

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced before R2006a**

## asianbyeqp

Price Asian option from Equal Probabilities binomial tree

### Syntax

```
Price = asianbyeqp(EQPTree,OptSpec,Strike,Settle,ExerciseDates)
Price = asianbyeqp( ____,AmericanOpt,AvgType,AvgPrice,AvgDate)
```

### Description

`Price = asianbyeqp(EQPTree,OptSpec,Strike,Settle,ExerciseDates)` prices Asian options using an Equal Probabilities binomial tree.

`Price = asianbyeqp( ____,AmericanOpt,AvgType,AvgPrice,AvgDate)` adds optional arguments for `AmericanOpt`, `AvgType`, `AvgPrice`, and `AvgDate`.

### Examples

#### Price a Floating-Strike Asian Option Using an EQP Equity Tree

This example shows how to price a floating-strike Asian option using an EQP equity tree by loading the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;

OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';

Price = asianbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price = 1.2724
```

## Input Arguments

### **EQPTree — Stock tree structure**

structure

Stock tree structure, specified by using `eqptree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values.

To compute the value of a floating-strike Asian option, `Strike` must be specified as NaN. Floating-strike Asian options are also known as average strike options.

Data Types: `double`

### **Settle — Settlement date or trade date**

serial date number | date character vector

Settlement date or trade date for the Asian option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every Asian option is set to the `ValuationDate` of the stock tree. The Asian argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a NINST-by-1 matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one ExerciseDates on the option expiry date.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

Data Types: double | char

### **AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

(Optional) Option type, specified as NINST-by-1 positive integer flags with values:

- 0 — European
- 1 — American

Data Types: double

### **AvgType — Average types**

arithmetic (default) | character vector with values of arithmetic or geometric

Average types, specified as arithmetic for arithmetic average, or geometric for geometric average.

Data Types: char

### **AvgPrice — Average price of underlying asset at Settle**

scalar

Average price of underlying asset at Settle, specified as a scalar.

---

**Note** Use this argument when AvgDate < Settle.

---



Data Types: double

**AvgDate — Date averaging period begins**

scalar

Date averaging period begins, specified as a scalar.

Data Types: char | double

## Output Arguments

**Price — Expected prices for Asian options at time 0**

vector

Expected prices for Asian options at time 0, returned as a NINST-by-1 vector. Pricing of Asian options is done using Hull-White (1993). Therefore, for these options there are no unique prices on the tree nodes except for the root node.

## References

- [1] Hull, J., and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Vol. 1, pp. 21-31.

## See Also

eqptree | instasian

## Topics

"Pricing Asian Options"

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Asian Option" on page 3-44

"Supported Equity Derivatives" on page 3-24

## External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced before R2006a**

## asianbyitt

Price Asian options using implied trinomial tree (ITT)

### Syntax

```
Price = asianbyitt(ITTTree,OptSpec,Strike,Settle,ExerciseDates)
Price = asianbyitt( ____,AmericanOpt,AvgType,AvgPrice,AvgDate)
```

### Description

`Price = asianbyitt(ITTTree,OptSpec,Strike,Settle,ExerciseDates)` prices Asian options using an implied trinomial tree (ITT).

`Price = asianbyitt( ____,AmericanOpt,AvgType,AvgPrice,AvgDate)` adds optional arguments for `AmericanOpt`, `AvgType`, `AvgPrice`, and `AvgDate`.

### Examples

#### Price a Floating-Strike Asian Option Using an ITT Equity Tree

This example shows how to price a floating-strike Asian option using an ITT equity tree by loading the file `deriv.mat`, which provides `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

```
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2006';
ExerciseDates = '01-Jan-2007';
```

```
Price = asianbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates)
```

```
Price = 1.0778
```

## Input Arguments

### **ITTree** — Stock tree structure

structure

Stock tree structure, specified by using `itttree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a cell array of character vectors.

Data Types: `char` | `cell`

### **Strike** — Option strike price value

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values.

To compute the value of a floating-strike Asian option, `Strike` must be specified as NaN. Floating-strike Asian options are also known as average strike options.

Data Types: `double`

### **Settle** — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the Asian option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every Asian option is set to the `ValuationDate` of the stock tree. The Asian argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

### **ExerciseDates** — Option exercise dates

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a `NINST-by-1` matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST-by-2` vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a `NINST-by-1` vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

### **AmericanOpt** — Option type

`0` European (default) | integer with values `0` or `1`

(Optional) Option type, specified as `NINST-by-1` positive integer flags with values:

- `0` — European
- `1` — American

Data Types: `double`

### **AvgType** — Average types

`arithmetic` (default) | character vector with values of `arithmetic` or `geometric`

Average types, specified as `arithmetic` for arithmetic average, or `geometric` for geometric average.

Data Types: `char`

### **AvgPrice** — Average price of underlying asset at Settle

scalar

Average price of underlying asset at `Settle`, specified as a scalar.

---

**Note** Use this argument when `AvgDate < Settle`.

---

Data Types: `double`

**AvgDate — Date averaging period begins**

scalar

Date averaging period begins, specified as a scalar.

Data Types: char | double

## Output Arguments

**Price — Expected prices for Asian options at time 0**

vector

Expected prices for Asian options at time 0, returned as a NINST-by-1 vector. Pricing of Asian options is done using Hull-White (1993). Therefore, for these options there are no unique prices on the tree nodes except for the root node.

## References

[1] Hull, J., and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Vol. 1, pp. 21-31.

## See Also

instasian | itttree

## Topics

"Pricing Asian Options"

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Asian Option" on page 3-44

"Supported Equity Derivatives" on page 3-24

## External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced in R2007a**

## asianbyls

Price European or American Asian options using Monte Carlo simulations

### Syntax

```
Price = asianbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,  
ExerciseDates)
```

```
Price = asianbyls( ____,Name,Value)
```

```
[Price,Paths,Times,Z] = asianbyls(RateSpec,StockSpec,OptSpec,Strike,  
Settle,ExerciseDates)
```

```
[Price,Paths,Times,Z] = asianbyls( ____,Name,Value)
```

### Description

`Price = asianbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)` returns fixed- and floating-strike Asian option prices using the Longstaff-Schwartz model. `asianbyls` computes prices of European and American Asian options.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

To compute the value of a floating-strike Asian option, `Strike` should be specified as `NaN`. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

`Price = asianbyls( ____,Name,Value)` adds optional name-value pair arguments.

`[Price,Paths,Times,Z] = asianbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)` returns fixed- and floating-strike Asian option `Price`, `Paths`, `Times`, and `Z` values using the Longstaff-Schwartz model. `asianbyls` computes prices of European and American Asian options.

`[Price,Paths,Times,Z] = asianbyls( ____,Name,Value)` adds optional name-value pair arguments.

## Examples

### Compute the Price of an Asian Option Using the Longstaff-Schwartz Model

Define the RateSpec.

```
Rates = 0.05;
StartDate = 'Jan-1-2013';
EndDate = 'Jan-1-2014';
RateSpec = intenvset('ValuationDate', StartDate, 'StartDates', StartDate, ...
    'EndDates', EndDate, 'Compounding', -1, 'Rates', Rates)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.2;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2000
    AssetPrice: 100
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the Asian 'call' option.

```
Settle = 'Jan-1-2013';  
ExerciseDates = 'Jan-1-2014';  
Strike = 110;  
OptSpec = 'call';
```

Compute the price for the European arithmetic average price for the Asian option using the Longstaff-Schwartz model.

```
NumTrials = 10000;  
NumPeriods = 100;  
AvgType = 'arithmetic';  
Antithetic = true;  
Price = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, ...  
    'NumTrials', NumTrials, 'NumPeriods', NumPeriods, 'Antithetic', Antithetic, 'AvgType', A  
Price = 1.9876
```

## Input Arguments

**RateSpec** — Interest-rate term structure  
structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec** — Stock specification for underlying asset  
structure

Stock specification for underlying asset, specified using StockSpec obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0.

Data Types: `struct`

**OptSpec** — Definition of option  
character vector with a value of 'call' or 'put'



Definition of option, specified as 'call' or 'put' using a character vector

Data Types: char

### **Strike — Option strike price value**

nonnegative scalar integer

Option strike price value, specified with a nonnegative scalar integer. To compute the value of a floating-strike Asian option, `Strike` should be specified as NaN. Floating-strike Asian options are also known as average strike options.

Data Types: double

### **Settle — Settlement or trade date**

date character vector | nonnegative scalar integer

Settlement or trade date for the Asian option, specified as a nonnegative scalar integer or date character vector. By default, `asianbyls` calculates the price of Asian options based on averages that start on the settlement date.

Data Types: double | char

### **ExerciseDates — Option exercise dates**

date character vector | nonnegative scalar integer

Option exercise dates, specified as a nonnegative scalar integer or date character vector:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a 1-by-1 vector of serial date numbers or cell array of date character vectors, the option can be exercised between `Settle` and the single listed `ExerciseDates`.

Data Types: double | char

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price = asianbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'NumTrials', NumTrials, 'NumPeriods', NumPeriods, 'Antithetic', Antithetic, 'AvgType', 'arithmetic')`

### **AmericanOpt — Option type**

`0` European (default) | scalar with value [`0`, `1`]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a NINST-by-1 positive integer scalar flags with values:

- `0` — European
- `1` — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: `single` | `double`

### **AvgType — Average types**

`arithmetic` (default) | character vector with values of `arithmetic` or `geometric`

Average types, specified as the comma-separated pair consisting of 'AvgType' and `arithmetic` for arithmetic average, or `geometric` for geometric average.

Data Types: `char`

### **AvgPrice — Average price of underlying asset at Settle**

numeric

Average price of underlying asset at `Settle`, specified as the comma-separated pair consisting of 'AvgPrice' and a scalar numeric value. The `AvgPrice` is assumed to be calculated in the time window starting at `AvgDate` and ending on `Settle`. In other words, the average is backward looking.

---

**Note** Use this argument when `AvgDate < Settle`.

---

Data Types: double

**AvgDate — Date averaging period begins**

serial date number

Date averaging period begins, specified as the comma-separated pair consisting of 'AvgDate' and a scalar serial date number.

Data Types: double

**NumTrials — Simulation trials**

1000 (default) | numeric

Simulation trials, specified as the comma-separated pair consisting of 'NumTrials' and a scalar number of independent sample paths.

Data Types: double

**NumPeriods — Simulation periods per trial**

100 (default) | numeric

Simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' scalar numeric value. NumPeriods is considered only when pricing European Asian options. For American Asian options, NumPeriods is equal to the number of exercise days during the life of the option.

Data Types: double

**Z — Dependent random variates**

nonnegative integer

Dependent random variates used to generate the Brownian motion vector (that is, Wiener processes) that drive the simulation, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-2-by-NumTrials 3-D time series array.

Data Types: single | double

**Antithetic — Indicator for antithetic sampling**

false (default) | logical flag with value of true or false

Logical flag to indicate antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

## Output Arguments

### **Price — Expected price of Asian option**

scalar

Expected price of the Asian option, returned as a 1-by-1 scalar.

### **Paths — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a  $(\text{NumPeriods} + 1)$ -by-1-by- $\text{NumTrials}$  3-D time series array. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

### **Times — Observation times associated with simulated paths**

vector

Observation times associated with the simulated paths, returned as a  $(\text{NumPeriods} + 1)$ -by-1 column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

### **Z — Dependent random variates**

vector

Dependent random variates, returned, if `Z` is specified as an optional input argument, the same value is returned. Otherwise, `Z` contains the random variates generated internally.

## See Also

`asianbycrr` | `asianbykv` | `asianbylevy` | `asiansensbyls` | `intenvset` | `stockspec`

## Topics

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

## External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 38 sec)

**Introduced in R2013b**

## asianbystt

Price Asian options using standard trinomial tree

### Syntax

```
Price = asianbystt(STTTree,OptSpec,Strike,Settle,ExerciseDates)
Price = asianbystt( ____,AmericanOpt,AvgType,AvgPrice,AvgDate)
```

### Description

Price = asianbystt(STTTree,OptSpec,Strike,Settle,ExerciseDates) prices Asian options using a standard trinomial (STT) tree.

Price = asianbystt( \_\_\_\_,AmericanOpt,AvgType,AvgPrice,AvgDate) prices Asian options using a standard trinomial (STT) tree with optional arguments for AmericanOpt, AvgType, AvgPrice, and AvgDate.

### Examples

#### Price an Asian Option Using the Standard Trinomial Tree Model

Create a RateSpec.

```
StartDates = 'Jan-1-2009';
EndDates = 'Jan-1-2013';
Rates = 0.035;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.8694
```

```

        Rates: 0.0350
        EndTimes: 4
        StartTimes: 0
        EndDates: 735235
        StartDates: 733774
        ValuationDate: 733774
        Basis: 1
        EndMonthRule: 1

```

Create a StockSpec.

```

AssetPrice = 85;
Sigma = 0.15;
StockSpec = stockspec(Sigma, AssetPrice)

```

```

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1500
    AssetPrice: 85
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []

```

Create an STTTree.

```

NumPeriods = 4;
TimeSpec = stttimespec(StartDates, EndDates, 4);
STTTree = stttree(StockSpec, RateSpec, TimeSpec)

```

```

STTTree = struct with fields:
    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [733774 734139 734504 734869 735235]
    STree: {1x5 cell}
    Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}

```

Define the Asian option and compute the price.

```

Settle = '01-Jan-2009';
ExerciseDates = [datenum('1/1/12'); datenum('1/1/13')];

```

```
OptSpec = 'call';  
Strike = 100;  
  
Price = asianbystt(STTTree, OptSpec, Strike, Settle, ExerciseDates)  
  
Price = 2×1  
  
    1.6905  
    2.6203
```

## Input Arguments

**STTTree** — Stock tree structure for standard trinomial tree structure

Stock tree structure for a standard trinomial tree, specified by using `stttree`.

Data Types: `struct`

**OptSpec** — Definition of option

character vector with value 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector.

Data Types: `char`

**Strike** — Option strike price value

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values. To compute the value of a floating-strike Asian option, `Strike` should be specified as NaN. Floating-strike Asian options are also known as average strike options.

Data Types: `double`

**Settle** — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the Asian option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.



---

**Note** The `Settle` date for every Asian option is set to the `ValuationDate` of the stock tree. The Asian argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a `nINST`-by-1 matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST`-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a `NINST`-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

### **AmericanOpt — Option type**

0 European (default) | scalar with values [0, 1]

Option type, specified as `NINST`-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

Data Types: `single` | `double`

### **AvgType — Average types**

`arithmetic` (default) | character vector with values of `arithmetic` or `geometric`

Average types, specified as `arithmetic` for arithmetic average, or `geometric` for geometric average.

Data Types: `char`

### **AvgPrice — Average price of underlying asset at Settle**

scalar

Average price of underlying asset at `Settle`, specified as a scalar.

---

**Note** Use this argument when `AvgDate < Settle`.

---

Data Types: `double`

### **AvgDate — Date averaging period begins**

scalar

Date averaging period begins, specified as a scalar.

Data Types: `double`

## **Output Arguments**

### **Price — Expected prices for Asian options at time 0**

matrix

Expected prices for Asian options at time 0, returned as a NINST-by-1 matrix. Pricing of Asian options is done using Hull-White (1993). Consequently, for these options there are no unique prices on the tree nodes with the exception of the root node.

## **References**

[1] Hull, J., and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Vol. 1, pp. 21-31.

## **See Also**

`instasian` | `sttprice` | `sttsens` | `stttimespec` | `stttree`

## **Topics**

"Asian Option" on page 3-44

"Supported Equity Derivatives" on page 3-24

## **External Websites**

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced in R2015b**

## asiansensbyls

Calculate price and sensitivities for European or American Asian options using Monte Carlo simulations

### Syntax

```
PriceSens = asiansensbyls(RateSpec,StockSpec,OptSpec,StrikeSettle,  
ExerciseDates)
```

```
PriceSens = asiansensbyls( ____,Name,Value)
```

```
[PriceSens,Path,Times,Z] = asiansensbyls(RateSpec,StockSpec,OptSpec,  
StrikeSettle,ExerciseDates)
```

```
[PriceSens,Path,Times,Z] = asiansensbyls( ____,Name,Value)
```

### Description

`PriceSens = asiansensbyls(RateSpec,StockSpec,OptSpec,StrikeSettle,ExerciseDates)` returns Asian option prices or sensitivities for fixed- and floating-strike Asian options using the Longstaff-Schwartz model. `asiansensbyls` supports European and American Asian options.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

To compute the value of a floating-strike Asian option, `Strike` should be specified as `NaN`. Fixed-strike Asian options are also known as average price options and floating-strike Asian options are also known as average strike options.

`PriceSens = asiansensbyls( ____,Name,Value)` returns Asian option prices or sensitivities for fixed- and floating-strike Asian options using optional name-value pair arguments and the Longstaff-Schwartz model.

`[PriceSens,Path,Times,Z] = asiansensbyls(RateSpec,StockSpec,OptSpec,StrikeSettle,ExerciseDates)` returns Asian option prices or sensitivities (`PriceSens`, `Path`, `Times`, and `Z`) for fixed- and floating-strike Asian options using the Longstaff-Schwartz model.

[PriceSens, Path, Times, Z] = asiansensbyls( \_\_\_\_, Name, Value) returns Asian option prices or sensitivities (PriceSens, Path, Times, and Z) for fixed- and floating-strike Asian options using optional name-value pair arguments and the Longstaff-Schwartz model.

## Examples

### Compute the Price and Sensitivities of an Asian Option Using the Longstaff-Schwartz Model

Define the RateSpec.

```
Rates = 0.05;
StartDate = 'Jan-1-2013';
EndDate = 'Jan-1-2014';
RateSpec = intenvset('ValuationDate', StartDate, 'StartDates', StartDate, ...
    'EndDates', EndDate, 'Compounding', -1, 'Rates', Rates)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.2;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2000
    AssetPrice: 100
```

```
DividendType: []  
DividendAmounts: 0  
ExDividendDates: []
```

Define the Asian 'call' option.

```
Settle = 'Jan-1-2013';  
ExerciseDates = 'Jan-1-2014';  
Strike = 110;  
OptSpec = 'call';
```

Compute the price for the European arithmetic average price and sensitivities for the Asian option using the Longstaff-Schwartz model.

```
NumTrials = 10000;  
NumPeriods = 100;  
AvgType = 'arithmetic';  
Antithetic = true;  
OutSpec = {'Price', 'Delta', 'Gamma'};  
PriceSens = asiansensbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates,  
    'NumTrials', NumTrials, 'NumPeriods', NumPeriods, 'Antithetic', Antithetic, 'AvgType',  
    AvgType, 'OutSpec', OutSpec)  
PriceSens = 1.9876
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with value 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector.

Data Types: `char`

### **Strike — Option strike price value**

nonnegative scalar integer

Option strike price value, specified with a nonnegative scalar integer. To compute the value of a floating-strike Asian option, `Strike` should be specified as `NaN`. Floating-strike Asian options are also known as average strike options.

Data Types: `double`

### **Settle — Settlement date or trade date**

nonnegative scalar integer | date character vector

Settlement date or trade date for the Asian option, specified as a nonnegative scalar integer or date character vector.

Data Types: `double` | `char`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a 1-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between `Settle` and the single listed `ExerciseDates`.

Data Types: `double` | `char`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: PriceSens =  
asiansensbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates  
, 'NumTrials', NumTrials, 'NumPeriods',  
NumPeriods, 'Antithetic', Antithetic, 'AvgType', AvgType, 'OutSpec',  
{'All'})
```

### AmericanOpt — Option type

0 European (default) | scalar with values [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: single | double

### AvgType — Average types

arithmetic (default) | character vector with values of arithmetic or geometric

Average types, specified as the comma-separated pair consisting of 'AvgType' and arithmetic for arithmetic average, or geometric for geometric average.

Data Types: char

### AvgPrice — Average price of underlying asset at Settle

numeric



Average price of underlying asset at `Settle`, specified as the comma-separated pair consisting of 'AvgPrice' and a scalar numeric value.

---

**Note** Use this argument when `AvgDate < Settle`.

---

Data Types: double

### **AvgDate — Date averaging period begins**

serial date number

Date averaging period begins, specified as the comma-separated pair consisting of 'AvgDate' and a scalar serial date number.

Data Types: double

### **NumTrials — Simulation trials**

1000 (default) | numeric

Simulation trials, specified as the comma-separated pair consisting of 'NumTrials' and a scalar number of independent sample paths.

Data Types: double

### **NumPeriods — Simulation periods per trial**

100 (default) | numeric

Simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a scalar numeric value. `NumPeriods` is considered only when pricing European Asian options. For American Asian options, `NumPeriod` is equal to the number of exercise days during the life of the option.

Data Types: double

### **Z — Dependent random variates**

nonnegative integer

Dependent random variates used to generate the Brownian motion vector (that is, Wiener processes) that drive the simulation, specified as the comma-separated pair consisting of 'Z' and a `NumPeriods-by-2-by-NumTrials` 3-D time series array.

Data Types: single | double

**Antithetic — Indicates antithetic sampling**

false (default) | logical flag with value of true or false

Indicates antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}

Data Types: char | cell

## Output Arguments

**PriceSens — Expected price or sensitivities of Asian option**

scalar

Expected price or sensitivities (defined by OutSpec) of the Asian option, returned as a 1-by-1 array.

**Path — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a (NumPeriods + 1)-by-2-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

**Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a  $(\text{NumPeriods} + 1)$ -by-1 column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

**Z — Dependent random variates**

vector

Dependent random variates, returned, if `Z` is specified as an optional input argument, the same value is returned. Otherwise, `Z` contains the random variates generated internally.

**See Also**

`asianbycrr` | `asianbykv` | `asianbylevy` | `asianbyls` | `intenvset` | `stockspec`

**Topics**

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

**External Websites**

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced in R2013b**

## asianbykv

Prices European geometric Asian options using Kemna-Vorst model

### Syntax

```
Price = asianbykv(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates)
```

### Description

Price = asianbykv(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates) returns prices of European geometric Asian options using the Kemna-Vorst model.

### Examples

#### Compute the Price of an Asian Option Using the Kemna-Vorst Model

Define the RateSpec.

```
StartDates = 'Jan-1-2013';  
EndDates = 'Jan-1-2014';  
Rates = 0.035;  
Basis = 1;  
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...  
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9656  
    Rates: 0.0350  
    EndTimes: 1  
    StartTimes: 0  
    EndDates: 735600
```

```

    StartDates: 735235
    ValuationDate: 735235
        Basis: 1
    EndMonthRule: 1

```

Define the `StockSpec` for the asset.

```

AssetPrice = 100;
Sigma = 0.15;
DivType = 'continuous';
DivAmounts = 0.03;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1500
    AssetPrice: 100
    DividendType: {'continuous'}
    DividendAmounts: 0.0300
    ExDividendDates: []

```

Define the Asian 'call' and 'put' options.

```

Strike = 102;
OptSpec = {'put'; 'call'};
Settle = 'Jan-1-2013';
Maturity = 'Apr-1-2013';

```

Compute the European geometric Average Price for the Asian option using the Kemna-Vorst model.

```
Price = asiansensbykv(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity)
```

```
Price = 2×1
```

```

    2.8881
    0.9210

```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

The annualized continuously compounded interest-rate term structure specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors

Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of character vectors.

Data Types: `cell` | `char`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values, specified with nonnegative integers using a NINST-by-1 vector.

Data Types: `single` | `double`

### **Settle — Settlement dates or trade dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates or trade dates for the Asian option, specified as a character vector or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

### **ExerciseDates — European option exercise dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

European option exercise dates, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates. For a European option, there is only one ExerciseDates on the option expiry date.

Data Types: double | char | cell

## **Output Arguments**

### **Price — Expected prices of an Asian option**

vector

Expected prices of the Asian option, returned as an NINST-by-1 vector.

## **See Also**

asianbycrr | asianbylevy | asianbyls | asiansensbykv | intenvset | stockspec

## **Topics**

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

## **External Websites**

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

## **Introduced in R2013b**

## asiansensbykv

Calculate prices or sensitivities of European geometric Asian options using Kemna-Vorst model

### Syntax

```
PriceSens = asiansensbykv(RateSpec,StockSpec,OptSpec,Strike,Settle,
ExerciseDates)
PriceSens = asiansensbykv( ____,Name,Value)
```

### Description

`PriceSens = asiansensbykv(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)` returns prices or sensitivities of European geometric Asian options using Kemna-Vorst model.

`PriceSens = asiansensbykv( ____,Name,Value)` adds optional name-value pair arguments.

### Examples

#### Compute the Price and Sensitivities of an Asian Option Using the Kemna-Vorst Model

Define the `RateSpec`.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.035;
Basis = 1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
```



```

Compounding: -1
  Disc: 0.9656
  Rates: 0.0350
  EndTimes: 1
  StartTimes: 0
  EndDates: 735600
  StartDates: 735235
ValuationDate: 735235
  Basis: 1
  EndMonthRule: 1

```

Define the `StockSpec` for the asset.

```

AssetPrice = 100;
Sigma = 0.15;
DivType = 'continuous';
DivAmounts = 0.03;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1500
    AssetPrice: 100
    DividendType: {'continuous'}
    DividendAmounts: 0.0300
    ExDividendDates: []

```

Define the Asian 'call' and 'put' options.

```

Strike = 102;
OptSpec = {'put'; 'call'};
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2014';

```

Compute the European geometric Average Price and sensitivities for the Asian option using the Kemna-Vorst model.

```

OutSpec = {'Price', 'Delta', 'Gamma'};
PriceSens = asiansensbykv(RateSpec, StockSpec, OptSpec, Strike, ...
Settle, ExerciseDates, 'OutSpec', OutSpec)

```

```
PriceSens = 2x1
```

4.3871  
2.5163

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

The annualized continuously compounded interest-rate term structure specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of character vectors.

Data Types: `cell` | `char`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values, specified with nonnegative integers using a NINST- by-1 vector.

Data Types: `single` | `double`

**Settle — Settlement dates or trade dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates or trade dates for the Asian option, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

**ExerciseDates — Option exercise dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

European option exercise dates, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates. For a European option, there is only one ExerciseDates on the option expiry date.

Data Types: double | char | cell

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: PriceSens =  
asiansensbykv(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'OutSpec', {'All'})

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'. | cell array of character vectors with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta' and 'All'.

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec as:

```
Example: OutSpec =  
{'delta','gamma','vega','lambda','rho','theta','price'}
```

Data Types: char | cell

## Output Arguments

### **PriceSens** — Expected prices or sensitivities of the Asian option

vector

Expected prices or sensitivities (defined by `OutSpec`) of the Asian option, returned as an 1-by-1 vector. If the `OutSpec` is not specified only price is returned.

## See Also

`asianbycrr` | `asianbykv` | `asianbylevy` | `asianbyls` | `intenvset` | `stockspec`

## Topics

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

## External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced in R2013b**

# asianbylevy

Price of European arithmetic Asian options using Levy model

## Syntax

```
Price = asianbylevy(RateSpec, StockSpec, OptSpec, Strike, Settle,
ExerciseDates)
```

## Description

Price = asianbylevy(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates) returns European arithmetic average pricing for Asian options using the Levy model.

## Examples

### Compute the Price of an Asian Option Using the Levy Model

Define the RateSpec.

```
Rates = 0.07;
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, 'EndDates',
EndDates, 'Rates', Rates, 'Compounding', -1)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9324
    Rates: 0.0700
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
```

```
ValuationDate: 735235
      Basis: 0
      EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 6.8;
Sigma = 0.14;
DivType = 'continuous';
DivAmounts = 0.09;
StockSpec = stockspect(Sigma, AssetPrice, DivType, DivAmounts)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1400
    AssetPrice: 6.8000
    DividendType: {'continuous'}
    DividendAmounts: 0.0900
    ExDividendDates: []
```

Define two options for 'call' and 'put'.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
Strike = 6.9;
OptSpec = {'call'; 'put'};
```

Compute the European arithmetic average price for the Asian option using the Levy model.

```
Price= asianbylevy(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity)
```

```
Price = 2×1
```

```
0.0944
0.2237
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be  $\emptyset$ .

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values, specified with nonnegative integers as a NINST-by-1 vector.

Data Types: `single` | `double`

### **Settle — Settlement dates or trade dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates or trade dates for the Asian option, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

### **ExerciseDates — Option exercise dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Option exercise dates, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates. For a European option, there is only one ExerciseDates on the option expiry date.

Data Types: double | char | cell

## **Output Arguments**

### **Price — Expected prices of Asian option**

vector

Expected prices of the Asian option, returned as a NINST-by-1 vector.

## **See Also**

asianbycrr | asianbykv | asianbyls | asiansensbylevy | intenvset | stockspec

## **Topics**

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

## **External Websites**

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced in R2013b**



# asiansensbylevy

Calculate prices or sensitivities of European arithmetic Asian options using Levy model

## Syntax

```
PriceSens = asiansensbylevy(RateSpec, StockSpec, OptSpec, StrikeSettle,
ExerciseDates)
PriceSens = asiansensbylevy( ____, Name, Value)
```

## Description

PriceSens = asiansensbylevy(RateSpec, StockSpec, OptSpec, StrikeSettle, ExerciseDates) returns European average pricing or sensitivities for arithmetic Asian options using the Levy model.

PriceSens = asiansensbylevy( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

## Examples

### Compute the Price and Sensitivities of an Asian Option Using the Levy Model

Define the RateSpec.

```
Rates = 0.07;
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, 'EndDates',
EndDates, 'Rates', Rates, 'Compounding', -1)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9324
```

```
        Rates: 0.0700
      EndTimes: 1
    StartTimes: 0
      EndDates: 735600
    StartDates: 735235
  ValuationDate: 735235
        Basis: 0
    EndMonthRule: 1
```

Define the `StockSpec` for the asset.

```
AssetPrice = 6.8;
Sigma = 0.14;
DivType = 'continuous';
DivAmounts = 0.09;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1400
    AssetPrice: 6.8000
    DividendType: {'continuous'}
    DividendAmounts: 0.0900
    ExDividendDates: []
```

Define two options for a 'call' and 'put'.

```
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2014';
Strike = 6.9;
OptSpec = {'call'; 'put'};
```

Compute the European arithmetic average price and sensitivities for the Asian option using the Levy model.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
PriceSens = asiansensbylevy(RateSpec, StockSpec, OptSpec, Strike, ...
    Settle, ExerciseDates, 'OutSpec', OutSpec)

PriceSens = 2×1

    0.1358
    0.2921
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values, specified with nonnegative integers using a NINST-by-1 vector.

Data Types: `single` | `double`

### **Settle — Settlement dates or trade dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates or trade dates for the Asian option, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

### **ExerciseDates — Option exercise dates**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Option exercise dates, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates. For a European option, there is only one ExerciseDates on the option expiry date.

Data Types: double | char | cell

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: PriceSens =  
asiansensbylevy(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'OutSpec', {'All'})

### **OutSpec — Define outputs**

{'Price'} (default) | character vector with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'. | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}

Data Types: char | cell

## Output Arguments

### **PriceSens** — Expected prices or sensitivities of Asian option

vector

Expected prices or sensitivities (defined by `OutSpec`) of the Asian option, returned as an 1-by-1 vector. If the `OutSpec` is not specified only the price is returned.

## See Also

`asianbycrr` | `asianbykv` | `asianbykv` | `asianbyls` | `intenvset` | `stockspec`

## Topics

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

## External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced in R2013b**

## asianbyhbm

Price European discrete arithmetic fixed Asian options using Haug, Haug, Margrabe model

### Syntax

```
Price = asianbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle,
ExerciseDates)
Price = asianbyhbm( ____, Name, Value)
```

### Description

Price = asianbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates) prices European discrete arithmetic fixed Asian options using the Haug, Haug, Margrabe model.

Price = asianbyhbm( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

### Examples

#### Price an Asian Option with Averaging Period Starting Before the Settle Date

Define the Asian option parameters.

```
AssetPrice = 100;
Strike = 95;
Rates = 0.1;
Sigma = 0.15;
Settle = 'Apr-1-2013';
Maturity = 'Oct-1-2013';
```

Create a RateSpec using the intenvset function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a StockSpec for the underlying asset using the stockspec function.

```
DividendType = 'Continuous';
DividendAmounts = 0.05;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price of the Asian option using the Haug, Haug, Margrabe approximation. Assume that the averaging period has started before the Settle date.

```
OptSpec = 'Call';
ExerciseDates = 'Oct-1-2013';
NumFixings = 12;
AvgDate = 'Jan-1-2013';
AvgPrice = 100;
```

```
Price = asianbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, ...
'NumFixings', NumFixings, 'AvgDate', AvgDate, 'AvgPrice', AvgPrice)
```

```
Price = 5.8216
```

### Price an Asian Option with Averaging Period Starting After the Settle Date

Define the Asian option parameters.

```
AssetPrice = 100;
Strike = 95;
Rates = 0.1;
Sigma = 0.15;
Settle = 'Apr-1-2013';
Maturity = 'Oct-1-2013';
```

Create a RateSpec using the intenvset function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a StockSpec for the underlying asset using the stockspec function.

```
DividendType = 'Continuous';
DividendAmounts = 0.05;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price of the Asian option using the Haug, Haug, Margrabe approximation. Assume that the averaging period starts after the `Settle` date.

```
OptSpec = 'Call';  
ExerciseDates = 'Oct-1-2013';  
NumFixings = 15;  
AvgDate = 'Jan-1-2013';  
  
Price = asianbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, ...  
    'NumFixings', NumFixings, 'AvgDate', AvgDate)  
  
Price = 1.3785e-07
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of option, specified as 'call' or 'put' using a character vector, cell array of character vectors, or string array.



Data Types: `char` | `cell` | `string`

### **Strike — Option strike price value**

nonnegative integer | vector of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 vector of strike price values.

Data Types: `double`

### **Settle — Settlement dates or trade dates**

serial date number | date character vector | `datetime` | string array

Settlement date or trade date for the Asian option, specified as a NINST-by-1 vector using serial date numbers, date character vectors, `datetime`, or string arrays.

Data Types: `double` | `char` | `datetime` | `string`

### **ExerciseDates — European option exercise dates**

serial date number | date character vector | `datetime` | string array

European option exercise dates, specified as a NINST-by-1 vector using serial date numbers, date character vectors, `datetimes`, or string arrays.

---

**Note** For a European option, there is only one `ExerciseDates` on the option expiry date.

---

Data Types: `double` | `char` | `datetime` | `string`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
asianbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'NumFixings', 15)
```

### **AvgPrice — Average price of underlying asset at the Settle date**

vector

Average price of underlying asset at the `Settle` date, specified as the comma-separated pair consisting of `'AvgPrice'` and a NINST-by-1 vector.

---

**Note** Use the `AvgPrice` argument when `AvgDate < Settle`.

---

Data Types: `double`

**AvgDate — Date averaging period begins**

`serial date number` | `date character vector` | `datetime` | `string array`

Date averaging period begins, specified as the comma-separated pair consisting of `'AvgDate'` and a NINST-by-1 vector using serial date numbers, date character vectors, datetimes, or string array.

Data Types: `char` | `double` | `datetime` | `string`

**NumFixings — Total number of fixings or averaging points**

`10 (default)` | `vector`

Total number of fixings or averaging points, specified as the comma-separated pair consisting of `'NumFixings'` and a NINST-by-1 vector.

Data Types: `double`

## Output Arguments

**Price — Expected prices for fixed Asian options**

`vector`

Expected prices for fixed Asian options, returned as a NINST-by-1 vector.

## Definitions

### Fixed Asian Options

Fixed Asian options have a specified strike.

The payoff at maturity for a fixed (average price) Asian option is:

- Fixed call (average price option):  $\max(0, S_{av} - X)$
- Fixed put (average price option):  $\max(0, X - S_{av})$

where:

$S_{av}$  is the arithmetic or geometric average price of underlying asset.

$S$  is the price at maturity of the underlying asset.

$X$  is the strike price.

## References

- [1] Haug, E. G. *The Complete Guide to Option Pricing Formulas*. McGraw-Hill Education, 2007.

## See Also

asianbycrr | asianbykv | asianbylevy | asianbyls | asianbytw |  
asiansensbyhbm | intenvset | stockspect

## Topics

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

## External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

## Introduced in R2018a

## asiansensbyhbm

Calculate price and sensitivities of European discrete arithmetic fixed Asian options using Haug, Haug, Margrabe model

### Syntax

```
PriceSens = asiansensbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle,
ExerciseDates)
PriceSens = asiansensbyhbm( ____, Name, Value)
```

### Description

PriceSens = asiansensbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates) calculates prices and sensitivities for European discrete arithmetic fixed Asian options using the Haug, Haug, Margrabe model.

PriceSens = asiansensbyhbm( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

### Examples

#### Compute Price and Sensitivities for Asian Option with Averaging Period Starting Before the Settle Date

Define the Asian option parameters.

```
AssetPrice = 100;
Strike = 95;
Rates = 0.1;
Sigma = 0.15;
Settle = 'Apr-1-2013';
Maturity = 'Oct-1-2013';
```

Create a RateSpec using the intenvset function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a StockSpec for the underlying asset using the stockspect function.

```
DividendType = 'Continuous';
DividendAmounts = 0.05;
```

```
StockSpec = stockspect(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price and sensitivities of the Asian option using the Haug, Haug, Margrabe approximation. Assume that the averaging period has started before the Settle date.

```
OptSpec = 'Call';
ExerciseDates = 'Oct-1-2013';
NumFixings = 12;
AvgDate = 'Jan-1-2013';
AvgPrice = 100;
OutSpec = {'Price', 'Delta', 'Gamma'};
```

```
[Price,Delta,Gamma] = asiansensbyhbm(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,
'NumFixings',NumFixings,'AvgDate',AvgDate,'AvgPrice',AvgPrice,'OutSpec',OutSpec)
```

```
Price = 5.8216
```

```
Delta = 0.5907
```

```
Gamma = 0.0143
```

### Compute Price and Sensitivities for Asian Option with Averaging Period Starting After the Settle Date

Define the Asian option parameters.

```
AssetPrice = 100;
Strike = 95;
Rates = 0.1;
Sigma = 0.15;
Settle = 'Apr-1-2013';
Maturity = 'Oct-1-2013';
```

Create a RateSpec using the intenvset function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...  
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a StockSpec for the underlying asset using the stockspec function.

```
DividendType = 'Continuous';  
DividendAmounts = 0.05;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price and sensitivities of the Asian option using the Haug, Haug, Margrabe approximation. Assume that the averaging period started after the Settle date.

```
OptSpec = 'Call';  
ExerciseDates = 'Oct-1-2013';  
NumFixings = 15;  
AvgDate = 'Jan-1-2013';  
OutSpec = {'Price', 'Delta', 'Gamma'};
```

```
[Price,Delta,Gamma] = asiansensbyhbm(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,  
'NumFixings',NumFixings,'AvgDate',AvgDate,'OutSpec',OutSpec)
```

```
Price = 1.3785e-07
```

```
Delta = 1.1438e-07
```

```
Gamma = 9.0830e-08
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types: struct

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of option, specified as 'call' or 'put' using a character vector, cell array of character vectors, or string array.

Data Types: `char` | `cell` | `string`

### **Strike — Option strike price value**

nonnegative integer | vector of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 vector of strike price values.

Data Types: `double`

### **Settle — Settlement dates or trade dates**

serial date number | date character vector | datetime | string array

Settlement date or trade date for the Asian option, specified as a NINST-by-1 vector using serial date numbers, date character vectors, datetimes, or string arrays.

Data Types: `double` | `char` | `datetime` | `string`

### **ExerciseDates — European option exercise dates**

serial date number | date character vector | datetime | string array

European option exercise dates, specified as a NINST-by-1 vector using serial date numbers, date character vectors, datetimes, or string arrays.

---

**Note** For a European option, there is only one `ExerciseDates` on the option expiry date.

---

Data Types: `double` | `char` | `datetime` | `string`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `PriceSens = asiansensbyhbm(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'OutSpec', {'All'}, 'NumFixings', 15)`

### OutSpec — Define outputs

`{'Price'}` (default) | character vector with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | cell array of character vectors with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | string array with values `"Price"`, `"Delta"`, `"Gamma"`, `"Vega"`, `"Lambda"`, `"Rho"`, `"Theta"`, and `"All"`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a `NOUT`-by-1 or 1-by-`NOUT` cell array of character vectors or string array with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output is `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` to include each sensitivity:

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: `char` | `cell` | `string`

### AvgPrice — Average price of underlying asset at the Settle date

vector

Average price of underlying asset at the `Settle` date, specified as the comma-separated pair consisting of `'AvgPrice'` and a `NINST`-by-1 vector.

---

**Note** Use the `AvgPrice` argument when `AvgDate < Settle`.

---



Data Types: double

### **AvgDate — Date averaging period begins**

character vector | serial date number | datetime | string array

Date averaging period begins, specified as the comma-separated pair consisting of 'AvgDate' and a NINST-by-1 vector using character vectors, serial date numbers, datetimes, or string arrays.

Data Types: char | double | datetime | string

### **NumFixings — Total number of fixings or averaging points**

10 (default) | vector

Total number of fixings or averaging points, specified as the comma-separated pair consisting of 'NumFixings' and a NINST-by-1 vector.

Data Types: double

## **Output Arguments**

### **PriceSens — Expected prices or sensitivities for fixed Asian options**

vector

Expected prices or sensitivities for fixed Asian options, returned as a NINST-by-1 vector. `asianbyhbm` calculates prices of European arithmetic fixed (average price) Asian options with discretely monitoring.

## **Definitions**

### **Fixed Asian Options**

Fixed Asian options have a specified strike.

The payoff at maturity for a fixed (average price) Asian option is:

- Fixed call (average price option):  $\max(0, S_{av} - X)$
- Fixed put (average price option):  $\max(0, X - S_{av})$

where:

$S_{av}$  is the arithmetic or geometric average price of underlying asset.

$S$  is the price at maturity of the underlying asset.

$X$  is the strike price.

### References

- [1] Haug, E. G. *The Complete Guide to Option Pricing Formulas*. McGraw-Hill Education, 2007.

### See Also

asianbycrr | asianbyhbm | asianbykv | asianbylevy | asianbyls | asianbytw |  
intenvset | stockspect

### Topics

“Pricing Asian Options”

“Asian Option” on page 3-44

“Supported Equity Derivatives” on page 3-24

### External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

### Introduced in R2018a

## asianbytw

Price European arithmetic fixed Asian options using Turnbull-Wakeman model

### Syntax

```
Price = asianbytw(RateSpec, StockSpec, OptSpec, Strike, Settle,
ExerciseDates)
Price = asianbytw( ____, Name, Value)
```

### Description

Price = asianbytw(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates) prices European arithmetic fixed Asian options using the Turnbull-Wakeman model.

Price = asianbytw( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

### Examples

#### Price an Asian Option with Averaging Period Before the Settle Date

Define the Asian option parameters.

```
AssetPrice = 100;
Strike = 95;
Rates = 0.1;
Sigma = 0.15;
Settle = 'Apr-1-2013';
Maturity = 'Oct-1-2013';
```

Create a RateSpec using the intenvset function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a `StockSpec` for the underlying asset using the `stockspec` function.

```
DividendType = 'Continuous';  
DividendAmounts = 0.05;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price of the Asian option using the Turnbull-Wakeman approximation. Assume that the averaging period has started before the `Settle` date.

```
OptSpec = 'Call';  
ExerciseDates = 'Oct-1-2013';  
AvgDate = 'Jan-1-2013';  
AvgPrice = 100;
```

```
Price = asianbytw(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, ...  
'AvgDate', AvgDate, 'AvgPrice', AvgPrice)
```

```
Price = 5.6731
```

### **Price an Asian Option with Averaging Period After the Settle Date**

Define the Asian option parameters.

```
AssetPrice = 100;  
Strike = 95;  
Rates = 0.1;  
Sigma = 0.15;  
Settle = 'Apr-1-2013';  
Maturity = 'Oct-1-2013';
```

Create a `RateSpec` using the `intenvset` function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...  
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a `StockSpec` for the underlying asset using the `stockspec` function.

```
DividendType = 'Continuous';  
DividendAmounts = 0.05;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price of the Asian option using the Turnbull-Wakeman approximation. Assume that the averaging period starts after the `Settle` date.

```
OptSpec = 'Call';
ExerciseDates = 'Oct-1-2013';
AvgDate = 'Jan-1-2013';

Price = asianbytw(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates, ...
'AvgDate',AvgDate)

Price = 1.0774e-08
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### StockSpec — Stock specification for underlying asset

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### OptSpec — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of option, specified as 'call' or 'put' using a character vector, cell array of character vectors, or string array.

Data Types: `char` | `cell` | `string`

**Strike — Option strike price value**

nonnegative integer | vector of nonnegative integer

Option strike price value, specified with a nonnegative integer using a NINST-by-1 vector of strike price values.

Data Types: `double`

**Settle — Settlement dates or trade dates**

serial date number | date character vector | `datetime` | string array

Settlement date or trade date for the Asian option, specified as a NINST-by-1 vector using serial date numbers, date character vectors, `datetimes`, or string arrays.

Data Types: `double` | `char` | `datetime` | `string`

**ExerciseDates — European option exercise dates**

serial date number | date character vector | `datetime` | string array

European option exercise dates, specified as a NINST-by-1 vector using serial date numbers, date character vectors, `datetimes`, or string arrays.

---

**Note** For a European option, there is only one `ExerciseDates` on the option expiry date.

---

Data Types: `double` | `char` | `datetime` | `string`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price =`

```
asianbytw(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'AvgPrice', 1500)
```

**AvgPrice — Average price of underlying asset at the Settle date**

vector

Average price of underlying asset at the `Settle` date, specified as the comma-separated pair consisting of `'AvgPrice'` and a NINST-by-1 vector.

---

**Note** Use the `AvgPrice` argument when `AvgDate < Settle`.

---

Data Types: `double`

### **AvgDate — Date averaging period begins**

`serial date number` | `date character vector` | `datetime` | `string array`

Date averaging period begins, specified as the comma-separated pair consisting of `'AvgDate'` and a NINST-by-1 vector using serial date numbers, date character vectors, datetimes, or string arrays.

Data Types: `char` | `double` | `datetime` | `string`

## **Output Arguments**

### **Price — Expected prices for fixed Asian options**

`vector`

Expected prices for Asian options, returned as a NINST-by-1 vector.

## **Definitions**

### **Fixed Asian Options**

Fixed Asian options have a specified strike.

The payoff at maturity for a fixed (average price) Asian option is:

- Fixed call (average price option):  $\max(0, S_{av} - X)$
- Fixed put (average price option):  $\max(0, X - S_{av})$

where:

$S_{av}$  is the arithmetic or geometric average price of underlying asset.

$S$  is the price at maturity of the underlying asset.

$X$  is the strike price.

### References

- [1] Turnbull, S. M. and L. M. Wakeman. "A Quick Algorithm for Pricing European Average Options." *Journal of Financial and Quantitative Analysis* Vol. 26(3).1991, pp. 377-389.

### See Also

asianbycrr | asianbyhbm | asianbykv | asianbylevy | asianbyls |  
asiansensbytw | intenvset | stockspect

### Topics

"Pricing Asian Options"

"Asian Option" on page 3-44

"Supported Equity Derivatives" on page 3-24

### External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

**Introduced in R2018a**



# asiansensbytw

Calculate price and sensitivities of European fixed arithmetic Asian options using Turnbull-Wakeman model

## Syntax

```
PriceSens = asiansensbytw(RateSpec,StockSpec,OptSpec,Strike,Settle,
ExerciseDates)
PriceSens = asiansensbytw( ____,Name,Value)
```

## Description

`PriceSens = asiansensbytw(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)` calculates prices and sensitivities for European fixed arithmetic Asian options using the Turnbull-Wakeman model.

`PriceSens = asiansensbytw( ____,Name,Value)` adds optional name-value pair arguments.

## Examples

### Compute Price and Sensitivities for Asian Option with Averaging Period Before the Settle Date

Define the Asian option parameters.

```
AssetPrice = 100;
Strike = 95;
Rates = 0.1;
Sigma = 0.15;
Settle = 'Apr-1-2013';
Maturity = 'Oct-1-2013';
```

Create a `RateSpec` using the `intenvset` function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...  
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a StockSpec for the underlying asset using the stockspec function.

```
DividendType = 'Continuous';  
DividendAmounts = 0.05;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price and sensitivities of the Asian option using the Turnbull-Wakeman approximation. Assume that the averaging period has started before the Settle date.

```
OptSpec = 'Call';  
ExerciseDates = 'Oct-1-2013';  
AvgDate = 'Jan-1-2013';  
AvgPrice = 100;  
OutSpec = {'Price', 'Delta', 'Gamma'};
```

```
[Price, Delta, Gamma] = asiandelta(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates,  
'AvgDate', AvgDate, 'AvgPrice', AvgPrice, 'OutSpec', OutSpec)
```

```
Price = 5.6731
```

```
Delta = 0.5995
```

```
Gamma = 0.0135
```

### **Compute Price and Sensitivities for Asian Option with Averaging Period After the Settle Date**

Define the Asian option parameters.

```
AssetPrice = 100;  
Strike = 95;  
Rates = 0.1;  
Sigma = 0.15;  
Settle = 'Apr-1-2013';  
Maturity = 'Oct-1-2013';
```

Create a RateSpec using the intenvset function.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

Create a StockSpec for the underlying asset using the stockspec function.

```
DividendType = 'Continuous';
DividendAmounts = 0.05;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);
```

Calculate the price and sensitivities of the Asian option using the Turnbull-Wakeman approximation. Assume that the averaging period starts after the Settle date.

```
OptSpec = 'Call';
ExerciseDates = 'Oct-1-2013';
AvgDate = 'Jan-1-2013';
OutSpec = {'Price', 'Delta', 'Gamma'};
```

```
[Price,Delta,Gamma] = asiansensbytw(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,
'AvgDate',AvgDate,'OutSpec',OutSpec)
```

```
Price = 1.0774e-08
```

```
Delta = 1.0380e-08
```

```
Gamma = 9.6246e-09
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types: struct

### StockSpec — Stock specification for underlying asset

structure

Stock specification for underlying asset, specified using StockSpec obtained from stockspec. For information on the stock specification, see stockspec.

stockspec can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in StockSpec, dividends are assumed to be 0.

Data Types: struct

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of option, specified as 'call' or 'put' using a character vector, cell array of character vectors, or string array.

Data Types: char | cell | string

**Strike — Option strike price value**

nonnegative integer | vector of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 vector of strike price values.

Data Types: double

**Settle — Settlement dates or trade dates**

serial date number | date character vector | datetime | string array

Settlement date or trade date for the Asian option, specified as a NINST-by-1 vector using serial date numbers, date character vectors, datetimes, or string arrays.

Data Types: double | char

**ExerciseDates — European option exercise dates**

serial date number | date character vector | datetime | string array

European option exercise dates, specified as a NINST-by-1 vector using serial date numbers, date character vectors, datetimes, or string arrays.

---

**Note** For a European option, there is only one ExerciseDates on the option expiry date.

---

Data Types: double | char | datetime | string

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `PriceSens = asiansensbytw(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'OutSpec', {'All'})`

### OutSpec — Define outputs

`Price` (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | string array with values "Price", "Delta", "Gamma", "Vega", "Lambda", "Rho", "Theta", and "All"

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors or string array with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

`OutSpec = {'All'}` specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying `OutSpec` to include each sensitivity:

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: `char` | `cell` | `string`

### AvgPrice — Average price of underlying asset at the Settle date

vector

Average price of underlying asset at the `Settle` date, specified as the comma-separated pair consisting of 'AvgPrice' and a NINST-by-1 vector.

---

**Note** Use the `AvgPrice` argument when `AvgDate < Settle`.

---

Data Types: `double`

**AvgDate — Date averaging period begins**

serial date number | date character vector | datetime | string array

Date averaging period begins, specified as the comma-separated pair consisting of 'AvgDate' and a NINST-by-1 vector using serial date numbers, date character vectors, datetimes, or string arrays.

Data Types: char | double | datetime | string

## Output Arguments

**PriceSens — Expected prices or sensitivities for fixed Asian options**

vector

Expected prices or sensitivities for fixed Asian options, returned as a NINST-by-1 vector. `asiansensbytw` calculates prices of European arithmetic fixed (average price) Asian options.

## Definitions

### Fixed Asian Options

Fixed Asian options have a specified strike.

The payoff at maturity for a fixed (average price) Asian option is:

- Fixed call (average price option):  $\max(0, S_{av} - X)$
- Fixed put (average price option):  $\max(0, X - S_{av})$

where:

$S_{av}$  is the arithmetic or geometric average price of underlying asset.

$S$  is the price at maturity of the underlying asset.

$X$  is the strike price.

## References

- [1] Turnbull, S. M. and L. M. Wakeman. "A Quick Algorithm for Pricing European Average Options." *Journal of Financial and Quantitative Analysis* Vol. 26(3).1991, pp. 377-389.

## See Also

asianbycrr | asianbyhbm | asianbykv | asianbylevy | asianbyls | asianbytw |  
intenvset | stockspect

## Topics

"Pricing Asian Options"

"Asian Option" on page 3-44

"Supported Equity Derivatives" on page 3-24

## External Websites

How to Price Asian Options Efficiently Using MATLAB (4 min 37 sec)

## Introduced in R2018a

## assetbyls

Determine price of asset-or-nothing digital options using Black-Scholes model

### Syntax

```
Price = assetbyls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
```

### Description

`Price = assetbyls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)` computes asset-or-nothing European digital options using the Black-Scholes option pricing model.

### Examples

#### Compute Asset-Or-Nothing Digital Option Prices Using the Black-Scholes Option Pricing Model

Consider two asset-or-nothing put options on a nondividend paying stock with a strike of 95 and 93 and expiring on January 30, 2009. On November 3, 2008 the stock is trading at 97.50. Using this data, calculate the price of the asset-or-nothing put options if the risk-free rate is 4.5% and the volatility is 22%. First, create the `RateSpec`.

```
Settle = 'Nov-3-2008';  
Maturity = 'Jan-30-2009';  
Rates = 0.045;  
Compounding = -1;  
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...  
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding)  
  
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9893
```



```
        Rates: 0.0450
        EndTimes: 0.2391
        StartTimes: 0
        EndDates: 733803
        StartDates: 733715
        ValuationDate: 733715
        Basis: 0
        EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 97.50;
Sigma = .22;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2200
    AssetPrice: 97.5000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the put options.

```
OptSpec = {'put'};
Strike = [95;93];
```

Calculate the price.

```
Paon = assetblys(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
```

```
Paon = 2×1
```

```
33.7666
26.9662
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as an NINST-by-1 vector.

Data Types: char | cell

### **Strike — Pay-off strike value**

vector

Pay-off strike value, specified as an NINST-by-1 vector.

Data Types: double

## **Output Arguments**

### **Price — Expected prices for asset-or-nothing option**

vector

Expected prices for asset-or-nothing option, returned as a NINST-by-1 vector.

## **See Also**

assetsensbyls | cashbyls | gapbyls | supersharebyls

## **Topics**

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**

## assetsensbyls

Determine price or sensitivities of asset-or-nothing digital options using Black-Scholes model

### Syntax

```
PriceSens = assetsensbyls(RateSpec,StockSpec,Settle,Maturity,  
OptSpec,Strike)  
PriceSens = assetsensbyls( ____,Name,Value)
```

### Description

`PriceSens = assetsensbyls(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike)` computes asset-or-nothing European digital options or sensitivities using the Black-Scholes option pricing model.

`PriceSens = assetsensbyls( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute Asset-Or-Nothing Digital Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model

Consider two asset-or-nothing put options on a nondividend paying stock with a strike of 95 and 93 and expiring on January 30, 2009. On November 3, 2008 the stock is trading at 97.50. Using this data, calculate the price and sensitivity of the asset-or-nothing put options if the risk-free rate is 4.5% and the volatility is 22%. First, create the `RateSpec`.

```
Settle = 'Nov-3-2008';  
Maturity = 'Jan-30-2009';  
Rates = 0.045;  
Compounding = -1;
```

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9893
    Rates: 0.0450
    EndTimes: 0.2391
    StartTimes: 0
    EndDates: 733803
    StartDates: 733715
    ValuationDate: 733715
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 97.50;
Sigma = .22;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2200
    AssetPrice: 97.5000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the put options.

```
OptSpec = {'put'};
Strike = [95;93];
```

Calculate the delta, price, and gamma.

```
OutSpec = { 'delta'; 'price'; 'gamma'};
[Delta, Price, Gamma] = assetsensbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)
```

```
Delta = 2×1
```

```
-3.0833
```

-2.8337

Price = 2×1

33.7666

26.9662

Gamma = 2×1

0.0941

0.1439

## Input Arguments

### **RateSpec** — Interest-rate term structure structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle** — Settlement or trade date serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as an NINST-by-1 vector.

Data Types: char | cell

### **Strike — Pay-off strike value**

vector

Pay-off strike value, specified as an NINST-by-1 vector.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Gamma,Delta] = assetsensbyls(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,'OutSpec',{'gamma'; 'delta'})`

### **OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.

Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}

Data Types: char | cell

## Output Arguments

### **PriceSens** — Expected prices or sensitivities for asset-or-nothing option

vector

Expected prices or sensitivities (defined using OutSpec) for asset-or-nothing option, returned as a NINST-by-1 vector.

## See Also

assetbybls | cashbybls | gapbybls | supersharebybls

## Topics

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**



# barrierbycrr

Price barrier option from Cox-Ross-Rubinstein binomial tree

## Syntax

```
[Price,PriceTree] = barrierbycrr(CRRTree,OptSpec,Strike,Settle,  
AmericanOpt,ExerciseDates,BarrierSpec,Barrier)  
[Price,PriceTree] = barrierbycrr( ____,Rebate,Options)
```

## Description

[Price,PriceTree] = barrierbycrr(CRRTree,OptSpec,Strike,Settle, AmericanOpt,ExerciseDates,BarrierSpec,Barrier) calculates prices for barrier options using a Cox-Ross-Rubinstein binomial tree.

[Price,PriceTree] = barrierbycrr( \_\_\_\_,Rebate,Options) adds optional arguments for Rebate and Options.

## Examples

### Price a Barrier Option Using a CRR Binomial Tree

This example shows how to price a barrier option using a CRR binomial tree by loading the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;  
  
OptSpec = 'Call';  
Strike = 105;  
Settle = '01-Jan-2003';  
ExerciseDates = '01-Jan-2006';  
AmericanOpt = 1;  
BarrierSpec = 'UI';
```

```
Barrier = 102;
```

```
Price = barrierbycrr(CRRTree, OptSpec, Strike, Settle, ...  
ExerciseDates, AmericanOpt, BarrierSpec, Barrier)
```

```
Price = 12.1272
```

## Input Arguments

### **CRRTree — Stock tree structure**

structure

Stock tree structure, specified by using `crrtree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of an option as 'call' or 'put', specified as a NINST-by-1 cell array of character vector values.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

integer

Option strike price value for a European or an American Option, specified as NINST-by-1 matrix of integers. Each row is the schedule for one option.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the barrier option, specified as a NINST-by-1 matrix of serial date numbers or date character vectors. The `Settle` date for every barrier is set to the `ValuationDate` of the stock tree. The barrier argument `Settle` is ignored.

Data Types: `double` | `char`

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or a date character vector:

- For a European option, use a 1-by-1 matrix of dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date which is the maturity of the instrument.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a NINST-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed date in `ExerciseDates`.

Data Types: double | char

**AmericanOpt — Option type**

Option type values 0 or 1

Option type, specified as NINST-by-1 matrix of integer flags with values:

- 0 — European
- 1 — American

Data Types: double

**BarrierSpec — Barrier option type**

character vector with values: 'UI', 'U0', 'DI', 'D0' | cell array of character vectors with values: 'UI', 'U0', 'DI', 'D0'

Barrier option type, specified as a character vector or an NINST-by-1 cell array of character vectors with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option.

- 'U0' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy/sell (call/put) the underlying security at the strike price as long as the underlying asset does not

go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually, with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: `char` | `cell`

**Barrier — Barrier level**

`numeric`

Barrier level, specified as a NINST-by-1 matrix of numeric values.

Data Types: `double`

**Rebate — Rebate value**

0 (default) | `numeric`

(Optional) Rebate value, specified as a NINST-by-1 matrix of numeric values. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

### **Options — Derivatives pricing options**

structure

(Optional) Derivatives pricing options, specified as structure that is created with derivset.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices for barrier options at time 0**

matrix

Expected prices for barrier options at time 0, returned as a NINST-by-1 matrix.

### **PriceTree — Structure with vector of barrier option prices at each node**

tree structure

Structure with a vector of barrier option prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.t0bs contains the observation times.

PriceTree.d0bs contains the observation dates.

## Definitions

### Barrier Option

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

### References

[1] Derman, E., I. Kani, D. Ergener and I. Bardhan. "Enhanced Numerical Methods for Options with Barriers." *Financial Analysts Journal*. (Nov.-Dec.), 1995, pp. 65-74.

### See Also

`crrtree` | `instbarrier`

### Topics

"Computing Prices Using CRR" on page 3-131

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Pricing European Call Options Using Different Equity Models"

"Barrier Option" on page 3-26

"Pricing Options Structure" on page B-2

"Supported Equity Derivatives" on page 3-24

**Introduced before R2006a**

# barrierbyeqp

Price barrier option from Equal Probabilities binomial tree

## Syntax

```
[Price,PriceTree] = barrierbyeqp(EQPTree,OptSpec,Strike,Settle,  
AmericanOpt,ExerciseDates,BarrierSpec,Barrier)  
[Price,PriceTree] = barrierbyeqp( ____,Rebate,Options)
```

## Description

[Price,PriceTree] = barrierbyeqp(EQPTree,OptSpec,Strike,Settle, AmericanOpt,ExerciseDates,BarrierSpec,Barrier) calculates prices for barrier options using an Equal Probabilities binomial tree.

[Price,PriceTree] = barrierbyeqp( \_\_\_\_,Rebate,Options) adds optional arguments for Rebate and Options.

## Examples

### Price a Barrier Option Using an EQP Equity Tree

This example shows how to price a barrier option using an EQP equity tree by loading the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;  
  
OptSpec = 'Call';  
Strike = 105;  
Settle = '01-Jan-2003';  
ExerciseDates = '01-Jan-2006';  
AmericanOpt = 1;  
BarrierSpec = 'UI';
```

```
Barrier = 102;
```

```
Price = barrierbyeqp(EQPTree, OptSpec, Strike, Settle, ...  
ExerciseDates, AmericanOpt, BarrierSpec, Barrier)
```

```
Price = 12.2632
```

## Input Arguments

### **EQPTree — Stock tree structure**

structure

Stock tree structure, specified by using `eqptree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of an option as 'call' or 'put', specified as a NINST-by-1 cell array of character vector values.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

numeric

Option strike price value for a European or an American Option, specified as NINST-by-1 matrix of numeric values. Each row is the schedule for one option.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the barrier option, specified as a NINST-by-1 matrix of serial date numbers or date character vectors. The `Settle` date for every barrier is set to the `ValuationDate` of the stock tree. The barrier argument `Settle` is ignored.

Data Types: `double` | `char`



**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or a date character vector:

- For a European option, use a 1-by-1 matrix of dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date which is the maturity of the instrument.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a NINST-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed date in `ExerciseDates`.

Data Types: double | char

**AmericanOpt — Option type**

option type with values 0 or 1

Option type, specified as NINST-by-1 matrix of flags with values:

- 0 — European
- 1 — American

Data Types: double

**BarrierSpec — Barrier option type**

character vector with values: 'UI', 'U0', 'DI', 'D0' | cell array of character vectors with values: 'UI', 'U0', 'DI', 'D0'

Barrier option type, specified as a character vector or an NINST-by-1 cell array of character vectors with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option.

- 'U0' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not

go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually, with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: `char` | `cell`

**Barrier — Barrier level**

`numeric`

Barrier level, specified as a NINST-by-1 matrix of numeric values.

Data Types: `double`

**Rebate — Rebate value**

0 (default) | `numeric`

(Optional) Rebate value, specified as a NINST-by-1 matrix of numeric values. For Knock-in options, the `Rebate` is paid at expiry. For Knock-out options, the `Rebate` is paid when the `Barrier` is reached.

Data Types: `double`

### **Options — Derivatives pricing options**

structure

(Optional) Derivatives pricing options, specified as a structure that is created with `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices for barrier options at time 0**

matrix

Expected prices for barrier options at time 0, returned as a NINST-by-1 matrix.

### **PriceTree — Structure with vector of barrier option prices at each node**

tree structure

Structure with a vector of barrier option prices at each node, returned as a tree structure.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

`PriceTree.PTree` contains the prices.

`PriceTree.t0bs` contains the observation times.

`PriceTree.d0bs` contains the observation dates.

## Definitions

### Barrier Option

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

### References

[1] Derman, E., I. Kani, D. Ergener and I. Bardhan. "Enhanced Numerical Methods for Options with Barriers." *Financial Analysts Journal*. (Nov.-Dec.), 1995, pp. 65-74.

### See Also

`eqptree` | `instbarrier`

### Topics

"Computing Prices Using CRR" on page 3-131

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Pricing European Call Options Using Different Equity Models"

"Barrier Option" on page 3-26

"Pricing Options Structure" on page B-2

"Supported Equity Derivatives" on page 3-24

### Introduced before R2006a

# barrierbyfd

Calculate barrier option prices using finite difference method

## Syntax

```
[Price,PriceGrid,AssetPrices,Times] = barrierbyfd(RateSpec,  
StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier)  
[Price,PriceGrid,AssetPrices,Times] = barrierbyfd( ____,Name,Value)
```

## Description

[Price,PriceGrid,AssetPrices,Times] = barrierbyfd(RateSpec, StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier) calculates European and American barrier option prices on a single underlying asset using the finite difference method. barrierbyfd assumes that the barrier is continuously monitored.

[Price,PriceGrid,AssetPrices,Times] = barrierbyfd( \_\_\_\_,Name,Value) adds optional name-value pair arguments. barrierbyfd assumes that the barrier is continuously monitored.

## Examples

### Price a Barrier Down and Out Call Option Using Finite Difference Method

Create a RateSpec.

```
AssetPrice = 50;  
Strike = 45;  
Rate = 0.035;  
Volatility = 0.30;  
Settle = '01-Jan-2015';  
Maturity = '01-Jan-2016';  
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...  
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', Basis)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9656  
    Rates: 0.0350  
    EndTimes: 1  
    StartTimes: 0  
    EndDates: 736330  
    StartDates: 735965  
    ValuationDate: 735965  
    Basis: 1  
    EndMonthRule: 1
```

Create a StockSpec.

```
StockSpec = stockspec(Volatility, AssetPrice)
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
    Sigma: 0.3000  
    AssetPrice: 50  
    DividendType: []  
    DividendAmounts: 0  
    ExDividendDates: []
```

Calculate the price of a European Down and Out call option using Finite Difference.

```
Barrier = 40;  
BarrierSpec = 'D0';  
OptSpec = 'Call';  
Price = barrierbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity,...  
BarrierSpec, Barrier)
```

```
Price = 8.5020
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values 'call' or 'put'

Definition of an option as 'call' or 'put', specified as a character vector or string array with values "call" or "put".

Data Types: `char` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

### **ExerciseDates** — Option exercise dates

date character vector | serial date number | datetime object

Option exercise dates, specified as a date character vector, a serial date number, or datetime object:

- For a European option, there is only one `ExerciseDates` on the option expiry date which is the maturity of the instrument.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: `double` | `char` | `cell` | `datetime`

### **BarrierSpec** — Barrier option type

character vector with values: `'UI'`, `'UO'`, `'DI'`, `'DO'`

Barrier option type, specified as a character vector with the following values:

- `'UI'` — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- `'UO'` — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually, with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- `'DI'` — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell



(call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'D0' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: `char`

### Barrier — Barrier level

numeric

Barrier level, specified as a scalar numeric value.

Data Types: `double`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price =`

```
barrierbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, Rebate, 1000)
```

**Rebate — Rebate value**

0 (default) | numeric

Rebate value, specified as the comma-separated pair consisting of 'Rebate' and a scalar numeric. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

**AssetGridSize — Size of asset grid used for a finite difference grid**

400 (default) | positive numeric

Size of the asset grid used for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar positive numeric.

Data Types: double

**TimeGridSize — Size of time grid used for finite difference grid**

100 (default) | positive numeric

Size of the time grid used for the finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a scalar positive numeric.

Data Types: double

**AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of the following values:

- 0 — European
- 1 — American

Data Types: logical

## Output Arguments

**Price — Expected prices for barrier options**

matrix

Expected prices for barrier options, returned as a NINST-by-1 matrix.

**PriceGrid — Grid containing prices calculated by finite difference method**

grid

Grid containing prices calculated by the finite difference method, returned as a grid that is two-dimensional with size `PriceGridSize*length(Times)`. The number of columns does not have to be equal to the `TimeGridSize`, because ex-dividend dates in the `StockSpec` are added to the time grid. The price for  $t = 0$  is contained in `PriceGrid(:, end)`.

**AssetPrices — Prices of asset defined by StockSpec**

vector

Prices of the asset defined by the `StockSpec` corresponding to the first dimension of `PriceGrid`, returned as a vector.

**Times — Times corresponding to second dimension of PriceGrid**

vector

Times corresponding to the second dimension of the `PriceGrid`, returned as a vector.

## Definitions

### Barrier Option

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

### References

- [1] Hull, J. *Options, Futures, and Other Derivatives*. Fourth Edition. Prentice Hall. 2000, pp. 646-649.
- [2] Aitsahlia, F, L. Imhof, and T.L. Lai. "Pricing and hedging of American knock-in options." *The Journal of Derivatives*. Vol. 11.3, 2004, pp. 44-50.

[3] Rubinstein M. and E. Reiner. “Breaking down the barriers.” *Risk*. Vol. 4(8), 1991, pp. 28-35.

## **See Also**

barrierbybls | barrierbyls | barriersensbybls | barriersensbyfd |  
barriersensbyls

## **Topics**

“Barrier Option” on page 3-26

“Supported Equity Derivatives” on page 3-24

**Introduced in R2016b**

# barriersensbyfd

Calculate barrier option prices or sensitivities using finite difference method

## Syntax

```
[PriceSens,PriceGrid,AssetPrices,Times] = barriersensbyfd(RateSpec,  
StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier)  
[PriceSens,PriceGrid,AssetPrices,Times] = barriersensbyfd( ____,  
Name,Value)
```

## Description

[PriceSens,PriceGrid,AssetPrices,Times] = barriersensbyfd(RateSpec, StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier) calculates European and American barrier option prices or sensitivities of a single underlying asset using the finite difference method. barrierbyfd assumes that the barrier is continuously monitored.

[PriceSens,PriceGrid,AssetPrices,Times] = barriersensbyfd( \_\_\_\_, Name,Value) adds optional name-value pair arguments. barriersensbyfd assumes that the barrier is continuously monitored.

## Examples

### Calculate Price and Sensitivities for a Barrier Down and Out Call Option Using Finite Difference Method

Create a RateSpec.

```
AssetPrice = 50;  
Strike = 45;  
Rate = 0.035;  
Volatility = 0.30;  
Settle = '01-Jan-2015';
```

```
Maturity = '01-Jan-2016';  
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',Settle,'StartDates',Settle,'EndDates',...  
Maturity,'Rates',Rate,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9656  
    Rates: 0.0350  
    EndTimes: 1  
    StartTimes: 0  
    EndDates: 736330  
    StartDates: 735965  
    ValuationDate: 735965  
    Basis: 1  
    EndMonthRule: 1
```

Create a StockSpec.

```
StockSpec = stockspec(Volatility,AssetPrice)
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
    Sigma: 0.3000  
    AssetPrice: 50  
    DividendType: []  
    DividendAmounts: 0  
    ExDividendDates: []
```

Calculate the Price, Delta, and Theta of a European Down and Out call option using the finite difference method.

```
Barrier = 40;  
BarrierSpec = 'D0';  
OptSpec = 'Call';  
OutSpec = {'price';'delta';'theta'};  
[Price, Delta, Theta] = barriersensbyfd(RateSpec,StockSpec,OptSpec,Strike,Settle,...  
Maturity, BarrierSpec,Barrier,'Outspec',OutSpec)  
  
Price = 8.5020  
Delta = 0.8569
```

Theta = -1.8502

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values 'call' or 'put'

Definition of an option as 'call' or 'put', specified as a character vector or string array with values "call" or "put".

Data Types: `char` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

**Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: double | char | datetime

**ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as a serial date number, a date character vector, or datetime object:

- For a European option, there is only one `ExerciseDates` on the option expiry date which is the maturity of the instrument.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: double | char | datetime

**BarrierSpec — Barrier option type**

character vector with values: 'UI', 'UO', 'DI', 'DO'

Barrier option type, specified as a character vector with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'UO' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually, with an up-



and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price, as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: `char`

### Barrier — Barrier level

numeric

Barrier level, specified as a scalar numeric.

Data Types: `double`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `PriceSens = barriersensbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, Rebate, 1000, AmericanOpt, 1)`

**Rebate — Rebate value**

0 (default) | numeric

Rebate value, specified as the comma-separated pair consisting of 'Rebate' and a scalar numeric. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

`OutSpec = {'All'}` specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: char | cell

**AssetGridSize — Size of asset grid used for finite difference grid**

400 (default) | positive numeric

Size of the asset grid used for a finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar positive numeric.

Data Types: double

**TimeGridSize — Size of time grid used for finite difference grid**

100 (default) | positive numeric

Size of the time grid used for a finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a scalar positive numeric.

Data Types: double

### **AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of the following values:

- 0 — European
- 1 — American

Data Types: logical

## **Output Arguments**

### **PriceSens — Expected prices or sensitivities values for barrier options**

matrix

Expected prices or sensitivities (defined using OutSpec) for barrier options, returned as a NINST-by-1 matrix.

### **PriceGrid — Grid containing prices calculated by finite difference method**

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with size PriceGridSize\*length(Times). The number of columns does not have to be equal to the TimeGridSize, because ex-dividend dates in the StockSpec are added to the time grid. The price for  $t = 0$  is contained in PriceGrid(:, end).

### **AssetPrices — Prices of the asset defined by StockSpec**

vector

Prices of the asset defined by the StockSpec corresponding to the first dimension of PriceGrid, returned as a vector.

### **Times — Times corresponding to second dimension of PriceGrid**

vector

Times corresponding to the second dimension of the `PriceGrid`, returned as a vector.

## Definitions

### Barrier Option

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

### References

- [1] Hull, J. *Options, Futures and Other Derivatives*. Fourth Edition. Prentice Hall, 2000, pp. 646-649.
- [2] Aitsahlia, F., L. Imhof, and T.L. Lai. "Pricing and hedging of American knock-in options." *The Journal of Derivatives*. Vol. 11.3 , 2004, pp. 44-50.
- [3] Rubinstein M. and E. Reiner. "Breaking down the barriers." *Risk*. Vol. 4(8), 1991, pp. 28-35.

### See Also

`barrierbybls` | `barrierbyfd` | `barrierbyls` | `barriersensbybls` | `barriersensbyls`

### Topics

"Barrier Option" on page 3-26  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2016b**

# dblbarrierbyfd

Calculate double barrier option price using finite difference method

## Syntax

```
[Price,PriceGrid,AssetPrices,Times] = dblbarrierbyfd(RateSpec,  
StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier)  
[Price,PriceGrid,AssetPrices,Times] = dblbarrierbyfd( ____,Name,Value)
```

## Description

[Price,PriceGrid,AssetPrices,Times] = dblbarrierbyfd(RateSpec, StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier) calculates a European or American call or put double barrier option price on a single underlying asset using the finite difference method. `dblbarrierbyfd` assumes that the barrier is continuously monitored.

[Price,PriceGrid,AssetPrices,Times] = dblbarrierbyfd( \_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Price an American Double Knock-Out Call Option with Rebate

Compute the price of an American double barrier option for a double knock-out (down and out-up and out) call option with a rebate using the following data:

```
Rate = 0.05;  
Settle = '01-Jun-2018';  
Maturity = '01-Dec-2018';  
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', Maturity)
```

Define a StockSpec.

```
AssetPrice = 100;  
Volatility = 0.25;  
StockSpec = stockspec(Volatility, AssetPrice);
```

Define the double barrier option.

```
LBarrier = 80;  
UBarrier = 130;  
Barrier = [UBarrier LBarrier];  
BarrierSpec = 'DKO';  
OptSpec = 'Call';  
Strike = 110;  
Rebate = 1;
```

Compute the price of an American option using finite differences.

```
Price = dblbarrierbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec)
```

```
Price = 4.0002
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset, specified by the StockSpec obtained from `stockspect`.

stockspec handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string scalar with values "call" or "put"

Definition of an option, specified as a character vector with a value of 'call' or 'put', or a string scalar with values "call" or "put".

Data Types: `char` | `string`

### **Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

### **Settle — Settlement or trade date**

date character vector | serial date number | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

### **ExerciseDates — Option exercise dates**

date character vector | serial date number | datetime object

Option exercise dates, specified as a date character vector, a serial date number, or datetime object.

- For a European option, the option expiry date has only one `ExerciseDates` value.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates. If only one non-`NaN` date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: `double` | `char` | `cell` | `datetime`

**BarrierSpec — Double barrier option type**

character vector with value of 'DKI' or 'DKO' | scalar string with value of "DKI" or "DKO"

Double barrier option type, specified as a character vector or string with one of the following values:

- 'DKI' — Double Knock-in

The 'DKI' option becomes effective when the price of the underlying asset reaches one of the barriers. It gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, if the underlying asset goes above or below the barrier levels during the life of the option.

- 'DKO' — Double Knock-out

The 'DKO' option gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, as long as the underlying asset remains between the barrier levels during the life of the option. This option terminates when the price of the underlying asset passes one of the barriers.

Option	Barrier Type	Payoff If Any Barrier Crossed	Payoff If Barriers Not Crossed
Call/Put	Double Knock-in	Standard Call/Put	Worthless
Call/Put	Double Knock-out	Worthless	Standard Call/Put

Data Types: char | string

**Barrier — Barrier level**

vector

Barrier level, specified as a 1-by-2 vector of numeric values, where the first column is the upper barrier (1)(UB) and the second column is the lower barrier (2)(LB). Barrier(1) must be greater than Barrier(2).

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes.



You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Price =  
dblbarrierbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, 'Rebate', [100, 100])

### **Rebate — Rebate value**

[0 0] for Double Knock-out or 0 for Double Knock-in (default) | vector | scalar numeric

Rebate value, specified as the comma-separated pair consisting of 'Rebate' and one of the following:

- For a Double Knock-out option, use a 1-by-2 vector of rebate values where the first column is the payout if the upper barrier(1)(UB) is hit and the second column is payout if the lower barrier(2)(LB) is hit. The rebate is paid when the barrier is reached.
- For a Double Knock-in option, use a scalar rebate value. The rebate is paid at expiry.

Data Types: double

### **AssetGridSize — Size of asset grid used for a finite difference grid**

400 (default) | positive scalar numeric

Size of the asset grid used for the finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a positive scalar numeric.

Data Types: double

### **TimeGridSize — Size of time grid used for finite difference grid**

100 (default) | positive scalar numeric

Size of the time grid used for the finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive scalar numeric.

---

**Note** The actual time grid may have a larger size because exercise and ex-dividend dates might be added to the grid from StockSpec.

---

Data Types: double

### **AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of the following values:

- 0 — European
- 1 — American

Data Types: logical

## Output Arguments

### **Price** — Expected prices for double barrier options

matrix

Expected prices for double barrier options, returned as a 1-by-1 matrix.

### **PriceGrid** — Grid containing prices

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with the size `AssetGridSize*TimeGridSize`. The number of columns does not have to be equal to the `TimeGridSize`, because exercise and ex-dividend dates in `StockSpec` are added to the time grid. `PriceGrid(:, end)` contains the price for `t = 0`.

### **AssetPrices** — Prices of asset defined by `StockSpec`

vector

Prices of the asset defined by the `StockSpec` corresponding to the first dimension of `PriceGrid`, returned as a vector.

### **Times** — Times corresponding to second dimension of `PriceGrid`

vector

Times corresponding to the second dimension of `PriceGrid`, returned as a vector.

## Definitions

### Double Barrier Option

A double barrier option is similar to the standard single barrier option except that it has two barrier levels: a lower barrier (LB) and an upper barrier (UB).

The payoff for a double barrier option depends on whether the underlying asset remains between the barrier levels during the life of the option. Double barrier options are less expensive than single barrier options as they have a higher knock-out probability. Because of this, double barrier options allow investors to reduce option premiums and match an investor's belief about the future movement of the underlying price process.

### References

- [1] Boyle, P., and Y. Tian. "An Explicit Finite Difference Approach to the Pricing of Barrier Options." *Applied Mathematical Finance*. Vol. 5, Number 1, 1998, pp. 17-43.
- [2] Hull, J. *Options, Futures, and Other Derivatives*. Fourth Edition. Upper Saddle River, NJ: Prentice Hall, 2000, pp. 646-649.
- [3] Rubinstein, M., and E. Reiner. "Breaking Down the Barriers." *Risk*. Vol. 4, Number 8, 1991, pp. 28-35.
- [4] Zvan, R., P. A. Forsyth and K. R. Vetzal. "PDE Methods for Pricing Barrier Options." *Journal of Economic Dynamics and Control*. Vol. 24, Number 11-12, 2000, pp. 1563-1590.

### See Also

dblbarrierbybls | dblbarriersensbybls | dblbarriersensbyfd

### Topics

"Double Barrier Option" on page 3-27

"Supported Equity Derivatives" on page 3-24

**Introduced in R2019a**

## dblbarriersensbyfd

Calculate double barrier option price and sensitivities using finite difference method

### Syntax

```
[PriceSens,PriceGrid,AssetPrices,Times] = dblbarriersensbyfd(
RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,
Barrier)
[PriceSens,PriceGrid,AssetPrices,Times] = dblbarriersensbyfd( ____,
Name,Value)
```

### Description

[PriceSens,PriceGrid,AssetPrices,Times] = dblbarriersensbyfd(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier) calculates a European or American call or put double barrier option price and sensitivities of a single underlying asset using the finite difference method. `dblbarrierbyfd` assumes that the barrier is continuously monitored.

[PriceSens,PriceGrid,AssetPrices,Times] = dblbarriersensbyfd( \_\_\_\_, Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Calculate Price and Sensitivities for an American Double Knock-Out Call Option with Rebate

Compute the price and sensitivities for an American double barrier option for a double knock-out (down and out-up and out) call option with a rebate using the following data:

```
Rate = 0.05;
Settle = '01-Jun-2018';
```

```
Maturity = '01-Dec-2018';
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', Maturity);
```

Define a StockSpec.

```
AssetPrice = 100;
Volatility = 0.25;
StockSpec = stockspect(Volatility, AssetPrice);
```

Define the double barrier option.

```
LBarrier = 80;
UBarrier = 130;
Barrier = [UBarrier LBarrier];
BarrierSpec = 'DK0';
OptSpec = 'Call';
Strike = 110;
Rebate = 1;
OutSpec = {'price'; 'vega'; 'theta'};
```

Compute the price and sensitivities for an American option using finite differences.

```
[Price, Vega, Theta] = dblbarriersensbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle);
```

```
Price = 4.0002
```

```
Vega = -1.9180e+03
```

```
Theta = -6.6509
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset, specified by the `StockSpec` obtained from `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | string scalar with values "call" or "put"

Definition of an option, specified as a character vector with a value of 'call' or 'put', or a string scalar with values "call" or "put".

Data Types: `char` | `string`

**Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

**Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

**ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as a serial date number, a date character vector, or datetime object.

- For a European option, the option expiry date has only one `ExerciseDates` value.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates. If only one non-`NaN` date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: `double` | `char` | `datetime`

### **BarrierSpec — Double barrier option type**

character vector with value of 'DKI' or 'DKO' | scalar string with value of "DKI" or "DKO"

Double barrier option type, specified as a character vector or string with one of the following values:

- 'DKI' — Double Knock-in

The 'DKI' option becomes effective when the price of the underlying asset reaches one of the barriers. It gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, if the underlying asset goes above or below the barrier levels during the life of the option.

- 'DKO' — Double Knock-out

The 'DKO' option gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, as long as the underlying asset remains between the barrier levels during the life of the option. This option terminates when the price of the underlying asset passes one of the barriers.

Option	Barrier Type	Payoff If Any Barrier Crossed	Payoff If Barriers Not Crossed
Call/Put	Double Knock-in	Standard Call/Put	Worthless
Call/Put	Double Knock-out	Worthless	Standard Call/Put

Data Types: `char` | `string`

### **Barrier — Barrier level**

vector

Barrier level, specified as a 1-by-2 vector of numeric values, where the first column is the upper barrier (1)(UB) and the second column is the lower barrier (2)(LB). Barrier(1) must be greater than Barrier(2).

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

```
Example: PriceSens =
dblbarriersensbyfd(RateSpec,StockSpec,OptSpec,Strike,Settle,Maturity
,BarrierSpec,Barrier,'OutSpec',
{'delta','gamma','vega','lambda','rho','theta','price'},'AmericanOpt
',1)
```

### OutSpec — Define outputs

`'Price'` (default) | character vector with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | cell array of character vectors with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | string array with values `"Price"`, `"Delta"`, `"Gamma"`, `"Vega"`, `"Lambda"`, `"Rho"`, `"Theta"`, and `"All"`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors or a string array with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

```
Example: OutSpec =
{'delta','gamma','vega','lambda','rho','theta','price'}
```

Data Types: char | cell | string

### Rebate — Rebate value

`[0 0]` for Double Knock-out or `0` for Double Knock-in (default) | vector | scalar numeric

Rebate value, specified as the comma-separated pair consisting of `'Rebate'` and one of the following:



- For a Double Knock-out option, use a 1-by-2 vector of rebate values where the first column is the payout if the upper barrier(1)(UB) is hit and the second column is payout if the lower barrier(2)(LB) is hit. The rebate is paid when the barrier is reached.
- For a Double Knock-in option, use a scalar rebate value. The rebate is paid at expiry.

Data Types: double

### **AssetGridSize — Size of asset grid used for finite difference grid**

400 (default) | positive scalar numeric

Size of the asset grid used for the finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a positive scalar numeric.

Data Types: double

### **TimeGridSize — Size of time grid used for finite difference grid**

100 (default) | positive scalar numeric

Size of the time grid used for the finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive scalar numeric.

---

**Note** The actual time grid may have a larger size because exercise and ex-dividend dates might be added to the grid from `StockSpec`.

---

Data Types: double

### **AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of the following values:

- 0 — European
- 1 — American

Data Types: logical

## Output Arguments

### **PriceSens** — Expected prices or sensitivities values for double barrier options

matrix

Expected prices or sensitivities (defined using `OutSpec`) for double barrier options, returned as a 1-by-NOUT matrix.

### **PriceGrid** — Grid containing prices

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with the size `AssetGridSize*TimeGridSize`. The number of columns does not have to be equal to the `TimeGridSize`, because exercise and ex-dividend dates in `StockSpec` are added to the time grid. `PriceGrid(:, end)` contains the price for  $t = 0$ .

### **AssetPrices** — Prices of the asset defined by `StockSpec`

vector

Prices of the asset defined by the `StockSpec` corresponding to the first dimension of `PriceGrid`, returned as a vector.

### **Times** — Times corresponding to second dimension of `PriceGrid`

vector

Times corresponding to the second dimension of the `PriceGrid`, returned as a vector.

## Definitions

### **Double Barrier Option**

A double barrier option is similar to the standard single barrier option except that it has two barrier levels: a lower barrier (LB) and an upper barrier (UB).

The payoff for a double barrier option depends on whether the underlying asset remains between the barrier levels during the life of the option. Double barrier options are less expensive than single barrier options as they have a higher knock-out probability. Because of this, double barrier options allow investors to reduce option premiums and match an investor's belief about the future movement of the underlying price process.

## References

- [1] Boyle, P., and Y. Tian. "An Explicit Finite Difference Approach to the Pricing of Barrier Options." *Applied Mathematical Finance*. Vol. 5, Number 1, 1998, pp. 17-43.
- [2] Hull, J. *Options, Futures, and Other Derivatives*. Fourth Edition. Upper Saddle River, NJ: Prentice Hall, 2000, pp. 646-649.
- [3] Rubinstein, M., and E. Reiner. "Breaking Down the Barriers." *Risk*. Vol. 4, Number 8, 1991, pp. 28-35.
- [4] Zvan, R., P. A. Forsyth and K. R. Vetzal. "PDE Methods for Pricing Barrier Options." *Journal of Economic Dynamics and Control*. Vol. 24, Number 11-12, 2000, pp. 1563-1590.

## See Also

dblbarrierbybls | dblbarrierbyfd | dblbarriersensbybls

## Topics

"Double Barrier Option" on page 3-27

"Supported Equity Derivatives" on page 3-24

**Introduced in R2019a**

## barrierbyls

Price European or American barrier options using Monte Carlo simulations

### Syntax

```
[Price,Paths,Times,Z] = barrierbyls(RateSpec,StockSpec,OptSpec,  
Strike,Settle,ExerciseDates,BarrierSpec,Barrier)  
[Price,Paths,Times,Z] = barrierbyls( ____,Name,Value)
```

### Description

[Price,Paths,Times,Z] = barrierbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier) calculates barrier option prices on a single underlying asset using the Longstaff-Schwartz model. barrierbyls computes prices of European and American barrier options.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

[Price,Paths,Times,Z] = barrierbyls( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price an American Barrier Down In Put Option

Compute the price of an American down in put option using the following data:

```
Rates = 0.0325;  
Settle = '01-Jan-2016';  
Maturity = '01-Jan-2017';  
Compounding = -1;  
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate',Settle,'StartDates',Settle,'EndDates',Maturity,
    'Rates',Rates,'Compounding',Compounding,'Basis',Basis)
```

```
RateSpec = struct with fields:
```

```
    FinObj: 'RateSpec'
    Compounding: -1
        Disc: 0.9680
        Rates: 0.0325
    EndTimes: 1
    StartTimes: 0
    EndDates: 736696
    StartDates: 736330
    ValuationDate: 736330
        Basis: 1
    EndMonthRule: 1
```

Define a StockSpec.

```
AssetPrice = 40;
Volatility = 0.20;
StockSpec = stockspec(Volatility,AssetPrice)
```

```
StockSpec = struct with fields:
```

```
    FinObj: 'StockSpec'
        Sigma: 0.2000
    AssetPrice: 40
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Calculate the price of an American barrier down in put option.

```
Strike = 45;
OptSpec = 'put';
Barrier = 35;
BarrierSpec = 'DI';
AmericanOpt = 1;
```

```
Price = barrierblys(RateSpec,StockSpec,OptSpec,Strike,Settle,Maturity,BarrierSpec,...
    Barrier,'NumTrials',2000,'AmericanOpt',AmericanOpt)
```

```
Price = 4.7306
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option as 'call' or 'put', specified as a character vector or string array with a value of "call" or "put".

Data Types: `char` | `string`

### **Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: double | char | datetime

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as a serial date number, a date character vector, or a datetime object:

- For a European option, there is only one `ExerciseDates` on the option expiry date which is the maturity of the instrument.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: double | char | cell

### **BarrierSpec — Barrier option type**

character vector with values: 'UI', 'UO', 'DI', 'DO'

Barrier option type, specified as a character vector with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option.

- 'UO' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes

below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually, the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: char

### Barrier — Barrier level

scalar numeric

Barrier level, specified as a scalar numeric.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Price =

```
barrierbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, Rebate, 1000)
```

### AmericanOpt — Option type

0 (European) (default) | values [0, 1]



Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: double

### **Rebate — Rebate value**

0 (default) | scalar numeric

Rebate value, specified as the comma-separated pair consisting of 'Rebate' and a scalar numeric. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

### **NumTrials — Number of independent sample paths**

1000 (default) | nonnegative integer

Number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of 'NumTrials' and a scalar nonnegative integer.

Data Types: double

### **NumPeriods — Number of simulation periods per trial**

100 (default) | nonnegative integer

Number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a scalar nonnegative integer.

Data Types: double

### **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-1-by-NumTrials 3-D time series array. The Z

value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: `double`

### **Antithetic — Indicator for antithetic sampling**

`false` (default) | scalar logical flag with value of `true` or `false`

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of `true` or `false`.

Data Types: `logical`

## **Output Arguments**

### **Price — Expected prices for barrier options**

`matrix`

Expected prices for barrier options, returned as a `NINST-by-1` matrix.

### **Paths — Simulated paths of correlated state variables**

`vector`

Simulated paths of correlated state variables, returned as a `NumPeriods + 1-by-1-by-NumTrials` 3-D time series array of simulated paths of correlated state variables. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

### **Times — Observation times associated with simulated paths**

`vector`

Observation times associated with simulated paths, returned as a `NumPeriods + 1-by-1` column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

### **Z — Time series array of dependent random variates**

`vector`

Time series array of dependent random variates, returned as a `NumPeriods-by-1-by-NumTrials` 3-D array when `Z` is specified as an input argument. If the `Z` input argument is not specified, then the `Z` output argument contains the random variates generated internally.

## Definitions

### Barrier Option

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by **Barrier**, during the life of the option.

### References

- [1] Hull, J. *Options, Futures and Other Derivatives*. Fourth Edition. Prentice Hall, 2000, pp. 646-649.
- [2] Aitsahlia, F., L. Imhof, and T.L. Lai. "Pricing and hedging of American knock-in options." *The Journal of Derivatives*. Vol. 11.3 , 2004, pp. 44-50.
- [3] Rubinstein M. and E. Reiner. "Breaking down the barriers." *Risk*. Vol. 4(8), 1991, pp. 28-35.

### See Also

barrierbybls | barrierbyfd | barriersensbybls | barriersensbyfd |  
barriersensbyls

### Topics

"Barrier Option" on page 3-26  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2016b**

## barriersensbyls

Calculate price and sensitivities for European or American barrier options using Monte Carlo simulations

### Syntax

```
[PriceSens,Paths,Times,Z] = barriersensbyls(RateSpec,StockSpec,  
OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier)  
[PriceSens,Paths,Times,Z] = barriersensbyls( ____,Name,Value)
```

### Description

[PriceSens,Paths,Times,Z] = barriersensbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier) calculates barrier option prices or sensitivities on a single underlying asset using the Longstaff-Schwartz model. barriersensbyls computes prices of European and American barrier options.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

[PriceSens,Paths,Times,Z] = barriersensbyls( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Delta and Gamma of an American Barrier Down In Put Option

Compute the price of an American down in put option using the following data:

```
Rates = 0.0325;  
Settle = '01-Jan-2016';  
Maturity = '01-Jan-2017';
```

```
Compounding = -1;
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate',Settle,'StartDates',Settle,'EndDates',Maturity,
    'Rates',Rates,'Compounding',Compounding,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9680
    Rates: 0.0325
    EndTimes: 1
    StartTimes: 0
    EndDates: 736696
    StartDates: 736330
    ValuationDate: 736330
    Basis: 1
    EndMonthRule: 1
```

Define a StockSpec.

```
AssetPrice = 40;
Volatility = 0.20;
StockSpec = stockspec(Volatility,AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2000
    AssetPrice: 40
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Calculate the delta and gamma of an American barrier down in put option.

```
Strike = 45;
OptSpec = 'put';
Barrier = 35;
BarrierSpec = 'DI';
AmericanOpt = 1;
```

```
OutSpec = {'delta','gamma'};

[Delta,Gamma] = barriersensbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,...
Maturity,BarrierSpec,Barrier,'NumTrials',2000,'AmericanOpt',AmericanOpt,'OutSpec',OutSpec);

Delta = -0.6346
Gamma = -0.3091
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option as 'call' or 'put', specified as a character vector or string array with values "call" or "put".

Data Types: `char` | `string`

**Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: double

**Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: double | char | datetime

**ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as a serial date number, a date character vector, or a datetime object:

- For a European option, there is only one `ExerciseDates` on the option expiry date which is the maturity of the instrument.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: double | char | cell

**BarrierSpec — Barrier option type**

character vector with values: 'UI', 'U0', 'DI', 'D0'

Barrier option type, specified as a character vector with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option.

- 'U0' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually, the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: char

**Barrier — Barrier level**

scalar numeric

Barrier level, specified as a scalar numeric.

Data Types: double



## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = barriersensbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, Rebate, 1000)`

### AmericanOpt — Option type

0 (European) (default) | values [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of the following values:

- 0 — European
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: double

### Rebate — Rebate value

0 (default) | numeric

Rebate value, specified as the comma-separated pair consisting of 'Rebate' and a scalar numeric. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

### NumTrials — Number of independent sample paths

1000 (default) | nonnegative integer

Number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of 'NumTrials' and a scalar nonnegative integer.

Data Types: double

**NumPeriods — Number of simulation periods per trial**

100 (default) | nonnegative integer

Number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a scalar nonnegative integer.

Data Types: double

**Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-1-by-NumTrials 3-D time series array. The Z value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: double

**Antithetic — Indicator for antithetic sampling**

false (default) | logical flag with value of true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a scalar value of true or false.

Data Types: logical

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.

Example: OutSpec =

```
{'delta','gamma','vega','lambda','rho','theta','price'}
```

Data Types: char | cell

## Output Arguments

### **PriceSens** — Expected prices or sensitivities for barrier options

matrix

Expected prices or sensitivities (defined using `OutSpec`) for barrier options, returned as a `NINST-by-1` matrix.

### **Paths** — Simulated paths of correlated state variables

vector

Simulated paths of correlated state variables, returned as a `NumPeriods + 1-by-1-by-NumTrials` 3-D time series array of simulated paths of correlated state variables. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

### **Times** — Observation times associated with simulated paths

vector

Observation times associated with simulated paths, returned as a `NumPeriods + 1-by-1` column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

### **Z** — Time series array of dependent random variates

vector

Time series array of dependent random variates, returned as a `NumPeriods-by-1-by-NumTrials` 3-D array when `Z` is specified as an input argument. If the `Z` input argument is not specified, then the `Z` output argument contains the random variates generated internally.

## Definitions

### **Barrier Option**

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

## References

- [1] Hull, J. *Options, Futures and Other Derivatives* Fourth Edition. Prentice Hall, 2000, pp. 646-649.
- [2] Aitsahlia, F., L. Imhof and T.L. Lai. "Pricing and hedging of American knock-in options." *The Journal of Derivatives*. Vol. 11.3, 2004, pp. 44-50.
- [3] Broadie, M., P. Glasserman and S. Kou. "A continuity correction for discrete barrier options." *Mathematical Finance*. Vol. 7.4 , 1997, pp. 3250-349.
- [4] Moon, K.S. "Efficient Monte Carlo algorithm for pricing barrier options." *Communications of the Korean Mathematical Society*. Vol 23.2, 2008 pp. 85-294.
- [5] Papatheodorou, B. "*Enhanced Monte Carlo methods for pricing and hedging exotic options*." University of Oxford thesis, 2005.
- [6] Rubinstein M. and E. Reiner. "Breaking down the barriers." *Risk*. Vol. 4(8), 1991, pp. 28-35.

## See Also

`barrierbybls` | `barrierbybls` | `barrierbyfd` | `barriersensbyfd` | `barriersensbyls`

## Topics

"Barrier Option" on page 3-26

"Supported Equity Derivatives" on page 3-24

## Introduced in R2016b

# barrierbybls

Price European barrier options using Black-Scholes option pricing model

## Syntax

```
Price = barrierbybls(RateSpec, StockSpec, OptSpec, Strike, Settle,
  ExerciseDates, BarrierSpec, Barrier)
Price = barrierbybls( ____, Name, Value)
```

## Description

`Price = barrierbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, BarrierSpec, Barrier)` calculates European barrier option prices using the Black-Scholes option pricing model.

`Price = barrierbybls( ____, Name, Value)` adds optional name-value pair arguments.

## Examples

### Price an European Barrier Down Out Call Option

Compute the price of an European barrier down out call option using the following data:

```
Rates = 0.035;
Settle = '01-Jan-2015';
Maturity = '01-jan-2016';
Compounding = -1;
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', Maturity,
  'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9656
    Rates: 0.0350
    EndTimes: 1
    StartTimes: 0
    EndDates: 736330
    StartDates: 735965
    ValuationDate: 735965
    Basis: 1
    EndMonthRule: 1
```

Define a StockSpec.

```
AssetPrice = 50;
Volatility = 0.30;
StockSpec = stockspec(Volatility, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
    AssetPrice: 50
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Calculate the price of an European barrier down out call option using the Black-Scholes option pricing model.

```
Strike = 50;
OptSpec = 'call';
Barrier = 45;
BarrierSpec = 'D0';
```

```
Price = barrierbybls(RateSpec, StockSpec, OptSpec, Strike, Settle,...
Maturity, BarrierSpec, Barrier)
```

```
Price = 4.4285
```

## Price European Barrier Down Out and Down In Call Options

Compute the price of European down out and down in call options using the following data:

```
Rates = 0.035;
Settle = '01-Jan-2015';
Maturity = '01-jan-2016';
Compounding = -1;
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', Maturity,
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
```

```
    FinObj: 'RateSpec'
    Compounding: -1
        Disc: 0.9656
        Rates: 0.0350
    EndTimes: 1
    StartTimes: 0
    EndDates: 736330
    StartDates: 735965
    ValuationDate: 735965
        Basis: 1
    EndMonthRule: 1
```

Define a StockSpec.

```
AssetPrice = 50;
Volatility = 0.30;
StockSpec = stockspec(Volatility, AssetPrice)
```

```
StockSpec = struct with fields:
```

```
    FinObj: 'StockSpec'
        Sigma: 0.3000
    AssetPrice: 50
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Calculate the price of European barrier down out and down in call options using the Black-Scholes Option Pricing model.

```
Strike = 50;  
OptSpec = 'Call';  
Barrier = 45;  
BarrierSpec = {'DO'; 'DI'};
```

```
Price = barrierbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec)
```

```
Price = 2×1
```

```
4.4285  
2.3301
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with values 'call' or 'put' | string array with values "call" or "put"



Definition of the option as 'call' or 'put', specified as an NINST-by-1 cell array of character vectors or string array with values 'call' or 'put' or "call" or "put".

Data Types: char | cell | string

### **Strike — Option strike price value**

numeric

Option strike price value, specified as an NINST-by-1 matrix of numeric values, where each row is the schedule for one option.

Data Types: double

### **Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as an NINST-by-1 matrix using serial date numbers, date character vectors, or datetime objects.

Data Types: double | char | datetime

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as an NINST-by-1 matrix of serial date numbers, date character vectors, or datetime objects.

---

**Note** For a European option, there is only one ExerciseDates on the option expiry date which is the maturity of the instrument.

---

Data Types: double | char | datetime

### **BarrierSpec — Barrier option type**

character vector with values: 'UI', 'UO', 'DI', 'DO'

Barrier option type, specified as an NINST-by-1 cell array of character vectors with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell

(call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option.

- 'U0' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-n

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually, the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: char | cell

**Barrier — Barrier level**

numeric

Barrier level, specified as NINST-by-1 matrix of numeric values.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `Price = barrierbybls(RateSpec,StockSpec,OptSpec,Strike,Settle,Maturity,BarrierSpec,Barrier,Rebate,1000)`

### Rebate — Rebate value

0 (default) | numeric

Rebate value, specified as the comma-separated pair consisting of 'Rebate' and NINST-by-1 matrix of numeric values. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

## Output Arguments

### Price — Expected prices for barrier options

matrix

Expected prices for barrier options at time 0, returned as a NINST-by-1 matrix.

## Definitions

### Barrier Option

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends

on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

### References

- [1] Hull, J. *Options, Futures and Other Derivatives* Fourth Edition. Prentice Hall, 2000, pp. 646-649.
- [2] Aitsahlia, F., L. Imhof, and T.L. Lai. "Pricing and hedging of American knock-in options." *The Journal of Derivatives*. Vol. 11.3, 2004, pp. 44-50.
- [3] Rubinstein M. and E. Reiner. "Breaking down the barriers." *Risk*. Vol. 4(8), 1991, pp. 28-35.

### See Also

`barrierbyfd` | `barrierbyls` | `barriersensbybls` | `barriersensbyfd` | `barriersensbyls`

### Topics

"Barrier Option" on page 3-26  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2016b**

## barriersensbybls

Calculate price or sensitivities for European barrier options using Black-Scholes option pricing model

### Syntax

```
PriceSens = barriersensbybls(RateSpec,StockSpec,OptSpec,Strike,  
Settle,ExerciseDates,BarrierSpec,Barrier)  
PriceSens = barriersensbybls( ____,Name,Value)
```

### Description

PriceSens = barriersensbybls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier) calculates European barrier option prices or sensitivities using the Black-Scholes option pricing model.

PriceSens = barriersensbybls( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Calculate Price and Sensitivities for European Barrier Down Out and Down In Call Options

Compute price of European barrier down out and down in call options using the following data:

```
Rates = 0.035;  
Settle = '01-Jan-2015';  
Maturity = '01-April-2015';  
Compounding = -1;  
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9913
    Rates: 0.0350
    EndTimes: 0.2500
    StartTimes: 0
    EndDates: 736055
    StartDates: 735965
    ValuationDate: 735965
    Basis: 1
    EndMonthRule: 1
```

Define a StockSpec.

```
AssetPrice = 19;
Volatility = 0.40;
DivType = 'Continuous';
DivAmount = 0.035;
StockSpec = stockspec(Volatility, AssetPrice, DivType, DivAmount)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.4000
    AssetPrice: 19
    DividendType: {'continuous'}
    DividendAmounts: 0.0350
    ExDividendDates: []
```

Calculate the price, delta, and gamma for European barrier down out and down in call options using the Black-Scholes option pricing model.

```
OptSpec = 'Call';
Strike = 20;
Barrier = 18;
BarrierSpec = {'DO'; 'DI'};
OutSpec = {'price', 'delta', 'gamma'};
```

```
[Price, Delta, Gamma] = barriersensbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, 'OutSpec', OutSpec)
```

Price = 2×1

0.6287  
0.4655

Delta = 2×1

0.6376  
-0.2036

Gamma = 2×1

0.0255  
0.0773

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option as 'call' or 'put', specified as an NINST-by-1 cell array of character vectors or string arrays with values "call" or "put".

Data Types: char | string | cell

**Strike — Option strike price value**

numeric

Option strike price value, specified as an NINST-by-1 matrix of numeric values.

Data Types: double

**Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as an NINST-by-1 matrix using serial date numbers, date character vectors, or datetime objects.

Data Types: double | char | datetime

**ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as an NINST-by-1 matrix of serial date numbers, date character vectors, or datetime objects.

---

**Note** For a European option, there is only one ExerciseDates on the option expiry date which is the maturity of the instrument.

---

Data Types: double | char | datetime

**BarrierSpec — Barrier option type**

character vector with values: 'UI', 'UO', 'DI', 'DO'

Barrier option type, specified as an NINST-by-1 cell array of character vectors with the following values:

- 'UI' — Up Knock-in



This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option.

- 'U0' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option.

- 'D0' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually, the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: char | cell

**Barrier — Barrier level**

numeric

Barrier level, specified as an NINST-by-1 matrix of numeric values.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price = barriersensbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, 'Rebate', 1000, 'OutSpec', 'Delta')`

### Rebate — Rebate values

0 (default) | numeric

Rebate values, specified as the comma-separated pair consisting of 'Rebate' and NINST-by-1 matrix of numeric values. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

### OutSpec — Define outputs

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

`OutSpec = {'All'}` specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: char | cell

## Output Arguments

### **PriceSens — Expected prices or sensitivities for barrier options**

matrix

Expected prices at time 0 or sensitivities (defined using `OutSpec`) for barrier options, returned as a NINST-by-1 matrix.

## Definitions

### **Barrier Option**

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

## References

- [1] Hull, J. *Options, Futures and Other Derivatives* Fourth Edition. Prentice Hall, 2000, pp. 646-649.
- [2] Aitsahlia, F., L. Imhof, and T.L. Lai. "Pricing and hedging of American knock-in options." *The Journal of Derivatives*. Vol. 11.3, 2004, pp. 44-50.
- [3] Rubinstein M. and E. Reiner. "Breaking down the barriers." *Risk*. Vol. 4(8), 1991, pp. 28-35.

## See Also

barrierbybls | barrierbyfd | barrierbyls | barriersensbyfd |  
barriersensbyls

## Topics

"Barrier Option" on page 3-26

“Supported Equity Derivatives” on page 3-24

**Introduced in R2016b**

## dblbarrierbybls

Price European double barrier options using Black-Scholes option pricing model

### Syntax

```
Price = dblbarrierbybls(RateSpec,StockSpec,OptSpec,Strike,Settle,
ExerciseDates,BarrierSpec,Barrier)
Price = dblbarrierbybls( ____,Name,Value)
```

### Description

Price = dblbarrierbybls(RateSpec,StockSpec,OptSpec,Strike,Settle, ExerciseDates,BarrierSpec,Barrier) calculates European double barrier option prices using the Black-Scholes option pricing model and the Ikeda and Kunitomo approximation.

Price = dblbarrierbybls( \_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Price a European Double Knock-Out Call Option

Compute the price of a European for a double knock-out (down and out-up and out) call option using the following data:

```
Rate = 0.05;
Settle = '01-Jun-2018';
Maturity = '01-Dec-2018';
Basis = 1;
```

Define a RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', Maturity)
```

Define a StockSpec.

```
AssetPrice = 100;  
Volatility = 0.25;  
StockSpec = stockspec(Volatility, AssetPrice);
```

Define the double barrier option.

```
LBarrier = 80;  
UBarrier = 130;  
Barrier = [UBarrier LBarrier];  
BarrierSpec = 'DK0';  
OptSpec = 'Call';  
Strike = 110;
```

Compute price of option using flat boundaries.

```
PriceFlat = dblbarrierbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, B  
PriceFlat = 1.1073
```

Compute price of option using two curved boundaries.

```
Curvature = [0.05 -0.05];  
PriceCurved = dblbarrierbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, B  
PriceCurved = 1.4548
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset, specified by the `StockSpec` obtained from `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors or string array with values 'call' or 'put' or "call" or "put".

Data Types: `char` | `cell` | `string`

### **Strike — Option strike price value**

matrix

Option strike price value, specified as an NINST-by-1 matrix of numeric values, where each row is the schedule for one option.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the double barrier option, specified as an NINST-by-1 matrix using serial date numbers, date character vectors, or datetime objects.

Data Types: `double` | `char` | `datetime`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as an NINST-by-1 matrix of serial date numbers, date character vectors, or datetime objects.

---

**Note** For a European option, the option expiry date has only one `ExerciseDates` value, which is the maturity of the instrument.

---

Data Types: double | char | datetime

**BarrierSpec — Double barrier option type**

cell array of character vectors with values of 'DKI' or 'DKO' | string array with values of "DKI" or "DKO"

Double barrier option type, specified as an NINST-by-1 cell array of character vectors or string array with the following values:

- 'DKI' — Double Knock-In

The 'DKI' option becomes effective when the price of the underlying asset reaches one of the barriers. It gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, if the underlying asset goes above or below the barrier levels during the life of the option.

- 'DKO' — Double Knock-Out

The 'DKO' option gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, as long as the underlying asset remains between the barrier levels during the life of the option. This option terminates when the price of the underlying asset passes one of the barriers.

Option	Barrier Type	Payoff If Any Barrier Crossed	Payoff If Barriers Not Crossed
Call/Put	Double Knock-in	Standard Call/Put	Worthless
Call/Put	Double Knock-out	Worthless	Standard Call/Put

Data Types: char | cell | string

**Barrier — Double barrier value**

numeric

Double barrier value, specified as NINST-by-1 matrix of numeric values, where each element is a 1-by-2 vector where the first column is Barrier(1)(UB) and the second column is Barrier(2)(LB). Barrier(1) must be greater than Barrier(2).

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes.



You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = dblbarrierbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, 'Curvature', [1,5])`

### **Curvature — Curvature levels of the upper and lower barriers**

`[]` (default) | matrix

Curvature levels of the upper and lower barriers, specified as the comma-separated pair consisting of 'Curvature' and an NINST-by-1 matrix, where each element is a 1-by-2 vector. The first column is the upper barrier curvature ( $d1$ ) and the second column is the lower barrier curvature ( $d2$ ).

- $d1 = d2 = 0$  corresponds to two flat boundaries.
- $d1 < 0 < d2$  corresponds to an exponentially growing lower boundary and an exponentially decaying upper boundary.
- $d1 > 0 > d2$  corresponds to a convex downward lower boundary and a convex upward upper boundary.

Data Types: double

## **Output Arguments**

### **Price — Expected prices for double barrier options**

matrix

Expected prices for double barrier options at time 0, returned as a NINST-by-1 matrix.

## **Definitions**

### **Double Barrier Option**

A double barrier option is similar to the standard single barrier option except that it has two barrier levels: a lower barrier (LB) and an upper barrier (UB).

The payoff for a double barrier option depends on whether the underlying asset remains between the barrier levels during the life of the option. Double barrier options are less

expensive than single barrier options as they have a higher knock-out probability. Because of this, double barrier options allow investors to reduce option premiums and match an investor's belief about the future movement of the underlying price process.

## **Ikeda and Kunitomo Approximation**

The analytical formulas of Ikeda and Kunitomo approach pricing as constrained by curved boundaries.

This approach has the advantage of covering barriers that are flat, have exponential growth or decay, or are concave. The Ikeda and Kunitomo model for pricing double barrier options focuses on calculating double knock-out barriers.

## **References**

- [1] Hull, J. *Options, Futures, and Other Derivatives*. Fourth Edition. Upper Saddle River, NJ: Prentice Hall, 2000.
- [2] Kunitomo, N., and M. Ikeda. "Pricing Options with Curved Boundaries." *Mathematical Finance*. Vol. 2, Number 4, 1992.
- [3] Rubinstein, M., and E. Reiner. "Breaking Down the Barriers." *Risk*. Vol. 4, Number 8, 1991, pp. 28-35.

## **See Also**

`dblbarrierbyfd` | `dblbarriersensbybls` | `dblbarriersensbyfd`

## **Topics**

"Double Barrier Option" on page 3-27

"Supported Equity Derivatives" on page 3-24

**Introduced in R2019a**

## dblbarriersensbybls

Calculate prices and sensitivities for European double barrier options using Black-Scholes option pricing model

### Syntax

```
PriceSens = dblbarriersensbybls(RateSpec,StockSpec,OptSpec,Strike,
Settle,ExerciseDates,BarrierSpec,Barrier)
PriceSens = dblbarriersensbybls( ____,Name,Value)
```

### Description

`PriceSens = dblbarriersensbybls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates,BarrierSpec,Barrier)` calculates European double barrier option prices and sensitivities using the Black-Scholes option pricing model and the Ikeda and Kunitomo approximation.

`PriceSens = dblbarriersensbybls( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Calculate Price and Sensitivities for a European Double Knock-Out Call Option

Compute the price and sensitivities for a European double knock-out (down and out-up and out) call option using the following data:

```
Rate = 0.05;
Settle = '01-Jun-2018';
Maturity = '01-Dec-2018';
Basis = 1;
```

Define a `RateSpec`.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', Maturity)
```

Define a StockSpec.

```
AssetPrice = 100;  
Volatility = 0.25;  
StockSpec = stockspect(Volatility, AssetPrice);
```

Define the double barrier option and sensitivities.

```
LBarrier = 80;  
UBarrier = 130;  
Barrier = [UBarrier LBarrier];  
BarrierSpec = 'DK0';  
OptSpec = 'Call';  
Strike = 110;  
OutSpec = {'Price', 'Delta', 'Gamma'};
```

Compute the price of the option using flat boundaries.

```
[PriceFlat, Delta, Gamma] = dblbarriersensbybls(RateSpec, StockSpec, OptSpec, Strike, S)
```

```
PriceFlat = 1.1073
```

```
Delta = 0.0411
```

```
Gamma = -0.0040
```

Compute the price of the option using two curved boundaries.

```
Curvature = [0.05 -0.05];  
[PriceCurved, Delta, Gamma] = dblbarriersensbybls(RateSpec, StockSpec, OptSpec, Strike, S, Curvature)
```

```
PriceCurved = 1.4548
```

```
Delta = 0.0620
```

```
Gamma = -0.0045
```

## Input Arguments

**RateSpec** — Interest-rate term structure  
structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec — Stock specification for underlying asset**  
structure

Stock specification for the underlying asset, specified by the `StockSpec` obtained from `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors or string array with values 'call' or 'put' or "call" or "put".

Data Types: `char` | `cell` | `string`

**Strike — Option strike price value**  
matrix

Option strike price value, specified as an NINST-by-1 matrix of numeric values, where each row is the schedule for one option.

Data Types: `double`

**Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the double barrier option, specified as an NINST-by-1 matrix using serial date numbers, date character vectors, or datetime objects.

Data Types: `double` | `char` | `datetime`

**ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as an NINST-by-1 matrix of serial date numbers, date character vectors, or datetime objects.

---

**Note** For a European option, the option expiry date has only one `ExerciseDates` value, which is the maturity of the instrument.

---

Data Types: `double` | `char` | `datetime`

**BarrierSpec — Double barrier option type**

cell array of character vectors with values of 'DKI' or 'DKO' | string array with values of "DKI" or "DKO"

Double barrier option type, specified as an NINST-by-1 cell array of character vectors or string array with the following values:

- 'DKI' — Double Knock-in

The 'DKI' option becomes effective when the price of the underlying asset reaches one of the barriers. It gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, if the underlying asset goes above or below the barrier levels during the life of the option.

- 'DKO' — Double Knock-out

The 'DKO' option gives the option holder the right but not the obligation to buy or sell the underlying security at the strike price, as long as the underlying asset remains between the barrier levels during the life of the option. This option terminates when the price of the underlying asset passes one of the barriers.

Option	Barrier Type	Payoff If Any Barrier Crossed	Payoff If Barriers Not Crossed
Call/Put	Double Knock-in	Standard Call/Put	Worthless
Call/Put	Double Knock-out	Worthless	Standard Call/Put

Data Types: `char` | `cell` | `string`

**Barrier — Double barrier value**

matrix

Double barrier value, specified as NINST-by-1 matrix of numeric values, where each element is a 1-by-2 vector where the first column is Barrier(1)(UB) and the second column is Barrier(2)(LB). Barrier(1) must be greater than Barrier(2).

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: PriceSens =  
dblbarriersensbybls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity, BarrierSpec, Barrier, 'Curvature', [1,5], 'OutSpec', 'Delta')

### Curvature — Curvature levels of the upper and lower barriers

[ ] (default) | matrix

Curvature levels of the upper and lower barriers, specified as the comma-separated pair consisting of 'Curvature' and an NINST-by-1 matrix of numeric values, where each element is a 1-by-2 vector. The first column is the upper barrier curvature ( $d1$ ) and the second column is the lower barrier curvature ( $d2$ ).

- $d1 = d2 = 0$  corresponds to two flat boundaries.
- $d1 < 0 < d2$  corresponds to an exponentially growing lower boundary and an exponentially decaying upper boundary.
- $d1 > 0 > d2$  corresponds to a convex downward lower boundary and a convex upward upper boundary.

Data Types: double

### OutSpec — Define outputs

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | string array with values "Price", "Delta", "Gamma", "Vega", "Lambda", "Rho", "Theta", and "All"

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.

Example: OutSpec =  
{'delta','gamma','vega','lambda','rho','theta','price'}

Data Types: char | cell

## Output Arguments

### **PriceSens — Expected prices or sensitivities for double barrier options**

matrix

Expected prices at time 0 or sensitivities (defined using OutSpec) for double barrier options, returned as a NINST-by-1 matrix.

## Definitions

### **Double Barrier Option**

A double barrier option is similar to the standard single barrier option except that it has two barrier levels: a lower barrier (LB) and an upper barrier (UB).

The payoff for a double barrier option depends on whether the underlying asset remains between the barrier levels during the life of the option. Double barrier options are less expensive than single barrier options as they have a higher knock-out probability. Because of this, double barrier options allow investors to reduce option premiums and match an investor's belief about the future movement of the underlying price process.

### **Ikeda and Kunitomo Approximation**

The analytical formulas of Ikeda and Kunitomo approach pricing as constrained by curved boundaries.



This approach has the advantage of covering barriers that are flat, have exponential growth or decay, or are concave. The Ikeda and Kunitomo model for pricing double barrier options focuses on calculating double knock-out barriers.

## References

- [1] Hull, J. *Options, Futures, and Other Derivatives*. Fourth Edition. Upper Saddle River, NJ: Prentice Hall, 2000.
- [2] Kunitomo, N., and M. Ikeda. "Pricing Options with Curved Boundaries." *Mathematical Finance*. Vol. 2, Number 4, 1992.
- [3] Rubinstein, M., and E. Reiner. "Breaking Down the Barriers." *Risk*. Vol. 4, Number 8, 1991, pp. 28-35.

## See Also

dblbarrierbybls | dblbarrierbyfd | dblbarriersensbyfd

## Topics

"Double Barrier Option" on page 3-27

"Supported Equity Derivatives" on page 3-24

**Introduced in R2019a**

## barrierbyitt

Price barrier options using implied trinomial tree (ITT)

### Syntax

```
[Price,PriceTree] = barrierbyitt(ITTTree,OptSpec,Strike,Settle,  
AmericanOpt,ExerciseDates,BarrierSpec,Barrier)  
[Price,PriceTree] = barrierbyitt( ____,Rebate,Options)
```

### Description

[Price,PriceTree] = barrierbyitt(ITTTree,OptSpec,Strike,Settle, AmericanOpt,ExerciseDates,BarrierSpec,Barrier) calculates prices for barrier options using implied trinomial tree (ITT).

[Price,PriceTree] = barrierbyitt( \_\_\_\_,Rebate,Options) adds optional arguments for Rebate and Options.

### Examples

#### Price a Barrier Option Using an ITT Tree

This example shows how to price a barrier option using an ITT tree by loading the file `deriv.mat`, which provides `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;  
  
OptSpec = 'Call';  
Strike = 85;  
Settle = '01-Jan-2006';  
ExerciseDates = '31-Dec-2008';  
AmericanOpt = 1;  
BarrierSpec = 'UI';
```

```
Barrier = 115;

Price = barrierbyitt(ITTTree,OptSpec,Strike,Settle,ExerciseDates,AmericanOpt,...
BarrierSpec,Barrier)

Price = 2.4074
```

## Input Arguments

### **ITTTree — Stock tree structure**

structure

Stock tree structure, specified by using `itttree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of an option as 'call' or 'put', specified as a NINST-by-1 cell array of character vector values.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

numeric

Option strike price value for a European or an American Option, specified as NINST-by-1 matrix of numeric values. Each row is the schedule for one option.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the barrier option, specified as a NINST-by-1 matrix of serial date numbers or date character vectors. The `Settle` date for every barrier is set to the `ValuationDate` of the stock tree. The barrier argument `Settle` is ignored.

Data Types: `double` | `char`

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or a date character vector:

- For a European option, use a NINST-by-1 matrix of dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date which is the maturity of the instrument.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is an NINST-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed date in `ExerciseDates`.

Data Types: double | char | cell

**AmericanOpt — Option type**

option type values 0 or 1

Option type, specified as NINST-by-1 flags with values:

- 0 — European
- 1 — American

Data Types: double

**BarrierSpec — Barrier option type**

character vector with values: 'UI', 'U0', 'DI', 'D0' | cell array of character vectors with values: 'UI', 'U0', 'DI', 'D0'

Barrier option type, specified as a character vector or a cell array of character vectors with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'U0' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually, with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: `char` | `cell`

### **Barrier — Barrier level**

numeric

Barrier level, specified as an NINST-by-1 matrix of numeric values.

Data Types: `double`

**Rebate — Rebate value**

0 (default) | numeric

(Optional) Rebate value, specified as a NINST-by-1 matrix of numeric values. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when the Barrier is reached.

Data Types: double

**Options — Derivatives pricing options**

structure

(Optional) Derivatives pricing options, specified as structure that is created with derivset.

Data Types: struct

## Output Arguments

**Price — Expected prices for barrier options at time 0**

vector

Expected prices for barrier options at time 0, returned as a NINST-by-1 vector.

**PriceTree — Structure with vector of barrier option prices at each node**

tree structure

Structure with a vector of barrier option prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.t0bs contains the observation times.

PriceTree.d0bs contains the observation dates.

## Definitions

### Barrier Option

A Barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

### References

- [1] Derman, E., I. Kani, D. Ergener and I. Bardhan. "Enhanced Numerical Methods for Options with Barriers." *Financial Analysts Journal*. (Nov.-Dec.), 1995, pp. 65-74.

### See Also

`instbarrier` | `itttree`

### Topics

"Computing Prices Using CRR" on page 3-131

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Pricing European Call Options Using Different Equity Models"

"Barrier Option" on page 3-26

"Pricing Options Structure" on page B-2

"Supported Equity Derivatives" on page 3-24

### Introduced in R2007a

## barrierbystt

Price barrier options using standard trinomial tree

### Syntax

```
[Price,PriceTree] = barrierbystt(STTTree,OptSpec,Strike,Settle,
ExerciseDates,AmericanOpt,BarrierSpec,Barrier)
[Price,PriceTree] = barrierbystt( ____,Name,Value)
```

### Description

[Price,PriceTree] = barrierbystt(STTTree,OptSpec,Strike,Settle, ExerciseDates,AmericanOpt,BarrierSpec,Barrier) prices barrier options using a standard trinomial (STT) tree.

[Price,PriceTree] = barrierbystt( \_\_\_\_,Name,Value) prices barrier options using a standard trinomial (STT) tree with an optional name-value pair argument for Rebate and Options.

### Examples

#### Price a Barrier Option Using the Standard Trinomial Tree Model

Create a RateSpec.

```
StartDates = 'Jan-1-2009';
EndDates = 'Jan-1-2013';
Rates = 0.035;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
```



```

Compounding: -1
  Disc: 0.8694
  Rates: 0.0350
  EndTimes: 4
  StartTimes: 0
  EndDates: 735235
  StartDates: 733774
ValuationDate: 733774
  Basis: 1
  EndMonthRule: 1

```

Create a StockSpec.

```

AssetPrice = 85;
Sigma = 0.15;
StockSpec = stockspec(Sigma, AssetPrice)

```

```

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1500
    AssetPrice: 85
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []

```

Create an STTtree.

```

NumPeriods = 4;
TimeSpec = stttimespec(StartDates, EndDates, 4);
STTtree = stttree(StockSpec, RateSpec, TimeSpec)

```

```

STTtree = struct with fields:
    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [733774 734139 734504 734869 735235]
    STree: {1x5 cell}
    Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}

```

Define the barrier option and compute the price.

```
Settle = '1/1/09';  
ExerciseDates = '1/1/12';  
OptSpec = 'call';  
Strike = 105;  
AmericanOpt = 1;  
BarrierSpec = 'UI';  
Barrier = 115;  
  
Price= barrierbystt(STTtree, OptSpec, Strike, Settle, ExerciseDates,...  
AmericanOpt, BarrierSpec, Barrier)  
  
Price = 3.7977
```

## Input Arguments

**STTtree** — Stock tree structure for standard trinomial tree structure

Stock tree structure for a standard trinomial tree, specified by using `stttree`.

Data Types: `struct`

**OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `char` | `cell`

**Strike** — European or American option strike price value

nonnegative numeric

European or American option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of nonnegative numeric values. Each row is the schedule for one option. To compute the value of a floating-strike barrier option, `Strike` should be specified as NaN. Floating-strike barrier options are also known as average strike options.

Data Types: `double`

**Settle** — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the barrier option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every barrier option is set to the `ValuationDate` of the stock tree. The barrier argument, `Settle`, is ignored.

---

Data Types: double | char | cell

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a NINST-by-1 matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a NINST-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char | cell

### **AmericanOpt — Option type**

scalar with values [0, 1]

Option type, specified as an NINST-by-1 matrix of flags with values:

- 0 — European
- 1 — American

Data Types: double

### **BarrierSpec — Barrier option type**

character vector with values: 'UI', 'UO', 'DI', 'DO' | cell array of character vectors with values: 'UI', 'UO', 'DI', 'DO'

Barrier option type, specified as a character vector or an NINST-by-1 cell array of character vectors with the following values:

- 'UI' — Up Knock-in

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'UO' — Up Knock-out

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price as long as the underlying asset does not go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually, with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock-in

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option. Note, `barrierbyfd` does not support American knock-in barrier options.

- 'DO' — Down Knock-up

This option gives the option holder the right, but not the obligation, to buy or sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: char | cell

### **Barrier — Barrier levels**

numeric

Barrier levels, specified as an NINST-by-1 matrix of numeric values.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
barrierbystt(STTTree, OptSpec, Strike, Settle, ExerciseDates, 1, 'UI', 115,
'Rebate', 25)
```

### **Rebate — Rebate values**

0 (default) | numeric

Rebate values, specified as the comma-separated pair consisting of 'Rebate' and a NINST-by-1 matrix of numeric values. For Knock-in options, the Rebate is paid at expiry. For Knock-out options, the Rebate is paid when theBarrier is reached.

Data Types: double

### **Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices for barrier options at time 0**

matrix

Expected prices for barrier options at time 0, returned as a NINST-by-1 matrix.

**PriceTree — Structure with vector of barrier option prices at each node**

tree structure

Structure with a vector of barrier option prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.tObs contains the observation times.

PriceTree.dObs contains the observation dates.

**References**

[1] Derman, E., I. Kani, D. Ergener and I. Bardhan. “Enhanced Numerical Methods for Options with Barriers.” *Financial Analysts Journal*. (Nov.-Dec.), 1995, pp. 65-74.

**See Also**

derivset | instbarrier | sttprice | sttsens | stttimespec | stttree

**Topics**

“Barrier Option” on page 3-26

“Supported Equity Derivatives” on page 3-24

**Introduced in R2015b**

# **basketbyju**

Price European basket options using Nengjiu Ju approximation model

## **Syntax**

Price =  
basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)

## **Description**

Price =  
basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)  
prices European basket options using the Nengjiu Ju approximation model.

## **Input Arguments**

### **RateSpec**

Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see `intenvset`.

### **BasketStockSpec**

BasketStock specification. For information on the basket of stocks specification, see `basketstockspec`.

### **OptSpec**

Character vector or 2-by-1 cell array of character vectors with values of 'call' or 'put'.

### **Strike**

Scalar for the option strike price.

## Settle

Scalar of the settlement or trade date specified as a character vector or serial date number.

## Maturity

Maturity date specified as a character vector or serial date number.

# Output Arguments

## Price

Price of the basket option.

# Examples

Find a European call basket option of two stocks. Assume that the stocks are currently trading at \$10 and \$11.50 with annual volatilities of 20% and 25%, respectively. The basket contains one unit of the first stock and one unit of the second stock. The correlation between the assets is 30%. On January 1, 2009, an investor wants to buy a 1-year call option with a strike price of \$21.50. The current annualized, continuously compounded interest rate is 5%. Use this data to compute the price of the call basket option with the Ju approximation model.

```
Settle = 'Jan-1-2009';
Maturity = 'Jan-1-2010';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', ...
    Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric, and
% have ones along the main diagonal.
Corr = [1 0.30; 0.30 1];

% Define BasketStockSpec
AssetPrice = [10;11.50];
Volatility = [0.2;0.25];
Quantity = [1;1];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

% Compute the price of the call basket option
OptSpec = {'call'};
```



```
Strike = 21.5;
PriceCorr30 = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)
```

This returns:

```
PriceCorr30 =

    2.12214
```

Compute the price of the basket instrument for these two stocks with a correlation of 60%. Then compare this cost to the total cost of buying two individual call options:

```
Corr = [1 0.60; 0.60 1];

% Define the new BasketStockSpec
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

% Compute the price of the call basket option with Correlation = -0.60
PriceCorr60 = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)
```

This returns:

```
PriceCorr60 =

    2.27566
```

The following table summarizes the sensitivity of the option to correlation changes. In general, the premium of the basket option decreases with lower correlation and increases with higher correlation.

Correlation	-0.60	-0.30	0	0.30	0.60
Premium	1.52830	1.76006	1.9527	2.1221	2.2756

Compute the cost of two vanilla 1-year call options using the Black-Scholes (BLS) model on the individual assets:

```
StockSpec = stockspec(Volatility, AssetPrice);
StrikeVanilla= [10;11.5];

PriceVanillaOption = optstockbybls(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, StrikeVanilla)
```

This returns:

```
PriceVanillaOption =

    1.0451
    1.4186
```

Find the total cost of buying two individual call options:

```
sum(PriceVanillaOption)
```

This returns:

```
ans=2.4637
```

The total cost of purchasing two individual call options is \$2.4637, compared to the maximum cost of the basket option of \$2.27 with a correlation of 60%.

## References

Nengjiu Ju. "Pricing Asian and Basket Options Via Taylor Expansion." *Journal of Computational Finance*. Vol. 5, 2002.

## See Also

basketsensbyju | basketstockspec

## Topics

"Equity Derivatives Using Closed-Form Solutions" on page 3-150

"Basket Option" on page 3-28

"Supported Equity Derivatives" on page 3-24

**Introduced in R2009b**

# basketbyls

Price European or American basket options using Monte Carlo simulations

## Syntax

```
[Price,Paths,Times,Z] = basketbyls(RateSpec,BasketStockSpec,OptSpec,  
Strike,Settle,ExerciseDates)  
[Price,Paths,Times,Z] = basketbyls( ____,Name,Value)
```

## Description

[Price,Paths,Times,Z] = basketbyls(RateSpec,BasketStockSpec,OptSpec,Strike,Settle,ExerciseDates) prices basket options using the Longstaff-Schwartz model.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

[Price,Paths,Times,Z] = basketbyls( \_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Prices Basket Options Using the Longstaff-Schwartz Model

Find an American call basket option of three stocks. The stocks are currently trading at \$35, \$40 and \$45 with annual volatilities of 12%, 15% and 18%, respectively. The basket contains 33.33% of each stock. Assume the correlation between all pair of assets is 50%. On May 1, 2009, an investor wants to buy a three-year call option with a strike price of \$42. The current annualized continuously compounded interest rate is 5%. Use this data to compute the price of the call basket option using the Longstaff-Schwartz model.

```
Settle = 'May-1-2009';
Maturity = 'May-1-2012';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates',...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric,
% and have ones along the main diagonal.
Corr = [1 0.50 0.50; 0.50 1 0.50; 0.50 0.50 1];

% Define BasketStockSpec
AssetPrice = [35;40;45];
Volatility = [0.12;0.15;0.18];
Quantity = [0.333;0.333;0.333];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

% Compute the price of the call basket option
OptSpec = {'call'};
Strike = 42;
AmericanOpt = 1; % American option
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity,...
'AmericanOpt',AmericanOpt)

Price = 5.4687
```

Increase the number of simulation trials to 2000 to give the following results:

```
NumTrial = 2000;
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity,...
'AmericanOpt',AmericanOpt,'NumTrials',NumTrial)

Price = 5.5501
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **BasketStockSpec — BasketStock specification**

structure

BasketStock specification, specified using `basketstockspec`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a character vector or a 2-by-1 cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

scalar numeric | vector

Option strike price value, specified as one of the following:

- For a European or Bermuda option, `Strike` is a scalar (European) or 1-by-NSTRIKES (Bermuda) vector of strike prices.
- For an American option, `Strike` is a scalar vector of the strike price.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as a scalar serial date number or date character vector.

Data Types: `double` | `char`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European or Bermuda option, `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between, or including, the pair of dates on that row. If there is only one non-NaN date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity, 'AmericanOpt', AmericanOpt, 'NumTrials', NumTrials)`

### **AmericanOpt** — Option type

0 (European/Bermuda) (default) | values [0, 1]

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and a NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: `double`

**NumPeriods — Number of simulation periods per trial**

100 (default) | nonnegative integer

Number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a scalar nonnegative integer.

---

**Note** NumPeriods is considered only when pricing European basket options. For American and Bermuda basket options, NumPeriod equals the number of exercise days during the life of the option.

---

Data Types: double

**NumTrials — Number of independent sample paths (simulation trials)**

1000 (default) | nonnegative integer

Number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of 'NumTrials' and a scalar nonnegative integer.

Data Types: double

**Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-NINST-by-NumTrials 3-D time series array. The Z value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: double

**Antithetic — Indicator for antithetic sampling**

false (default) | scalar logical flag with value of true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

## Output Arguments

### **Price — Expected prices for basket option**

matrix

Expected prices for basket option, returned as a `NINST-by-1` matrix.

### **Paths — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a `NumPeriods + 1-by-1-by-NumTrials` 3-D time series array of simulated paths of correlated state variables. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

### **Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a `NumPeriods + 1-by-1` column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

### **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, returned as a `NumPeriods-by-1-by-NumTrials` 3-D array when `Z` is specified as an input argument. If the `Z` input argument is not specified, then the `Z` output argument contains the random variates generated internally.

## References

- [1] Longstaff, F.A., and E.S. Schwartz. "Valuing American Options by Simulation: A Simple Least-Squares Approach." *The Review of Financial Studies*. Vol. 14, No. 1, Spring 2001, pp. 113-147.

## See Also

`basketsensbyls` | `basketstockspec`



## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Basket Option” on page 3-28

“Supported Equity Derivatives” on page 3-24

## **Introduced in R2009b**

## basketsensbyju

Determine European basket options price or sensitivities using Nengjiu Ju approximation model

### Syntax

```
PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, Maturity)
PriceSens = basketsensbyju( ____, Name, Value)
```

### Description

`PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)` calculates prices or sensitivities for basket options using the Nengjiu Ju approximation model.

`PriceSens = basketsensbyju( ____, Name, Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Calculate Prices and Sensitivities for Basket Options Using the Nengjiu Ju Approximation Model

Find a European call basket option of five stocks. Assume that the basket contains:

- 5% of the first stock trading at \$110
- 15% of the second stock trading at \$75
- 20% of the third stock trading at \$40
- 25% of the fourth stock trading at \$125

- 35% of the fifth stock trading at \$92

These stocks have annual volatilities of 20% and the correlation between the assets is zero. On May 1, 2009, an investor wants to buy a 1-year call option with a strike price of \$90. The current annualized, continuously compounded interest is 5%. Use this data to compute price and delta of the call basket option with the Ju approximation model.

```

Settle = 'May-1-2009';
Maturity = 'May-1-2010';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', ...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric, and
% have ones along the main diagonal.
NumInst = 5;
InstIdx = ones(NumInst,1);
Corr = diag(ones(5,1), 0);

% Define BasketStockSpec
AssetPrice = [110; 75; 40; 125; 92];
Volatility = 0.2;
Quantity = [0.05; 0.15; 0.2; 0.25; 0.35];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

% Compute the price of the call basket option. Calculate also the delta
% of the first stock.
OptSpec = {'call'};
Strike = 90;
OutSpec = {'Price', 'Delta'};
UndIdx = 1; % First element in the basket
[Price, Delta] = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ..
Maturity, 'OutSpec', OutSpec, 'UndIdx', UndIdx)

Price = 5.1610
Delta = 0.0297

Compute Delta with respect to the second asset:

UndIdx = 2; % Second element in the basket
OutSpec = {'Delta'};

```

```
Delta = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity, .  
'OutSpec', OutSpec, 'UndIdx', UndIdx)
```

```
Delta = 0.0906
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **BasketStockSpec — BasketStock specification**

structure

BasketStock specification, specified using `basketstockspec`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a character vector or a 2-by-1 cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

scalar numeric | vector

Option strike price value, specified as one of the following:

- For a European or Bermuda option, `Strike` is a scalar (European) or 1-by-NSTRIKES (Bermuda) vector of strike prices.
- For an American option, `Strike` is a scalar vector of the strike price.

Data Types: `double`

**Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as a scalar serial date number or date character vector.

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as a scalar serial date number or date character vector.

Data Types: double | char

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity, 'OutSpec', 'delta')`

**OutSpec — Define outputs**

`{'Price'}` (default) | character vector with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | cell array of character vectors with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: char | cell

### **UndIdx — Index of the underlying instrument to compute the sensitivity**

[ ] (default) | scalar numeric

Index of the underlying instrument to compute the sensitivity, specified as the comma-separated pair consisting of 'UndIdx' and a scalar numeric.

Data Types: double

## **Output Arguments**

### **PriceSens — Expected prices or sensitivities for basket option**

matrix

Expected prices or sensitivities (defined using OutSpec) for basket option, returned as a NINST-by-1 matrix.

## **References**

[1] Nengjiu Ju. “Pricing Asian and Basket Options Via Taylor Expansion.” *Journal of Computational Finance*. Vol. 5, 2002.

## **See Also**

basketbyju | basketstockspec

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Basket Option” on page 3-28

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009b**

# basketsensbyls

Calculate price and sensitivities for European or American basket options using Monte Carlo simulations

## Syntax

```
[PriceSens,Paths,Times,Z] = basketsensbyls(RateSpec,BasketStockSpec,
OptSpec,Strike,Settle,ExerciseDates)
[PriceSens,Paths,Times,Z] = basketsensbyls( ____,Name,Value)
```

## Description

[PriceSens,Paths,Times,Z] = basketsensbyls(RateSpec,BasketStockSpec, OptSpec,Strike,Settle,ExerciseDates) calculates price and sensitivities for European or American basket options using the Longstaff-Schwartz model.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

[PriceSens,Paths,Times,Z] = basketsensbyls( \_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Calculate Price and Sensitivities for Basket Options Using the Longstaff-Schwartz Model

Find a European put basket option of two stocks. The basket contains 50% of each stock. The stocks are currently trading at \$90 and \$75, with annual volatilities of 15%. Assume that the correlation between the assets is zero. On May 1, 2009, an investor wants to buy a one-year put option with a strike price of \$80. The current annualized, continuously compounded interest is 5%. Use this data to compute price and delta of the put basket option with the Longstaff-Schwartz approximation model.

```
Settle = 'May-1-2009';
Maturity = 'May-1-2010';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates',...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric,
% and have ones along the main diagonal.
NumInst = 2;
InstIdx = ones(NumInst,1);
Corr = diag(ones(NumInst,1), 0);

% Define BasketStockSpec
AssetPrice = [90; 75];
Volatility = 0.15;
Quantity = [0.50; 0.50];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

% Compute the price of the put basket option. Calculate also the delta
% of the first stock.
OptSpec = {'put'};
Strike = 80;
OutSpec = {'Price', 'Delta'};
UndIdx = 1; % First element in the basket

[PriceSens, Delta] = basketsensbyls(RateSpec, BasketStockSpec, OptSpec,...
Strike, Settle, Maturity, 'OutSpec', OutSpec, 'UndIdx', UndIdx)

PriceSens = 0.9822

Delta = -0.0995

Compute the Price and Delta of the basket with a correlation of -20%:

NewCorr = [1 -0.20; -0.20 1];

% Define the new BasketStockSpec.
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, NewCorr);

% Compute the price and delta of the put basket option.
[PriceSens, Delta] = basketsensbyls(RateSpec, BasketStockSpec, OptSpec,...
Strike, Settle, Maturity, 'OutSpec', OutSpec, 'UndIdx', UndIdx)
```



PriceSens = 0.7814

Delta = -0.0961

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **BasketStockSpec — BasketStock specification**

structure

`BasketStock` specification, specified using `basketstockspec`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a character vector or a 2-by-1 cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

scalar numeric | vector

Option strike price value, specified as one of the following:

- For a European or Bermuda option, `Strike` is a scalar (European) or 1-by-NSTRIKES (Bermuda) vector of strike prices.
- For an American option, `Strike` is a scalar vector of the strike price.

Data Types: `double`

**Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as a scalar serial date number or date character vector.

Data Types: double | char

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European or Bermuda option, `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between, or including, the pair of dates on that row. If there is only one non-NaN date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: double | char | cell

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec,  
Strike, Settle, Maturity, 'AmericanOpt', AmericanOpt, 'NumTrials', NumTrials,  
'OutSpec', 'delta')
```

**AmericanOpt — Option type**

0 (European/Bermuda) (default) | values [0, 1]

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and a NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda

- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: double

**NumPeriods — Number of simulation periods per trial**

100 (default) | nonnegative integer

Number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a scalar nonnegative integer.

---

**Note** NumPeriods is considered only when pricing European basket options. For American and Bermuda basket options, NumPeriod equals the number of exercise days during the life of the option.

---

Data Types: double

**NumTrials — Number of independent sample paths (simulation trials)**

1000 (default) | nonnegative integer

Number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of 'NumTrials' and a scalar nonnegative integer.

Data Types: double

**Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-NINST-by-NumTrials 3-D time series array. The Z value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: double

**Antithetic — Indicator for antithetic sampling**

false (default) | scalar logical flag with value of true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.

Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}

Data Types: char | cell

**UndIdx — Index of the underlying instrument to compute the sensitivity**

[] (default) | scalar numeric

Index of the underlying instrument to compute the sensitivity, specified as the comma-separated pair consisting of 'UndIdx' and a scalar numeric.

Data Types: double

## Output Arguments

**PriceSens — Expected prices or sensitivities for basket option**

matrix

Expected prices or sensitivities (defined using OutSpec) for basket option, returned as a NINST-by-1 matrix.

## **Paths — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a `NumPeriods + 1`-by-`1`-by-`NumTrials` 3-D time series array of simulated paths of correlated state variables. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

## **Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a `NumPeriods + 1`-by-`1` column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

## **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, returned as a `NumPeriods`-by-`1`-by-`NumTrials` 3-D array when `Z` is specified as an input argument. If the `Z` input argument is not specified, then the `Z` output argument contains the random variates generated internally.

## **References**

- [1] Longstaff, F.A., and E.S. Schwartz. "Valuing American Options by Simulation: A Simple Least-Squares Approach." *The Review of Financial Studies*. Vol. 14, No. 1, Spring 2001, pp. 113-147.

## **See Also**

`basketbyls` | `basketstockspec`

## **Topics**

"Equity Derivatives Using Closed-Form Solutions" on page 3-150

"Basket Option" on page 3-28

"Supported Equity Derivatives" on page 3-24

**Introduced in R2009b**

## **basketstockspec**

Specify basket stock structure using Longstaff-Schwartz model

### **Syntax**

```
BasketStockSpec = basketstockspec(Sigma,AssetPrice,Quantity,  
Correlation)  
BasketStockSpec = basketstockspec( ____,Name,Value)
```

### **Description**

`BasketStockSpec = basketstockspec(Sigma,AssetPrice,Quantity,Correlation)` creates a basket stock structure.

`BasketStockSpec = basketstockspec( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### **Examples**

#### **Create a Basket Stock Structure for Three Stocks**

Find a basket option of three stocks. The stocks are currently trading at \$56, \$92 and \$125 with annual volatilities of 20%, 12% and 15%, respectively. The basket option contains 25% of the first stock, 40% of the second stock, and 35% of the third. The first stock provides a continuous dividend of 1%, while the other two provide no dividends. The correlation between the first and second asset is 30%, between the second and third asset 11%, and between the first and third asset 16%. Use this data to create the `BasketStockSpec` structure:

```
AssetPrice = [56;92;125];  
Sigma = [0.20;0.12;0.15];
```

```
% Create the Correlation matrix. Correlation matrices are symmetric and
```

```

% have ones along the main diagonal.
NumInst = 3;
Corr = zeros(NumInst,1);
Corr(1,2) = .30;
Corr(2,3) = .11;
Corr(1,3) = .16;
Corr = triu(Corr,1) + tril(Corr',-1) + diag(ones(NumInst,1), 0);

% Define dividends
DivType = cell(NumInst,1);
DivType{1}='continuous';
DivAmounts = cell(NumInst,1);
DivAmounts{1} = 0.01;

Quantity = [0.25; 0.40; 0.35];

BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Corr, ...
'DividendType', DivType, 'DividendAmounts', DivAmounts)

BasketStockSpec = struct with fields:
    FinObj: 'BasketStockSpec'
    Sigma: [3x1 double]
    AssetPrice: [3x1 double]
    Quantity: [3x1 double]
    Correlation: [3x3 double]
    DividendType: {3x1 cell}
    DividendAmounts: {3x1 cell}
    ExDividendDates: {3x1 cell}

```

Examine the BasketStockSpec structure.

BasketStockSpec.Correlation

ans = 3x3

1.0000	0.3000	0.1600
0.3000	1.0000	0.1100
0.1600	0.1100	1.0000

### Create a Basket Stock Structure for Two Stocks

Find a basket option of two stocks. The stocks are currently trading at \$60 and \$55 with volatilities of 30% per annum. The basket option contains 50% of each stock. The first stock provides a cash dividend of \$0.25 on May 1, 2009 and September 1, 2009. The second stock provides a continuous dividend of 3%. The correlation between the assets is 40%. Use this data to create the structure `BasketStockSpec`:

```
AssetPrice = [60;55];
Sigma = [0.30;0.30];

% Create the Correlation matrix. Correlation matrices are symmetric and
% have ones along the main diagonal.
Correlation = [1 0.40;0.40 1];

% Define dividends
NumInst = 2;
DivType = cell(NumInst,1);
DivType{1}='cash';
DivType{2}='continuous';

DivAmounts = cell(NumInst,1);
DivAmounts{1} = [0.25 0.25];
DivAmounts{2} = 0.03;

ExDates = cell(NumInst,1);
ExDates{1} = {'May-1-2009' 'Sept-1-2009'};

Quantity = [0.5; 0.50];

BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation, ...
'DividendType', DivType, 'DividendAmounts', DivAmounts, 'ExDividendDates',ExDates)

BasketStockSpec = struct with fields:
    FinObj: 'BasketStockSpec'
    Sigma: [2x1 double]
    AssetPrice: [2x1 double]
    Quantity: [2x1 double]
    Correlation: [2x2 double]
    DividendType: {2x1 cell}
    DividendAmounts: {2x1 cell}
    ExDividendDates: {2x1 cell}
```

Examine the `BasketStockSpec` structure.



BasketStockSpec.DividendType

```
ans = 2x1 cell array
      {'cash'      }
      {'continuous'}
```

## Input Arguments

### **Sigma** — Annual price volatility of the underlying security

vector in decimals

Annual price volatility of the underlying security, specified as an NINST-by-1 vector in decimals.

Data Types: double

### **AssetPrice** — Underlying asset price values at time 0

vector

Underlying asset price values at time 0, specified as a NINST-by-1 vector.

Data Types: double

### **Quantity** — Quantities of the instruments contained in the basket

vector

Quantities of the instruments contained in the basket, specified as an NINST-by-1 vector.

Data Types: double

### **Correlation** — Correlation values

matrix

Correlation values, specified as an NINST-by-1 matrix.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

```
Example: BasketStockSpec =  
basketstockspec(Sigma, AssetPrice, Quantity, Correlation, 'DividendType'  
, DivType, 'DividendAmounts', DivAmounts)
```

**DividendAmounts — Dividend amounts for basket instruments**

row vector

Dividend amounts for basket instruments, specified as the comma-separated pair consisting of 'DividendAmounts' and an NINST-by-1 cell array. Each element of the cell array is a 1-by-NDIV row vector of cash dividends or a scalar representing a continuous annualized dividend yield for the corresponding instrument.

Data Types: double

**DividendType — Stock dividend type**

cell array of character vectors

Stock dividend type, specified as the comma-separated pair consisting of 'DividendType' and an NINST-by-1 cell array of character vectors specifying each stock's dividend type. Dividend type must be either `cash` for actual dollar dividends or `continuous` for continuous dividend yield.

Data Types: char | cell

**ExDividendDates — Ex-dividend dates for the basket instruments**

cell array

Ex-dividend dates for the basket instruments, specified as the comma-separated pair consisting of 'ExDividendDates' and an NINST-by-1 cell array specifying the ex-dividend dates for the basket instruments. Each row is a 1-by-NDIV matrix of ex-dividend dates for cash type. For rows that correspond to basket instruments with continuous dividend type, the cell is empty. If none of the basket instruments pay continuous dividends, do not specify ExDividendDates.

Data Types: cell

## Output Arguments

### **BasketStockSpec — Structure encapsulating the properties of a basket stock structure**

structure

Structure encapsulating the properties of a basket stock structure, returned as a structure.

## See Also

[basketbyju](#) | [basketbyls](#) | [basketsensbyju](#) | [basketsensbyls](#) | [intenvset](#) | [stockspec](#)

## Topics

“Portfolio Creation” on page 1-8

“Hedging Functions” on page 4-4

“Instrument Constructors” on page 1-20

“Supported Equity Derivatives” on page 3-24

## Introduced in R2009b

## bdtpprice

Instrument prices from Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = bdtpprice(BDTree,InstSet)
[Price,PriceTree] = bdtpprice(___,Options)
```

### Description

`[Price,PriceTree] = bdtpprice(BDTree,InstSet)` computes arbitrage-free prices for instruments using an interest-rate tree created with `bdtree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`bdtpprice` handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See `instadd` to construct defined types.

`[Price,PriceTree] = bdtpprice(___,Options)` adds an optional input argument for `Options`.

### Examples

#### Price a Cap and Bond Instruments Contained in an Instrument Set

Load the BDT tree and instruments from the data file `deriv.mat` to price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
BDTSubSet = instselect(BDTInstSet,'Type', {'Bond', 'Cap'});
```

```
instdisp(BDTSubSet)
```

```
instdisp(BDTSubSet)
Index Type CouponRate Settle
```

```
Maturity
```

```
Period Basis EndMonthRule IssueDate FirstCouponDate LastCouponDate St
```

```

1 Bond 0.1 01-Jan-2000 01-Jan-2003 1 NaN NaN NaN NaN NaN NaN
2 Bond 0.1 01-Jan-2000 01-Jan-2004 2 NaN NaN NaN NaN NaN NaN

```

```

Index Type Strike Settle Maturity CapReset Basis Principal Name Quantity
3 Cap 0.15 01-Jan-2000 01-Jan-2004 1 NaN NaN 15% Cap 30

```

Price the bond and cap.

```
[Price, PriceTree] = bdtprice(BDTTree, BDTSubSet)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

> In checktree at 289

In bdtprice at 85

Price =

```

95.5030
93.9079
1.4375

```

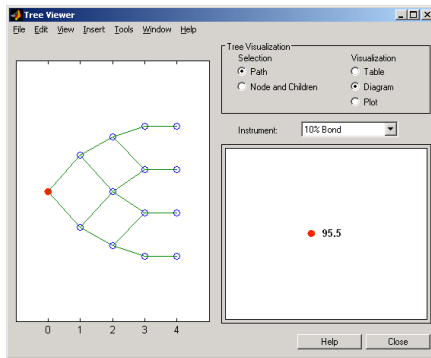
PriceTree =

```

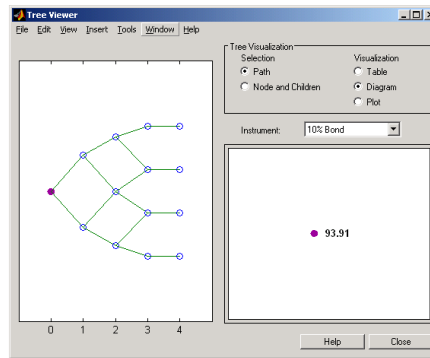
FinObj: 'BDTPriceTree'
PTree: {[3x1 double] [3x2 double] [3x3 double] [3x4 double] [3x4 double]}
AITree: {[3x1 double] [3x2 double] [3x3 double] [3x4 double] [3x4 double]}
tObs: [0 1 2 3 4]

```

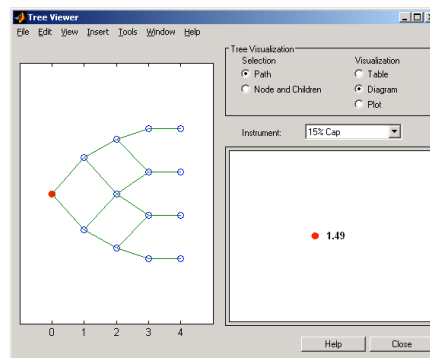
You can use the `treeview` function to see the prices of these three instruments along the price tree.



First 10% Bond (Maturity 2003)



Second 10% Bond (Maturity 2004)



15% Cap

## Price Multi-Stepped Coupon Bonds

Price the following multi-stepped coupon bonds using the following data:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

```
% Create RateSpec
```

```

RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07};

% Display the instrument portfolio
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)

```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	[Cell]	01-Jan-2010	01-Jan-2011	1	0	1	NaN
2	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
3	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
4	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build a BDTTtree to price the stepped coupon bonds. Assume the volatility to be 10%

```

Sigma = 0.1;
BDTTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RS, BDTTTimeSpec);

% Compute the price of the stepped coupon bonds
PBDT = bdtprice(BDTT, ISet)

```

PBDT = 4×1

```

100.6763
100.7368
100.9266
101.0115

```

### Price a Portfolio of Stepped Callable Bonds and Stepped Vanilla Bonds

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data: The data for the interest rate term structure is as follows:

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year

% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);

% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);

% Display the instrument portfolio
instdisp(ISet)

```

Index	Type	CouponRate	Settle	Maturity	OptSpec	Strike	ExerciseDates
1	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2012	call	100	01-Jan-2011
2	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2013	call	100	01-Jan-2011
3	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2014	call	100	01-Jan-2011

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
4	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
5	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
6	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build a BDTTtree and price the instruments. Build the tree Assume the volatility to be 10%

```

Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);

```



```

BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);

%The first three rows corresponds to the price of the stepped callable bonds and the
%last three rows corresponds to the price of the stepped vanilla bonds.
PBDT = bdtprice(BDTT, ISet)

PBDT = 6×1

    100.4799
    100.3228
    100.0840
    100.7368
    100.9266
    101.0115

```

### Price a Portfolio with Range Notes and a Floating Rate Note

Compute the price of a portfolio with range notes and a floating rate note using the following data: The data for the interest rate term structure is as follows:

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];

```

```

% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates = [0.048 0.059; 0.055 0.068 ; 0.07 0.09];

% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);

% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);

% Display the portfolio instrument
instdisp(InstSet)

```

Index	Type	Spread	Settle	Maturity	RateSched	FloatReset	Basis	Princ
1	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100
2	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRu
3	Float	200	01-Jan-2011	01-Jan-2014	1	0	100	1

Build a BDTTtree and price the instruments. Build the tree Assume the volatility to be 10%.

```

Sigma = 0.1;
BDTTS = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTV, RS, BDTTS);

```

```

% Price the portfolio
Price = bdtprice(BDTT, InstSet)

```

```
Price = 3×1
```

```

100.2841
 98.0757
105.5147

```

### Create a Float-Float Swap and Price with bdtprice

Use `instswap` to create a float-float swap and price the swap with `bdtprice`.

```

RateSpec = intenvset('Rates',.05,'StartDate',today,'EndDate',datemnth(today,60));
IS = instswap([.02 .03],today,datemnth(today,60),[], [], [], [1 1]);
VolSpec = bdtvolspec(today,datemnth(today,[10 60]),[.01 .02]);
TimeSpec = bdttimespec(today,cfdates(today,datemnth(today,60),1));
BDTTree = bdttree(VolSpec,RateSpec,TimeSpec);
bdtprice(BDTTree,IS)

ans = -4.3220

```

## Price Multiple Swaps with bdtprice

Use `instswap` to create multiple swaps and price the swaps with `bdtprice`.

```

RateSpec = intenvset('Rates',.05,'StartDate',today,'EndDate',datemnth(today,60));
IS = instswap([.03 .02],today,datemnth(today,60),[], [], [], [1 1]);
IS = instswap(IS,[200 300],today,datemnth(today,60),[], [], [], [0 0]);
IS = instswap(IS,[.08 300],today,datemnth(today,60),[], [], [], [1 0]);
VolSpec = bdtvolspec(today,datemnth(today,[10 60]),[.01 .02]);
TimeSpec = bdttimespec(today,cfdates(today,datemnth(today,60),1));
BDTTree = bdttree(VolSpec,RateSpec,TimeSpec);
bdtprice(BDTTree,IS)

ans =
    4.3220
   -4.3220
   -0.2701

```

## Input Arguments

### BDTTree — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdttree`.

Data Types: `struct`

### InstSet — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options — Derivatives pricing options structure**

`structure`

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Price for each instrument at time 0**

`vector`

Price for each instrument at time 0, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

Related single-type pricing functions are:

- `bondbybd`t — Price a bond from a BDT tree.
- `capbybd`t — Price a cap from a BDT tree.
- `cfbybd`t — Price an arbitrary set of cash flows from a BDT tree.
- `fixedbybd`t — Price a fixed-rate note from a BDT tree.
- `floatbybd`t — Price a floating-rate note from a BDT tree.
- `floorbybd`t — Price a floor from a BDT tree.
- `optbndbybd`t — Price a bond option from a BDT tree.
- `optfloatbybd`t — Price a floating-rate note with an option from a BDT tree.
- `optemfloatbybd`t — Price a floating-rate note with an embedded option from a BDT tree.
- `optembndbybd`t — Price a bond with embedded option by a BDT tree.
- `rangefloatbybd`t — Price range floating note using a BDT tree.
- `swapbybd`t — Price a swap from a BDT tree.

- `swaptionbybdt` — Price a swaption from a BDT tree.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## **See Also**

`bdtrens` | `bdttree` | `instadd` | `intenvprice` | `intenvsens`

## **Topics**

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bdtsens

Instrument prices and sensitivities from Black-Derman-Toy interest-rate tree

### Syntax

```
[Delta,Gamma,Vega,Price] = bdtsens(BDTree,InstSet)
[Delta,Gamma,Vega,Price] = bdtsens( ____,Options)
```

### Description

`[Delta,Gamma,Vega,Price] = bdtsens(BDTree,InstSet)` computes instrument sensitivities and prices for instruments using an interest-rate tree created with the `bdtree` function. All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

`bdtsens` handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See `instadd` for information on instrument types.

`[Delta,Gamma,Vega,Price] = bdtsens( ____,Options)` adds an optional input argument for `Options`.

### Examples

#### Compute Instrument Sensitivities and Prices for Cap and Bond instruments

Load the tree and instruments from the `deriv.mat` data file.

```
load deriv.mat;
BDTSubSet = instselect(BDTInstSet,'Type', {'Bond', 'Cap'});
```

```
instdisp(BDTSubSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.1	01-Jan-2000	01-Jan-2003	1	NaN	NaN	NaN

```

2      Bond 0.1      01-Jan-2000  01-Jan-2004  2      NaN  NaN      NaN
Index Type Strike Settle      Maturity      CapReset Basis Principal Name      Quant
3      Cap 0.15  01-Jan-2000  01-Jan-2004  1      NaN  NaN      15% Cap 30

```

Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.

```
[Delta, Gamma] = bdtsens(BDTree, BDSubSet)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

```
Delta = 3×1
```

```

-232.6681
-281.0517
 63.8102

```

```
Gamma = 3×1
103 ×
```

```

0.8037
1.1819
1.8535

```

## Input Arguments

### **BDTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdtree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Delta — Rate of change of instruments prices with respect to changes in interest rate**

vector

Rate of change of instruments prices with respect to changes in the interest rate, returned as a NINST-by-1 vector of deltas. `Delta` is computed by finite differences in calls to `bdttree`.

---

**Note** `Delta` is calculated based on yield shifts of 100 basis points.

---

### **Gamma — Rate of change of instruments deltas with respect to changes in interest rate**

vector

Rate of change of instruments deltas with respect to changes in the interest rate, returned as a NINST-by-1 vector of gammas. `Gamma` is computed by finite differences in calls to `bdttree`.

---

**Note** `Gamma` is calculated based on yield shifts of 100 basis points.

---

### **Vega — Rate of change of instruments prices with respect to changes in volatility**

vector

Rate of change of instruments prices with respect to changes in the volatility, returned as a NINST-by-1 vector of vegas. Volatility is  $\sigma(t, T)$  of the interest rate. `Vega` is computed by finite differences in calls to `bdttree`. For information on the volatility process, see `bdtvolspec`.



---

**Note** Vega is calculated based on 1% shift in the volatility process.

---

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

## **See Also**

bdtpprice | bdttree | bdtvolspec | instadd

## **Topics**

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## **bdttimespec**

Specify time structure for Black-Derman-Toy interest-rate tree

### **Syntax**

```
TimeSpec = bdttimespec(ValuationDate,Maturity)
TimeSpec = bdttimespec( ____,Compounding)
```

### **Description**

`TimeSpec = bdttimespec(ValuationDate,Maturity)` sets the number of levels and node times for a `bdttree` and determines the mapping between dates and time for rate quoting.

`TimeSpec = bdttimespec( ____,Compounding)` adds the optional argument `Compounding`.

### **Examples**

#### **Specify a Five-Period Tree with Annual Nodes**

This example shows how to specify a five-period tree with annual nodes and use annual compounding to report rates.

```
Compounding = 1;
ValuationDate = '01-01-2000';
Maturity = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];

TimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)

TimeSpec = struct with fields:
    FinObj: 'BDTTimeSpec'
    ValuationDate: 730486
```

```

    Maturity: [5x1 double]
    Compounding: 1
    Basis: 0
    EndMonthRule: 1

```

## Input Arguments

### ValuationDate — Pricing date and first observation in the tree

serial date number | character vector date

Pricing date and first observation in the tree, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### Maturity — Dates marking the cash flow dates of the tree

serial date number | date character vector

Dates marking the cash flow dates of the tree, specified as NLEVELS-by-1 vector of serial date numbers or date character vectors. Cash flows with these maturities fall on tree nodes. **Maturity** should be in increasing order.

Data Types: double | char | cell

### Compounding — Rate at which the input zero rates were compounded when annualized

1 (default) | integer with value of 1, 2, 3, 4, 6, 12, 365, or -1

(Optional) Rate at which the input zero rates were compounded when annualized, specified as a scalar integer value.

- If Compounding = 1, 2, 3, 4, 6, 12:

$Disc = (1 + Z/F)^{-T}$ , where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units; for example,  $T = F$  is one year.

- If Compounding = 365:

$Disc = (1 + Z/F)^{-T}$ , where F is the number of days in the basis year and T is a number of days elapsed computed by basis.

- If Compounding = -1:

$Disc = \exp(-T*Z)$ , where T is time in years.

Data Types: double

## Output Arguments

**TimeSpec** — Specification for the time layout for `bdttree` structure

Specification for the time layout for `bdttree`, returned as a structure. The state observation dates are `[ValuationDate; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity(end)`.

## See Also

`bdtprice` | `bdttree` | `bdtvolspec` | `instadd`

## Topics

“Specifying the Time Structure (TimeSpec)” on page 2-85

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# bdttree

Build Black-Derman-Toy interest-rate tree

## Syntax

```
BDTTree = bdttree(VolSpec,RateSpec,TimeSpec)
```

## Description

`BDTTree = bdttree(VolSpec,RateSpec,TimeSpec)` creates a structure containing time and interest-rate information on a recombining tree.

## Examples

### Create a BDTTree

Using the data provided, create a BDT volatility specification (using `bdtvolspec`), rate specification (using `intenvset`), and tree time layout specification (using `bdttimespec`). Then use these specifications to create a BDT tree with `bdttree`.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];

RateSpec = intenvset('Compounding', Compounding,...
    'ValuationDate', ValuationDate,...
    'StartDates', StartDate,...
    'EndDates', EndDates,...
    'Rates', Rates);

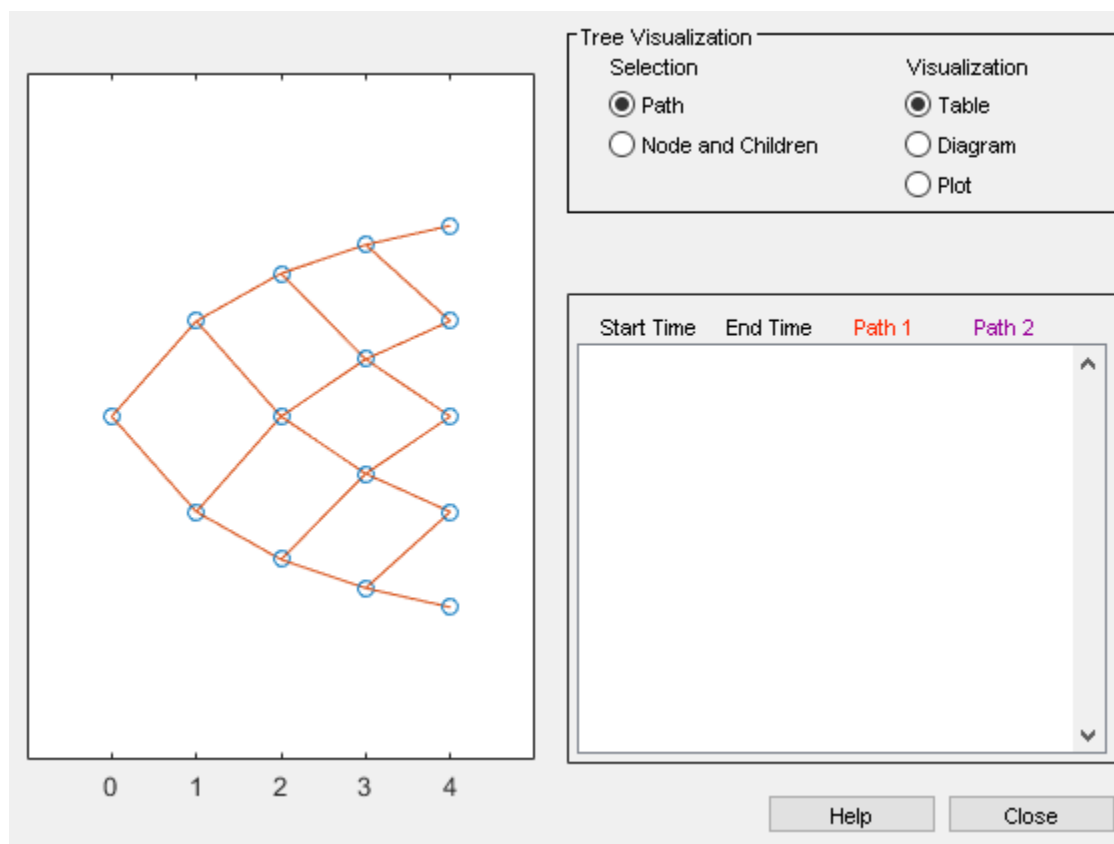
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
```

```
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)

BDTTree = struct with fields:
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [730486 730852 731217 731582 731947]
    TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
    CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
    FwdTree: {1x5 cell}
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(BDTTree)
```



## Input Arguments

### **VolSpec – Volatility process specification**

structure

Volatility process specification, specified using the VolSpec output obtained from bdtvolspec.

Data Types: struct

### **RateSpec – Interest-rate specification for initial risk-free rate curve**

structure

Interest-rate specification for initial rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **TimeSpec — Time tree layout specification**

structure

Time tree layout specification, specified using the `TimeSpec` output obtained from `bdttimespec`. The `TimeSpec` defines the observation dates of the BDT tree and the Compounding rule for date to time mapping and price-yield formulas.

Data Types: `struct`

## **Output Arguments**

### **BDTTree — Time and interest-rate information of a recombining tree**

structure

Time and interest-rate information of a recombining tree, returned as a structure.

## **See Also**

`bdtprice` | `bdttimespec` | `bdttree` | `bdtvolspec` | `instadd` | `intenvset`

## **Topics**

“Specifying the Interest-Rate Term Structure (`RateSpec`)” on page 2-84

“Specifying the Time Structure (`TimeSpec`)” on page 2-85

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# bdtvolspec

Specify Black-Derman-Toy interest-rate volatility process

## Syntax

```
VolSpec = bdtvolspec(ValuationDate,VolDates,VolCurve)
VolSpec = bdtvolspec( ____,InterpMethod)
```

## Description

`VolSpec = bdtvolspec(ValuationDate,VolDates,VolCurve)` creates a structure specifying the volatility for `bdttree`.

`VolSpec = bdtvolspec( ____,InterpMethod)` adds the optional argument `InterpMethod`.

## Examples

### Create a BDT Volatility Specification

This example shows how to create a BDT volatility specification (`VolSpec`) using the following data.

```
ValuationDate = '01-01-2000';
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Volatility = [.2; .19; .18; .17; .16];
```

```
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)
```

```
BDTVolSpec = struct with fields:
    FinObj: 'BDTVolSpec'
    ValuationDate: 730486
    VolDates: [5x1 double]
    VolCurve: [5x1 double]
```

VolInterpMethod: 'linear'

## Input Arguments

### **ValuationDate — Observation date of the investment horizon**

serial date number | character vector date

Observation date of the investment horizon, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### **VolDates — Number of points of yield volatility end dates**

serial date number | date character vector

Number of points of yield volatility end dates, specified as a NPOINTS-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **VolCurve — Yield volatility values**

decimal

Yield volatility values, specified as a NPOINTS-by-1 vector of decimal values. The term structure of `VolCurve` is the yield volatility represented by the value of the volatility of the yield from time  $t = 0$  to time  $t + i$ , where  $i$  is any point within the volatility curve.

Data Types: double

### **InterpMethod — Interpolation method**

'linear' (default) | character vector with values supported by `interp1`

(Optional) Interpolation method, specified as a character vector with values supported by `interp1`.

Data Types: char

## Output Arguments

**VolSpec** — Specification for the volatility model for `bdttree` structure

Structure specifying the volatility model for `bdttree`.

## See Also

`bdttree` | `interp1`

## Topics

“Specifying the Volatility Model (VolSpec)” on page 2-82

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bkprice

Instrument prices from Black-Karasinski interest-rate tree

### Syntax

```
[Price,PriceTree] = bkprice(BKTree,InstSet)
[Price,PriceTree] = bkprice( ____,Options)
```

### Description

[Price,PriceTree] = bkprice(BKTree,InstSet) computes arbitrage-free prices for instruments using an interest-rate tree created with `bktree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`bkprice` handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See `instadd` to construct defined types.

[Price,PriceTree] = bkprice( \_\_\_\_,Options) adds an optional input argument for `Options`.

### Examples

#### Price Cap and Bond Instruments in the Instrument Set

Load the BK tree and instruments from the data file `deriv.mat`. Price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
BKSubSet = instselect(BKInstSet,'Type', {'Bond', 'Cap'});
```

```
instdisp(BKSubSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.03	01-Jan-2004	01-Jan-2007	1	0	1	NaN

---

```

2      Bond 0.03      01-Jan-2004      01-Jan-2008      1      0      1      NaN
Index Type Strike Settle      Maturity      CapReset Basis Principal Name      Quanti
3      Cap 0.04      01-Jan-2004      01-Jan-2008      1      0      100      4% Cap 10

```

```
[Price, PriceTree] = bkprice(BKTree, BKSubSet)
```

```
Price = 3x1
```

```

98.1096
95.6734
2.2706

```

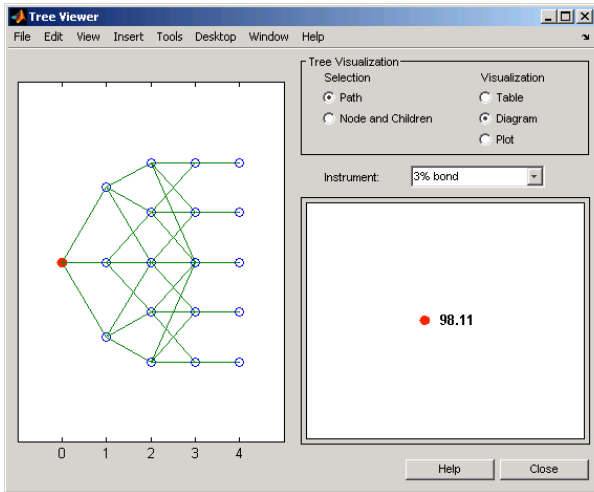
```
PriceTree = struct with fields:
```

```

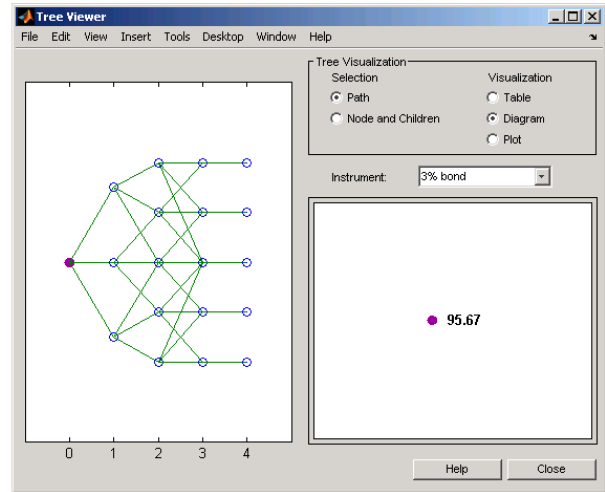
  FinObj: 'BKPriceTree'
  PTree: {1x5 cell}
  AITree: {1x5 cell}
  tObs: [0 1 2 3 4]
  Connect: {[2] [2 3 4] [2 2 3 4 4]}
  Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

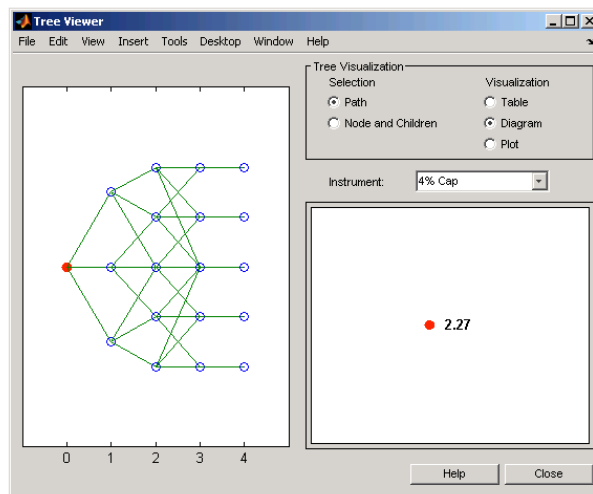
You can use `treeview` to see the prices of these three instruments along the price tree.



First 3% Bond (Maturity 2007)



Second 3% Bond (Maturity 2008)



4% Cap

## Price Multi-Stepped Coupon Bonds

Price the following multi-stepped coupon bonds using the following data:

*% The data for the interest rate term structure is as follows:*

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

*% Create RateSpec*

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

*% Create a portfolio of stepped coupon bonds with different maturities*

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07};
```

```
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	[Cell]	01-Jan-2010	01-Jan-2011	1	0	1	NaN
2	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
3	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
4	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build the BKTtree with the following data:

```
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);
```

Compute the price of the stepped coupon bonds.

```
PBK = bkprice(BKT, ISet)
```

```
PBK = 4×1

100.6763
100.7368
100.9266
101.0115
```

## Price a Portfolio of Stepped Callable Bonds and Stepped Vanilla Bonds

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014'
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; % Callable in one year

% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);

% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);
```



```
% Display the instrument portfolio
instdisp(ISet)
```

Index	Type	CouponRate	Settle	Maturity	OptSpec	Strike	ExerciseDates
1	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2012	call	100	01-Jan-2011
2	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2013	call	100	01-Jan-2011
3	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2014	call	100	01-Jan-2011

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
4	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
5	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
6	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build the BKTtree with the following data:

```
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);
```

Compute the price, where the first three rows of the output corresponds to the price of the stepped callable bonds and the last three rows corresponds to the price of the stepped vanilla bonds.

```
PBK = bkprice(BKT, ISet)
```

```
PBK = 6×1
```

```
100.6729
100.6763
100.6763
100.7368
100.9266
101.0115
```

## Price a Portfolio of Range Notes and Floating-Rate Notes

Price a portfolio of range notes and floating-rate notes using the following data:

```
% The data for the interest rate term structure is as follows:
```

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
```

```
% Create RateSpec
```

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', ...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

```
% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
```

```
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
```

```
% First Range Note:
```

```
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];
```

```
% Second Range Note:
```

```
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates = [0.048 0.059; 0.055 0.068 ; 0.07 0.09];
```

```
% Create InstSet
```

```
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);
```

```
% Add a floating-rate note
```

```
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);
```

```
% Display the portfolio instrument
```

```
instdisp(InstSet)
```

Index	Type	Spread	Settle	Maturity	RateSched	FloatReset	Basis	Princ
1	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100
2	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100
Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRu
3	Float	200	01-Jan-2011	01-Jan-2014	1	0	100	1

Build the BKTtree with the following data:

```
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;
```

```
BKVS = bkvolspec(RS.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
BKTS = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVS, RS, BKTS);
```

Price the portfolio.

```
Price = bkprice(BKT, InstSet)
```

```
Price = 3×1
```

```
105.5147
101.4805
105.5147
```

## Input Arguments

### **BKTtree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options** — Derivatives pricing options structure

structure

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## Output Arguments

### **Price** — Price for each instrument at time 0

vector

Price for each instrument at time 0, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

Related single-type pricing functions are:

- `bondbybk` — Price a bond from a Black-Karasinski tree.
- `capbybk` — Price a cap from a Black-Karasinski tree.
- `cfbybk` — Price an arbitrary set of cash flows from a Black-Karasinski tree.
- `fixedbybk` — Price a fixed-rate note from a Black-Karasinski tree.
- `floatbybk` — Price a floating-rate note from a Black-Karasinski tree.
- `floorbybk` — Price a floor from a Black-Karasinski tree.
- `optbndbybk` — Price a bond option from a Black-Karasinski tree.
- `optembndbybk` — Price a bond with embedded option by a Black-Karasinski tree.
- `optfloatbybk` — Price a floating-rate note with an option from a Black-Karasinski tree.
- `optemfloatbybk` — Price a floating-rate note with an embedded option from a Black-Karasinski tree.
- `rangefloatbybk` — Price range floating note from a Black-Karasinski tree.
- `swapbybk` — Price a swap from a Black-Karasinski tree.
- `swaptionbybk` — Price a swaption from a Black-Karasinski tree.

### **PriceTree** — Tree structure of instrument prices

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## See Also

`bdttree` | `bksens` | `instadd` | `intenvprice` | `intenvsens`

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bksens

Instrument prices and sensitivities from Black-Karasinski interest-rate tree

### Syntax

```
[Delta,Gamma,Vega,Price] = bksens(BKTree,InstSet)
[Delta,Gamma,Vega,Price] = bksens( ____,Options)
```

### Description

`[Delta,Gamma,Vega,Price] = bksens(BKTree,InstSet)` computes instrument sensitivities and prices for instruments using an interest-rate tree created with the `bktree` function. All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

`bksens` handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See `instadd` for information on instrument types.

`[Delta,Gamma,Vega,Price] = bksens( ____,Options)` adds an optional input argument for `Options`.

### Examples

#### Compute Instrument Sensitivities and Prices for Cap and Bond Instruments

Load the tree and instruments from the `deriv.mat` data file.

```
load deriv.mat;
BKSubSet = instselect(BKInstSet,'Type', {'Bond', 'Cap'});
```

```
instdisp(BKSubSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.03	01-Jan-2004	01-Jan-2007	1	0	1	NaN

2	Bond	0.03	01-Jan-2004	01-Jan-2008	1	0	1		NaN
Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal	Name	Quantity
3	Cap	0.04	01-Jan-2004	01-Jan-2008	1	0	100	4% Cap	10

Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.

```
[Delta, Gamma] = bksens(BKTree, BKSubSet)
```

```
Delta = 3×1
```

```
-285.7151
-365.7048
 189.5319
```

```
Gamma = 3×1
103 ×
```

```
0.8456
1.4345
6.9999
```

## Input Arguments

### **BKTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

**Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, created using `derivset`.Data Types: `struct`

## Output Arguments

**Delta — Rate of change of instruments prices with respect to changes in interest rate**

vector

Rate of change of instruments prices with respect to changes in the interest rate, returned as a NINST-by-1 vector of deltas. Delta is computed by finite differences in calls to `bktree`.

---

**Note** Delta is calculated based on yield shifts of 100 basis points.

---

**Gamma — Rate of change of instruments deltas with respect to changes in interest rate**

vector

Rate of change of instruments deltas with respect to changes in the interest rate, returned as a NINST-by-1 vector of gammas. Gamma is computed by finite differences in calls to `bktree`.

---

**Note** Gamma is calculated based on yield shifts of 100 basis points.

---

**Vega — Rate of change of instruments prices with respect to changes in volatility**

vector

Rate of change of instruments prices with respect to changes in the volatility, returned as a NINST-by-1 vector of vegas. Volatility is  $\sigma(t, T)$  of the interest rate. Vega is computed by finite differences in calls to `bktree`. For information on the volatility process, see `bkvolspec`.



---

**Note** Vega is calculated based on 1% shift in the volatility process.

---

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

## **See Also**

bkprice | bktree | bkvolspec | instadd

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bktimespec

Specify time structure for Black-Karasinski tree

### Syntax

```
TimeSpec = bktimespec(ValuationDate,Maturity)
TimeSpec = bktimespec( ____,Compounding)
```

### Description

`TimeSpec = bktimespec(ValuationDate,Maturity)` sets the number of levels and node times for a `bktree` and determines the mapping between dates and time for rate quoting.

`TimeSpec = bktimespec( ____,Compounding)` adds the optional argument `Compounding`.

### Examples

#### Specify a Four-Period Tree with Annual Nodes

This example shows how to specify a four-period tree with annual nodes using annual compounding to report rates.

```
ValuationDate = 'Jan-1-2004';
Maturity = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
Compounding = 1;
TimeSpec = bktimespec(ValuationDate, Maturity, Compounding)

TimeSpec = struct with fields:
    FinObj: 'BKTimeSpec'
    ValuationDate: 731947
    Maturity: [4x1 double]
```

```

Compounding: 1
      Basis: 0
EndMonthRule: 1

```

## Input Arguments

### ValuationDate — Pricing date and first observation in the tree

serial date number | character vector date

Pricing date and first observation in the tree, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### Maturity — Dates marking the cash flow dates of the tree

serial date number | date character vector

Dates marking the cash flow dates of the tree, specified as NLEVELS-by-1 vector of serial date numbers or date character vectors. Cash flows with these maturities fall on tree nodes. **Maturity** should be in increasing order.

Data Types: double | char | cell

### Compounding — Rate at which the input zero rates were compounded when annualized

1 (default) | integer with value of 1, 2, 3, 4, 6, 12, 365, or -1

(Optional) Rate at which the input zero rates were compounded when annualized, specified as a scalar integer value.

- If **Compounding** = 1, 2, 3, 4, 6, 12:

$Disc = (1 + Z/F)^{-T}$ , where  $F$  is the compounding frequency,  $Z$  is the zero rate, and  $T$  is the time in periodic units; for example,  $T = F$  is one year.

- If **Compounding** = 365:

$Disc = (1 + Z/F)^{-T}$ , where  $F$  is the number of days in the basis year and  $T$  is a number of days elapsed computed by basis.

- If **Compounding** = -1:

$\text{Disc} = \exp(-T*Z)$ , where T is time in years.

Data Types: double

## Output Arguments

**TimeSpec** — Specification for the time layout for `bktree` structure

Specification for the time layout for `bktree`, returned as a structure. The state observation dates are `[ValuationDate; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity(end)`.

## See Also

`bksens` | `bktree` | `bkvolspec`

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Specifying the Time Structure (TimeSpec)” on page 2-85

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# bktree

Build Black-Karasinski interest-rate tree

## Syntax

```
BKTree = bktree(VolSpec,RateSpec,TimeSpec)  
BKTree = bktree( ____,Name,Value)
```

## Description

`BKTree = bktree(VolSpec,RateSpec,TimeSpec)` creates a structure containing time and interest-rate information on a recombining tree.

`BKTree = bktree( ____,Name,Value)` adds additional name-value pair arguments.

## Examples

### Create a BKTree

Using the data provided, create a BK volatility specification (using `bkvolspec`), rate specification (using `intenvset`), and tree time layout specification (using `bktimespec`). Then use these specifications to create a BK tree using `bktree`.

```
Compounding = -1;  
ValuationDate = '01-01-2004';  
StartDate = ValuationDate;  
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';  
            '12-31-2007'];  
VolCurve = 0.01;  
AlphaDates = '01-01-2008';  
AlphaCurve = 0.1;  
Rates = [0.0275; 0.0312; 0.0363; 0.0415];  
  
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve,...
```

```
AlphaDates, AlphaCurve);

RateSpec = intenvset('Compounding', Compounding,...
    'ValuationDate', ValuationDate,...
    'StartDates', ValuationDate,...
    'EndDates', VolDates,...
    'Rates', Rates);

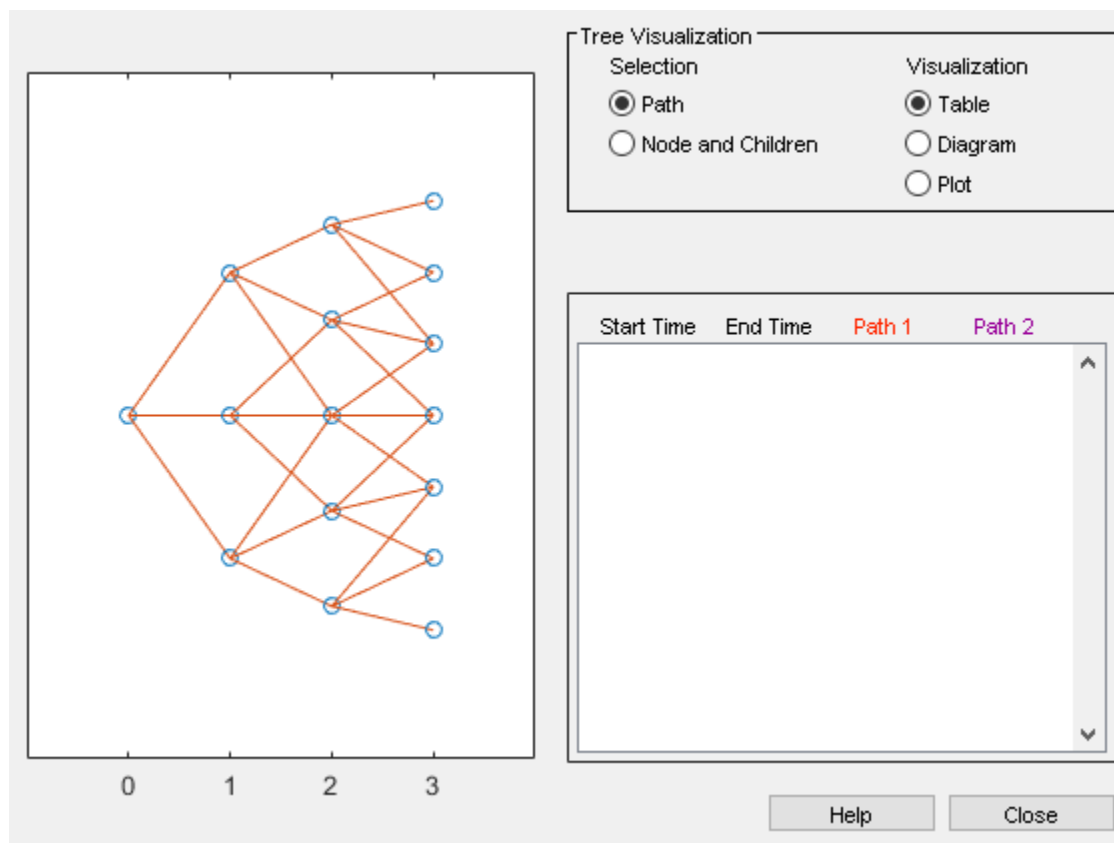
BKTimeSpec = bktimespec(ValuationDate, VolDates, Compounding);

BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec)

BKTree = struct with fields:
    FinObj: 'BKFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    t0bs: [0 0.9973 1.9973 2.9973]
    d0bs: [731947 732312 732677 733042]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [3.9973]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0278] [1.0361 1.0355 1.0349] [1x5 double] [1x7 double]}
```

Use `treeview` to observe the tree you have created.

```
treeview(BKTree)
```



## Input Arguments

### **VolSpec – Volatility process specification**

structure

Volatility process specification, specified using the VolSpec output obtained from `bdtvolspec`.

Data Types: `struct`

### **RateSpec – Interest-rate specification for initial risk-free rate curve**

structure

Interest-rate specification for initial rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **TimeSpec — Time tree layout specification**

structure

Time tree layout specification, specified using the `TimeSpec` output obtained from `bdttimespec`. The `TimeSpec` defines the observation dates of the BK tree and the Compounding rule for date to time mapping and price-yield formulas.

Data Types: `struct`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec, 'Method', 'HW1996')`

### **Method — Hull-White method upon which the tree-node connectivity algorithm is based**

'HW2000' (default) | character vector with values 'HW2000' or 'HW1996'

Hull-White method upon which the tree-node connectivity algorithm is based, specified as a character vector with a value of 'HW2000' or 'HW1996'.

`bktree` supports two tree-node connectivity algorithms. `HW1996` is based on the original paper published in the *Journal of Derivatives*, and `HW2000` is the general version of the algorithm, as specified in the paper published in August 2000.

Data Types: `char`

## **Output Arguments**

### **BKTree — Time and interest-rate information of a recombining tree**

structure



Time and interest-rate information of a recombining tree, returned as a structure.

## References

- [1] Hull, J., and A. White. "Using Hull-White Interest Rate Trees." *Journal of Derivatives*. 1996.
- [2] Hull, J., and A. White. "*The General Hull-White Model and Super Calibration*." August 2000.

## See Also

[bkprice](#) | [bksens](#) | [bktimespec](#) | [bkvolspec](#) | [intenvset](#)

## Topics

- "Pricing Using Interest-Rate Tree Models" on page 2-99
- "Calibrating Hull-White Model Using Market Data" on page 2-111
- "Pricing Options Structure" on page B-2
- "Understanding Interest-Rate Tree Models" on page 2-79
- "Supported Interest-Rate Instruments" on page 2-2

**Introduced before R2006a**

## bkvolspec

Specify Black-Karasinski interest-rate volatility process

### Syntax

```
VolSpec = bdtvolspec(ValuationDate,VolDates,VolCurve,AlphaDates,  
AlphaCurve)  
VolSpec = bdtvolspec( ____,InterpMethod)
```

### Description

`VolSpec = bdtvolspec(ValuationDate,VolDates,VolCurve,AlphaDates,AlphaCurve)` creates a structure specifying the volatility for `bktree`.

`VolSpec = bdtvolspec( ____,InterpMethod)` adds the optional argument `InterpMethod`.

### Examples

#### Create a Black-Karasinski Volatility Specification

This example shows how to create a Black-Karasinski volatility specification (`VolSpec`) using the following data.

```
ValuationDate = '01-01-2004';  
StartDate = ValuationDate;  
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';  
'12-31-2007'];  
VolCurve = 0.01;  
AlphaDates = '01-01-2008';  
AlphaCurve = 0.1;  
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve,...  
AlphaDates, AlphaCurve)
```

```
BKVolSpec = struct with fields:
    FinObj: 'BKVolSpec'
    ValuationDate: 731947
    VolDates: [4x1 double]
    VolCurve: [4x1 double]
    AlphaCurve: 0.1000
    AlphaDates: 733408
    VolInterpMethod: 'linear'
```

## Input Arguments

### **ValuationDate — Observation date of the investment horizon**

serial date number | character vector date

Observation date of the investment horizon, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### **VolDates — Number of points of yield volatility end dates**

serial date number | date character vector

Number of points of yield volatility end dates, specified as a NPOINTS-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **VolCurve — Yield volatility values**

decimal

Yield volatility values, specified as a NPOINTS-by-1 vector of decimal values. The term structure of `VolCurve` is the yield volatility represented by the value of the volatility of the yield from time  $t = 0$  to time  $t + i$ , where  $i$  is any point within the volatility curve.

Data Types: double

### **AlphaDates — Mean reversion end dates**

serial date number | date character vector

Mean reversion end dates, specified as a NPOINTS-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**AlphaCurve — Positive mean reversion values**

positive decimal

Positive mean reversion values, specified as a NPOINTS-by-1 vector of positive decimal values.

Data Types: double

**InterpMethod — Interpolation method**

'linear' (default) | character vector with values supported by interp1

(Optional) Interpolation method, specified as a character vector with values supported by interp1.

Data Types: char

## Output Arguments

**VolSpec — Specification for the volatility model for bktree**

structure

Structure specifying the volatility model for bktree.

## See Also

bkprice | bktimespec | bktree | interp1

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Specifying the Volatility Model (VolSpec)” on page 2-82

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# bondbybdt

Price bond from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price,PriceTree] = bondbybdt(BDTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = bondbybdt( ____,Name,Value)
```

## Description

[Price,PriceTree] = bondbybdt(BDTree,CouponRate,Settle,Maturity) prices bond from a Black-Derman-Toy interest-rate tree. bondbybdt computes prices of vanilla bonds, stepped coupon bonds and amortizing bonds.

[Price,PriceTree] = bondbybdt( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Bond Using a BDT Tree

Price a 10% bond using a BDT interest-rate tree.

Load `deriv.mat`, which provides `BDTree`. The `BDTree` structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.10;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Period = 1;
```

Use `bondbybdt` to compute the price of the bond.

```
Price = bondbybdt(BDTree, CouponRate, Settle, Maturity, Period)
Price = 95.5030
```

### Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, 'EndDates', ..
EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x1 double]
    Rates: [4x1 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734139
    ValuationDate: 734139
    Basis: 0
    EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};;
Period = 1;
```

Build the BDT tree and assume the volatility to be 10% using the following market data:

```
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec)

BDTT = struct with fields:
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734139 734504 734869 735235]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFwdT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    FwdTree: {1x4 cell}
```

Compute the price of the stepped coupon bonds.

```
PBDT= bondbybdt(BDTT, CouponRate, Settle,Maturity , Period)
```

```
PBDT = 4x1
```

```
100.7246
100.0945
101.5900
102.0820
```

## Price Two Bonds with Amortization Schedules

Price two bonds with amortization schedules using the Face input argument to define the schedule.

Define the interest-rate term structure.

```
Rates = 0.035;
ValuationDate = '1-Nov-2011';
StartDates = ValuationDate;
EndDates = '1-Nov-2017';
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

Create the bond instrument. The bonds have a coupon rate of 4% and 3.85%, a period of one year, and mature on 1-Nov-2017.

```
CouponRate = [0.04; 0.0385];  
Settle = '1-Nov-2011';  
Maturity = '1-Nov-2017';  
Period = 1;
```

Define the amortizing schedule.

```
Face = {'1-Nov-2015' 100; '1-Nov-2016' 85; '1-Nov-2017' 70};  
{ '1-Nov-2015' 100; '1-Nov-2016' 90; '1-Nov-2017' 80};
```

Build the BDT tree and assume the volatility to be 10%.

```
MatDates = {'1-Nov-2012'; '1-Nov-2013'; '1-Nov-2014'; '1-Nov-2015'; '1-Nov-2016'; '1-Nov-2017'};  
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);  
Volatility = 0.1;  
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1, length(MatDates)));  
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of the amortizing bonds.

```
Price = bondbybdt(BDTT, CouponRate, Settle, Maturity, 'Period', Period, ...  
'Face', Face)
```

```
Price = 2×1
```

```
102.4791  
101.7786
```

## Input Arguments

### **BDTtree** — Interest-rate structure

structure

Interest-rate tree structure, created by `bdttree`



Data Types: struct

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

### **Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The **Settle** date for every bond is set to the **ValuationDate** of the BDT tree. The bond argument **Settle** is ignored.

Data Types: char | double

### **Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each bond.

Data Types: char | double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1, Value1, . . . , NameN, ValueN**.

Example: `[Price, PriceTree] = bondbybdt(BDTree, CouponRate, Settle, Maturity, 'Period', 4, 'Face', 10000)`

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and an NINST-by-1 vector. Values for Period are 1, 2, 3, 4, 6, and 12.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **IssueDate — Bond issue date**

serial nonnegative date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial nonnegative date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial nonnegative date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**StartDate — Forward starting date of payments**

Settle date (default) | serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or an NINST-by-1 cell array of face values or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

**Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: struct

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

**BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if

`BusinessDayConvention` is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: `double`

## **Output Arguments**

### **Price — Expected bond prices at time 0**

vector

Expected bond prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## See Also

`bdtprice` | `bdttree` | `cfamounts` | `instbond`

## **Topics**

“Computing Instrument Sensitivities” on page 2-108

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Understanding the Interest-Rate Term Structure” on page 2-55

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## **Introduced before R2006a**

## blackvolbyrebonato

Compute Black volatility for LIBOR Market Model using Rebonato formula

### Syntax

```
outVol = blackvolbyrebonato(ZeroCurve,VolFunc,CorrMat,ExerciseDate,  
Maturity)  
outVol = blackvolbyrebonato( ____,Name,Value)
```

### Description

`outVol = blackvolbyrebonato(ZeroCurve,VolFunc,CorrMat,ExerciseDate, Maturity)` computes the Black volatility for a swaption using a LIBOR Market Model.

`outVol = blackvolbyrebonato( ____,Name,Value)` adds optional name-value pair arguments.

### Examples

#### Price Swaption for LIBOR Market Model Using the Rebonato Formula

Define the input maturity and tenor for a LIBOR Market Model (LMM) specified by the cell array of volatility function handles, and a correlation matrix for the LMM.

```
Settle = datenum('11-Aug-2004');  
  
% Zero Curve  
CurveTimes = (1:10)';  
CurveDates = daysadd(Settle,360*CurveTimes,1);  
  
ZeroRates = [0.03 0.033 0.036 0.038 0.04 0.042 0.043 0.044 0.045 0.046]';  
  
% Construct an IRCurve  
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
```



```

LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -.02 .7 .14];

numRates = length(ZeroRates);
VolFunc(1:numRates-1) = {@(t) LMMVolFunc(LMMVolParams,t)};

Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
CorrMat = CorrFunc(meshgrid(1:numRates-1)',meshgrid(1:numRates-1),Beta);

ExerciseDate = datenum('11-Aug-2009');
Maturity = daysadd(ExerciseDate,360*[3;4],1);

Vol = blackvolbyrebonato(irdc,VolFunc,CorrMat,ExerciseDate,Maturity,'Period',1)

Vol = 2x1

    0.2210
    0.2079

```

## Input Arguments

### **ZeroCurve** — Zero-curve for LiborMarketModel model

structure

Zero-curve for the LiborMarketModel, specified using IRDataCurve or RateSpec.

Data Types: struct

### **VolFunc** — Function handle for volatility

cell array of function handles

Function handle for volatility, specified by a NumRates-by-1 cell array of function handles. Each function handle must take time as an input and return a scalar volatility

Data Types: cell | function\_handle

### **CorrMat** — Correlation matrix

vector

Correlation matrix, specified by NumRates-by-NumRates.

Data Types: `single` | `double`

**ExerciseDate — Swaption exercise date**

serial date number | vector of serial date numbers | date character vector

Swaption exercise dates, specified by a `NumSwaptions-by-1` vector of serial date numbers or date character vectors.

Data Types: `single` | `double` | `char` | `cell`

**Maturity — Swap maturity date**

serial date number | vector of serial date numbers | date character vector

Swap maturity dates, specified using a `NumSwaptions-by-1` vector of serial date numbers or date character vectors.

Data Types: `single` | `double` | `char` | `cell`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Vol =`

```
blackvolbyrebonato(irdc, VolFunc, CorrMat, ExerciseDate, Maturity, 'Period', 1)
```

**Period — Compounding frequency of curve and reset of swaptions**

2 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Compounding frequency of curve and reset of swaptions, specified as the comma-separated pair consisting of `'Period'` and a positive integer for the values 1, 2, 4, 6, 12 in a `NumSwaptions-by-1` vector.

Data Types: `single` | `double`

## Output Arguments

### outVol — Black volatility for specified swaption

scalar | vector

Black volatility, returned as a vector for the specified swaptions.

## Algorithms

The Rebonato approximation formula relates the Black volatility for a European swaption, given a set of volatility functions and a correlation matrix

$$(v_{\alpha, \beta}^{LFM})^2 = \sum_{i, j = \alpha + 1}^{\beta} \frac{w_i(0)w_j(0)F_i(0)F_j(0)\rho_{i, j}}{S_{\alpha, \beta}(0)^2} \int_0^{T_{\alpha}} \sigma_i(t)\sigma_j(t)dt$$

where:

$$w_i(t) = \frac{\tau_i P(t, T_i)}{\sum_{k = \alpha + 1}^{\beta} \tau_k P(t, t_k)}$$

## References

- [1] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

## See Also

LiborMarketModel

## Topics

- “Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123  
 “Pricing Bermudan Swaptions with Monte Carlo Simulation”  
 “Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# blackvolbysabr

Calculate implied Black volatility using SABR model

## Syntax

```
outVol = blackvolbysabr(Alpha,Beta,Rho,Nu,Settle,ExerciseDate,  
ForwardValue,Strike)  
outVol = blackvolbysabr( ____,Name,Value)
```

## Description

`outVol = blackvolbysabr(Alpha,Beta,Rho,Nu,Settle,ExerciseDate,ForwardValue,Strike)` calculates the implied Black volatility using the SABR stochastic volatility model.

`outVol = blackvolbysabr( ____,Name,Value)` adds optional name-value pair arguments.

## Examples

### Compute the Implied Black Volatility Using the SABR Model

Define the model parameters and option data.

```
ForwardRate = 0.0357;  
Strike = 0.03;  
Alpha = 0.036;  
Beta = 0.5;  
Rho = -0.25;  
Nu = 0.35;
```

```
Settle = datenum('15-Sep-2013');  
ExerciseDate = datenum('15-Sep-2015');
```

Compute the Black volatility using the SABR model.

```
ComputedVols = blackvolbysabr(Alpha, Beta, Rho, Nu, Settle, ...  
ExerciseDate, ForwardRate, Strike)
```

```
ComputedVols = 0.2122
```

### Compute the Shifted Black Volatility Using the Shifted SABR Model

Define the model parameters and option data with a negative strike.

```
ForwardRate = 0.0002;  
Strike = -0.001; % -0.1% strike.  
Alpha = 0.01;  
Beta = 0.5;  
Rho = -0.1;  
Nu = 0.15;  
Shift = 0.005; % 0.5 percent shift
```

```
Settle = datenum('1-Mar-2016');  
ExerciseDate = datenum('1-Mar-2017');
```

Compute the Shifted Black volatility using the Shifted SABR model.

```
ComputedVols = blackvolbysabr(Alpha, Beta, Rho, Nu, Settle, ...  
ExerciseDate, ForwardRate, Strike, 'Shift', Shift)
```

```
ComputedVols = 0.1518
```

## Input Arguments

### Alpha — Current SABR volatility

scalar

Current SABR volatility, specified as a scalar.

Data Types: double

### Beta — SABR constant elasticity of variance (CEV) exponent

scalar

SABR CEV exponent, specified as a scalar.

Data Types: double

**Rho — Correlation between forward value and volatility**

scalar

Correlation between forward value and volatility, specified as a scalar.

Data Types: double

**Nu — Volatility of volatility**

scalar

Volatility of volatility, specified as a scalar.

Data Types: double

**Settle — Settlement date**

scalar for serial nonnegative date number | scalar for date character vector

Settlement date, specified as a scalar using a serial nonnegative date number or date character vector.

Data Types: double | char

**ExerciseDate — Option exercise date**

scalar for serial nonnegative date number | scalar for date character vector

Option exercise date, specified as a scalar using a serial nonnegative date number or date character vector.

Data Types: double | char

**ForwardValue — Current forward value of underlying asset**

scalar | vector

Current forward value of the underlying asset, specified as a scalar or vector of size NumVols-by-1.

Data Types: double

**Strike — Option strike price values**

scalar | vector

Option strike price values, specified as a scalar value or a vector of size NumVols-by-1.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `outVol = blackvolbysabr(Alpha,Beta,Rho,Nu,Settle,ExerciseDate,ForwardValue,Strike,'Basis',2,'Model','Obloj2008')`

### **Basis** — Day-count basis of instrument

0 (actual/actual) (default) | positive integers of the set [1...13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer of the set [1...13].

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double



**Model — Version of SABR model**

'Hagan2002' (default) | value 'Obloj2008'

Version of SABR model, specified as the comma-separated pair consisting of 'Model' and one of the following values:

- 'Hagan2002' — Original version by Hagan et al. (2002)
- 'Obloj2008' — Version by Obloj (2008)

Data Types: char

**Shift — Shift in decimals for shifted SABR model**

0 (no shift) (default) | positive decimal

Shift in decimals for the shifted SABR model (to be used with the Shifted Black model), specified as the comma-separated pair consisting of 'Shift' and a scalar positive decimal value. Set this parameter to a positive shift in decimals to add a positive shift to `ForwardValue` and `Strike`, which effectively sets a negative lower bound for `ForwardValue` and `Strike`. For example, a `Shift` value of 0.01 is equal to a 1% shift.

Data Types: double

## Output Arguments

**outVol — Implied Black volatility computed by SABR model**

scalar | vector

Implied Black volatility computed by SABR model, returned as a scalar or vector of size `NumVols-by-1`.

## Algorithms

The SABR stochastic volatility model treats the underlying forward  $\hat{F}$  and volatility  $\hat{\alpha}$  as separate random processes, which are related with correlation  $\rho$ :

$$\begin{aligned}d\widehat{F} &= \widehat{\alpha}\widehat{F}^\beta dW_1 \\d\widehat{\alpha} &= v\widehat{\alpha} dW_2 \\dW_1 dW_2 &= \rho dt \\ \widehat{F}(0) &= F \\ \widehat{\alpha}(0) &= \alpha\end{aligned}$$

where

- $\widehat{F}$  is the underlying forward (a variable).
- $F$  is the current underlying forward (a constant).
- $\widehat{\alpha}$  is the SABR volatility (a variable).
- $\alpha$  is the current SABR volatility (a constant).
- $\beta$  is the SABR constant elasticity of variance (CEV) exponent.
- $v$  is the volatility of volatility.
- $dW_1$  is Brownian motion.
- $dW_2$  is Brownian motion.
- $\rho$  is the correlation between forward value and volatility.

In contrast, Black's lognormal model assumes a constant volatility,  $\sigma_B$ .

$$d\widehat{F} = \sigma_B \widehat{F} dW$$

Hagan et al. (2002) derived the following closed-form approximation of implied Black lognormal volatility ( $\sigma_B$ ) for the SABR model

$$\sigma_B(F, K) = \frac{\alpha \left\{ 1 + \left[ \frac{(1-\beta)^2}{24} \frac{\alpha^2}{(FK)^{1-\beta}} + \frac{1}{4} \frac{\rho\beta v\alpha}{(FK)^{(1-\beta)/2}} + \frac{2-3\rho^2}{24} v^2 \right] T + \dots \right\}}{(FK)^{(1-\beta)/2} \left\{ 1 + \frac{(1-\beta)^2}{24} \log^2(F/K) + \frac{(1-\beta)^4}{1920} \log^4(F/K) + \dots \right\}} \left( \frac{z}{x(z)} \right)$$

$$z = \frac{v}{\alpha} (F \log(F/K))$$

$$x(z) = \log \left\{ \frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho} \right\}$$

where

- $F$  is the current forward value of the underlying.
- $\alpha$  is the current SABR volatility.
- $K$  is the strike value.
- $T$  is the time to option maturity.

Obloj (2008) advocated the following closed-form approximation of implied Black lognormal volatility for the SABR model (for  $\beta < 1$ )

$$\sigma_B(F, K) = \frac{v \log(F/K)}{x(z)} \left\{ 1 + \left[ \frac{(1-\beta)^2}{24} \frac{\alpha^2}{(FK)^{1-\beta}} + \frac{1}{4} \frac{\rho\beta v\alpha}{(FK)^{(1-\beta)/2}} + \frac{2-3\rho^2}{24} v^2 \right] T + \dots \right\}$$

$$z = \frac{v F^{(1-\beta)} - K^{(1-\beta)}}{\alpha \frac{1-\beta}{1-\beta}}$$

$$x(z) = \log \left\{ \frac{\sqrt{1-2\rho z + z^2} + z - \rho}{1-\rho} \right\}$$

These expressions can be simplified in special situations, such as the at-the-money ( $F = K$ ) and stochastic lognormal ( $\beta = 1$ ) cases [1,2].

## References

- [1] Hagan, P. S., D. Kumar, A.S. Lesniewski, and D.E. Woodward. "Managing Smile Risk." Wilmott Magazine, September, pp. 84-108, 2002.
- [2] Obloj, J. "Fine-tune your smile: Correction to Hagan et. al." Wilmott Magazine, 2008.

## See Also

optsensbysabr | swaptionbyblk | swaptionbynormal

## Topics

- "Calibrate the SABR Model" on page 2-36
- "Price a Swaption Using the SABR Model" on page 2-42
- "Price Swaptions with Negative Strikes Using the Shifted SABR Model" on page 2-27
- "Work with Negative Interest Rates" on page 2-22
- "Supported Interest-Rate Instruments" on page 2-2

**External Websites**

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

**Introduced in R2014a**

# bondbybk

Price bond from Black-Karasinski interest-rate tree

## Syntax

```
[Price,PriceTree] = bondbybk(BKTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = bondbybk( ____,Name,Value)
```

## Description

[Price,PriceTree] = bondbybk(BKTree,CouponRate,Settle,Maturity) prices bond from a Black-Karasinski interest-rate tree. bondbybk computes prices of vanilla bonds, stepped coupon bonds and amortizing bonds.

[Price,PriceTree] = bondbybk( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Bond Using a BK Tree

Price a 4% bond using a Black-Karasinski interest-rate tree.

Load `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.04;
Settle = '01-Jan-2004';
Maturity = '31-Dec-2008';
```

Use `bondbybk` to compute the price of the bond.

```
Price = bondbybk(BKTree, CouponRate, Settle, Maturity)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

```
Price = 98.0300
```

### **Price a Stepped Coupon Bond**

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];  
ValuationDate = 'Jan-1-2010';  
StartDates = ValuationDate;  
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; ...  
            'Jan-1-2013'; 'Jan-1-2014'};  
Compounding = 1;
```

Create the RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
             'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: [4x1 double]  
    Rates: [4x1 double]  
    EndTimes: [4x1 double]  
    StartTimes: [4x1 double]  
    EndDates: [4x1 double]  
    StartDates: 734139  
    ValuationDate: 734139  
    Basis: 0  
    EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';  
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
```

```
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};
Period = 1;
```

Build the BK tree using the following market data:

```
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);
```

Compute the price of the stepped coupon bonds.

```
PBK= bondbybk(BKT, CouponRate, Settle, Maturity , Period)
```

```
PBK = 4×1
```

```
100.7246
100.0945
101.5900
102.0820
```

## Price a Bond with an Amortization Schedule

Price a bond with an amortization schedule using the Face input argument to define the schedule.

Define the interest-rate term structure.

```
Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates= '1-Jan-2017';
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
        Disc: 0.6853  
        Rates: 0.0650  
    EndTimes: 6  
    StartTimes: 0  
    EndDates: 736696  
    StartDates: 734504  
    ValuationDate: 734504  
        Basis: 0  
    EndMonthRule: 1
```

Create the bond instrument. The bond has a coupon rate of 7%, a period of one year, and matures on 1-Jan-2017.

```
CouponRate = 0.07;  
Settle = '1-Jan-2011';  
Maturity = '1-Jan-2017';  
Period = 1;  
Face = {'1-Jan-2015' 100; '1-Jan-2016' 90; '1-Jan-2017' 80};
```

Build the BK tree with the following market data:

```
VolDates = ['1-Jan-2012'; '1-Jan-2013'; ...  
'1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'];  
VolCurve = 0.01;  
AlphaDates = '01-01-2017';  
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...  
AlphaDates, AlphaCurve);  
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);  
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of the amortizing bond.

```
Price = bondbybk(BKT, CouponRate, Settle, Maturity, 'Period', Period, ...  
'Face', Face)
```

```
Price = 102.3155
```



Compare the results with price of a vanilla bond.

```
PriceVanilla = bondbybk(BKT, CouponRate, Settle, Maturity, Period)
```

```
PriceVanilla = 102.4205
```

## Input Arguments

### **BKTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `bktree`

Data Types: `struct`

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an `NINST-by-1` decimal annual rate or `NINST-by-1` cell array, where each element is a `NumDates-by-2` cell array. The first column of the `NumDates-by-2` cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or `NINST-by-1` vector of serial date numbers or date character vectors.

The `Settle` date for every bond is set to the `ValuationDate` of the BK tree. The bond argument `Settle` is ignored.

Data Types: `char` | `double`

### **Maturity** — Maturity date

serial date number | character vector

Maturity date, specified as a `NINST-by-1` vector of serial date numbers or date character vectors representing the maturity date for each bond.

Data Types: char | double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = bondbybk(BKTree, CouponRate, Settle, Maturity, 'Period', 4, 'Face', 10000)`

### Period — Coupons per year

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of `'Period'` and a NINST-by-1 vector. Values for `Period` are 1, 2, 3, 4, 6, and 12.

Data Types: double

### Basis — Day-count basis

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of `'Basis'` and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **IssueDate — Bond issue date**

serial nonnegative date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial nonnegative date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial nonnegative date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**StartDate — Forward starting date of payments**

Settle date (default) | serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or an NINST-by-1 cell array of face values or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

**Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with derivset.

Data Types: struct

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: double

## Output Arguments

### **Price — Expected bond prices at time 0**

vector

Expected bond prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## Definitions

### **Vanilla Bond**

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

## **Stepped Coupon Bond**

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

## **Bond with an Amortization Schedule**

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## **See Also**

[bkprice](#) | [bktree](#) | [cfamounts](#) | [hwprice](#) | [hwtree](#) | [instbond](#)

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding the Interest-Rate Term Structure” on page 2-55

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## **bondbyhjm**

Price bond from Heath-Jarrow-Morton interest-rate tree

### **Syntax**

```
[Price,PriceTree] = bondbyhjm(HJMTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = bondbyhjm( ____,Name,Value)
```

### **Description**

[Price,PriceTree] = bondbyhjm(HJMTree,CouponRate,Settle,Maturity) prices bond from a Heath-Jarrow-Morton interest-rate tree. bondbyhjm computes prices of vanilla bonds, stepped coupon bonds and amortizing bonds.

[Price,PriceTree] = bondbyhjm( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### **Examples**

#### **Price a Bond Using an HJM Tree**

Price a 4% bond using an HJM interest-rate tree.

Load `deriv.mat`, which provides HJMTree. The HJMTree structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use bondbyhjm to compute the price of the bond.



```
Price = bondbyhjm(HJMTree, CouponRate, Settle, Maturity)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

```
Price = 97.5280
```

## Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};
Period = 1;
```

Build the HJM tree using the following market data:

```
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RS, HJMTimeSpec);
```

Compute the price of the stepped coupon bonds.

```
PHJM= bondbyhjm(HJMT, CouponRate, Settle, Maturity , Period)
```

```
PHJM = 4×1
```

```
100.7246
100.0945
101.5900
102.0820
```

### Price a Bond with an Amortization Schedule

Price a bond with an amortization schedule using the Face input argument to define the schedule.

Define the interest-rate term structure.

```
Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates= '1-Jan-2017';
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: 0.6853
    Rates: 0.0650
    EndTimes: 6
    StartTimes: 0
    EndDates: 736696
    StartDates: 734504
    ValuationDate: 734504
    Basis: 0
    EndMonthRule: 1
```

Create the bond instrument. The bond has a coupon rate of 7%, a period of one year, and matures on 1-Jan-2017.

```
CouponRate = 0.07;
Settle = '1-Jan-2011';
```

```
Maturity = '1-Jan-2017';
Period = 1;
Face = {'1-Jan-2015' 100; '1-Jan-2016' 90; '1-Jan-2017' 80};
```

Build the HJM tree using the following market data:

```
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015';...
'Jan-1-2016'; 'Jan-1-2017'};
HJMTimeSpec = hjmtimespec(ValuationDate, MaTree);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec);
```

Compute the price of the amortizing bond.

```
Price = bondbyhjm(HJMT, CouponRate, Settle, Maturity, 'Period',...
Period, 'Face' , Face)
```

```
Price = 102.3155
```

Compare the results with price of a vanilla bond.

```
PriceVanilla = bondbyhjm(HJMT, CouponRate, Settle, Maturity, Period)
```

```
PriceVanilla = 102.4205
```

## Input Arguments

### HJMTree — Interest-rate structure

structure

Interest-rate tree structure, created by `hjm tree`

Data Types: `struct`

### CouponRate — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

**Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The **Settle** date for every bond is set to the **ValuationDate** of the HJM tree. The bond argument **Settle** is ignored.

Data Types: char | double

**Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each bond.

Data Types: char | double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1, Value1, . . . , NameN, ValueN**.

Example: `[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle, Maturity, 'Period', 4, 'Face', 10000)`

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of **'Period'** and a NINST-by-1 vector. Values for **Period** are 1, 2, 3, 4, 6, and 12.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**IssueDate — Bond issue date**

serial nonnegative date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial nonnegative date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial nonnegative date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**StartDate — Forward starting date of payments**

Settle date (default) | serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or an NINST-by-1 cell array of face values or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

### **Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: struct

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.

- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: `double`

## **Output Arguments**

### **Price — Expected bond prices at time 0**

vector

Expected bond prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PBush` contains the clean prices.
- `PriceTree.AIBush` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.



## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## See Also

[cfamounts](#) | [hjmtime](#) | [hjmprice](#) | [hjmtree](#) | [instbond](#)

## Topics

“Computing Instrument Prices” on page 2-99

“Understanding the Interest-Rate Term Structure” on page 2-55

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## **bondbyhw**

Price bond from Hull-White interest-rate tree

### **Syntax**

```
[Price,PriceTree] = bondbyhw(HWTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = bondbyhw( ____,Name,Value)
```

### **Description**

[Price,PriceTree] = bondbyhw(HWTree,CouponRate,Settle,Maturity) prices bond from a Hull-White interest-rate tree. bondbyhw computes prices of vanilla bonds, stepped coupon bonds and amortizing bonds.

[Price,PriceTree] = bondbyhw( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### **Examples**

#### **Price a Bond Using the HW Tree**

Price a 4% bond using a Hull-White interest-rate tree.

Load `deriv.mat`, which provides `HWTtree`. The `HWTtree` structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.04;
Settle = '01-Jan-2004';
Maturity = '31-Dec-2008';
```

Use `bondbyhw` to compute the price of the bond.

```
Price = bondbyhw(HWTTree, CouponRate, Settle, Maturity)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

```
Price = 98.0483
```

## Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x1 double]
    Rates: [4x1 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734139
    ValuationDate: 734139
    Basis: 0
    EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};;
Period = 1;
```

Build the HW tree using the following market data:

```
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];  
VolCurve = 0.01;  
AlphaDates = '01-01-2014';  
AlphaCurve = 0.1;
```

```
HWVolSpec = hwwolspec(RS.ValuationDate, VolDates, VolCurve, ...  
AlphaDates, AlphaCurve);  
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);  
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);
```

Compute the price of the stepped coupon bonds.

```
PHW= bondbyhw(HWT, CouponRate, Settle,Maturity , Period)
```

```
PHW = 4×1
```

```
100.7246  
100.0945  
101.5900  
102.0820
```

### **Price Two Bonds with Amortization Schedules**

Price two bonds with amortization schedules using the Face input argument to define the schedules.

Define the interest rate term structure.

```
Rates = 0.035;  
ValuationDate = '1-Nov-2011';  
StartDates = ValuationDate;  
EndDates = '1-Nov-2017';  
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

Create the bond instrument. The bonds have a coupon rate of 4% and 3.85%, a period of one year, and mature on 1-Nov-2017.

```
CouponRate = [0.04; 0.0385];
Settle = '1-Nov-2011';
Maturity = '1-Nov-2017';
Period = 1;
```

Define the amortizing schedule.

```
Face = {{ '1-Nov-2015' 100; '1-Nov-2016' 85; '1-Nov-2017' 70};
        { '1-Nov-2015' 100; '1-Nov-2016' 90; '1-Nov-2017' 80}};
```

Build the HW tree and assume the volatility to be 10%.

```
VolDates = [ '1-Nov-2012'; '1-Nov-2013'; '1-Nov-2014'; '1-Nov-2015'; '1-Nov-2016'; '1-Nov-2017'];
VolCurve = 0.1;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;
```

```
HWVolSpec = hwwolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of the amortizing bonds.

```
Price = bondbyhw(HWT, CouponRate, Settle, Maturity, 'Period', Period, ...
    'Face', Face)
```

```
Price = 2×1
```

```
102.4791
101.7786
```

## Input Arguments

### HWTtree — Interest-rate structure

structure

Interest-rate tree structure, created by hwtree.

Data Types: struct

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

**Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The **Settle** date for every bond is set to the **ValuationDate** of the HW tree. The bond argument **Settle** is ignored.

Data Types: char | double

**Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each bond.

Data Types: char | double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1**, **Value1**, ..., **NameN**, **ValueN**.

Example: [Price,PriceTree] =  
bondbyhw(HWTree,CouponRate,Settle,Maturity,'Period',4,'Face',10000)

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector. Values for Period are 1, 2, 3, 4, 6, and 12.

Data Types: double

### **Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**IssueDate — Bond issue date**

serial nonnegative date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial nonnegative date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial nonnegative date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char



**StartDate — Forward starting date of payments**

Settle date (default) | serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or an NINST-by-1 cell array of face values or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

**Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: struct

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

**BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if

`BusinessDayConvention` is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: `double`

## **Output Arguments**

### **Price — Expected bond prices at time 0**

vector

Expected bond prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

## **Bond with an Amortization Schedule**

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## **See Also**

`bkprice` | `bktree` | `cfamounts` | `hwprice` | `hwtree` | `instbond`

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding the Interest-Rate Term Structure” on page 2-55

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# bondbycir

Price bond from Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = bondbycir(CIRTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = bondbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = bondbycir(CIRTree,CouponRate,Settle,Maturity) prices bond from a Cox-Ingersoll-Ross (CIR) interest-rate tree. bondbycir computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = bondbycir( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Bond Using a CIR Interest-Rate Tree

Define the CouponRate for a bond.

```
CouponRate = 0.035;
```

Create a RateSpec using the intenvset function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2021';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [736696 737061 737426 737791]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

Price the bond.

```
[Price,PriceTree] = bondbycir(CIRT,CouponRate,Settle,Maturity)

Price = 94.0880

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    tObs: [0 1 2 3 4]
    dObs: [736696 737061 737426 737791 738157]
    PTree: {1x5 cell}
    AITree: {[0] [0 0 0] [0 0 0 0 0] [0 0 0 0 0 0 0] [0 0 0 0 0 0 0 0]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
```

## Input Arguments

### **CIRTree — Interest-rate structure**

structure

Interest-rate tree structure, created by `cirtree`.

Data Types: `struct`

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an `NINST`-by-1 decimal annual rate or `NINST`-by-1 cell array, where each element is a `NumDates`-by-2 cell array. The first column of the `NumDates`-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle — Settlement date**

serial date number | character vector | string array | datetime

Settlement date, specified either as a scalar or `NINST`-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

The `Settle` date for every bond is set to the `ValuationDate` of the CIR tree. The bond argument `Settle` is ignored.

Data Types: `char` | `double` | `string` | `datetime`

### **Maturity — Maturity date**

serial date number | character vector | string array | datetime

Maturity date, specified as a `NINST`-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays representing the maturity date for each bond.

Data Types: `char` | `double` | `string` | `datetime`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = bondbycir(CIRTree, CouponRate, Settle, Maturity, 'Period', 4, 'Face', 10000)`

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and an NINST-by-1 vector. Values for Period are 1, 2, 3, 4, 6, and 12.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double



### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when `Maturity` is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **IssueDate — Bond issue date**

serial date number | date character vector | string array | datetime

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial date number, date character vector, string array, or datetime array.

Data Types: double | char | string | datetime

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector | string array | datetime

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial date number, date character vector, string array, or datetime array.

When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: double | char | string | datetime

### **LastCouponDate — Irregular last coupon date**

serial nonnegative date number | date character vector | string array | datetime

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial date number, date character vector, string array, or datetime array.

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: `double` | `char` | `string` | `datetime`

### **StartDate — Forward starting date of payments**

`Settle` date (default) | serial date number | date character vector | string array | `datetime`

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of '`StartDate`' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or `datetime` arrays.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: `char` | `double` | `string` | `datetime`

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of '`Face`' and a NINST-by-1 vector of nonnegative face values or a NINST-by-1 cell array of face values or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

Data Types: `cell` | `double`

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

`false` (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of '`AdjustCashFlowsBasis`' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: `logical`

### **BusinessDayConvention — Business day conventions**

`actual` (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

## **Output Arguments**

### **Price — Expected bond prices at time 0**

vector

Expected bond prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.tObs` contains the observation times.
- `PriceTree.dObs` contains the observation dates.
- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.

- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

capbycir | cfbycir | fixedbycir | floatbycir | floorbycir | instbond |  
oasbycir | optbndbycir | optembndbycir | optemfloatbycir | optfloatbycir |  
rangefloatbycir | swapbycir | swaptionbycir

## Topics

"Pricing Using Interest-Rate Tree Models" on page 2-99  
"Calibrating Hull-White Model Using Market Data" on page 2-111  
"Understanding the Interest-Rate Term Structure" on page 2-55  
"Pricing Options Structure" on page B-2  
"Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

## bondbyzero

Price bond from set of zero curves

### Syntax

```
[Price,DirtyPrice,CFlowAmounts,CFlowDates] = bondbyzero(RateSpec,  
CouponRate,Settle,Maturity)  
[Price,DirtyPrice,CFlowAmounts,CFlowDates] = bondbyzero( ____,  
Name,Value)
```

### Description

[Price,DirtyPrice,CFlowAmounts,CFlowDates] = bondbyzero(RateSpec, CouponRate,Settle,Maturity) prices a bond from a set of zero curves. bondbyzero computes prices of vanilla bonds, stepped coupon bonds and amortizing bonds.

[Price,DirtyPrice,CFlowAmounts,CFlowDates] = bondbyzero( \_\_\_\_, Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a Vanilla Bond

Price a 4% bond using a zero curve.

Load `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure, needed to price the bond.

```
load deriv.mat;  
CouponRate = 0.04;  
Settle = '01-Jan-2000';  
Maturity = '01-Jan-2004';  
Price = bondbyzero(ZeroRateSpec, CouponRate, Settle, Maturity)  
  
Price = 97.5334
```

## Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define data for the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x1 double]
    Rates: [4x1 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734139
    ValuationDate: 734139
    Basis: 0
    EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};
Period = 1;
```

Compute the price of the stepped coupon bonds.

```
PZero= bondbyzero(RS, CouponRate, Settle, Maturity ,Period)
```

```
PZero = 4×1  
  
100.7246  
100.0945  
101.5900  
102.0820
```

### Price a Bond with an Amortizing Schedule

Price a bond with an amortizing schedule using the Face input argument to define the schedule.

Define data for the interest-rate term structure.

```
Rates = 0.065;  
ValuationDate = '1-Jan-2011';  
StartDates = ValuationDate;  
EndDates= '1-Jan-2017';  
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: 0.6853  
    Rates: 0.0650  
    EndTimes: 6  
    StartTimes: 0  
    EndDates: 736696  
    StartDates: 734504  
    ValuationDate: 734504  
    Basis: 0  
    EndMonthRule: 1
```

Create and price the amortizing bond instrument. The bond has a coupon rate of 7%, a period of one year, and matures on 1-Jan-2017.



```

CouponRate = 0.07;
Settle = '1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
Face = {'1-Jan-2015' 100; '1-Jan-2016' 90; '1-Jan-2017' 80};
Price = bondbyzero(RateSpec, CouponRate, Settle, Maturity, 'Period', ...
Period, 'Face', Face)

Price = 102.3155

```

Compare the results with price of a vanilla bond.

```

PriceVanilla = bondbyzero(RateSpec, CouponRate, Settle, Maturity, Period)

PriceVanilla = 102.4205

```

Price both the amortizing and vanilla bonds.

```

Face = {'1-Jan-2015' 100; '1-Jan-2016' 90; '1-Jan-2017' 80};
      100};
PriceBonds = bondbyzero(RateSpec, CouponRate, Settle, Maturity, 'Period', ...
      Period, 'Face', Face)

PriceBonds = 2×1

    102.3155
    102.4205

```

### Price a Bond in a Holding Period

When a bond is first issued, it can be priced with `bondbyzero` on that day by setting the `Settle` date to the issue date. Later on, if the bond needs to be traded someday between the issue date and the maturity date, its new price can be computed by updating the `Settle` date, as well as the `RateSpec` input.

Note that the bond's price is determined by its remaining cash flows and the zero-rate term structure, which can both change as the bond matures. While `bondbyzero` automatically updates the bond's remaining cash flows with respect to the new `Settle` date, you must supply a new `RateSpec` input in order to reflect the new zero-rate term structure for that new `Settle` date.

Use the following Bond information.

```
IssueDate = datenum('20-May-2014');
CouponRate = 0.01;
Maturity = datenum('20-May-2019');
```

Determine the bond price on 20-May-2014.

```
Settle1 = datenum('20-May-2014');
ZeroDates1 = datemnth(Settle1,12*[1 2 3 5 7 10 20]');
ZeroRates1 = [0.23 0.63 1.01 1.60 2.01 2.27 2.79]'/100;
RateSpec1 = intenvset('StartDate',Settle1,'EndDates',ZeroDates1,'Rates',ZeroRates1);
[Price1,~,CFlowAmounts1,CFlowDates1] = bondbyzero(RateSpec1,...
    CouponRate,Settle1,Maturity,'IssueDate',IssueDate);
Price1
```

```
Price1 = 97.1899
```

Determine the bond price on 10-Aug-2015.

```
Settle2 = datenum('10-Aug-2015');
ZeroDates2 = datemnth(Settle2,12*[1 2 3 5 7 10 20]');
ZeroRates2 = [0.40 0.73 1.09 1.62 1.98 2.24 2.58]'/100;
RateSpec2 = intenvset('StartDate',Settle2,'EndDates',ZeroDates2,'Rates',ZeroRates2);
[Price2,~,CFlowAmounts2,CFlowDates2] = bondbyzero(RateSpec2,...
    CouponRate,Settle2,Maturity,'IssueDate',IssueDate);
Price2
```

```
Price2 = 98.9384
```

### Price Three Bonds Using Two Different Curves

To price three bonds using two different curves, define the RateSpec:

```
StartDates = '01-April-2016';
EndDates = ['01-April-2017'; '01-April-2018'; '01-April-2019'; '01-April-2020'];
Rates = [[0.0356;0.041185;0.04489;0.047741],[0.0325;0.0423;0.0437;0.0465]];
RateSpec = intenvset('Rates',Rates,'StartDates',StartDates,...
    'EndDates',EndDates,'Compounding',1)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
```

```

        Disc: [4x2 double]
        Rates: [4x2 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: 736421
ValuationDate: 736421
        Basis: 0
        EndMonthRule: 1

```

Price three bonds with the same Maturity and different coupons.

```

Settle = '01-April-2016';
Maturity = '01-April-2020';
Price = bondbyzero(RateSpec,[0.025;0.028;0.035],Settle,Maturity)

```

Price = 3x2

```

92.0766    92.4888
93.1680    93.5823
95.7145    96.1338

```

### Price a Vanilla Bond Using the Optional Input Argument AdjustCashFlowsBasis

To adjust the cash flows according to the accrual amount, use the optional input argument AdjustCashFlowsBasis when calling bondbyzero.

Use the following data to define the interest-rate term structure and to create a RateSpec.

```

Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates= '1-Jan-2017';
Compounding = 1;
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',StartDates,...
'EndDates', EndDates,'Rates',Rates,'Compounding',Compounding);
CouponRate = 0.07;
Settle = '1-Jan-2011';
Maturity = '1-Jan-2017';

```

```

Period = 1;
Face = {'1-Jan-2015' 100; '1-Jan-2016' 90; '1-Jan-2017' 80};

```

Use `cfamounts` and cycle through the `Basis` of 0 to 13, using the optional argument `AdjustCashFlowsBasis` to determine the cash flow amounts for accrued interest due at settlement.

```

AdjustCashFlowsBasis = true;
CFlowAmounts = cfamounts(CouponRate,Settle,Maturity, 'Period',Period, 'Basis',0:13, 'Adj

```

```
CFlowAmounts = 14x7
```

0	7.0000	7.0000	7.0000	7.0000	7.0000	107.0000
0	7.0000	7.0000	7.0000	7.0000	7.0000	107.0000
0	7.0972	7.1167	7.0972	7.0972	7.0972	107.1167
0	7.0000	7.0192	7.0000	7.0000	7.0000	107.0192
0	7.0000	7.0000	7.0000	7.0000	7.0000	107.0000
0	7.0000	7.0000	7.0000	7.0000	7.0000	107.0000
0	7.0000	7.0000	7.0000	7.0000	7.0000	107.0000
0	7.0000	7.0000	7.0000	7.0000	7.0000	107.0000
0	7.0000	7.0000	7.0000	7.0000	7.0000	107.0000
0	7.0972	7.1167	7.0972	7.0972	7.0972	107.1167
:						

Notice that the cash flow amounts have been adjusted according to `Basis`.

Price a vanilla bond using the input argument `AdjustCashFlowsBasis`.

```
PriceVanilla = bondbyzero(RateSpec,CouponRate,Settle,Maturity, 'Period',Period, 'Basis',C
```

```
PriceVanilla = 14x1
```

102.4205
102.4205
102.9216
102.4506
102.4205
102.4205
102.4205
102.4205
102.4205
102.4205
102.9216
:

## Input Arguments

### **RateSpec** — Interest-rate structure

structure

Interest-rate structure, specified using `intenvset` to create a `RateSpec` for an annualized zero rate term structure.

Data Types: `struct`

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an `NINST`-by-1 decimal annual rate or `NINST`-by-1 cell array, where each element is a `NumDates`-by-2 cell array. The first column of the `NumDates`-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or `NINST`-by-1 vector of serial date numbers or date character vectors.

`Settle` must be earlier than `Maturity`.

Data Types: `char` | `double`

### **Maturity** — Maturity date

serial date number | character vector

Maturity date, specified as a `NINST`-by-1 vector of serial date numbers or date character vectors representing the maturity date for each bond.

Data Types: `char` | `double`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = bondbyzero(RateSpec, CouponRate, Settle, Maturity, 'Period', 4, 'Face', 10000)`

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector. Values for Period are 1, 2, 3, 4, 6, and 12.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **IssueDate — Bond issue date**

serial nonnegative date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial nonnegative date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial nonnegative date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial nonnegative date number or date character vector.

Data Types: double | char

### **StartDate — Forward starting date of payments**

Settle date (default) | serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

### **Face — Face value**

100 (default) | scalar of nonnegative value | cell array of nonnegative values

Face value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 scalar of nonnegative face values or an NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is the associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

### **Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with derivset.

Data Types: struct

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-



business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

## **Output Arguments**

### **Price — Fixed-rate note prices**

matrix

Floating-rate note prices, returned as a (NINST) by number of curves (NUMCURVES) matrix. Each column arises from one of the zero curves.

### **DirtyPrice — Dirty bond price**

matrix

Dirty bond price (clean + accrued interest), returned as a NINST- by-NUMCURVES matrix. Each column arises from one of the zero curves.

### **CFlowAmounts — Cash flow amounts**

matrix

Cash flow amounts, returned as a NINST- by-NUMCFS matrix of cash flows for each bond.

### **CFlowDates — Cash flow dates**

matrix

Cash flow dates, returned as a NINST- by-NUMCFS matrix of payment dates for each bond.

## **Definitions**

### **Vanilla Bond**

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### **Stepped Coupon Bond**

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### **Bond with an Amortization Schedule**

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## **See Also**

`cfamounts` | `cfbyzero` | `fixedbyzero` | `floatbyzero` | `swapbyzero`

## **Topics**

“Pricing Using Interest-Rate Term Structure” on page 2-72

“Understanding the Interest-Rate Term Structure” on page 2-55

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bushpath

Extract entries from node of bushy tree

### Syntax

Values = bushpath(Tree,BranchList)

### Arguments

Tree	Bushy tree.
BranchList	Number of paths (NUMPATHS) by path length (PATHLENGTH) matrix containing the sequence of branchings.

### Description

Values = bushpath(Tree,BranchList) extracts entries of a node of a bushy tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number 1, the second-to-top is 2, and so on. Set the branch sequence to zero to obtain the entries at the root node.

Values is a number of values (NUMVALS)-by-NUMPATHS matrix containing the retrieved entries of a bushy tree.

### Examples

Create an HJM tree by loading the example file.

```
load deriv.mat;
```

Then

```
FwdRates = bushpath(HJMTree.FwdTree, [1 2 1])
```

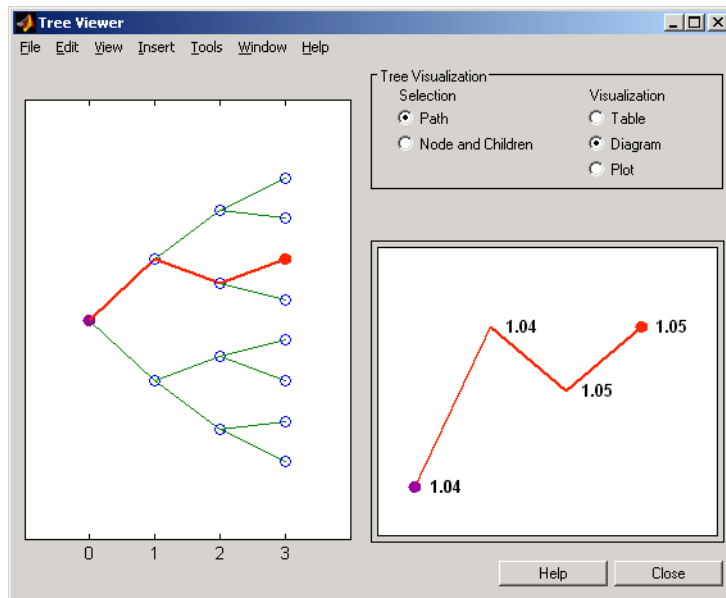
returns the rates at the tree nodes located by taking the up branch, then the down branch, and finally the up branch again.

```
FwdRates =
```

```
1.0356
1.0364
1.0526
1.0463
```

You can visualize this with the `treeviewer` function.

```
treeviewer(HJMTTree)
```



## See Also

`bushshape` | `mkbush`

## Topics

“Graphical Representation of Trees” on page 2-157

**Introduced before R2006a**

## bushshape

Retrieve shape of bushy tree

### Syntax

```
[NumLevels,NumChild,NumPos,NumStates,Trim] = bushshape(Tree)
```

### Arguments

Tree	Bushy tree.
------	-------------

### Description

[NumLevels,NumChild,NumPos,NumStates,Trim] = bushshape(Tree) returns information on a bushy tree's shape.

NumLevels is the number of time levels of the tree.

NumChild is a 1-by-number of levels (NUMLEVELS) vector with the number of branches (children) of the nodes in each level.

NumPos is a 1-by-NUMLEVELS vector containing the length of the state vectors in each level.

NumStates is a 1-by-NUMLEVELS vector containing the number of state vectors in each level.

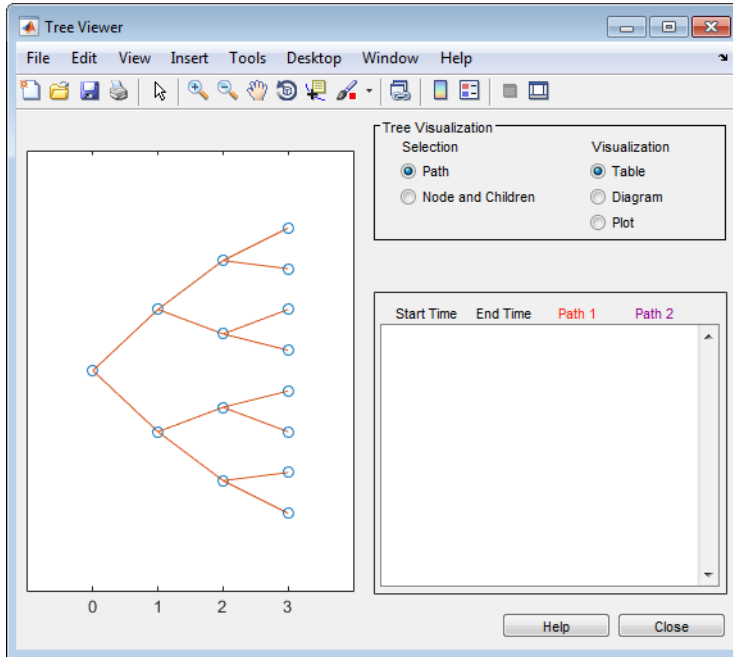
Trim is 1 if NumPos decreases by 1 when moving from one time level to the next. Otherwise, it is 0.

### Examples

Create an HJM tree by loading the example file.

```
load deriv.mat;
```

With `treeviewer` you can see the general shape of the HJM interest-rate tree.



With this tree

```
[NumLevels, NumChild, NumPos, NumStates, Trim] = ...
bushshape(HJMTree.FwdTree)
```

returns

```
NumLevels =
    4

NumChild =
    2    2    2    0

NumPos =
    4    3    2    1

NumStates =
```



```
      1      2      4      8
Trim =
  1
```

You can recreate this tree using the `mkbush` function.

```
Tree = mkbush(NumLevels, NumChild(1), NumPos(1), Trim);
Tree = mkbush(NumLevels, NumChild, NumPos);
```

## See Also

`bushpath` | `mkbush`

## Topics

“Graphical Representation of Trees” on page 2-157

**Introduced before R2006a**

## capbybdt

Price cap instrument from Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = capbybdt(BDTree,Strike,Settle,Maturity)
[Price,PriceTree] = capbybdt( ____,CapReset,Basis,Principal,Options)
```

### Description

[Price,PriceTree] = capbybdt(BDTree,Strike,Settle,Maturity) computes the price of a cap instrument from a Black-Derman-Toy interest-rate tree. capbybdt computes prices of vanilla caps and amortizing caps.

[Price,PriceTree] = capbybdt( \_\_\_\_,CapReset,Basis,Principal,Options) adds optional arguments.

### Examples

#### Price a 3% Cap Instrument Using a BDT Interest-Rate Tree

Load the file `deriv.mat`, which provides `BDTree`. The `BDTree` structure contains the time and interest-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use `capbybdt` to compute the price of the cap instrument.

```
Price = capbybdt(BDTree, Strike, Settle, Maturity)
```

Price = 28.4001

### Price a 10% Cap Instrument Using a BDT Interest-Rate Tree

Set the required arguments for the three specifications required to create a BDT tree.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
```

Create the specifications.

```
RateSpec = intenvset('Compounding', Compounding,...
'ValuationDate', ValuationDate,...
'StartDates', StartDate,...
'EndDates', EndDates,...
'Rates', Rates);
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
```

Create the BDT tree from the specifications.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
```

BDTTree = struct with fields:

```
  FinObj: 'BDTFwdTree'
  VolSpec: [1x1 struct]
  TimeSpec: [1x1 struct]
  RateSpec: [1x1 struct]
  tObs: [0 1 2 3 4]
  dObs: [730486 730852 731217 731582 731947]
  TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
  CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
  FwdTree: {1x5 cell}
```

Set the cap arguments. Remaining arguments will use defaults.

```
CapStrike = 0.10;
Settlement = ValuationDate;
```

```
Maturity = '01-01-2002';  
CapReset = 1;
```

Use `capbybdt` to find the price of the cap instrument.

```
Price = capbybdt(BDTree, CapStrike, Settlement, Maturity, ...  
CapReset)
```

```
Price = 1.7169
```

### Compute the Price of an Amortizing Cap Using the BDT Model

Define the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];  
ValuationDate = '15-Nov-2011';  
StartDates = ValuationDate;  
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014' ; '15-Nov-2015'; '15-Nov-2016'};  
Compounding = 1;  
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: [5x1 double]  
    Rates: [5x1 double]  
    EndTimes: [5x1 double]  
    StartTimes: [5x1 double]  
    EndDates: [5x1 double]  
    StartDates: 734822  
    ValuationDate: 734822  
    Basis: 0  
    EndMonthRule: 1
```

Define the cap instrument.

```
Settle = '15-Nov-2011';  
Maturity = '15-Nov-2015';  
Strike = 0.04;  
CapReset = 1;  
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};
```

Build the BDT Tree.

```
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility*ones(1,length(EndDates)))
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)

BDTTree = struct with fields:
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [734822 735188 735553 735918 736283]
    TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
    CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
    FwdTree: {1x5 cell}
```

Price the amortizing cap.

```
Basis = 0;
Price = capbybdt(BDTTree, Strike, Settle, Maturity, CapReset, Basis, Principal)

Price = 1.4042
```

## Input Arguments

### **BDTTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdttree`.

Data Types: `struct`

### **Strike** — Rate at which cap is exercised

decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

**Settle — Settlement date for cap**

serial date number | date character vector | cell array of date character vectors

Settlement date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The **Settle** date for every cap is set to the **ValuationDate** of the BDT tree. The cap argument **Settle** is ignored.

Data Types: double | char | cell

**Maturity — Maturity date for cap**

serial date number | date character vector | cell array of date character vectors

Maturity date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**CapReset — Reset frequency payment per year**

1 (default) | numeric

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)

- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal — Notional principal amount**

100 (default) | `numeric`

(Optional) Notional principal amount, specified as a `NINST`-by-1 of notional principal amounts, or a `NINST`-by-1 cell array, where each element is a `NumDates`-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: `double` | `cell`

### **Options — Derivatives pricing options structure**

`structure`

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected price of cap at time 0**

`vector`

Expected price of the cap at time 0, returned as a `NINST`-by-1 vector.

### **PriceTree — Tree structure with values of cap at each node**

`vector`

Tree structure with values of the cap at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains cap prices.
- `PriceTree.t0bs` contains the observation times.

### See Also

`bdttree` | `capbynormal` | `cfbybdt` | `floorbybdt` | `swapbybdt`

### Topics

“Computing Instrument Prices” on page 2-99

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# capbybk

Price cap instrument from Black-Karasinski interest-rate tree

## Syntax

```
[Price,PriceTree] = capbybk(BKTree,Strike,Settle,Maturity)
[Price,PriceTree] = capbybk( ____,CapReset,Basis,Principal,Options)
```

## Description

[Price,PriceTree] = capbybk(BKTree,Strike,Settle,Maturity) computes the price of a cap instrument from a Black-Karasinski interest-rate tree. capbybk computes prices of vanilla caps and amortizing caps.

[Price,PriceTree] = capbybk( \_\_\_\_,CapReset,Basis,Principal,Options) adds optional arguments.

## Examples

### Price a 3% Cap Instrument Using a Black-Karasinski Interest-Rate Tree

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2004';
Maturity = '01-Jan-2007';
```

Use `capbybk` to compute the price of the cap instrument.

```
Price = capbybk(BKTree, Strike, Settle, Maturity)
```

```
Price = 2.0965
```

### Compute the Price of an Amortizing and Vanilla Caps Using the BK Model

Load `deriv.mat` to specify the `BKTree` and then define the cap instrument.

```
load deriv.mat;
Settle = '01-Jan-2004';
Maturity = '01-Jan-2008';
Strike = 0.05;
CapReset = 1;
Principal ={'01-Jan-2005' 100;'01-Jan-2006' 60;'01-Jan-2007' 30;'01-Jan-2008' 30};...
          100};
```

Price the amortizing and vanilla caps.

```
Basis = 1;
Price = capbybk(BKTree, Strike, Settle, Maturity, CapReset, Basis, Principal)
```

```
Price = 2×1
```

```
    0.2226
    0.7422
```

## Input Arguments

### **BKTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **Strike** — Rate at which cap is exercised

decimal

Rate at which cap is exercised, specified as a `NINST-by-1` vector of decimal values.

Data Types: `double`

**Settle — Settlement date for cap**

serial date number | date character vector | cell array of date character vectors

Settlement date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The **Settle** date for every cap is set to the **ValuationDate** of the BK tree. The cap argument **Settle** is ignored.

Data Types: double | char | cell

**Maturity — Maturity date for cap**

serial date number | date character vector | cell array of date character vectors

Maturity date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**CapReset — Reset frequency payment per year**

1 (default) | numeric

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)

- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal — Notional principal amount**

100 (default) | `numeric`

(Optional) Notional principal amount, specified as a `NINST`-by-1 of notional principal amounts, or a `NINST`-by-1 cell array, where each element is a `NumDates`-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: `double` | `cell`

### **Options — Derivatives pricing options structure**

`structure`

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected price of cap at time 0**

`vector`

Expected price of the cap at time 0, returned as a `NINST`-by-1 vector.

### **PriceTree — Tree structure with values of cap at each node**

`vector`

Tree structure with values of the cap at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains cap prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## See Also

`bktree` | `capbynormal` | `cfbybk` | `floorbybk` | `swapbybk`

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## capbyblk

Price caps using Black option pricing model

### Syntax

```
[CapPrice,Caplets] = capbyblk(RateSpec,Strike,Settle,Maturity,Volatility)
[CapPrice,Caplets] = capbyblk( ____,Name,Value)
```

### Description

[CapPrice,Caplets] = capbyblk(RateSpec,Strike,Settle,Maturity,Volatility) price caps using the Black option pricing model. capbyblk computes prices of vanilla caps and amortizing caps.

[CapPrice,Caplets] = capbyblk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a Cap Using the Black Option Pricing Model

Consider an investor who gets into a contract that caps the interest rate on a \$100,000 loan at 8% quarterly compounded for 3 months, starting on January 1, 2009. Assuming that on January 1, 2008 the zero rate is 6.9394% continuously compounded and the volatility is 20%, use this data to compute the cap price. First, calculate the RateSpec:

```
ValuationDate = 'Jan-01-2008';
EndDates = 'April-01-2010';
Rates = 0.069394;
Compounding = -1;
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, ...
```

```
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
```

```
    FinObj: 'RateSpec'
    Compounding: -1
        Disc: 0.8554
        Rates: 0.0694
        EndTimes: 2.2500
    StartTimes: 0
        EndDates: 734229
    StartDates: 733408
    ValuationDate: 733408
        Basis: 1
    EndMonthRule: 1
```

Compute the price of the cap.

```
Settle = 'Jan-01-2009'; % cap starts in a year
```

```
Maturity = 'April-01-2009';
```

```
Volatility = 0.20;
```

```
CapRate = 0.08;
```

```
CapReset = 4;
```

```
Principal=100000;
```

```
CapPrice = capbyblk(RateSpec, CapRate, Settle, Maturity, Volatility, ...
'Reset', CapReset, 'ValuationDate', ValuationDate, 'Principal', Principal, ...
'Basis', Basis)
```

```
CapPrice = 51.6125
```

## Price a Cap Using a Different Curve to Generate the Future Forward Rates

Define the OIS and Libor rates.

```
Settle = datenum('15-Mar-2013');
```

```
CurveDates = daysadd(Settle, 360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10], 1);
```

```
OISRates = [.0018 .0019 .0021 .0023 .0031 .006 .011 .017 .021 .026 .03]';
```

```
LiborRates = [.0045 .0047 .005 .0055 .0075 .0109 .0162 .0216 .0262 .0309 .0348]';
```

Create an associated RateSpec for the OIS and Libor curves.

```
OISCurve = intenvset('Rates',OISRates,'StartDate',Settle,'EndDates',CurveDates,'Compou
LiborCurve = intenvset('Rates',LiborRates,'StartDate',Settle,'EndDates',CurveDates,'Co
```

Define the Cap instruments.

```
Maturity = {'15-Mar-2018';'15-Mar-2020'};
Strike = [0.04;0.05];
BlackVol = 0.2;
```

Price the cap instruments using the term structure OISCurve both for discounting the cash flows and generating future forward rates.

```
[Price, Caplets] = capbyblk(OISCurve, Strike, Settle, Maturity, BlackVol)
```

```
Price = 2×1
```

```
0.7472
0.9890
```

```
Caplets = 2×7
```

```
0 0.0000 0.0033 0.2996 0.4443 NaN NaN
0 0.0000 0.0003 0.1134 0.2112 0.2292 0.4349
```

Price the cap instruments using the term structure LiborCurve to generate future forward rates. The term structure OISCurve is used for discounting the cash flows.

```
[PriceLC, CapletsLC] = capbyblk(OISCurve, Strike, Settle, Maturity, BlackVol,'Projecti
```

```
PriceLC = 2×1
```

```
1.3293
1.6329
```

```
CapletsLC = 2×7
```

```
0 0.0000 0.0337 0.4250 0.8706 NaN NaN
0 0.0000 0.0052 0.1767 0.4849 0.3663 0.5998
```



## Compute the Price of Two Amortizing Caps Using the Black Model

Define the RateSpec.

```
Rates = [0.0358; 0.0421; 0.0473; 0.0527; 0.0543];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014' ; '15-Nov-2015'; '15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
    ValuationDate: 734822
    Basis: 0
    EndMonthRule: 1
```

Define the cap instruments.

```
Settle = '15-Nov-2011';
Maturity = '15-Nov-2015';
Strike = [0.03; 0.035];
Reset = 1;
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};
```

Price the amortizing caps.

```
Volatility = 0.10;
Price = capbyblk (RateSpec, Strike, Settle, Maturity, Volatility, ...
    'Reset', Reset, 'Principal', Principal)
```

```
Price = 2x1

    3.0339
    2.0141
```

### Price a Cap Using the Shifted Black Model

Create the RateSpec.

```
ValuationDate = 'Mar-01-2016';  
EndDates = {'Mar-01-2017'; 'Mar-01-2018'; 'Mar-01-2019'; 'Mar-01-2020'; 'Mar-01-2021'};  
Rates = [-0.21; -0.12; 0.01; 0.10; 0.20]/100;  
Compounding = 1;  
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',ValuationDate, ...  
'EndDates',EndDates,'Rates',Rates,'Compounding',Compounding,'Basis',Basis)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: [5x1 double]  
    Rates: [5x1 double]  
    EndTimes: [5x1 double]  
    StartTimes: [5x1 double]  
    EndDates: [5x1 double]  
    StartDates: 736390  
    ValuationDate: 736390  
    Basis: 1  
    EndMonthRule: 1
```

Price the cap with a negative strike using the Shifted Black model.

```
Settle = 'Jun-01-2016'; % Cap starts in 3 months.  
Maturity = 'Sep-01-2016';  
ShiftedBlackVolatility = 0.31;  
CapRate = -0.003; % -0.3 percent strike.  
CapReset = 4;  
Principal = 100000;  
Shift = 0.01; % 1 percent shift.
```

```
CapPrice = capbyblk(RateSpec,CapRate,Settle,Maturity,ShiftedBlackVolatility,...  
'Reset',CapReset,'ValuationDate',ValuationDate,'Principal',Principal,...  
'Basis',Basis,'Shift',Shift)
```

```
CapPrice = 26.0733
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **Strike — Rate at which cap is exercised**

decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

### **Settle — Settlement date for cap**

serial date number | date character vector

Settlement date for the cap, specified as a serial date number or a date character vector.

Data Types: `double` | `char`

### **Maturity — Maturity date for cap**

serial date number | date character vector | cell array of date character vectors

Maturity date for the cap, specified as a serial date number or date character vector.

Data Types: `double` | `char`

### **Volatility — Volatilities values**

numeric

Volatilities values, specified as a NINST-by-1 vector of numeric values.

The `Volatility` input is not intended for volatility surfaces or cubes. If you specify a matrix for the `Volatility` input, `capbyblk` internally converts it into a vector. `capbyblk` assumes that the volatilities specified in the `Volatility` input are flat volatilities, which are applied equally to each of the caplets.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, Volatility, 'Reset', CapReset, 'Principal', 100000, 'Basis', 7)`

### **Reset — Reset frequency payment per year**

1 (default) | numeric

Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as a NINST-by-1 vector or a NINST-by-1 cell array. When `Principal` is a NINST-by-1 cell array, each element is a NumDates-by-2 cell array, where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: double | cell

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360

- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **ProjectionCurve — Rate curve used in generating future forward rates**

if `ProjectionCurve` is not specified, then `RateSpec` is used both for discounting cash flows and projecting future forward rates (default) | structure

The rate curve to be used in generating the future forward rates. This structure must be created using `intenvset`. Use this optional input if the forward curve is different from the discount curve.

Data Types: `struct`

### **Shift — Shift in decimals for shifted Black model**

0 (no shift) (default) | positive decimal

Shift in decimals for the shifted Black model, specified using a scalar or `NINST-by-1` vector of rate shifts in positive decimals. Set this parameter to a positive rate shift in decimals to add a positive shift to the forward rate and strike, which effectively sets a negative lower bound for the forward rate. For example, a `Shift` of `0.01` is equal to a 1% shift.

Data Types: `double`

## Output Arguments

### CapPrice — Expected price of cap

vector

Expected price of the cap, returned as a NINST-by-1 vector.

### Caplets — Caplets

array

Caplets, returned as a NINST-by-NCF array of caplets, padded with NaNs.

## Definitions

### Shifted Black

The Shifted Black model is essentially the same as the Black's model, except that it models the movements of  $(F + Shift)$  as the underlying asset, instead of  $F$  (which is the forward rate in the case of caplets).

This model allows negative rates, with a fixed negative lower bound defined by the amount of shift; that is, the zero lower bound of Black's model has been shifted.

## Algorithms

### Black Model

$$dF = \sigma_{Black} F dw$$

$$call = e^{-\nu T} [FN(d_1) - KN(d_2)]$$

$$put = e^{-\nu T} [KN(-d_2) - FN(-d_1)]$$

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \left(\frac{\sigma_B^2}{2}\right)T}{\sigma_B \sqrt{T}}, \quad d_2 = d_1 - \sigma_B \sqrt{T}$$

$$\sigma_B = \sigma_{Black}$$

Where  $F$  is the forward value and  $K$  is the strike.

## Shifted Black Model

$$dF = \sigma_{\text{Shifted\_Black}}(F + \text{Shift})dw$$

$$\text{call} = e^{-\nu T}[(F + \text{Shift})N(d_{s1}) - (K + \text{Shift})N(d_{s2})]$$

$$\text{put} = e^{-\nu T}[(K + \text{Shift})N(-d_{s2}) - (F + \text{Shift})N(-d_{s1})]$$

$$d_{s1} = \frac{\ln\left(\frac{F + \text{Shift}}{K + \text{Shift}}\right) + \left(\frac{\sigma_{sB}^2}{2}\right)T}{\sigma_{sB}\sqrt{T}}, \quad d_{s2} = d_{s1} - \sigma_{sB}\sqrt{T}$$

$$\sigma_{sB} = \sigma_{\text{Shifted\_Black}}$$

Where  $F+\text{Shift}$  is the forward value and  $K+\text{Shift}$  is the strike for the shifted version.

## See Also

capbynornal | floorbyblk | intenvset

## Topics

“Work with Negative Interest Rates” on page 2-22

“Supported Interest-Rate Instruments” on page 2-2

## External Websites

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

## Introduced in R2009a

## capbycir

Price cap instrument from Cox-Ingersoll-Ross interest-rate tree

### Syntax

```
[Price,PriceTree] = capbycir(CIRTree,Strike,Settle,Maturity)
[Price,PriceTree] = capbycir( ____,Name,Value)
```

### Description

[Price,PriceTree] = capbycir(CIRTree,Strike,Settle,Maturity) computes the price of a cap instrument from a Cox-Ingersoll-Ross (CIR) interest-rate tree. capbycir computes prices of vanilla caps and amortizing caps using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = capbycir( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a Cap Using a CIR Interest-Rate Tree

Define the Strike for a cap.

```
Strike = 0.03;
```

Create a RateSpec using the intenvset function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```



Create a CIR tree.

```

NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2021';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolSpec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [736696 737061 737426 737791]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

Price the 3% cap.

```

[Price,PriceTree] = capbycir(CIRT,Strike,Settle,Maturity)

Price = 7.9081

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    tObs: [0 1 2 3 4]
    PTree: {1x5 cell}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

## Input Arguments

**CIRTree** — Interest-rate tree structure  
structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

**Strike — Rate at which cap is exercised**

decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

**Settle — Settlement date for cap**

serial date number | date character vector | cell array of date character vectors | string array | `datetime`

Settlement date for the cap, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or `datetime` arrays. The `Settle` date for every cap is set to the `ValuationDate` of the CIR tree. The cap argument `Settle` is ignored.

Data Types: `double` | `char` | `cell` | `string` | `datetime`

**Maturity — Maturity date for cap**

serial date number | date character vector | cell array of date character vectors | string array | `datetime`

Maturity date for the cap, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or `datetime` arrays.

Data Types: `double` | `char` | `cell` | `string` | `datetime`

**Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = capbycir(CIRTree, CouponRate, Settle, Maturity, 'Basis', 3)`

**CapReset — Reset frequency payment per year**

1 (default) | numeric

Reset frequency payment per year, specified as the comma-separated pair consisting of 'CapReset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 of notional principal amounts or a NINST-by-1 cell array.

For the `NINST-by-1` cell array, each element is a `NumDates-by-2` cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: `double` | `cell`

## Output Arguments

### **Price** — Expected price of cap at time 0

vector

Expected price of the cap at time 0, returned as a `NINST-by-1` vector.

### **PriceTree** — Tree structure with values of cap at each node

vector

Tree structure with values of the cap at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains cap prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

- [3] Hirsa, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | fixedbycir | floatbycir | floorbycir | instcap |  
oasbycir | optbndbycir | optembndbycir | optemfloatbycir | optfloatbycir |  
rangefloatbycir | swapbycir | swaptionbycir

## Topics

"Pricing Using Interest-Rate Tree Models" on page 2-99  
"Understanding Interest-Rate Tree Models" on page 2-79  
"Pricing Options Structure" on page B-2  
"Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

## capbyhjm

Price cap instrument from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = capbyhjm(HJMTree,Strike,Settle,Maturity)
[Price,PriceTree] = capbyhjm( ____,CapReset,Basis,Principal,Options)
```

### Description

[Price,PriceTree] = capbyhjm(HJMTree,Strike,Settle,Maturity) computes the price of a cap instrument from a Heath-Jarrow-Morton interest-rate tree. capbyhjm computes prices of vanilla caps and amortizing caps.

[Price,PriceTree] = capbyhjm( \_\_\_\_,CapReset,Basis,Principal,Options) adds optional arguments.

### Examples

#### Price a 3% Cap Instrument Using an HJM Forward-Rate Tree

Load the file `deriv.mat`, which provides HJMTree. The HJMTree structure contains the time and forward-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use capbyhjm to compute the price of the cap instrument.

```
Price = capbyhjm(HJMTree, Strike, Settle, Maturity)
```

```
Price = 6.2831
```

### Compute the Price of an Amortizing Cap Using the HJM Model

Load `deriv.mat` to specify the HJMTree and then define the cap instrument.

```
load deriv.mat;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
Strike = 0.045;
CapReset = 1;
Principal = {'01-Jan-2001' 100; '01-Jan-2002' 80; '01-Jan-2003' 70; '01-Jan-2004' 30};
```

Price the amortizing cap.

```
Basis = 1;
Price = capbyhjm(HJMTree, Strike, Settle, Maturity, CapReset, Basis, Principal)
```

```
Price = 1.4588
```

## Input Arguments

### HJMTree — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hjm tree`.

Data Types: `struct`

### Strike — Rate at which cap is exercised

decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

### Settle — Settlement date for cap

serial date number | date character vector | cell array of date character vectors

Settlement date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The `Settle` date for every cap is set to the `ValuationDate` of the HJM tree. The cap argument `Settle` is ignored.

Data Types: `double` | `char` | `cell`

**Maturity — Maturity date for cap**

`serial date number` | `date character vector` | `cell array of date character vectors`

Maturity date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**CapReset — Reset frequency payment per year**

`1` (default) | `numeric`

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: `double`

**Basis — Day-count basis of instrument**

`0` (actual/actual) (default) | `integer from 0 to 13`

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)



- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal — Notional principal amount**

100 (default) | `numeric`

(Optional) Notional principal amount, specified as a NINST-by-1 of notional principal amounts, or a NINST-by-1 cell array, where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: `double` | `cell`

### **Options — Derivatives pricing options structure**

`structure`

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected price of cap at time 0**

`vector`

Expected price of the cap at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure with values of cap at each node**

`vector`

Tree structure with values of the cap at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.t0bs` contains the observation times.

- `PriceTree.PBush` contains the clean prices.

### See Also

`capbynormal` | `cfbyhjm` | `floorbyhjm` | `hjmtree` | `swapbyhjm`

### Topics

“Computing Instrument Prices” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## capbyhw

Price cap instrument from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = capbyhw(HWTree,Strike,Settle,Maturity)
[Price,PriceTree] = capbyhw( ____,CapReset,Basis,Principal,Options)
```

### Description

`[Price,PriceTree] = capbyhw(HWTree,Strike,Settle,Maturity)` computes the price of a cap instrument from a Hull-White interest-rate tree. `capbyhw` computes prices of vanilla caps and amortizing caps.

`[Price,PriceTree] = capbyhw( ____,CapReset,Basis,Principal,Options)` adds optional arguments.

### Examples

#### Price a 3% Cap Instrument Using a Hull-White Interest-Rate Tree

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2004';
Maturity = '01-Jan-2007';
```

Use `capbyhw` to compute the price of the cap instrument.

```
Price = capbyhw(HWTree, Strike, Settle, Maturity)
```

```
Price = 2.3090
```

### Compute the Price of an Amortizing and Vanilla Caps Using the HW Model

Define the RateSpec.

```
Rates = [0.035; 0.042; 0.047; 0.052; 0.054];
ValuationDate = '01-April-2014';
StartDates = ValuationDate;
EndDates = {'01-April-2019'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: 737516
    StartDates: 735690
    ValuationDate: 735690
    Basis: 0
    EndMonthRule: 1
```

Define the cap instruments.

```
Settle = '01-April-2014';
Maturity = '01-April-2018';
Strike = 0.055;
CapReset = 1;
Principal = {'01-April-2015' 100; '01-April-2016' 60; '01-April-2017' 40; '01-April-2018'
    100};
```

Build the HW Tree.

```
VolDates = ['01-April-2015'; '01-April-2016'; '01-April-2017'; '01-April-2018'];
VolCurve = 0.05;
AlphaDates = '01-April-2018';
```

```

AlphaCurve = 0.10;

HWVolSpec = hwwolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
                    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWTTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)

HWTTree = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [735690 736055 736421 736786]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0350] [1.1300 1.0363 0.9503] [1x5 double] [1x7 double]}

```

Price the amortizing and vanilla caps.

```

Basis = 0;
Price = capbyhw(HWTTree, Strike, Settle, Maturity, CapReset, Basis, Principal)

Price = 2x1

    1.6754
    4.6149

```

## Input Arguments

### **HWTTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: `struct`

### **Strike** — Rate at which cap is exercised

decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: double

**Settle — Settlement date for cap**

serial date number | date character vector | cell array of date character vectors

Settlement date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The **Settle** date for every cap is set to the **ValuationDate** of the HW tree. The cap argument **Settle** is ignored.

Data Types: double | char | cell

**Maturity — Maturity date for cap**

serial date number | date character vector | cell array of date character vectors

Maturity date for the cap, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**CapReset — Reset frequency payment per year**

1 (default) | numeric

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

(Optional) Notional principal amount, specified as a NINST-by-1 of notional principal amounts, or a NINST-by-1 cell array, where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: double | cell

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected price of cap at time 0**

vector

Expected price of the cap at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure with values of cap at each node**

vector

Tree structure with values of the cap at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains cap prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## See Also

`capbynormal` | `cfbyhw` | `floorbyhw` | `hwtree` | `swapbyhw`

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



## capbylg2f

Price cap using Linear Gaussian two-factor model

### Syntax

```
CapPrice = capbylg2f(ZeroCurve,a,b,sigma,eta,rho,Strike,Maturity)
CapPrice = capbylg2f(____,Name,Value)
```

### Description

CapPrice = capbylg2f(ZeroCurve,a,b,sigma,eta,rho,Strike,Maturity) returns cap price for a two-factor additive Gaussian interest-rate model.

CapPrice = capbylg2f(\_\_\_\_,Name,Value) adds optional name-value pair arguments.

---

**Note** Use the optional name-value pair argument, `Notional`, to pass a schedule to compute the price for an amortizing cap.

---

## Examples

### Price a Cap Using a Linear Gaussian Two-Factor Model

Define the ZeroCurve, a, b, sigma, eta, and rho parameters to price the cap.

```
Settle = datenum('15-Dec-2007');

ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
CurveDates = daysadd(Settle,360*ZeroTimes);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
```

```
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;

CapMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);

Strike = [0.035 0.037 0.038 0.039 0.040 0.042 0.044 0.046 0.047 0.047 0.047]';

Price = capbylg2f(irdc,a,b,sigma,eta,rho,Strike,CapMaturity)

Price = 11x1

    0.0316
    0.3225
    0.7761
    1.3240
    1.9394
    3.1247
    4.8451
    7.3752
    9.8582
   11.4673
    :
```

### **Price an Amortizing Cap Using a Linear Gaussian Two-Factor Model**

Define the ZeroCurve, a, b, sigma, eta, rho, and Notional parameters for the amortizing cap.

```
Settle = datenum('15-Dec-2007');
% Define ZeroCurve
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
CurveDates = daysadd(Settle,360*ZeroTimes);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

% Define a, b, sigma, eta, and rho
a = .07;
```

```

b = .5;
sigma = .01;
eta = .006;
rho = -.7;

% Define the amortizing caps
CapMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);
Strike = [0.035 0.037 0.038 0.039 0.040 0.042 0.044 0.046 0.047 0.047 0.047]';
Notional = {'15-Dec-2010' 100;'15-Dec-2014' 70;'15-Dec-2022' 40;'15-Dec-2037' 10}};

% Price the amortizing caps
Price = capbylg2f(irdc,a,b,sigma,eta,rho,Strike,CapMaturity, 'Notional', Notional)

Price = 11x1

    0.0316
    0.3225
    0.7761
    1.1313
    1.5362
    2.3213
    2.8297
    3.6878
    3.7297
    3.8906
    :

```

## Input Arguments

### **ZeroCurve** — Zero-curve for Linear Gaussian two-factor model

structure

Zero-curve for the Linear Gaussian two-factor model, specified using `IRDataCurve` or `RateSpec`.

Data Types: `struct`

### **a** — Mean reversion for first factor for Linear Gaussian two-factor model

scalar numeric

Mean reversion for first factor for the Linear Gaussian two-factor model, specified as a scalar numeric.

Data Types: `single` | `double`

**b — Mean reversion for second factor for Linear Gaussian two-factor model**

scalar numeric

Mean reversion for second factor for the Linear Gaussian two-factor model, specified as a scalar numeric.

Data Types: `single` | `double`

**sigma — Volatility for first factor for Linear Gaussian two-factor model**

scalar numeric

Volatility for first factor for the Linear Gaussian two-factor model, specified as a scalar numeric.

Data Types: `single` | `double`

**eta — Volatility for second factor for Linear Gaussian two-factor model**

scalar numeric

Volatility for second factor for the Linear Gaussian two-factor model, specified as a scalar numeric.

Data Types: `single` | `double`

**rho — Scalar correlation of the factors**

scalar numeric

Scalar correlation of the factors, specified as a scalar numeric.

Data Types: `single` | `double`

**Strike — Cap strike price**

nonnegative integer | vector of nonnegative integers

Cap strike price, specified as a nonnegative integer using a NumCaps-by-1 vector.

Data Types: `single` | `double`

**Maturity — Cap maturity date**

serial date number | vector of serial date numbers | date character vector

Cap maturity date, specified using a NumCaps-by-1 vector of serial date numbers or date character vectors.

Data Types: single | double | char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price = capbylg2f(irdc,a,b,sigma,eta,rho,Strike,CapMaturity,'Reset',1,'Notional',100)`

### Reset — Frequency of cap payments per year

2 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1,2,3,4,6,12]

Frequency of cap payments per year, specified as the comma-separated pair consisting of 'Reset' and a positive integers for the values [1,2,4,6,12] in a NumCaps-by-1 vector.

Data Types: single | double

### Notional — Notional value of cap

100 (default) | nonnegative integer | vector of nonnegative integers

Notional value of cap, specified as the comma-separated pair consisting of 'Notional' and a NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is the associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: single | double

## Output Arguments

### CapPrice — Cap price

scalar | vector

Expected prices of cap, returned as a scalar or an NumCaps-by-1 vector.

## Algorithms

The following defines the two-factor additive Gaussian interest rate model, given the ZeroCurve, a, b, sigma, eta, and rho parameters:

$$r(t) = x(t) + y(t) + \phi(t)$$

$$dx(t) = -a(x)(t)dt + \sigma(dW_1(t), x(0) = 0$$

$$dy(t) = -b(y)(t)dt + \eta(dW_2(t), y(0) = 0$$

where  $dW_1(t)dW_2(t) = \rho dt$  is a two-dimensional Brownian motion with correlation  $\rho$  and  $\phi$  is a function chosen to match the initial zero curve.

## References

- [1] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

## See Also

LinearGaussian2F | floorbylg2f | swaptionbylg2f

## Topics

- “Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123  
“Pricing Bermudan Swaptions with Monte Carlo Simulation”

**Introduced in R2013a**

# capbynormal

Price caps using Normal or Bachelier pricing model

## Syntax

```
[CapPrice,Caplets] = capbynormal(RateSpec,Strike,Settle,Maturity,
Volatility)
[CapPrice,Caplets] = capbynormal( ____,Name,Value)
```

## Description

[CapPrice,Caplets] = capbynormal(RateSpec,Strike,Settle,Maturity, Volatility) prices caps using the Normal (Bachelier) pricing model for negative rates. capbynormal computes prices of vanilla caps and amortizing caps.

[CapPrice,Caplets] = capbynormal( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a Cap Using Normal Model for Negative Rates

Consider an investor who gets into a contract that caps the interest rate on a \$100,000 loan at -.08% quarterly compounded for 3 months, starting on January 1, 2009. Assuming that on January 1, 2008 the zero rate is .069394% continuously compounded and the volatility is 20%, use this data to compute the cap price. First, calculate the RateSpec, and then use capbynormal to compute the CapPrice.

```
ValuationDate = 'Jan-01-2008';
EndDates = 'April-01-2010';
Rates = 0.0069394;
Compounding = -1;
Basis = 1;
```

```

RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

Settle = 'Jan-01-2009'; % cap starts in a year
Maturity = 'April-01-2009';
Volatility = 0.20;
CapRate = -0.008;
CapReset = 4;
Principal=100000;

CapPrice = capbynormal(RateSpec, CapRate, Settle, Maturity, Volatility,...
'Reset',CapReset,'ValuationDate',ValuationDate,'Principal', Principal,...
'Basis', Basis)

CapPrice = 2.1682e+03

```

### Price a Cap Using capbynormal and Compare to capbyblk

Define the RateSpec.

```

Settle = datenum('20-Jan-2016');
ZeroTimes = [.5 1 2 3 4 5 7 10 20 30]';
ZeroRates = [0.0052 0.0055 0.0061 0.0073 0.0094 0.0119 0.0168 0.0222 0.0293 0.0307]';
ZeroDates = datemnth(Settle,12*ZeroTimes);
RateSpec = intenvset('StartDate',Settle,'EndDates',ZeroDates,'Rates',ZeroRates)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
        Disc: [10x1 double]
        Rates: [10x1 double]
    EndTimes: [10x1 double]
    StartTimes: [10x1 double]
    EndDates: [10x1 double]
    StartDates: 736349
    ValuationDate: 736349
        Basis: 0
    EndMonthRule: 1

```

Define the cap instrument and price with capbyblk.



```

ExerciseDate = datenum('20-Jan-2026');

[~,ParSwapRate] = swapyzero(RateSpec,[NaN 0],Settle,ExerciseDate)

ParSwapRate = 0.0216

Strike = .01;
BlackVol = .3;
NormalVol = BlackVol*ParSwapRate;

Price = capbyblk(RateSpec,Strike,Settle,ExerciseDate,BlackVol)

Price = 11.8693

```

Price the cap instrument using `capbnormal`.

```

Price_Normal = capbnormal(RateSpec,Strike,Settle,ExerciseDate,NormalVol)

Price_Normal = 12.5495

```

Price the cap instrument using `capbnormal` for a negative strike.

```

Price_Normal = capbnormal(RateSpec,-.005,Settle,ExerciseDate,NormalVol)

Price_Normal = 24.4816

```

## Input Arguments

### **RateSpec — Interest-rate term structure** structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **Strike — Rate at which cap is exercised** decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: double

**Settle — Settlement date for cap**

serial date number | date character vector | datetime object | string object

Settlement date for the cap, specified as a NINST-by-1 vector of serial date numbers, date character vectors, datetime objects, or string objects.

Data Types: double | char | datetime | string

**Maturity — Maturity date for cap**

serial date number | date character vector | datetime object | string object

Maturity date for the cap, specified as a NINST-by-1 vector of serial date numbers, date character vectors, datetime objects, or string objects.

Data Types: double | char | datetime | string

**Volatility — Normal volatilities values**

numeric

Normal volatilities values, specified as a NINST-by-1 vector of numeric values.

For more information on the Normal model, see “Work with Negative Interest Rates” on page 2-22.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[CapPrice, Caplets] = capbynormal(RateSpec, Strike, Settle, Maturity, Volatility, 'Reset', CapReset, 'Principal', 100000, 'Basis', 7)`

**Reset — Reset frequency payment per year**

1 (default) | numeric

Reset frequency payment per year, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector.

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 of notional principal amounts, or a NINST-by-1 cell array. Each element in the NINST-by-1 cell array is a NumDates-by-2 cell array, where the first column is dates, and the second column is the associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: double | cell

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of instrument representing the basis used when annualizing the input forward rate, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers. Values are:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**ValuationDate — Observation date of investment horizon**

if `ValuationDate` is not specified, then `Settle` is used (default) | serial date number | date character vector | datetime object | string object

Observation date of the investment horizon, specified as the comma-separated pair consisting of 'ValuationDate' and a serial date number, date character vector, datetime object, or string array.

Data Types: double | char | datetime | string

**ProjectionCurve — Rate curve used in generating future cash flows**

if `ProjectionCurve` is not specified, then `RateSpec` is used both for discounting cash flows and projecting future cash flows (default) | structure

The rate curve to be used in projecting the future cash flows, specified as the comma-separated pair consisting of 'ProjectionCurve' and rate curve structure. This structure must be created using `intenvset`. Use this optional input if the forward curve is different from the discount curve.

Data Types: struct

## Output Arguments

**CapPrice — Expected price of cap**

vector

Expected price of the cap, returned as a NINST-by-1 vector.

**Caplets — Caplets**

array

Caplets, returned as a NINST-by-NCF array of caplets, padded with NaNs.

## See Also

`capbyblk` | `floorbynormal` | `intenvset` | `swaptionbynormal`

## Topics

“Calibrating Caplets Using the Normal (Bachelier) Model”

“Work with Negative Interest Rates” on page 2-22  
“Supported Interest-Rate Instruments” on page 2-2

## **External Websites**

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

**Introduced in R2017a**

## capvolstrip

Strip caplet volatilities from flat cap volatilities

### Syntax

```
[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip(ZeroCurve,  
CapSettle, CapMaturity, CapVolatility)  
[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip( ____,  
Name, Value)
```

### Description

[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip(ZeroCurve, CapSettle, CapMaturity, CapVolatility) strips caplet volatilities from the flat cap volatilities by using the bootstrapping method. The function interpolates the cap volatilities on each caplet payment date before stripping the caplet volatilities.

[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip( \_\_\_\_, Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Stripping Caplet Volatilities from At-The-Money (ATM) Caps

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('23-Jun-2015');  
ZeroRates = [0.01 0.09 0.30 0.70 1.07 1.71]/100;  
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);  
ZeroCurve = IRDataCurve('Zero', ValuationDate, CurveDates, ZeroRates)
```

```
ZeroCurve =  
Type: Zero
```

```

        Settle: 736138 (23-Jun-2015)
    Compounding: 2
        Basis: 0 (actual/actual)
    InterpMethod: linear
        Dates: [6x1 double]
        Data: [6x1 double]

```

Define the ATM cap volatility data.

```

CapSettle = datenum('25-Jun-2015');
CapMaturity = datenum({'27-Jun-2016'; '26-Jun-2017'; '25-Jun-2018'; ...
    '25-Jun-2019'; '25-Jun-2020'});
CapVolatility = [0.29;0.38;0.42;0.40;0.38];

```

Strip caplet volatilities from ATM caps.

```

[CapletVols, CapletPaymentDates, ATMCapStrikes] = capvolstrip(ZeroCurve, ...
    CapSettle, CapMaturity, CapVolatility);

```

```

PaymentDates = cellstr(datestr(CapletPaymentDates));
format;
table(PaymentDates, CapletVols, ATMCapStrikes)

```

```

ans=9x3 table
    PaymentDates    CapletVols    ATMCapStrikes
    _____    _____    _____
    '27-Jun-2016'    0.29         0.0052014
    '27-Dec-2016'    0.34657     0.0071594
    '26-Jun-2017'    0.41404     0.0091175
    '26-Dec-2017'    0.42114     0.010914
    '25-Jun-2018'    0.45297     0.012698
    '26-Dec-2018'    0.37257     0.014222
    '25-Jun-2019'    0.36184     0.015731
    '26-Dec-2019'    0.3498      0.017262
    '25-Jun-2020'    0.33668     0.018774

```

### Stripping Caplet Volatilities from Caps with the Same Strikes

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('17-Feb-2015');
ZeroRates = [0.02 0.07 0.25 0.70 1.10 1.62]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero',ValuationDate,CurveDates,ZeroRates)
```

```
ZeroCurve =
    Type: Zero
    Settle: 736012 (17-Feb-2015)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [6x1 double]
    Data: [6x1 double]
```

Define the cap volatility data.

```
CapSettle = datenum('19-Feb-2015');
CapMaturity = datenum({'19-Feb-2016';'21-Feb-2017';'20-Feb-2018'; ...
    '19-Feb-2019';'19-Feb-2020'});
CapVolatility = [0.44;0.45;0.44;0.41;0.39];
CapStrike = 0.013;
```

Strip caplet volatilities from caps with the same strike.

```
[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip(ZeroCurve, ...
    CapSettle, CapMaturity, CapVolatility, 'Strike', CapStrike);
```

```
PaymentDates = cellstr(datestr(CapletPaymentDates));
format;
table(PaymentDates, CapletVols, CapStrikes)
```

ans=9x3 table

PaymentDates	CapletVols	CapStrikes
'19-Feb-2016'	0.44	0.013
'19-Aug-2016'	0.44495	0.013
'21-Feb-2017'	0.45256	0.013
'21-Aug-2017'	0.43835	0.013
'20-Feb-2018'	0.42887	0.013
'20-Aug-2018'	0.38157	0.013
'19-Feb-2019'	0.35237	0.013
'19-Aug-2019'	0.3525	0.013
'19-Feb-2020'	0.33136	0.013



## Stripping Caplet Volatilities Using Manually Specified Caplet Dates

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('06-Mar-2015');
ZeroRates = [0.01 0.08 0.27 0.73 1.16 1.70]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero',ValuationDate,CurveDates,ZeroRates)
```

```
ZeroCurve =
    Type: Zero
    Settle: 736029 (06-Mar-2015)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [6x1 double]
    Data: [6x1 double]
```

Define the cap volatility data.

```
CapSettle = datenum('06-Mar-2015');
CapMaturity = datenum({'07-Mar-2016';'06-Mar-2017';'06-Mar-2018'; ...
    '06-Mar-2019';'06-Mar-2020'});
CapVolatility = [0.43;0.44;0.44;0.43;0.41];
CapStrike = 0.011;
```

Specify quarterly and semiannual dates.

```
CapletDates = [cfdates(CapSettle, '06-Mar-2016', 4) ...
    cfdates('06-Mar-2016', '06-Mar-2020', 2)];
CapletDates(~isbusday(CapletDates)) = ...
    busdate(CapletDates(~isbusday(CapletDates)), 'modifiedfollow');
```

Strip caplet volatilities using specified CapletDates.

```
[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip(ZeroCurve, ...
    CapSettle, CapMaturity, CapVolatility, 'Strike', CapStrike, ...
    'CapletDates', CapletDates);

PaymentDates = cellstr(datestr(CapletPaymentDates));
format;
table(PaymentDates, CapletVols, CapStrikes)
```

```
ans=11x3 table
  PaymentDates    CapletVols    CapStrikes
  _____    _____    _____
  '08-Sep-2015'    0.43         0.011
  '07-Dec-2015'    0.42999     0.011
  '07-Mar-2016'    0.43         0.011
  '06-Sep-2016'    0.43538     0.011
  '06-Mar-2017'    0.44396     0.011
  '06-Sep-2017'    0.43999     0.011
  '06-Mar-2018'    0.44001     0.011
  '06-Sep-2018'    0.41934     0.011
  '06-Mar-2019'    0.40985     0.011
  '06-Sep-2019'    0.36818     0.011
  '06-Mar-2020'    0.34657     0.011
```

### Stripping Caplet Volatilities from Caps Using the Shifted Black Model

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('1-Mar-2016');
ZeroRates = [-0.38 -0.25 -0.21 -0.12 0.01 0.2]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero',ValuationDate,CurveDates,ZeroRates)
```

```
ZeroCurve =
    Type: Zero
    Settle: 736390 (01-Mar-2016)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [6x1 double]
    Data: [6x1 double]
```

Define the cap volatility (Shifted Black) data.

```
CapSettle = datenum('1-Mar-2016');
CapMaturity = datenum({'1-Mar-2017';'1-Mar-2018';'1-Mar-2019'; ...
    '2-Mar-2020';'1-Mar-2021'});
CapVolatility = [0.35;0.40;0.37;0.34;0.32]; % Shifted Black volatilities
```

```
Shift = 0.01; % 1 percent shift.
CapStrike = -0.001; % -0.1 percent strike.
```

Strip caplet volatilities from caps using the Shifted Black Model.

```
[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip(ZeroCurve, ...
CapSettle,CapMaturity,CapVolatility,'Strike',CapStrike,'Shift',Shift);
```

```
PaymentDates = string(datestr(CapletPaymentDates));
format;
table(PaymentDates,CapletVols,CapStrikes)
```

```
ans=9x3 table
```

PaymentDates	CapletVols	CapStrikes
"01-Mar-2017"	0.35	-0.001
"01-Sep-2017"	0.39129	-0.001
"01-Mar-2018"	0.4335	-0.001
"04-Sep-2018"	0.35284	-0.001
"01-Mar-2019"	0.3255	-0.001
"03-Sep-2019"	0.3011	-0.001
"02-Mar-2020"	0.27266	-0.001
"01-Sep-2020"	0.27698	-0.001
"01-Mar-2021"	0.25697	-0.001

## Stripping Caplet Volatilities from Caps Using Normal Model

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('1-Jun-2018');
ZeroRates = [-0.38 -0.25 -0.21 -0.12 0.01 0.2]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero',ValuationDate,CurveDates,ZeroRates)
```

```
ZeroCurve =
    Type: Zero
    Settle: 737212 (01-Jun-2018)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
```

```
Dates: [6x1 double]
Data: [6x1 double]
```

Define the normal cap volatility data.

```
CapSettle = datenum('1-Jun-2018');
CapMaturity = datenum({'3-Jun-2019'; '1-Jun-2020'; '1-Jun-2021'; ...
    '1-Jun-2022'; '1-Jun-2023'});
CapVolatility = [0.0057; 0.0059; 0.0057; 0.0053; 0.0051]; % Normal volatilities
CapStrike = -0.002; % -0.2 percent strike.
```

Strip caplet volatilities from caps using the Normal (Bachelier) model.

```
[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip(ZeroCurve, ...
    CapSettle, CapMaturity, CapVolatility, 'Strike', CapStrike, 'Model', 'normal');

PaymentDates = string(datestr(CapletPaymentDates));
format;
table(PaymentDates, CapletVols, CapStrikes)
```

ans=9x3 table

PaymentDates	CapletVols	CapStrikes
"03-Jun-2019"	0.0057	-0.002
"02-Dec-2019"	0.0058686	-0.002
"01-Jun-2020"	0.0060472	-0.002
"01-Dec-2020"	0.0055705	-0.002
"01-Jun-2021"	0.0053912	-0.002
"01-Dec-2021"	0.0047404	-0.002
"01-Jun-2022"	0.004357	-0.002
"01-Dec-2022"	0.0046481	-0.002
"01-Jun-2023"	0.0044477	-0.002

## Input Arguments

### ZeroCurve — Zero rate curve

RateSpec object | IRDataCurve object

Zero rate curve, specified using a RateSpec or IRDataCurve object containing the zero rate curve for discounting according to its day count convention. If you do not specify the

optional argument `ProjectionCurve`, the function uses `ZeroCurve` to compute the underlying forward rates as well. The observation date of the `ZeroCurve` specifies the valuation date. For more information on creating a `RateSpec`, see `intenvset`. For more information on creating an `IRDataCurve` object, see `IRDataCurve`.

Data Types: `struct`

### **CapSettle — Common cap settle date**

serial date number | date character vector

Common cap settle date, specified as a scalar serial date number or date character vector. The `CapSettle` date cannot be earlier than the `ZeroCurve` valuation date.

Data Types: `double` | `char`

### **CapMaturity — Cap maturity dates**

serial date numbers | date character vectors

Cap maturity dates, specified using serial date numbers or cell array of date character vectors as a `NCap-by-1` vector.

Data Types: `double` | `char` | `cell`

### **CapVolatility — Flat cap volatilities**

vector of positive decimals

Flat cap volatilities, specified as an `NCap-by-1` vector of positive decimals.

Data Types: `double`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[CapletVols, CapletPaymentDates, CapStrikes] = capvolstrip(ZeroCurve, CapSettle, CapMaturity, CapVolatility, 'Strike', . . . 2)`

### **Strike — Cap strike rate**

If not specified, all caps are at-the-money and the function computes the ATM strike for each cap maturing on each caplet payment date (default) | scalar decimal | vector

Cap strike rate, specified as the comma-separated pair consisting of 'Strike' and a scalar decimal value or an NCapletVols-by-1 vector. Use Strike as a scalar to specify a single strike that applies equally to all caps. Or, specify an NCapletVols-by-1 vector of strikes for the caps.

Data Types: double

**CapletDates — Caplet reset and payment dates**

if not specified, the default is to automatically generate periodic caplet dates (default) | serial date numbers | date character vectors

Caplet reset and payment dates, specified as the comma-separated pair consisting of 'CapletDates' and an NCapletDates-by-1 vector using serial date numbers or a cell array of date character vectors.

Use CapletDates to manually specify all caplet reset and payment dates. For example, some date intervals may be quarterly, while others may be semiannual. All dates must be later than CapSettle and cannot be later than the last CapMaturity date. Dates are adjusted according to the BusDayConvention and Holidays inputs.

If CapletDates is not specified, the default is to automatically generate periodic caplet dates after CapSettle based on the last CapMaturity date as the reference date, using the following optional inputs: Reset, EndMonthRule, BusDayConvention, and Holidays.

Data Types: double | char | cell

**Reset — Frequency of periodic payments per year within a cap**

2 (default) | positive scalar integer with values 1,2, 3, 4, 6, or 12

Frequency of periodic payments per year within a cap, specified as the comma-separated pair consisting of 'Reset' and a positive scalar integer with values 1,2, 3, 4, 6, or 12.

---

**Note** If you specify CapletDates, the function ignores the input for Reset.

---

Data Types: double

**EndMonthRule — End-of-month rule flag for generating caplet dates**

1 (in effect) (default) | scalar nonnegative integer [0, 1]

End-of-month rule flag for generating caplet dates, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar nonnegative integer [0, 1].

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **BusDayConvention — Business day conventions**

'modifiedfollow' (default) | character vector with values 'actual', 'follow', 'modifiedfollow', 'previous', 'modifiedprevious'

Business day conventions, specified as the comma-separated pair consisting of 'BusDayConvention' and a character vector. Use this argument to specify how the function treats non-business days, which are days on which businesses are not open (such as weekends and statutory holidays).

- 'actual' — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- 'follow' — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- 'modifiedfollow' — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However, if the following business day is in a different month, the previous business day is adopted instead.
- 'previous' — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- 'modifiedprevious' — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However, if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | vector of MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and NHolidays-by-1 vector of MATLAB date numbers.

Data Types: double

**ProjectionCurve — Rate curve for computing underlying forward rates**

if not specified, the default is to use the `ZeroCurve` input for computing the underlying forward rates (default) | `RateSpec` object | `IRDataCurve` object

Rate curve for computing underlying forward rates, specified as the comma-separated pair consisting of 'ProjectionCurve' and a `RateSpec` object or `IRDataCurve` object. For more information on creating a `RateSpec`, see `intenvset`. For more information on creating an `IRDataCurve` object, see `IRDataCurve`.

Data Types: `struct`

**MaturityInterpMethod — Method for interpolating the cap volatilities on each caplet maturity date before stripping the caplet volatilities**

'linear' (default) | character vector with values: 'linear', 'nearest', 'next', 'previous', 'spline', 'pchip'

Method for interpolating the cap volatilities on each caplet maturity date before stripping the caplet volatilities, specified as the comma-separated pair consisting of 'MaturityInterpMethod' and a character vector with values: 'linear', 'nearest', 'next', 'previous', 'spline', or 'pchip'.

- 'linear' — Linear interpolation. The interpolated value at a query point is based on linear interpolation of the values at neighboring grid points in each respective dimension. This is the default interpolation method.
- 'nearest' — Nearest neighbor interpolation. The interpolated value at a query point is the value at the nearest sample grid point.
- 'next' — Next neighbor interpolation. The interpolated value at a query point is the value at the next sample grid point.
- 'previous' — Previous neighbor interpolation. The interpolated value at a query point is the value at the previous sample grid point.
- 'spline' — Spline interpolation using not-a-knot end conditions. The interpolated value at a query point is based on a cubic interpolation of the values at neighboring grid points in each respective dimension.
- 'pchip' — Shape-preserving piecewise cubic interpolation. The interpolated value at a query point is based on a shape-preserving piecewise cubic interpolation of the values at neighboring grid points.

For more information on interpolation methods, see `interp1`.



---

**Note** The function uses constant extrapolation to calculate volatilities falling outside the range of user-supplied data.

---

Data Types: char

**Limit — Upper bound of implied volatility search interval**

10 (or 1000% per annum) (default) | positive scalar decimal

Upper bound of implied volatility search interval, specified as the comma-separated pair consisting of 'Limit' and a positive scalar decimal.

Data Types: double

**Tolerance — Implied volatility search termination tolerance**

1e-5 (default) | positive numeric scalar

Implied volatility search termination tolerance, specified as the comma-separated pair consisting of 'Tolerance' and a positive numeric scalar.

Data Types: double

**OmitFirstCaplet — Flag to omit the first caplet payment in the caps**

true (always omit the first caplet) (default) | logical

Flag to omit the first caplet payment in the caps, specified as the comma-separated pair consisting of 'OmitFirstCaplet' and a scalar logical.

If the caps are spot-starting, the first caplet payment is omitted. If the caps are forward-starting, the first caplet payment is included. Regardless of the status of the caps, if you set this logical to false, then the function includes the first caplet payment.

In general, “spot lag” is the delay between the fixing date and the effective date for LIBOR-like indices. “Spot lag” determines whether a cap is spot-starting or forward-starting (Corb, 2012). Caps are considered to be spot-starting if they settle within “spot lag” business days after the valuation date. Those that settle later are considered to be forward-starting. The first caplet is omitted if caps are spot-starting, while it is included if they are forward-starting (Tuckman, 2012).

Data Types: logical

**Shift — Shift in decimals for shifted SABR model**

0 (no shift) (default) | positive scalar decimal

Shift in decimals for the shifted SABR model (to be used with the Shifted Black model), specified as the comma-separated pair consisting of 'Shift' and a positive scalar decimal value. Set this parameter to a positive shift in decimals to add a positive shift to the forward rate and strike, which effectively sets a negative lower bound for the forward rate and strike. For example, a Shift value of 0.01 is equal to a 1% shift.

Data Types: double

### Model — Model used for implied volatility

'lognormal' (default) | character vector with value of 'lognormal' or 'normal' | string scalar with value of "lognormal" or "normal"

Model used for the implied volatility calculation, specified as the comma-separated pair consisting of 'Model' and a scalar character vector or string scalar with one of the following values:

- 'lognormal' - Implied Black (no shift) or Shifted Black volatility.
- 'normal' - Implied Normal (Bachelier) volatility. If you specify 'normal', Shift must be zero.

The capvolstrip function supports three volatility types.

'Model' Value	'Shift' Value	Volatility Type
'lognormal'	Shift = 0	Black
'lognormal'	Shift > 0	Shifted Black
'normal'	Shift = 0	Normal (Bachelier)

Data Types: char | string

## Output Arguments

### CapletVols — Stripped caplet volatilities

vector of decimals

Stripped caplet volatilities, returned as an NCapletVols-by-1 vector of decimals.

---

**Note** capvolstrip can output NaNs for some caplet volatilities. You might encounter this output if no volatility matches the caplet price implied by the user-supplied cap data.

---

**CapletPaymentDates — Payment dates**

vector of date numbers

Payment dates (in date numbers), returned as an `NCapletVols`-by-1 vector of date numbers corresponding to `CapletVols`.

**CapStrikes — Cap strikes**

vector of decimals

Cap strikes, returned as an `NCapletVols`-by-1 vector of strikes in decimals for caps maturing on the corresponding `CapletPaymentDates`. `CapStrikes` are the same as the strikes of the corresponding caplets that have been stripped.

## Limitations

When bootstrapping the caplet volatilities from ATM caps, the function reuses the caplet volatilities stripped from the shorter maturity caps in the longer maturity caps without adjusting for the difference in strike. `capvolstrip` follows the simplified approach described in Gatarek, 2006.

## Definitions

### At-The-Money

A cap or floor is at-the-money (ATM) if its strike is equal to the forward swap rate.

The forward swap rate is the fixed rate of a swap that makes the present value of the floating leg equal to that of the fixed leg. In comparison, a caplet or floorlet is ATM if its strike is equal to the forward rate (not the forward swap rate). In general (except over a single period), the forward rate is not equal to the forward swap rate. So, to be precise, the individual caplets in an ATM cap have slightly different moneyness and are only approximately ATM (Alexander, 2003).

In addition, the swap rate changes with swap maturity. Similarly, the ATM cap strike also changes with cap maturity, so the ATM cap strikes are computed for each cap maturity before stripping the caplet volatilities. As a result, when stripping the caplet volatilities from the ATM caps with increasing maturities, the ATM strikes of consecutive caps are different.

## References

- [1] Alexander, C. "Common Correlation and Calibrating the Lognormal Forward Rate Model." *Wilmott Magazine*, 2003.
- [2] Corb, H. *Interest Rate Swaps and Other Derivatives*. Columbia Business School Publishing, 2012.
- [3] Gatarek, D., P. Bachert, and R. Maksymiuk. *The LIBOR Market Model in Practice*. Chichester, UK: Wiley, 2006.
- [4] Tuckman, B., and Serrat, A. *Fixed Income Securities: Tools for Today's Markets*. Hoboken, NJ: Wiley, 2012.

## See Also

capbyblk | capbynormal | floorvolstrip | intenvset | interp1

## Topics

"Price Swaptions with Negative Strikes Using the Shifted SABR Model" on page 2-27  
"Work with Negative Interest Rates" on page 2-22

## External Websites

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

## Introduced in R2016a

## cashbyls

Determine price of cash-or-nothing digital options using Black-Scholes model

### Syntax

```
Price = cashbyls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike,
Payoff)
```

### Description

Price = cashbyls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff) computes the price for cash-or-nothing European digital options using the Black-Scholes option pricing model.

### Examples

#### Compute Cash-or-Nothing Option Prices Using the Black-Scholes Option Pricing Model

Consider a European call and put cash-or-nothing options on a futures contract with and exercise strike price of \$90, a fixed payoff of \$10 that expires on October 1, 2008. Assume that on January 1, 2008, the contract trades at \$110, and has a volatility of 25% per annum and the risk-free rate is 4.5% per annum. Using this data, calculate the price of the call and put cash-or-nothing options on the futures contract. First, create the RateSpec:

```
Settle = 'Jan-1-2008';
Maturity = 'Oct-1-2008';
Rates = 0.045;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9668
    Rates: 0.0450
    EndTimes: 0.7500
    StartTimes: 0
    EndDates: 733682
    StartDates: 733408
    ValuationDate: 733408
    Basis: 1
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 110;
Sigma = .25;
DivType = 'Continuous';
DivAmount = Rates;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmount)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2500
    AssetPrice: 110
    DividendType: {'continuous'}
    DividendAmounts: 0.0450
    ExDividendDates: []
```

Define the call and put options.

```
OptSpec = {'call'; 'put'};
Strike = 90;
Payoff = 10;
```

Calculate the prices.

```
Pcon = cashbyb1s(RateSpec, StockSpec, Settle,...
    Maturity, OptSpec, Strike, Payoff)
```

```
Pcon = 2×1
```

```
7.6716
```

---

1.9965

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as an NINST-by-1 vector.

Data Types: char | cell

### **Strike — Strike price value**

vector

Strike price value, specified as an NINST-by-1 vector.

Data Types: double

### **Payoff — Payoff values**

vector

Payoff values (or the amount to be paid at expiration), specified as an NINST-by-1 vector.

Data Types: double

## **Output Arguments**

### **Price — Expected prices for cash-or-nothing option**

vector

Expected prices for cash-or-nothing option, returned as a NINST-by-1 vector.

## **See Also**

assetbybls | cashsensbybls | gapbybls | supersharebybls

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**



# cashsensbyls

Determine price or sensitivities of cash-or-nothing digital options using Black-Scholes model

## Syntax

```
PriceSens = cashsensbyls(RateSpec,StockSpec,Settle,Maturity,OptSpec,  
Strike,Payoff)  
PriceSens = cashsensbyls( ____,Name,Value)
```

## Description

`PriceSens = cashsensbyls(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,Payoff)` computes the price or sensitivities for cash-or-nothing European digital options using the Black-Scholes option pricing model.

`PriceSens = cashsensbyls( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute Cash-or-Nothing Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model

Consider a European call and put cash-or-nothing options on a futures contract with an exercise price of \$90, and a fixed payoff of \$10 that expires on October 1, 2008. Assume that on January 1, 2008 the contract trades at \$110, and has a volatility of 25% per annum and the risk-free rate is 4.5% per annum. Using this data, calculate the price and sensitivity of the call and put cash-or-nothing options on the futures contract. First, create the `RateSpec`:

```
Settle = 'Jan-1-2008';  
Maturity = 'Oct-1-2008';  
Rates = 0.045;
```

```
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9668
    Rates: 0.0450
    EndTimes: 0.7500
    StartTimes: 0
    EndDates: 733682
    StartDates: 733408
    ValuationDate: 733408
    Basis: 1
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 110;
Sigma = .25;
DivType = 'Continuous';
DivAmount = Rates;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmount)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2500
    AssetPrice: 110
    DividendType: {'continuous'}
    DividendAmounts: 0.0450
    ExDividendDates: []
```

Define the call and put options.

```
OptSpec = {'call'; 'put'};
Strike = 90;
Payoff = 10;
```

Compute the gamma, theta, and price.

```
OutSpec = { 'gamma'; 'theta'; 'price'};
[Gamma, Theta, Price] = cashsensbyls(RateSpec, StockSpec, ...
Settle, Maturity, OptSpec, Strike, Payoff, 'OutSpec', OutSpec)
```

```
Gamma = 2×1
```

```
-0.0050
 0.0050
```

```
Theta = 2×1
```

```
-2.2489
 1.8139
```

```
Price = 2×1
```

```
7.6716
 1.9965
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### StockSpec — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as an NINST-by-1 vector.

Data Types: `char` | `cell`

**Strike — Strike price value**

vector

Strike price value, specified as an NINST-by-1 vector.

Data Types: `double`

**Payoff — Payoff values**

vector

Payoff values (or the amount to be paid at expiration), specified as an NINST-by-1 vector.

Data Types: `double`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Gamma, Theta, Price] = cashsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff, 'OutSpec', {'gamma'; 'theta'; 'price'})`

### **OutSpec — Define outputs**

`{'Price'}` (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

`OutSpec = {'All'}` specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: `char` | `cell`

## **Output Arguments**

**PriceSens — Expected prices or sensitivities for cash-or-nothing option**  
vector

Expected prices or sensitivities (defined using `OutSpec`) for cash-or-nothing option, returned as a NINST-by-1 vector.

## **See Also**

`cashbybls`

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**

## cbondbycrr

Price convertible bonds from CRR binomial tree

### Syntax

```
Price = cbondbycrr(CRRTree,CouponRate,Settle,Maturity,ConvRatio)
[Price,PriceTree] = cbondbycrr(CRRTree,CouponRate,Settle,Maturity,
ConvRatio)
[Price,PriceTree,EquityTree,DebtTree] = cbondbycrr( ____,Name,Value)
```

### Description

`Price = cbondbycrr(CRRTree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds from a CRR binomial tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree] = cbondbycrr(CRRTree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds from a CRR binomial tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree,EquityTree,DebtTree] = cbondbycrr( ____,Name,Value)` prices convertible bonds from a CRR binomial tree using a credit spread or incorporating the risk of bond default.

To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair input argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair input arguments `DefaultProbability` and `RecoveryRate`.

## Examples

### Price Convertible Bond Using a CRR Tree

Price a convertible bond using the following data for the interest-rate term structure:

```
StartDates = 'Jan-1-2014';
EndDates = 'Jan-1-2015';
Rates = 0.1;
Basis = 1;
```

Create the RateSpec and StockSpec.

```
Sigma = 0.3;
Price = 50;
```

```
RateSpec = intenvset('ValuationDate',StartDates,'StartDates',StartDates,'EndDates',EndDates,
'Rates',Rates,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9048
    Rates: 0.1000
    EndTimes: 1
    StartTimes: 0
    EndDates: 735965
    StartDates: 735600
    ValuationDate: 735600
    Basis: 1
    EndMonthRule: 1
```

```
StockSpec = stockspec(Sigma,Price)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
    AssetPrice: 50
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Create the CRR tree for the equity.

```
Settle = '1-Jan-2014';
Maturity = '1-Oct-2014';
NumSteps = 3;
TimeSpec = crrtimespec(Settle,Maturity,NumSteps);
CRRT = crrtree(StockSpec,RateSpec,TimeSpec)
```



```

CRRT = struct with fields:
    FinObj: 'BinStockTree'
    Method: 'CRR'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 0.2491 0.4982 0.7473]
        dObs: [735600 735691 735782 735873]
    STree: {1x4 cell}
    UpProbs: [0.5465 0.5465 0.5465]

```

Define and price the convertible bond.

```

CouponRate = 0;
Period = 1;
ConvRatio = 2;
CallExDates = '1-Oct-2014';
CallStrike = 115;
AmericanCall = 1;
Spread = 0.05;

```

```

[Price,PriceTree,EqtTree,DbtTree] = cbondbycrr(CRRT,CouponRate,Settle,Maturity,ConvRatio,
'Period',Period,'Spread',Spread,'CallExDates',CallExDates,'CallStrike',CallStrike,'Amer

```

```

Price = 104.9490

```

```

PriceTree = struct with fields:
    FinObj: 'BinPriceTree'
    PTree: {1x4 cell}
        tObs: [0 0.2491 0.4982 0.7473]
        dObs: [735600 735691 735782 735873]

```

```

EqTree = struct with fields:
    FinObj: 'BinPriceTree'
    PTree: {1x4 cell}
        tObs: [0 0.2491 0.4982 0.7473]
        dObs: [735600 735691 735782 735873]

```

```

DbtTree = struct with fields:
    FinObj: 'BinPriceTree'
    PTree: {[28.4278] [0 65.0790] [0 43.6821 96.3327] [1x4 double]}
        tObs: [0 0.2491 0.4982 0.7473]

```

```
dObs: [735600 735691 735782 735873]
```

### **Price a Convertible Bond Using a CRR Tree and Incorporate Default Risk Using DefaultProbability and RecoveryRate**

Create the interest-rate term structure RateSpec.

```
StartDates = 'Jan-1-2014';  
EndDates = 'Jan-1-2016';  
Rates = 0.025;  
Basis = 1;  
RateSpec = intenvset('ValuationDate',StartDates,'StartDates',...  
StartDates,'EndDates',EndDates,'Rates',Rates,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9512  
    Rates: 0.0250  
    EndTimes: 2  
    StartTimes: 0  
    EndDates: 736330  
    StartDates: 735600  
    ValuationDate: 735600  
    Basis: 1  
    EndMonthRule: 1
```

Create the StockSpec.

```
AssetPrice = 110;  
Sigma = 0.22;  
Div = 0.02;  
StockSpec = stockspecc(Sigma,AssetPrice,'continuous',Div)
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
    Sigma: 0.2200  
    AssetPrice: 110  
    DividendType: {'continuous'}  
    DividendAmounts: 0.0200
```

```
ExDividendDates: []
```

Create the CRR tree for the equity.

```
Settle = '1-Jan-2014';
Maturity = '1-Oct-2014';
NumSteps = 3;
TimeSpec = crrtimespec(Settle,Maturity,NumSteps);
CRRT = crrtree(StockSpec,RateSpec,TimeSpec)
```

```
CRRT = struct with fields:
    FinObj: 'BinStockTree'
    Method: 'CRR'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 0.2491 0.4982 0.7473]
        dObs: [735600 735691 735782 735873]
    STree: {1x4 cell}
    UpProbs: [0.4782 0.4782 0.4782]
```

Define and price the convertible bond using the optional `DefaultProbability` and `RecoveryRate` arguments.

```
CouponRate = 0;
Period = 1;
ConvRatio = 2;
CallExDates = '1-Oct-2014';
CallStrike = 115;
AmericanCall = 1;
DefaultProbability = .30;
RecoveryRate = .82;
```

```
[Price,PriceTree,EqTree,DbtTree] = cbondbycrr(CRRT,CouponRate,Settle,Maturity,ConvRatio,
'Period',Period,'CallExDates',CallExDates,'CallStrike',CallStrike,'AmericanCall',AmericanCall,
'DefaultProbability',DefaultProbability,'RecoveryRate',RecoveryRate)
```

```
Price = 220
```

```
PriceTree = struct with fields:
    FinObj: 'BinPriceTree'
    PTree: {1x4 cell}
    tObs: [0 0.2491 0.4982 0.7473]
```

```
dObs: [735600 735691 735782 735873]
```

```
EqTree = struct with fields:  
  FinObj: 'BinPriceTree'  
  PTree: {1x4 cell}  
  tObs: [0 0.2491 0.4982 0.7473]  
  dObs: [735600 735691 735782 735873]
```

```
DbtTree = struct with fields:  
  FinObj: 'BinPriceTree'  
  PTree: {[0] [0 0] [0 0 0] [0 0 0 0]}  
  tObs: [0 0.2491 0.4982 0.7473]  
  dObs: [735600 735691 735782 735873]
```

## Input Arguments

### **CRRTree** — Stock tree structure

structure

Stock tree structure, specified by using `crرتree`.

Data Types: `struct`

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date

serial date number | date character vector

Settlement date, specified as an NINST-by-1 scalar using a serial date number or date character vector.

---

**Note** The `Settle` date for every convertible bond is set to the `ValuationDate` of the CRR stock tree. The bond argument, `Settle`, is ignored.

---

Data Types: double | char

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 scalar using a serial date number or date character vector.

Data Types: double | char

### **ConvRatio — Number of shares convertible to one bond**

nonnegative number

Number of shares convertible to one bond, specified as an NINST-by-1 with a nonnegative number.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: [Price, PriceTree, EquityTree, DebtTree] =
cbondbycrr(CRRT, CouponRate, Settle, Maturity,
ConvRatio, 'Spread', Spread, 'CallExDates', CallExDates, 'CallStrike', CallStrike, 'AmericanCall', 1)
```

### **Spread — Number of basis points over the reference rate**

0 (default) | vector

Number of basis points over the reference rate, specified as the comma-separated pair consisting of 'Spread' and a NINST-by-1 vector.

---

**Note** To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. To incorporate default risk into the algorithm,

specify the optional input arguments `DefaultProbability` and `RecoveryRate`. Do not use `Spread` with `DefaultProbability` and `RecoveryRate`.

---

Data Types: double

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: char | double

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or a NINST-by-1 cell array, where each element is

a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is the associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

### **CallStrike — Call strike price for European, Bermuda, or American option**

nonnegative integer | vector of nonnegative integers

Call strike price for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallStrike' and one of the following values:

- For a European call option — NINST-by-1 vector of nonnegative integers
- For a Bermuda call option — NINST-by-NSTRIKES matrix of call strike price values, where each row is the schedule for one call option. If a call option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American call option — NINST-by-1 vector of strike price values for each option.

Data Types: single | double

### **CallExDates — Call exercise date for European, Bermuda, or American option**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Call exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates, where each row is the schedule for one option. For a European option, there is only one CallExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the call option can be exercised on any tree date between or including the pair of dates on that row. If CallExDates is NINST-by-1, the option can be exercised between the ValuationDate of the CRR stock tree and the single listed CallExDate.

Data Types: char | cell | double

**AmericanCall — Call option type indicator**

0 if AmericanCall is NaN or not entered (default) | scalar | vector of positive integers [0, 1]

Call option type, specified as the comma-separated pair consisting of 'AmericanCall' and a NINST-by-1 vector of positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanCall is 0 for each European or Bermuda option.
- For an American option — AmericanCall is 1 for each American option. The AmericanCall argument is required to invoke American exercise rules.

Data Types: single | double

**PutStrike — Put strike values for European, Bermuda, or American option**

positive integer | vector of positive integers

Put strike values for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutStrike' and one of the following values:

- For a European put option — NINST-by-1 vector of nonnegative integers
- For a Bermuda put option — NINST-by-NSTRIKES matrix of strike price values where each row is the schedule for one option. If a put option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American put option — NINST-by-1 vector of strike price values for each option.

Data Types: single | double

**PutExDates — Put exercise date for European, Bermuda, or American option**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Put exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates where each row is the schedule for one option. For a European option, there is only one PutExDate on the option expiry date.



- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the put option can be exercised on any tree date between or including the pair of dates on that row. If PutExDates is NINST-by-1, the put option can be exercised between the ValuationDate of the CRR stock tree and the single listed PutExDate.

Data Types: double | char | cell

### **AmericanPut — Put option type indicator**

0 if AmericanPut is NaN or not entered (default) | positive integer [0, 1] | vector of positive integers [0, 1]

Put option type, specified as the comma-separated pair consisting of 'AmericanPut' and a NINST-by-1 vector of positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanPut is 0 for each European or Bermuda option.
- For an American option — AmericanPut is 1 for each American option. The AmericanPut argument is required to invoke American exercise rules.

Data Types: single | double

### **ConvDates — Convertible dates**

MaturityDate (default) | serial date number | date character vector

Convertible dates, specified as the comma-separated pair consisting of 'ConvDates' and a NINST-by-1 or NINST-by-2 matrix of serial date numbers or date character vectors. If ConvDates is not specified, the bond is always convertible until maturity.

For each instrument, the bond can be converted on any tree date between or including the pair of dates on that row.

If ConvDates is NINST-by-1, the bond can be converted between the ValuationDate of the CRR stock tree and the single listed ConvDates.

Data Types: char | single | double

### **DefaultProbability — Annual probability of default rate**

0 (default) | nonnegative decimal

Annual probability of default rate, specified as the comma-separated pair consisting of 'DefaultProbability' and a NINST-by-1 nonnegative decimal value.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. Do not use `DefaultProbability` and `RecoveryRate` with `Spread`.

---

Data Types: `single` | `double`

**RecoveryRate — Recovery rate**

1 (default) | nonnegative decimal

Recovery rate, specified as the comma-separated pair consisting of 'RecoveryRate' and a NINST-by-1 nonnegative decimal.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. Do not use `DefaultProbability` and `RecoveryRate` with `Spread`.

---

Data Types: `single` | `double`

## Output Arguments

**Price — Expected price at time 0**

array

Expected price at time 0, returned as an NINST-by-1 array.

**PriceTree — Structure with vector of convertible bond prices at each node**

tree structure

Structure with a vector of convertible bond prices at each node, returned as a tree structure.

**EquityTree — Structure with vector of convertible bond equity component at each node**

tree structure

Structure with a vector of convertible bond equity component at each node, returned as a tree structure.

### **DebtTree — Structure with vector of convertible bond debt component at each node**

tree structure

Structure with a vector of convertible bond debt component at each node, returned as a tree structure.

## **Definitions**

### **Callable Convertible**

A convertible bond that is callable by the issuer. The issuer of the bond forces conversion, removing the advantage that conversion is at the discretion of the bondholder.

Upon call, the bondholder can either convert the bond or redeem at the call price. This option enables the issuer to control the price of the convertible bond and, if necessary, refinance the debt with a new cheaper one.

### **Puttable Convertible**

A convertible bond with a put feature allows the bondholder to sell back the bond at a premium on a specific date.

This option protects the holder against rising interest rates by reducing the year to maturity.

## **Algorithms**

`cbondbycrr`, `cbondbyeqp`, `cbondbyitt`, and `cbondbystt` return price information in the form of a price vector and a price tree. These functions implement the risk in the form of either a credit spread or incorporating the risk of bond default. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair arguments `DefaultProbability` and `RecoveryRate`.

## References

- [1] Tsiveriotis, K., and C. Fernandes. "Valuing Convertible Bonds with Credit Risk." *Journal of Fixed Income*. Vol. 8, 1998, pp. 95-102.
- [2] Hull, J. *Options, Futures and Other Derivatives*. Fourth Edition. Prentice Hall, 2000, pp. 646-649.

## See Also

cbondbyeqp | crrsens | crrtree | eqpprice | eqpsens | instadd | instcbond |  
instdisp | intenvset | stockspect

## Topics

"Convertible Bond" on page 2-3

**Introduced in R2015a**

## cbondbyeqp

Price convertible bonds from EQP binomial tree

### Syntax

```
Price = cbondbyeqp(EQPTree,CouponRate,Settle,Maturity,ConvRatio)
[Price,PriceTree] = cbondbyeqp(EQPTree,CouponRate,Settle,Maturity,
ConvRatio)
[Price,PriceTree,EquityTree,DebtTree] = cbondbyeqp( ____,Name,Value)
```

### Description

`Price = cbondbyeqp(EQPTree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds from an EQP binomial tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree] = cbondbyeqp(EQPTree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds from an EQP binomial tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree,EquityTree,DebtTree] = cbondbyeqp( ____,Name,Value)` prices convertible bonds from an EQP binomial tree using a credit spread or incorporating the risk of bond default.

To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair input argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair input arguments `DefaultProbability` and `RecoveryRate`.

### Examples

#### Price Convertible Bond Using an EQP Tree

Create the interest-rate term structure `RateSpec`.

```
StartDates = 'Jan-1-2014';
EndDates = 'Jan-1-2016';
Rates = 0.025;
Basis = 1;
RateSpec = intenvset('ValuationDate',StartDates,'StartDates',StartDates,'EndDates',EndDates,
'Rates',Rates,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0250
    EndTimes: 2
    StartTimes: 0
    EndDates: 736330
    StartDates: 735600
    ValuationDate: 735600
    Basis: 1
    EndMonthRule: 1
```

Create the StockSpec.

```
AssetPrice = 110;
Sigma = 0.22;
Div = 0.02;
StockSpec = stockspec(Sigma,AssetPrice,'continuous',Div)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2200
    AssetPrice: 110
    DividendType: {'continuous'}
    DividendAmounts: 0.0200
    ExDividendDates: []
```

Create the EQP tree for the equity.

```
NumSteps = 6;
TimeSpec = eqptimespec(StartDates,EndDates,NumSteps);
EQPTree = eqptree(StockSpec,RateSpec,TimeSpec)
```

```
EQPTree = struct with fields:
    FinObj: 'BinStockTree'
```

```

Method: 'EQP'
StockSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
    dObs: [735600 735721 735843 735965 736086 736208 736330]
    STree: {1x7 cell}
UpProbs: [0.5000 0.5000 0.5000 0.5000 0.5000 0.5000]

```

Define the convertible bond. The convertible bond can be called starting on Jan 1, 2015 with a strike price of 125.

```

Settle = 'Jan-1-2014';
Maturity = 'Jan-1-2016';
CouponRate = 0.03;
CallStrike = 125;
Period = 1;
CallExDates = [datenum('Jan-1-2015') datenum('Jan-1-2016')];
ConvRatio = 1.5;

```

Price the convertible bond.

```
Spread = 0.045;
```

```

[Price,PriceTree,EqtTre,DbtTree] = cbondbyeqp(EQPTree,CouponRate,Settle,...
Maturity,ConvRatio,'Period',Period,'Spread',Spread,'CallExDates',...
CallExDates,'CallStrike',CallStrike,'AmericanCall',1)

```

```
Price = 165
```

```
PriceTree = struct with fields:
```

```

    FinObj: 'BinPriceTree'
    PTree: {1x7 cell}
    tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
    dObs: [735600 735721 735843 735965 736086 736208 736330]

```

```
EqtTre = struct with fields:
```

```

    FinObj: 'BinPriceTree'
    PTree: {1x7 cell}
    tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
    dObs: [735600 735721 735843 735965 736086 736208 736330]

```

```

DbtTree = struct with fields:
  FinObj: 'BinPriceTree'
  PTree: {1x7 cell}
  tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
  dObs: [735600 735721 735843 735965 736086 736208 736330]

```

### Price a Convertible Bond Using an EQP Tree and Incorporate Default Risk Using DefaultProbability and RecoveryRate

Create the interest-rate term structure RateSpec.

```

StartDates = 'Jan-1-2014';
EndDates = 'Jan-1-2016';
Rates = 0.025;
Basis = 1;
RateSpec = intenvset('ValuationDate',StartDates,'StartDates',...
StartDates,'EndDates',EndDates,'Rates',Rates,'Compounding',-1,'Basis',Basis)

```

```

RateSpec = struct with fields:
  FinObj: 'RateSpec'
  Compounding: -1
  Disc: 0.9512
  Rates: 0.0250
  EndTimes: 2
  StartTimes: 0
  EndDates: 736330
  StartDates: 735600
  ValuationDate: 735600
  Basis: 1
  EndMonthRule: 1

```

Create the StockSpec.

```

AssetPrice = 110;
Sigma = 0.22;
Div = 0.02;
StockSpec = stockspec(Sigma,AssetPrice,'continuous',Div)

StockSpec = struct with fields:
  FinObj: 'StockSpec'

```



```

        Sigma: 0.2200
        AssetPrice: 110
        DividendType: {'continuous'}
        DividendAmounts: 0.0200
        ExDividendDates: []

```

Create the EQP tree for the equity.

```

NumSteps = 6;
TimeSpec = eqptimespec(StartDates,EndDates,NumSteps);
EQPTree = eqptree(StockSpec,RateSpec,TimeSpec)

```

```

EQPTree = struct with fields:
    FinObj: 'BinStockTree'
    Method: 'EQP'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
        dObs: [735600 735721 735843 735965 736086 736208 736330]
    STree: {1x7 cell}
    UpProbs: [0.5000 0.5000 0.5000 0.5000 0.5000 0.5000]

```

Define and price the convertible bond using the optional `DefaultProbability` and `RecoveryRate` arguments.

```

Settle = 'Jan-1-2014';
Maturity = 'Jan-1-2016';
CouponRate = 0.03;
CallStrike = 125;
Period = 1;
CallExDates = [datenum('Jan-1-2015') datenum('Jan-1-2016')];
ConvRatio = 1.5;
DefaultProbability = .30;
RecoveryRate = .82;

```

```

[Price,PriceTree,EqTree,DbtTree] = cbondbyeqp(EQPTree,CouponRate,Settle,...
Maturity,ConvRatio,'Period',Period,'CallExDates',...
CallExDates,'CallStrike',CallStrike,'AmericanCall',1,...
'DefaultProbability',DefaultProbability,'RecoveryRate',RecoveryRate)

```

```

Price = 165

```

```
PriceTree = struct with fields:
  FinObj: 'BinPriceTree'
  PTree: {1x7 cell}
  tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
  dObs: [735600 735721 735843 735965 736086 736208 736330]
```

```
EqTree = struct with fields:
  FinObj: 'BinPriceTree'
  PTree: {1x7 cell}
  tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
  dObs: [735600 735721 735843 735965 736086 736208 736330]
```

```
DbtTree = struct with fields:
  FinObj: 'BinPriceTree'
  PTree: {1x7 cell}
  tObs: [0 0.3333 0.6667 1 1.3333 1.6667 2]
  dObs: [735600 735721 735843 735965 736086 736208 736330]
```

## Input Arguments

### **EQPTree** — Stock tree structure

structure

Stock tree structure, specified by using `eqptree`.

Data Types: `struct`

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date

serial date number | date character vector

Settlement date, specified as an NINST-by-1 scalar using a serial date number or date character vector.

---

**Note** The `Settle` date for every convertible bond is set to the `ValuationDate` of the EQP stock tree. The bond argument, `Settle`, is ignored.

---

Data Types: double | char

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 scalar using a serial date number or date character vector.

Data Types: double | char

### **ConvRatio — Number of shares convertible to one bond**

nonnegative number

Number of shares convertible to one bond, specified as an NINST-by-1 with a nonnegative number.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: [Price, PriceTree, EquityTree, DebtTree] =
cbondbyeqp(EQPT, CouponRate, Settle, Maturity,
ConvRatio, 'Spread', Spread, 'CallExDates', CallExDates,
'CallStrike', CallStrike, 'AmericanCall', 1)
```

### **Spread — Number of basis points over the reference rate**

0 (default) | vector

Number of basis points over the reference rate, specified as the comma-separated pair consisting of `'Spread'` and a NINST-by-1 vector.

**Note** To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. Do not use `Spread` with `DefaultProbability` and `RecoveryRate`.

---

Data Types: double

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: char | double

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or an NINST-by-1 cell array where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is the associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

### **CallStrike** — Call strike price for European, Bermuda, or American option

nonnegative integer | vector of nonnegative integers

Call strike price for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallStrike' and one of the following values:

- For a European call option — NINST-by-1 vector of nonnegative integers
- For a Bermuda call option — NINST-by-NSTRIKES matrix of strike price values, where each row is the schedule for one call option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American call option — NINST-by-1 vector of strike price values for each call option.

Data Types: single | double

### **CallExDates** — Call exercise date for European, Bermuda, or American option

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Call exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates, where each row is the schedule for one call option. For a European option, there is only one CallExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the call option can be exercised on any tree date between or including the pair of dates on that row. If CallExDates is NINST-by-1, the call option can be exercised between the ValuationDate of the EQP stock tree and the single listed CallExDate.

Data Types: double | char | cell

**AmericanCall** — Call option type indicator

0 if AmericanCall is NaN or not entered (default) | scalar | vector of positive integers [0, 1]

Call option type, specified as the comma-separated pair consisting of 'AmericanCall' and a NINST-by-1 vector with positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanCall is 0 for each European or Bermuda option.
- For an American option — AmericanCall is 1 for each American option. The AmericanCall argument is required to invoke American exercise rules.

Data Types: single | double

**PutStrike** — Put strike values for European, Bermuda, or American option

positive integer | vector of positive integers

Put strike values for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutStrike' and one of the following values:

- For a European put option — NINST-by-1 vector of nonnegative integers
- For a Bermuda put option — NINST-by-NSTRIKES matrix of put strike price values, where each row is the schedule for one put option. If a put option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American put option — NINST-by-1 vector of strike price values for each put option.

Data Types: single | double

**PutExDates** — Put exercise date for European, Bermuda, or American option

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Put exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.

- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates, where each row is the schedule for one put option. For a European option, there is only one PutExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the put option can be exercised on any tree date between or including the pair of dates on that row. If PutExDates is NINST-by-1, the put option can be exercised between the ValuationDate of the EQP stock tree and the single listed PutExDate.

Data Types: double | char | cell

### **AmericanPut — Put option type indicator**

0 if AmericanPut is NaN or not entered (default) | positive integer [0, 1] | vector of positive integers [0, 1]

Put option type, specified as the comma-separated pair consisting of 'AmericanPut' and a NINST-by-1 vector of positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanPut is 0 for each European or Bermuda option.
- For an American option — AmericanPut is 1 for each American option. The AmericanPut argument is required to invoke American exercise rules.

Data Types: single | double

### **ConvDates — Convertible dates**

MaturityDate (default) | serial date number | date character vector

Convertible dates, specified as the comma-separated pair consisting of 'ConvDates' and a NINST-by-1 or NINST-by-2 matrix of serial date numbers or date character vectors. If ConvDates is not specified, the bond is always convertible until maturity.

For each instrument, the bond can be converted on any tree date between or including the pair of dates on that row.

If ConvDates is NINST-by-1, the bond can be converted between the ValuationDate of the EQP stock tree and the single listed ConvDates.

Data Types: char | single | double

### **DefaultProbability — Annual probability of default rate**

0 (default) | nonnegative decimal

Annual probability of default rate, specified as the comma-separated pair consisting of 'DefaultProbability' and a NINST-by-1 nonnegative decimal value.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. Do not use `DefaultProbability` and `RecoveryRate` with `Spread`.

---

Data Types: `single` | `double`

### **RecoveryRate — Recovery rate**

1 (default) | nonnegative decimal

Recovery rate, specified as the comma-separated pair consisting of 'RecoveryRate' and a NINST-by-1 nonnegative decimal value.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. Do not use `DefaultProbability` and `RecoveryRate` with `Spread`.

---

Data Types: `single` | `double`

## **Output Arguments**

### **Price — Expected price at time 0**

array

Expected price at time 0, returned as an NINST-by-1 array.

### **PriceTree — Structure with vector of convertible bond prices at each node**

tree structure

Structure with a vector of convertible bond prices at each node, returned as a tree structure.



**EquityTree — Structure with vector of convertible bond equity component at each node**

tree structure

Structure with a vector of convertible bond equity component at each node, returned as a tree structure.

**DebtTree — Structure with vector of convertible bond debt component at each node**

tree structure

Structure with a vector of convertible bond debt component at each node, returned as a tree structure.

## Definitions

### Callable Convertible

A convertible bond that is callable by the issuer. The issuer of the bond forces conversion, removing the advantage that conversion is at the discretion of the bondholder.

Upon call, the bondholder can either convert the bond or redeem at the call price. This option enables the issuer to control the price of the convertible bond and, if necessary, refinance the debt with a new cheaper one.

### Puttable Convertible

A convertible bond with a put feature allows the bondholder to sell back the bond at a premium on a specific date.

This option protects the holder against rising interest rates by reducing the year to maturity.

## Algorithms

`cbondbycrr`, `cbondbyeqp`, `cbondbyitt`, and `cbondbystt` return price information in the form of a price vector and a price tree. These functions implement the risk in the form

of either a credit spread or incorporating the risk of bond default. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair arguments `DefaultProbability` and `RecoveryRate`.

### References

- [1] Tsiveriotis, K., and C. Fernandes. "Valuing Convertible Bonds with Credit Risk." *Journal of Fixed Income*. Vol. 8, 1998, pp. 95-102.
- [2] Hull, J. *Options, Futures and Other Derivatives*. Fourth Edition. Prentice Hall, 2000, pp. 646-649.

### See Also

`cbondbycrr` | `crrsens` | `eqpprice` | `eqpsens` | `eqptree` | `instadd` | `instcbond` | `instdisp` | `intenvset` | `stockspec`

### Topics

"Convertible Bond" on page 2-3

**Introduced in R2015a**

## cbondbyitt

Price convertible bonds from ITT trinomial tree

### Syntax

```
Price = cbondbyitt(ITTTree,CouponRate,Settle,Maturity,ConvRatio)
[Price,PriceTree] = cbondbyitt(ITTTree,CouponRate,Settle,Maturity,
ConvRatio)
[Price,PriceTree,EquityTree,DebtTree] = cbondbyitt( ____,Name,Value)
```

### Description

`Price = cbondbyitt(ITTTree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds from an ITT trinomial tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree] = cbondbyitt(ITTTree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds from an ITT trinomial tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree,EquityTree,DebtTree] = cbondbyitt( ____,Name,Value)` prices convertible bonds from an ITT trinomial tree using a credit spread or incorporating the risk of bond default.

To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair input argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair input arguments `DefaultProbability` and `RecoveryRate`.

### Examples

#### Price a Convertible Bond Using an ITT Tree

Price a convertible bond using the following data for an `ITTTree` from `deriv.mat`:

```
load deriv.mat
```

Use `cbondbyitt` to price a convertible bond using an ITT trinomial tree.

```
CouponRate = 0.05;  
Settle = 'Jan-1-2006';  
Maturity = 'Jan-1-2008';  
Period = 1;  
CallStrike = 65;  
CallExDates = 'Jan-1-2007';  
ConvRatio = 1;  
Spread = 0.015;
```

```
[Price,PriceTree,EqTree,DbtTree] = cbondbyitt(ITTree,CouponRate,Settle,Maturity,ConvR  
'Period',Period,'Spread',Spread,'CallExDates',CallExDates,'CallStrike',CallStrike,'Ame
```

```
Price = 58.9170
```

```
PriceTree = struct with fields:
```

```
  FinObj: 'TrinPriceTree'  
  PTree: {1x5 cell}  
  tObs: [0 1 2 3 4]  
  dObs: [732678 733043 733408 733773 734139]
```

```
EqTree = struct with fields:
```

```
  FinObj: 'TrinPriceTree'  
  PTree: {1x5 cell}  
  tObs: [0 1 2 3 4]  
  dObs: [732678 733043 733408 733773 734139]
```

```
DbtTree = struct with fields:
```

```
  FinObj: 'TrinPriceTree'  
  PTree: {1x5 cell}  
  tObs: [0 1 2 3 4]  
  dObs: [732678 733043 733408 733773 734139]
```

### **Price a Convertible Bond Using an ITT Tree and Incorporate Default Risk Using DefaultProbability and RecoveryRate**

Price a convertible bond using the following data for an ITTtree from `deriv.mat`.

```
load deriv.mat
```

Use `cbondbyitt` to price a convertible bond using an ITT trinomial tree with the optional `DefaultProbability` and `RecoveryRate` arguments.

```
CouponRate = 0.05;
Settle = 'Jan-1-2006';
Maturity = 'Jan-1-2008';
Period = 1;
CallStrike = 65;
CallExDates = 'Jan-1-2007';
ConvRatio = 1;
DefaultProbability = .30;
RecoveryRate = .82;
```

```
[Price,PriceTree,EqTree,DbtTree] = cbondbyitt(ITTree,CouponRate,Settle,Maturity,ConvR
'Period',Period,'CallExDates',CallExDates,'CallStrike',CallStrike,'AmericanCall',1,..
'DefaultProbability',DefaultProbability,'RecoveryRate',RecoveryRate)
```

```
Price = 50.6487
```

```
PriceTree = struct with fields:
```

```
  FinObj: 'TrinPriceTree'
  PTree: {1x5 cell}
  tObs: [0 1 2 3 4]
  dObs: [732678 733043 733408 733773 734139]
```

```
EqTree = struct with fields:
```

```
  FinObj: 'TrinPriceTree'
  PTree: {1x5 cell}
  tObs: [0 1 2 3 4]
  dObs: [732678 733043 733408 733773 734139]
```

```
DbtTree = struct with fields:
```

```
  FinObj: 'TrinPriceTree'
  PTree: {1x5 cell}
  tObs: [0 1 2 3 4]
  dObs: [732678 733043 733408 733773 734139]
```

## Input Arguments

### **ITTTree** — Stock tree structure

structure

Stock tree structure, specified by using `itttree`.

Data Types: `struct`

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date

scalar for serial nonnegative date number | scalar for date character vector

Settlement date, specified as an NINST-by-1 scalar using a serial nonnegative date number or date character vector.

---

**Note** The `Settle` date for every convertible bond is set to the `ValuationDate` of the standard trinomial (STT) stock tree. The bond argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

### **Maturity** — Maturity date

scalar for serial nonnegative date number | scalar for date character vector

Maturity date, specified as an NINST-by-1 scalar using a serial nonnegative date number or date character vector.

Data Types: `double` | `char`

### **ConvRatio** — Number of shares convertible to one bond

nonnegative number

Number of shares convertible to one bond, specified as an NINST-by-1 with a nonnegative number.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

```
Example: [Price, PriceTree, EquityTree, DebtTree] =
cbondbyitt(ITTree, CouponRate, Settle, Maturity,
ConvRatio, 'Spread', Spread, 'CallExDates', CallExDates, 'CallStrike', Cal
lStrike, 'AmericanCall', 1)
```

### Spread — Number of basis points over the reference rate

0 (default) | vector

Number of basis points over the reference rate, specified as the comma-separated pair consisting of 'Spread' and a NINST-by-1 vector.

---

**Note** To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. Do not use `Spread` with `DefaultProbability` and `RecoveryRate`.

---

Data Types: double

### Period — Coupons per year

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### IssueDate — Bond issue date

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or a NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is the associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

**CallStrike — Call strike price for European, Bermuda, or American option**

nonnegative integer | vector of nonnegative integers

Call strike price for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallStrike' and one of the following values:

- For a European call option — NINST-by-1 vector of nonnegative integers.
- For a Bermuda call option — NINST-by-NSTRIKES matrix of call strike price values, where each row is the schedule for one call option. If a call option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.



- For an American call option — NINST-by-1 vector of strike price values for each option.

Data Types: single | double

**CallExDates — Call exercise date for European, Bermuda, or American option**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Call exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates, where each row is the schedule for one option. For a European option, there is only one CallExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the call option can be exercised on any tree date between or including the pair of dates on that row. If CallExDates is NINST-by-1, the option can be exercised between the ValuationDate of the ITT stock tree and the single listed CallExDate.

Data Types: double | char | cell

**AmericanCall — Call option type indicator**

0 if AmericanCall is NaN or not entered (default) | positive integer [0, 1] | vector of positive integers [0, 1]

Call option type, specified as the comma-separated pair consisting of 'AmericanCall' and a NINST-by-1 vector of positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanCall is 0 for each European or Bermuda option.
- For an American option — AmericanCall is 1 for each American option. The AmericanCall argument is required to invoke American exercise rules.

Data Types: single | double

**PutStrike — Put strike values for European, Bermuda, or American option**

positive integer | vector of positive integers

Put strike values for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutStrike' and one of the following values:

- For a European put option — NINST-by-1 vector of nonnegative integers.
- For a Bermuda put option — NINST-by-NSTRIKES matrix of strike price values where each row is the schedule for one option. If a put option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American put option — NINST-by-1 vector of strike price values for each option.

Data Types: `single` | `double`

### **PutExDates — Put exercise date for European, Bermuda, or American option**

`serial date number` | `vector of serial date numbers` | `date character vector` | `cell array of date character vectors`

Put exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates where each row is the schedule for one option. For a European option, there is only one PutExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the put option can be exercised on any tree date between or including the pair of dates on that row. If PutExDates is NINST-by-1, the put option can be exercised between the ValuationDate of the ITT stock tree and the single listed PutExDate.

Data Types: `double` | `char` | `cell`

### **AmericanPut — Put option type indicator**

`0` if AmericanPut is NaN or not entered (default) | `positive integer [0, 1]` | `vector of positive integers [0, 1]`

Put option type, specified as the comma-separated pair consisting of 'PutExDates' and a NINST-by-1 vector of positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanPut is 0 for each European or Bermuda option.

- For an American option — `AmericanPut` is 1 for each American option. The `AmericanPut` argument is required to invoke American exercise rules.

Data Types: `single` | `double`

### **ConvDates — Convertible dates**

`MaturityDate` (default) | serial date number | date character vector

Convertible dates, specified as the comma-separated pair consisting of `'ConvDates'` and a NINST-by-1 or NINST-by-2 matrix of serial date numbers or date character vectors. If `ConvDates` is not specified, the bond is always convertible until maturity.

For each instrument, the bond can be converted on any tree date between or including the pair of dates on that row.

If `ConvDates` is NINST-by-1, the bond can be converted between the `ValuationDate` of the ITT stock tree and the single listed `ConvDates`.

Data Types: `char` | `single` | `double`

### **DefaultProbability — Annual probability of default rate**

0 (default) | nonnegative decimal

Annual probability of default rate, specified as the comma-separated pair consisting of `'DefaultProbability'` and a NINST-by-1 nonnegative decimal value.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. Do not use `DefaultProbability` and `RecoveryRate` with `Spread`.

---

Data Types: `single` | `double`

### **RecoveryRate — Recovery rate**

1 (default) | nonnegative decimal

Recovery rate, specified as the comma-separated pair consisting of `'RecoveryRate'` and a NINST-by-1 nonnegative decimal value.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit

spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. Do not use `DefaultProbability` and `RecoveryRate` with `Spread`.

---

Data Types: `single` | `double`

## Output Arguments

### **Price — Expected price at time 0**

array

Expected price at time 0, returned as an NINST-by-1 array.

### **PriceTree — Structure with vector of convertible bond prices at each node**

tree structure

Structure with a vector of convertible bond prices at each node, returned as a tree structure.

### **EquityTree — Structure with vector of convertible bond equity component at each node**

tree structure

Structure with a vector of convertible bond equity components at each node, returned as a tree structure.

### **DebtTree — Structure with vector of convertible bond debt component at each node**

tree structure

Structure with a vector of convertible bond debt components at each node, returned as a tree structure.

## Definitions

### **Callable Convertible**

A convertible bond that is callable by the issuer. The issuer of the bond forces conversion, removing the advantage that conversion is at the discretion of the bondholder.

Upon call, the bondholder can either convert the bond or redeem at the call price. This option enables the issuer to control the price of the convertible bond and, if necessary, refinance the debt with a new cheaper bond.

## Puttable Convertible

A convertible bond with a put feature allows the bondholder to sell back the bond at a premium on a specific date.

This option protects the holder against rising interest rates by reducing the year to maturity.

## Algorithms

`cbondbycrr`, `cbondbyeqp`, `cbondbyitt`, and `cbondbystt` return price information in the form of a price vector and a price tree. These functions implement the risk in the form of either a credit spread or incorporating the risk of bond default. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair arguments `DefaultProbability` and `RecoveryRate`.

## References

- [1] Tsiveriotis, K., and C. Fernandes. "Valuing Convertible Bonds with Credit Risk." *Journal of Fixed Income*. Vol 8, 1998, pp. 95-102.
- [2] Hull, J. *Options, Futures and Other Derivatives*. Fourth Edition. Prentice Hall, 2000, pp. 646-649.

## See Also

`cbondbystt` | `instadd` | `instcbond` | `instdisp` | `intenvset` | `ittprice` | `ittsens` | `itttree` | `stockspec`

## Topics

"Convertible Bond" on page 2-3

**Introduced in R2015b**

# cbondbystt

Price convertible bonds from standard trinomial tree

## Syntax

```
Price = cbondbystt(STTtree,CouponRate,Settle,Maturity,ConvRatio)
[Price,PriceTree] = cbondbystt(STTtree,CouponRate,Settle,Maturity,
ConvRatio)
[Price,PriceTree,EquityTree,DebtTree] = cbondbystt( ____,Name,Value)
```

## Description

`Price = cbondbystt(STTtree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds using a standard trinomial (STT) tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree] = cbondbystt(STTtree,CouponRate,Settle,Maturity,ConvRatio)` prices convertible bonds using a standard trinomial (STT) tree using the Tsiveriotis and Fernandes method.

`[Price,PriceTree,EquityTree,DebtTree] = cbondbystt( ____,Name,Value)` prices convertible bonds from a standard trinomial (STT) tree using a credit spread or incorporating the risk of bond default.

To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair input argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair input arguments `DefaultProbability` and `RecoveryRate`.

## Examples

### Price a Convertible Bond Using a STTtree

Create a `RateSpec`.

```
StartDates = 'Jan-1-2015';
EndDates = 'Jan-1-2020';
Rates = 0.025;
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',StartDates,'StartDates',StartDates,...
'EndDates',EndDates,'Rates',Rates,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.8825
    Rates: 0.0250
    EndTimes: 5
    StartTimes: 0
    EndDates: 737791
    StartDates: 735965
    ValuationDate: 735965
    Basis: 1
    EndMonthRule: 1
```

Create a StockSpec.

```
AssetPrice = 80;
Sigma = 0.12;
StockSpec = stockspec(Sigma,AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1200
    AssetPrice: 80
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Create a STTtree.

```
TimeSpec = stttimespec(StartDates, EndDates, 20);
STTtree = stttree(StockSpec, RateSpec, TimeSpec)
```

```
STTtree = struct with fields:
    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
```



```

TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [1x21 double]
    dObs: [1x21 double]
STree: {1x21 cell}
Probs: {1x20 cell}

```

Define the convertible bond. The convertible bond can be called starting on Jan 1, 2016 with a strike price of 95.

```

CouponRate = 0.03;
Settle = 'Jan-1-2015';
Maturity = 'April-1-2018';
Period = 1;
CallStrike = 95;
CallExDates = [datenum('Jan-1-2016') datenum('April-1-2018')];
ConvRatio = 1;
Spread = 0.025;

```

Price the convertible bond using the standard trinomial tree model.

```

[Price,PriceTree,EqTree,DbtTree] = cbondbystt(STTree,CouponRate,Settle,Maturity,ConvRat
'Period',Period,'Spread',Spread,'CallExDates',CallExDates,'CallStrike',CallStrike,'Ame

```

```

Price = 90.2511

```

```

PriceTree = struct with fields:

```

```

    FinObj: 'TrinPriceTree'
    PTree: {1x21 cell}
    tObs: [1x21 double]
    dObs: [1x21 double]

```

```

EqTree = struct with fields:

```

```

    FinObj: 'TrinPriceTree'
    PTree: {1x21 cell}
    tObs: [1x21 double]
    dObs: [1x21 double]

```

```

DbtTree = struct with fields:

```

```

    FinObj: 'TrinPriceTree'
    PTree: {1x21 cell}
    tObs: [1x21 double]

```

```
dObs: [1x21 double]
```

### Spread Effect Analysis for a Convertible Bond Using a STTtree

This example demonstrates the spread effect analysis of a 4% coupon convertible bond, callable at 110 at end of the second year, maturing in five years, with spreads of 0, 50, 100, and 150 BP.

Define the RateSpec.

```
StartDates = '1-Apr-2015';  
EndDates = '1-Apr-2020';  
Rates = 0.05;  
Compounding = -1;  
Basis = 1;  
RateSpec = intenvset('StartDates',StartDates,'EndDates',EndDates,'Rates',Rates,...  
    'Compounding',Compounding,'Basis',Basis)
```

RateSpec = struct with fields:

```
    FinObj: 'RateSpec'  
    Compounding: -1  
        Disc: 0.7788  
        Rates: 0.0500  
    EndTimes: 5  
    StartTimes: 0  
    EndDates: 737882  
    StartDates: 736055  
    ValuationDate: 736055  
        Basis: 1  
    EndMonthRule: 1
```

Define the convertible bond data.

```
Settle = '1-Apr-2015';  
Maturity = '1-Apr-2020';  
CouponRate = 0.04;  
CallStrike = 110;  
CallExDates = [datenum('1-Apr-2017') datenum(Maturity)];  
ConvRatio = 1;  
AmericanCall = 1;
```

```

Sigma = 0.3;
Spreads = 0:0.005:0.015;
Prices = 40:10:140;
convprice = zeros(length(Prices),length(Spreads));

```

Define the TimeSpec for the Standard Trinomial Tree, create an STTTtree using stttree, and price the convertible bond using cbondbystt.

```

NumSteps = 200;
TimeSpec = stttimespec(StartDates, EndDates, NumSteps);

for PriceIdx = 1:length(Prices)
    StockSpec = stockspec(Sigma, Prices(PriceIdx));
    STTT = stttree(StockSpec, RateSpec, TimeSpec);
    convprice(PriceIdx,:) = cbondbystt(STTT, CouponRate, Settle, Maturity, ConvRatio,
    'Spread', Spreads(:), 'CallExDates', CallExDates, 'CallStrike', CallStrike, ...
    'AmericanCall', AmericanCall);
end

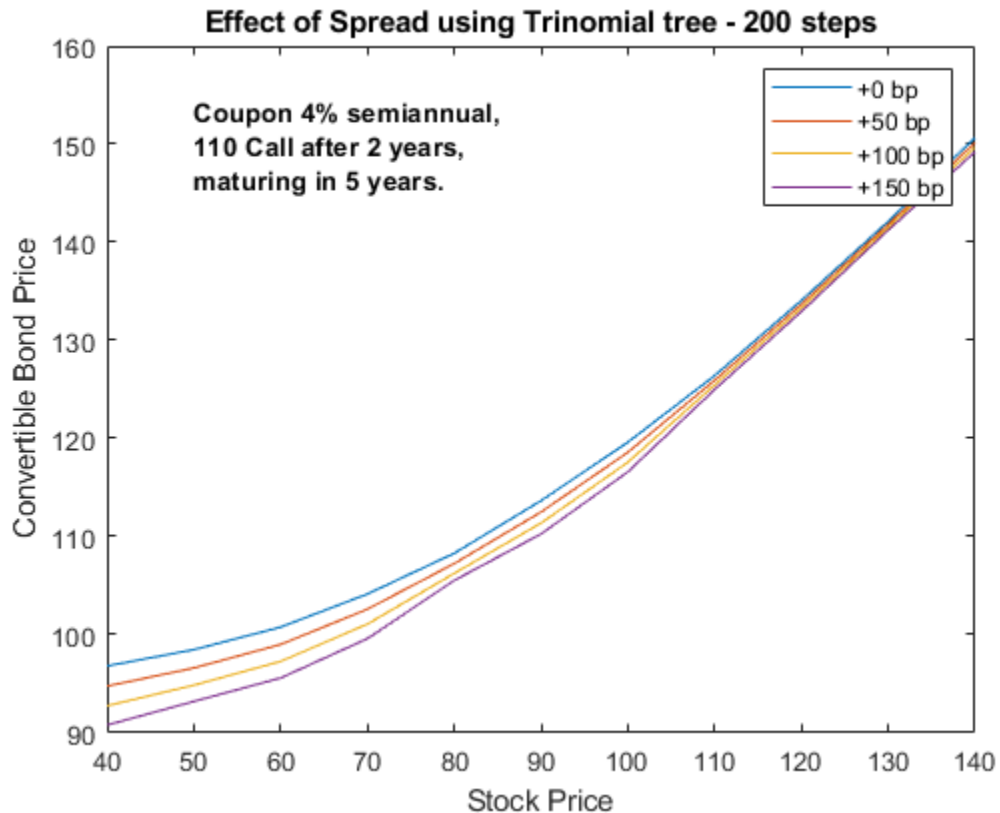
```

Plot the spread effect analysis for the convertible bond.

```

stock = repmat(Prices',1,length(Spreads));
plot(stock,convprice);
legend({'+0 bp'; '+50 bp'; '+100 bp'; '+150 bp'});
title ('Effect of Spread using Trinomial tree - 200 steps')
xlabel('Stock Price');
ylabel('Convertible Bond Price');
text(50, 150, ['Coupon 4% semiannual,', sprintf('\n'), ...
    '110 Call after 2 years,' sprintf('\n'), ...
    'maturing in 5 years.'],'fontweight','Bold')

```



### Price a Convertible Bond Using an STT Tree and Incorporate Default Risk Using DefaultProbability and RecoveryRate

Create the interest-rate term structure RateSpec.

```
StartDates = 'Jan-1-2015';
EndDates = 'Jan-1-2020';
Rates = 0.025;
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',StartDates,'StartDates',StartDates,...
'EndDates',EndDates,'Rates',Rates,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.8825
    Rates: 0.0250
    EndTimes: 5
    StartTimes: 0
    EndDates: 737791
    StartDates: 735965
    ValuationDate: 735965
    Basis: 1
    EndMonthRule: 1
```

Create the StockSpec.

```
AssetPrice = 80;
Sigma = 0.12;
StockSpec = stockspec(Sigma,AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1200
    AssetPrice: 80
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Create the STT tree for the equity.

```
TimeSpec = stttimespec(StartDates, EndDates, 20);
STTtree = stttree(StockSpec, RateSpec, TimeSpec)
```

```
STTtree = struct with fields:
    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [1x21 double]
    dObs: [1x21 double]
    STree: {1x21 cell}
    Probs: {1x20 cell}
```

Define and price the convertible bond using the optional `DefaultProbability` and `RecoveryRate` arguments.

```
CouponRate = 0.03;
Settle = 'Jan-1-2015';
Maturity = 'April-1-2018';
Period = 1;
CallStrike = 95;
CallExDates = [datenum('Jan-1-2016') datenum('April-1-2018')];
ConvRatio = 1;
DefaultProbability = .30;
RecoveryRate = .82;
```

```
[Price,PriceTree,EqtTre,DbtTree] = cbondbystt(STTTree,CouponRate,Settle,Maturity,ConvRatio,
'Period',Period,'CallExDates',CallExDates,'CallStrike',CallStrike,'AmericanCall',1,...
'DefaultProbability',DefaultProbability,'RecoveryRate',RecoveryRate)
```

```
Price = 80
```

```
PriceTree = struct with fields:
  FinObj: 'TrinPriceTree'
  PTree: {1x21 cell}
         tObs: [1x21 double]
         dObs: [1x21 double]
```

```
EqtTre = struct with fields:
  FinObj: 'TrinPriceTree'
  PTree: {1x21 cell}
         tObs: [1x21 double]
         dObs: [1x21 double]
```

```
DbtTree = struct with fields:
  FinObj: 'TrinPriceTree'
  PTree: {1x21 cell}
         tObs: [1x21 double]
         dObs: [1x21 double]
```

## Input Arguments

### **STTtree** — Stock tree structure for standard trinomial tree structure

Stock tree structure for a standard trinomial tree, specified by using `stttree`.

Data Types: `struct`

### **CouponRate** — Bond coupon rate positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date scalar for serial nonnegative date number | scalar for date character vector

Settlement date, specified as an NINST-by-1 scalar using a serial nonnegative date number or date character vector.

---

**Note** The `Settle` date for every convertible bond is set to the `ValuationDate` of the standard trinomial (STT) stock tree. The bond argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

### **Maturity** — Maturity date scalar for serial nonnegative date number | scalar for date character vector

Maturity date, specified as an NINST-by-1 scalar using a serial nonnegative date number or date character vector.

Data Types: `double` | `char`

### **ConvRatio** — Number of shares convertible to one bond nonnegative number

Number of shares convertible to one bond, specified as an NINST-by-1 with a nonnegative number.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

```
Example: [Price, PriceTree, EquityTree, DebtTree] =  
cbondbystt(STTTree, CouponRate, Settle, Maturity, ConvRatio, 'Spread', Spr  
ead, 'CallExDates', CallExDates, 'CallStrike', CallStrike, 'AmericanCall'  
, 1)
```

### **Spread — Number of basis points over the reference rate**

0 (default) | vector

Number of basis points over the reference rate, specified as the comma-separated pair consisting of 'Spread' and a NINST-by-1 vector.

---

**Note** To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. Do not use `Spread` with `DefaultProbability` and `RecoveryRate`.

---

Data Types: double

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector



Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using a serial date number or date character vector.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 scalar using a serial date number or date character vector.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 using a serial date number or date character vector.

Data Types: double | char

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of nonnegative face values or a NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is the associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

### **CallStrike — Call strike price for European, Bermuda, or American option**

nonnegative integer | vector of nonnegative integers

Call strike price for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallStrike' and one of the following values:

- For a European call option — NINST-by-1 vector of nonnegative integers
- For a Bermuda call option — NINST-by-NSTRIKES matrix of call strike price values, where each row is the schedule for one call option. If a call option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.

- For an American call option — NINST-by-1 vector of strike price values for each option.

Data Types: single | double

**CallExDates — Call exercise date for European, Bermuda, or American option**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Call exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates, where each row is the schedule for one option. For a European option, there is only one CallExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the call option can be exercised on any tree date between or including the pair of dates on that row. If CallExDates is NINST-by-1, the option can be exercised between the ValuationDate of the STT stock tree and the single listed CallExDate.

Data Types: double | char | cell

**AmericanCall — Call option type indicator**

0 if AmericanCall is NaN or not entered (default) | positive integer [0, 1] | vector of positive integers [0, 1]

Call option type, specified as the comma-separated pair consisting of 'AmericanCall' and a NINST-by-1 vector of positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanCall is 0 for each European or Bermuda option.
- For an American option — AmericanCall is 1 for each American option. The AmericanCall argument is required to invoke American exercise rules.

Data Types: single | double

**PutStrike — Put strike values for European, Bermuda, or American option**

positive integer | vector of positive integers

Put strike values for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutStrike' and one of the following values:

- For a European put option — NINST-by-1 vector of nonnegative integers.
- For a Bermuda put option — NINST-by-NSTRIKES matrix of strike price values where each row is the schedule for one option. If a put option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American put option — NINST-by-1 vector of strike price values for each option.

Data Types: single | double

### **PutExDates — Put exercise date for European, Bermuda, or American option**

serial date nonnegative number | vector of serial date numbers | date character vector | cell array of date character vectors

Put exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates where each row is the schedule for one option. For a European option, there is only one PutExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the put option can be exercised on any tree date between or including the pair of dates on that row. If PutExDates is NINST-by-1, the put option can be exercised between the ValuationDate of the STT stock tree and the single listed PutExDate.

Data Types: double | char | cell

### **AmericanPut — Put option type indicator**

0 if AmericanPut is NaN or not entered (default) | positive integer [0, 1] | vector of positive integers [0, 1]

Put option type, specified as the comma-separated pair consisting of 'AmericanPut' and a NINST-by-1 vector of positive integer flags with values 0 or 1.

- For a European or Bermuda option — AmericanPut is 0 for each European or Bermuda option.

- For an American option — `AmericanPut` is 1 for each American option. The `AmericanPut` argument is required to invoke American exercise rules.

Data Types: `single` | `double`

### **ConvDates — Convertible dates**

`MaturityDate` (default) | serial date number | date character vector

Convertible dates, specified as the comma-separated pair consisting of `'ConvDates'` and a NINST-by-1 or NINST-by-2 matrix of serial date numbers or date character vectors. If `ConvDates` is not specified, the bond is always convertible until maturity.

For each instrument, the bond can be converted on any tree date between or including the pair of dates on that row.

If `ConvDates` is NINST-by-1, the bond can be converted between the `ValuationDate` of the standard trinomial (STT) stock tree and the single listed `ConvDates`.

Data Types: `single` | `double` | `char`

### **DefaultProbability — Annual probability of default rate**

0 (default) | nonnegative decimal

Annual probability of default rate, specified as the comma-separated pair consisting of `'DefaultProbability'` and a NINST-by-1 nonnegative decimal value.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional input argument `Spread`. Do not use `DefaultProbability` and `RecoveryRate` with `Spread`.

---

Data Types: `single` | `double`

### **RecoveryRate — Recovery rate**

1 (default) | nonnegative decimal

Recovery rate, specified as the comma-separated pair consisting of `'RecoveryRate'` and a NINST-by-1 nonnegative decimal value.

---

**Note** To incorporate default risk into the algorithm, specify the optional input arguments `DefaultProbability` and `RecoveryRate`. To incorporate the risk in the form of credit

---

spread (Tsiveriotis-Fernandes method), use the optional input argument Spread. Do not use DefaultProbability and RecoveryRate with Spread.

---

Data Types: single | double

## Output Arguments

### **Price — Expected price at time 0**

array

Expected price at time 0, returned as an NINST-by-1 array.

### **PriceTree — Structure with vector of convertible bond prices at each node**

tree structure

Structure with a vector of convertible bond prices at each node, returned as a tree structure.

### **EquityTree — Structure with vector of convertible bond equity component at each node**

tree structure

Structure with a vector of convertible bond equity components at each node, returned as a tree structure.

### **DebtTree — Structure with vector of convertible bond debt component at each node**

tree structure

Structure with a vector of convertible bond debt components at each node, returned as a tree structure.

## Definitions

### **Callable Convertible**

A convertible bond that is callable by the issuer. The issuer of the bond forces conversion, removing the advantage that conversion is at the discretion of the bondholder.

Upon call, the bondholder can either convert the bond or redeem at the call price. This option enables the issuer to control the price of the convertible bond and, if necessary, refinance the debt with a new cheaper bond.

## Puttable Convertible

A convertible bond with a put feature allows the bondholder to sell back the bond at a premium on a specific date.

This option protects the holder against rising interest rates by reducing the year to maturity.

## Algorithms

`cbondbycrr`, `cbondbyeqp`, `cbondbyitt`, and `cbondbystt` return price information in the form of a price vector and a price tree. These functions implement the risk in the form of either a credit spread or incorporating the risk of bond default. To incorporate the risk in the form of credit spread (Tsiveriotis-Fernandes method), use the optional name-value pair argument `Spread`. To incorporate default risk into the algorithm, specify the optional name-value pair arguments `DefaultProbability` and `RecoveryRate`.

## References

- [1] Tsiveriotis, K., and C. Fernandes. "Valuing Convertible Bonds with Credit Risk." *Journal of Fixed Income*. Vol 8, 1998, pp. 95-102.
- [2] Hull, J. *Options, Futures and Other Derivatives*. Fourth Edition. Prentice Hall, 2000, pp. 646-649.

## See Also

`cbondbycrr` | `cbondbyeqp` | `instadd` | `instcbond` | `instdisp` | `intenvset` | `stockspec` | `sttprice` | `sttsens` | `stttree`

## Topics

"Convertible Bond" on page 2-3

**Introduced in R2015b**

## cfbybdt

Price cash flows from Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = cfbybdt(BDTree,CFlowAmounts,CFlowDates,Settle)
[Price,PriceTree] = cfbybdt( ____,Basis,Options)
```

### Description

[Price,PriceTree] = cfbybdt(BDTree,CFlowAmounts,CFlowDates,Settle) prices cash flows from a Black-Derman-Toy interest-rate tree.

[Price,PriceTree] = cfbybdt( \_\_\_\_,Basis,Options) adds optional arguments.

### Examples

#### Price a Portfolio Containing Two Cash Flow Instruments

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004.

Load the file `deriv.mat`, which provides `BDTree`. The `BDTree` structure contains the time and interest-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `BDTree` is January 1, 2000 (date number 730486).

```
BDTree.RateSpec.ValuationDate
```

```
ans = 730486
```

Provide values for the other required arguments.



```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [730852, NaN, 731582, 731947;
              730852, 731217, 731582, 731947];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbybdt(BDTree, CFlowAmounts, ...
CFlowDates, BDTree.RateSpec.ValuationDate)
```

```
Price = 2×1
```

```
74.0112
74.3671
```

```
PriceTree = struct with fields:
  FinObj: 'BDTreePriceTree'
  tObs: [0 1 2 3 4]
  PTree: {1×5 cell}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

## Input Arguments

### **BDTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdtree`.

Data Types: `struct`

### **CFlowAmounts** — Cash flow amounts

matrix

Cash flow amounts, specified as a Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

Data Types: `double`

**CFlowDates — Cash flow dates**

matrix

Cash flow dates, specified as NINST-by-MOSTCFS matrix. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector

Settlement date, specified as a vector of serial date numbers or a date character vectors. The **Settle** date for every cash flow is set to the **ValuationDate** of the BDT tree. The cash flow argument, **Settle**, is ignored.

Data Types: double | char

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis of the instrument, specified as a vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices at time 0**

vector

Expected prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## **See Also**

`bdtprice` | `bdttree` | `cfamounts` | `instcf`

## **Topics**

“Computing Instrument Prices” on page 2-99

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## cfbybk

Price cash flows from Black-Karasinski interest-rate tree

### Syntax

```
[Price,PriceTree] = cfbybk(BKTree,CFlowAmounts,CFlowDates,Settle)
[Price,PriceTree] = cfbybk( ____,Basis,Options)
```

### Description

[Price,PriceTree] = cfbybk(BKTree,CFlowAmounts,CFlowDates,Settle) prices cash flows from a Black-Karasinski interest-rate tree.

[Price,PriceTree] = cfbybk( \_\_\_\_,Basis,Options) adds optional arguments.

### Examples

#### Price a Portfolio Containing Two Cash Flow Instruments

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2005 to January 1, 2009.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `BKTree` is January 1, 2004 (date number 731947).

```
BKTree.RateSpec.ValuationDate
```

```
ans =
```

```
731947
```

Provide values for the other required arguments.

```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [732678, NaN, 733408,733774;
              732678, 733034, 733408, 734774];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbybk(BKTree, CFlowAmounts, CFlowDates,...
BKTree.RateSpec.ValuationDate)
```

```
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
> In cfbytrintree (line 88)
   In cfbybk (line 75)
```

```
Price =
```

```
    93.3600
    81.6218
```

```
PriceTree =
```

```
struct with fields:
```

```
    FinObj: 'BKPriceTree'
    PTree: {[2×1 double] [2×3 double] [2×5 double] [2×5 double] [2×5 double]}
    tObs: [0 1 2 3 4]
    Connect: {[2] [2 3 4] [2 2 3 4 4]}
    Probs: {[3×1 double] [3×3 double] [3×5 double]}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

## Input Arguments

### **BKTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

**CFlowAmounts — Cash flow amounts**

matrix

Cash flow amounts, specified as a Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

Data Types: double

**CFlowDates — Cash flow dates**

matrix

Cash flow dates, specified as NINST-by-MOSTCFS matrix. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector

Settlement date, specified as a vector of serial date numbers or a date character vectors. The Settle date for every cash flow is set to the ValuationDate of the BK tree. The cash flow argument, Settle, is ignored.

Data Types: double | char

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis of the instrument, specified as a vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)

- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices at time 0**

vector

Expected prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.

- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

### See Also

`bkprice` | `bktree` | `cfamounts` | `instcf`

### Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# cfbycir

Price cash flows from Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = cfbycir(CIRTree,CFlowAmounts,CFlowDates,Settle)
[Price,PriceTree] = cfbycir( ____,Basis)
```

## Description

[Price,PriceTree] = cfbycir(CIRTree,CFlowAmounts,CFlowDates,Settle) prices cash flows from a Cox-Ingersoll-Ross (CIR) interest-rate tree using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = cfbycir( \_\_\_\_,Basis) adds an optional argument for Basis.

## Examples

### Price a Portfolio Containing Two Cash Flow Instruments

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2017 to June 1, 2020.

Load the file `deriv.mat`, which provides `CIRTree`. The `CIRTree` structure contains the time and interest-rate information required to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in the `CIRTree` is January 1, 2017 (serial date number 736696).

```
CIRTree.RateSpec.ValuationDate
```

```
ans = 736696
```

Provide values for the other required arguments.

```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];  
CFlowDates = [736847,NaN,737061,737212;  
              737426,737577,737791,737943];
```

Compute the prices of the two cash flow instruments.

```
warning('off')  
[Price,PriceTree] = cfbycir(CIRTree, CFlowAmounts, CFlowDates,...  
CIRTree.RateSpec.ValuationDate)
```

```
Price = 2×1
```

```
109.6845  
98.7246
```

```
PriceTree = struct with fields:
```

```
  FinObj: 'CIRPriceTree'  
  PTree: {1×5 cell}  
  tObs: [0 1 2 3 4]  
  Connect: {[3×1 double] [3×3 double] [3×5 double]}  
  Probs: {[3×1 double] [3×3 double] [3×5 double]}
```

## Input Arguments

### **CIRTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **CFlowAmounts** — Cash flow amounts

matrix

Cash flow amounts, specified as a number of instruments (`NINST`) by maximum number of cash flows (`MOSTCFS`) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than `MOSTCFS` cash flows, the end of the row is padded with NaNs.

Data Types: double

### **CFlowDates — Cash flow dates**

matrix

Cash flow dates, specified as NINST-by-MOSTCFS matrix. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.

Data Types: double

### **Settle — Settlement date**

serial date number | date character vector | string array | datetime

Settlement date, specified as a vector of serial date numbers, date character vectors, string arrays, or datetime arrays. The Settle date for every cash flow is set to the ValuationDate of the CIR tree. The cash flow argument Settle is ignored.

Data Types: double | char | string | datetime

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis of the instrument, specified as a vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

## Output Arguments

### **Price — Expected prices at time 0**

vector

Expected prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.

- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | cfbycir | fixedbycir | floatbycir | floorbycir | instcf  
| oasbycir | optbndbycir | optembndbycir | optemfloatbycir | optfloatbycir |  
rangefloatbycir | swapbycir | swaptionbycir

## Topics

"Pricing Using Interest-Rate Tree Models" on page 2-99  
"Pricing Options Structure" on page B-2  
"Understanding Interest-Rate Tree Models" on page 2-79  
"Supported Interest-Rate Instruments" on page 2-2

**Introduced in R2018a**

## cfbyhjm

Price cash flows from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = cfbyhjm(HJMTtree,CFlowAmounts,CFlowDates,Settle)
[Price,PriceTree] = cfbyhjm( ____,Basis,Options)
```

### Description

[Price,PriceTree] = cfbyhjm(HJMTtree,CFlowAmounts,CFlowDates,Settle) prices cash flows from a Heath-Jarrow-Morton interest-rate tree.

[Price,PriceTree] = cfbyhjm( \_\_\_\_,Basis,Options) adds optional arguments.

### Examples

#### Price a Portfolio Containing Two Cash Flow Instruments

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004.

Load the file `deriv.mat`, which provides `HJMTtree`. The `HJMTtree` structure contains the time and interest-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `HJMTtree` is January 1, 2000 (date number 730486).

```
HJMTtree.RateSpec.ValuationDate
```

```
ans = 730486
```

Provide values for the other required arguments.

```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [730852, NaN, 731582, 731947;
              730852, 731217, 731582, 731947];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbyhjm(HJMTree, CFlowAmounts, ...
CFlowDates, HJMTree.RateSpec.ValuationDate)
```

```
Price = 2×1
```

```
96.7805
97.2188
```

```
PriceTree = struct with fields:
  FinObj: 'HJMPriceTree'
  tObs: [0 1 2 3 4]
  PBush: {1x5 cell}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

## Input Arguments

**HJMTree** — Interest-rate tree structure  
structure

Interest-rate tree structure, specified by using `hjmtree`.

Data Types: `struct`

**CFlowAmounts** — Cash flow amounts  
matrix

Cash flow amounts, specified as a Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

Data Types: `double`

**CFlowDates — Cash flow dates**

matrix

Cash flow dates, specified as NINST-by-MOSTCFS matrix. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector

Settlement date, specified as a vector of serial date numbers or a date character vectors. The **Settle** date for every cash flow is set to the **ValuationDate** of the HJM tree. The cash flow argument, **Settle**, is ignored.

Data Types: double | char

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis of the instrument, specified as a vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .



Data Types: `double`

**Options — Derivatives pricing options structure**  
structure

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## Output Arguments

**Price — Expected prices at time 0**  
vector

Expected prices at time 0, returned as a NINST-by-1 vector.

**PriceTree — Tree structure of instrument prices**  
structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.tObs` contains the observation times.
- `PriceTree.PBush` contains the clean prices.

## See Also

`cfamounts` | `hjmprice` | `hjmtree` | `instcf`

## Topics

“Computing Instrument Prices” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## cfbyhw

Price cash flows from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = cfbyhw(HWTree,CFlowAmounts,CFlowDates,Settle)
[Price,PriceTree] = cfbyhw( ____,Basis,Options)
```

### Description

[Price,PriceTree] = cfbyhw(HWTree,CFlowAmounts,CFlowDates,Settle) prices cash flows from a Hull-White interest-rate tree.

[Price,PriceTree] = cfbyhw( \_\_\_\_,Basis,Options) adds optional arguments.

### Examples

#### Price a Portfolio Containing Two Cash Flow Instruments

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2005 to January 1, 2009.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `HWTree` is January 1, 2004 (date number 731947).

```
HWTree.RateSpec.ValuationDate
```

```
ans =
```

```
731947
```

Provide values for the other required arguments.

```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [732678, NaN, 733408, 733774;
              732678, 733034, 733408, 734774];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbyhw(HWTree, CFlowAmounts, CFlowDates,...
HWTree.RateSpec.ValuationDate)
```

```
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
> In cfbytrintree (line 88)
   In cfbyhw (line 75)
```

```
Price =
```

```
    93.3789
    81.7651
```

```
PriceTree =
```

```
struct with fields:
```

```
    FinObj: 'HWPriceTree'
    PTree: {[2×1 double] [2×3 double] [2×5 double] [2×5 double] [2×5 double]}
    tObs: [0 1 2 3 4]
    Connect: {[2] [2 3 4] [2 2 3 4 4]}
    Probs: {[3×1 double] [3×3 double] [3×5 double]}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

## Input Arguments

### **HWTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: `struct`

**CFlowAmounts — Cash flow amounts**

matrix

Cash flow amounts, specified as a Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

Data Types: double

**CFlowDates — Cash flow dates**

matrix

Cash flow dates, specified as NINST-by-MOSTCFS matrix. Each entry contains the serial date number of the corresponding cash flow in CFlowAmounts.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector

Settlement date, specified as a vector of serial date numbers or a date character vectors. The Settle date for every cash flow is set to the ValuationDate of the HW tree. The cash flow argument, Settle, is ignored.

Data Types: double | char

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis of the instrument, specified as a vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)

- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices at time 0**

vector

Expected prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.

- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

### See Also

`cfamounts` | `hwprice` | `hwtree` | `instcf`

### Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## cfbyzero

Price cash flows from set of zero curves

### Syntax

```
Price = cfbyzero(RateSpec,CFlowAmounts,CFlowDates,Settle)
Price = cfbyzero( ____,Basis)
```

### Description

`Price = cfbyzero(RateSpec,CFlowAmounts,CFlowDates,Settle)` prices cash flows from a set of zero curves.

`Price = cfbyzero( ____,Basis)` adds an optional argument.

### Examples

#### Compute the Price and Sensitivity From the Interest-Rate Term Structure

This example shows how to price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004. Load the file `deriv.mat`, which provides `ZeroRateSpec`. The `ZeroRateSpec` structure contains the interest-rate information needed to price the instruments.

```
load deriv.mat
CFlowAmounts = [5 NaN 5.5 105;5 0 6 105];
CFlowDates = [730852, NaN, 731582,731947;
              730852, 731217, 731582, 731947];
Settle = 730486;
Price = cfbyzero(ZeroRateSpec, CFlowAmounts, CFlowDates, Settle)
```

```
Price = 2×1
```

```
    96.7804
```

97.2187

## Input Arguments

### **RateSpec — Annualized zero rate term structure**

structure

Annualized zero rate term structure, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **CFlowAmounts — Cash flow amounts**

matrix

Cash flow amounts, specified as a Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

Data Types: `double`

### **CFlowDates — Cash flow dates**

matrix

Cash flow dates, specified as NINST-by-MOSTCFS matrix. Each entry contains the serial date number of the corresponding cash flow in `CFlowAmounts`.

Data Types: `double`

### **Settle — Settlement date on which cash flows are priced**

serial date number | date character vector

Settlement date on which the cash flows are priced, specified using a scalar or NINST-by-1 vector of serial date numbers or date character vectors of the same value which represent the settlement date for each cash flow. `Settle` must be earlier than `Maturity`.

Data Types: `double` | `char`

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13



(Optional) Day-count basis of the instrument, specified as a vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

## Output Arguments

### Price — Cash flow prices

matrix

Cash flow prices, returned as a NINST-by-NUMCURVES matrix where each column arises from one of the zero curves.

## See Also

bondbyzero | fixedbyzero | floatbyzero | swapbyzero

## Topics

“Pricing Using Interest-Rate Term Structure” on page 2-72

“Understanding the Interest-Rate Term Structure” on page 2-55  
“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# chooserbybls

Price European simple chooser options using Black-Scholes model

## Syntax

Price = chooserbybls(RateSpec, StockSpec, Settle, Maturity, Strike)

## Arguments

RateSpec	The annualized continuously compounded rate term structure. For information on the interest rate specification, see <code>intenvset</code> .
StockSpec	Stock specification. See <code>stockspec</code> .
Settle	NINST-by-1 vector of settlement or trade dates.
Maturity	NINST-by-1 vector of maturity dates.
Strike	NINST-by-1 vector of strike price values.
ChooseDate	NINST-by-1 vector of chooser dates.

## Description

Price = chooserbybls(RateSpec, StockSpec, Settle, Maturity, Strike) computes the price for European simple chooser options using the Black-Scholes model.

Price is a NINST-by-1 vector of expected prices.

---

**Note** Only dividends of type `continuous` can be considered for choosers.

---

## Examples

### Price European Simple Chooser Options Using the Black-Scholes Model

Consider a European chooser option with an exercise price of \$60 on June 1, 2007. The option expires on December 2, 2007. Assume the underlying stock provides a continuous dividend yield of 5% per annum, is trading at \$50, and has a volatility of 20% per annum. The annualized continuously compounded risk-free rate is 10% per annum. Assume that the choice must be made on August 31, 2007. Using this data:

```
AssetPrice = 50;  
Strike = 60;  
Settlement = 'Jun-1-2007';  
Maturity = 'Dec-2-2007';  
ChooseDate = 'Aug-31-2007';  
RiskFreeRate = 0.1;  
Sigma = 0.20;  
Yield = 0.05
```

```
Yield = 0.0500
```

Define the RateSpec and StockSpec.

```
RateSpec = intenvset('Compounding', -1, 'Rates', RiskFreeRate, 'StartDates', ...  
Settlement, 'EndDates', Maturity);  
StockSpec = stockspec(Sigma, AssetPrice, 'continuous', Yield);
```

Price the chooser option.

```
Price = chooserbybls(RateSpec, StockSpec, Settlement, Maturity, ...  
Strike, ChooseDate)
```

```
Price = 8.9308
```

## References

Rubinstein, Mark. "Options for the Undecided." *Risk*. Vol. 4, 1991.

## See Also

blsprice | intenvset

## **Topics**

“Pricing Using the Black-Scholes Model” on page 3-154

“Chooser Option” on page 3-29

“Supported Equity Derivatives” on page 3-24

## **Introduced in R2008b**

## **cirprice**

Instrument prices from Cox-Ingersoll-Ross interest-rate model

### **Syntax**

```
Price = cirprice(CIRTree,InstSet)
[Price,PriceTree] = cirprice(CIRTree,InstSet)
```

### **Description**

`Price = cirprice(CIRTree,InstSet)` computes prices for instruments using a Cox-Ingersoll-Ross (CIR) interest rate tree created with `cirtree`. The CIR tree uses a CIR++ model with the Nawalka-Beliaeva (NB) approach.

`cirprice` handles the following instrument type values: 'Bond', 'CashFlow', 'OptBond', 'Fixed', 'Float', 'Cap', 'Floor', 'Swap', 'Swaption', 'RangeFloat', 'OptFloat', 'OptEmFloat'.

`[Price,PriceTree] = cirprice(CIRTree,InstSet)` returns the optional output for `PriceTree`.

### **Examples**

#### **Price Bonds Using a CIR Interest-Rate Tree**

Define two bond instruments.

```
CouponRate= [0.035;0.04];
Settle= 'Jan-1-2017';
Maturity = 'Jan-1-2019';
Period = 1;
InstSet = instbond(CouponRate, Settle, Maturity, Period)

InstSet = struct with fields:
    FinObj: 'Instruments'
```

```

IndexTable: [1x1 struct]
    Type: {'Bond'}
    FieldName: {{11x1 cell}}
    FieldClass: {{11x1 cell}}
    FieldData: {{11x1 cell}}

```

```
instdisp(InstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.035	01-Jan-2017	01-Jan-2019	1	0	1	NaN
2	Bond	0.04	01-Jan-2017	01-Jan-2019	1	0	1	NaN

Create a RateSpec using the `intenvset` function.

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End

```

Create a CIR tree.

```

NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2019';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

```

```
CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)
```

```

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 0.5000 1 1.5000]
    dObs: [736696 736878 737061 737243]
    FwdTree: {[1.0173] [1.0276 1.0175 1.0097] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}

```

```
Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

Price the bonds.

```
Price = cirprice(CIRT,InstSet)
```

```
Price = 2x1
```

```
98.6793  
99.6228
```

## Input Arguments

### **CIRTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

## Output Arguments

### **Price** — Expected floating-rate note prices at time 0

vector

Expected floating-rate note prices at time 0, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.



## PriceTree — Tree structure of instrument prices structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:

- PriceTree.PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | cfbycir | cirsens | fixedbycir | floatbycir |  
 floorbycir | oasbycir | optbndbycir | optembndbycir | optemfloatbycir |  
 optfloatbycir | rangefloatbycir | swapbycir | swaptionbycir

## Topics

- "Computing Instrument Prices" on page 2-99  
 "Understanding Interest-Rate Tree Models" on page 2-79  
 "Pricing Options Structure" on page B-2  
 "Supported Interest-Rate Instruments" on page 2-2

**Introduced in R2018a**

## cirsens

Instrument sensitivities and prices from Cox-Ingersoll-Ross interest-rate model

### Syntax

```
[Delta,Gamma,Vega,Price] = cirsens(CIRTree,InstSet)
```

### Description

[Delta,Gamma,Vega,Price] = cirsens(CIRTree,InstSet) computes dollar sensitivities and prices for instruments using a Cox-Ingersoll-Ross (CIR) interest rate tree created with cirtree. The CIR tree uses a CIR++ model with the Nawalka-Beliaeva (NB) approach.

---

**Note** All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

cirsens handles the following instrument type values: 'Bond', 'CashFlow', 'OptBond', 'Fixed', 'Float', 'Cap', 'Floor', 'Swap', 'Swaption', 'RangeFloat', 'OptFloat', 'OptEmFloat'.

### Examples

#### Compute Instrument Sensitivities Using a CIR Interest-Rate Tree

Define and set up two bond instruments. Compute Delta and Gamma for the bond instruments contained in the instrument set.

```
CouponRate= [0.035;0.04];  
Settle= 'Jan-1-2017';  
Maturity = 'Jan-1-2019';  
Period = 1;  
InstSet = instbond(CouponRate, Settle, Maturity, Period)
```

```

InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {'Bond'}
    FieldName: {{11x1 cell}}
    FieldClass: {{11x1 cell}}
    FieldData: {{11x1 cell}}

```

```

instdisp(InstSet)

```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.035	01-Jan-2017	01-Jan-2019	1	0	1	NaN
2	Bond	0.04	01-Jan-2017	01-Jan-2019	1	0	1	NaN

Create a RateSpec using the `intenvset` function.

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End

```

Create a CIR tree.

```

NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2019';
CIRTimeSpec = cirtimespec(Settle, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

```

```

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

```

```

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tobs: [0 0.5000 1 1.5000]
        dobs: [736696 736878 737061 737243]

```

```
FwdTree: {[1.0173] [1.0276 1.0175 1.0097] [1x5 double] [1x7 double]}
Connect: {[3x1 double] [3x3 double] [3x5 double]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

Calculate the Delta and Gamma sensitivities for the two bonds.

```
[Delta, Gamma] = cirsens(CIRT,InstSet)
```

```
Delta = 2x1
```

```
-186.1885
-187.5390
```

```
Gamma = 2x1
```

```
532.8675
536.3132
```

## Input Arguments

### **CIRTtree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

## Output Arguments

### **Delta — Rate of change of instruments prices with respect to changes in interest rate**

vector

Rate of change of instruments prices with respect to changes in the interest rate, returned as a NINST-by-1 vector of deltas. Delta is computed by finite differences in calls to `cirtree`.

---

**Note** Delta is calculated based on yield shifts of 100 basis points.

---

### **Gamma — Rate of change of instruments deltas with respect to changes in interest rate**

vector

Rate of change of instruments deltas with respect to changes in the interest rate, returned as a NINST-by-1 vector of gammas. Gamma is computed by finite differences in calls to `cirtree`.

---

**Note** Gamma is calculated based on yield shifts of 100 basis points.

---

### **Vega — Rate of change of instruments prices with respect to changes in volatility**

vector

Rate of change of instruments prices with respect to changes in the volatility, returned as a NINST-by-1 vector of vegas. Volatility is  $\Sigma(t, T)$  of the interest rate. Vega is computed by finite differences in calls to `cirtree`. For information on the volatility process, see `cirvolspec`.

---

**Note** Vega is calculated based on 1% shift in the volatility process.

---

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

[bondbycir](#) | [capbycir](#) | [cfbycir](#) | [cirprice](#) | [fixedbycir](#) | [floatbycir](#) | [floorbycir](#) | [oasbycir](#) | [optbndbycir](#) | [optembndbycir](#) | [optemfloatbycir](#) | [optfloatbycir](#) | [rangefloatbycir](#) | [swapbycir](#) | [swaptionbycir](#)

## Topics

- "Computing Instrument Prices" on page 2-99
- "Understanding Interest-Rate Tree Models" on page 2-79
- "Pricing Options Structure" on page B-2
- "Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

## classfin

Create financial structure or return financial structure class name

### Syntax

```
Obj = classfin(ClassName)
Obj = classfin(Struct, ClassName)
ClassName = classfin(Obj)
```

### Arguments

ClassName	Character vector containing the name of a financial structure class.
Struct	MATLAB structure to be converted into a financial structure.
Obj	Name of a financial structure.

### Description

`Obj = classfin(ClassName)` and `Obj = classfin(Struct, ClassName)` create a financial structure of class `ClassName`.

`ClassName = classfin(Obj)` returns a character vector containing a financial structure's class name.

### Examples

#### Create a Financial Structure

This example shows how to create a financial structure `HJMTimeSpec` and complete its fields. (Typically, the function `hjmtimespec` is used to create `HJMTimeSpec` structures).

```
TimeSpec = classfin('HJMTimeSpec');
TimeSpec.ValuationDate = datenum('Dec-10-1999');
```



```

TimeSpec.Maturity = datenum('Dec-10-2002');
TimeSpec.Compounding = 2;
TimeSpec.Basis = 0;
TimeSpec.EndMonthRule = 1;
TimeSpec

```

```

TimeSpec = struct with fields:
    FinObj: 'HJMTimeSpec'
    ValuationDate: 730464
    Maturity: 731560
    Compounding: 2
    Basis: 0
    EndMonthRule: 1

```

### Convert an Existing MATLAB Structure into a Financial Structure

This example shows how to convert an existing MATLAB structure into a financial structure.

```

TSpec.ValuationDate = datenum('Dec-10-1999');
TSpec.Maturity = datenum('Dec-10-2002');
TSpec.Compounding = 2;
TSpec.Basis = 0;
TSpec.EndMonthRule = 0;
TimeSpec = classfin(TSpec, 'HJMTimeSpec')

```

```

TimeSpec = struct with fields:
    ValuationDate: 730464
    Maturity: 731560
    Compounding: 2
    Basis: 0
    EndMonthRule: 0
    FinObj: 'HJMTimeSpec'

```

### Return a Character Vector Containing a Financial Structure's Class Name

This example shows how to obtain a character vector containing a financial structure's class name.

```
load deriv.mat
ClassName = classfin(HJMTree)

ClassName =
'HJMFwdTree'
```

### See Also

isafin

### Topics

“Portfolio Creation” on page 1-8  
“Instrument Constructors” on page 1-20  
“Hedging” on page 4-2

**Introduced before R2006a**

# cirtimespec

Specify time structure for Cox-Ingersoll-Ross tree

## Syntax

```
TimeSpec = cirtimespec(ValuationDate,Maturity,NumPeriods)
TimeSpec = cirtimespec( ____,Name,Value)
```

## Description

`TimeSpec = cirtimespec(ValuationDate,Maturity,NumPeriods)` creates a time spec for a Cox-Ingersoll-Ross (CIR) tree.

`TimeSpec = cirtimespec( ____,Name,Value)` adds additional name-value pair arguments.

## Examples

### Set the Number of Levels and Node Times for a CIR Tree

Set the number of levels and node times for an CIR tree by specifying a four-period tree with time steps of 1 year.

```
ValuationDate = 'Jan-1-2017';
Maturity = '01-Jan-2021';
NumPeriods = 4
```

```
NumPeriods = 4
```

```
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods)
```

```
CIRTimeSpec = struct with fields:
    FinObj: 'CIRTimeSpec'
    ValuationDate: 736696
    Maturity: 738157
```

```
    NumPeriods: 4
    Compounding: 1
    Basis: 0
    EndMonthRule: 1
    tObs: [0 1 2 3 4]
    dObs: [736696 737061 737426 737791 738157]
```

## Input Arguments

### **ValuationDate** — Date marking the pricing date and first observation tree

serial date number | date character vector

Date marking the pricing date and first observation in the tree, specified as a scalar using a serial date number or date character vector.

Data Types: double | char

### **Maturity** — Date marking the depth of tree

serial date number | date character vector

Date marking the depth of the tree, specified as a scalar using a serial date number or date character vector.

Data Types: double | char

### **NumPeriods** — Determines how many time steps are in tree

nonnegative integer

Determines how many time steps are in tree, specified as a scalar using a nonnegative integer value.

Data Types: double

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: TimeSpec =  
cirtimespec(Valuationdate,Maturity,NumPeriods,'Basis',3)

**Compounding — Frequency at which the rates are compounded when annualized**

1 (default) | integer with value of 1, 2, 3, 4, 6, or 12

Frequency at which the rates are compounded when annualized, specified as the comma-separated pair consisting of 'Compounding' and a scalar value:

- 1 — Annual compounding
- 2 — Semiannual compounding
- 3 — Compounding three times per year
- 4 — Quarterly compounding
- 6 — Bimonthly compounding
- 12 — Monthly compounding

Data Types: double

**Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis, specified as the comma-separated pair consisting of 'Basis' and a scalar value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when `Maturity` is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a scalar.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: `logical`

## Output Arguments

### **TimeSpec — Time layout for CIR tree**

structure

Time layout for the `CIRtree`, returned as a structure.

## See Also

`cirtree`

**Introduced in R2018a**

# cirvolspec

Specify Cox-Ingersoll-Ross interest-rate volatility process

## Syntax

```
VolSpec = cirvolspec(Sigma,Alpha,Theta)
```

## Description

`VolSpec = cirvolspec(Sigma,Alpha,Theta)` creates a Cox-Ingersoll-Ross (CIR) `VolSpec`.

## Examples

### Create a Structure Specifying the Volatility for `cirtree`

Create a Cox-Ingersoll-Ross volatility specification (`CIRVolSpec`) using the following data.

```
Alpha = 0.03;  
Theta = 0.02;  
Sigma = 0.1;  
CIRVolSpec = cirvolspec(Sigma,Alpha,Theta)
```

```
CIRVolSpec = struct with fields:  
  FinObj: 'CIRVolSpec'  
  Sigma: 0.1000  
  Alpha: 0.0300  
  Theta: 0.0200
```

## Input Arguments

### **Sigma — Volatility**

numeric

Volatility, specified as a scalar using a numeric value.

Data Types: double

### **Alpha — Mean reversion speed**

numeric

Mean reversion speed, specified as a scalar using a numeric value.

Data Types: double

### **Theta — Mean reversion level or long-term mean of short rate**

numeric

Mean reversion level or long-term mean of the short rate, specified as a scalar using a numeric value.

Data Types: double

## Output Arguments

### **VolSpec — Volatility model for CIR tree**

structure

Volatility model for the CIRtree, returned as a structure.

## See Also

cirtree

**Introduced in R2018a**



# cirtree

Build a Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
CIRTree = cirvolspec(VolSpec,RateSpec,TimeSpec)
```

## Description

`CIRTree = cirvolspec(VolSpec,RateSpec,TimeSpec)` builds a Cox-Ingersoll-Ross (CIR) interest-rate tree. The CIR tree uses a CIR++ model with the Nawalka-Beliaeva (NB) approach.

## Examples

### Create a CIR Tree

Create a RateSpec using the `intenvset` function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2021';
CIRTimeSpec = cirtimespec(Settle, Maturity, NumPeriods);
```

```
CIRVolSpec = cirvolSpec(Sigma, Alpha, Theta);  
CIRT = cirtree(CIRVolSpec, RateSpec, CIRTTimeSpec)  
CIRT = struct with fields:  
    FinObj: 'CIRFwdTree'  
    VolSpec: [1x1 struct]  
    TimeSpec: [1x1 struct]  
    RateSpec: [1x1 struct]  
    tObs: [0 1 2 3]  
    dObs: [736696 737061 737426 737791]  
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double] [1x7 double]}  
    Connect: {[3x1 double] [3x3 double] [3x5 double]}  
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

## Input Arguments

### **VolSpec — Volatility process specification**

structure

Volatility process specification, specified using the VolSpec output obtained from `cirvolSpec`.

Data Types: `struct`

### **RateSpec — Interest-rate specification for initial risk-free rate curve**

structure

Interest-rate specification for initial risk-free rate curve, specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **TimeSpec — Time tree layout specification**

structure

Time tree layout specification, specified using the TimeSpec output obtained from `cirtimespec`.

Data Types: `struct`

## Output Arguments

**CIRTree** — Time and interest-rate information of a recombining tree  
structure

Time and interest-rate information of a recombining tree, returned as a structure.

## See Also

`cirtimespec` | `cirvolspec`

**Introduced in R2018a**

## compoundbycrr

Price compound option from Cox-Ross-Rubinstein binomial tree

### Syntax

```
[Price,PriceTree] = compoundbycrr(CRRTree,UOptSpec,UStrike,USettle,  
UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates)  
[Price,PriceTree] = compoundbycrr( ____,CAmericanOpt)
```

### Description

[Price,PriceTree] = compoundbycrr(CRRTree,UOptSpec,UStrike,USettle, UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates) prices compound options from a Cox-Ross-Rubinstein binomial tree.

[Price,PriceTree] = compoundbycrr( \_\_\_\_,CAmericanOpt) adds an optional argument for CAmericanOpt.

### Examples

#### Price a Compound Option Using a CRR Binomial Tree

This example shows how to price a compound option using a CRR binomial tree by loading the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat  
  
UOptSpec = 'Call';  
UStrike = 130;  
USettle = '01-Jan-2003';  
UExerciseDates = '01-Jan-2006';  
UAmericanOpt = 1;  
COptSpec = 'Put';
```

```
CStrike = 5;
CSettle = '01-Jan-2003';
CExerciseDates = '01-Jan-2005';

Price = compoundbycrr(CRRTree, UOptSpec, UStrike, USettle, ...
    UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
    CExerciseDates)

Price = 2.8482
```

## Input Arguments

### **CRRTree** — Stock tree structure

structure

Stock tree structure, specified by using `crrtree`.

Data Types: `struct`

### **UOptSpec** — Definition of underlying option

character vector with value 'call' or 'put'

Definition of underlying option, specified as 'call' or 'put' using a character vector.

Data Types: `char`

### **UStrike** — Underlying option strike price value

nonnegative integer

Underlying option strike price value, specified with a nonnegative integer using a 1-by-1 vector.

Data Types: `double`

### **USettle** — Underlying option settlement date or trade date

serial date number | date character vector

Underlying option settlement date or trade date, specified as a 1-by-1 vector using a serial date number or character vector.

Data Types: `double` | `char`

**UExerciseDates — Underlying option exercise date**

serial date number | date character vector

Underlying option exercise date, specified as a serial date number or date character vector:

- For a European option, use a 1-by-1 vector of the underlying exercise date. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a 1-by-2 vector of the underlying exercise date boundaries. The option can be exercised on any tree date. If only one non-`NaN` date is listed, or if `ExerciseDates` is 1-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

**UAmericanOpt — Underlying option type**

0 European (default) | scalar with values 0 or 1

Underlying option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

If `UAmericanOpt` is a `NaN` or is unspecified, the option is a European option.

Data Types: double

**COptSpec — Definition of compound option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of compound option, specified as 'call' or 'put' using a character vector or a cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

**CStrike — Compound option strike price values**

nonnegative integers

Compound option strike price values for a European and American option, specified with a nonnegative integer using a NINST-by-1 matrix. Each row is the schedule for one option.

Data Types: double

### **CSettle — Compound option settlement date or trade date**

serial date number | date character vector

Compound option settlement date or trade date, specified as a 1-by-1 vector using a serial date number or date character vector.

Data Types: double | char

### **CExerciseDates — Compound option exercise dates**

serial date number | date character vector

Compound option exercise dates, specified as serial date numbers or date character vectors:

- For a European option, use a `aNINST`-by-1 matrix of the compound exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST`-by-2 vector of the compound exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

### **CAmericanOpt — Compound option type**

0 European (default) | scalar with values 0 or 1

(Optional) Compound option type, specified as `NINST`-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

If `CAmericanOpt` is a `NaN` or is unspecified, the option is a European option.

Data Types: double

## Output Arguments

### **Price** — Expected prices for compound options at time 0

vector

Expected prices for compound options at time 0, returned as a NINST-by-1 vector.

### **PriceTree** — Structure with vector of compound option prices at each node

tree structure

Structure with a vector of compound option prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.t0bs contains the observation times.

PriceTree.d0bs contains the observation dates.

## References

[1] Rubinstein, Mark. “Double Trouble.” *Risk*. Vol. 5, 1991, p. 73.

## See Also

`crrtree` | `instcompound`

## Topics

“Computing Prices Using CRR” on page 3-131

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Compound Option” on page 3-30

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**



# compoundbyeqp

Price compound option from Equal Probabilities binomial tree

## Syntax

```
[Price,PriceTree] = compoundbyeqp(EQPTree,UOptSpec,UStrike,USettle,
UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates)
[Price,PriceTree] = compoundbyeqp( ____,CAmericanOpt)
```

## Description

[Price,PriceTree] = compoundbyeqp(EQPTree,UOptSpec,UStrike,USettle, UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates) prices compound options from a Equal Probabilities binomial tree.

[Price,PriceTree] = compoundbyeqp( \_\_\_\_,CAmericanOpt) adds an optional argument for CAmericanOpt.

## Examples

### Price a Compound Option Using an EQP Equity Tree

This example shows how to price a compound option using a EQP equity tree by loading the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2003';
UExerciseDates = '01-Jan-2006';
UAmericanOpt = 1;
COptSpec = 'Put';
CStrike = 5;
```

```
CSettle = '01-Jan-2003';
CExerciseDates = '01-Jan-2005';

Price = compoundbyeqp(EQPTree, UOptSpec, UStrike, USettle, ...
    UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
    CExerciseDates)

Price = 3.3931
```

## Input Arguments

### **EQPTree** — Stock tree structure

structure

Stock tree structure, specified by using `eqptree`.

Data Types: `struct`

### **UOptSpec** — Definition of underlying option

character vector with value 'call' or 'put'

Definition of underlying option, specified as 'call' or 'put' using a character vector.

Data Types: `char`

### **UStrike** — Underlying option strike price value

nonnegative integer

Underlying option strike price value, specified with a nonnegative integer using a 1-by-1 vector.

Data Types: `double`

### **USettle** — Underlying option settlement date or trade date

serial date number | date character vector

Underlying option settlement date or trade date, specified as a 1-by-1 vector using a serial date number or character vector.

Data Types: `double` | `char`

### **UExerciseDates** — Underlying option exercise date

serial date number | date character vector

Underlying option exercise date, specified as a serial date number or date character vector:

- For a European option, use a 1-by-1 vector of the underlying exercise date. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a 1-by-2 vector of the underlying exercise date boundaries. The option can be exercised on any tree date. If only one non-NaN date is listed, or if `ExerciseDates` is 1-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

### **UAmericanOpt — Underlying option type**

0 European (default) | scalar with values 0 or 1

Underlying option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

If `UAmericanOpt` is a NaN or is unspecified, the option is a European option.

Data Types: double

### **COptSpec — Definition of compound option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of compound option, specified as 'call' or 'put' using a character vector or a cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

### **CStrike — Compound option strike price values**

nonnegative integers

Compound option strike price values for a European and American option, specified with a nonnegative integer using a NINST-by-1 matrix. Each row is the schedule for one option.

Data Types: double

### **CSettle — Compound option settlement date or trade date**

serial date number | date character vector

Compound option settlement date or trade date, specified as a 1-by-1 vector using a serial date number or date character vector.

Data Types: double | char

### **CExerciseDates — Compound option exercise dates**

serial date number | date character vector

Compound option exercise dates, specified as serial date numbers or date character vectors:

- For a European option, use a NINST-by-1 matrix of the compound exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a NINST-by-2 vector of the compound exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

### **CAmericanOpt — Compound option type**

0 European (default) | scalar with values 0 or 1

(Optional) Compound option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

If `CAmericanOpt` is a NaN or is unspecified, the option is a European option.

Data Types: double

## **Output Arguments**

### **Price — Expected prices for compound options at time 0**

vector

Expected prices for compound options at time 0, returned as a NINST-by-1 vector.

## **PriceTree — Structure with vector of compound option prices at each node**

tree structure

Structure with a vector of compound option prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.t0bs contains the observation times.

PriceTree.d0bs contains the observation dates.

## **References**

[1] Rubinstein, Mark. "Double Trouble." *Risk*. Vol. 5, 1991, p. 73.

## **See Also**

eqptree | instcompound

## **Topics**

"Computing Prices Using CRR" on page 3-131

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Compound Option" on page 3-30

"Supported Equity Derivatives" on page 3-24

**Introduced before R2006a**

## compoundbyitt

Price compound option from implied trinomial tree (ITT)

### Syntax

```
[Price,PriceTree] = compoundbyitt(ITTree,UOptSpec,UStrike,USettle,  
UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates)  
[Price,PriceTree] = compoundbyitt( ____,CAmericanOpt)
```

### Description

```
[Price,PriceTree] = compoundbyitt(ITTree,UOptSpec,UStrike,USettle,  
UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates)
```

prices compound options from a Equal Probabilities binomial tree.

`[Price,PriceTree] = compoundbyitt( ____,CAmericanOpt)` adds an optional argument for `CAmericanOpt`.

### Examples

#### Price a Compound Option Using an ITT Tree

This example shows how to price a compound option using a ITT tree by loading the file `deriv.mat`, which provides `ITTree`. The `ITTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat  
  
UOptSpec = 'Call';  
UStrike = 99;  
USettle = '01-Jan-2006';  
UExerciseDates = '01-Jan-2010';  
UAmericanOpt = 1;  
COptSpec = 'Put';
```

```
CStrike = 5;
CSettle = '01-Jan-2006';
CExerciseDates = '01-Jan-2010';

Price = compoundbyitt(ITTTree, UOptSpec, UStrike, USettle, ...
    UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
    CExerciseDates)

Price = 2.7271
```

## Input Arguments

### **ITTTree** — Stock tree structure

structure

Stock tree structure, specified by using `itttree`.

Data Types: `struct`

### **UOptSpec** — Definition of underlying option

character vector with value 'call' or 'put'

Definition of underlying option, specified as 'call' or 'put' using a character vector.

Data Types: `char`

### **UStrike** — Underlying option strike price value

nonnegative integer

Underlying option strike price value, specified with a nonnegative integer using a 1-by-1 vector.

Data Types: `double`

### **USettle** — Underlying option settlement date or trade date

serial date number | date character vector

Underlying option settlement date or trade date, specified as a 1-by-1 vector using a serial date number or character vector.

Data Types: `double` | `char`

**UExerciseDates — Underlying option exercise date**

serial date number | date character vector

Underlying option exercise date, specified as a serial date number or date character vector:

- For a European option, use a 1-by-1 vector of the underlying exercise date. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a 1-by-2 vector of the underlying exercise date boundaries. The option can be exercised on any tree date. If only one non-`NaN` date is listed, or if `ExerciseDates` is 1-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

**UAmericanOpt — Underlying option type**

0 European (default) | scalar with values 0 or 1

Underlying option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

If `UAmericanOpt` is a `NaN` or is unspecified, the option is a European option.

Data Types: double

**COptSpec — Definition of compound option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of compound option, specified as 'call' or 'put' using a character vector or a cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

**CStrike — Compound option strike price values**

nonnegative integers

Compound option strike price values for a European and American option, specified with a nonnegative integer using a NINST-by-1 matrix. Each row is the schedule for one option.



Data Types: double

### **CSettle — Compound option settlement date or trade date**

serial date number | date character vector

Compound option settlement date or trade date, specified as a 1-by-1 vector using a serial date number or date character vector.

Data Types: double | char

### **CExerciseDates — Compound option exercise dates**

serial date number | date character vector

Compound option exercise dates, specified as serial date numbers or date character vectors:

- For a European option, use a `aNINST`-by-1 matrix of the compound exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST`-by-2 vector of the compound exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

### **CAmericanOpt — Compound option type**

0 European (default) | scalar with values 0 or 1

(Optional) Compound option type, specified as `NINST`-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

If `CAmericanOpt` is a `NaN` or is unspecified, the option is a European option.

Data Types: double

## Output Arguments

### **Price** — Expected prices for compound options at time 0

vector

Expected prices for compound options at time 0, returned as a NINST-by-1 vector.

### **PriceTree** — Structure with vector of compound option prices at each node

tree structure

Structure with a vector of compound option prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.t0bs contains the observation times.

PriceTree.d0bs contains the observation dates.

## References

[1] Rubinstein, Mark. “Double Trouble.” *Risk*. Vol. 5, 1991, p. 73.

## See Also

instcompound | itttree

## Topics

“Computing Prices Using CRR” on page 3-131

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Compound Option” on page 3-30

“Supported Equity Derivatives” on page 3-24

## Introduced in R2007a

# compoundbystt

Price compound options using standard trinomial tree

## Syntax

```
[Price,PriceTree] = compoundbystt(STTTree,UOptSpec,UStrike,USettle,
UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates)
[Price,PriceTree] = compoundbystt( ____,Name,Value)
```

## Description

[Price,PriceTree] = compoundbystt(STTTree,UOptSpec,UStrike,USettle,UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates) prices compound options using a standard trinomial (STT) tree.

[Price,PriceTree] = compoundbystt( \_\_\_\_,Name,Value) adds an optional name-value pair argument for CAmericanOpt.

## Examples

### Price a Compound Option Using the Standard Trinomial Tree Model

Create a RateSpec.

```
StartDates = 'Jan-1-2009';
EndDates = 'Jan-1-2013';
Rates = 0.035;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
```

```
Compounding: -1
  Disc: 0.8694
  Rates: 0.0350
  EndTimes: 4
  StartTimes: 0
  EndDates: 735235
  StartDates: 733774
ValuationDate: 733774
  Basis: 1
  EndMonthRule: 1
```

Create a `StockSpec`.

```
AssetPrice = 85;
Sigma = 0.15;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
  FinObj: 'StockSpec'
  Sigma: 0.1500
  AssetPrice: 85
  DividendType: []
  DividendAmounts: 0
  ExDividendDates: []
```

Create an `STTree`.

```
NumPeriods = 4;
TimeSpec = stttimespec(StartDates, EndDates, 4);
STTree = stttree(StockSpec, RateSpec, TimeSpec)
```

```
STTree = struct with fields:
  FinObj: 'STStockTree'
  StockSpec: [1x1 struct]
  TimeSpec: [1x1 struct]
  RateSpec: [1x1 struct]
  tObs: [0 1 2 3 4]
  dObs: [733774 734139 734504 734869 735235]
  STree: {1x5 cell}
  Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
```

Define the compound option and compute the price.

```

USettle = '1/1/09';
UExerciseDates = '1/1/12';
UOptSpec = 'call';
UStrike = 95;
UAmericanOpt = 1;
CSettle = '1/1/09';
CExerciseDates = '1/1/11';
COptSpec = 'put';
CStrike = 5;
CAmericanOpt = 1;

```

```

Price= compoundbystt(STTtree, UOptSpec, UStrike, USettle, UExerciseDates,...
UAmericanOpt, COptSpec, CStrike, CSettle,CExerciseDates, CAmericanOpt)

```

```
Price = 1.7090
```

## Input Arguments

**STTtree** — Stock tree structure for standard trinomial tree structure

Stock tree structure for standard trinomial tree, specified by using `stttree`.

Data Types: `struct`

**UOptSpec** — Definition of underlying option character vector with value 'call' or 'put'

Definition of underlying option, specified as 'call' or 'put' using a character vector.

Data Types: `char`

**UStrike** — Underlying option strike price value nonnegative integer

Underlying option strike price value, specified with a nonnegative integer using a 1-by-1 vector.

Data Types: `double`

**USettle** — Underlying option settlement date or trade date serial date number | date character vector

Underlying option settlement date or trade date, specified as a 1-by-1 vector using a serial date number or character vector.

Data Types: double | char

### **UExerciseDates — Underlying option exercise date**

serial date number | date character vector

Underlying option exercise date, specified as a serial date number or date character vector:

- For a European option, use a 1-by-1 vector of the underlying exercise date. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a 1-by-2 vector of the underlying exercise date boundaries. The option can be exercised on any tree date. If only one non-`NaN` date is listed, or if `ExerciseDates` is 1-by-1, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

### **UAmericanOpt — Underlying option type**

0 European (default) | scalar with values 0 or 1

Underlying option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

If `UAmericanOpt` is a `NaN` or is unspecified, the option is a European option.

Data Types: single | double

### **C0ptSpec — Definition of compound option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of compound option, specified as 'call' or 'put' using a character vector or a cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

### **CStrike — Compound option strike price values**

nonnegative integers

Compound option strike price values for a European and American option, specified with a nonnegative integer using a `NINST-by-1` matrix. Each row is the schedule for one option.

Data Types: `double`

### **CSettle — Compound option settlement date or trade date**

`serial date number` | `date character vector`

Compound option settlement date or trade date, specified as a `1-by-1` vector using a serial date number or date character vector.

Data Types: `double` | `char`

### **CExerciseDates — Compound option exercise dates**

`serial date number` | `date character vector`

Compound option exercise dates, specified as serial date numbers or date character vectors:

- For a European option, use a `nINST-by-1` matrix of the compound exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST-by-2` vector of the compound exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST-by-1`, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
compoundbystt(STTtree,UOptSpec,UStrike,USettle,UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates,'CAmericanOpt',1)
```

**CAmericanOpt — Compound option type**

0 European (default) | scalar with values [0,1]

Compound option type, specified as the comma-separated pair consisting of 'CAmericanOpt' and a NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

Data Types: single | double

## Output Arguments

**Price — Expected prices for compound options at time 0**

vector

Expected prices for compound options at time 0, returned as a NINST-by-1 vector.

**PriceTree — Structure with vector of compound option prices at each node**

tree structure

Structure with a vector of compound option prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.tObs contains the observation times.

PriceTree.dObs contains the observation dates.

## See Also

instcompound | sttprice | sttsens | stttimespec | stttree

## Topics

“Compound Option” on page 3-30



“Supported Equity Derivatives” on page 3-24

**Introduced in R2015b**

## crrprice

Instrument prices from Cox-Ross-Rubinstein tree

### Syntax

```
[Price,PriceTree] = crrprice(CRRTree,InstSet)
[Price,PriceTree] = crrprice( ____,Options)
```

### Description

[Price,PriceTree] = crrprice(CRRTree,InstSet) computes stock option prices using a CRR binomial tree created with `crrtree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`crrprice` handles instrument types: 'Asian', 'Barrier', 'Compound', 'CBond', 'Lookback', 'OptStock'. See `instadd` to construct defined types.

[Price,PriceTree] = crrprice( \_\_\_\_,Options) adds an optional input argument for `Options`.

### Examples

#### Price Barrier and Lookback Options in the Instrument Set

Load the CRR tree and instruments from the data file `deriv.mat`. Price the barrier and lookback options contained in the instrument set.

```
load deriv.mat;
CRRSubSet = instselect(CRRInstSet,'Type', ...
{'Barrier', 'Lookback'});
```

```
instdisp(CRRSubSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Bar
1	Barrier	call	105	01-Jan-2003	01-Jan-2006	1	ui	102

---

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quant
2	Lookback	call	115	01-Jan-2003	01-Jan-2006	0	Lookback1	7
3	Lookback	call	115	01-Jan-2003	01-Jan-2007	0	Lookback2	9

Price the barrier and lookback options.

```
[Price, PriceTree] = crrprice(CRRTree,CRRSubSet)
```

```
Price = 3x1
```

```
12.1272
```

```
7.6015
```

```
11.7772
```

```
PriceTree = struct with fields:
```

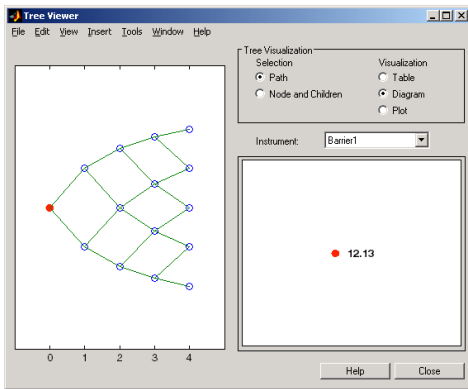
```
FinObj: 'BinPriceTree'
```

```
PTree: {1x5 cell}
```

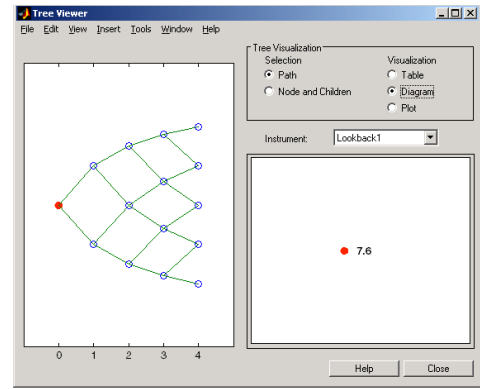
```
tObs: [0 1 2 3 4]
```

```
dObs: [731582 731947 732313 732678 733043]
```

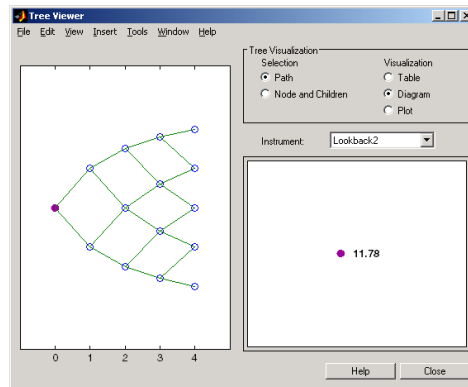
You can use `treeview` to see the prices of these three instruments along the price tree.



Barrier1



Lookback1



Lookback2

## Input Arguments

**CRRTree — Stock price tree structure**  
structure

Stock price tree structure, specified by using `crrtree`.

Data Types: `struct`

### **InstSet — Instrument variable**

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Price for each instrument**

vector

Price for each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned in that entry.

Related single-type pricing functions are:

- `asianbycrr`: Price an Asian option from a CRR tree.
- `barrierbycrr`: Price a barrier option from a CRR tree.
- `cbondbycrr`: Price convertible bonds from a CRR tree.
- `compoundbycrr`: Price a compound option from a CRR tree.
- `lookbackbycrr`: Price a lookback option from a CRR tree.
- `optstockbycrr`: Price an American, Bermuda, or European option from a CRR tree.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.t0bs` contains the observation times.
- `PriceTree.d0bs` contains the observation dates.

### See Also

`cbondbycrr` | `crrsens` | `crrtree` | `instadd` | `instcbond`

### Topics

“Computing Prices Using CRR” on page 3-131

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## crrsens

Instrument prices and sensitivities from Cox-Ross-Rubinstein tree

### Syntax

```
[Delta,Gamma,Vega,Price] = crrsens(CRRTree,InstSet)
[Delta,Gamma,Vega,Price] = crrsens( ____,Options)
```

### Description

`[Delta,Gamma,Vega,Price] = crrsens(CRRTree,InstSet)` computes instrument sensitivities and prices for instruments using a binomial tree created with the `crrtree` function. All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

`crrsens` handles instrument types: 'Asian', 'Barrier', 'Compound', 'CBond', 'Lookback', 'OptStock'. See `instadd` for information on instrument types.

`[Delta,Gamma,Vega,Price] = crrsens( ____,Options)` adds an optional input argument for `Options`.

### Examples

#### Compute Sensitivities for Barrier and Lookback Instruments Using a `crrtree`

Load the CRR tree and instruments from the data file `deriv.mat`. Compute the Delta and Gamma sensitivities of the barrier and lookback options contained in the instrument set.

```
load deriv.mat;
CRRSubSet = instselect(CRRInstSet,'Type', ...
{'Barrier', 'Lookback'});
instdisp(CRRSubSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Bar
1	Barrier	call	105	01-Jan-2003	01-Jan-2006	1	ui	102
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quant
2	Lookback	call	115	01-Jan-2003	01-Jan-2006	0	Lookback1	7
3	Lookback	call	115	01-Jan-2003	01-Jan-2007	0	Lookback2	9

Obtain the Delta and Gamma for the barrier and lookback options contained in the instrument set.

```
[Delta, Gamma] = crrsens(CRRTree, CRRSubSet)
```

```
Delta = 3×1
```

```
    0.6885
    0.6049
    0.8187
```

```
Gamma = 3×1
```

```
    0.0310
   -0.0000
         0
```

## Input Arguments

### **CRRTree** — Stock tree structure

structure

Stock tree structure, specified by using `crrtree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.



Data Types: `struct`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Delta — Rate of change of instruments prices with respect to changes in the stock price**

vector

Rate of change of instruments prices with respect to changes in the stock price, returned as a NINST-by-1 vector of deltas.

For path-dependent options ('Lookback' and 'Asian'), Delta and Gamma are computed by finite differences in calls to `crrprice`. For the rest of the options ('OptStock', 'Barrier', 'CBond', and 'Compound'), Delta and Gamma are computed from the `CRRTree` and the corresponding option price tree.

### **Gamma — Rate of change of instruments deltas with respect to changes in stock price**

vector

Rate of change of instruments deltas with respect to changes in the stock price, returned as a NINST-by-1 vector of gammas.

For path-dependent options ('Lookback' and 'Asian'), Delta and Gamma are computed by finite differences in calls to `crrprice`. For the rest of the options ('OptStock', 'Barrier', 'CBond', and 'Compound'), Delta and Gamma are computed from the `CRRTree` and the corresponding option price tree.

### **Vega — Rate of change of instruments prices with respect to changes in volatility of the stock**

vector

Rate of change of instruments prices with respect to changes in the volatility of the stock, returned as a NINST-by-1 vector of vegas. Vega is computed by finite differences in calls to `crrtree`.

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned in that entry.

### **References**

[1] Chriss, Neil. *Black-Scholes and Beyond: Option Pricing Models*. McGraw-Hill, 1996, pp 308-312.

### **See Also**

`cbondbycrr` | `crrprice` | `crrtree` | `instcbond`

### **Topics**

“Computing Prices Using CRR” on page 3-131

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

# crrtimespec

Specify time structure for Cox-Ross-Rubinstein tree

## Syntax

```
TimeSpec = crrtimespec(ValuationDate,Maturity,NumPeriods)
```

## Description

`TimeSpec = crrtimespec(ValuationDate,Maturity,NumPeriods)` sets the number of levels and node times for a CRR binomial tree (`crrtree`).

## Examples

### Set the Number of Levels and Node Times for a CRR Binomial Tree

This example shows how to specify a four-period CRR tree with time steps of 1 year.

```
ValuationDate = '1-July-2002';  
Maturity = '1-July-2006';  
TimeSpec = crrtimespec(ValuationDate, Maturity, 4)  
  
TimeSpec = struct with fields:  
    FinObj: 'BinTimeSpec'  
    ValuationDate: 731398  
    Maturity: 732859  
    NumPeriods: 4  
    Basis: 0  
    EndMonthRule: 1  
    tObs: [0 1 2 3 4]  
    dObs: [731398 731763 732128 732493 732859]
```

## Input Arguments

### **ValuationDate** — Pricing date and first observation in the tree

serial date number | character vector date

Pricing date and first observation in the `crmtree`, specified as a scalar date using a serial date number or date character vector.

Data Types: `double` | `char`

### **Maturity** — Date marking the depth of the CRR stock tree

serial date number | date character vector

Date marking the depth of the `crmtree` binomial tree, specified as scalar serial date number or date character vector.

Data Types: `double` | `char`

### **NumPeriods** — Number of time steps in the CRR stock tree

integer

Number of time steps in the `crmtree` binomial tree, specified as scalar integer value.

Data Types: `double`

## Output Arguments

### **TimeSpec** — Specification for the time layout for `crmtree`

structure

Specification for the time layout for `crmtree`, returned as a structure.

## See Also

`crmtree` | `stockspec`

## Topics

“Building Equity Binary Trees” on page 3-3

“Examining Equity Trees” on page 3-18

“Understanding Equity Trees” on page 3-2

“Differences Between CRR and EQP Tree Structures” on page 3-22  
“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## crrtree

Build Cox-Ross-Rubinstein stock tree

### Syntax

```
CRRTree = crrtree(StockSpec,RateSpec,TimeSpec)
```

### Description

CRRTree = crrtree(StockSpec,RateSpec,TimeSpec) builds a Cox-Ross-Rubinstein stock tree.

### Examples

#### Create a CRR Tree

Using the data provided, create a stock specification (StockSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create a CRR tree with crrtree.

```
Sigma = 0.20;  
AssetPrice = 50;  
DividendType = 'cash';  
DividendAmounts = [0.50; 0.50; 0.50; 0.50];  
ExDividendDates = {'03-Jan-2003'; '01-Apr-2003'; '05-July-2003';  
'01-Oct-2003'};  
  
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...  
DividendAmounts, ExDividendDates);  
  
RateSpec = intenvset('Rates', 0.05, 'StartDates', ...  
'01-Jan-2003', 'EndDates', '31-Dec-2003');  
  
ValuationDate = '1-Jan-2003';  
Maturity = '31-Dec-2003';
```

```
TimeSpec = crrtimespec(ValuationDate, Maturity, 4);
```

```
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)
```

Warning: RateSpec was not created with continuous compounding. Compounding will be set to continuous while leaving discount factors unaltered. This will result in the recalculation of the interest rates.

```
CRRTree =
```

```
struct with fields:
```

```
    FinObj: 'BinStockTree'
    Method: 'CRR'
    StockSpec: [1×1 struct]
    TimeSpec: [1×1 struct]
    RateSpec: [1×1 struct]
        tObs: [0 0.2493 0.4986 0.7479 0.9972]
        dObs: [731582 731673 731764 731855 731946]
    STree: {1×5 cell}
    UpProbs: [0.5370 0.5370 0.5370 0.5370]
```

Use `treeview` to observe the tree you have created.

## Input Arguments

### StockSpec — Stock specification

structure

Stock specification, specified by the `StockSpec` obtained from `stockspec`. See `stockspec` for information on creating a stock specification.

Data Types: `struct`

### RateSpec — Interest-rate specification for initial risk-free rate curve

structure

Interest-rate specification for initial risk-free rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

**Note** The standard CRR tree assumes a constant interest rate, but `RateSpec` allows you to specify an interest-rate curve with varying rates. If you specify variable interest rates, the resulting tree is not a standard CRR tree.

---

Data Types: `struct`

### **TimeSpec — Tree time layout specification**

structure

Tree time layout specification, specified by the `TimeSpec` obtained from `crrtimespec`. The `TimeSpec` defines the observation dates of the CRR binomial tree. See `crrtimespec` for information on the tree structure.

Data Types: `struct`

## **Output Arguments**

### **CRRTree — CRR binomial tree**

structure

CRR binomial tree, returned as a structure specifying the time layout for the tree.

## **See Also**

`crrtimespec` | `intenvset` | `stockspec`

## **Topics**

“Building Equity Binary Trees” on page 3-3

“Examining Equity Trees” on page 3-18

“Understanding Equity Trees” on page 3-2

“Differences Between CRR and EQP Tree Structures” on page 3-22

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**



## cvtree

Convert inverse-discount tree to interest-rate tree

### Syntax

```
RateTree = cvtree(Tree)
```

### Arguments

Tree	Heath-Jarrow-Morton, Black-Derman-Toy, Hull-White, Black-Karasinski, or Cox-Ingersoll-Ross tree structure using inverse-discount notation for forward rates.
------	--

### Description

`RateTree = cvtree(Tree)` converts a tree structure using inverse-discount notation to a tree structure using rate notation for forward rates.

### Examples

Convert a Hull-White tree using inverse-discount notation to a Hull-White tree displaying interest-rate notation.

```
load deriv.mat;
```

```
HWTtree
```

```
HWTtree =
```

```
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
```

```
d0bs: [731947 732313 732678 733043]
CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}
Connect: {[2] [2 3 4] [2 2 3 4 4]}
FwdTree: {1x4 cell}
```

```
HWTree.FwdTree{1}
```

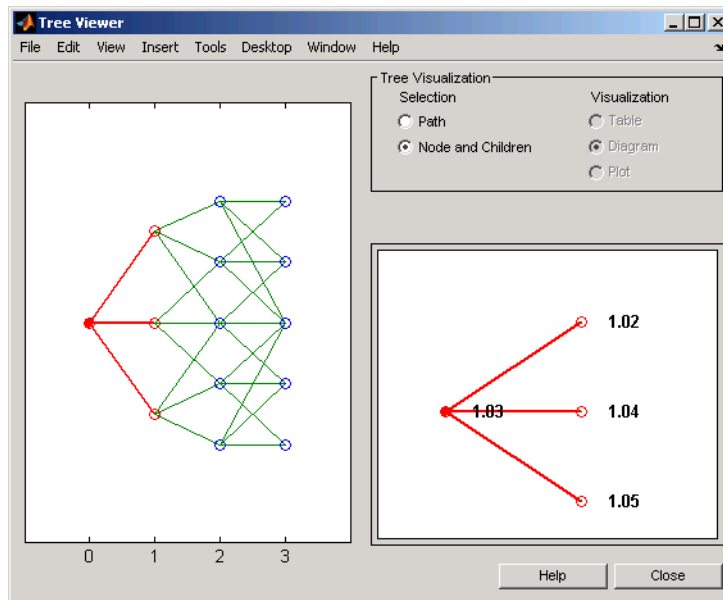
```
ans =
    1.0279
```

```
HWTree.FwdTree{2}
```

```
ans =
    1.0528    1.0356    1.0186
```

Use `treeviewer` to display the path of interest rates expressed in inverse-discount notation.

```
treeviewer(HWTree)
```



Use `cvtree` to convert the inverse-discount notation to interest-rate notation.

```
RTree = cvtree(HWTree)
```

```
RTree =  
    FinObj: 'HWRateTree'  
    VolSpec: [1x1 struct]  
    TimeSpec: [1x1 struct]  
    RateSpec: [1x1 struct]  
        tObs: [0 1 2 3]  
        dObs: [731947 732313 732678 733043]  
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}  
    Probs: {[3x1 double] [3x3 double] [3x5 double]}  
    Connect: {[2] [2 3 4] [2 2 3 4 4]}  
    RateTree: {1x4 cell}
```

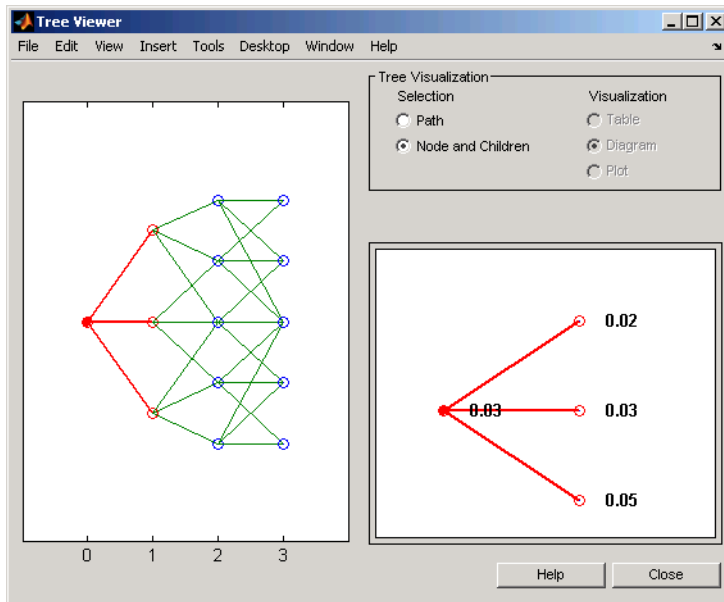
```
RTree.RateTree{1}
```

```
ans =  
    0.0275
```

```
RTree.RateTree{2}
```

```
ans =  
    0.0514    0.0349    0.0185
```

Now use `treeviewer` to display the converted tree, showing the path of interest rates expressed as forward rates.



## See Also

[disc2rate](#) | [rate2disc](#)

## Topics

“Graphical Representation of Trees” on page 2-157

**Introduced before R2006a**

## date2time

Time and frequency from dates

### Syntax

[Times,F] = date2time(Settle,Dates,Compounding,Basis,EndMonthRule)

### Arguments

Settle	Settlement date. A vector of serial date numbers or date character vectors.
Dates	Vector of dates corresponding to the compounding value.
Compounding	<p>(Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:</p> <p>Compounding = 1, 2, 3, 4, 6, 12 (Default = 2.)</p> <p>Disc = <math>(1 + Z/F)^{-T}</math>, where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units; for example, T = F is one year.</p> <p>Compounding = 365</p> <p>Disc = <math>(1 + Z/F)^{-T}</math>, where F is the number of days in the basis year and T is a number of days elapsed computed by basis.</p> <p>Compounding = -1</p> <p>Disc = <math>\exp(-T*Z)</math>, where T is time in years.</p>

<p><b>Basis</b></p>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>EndMonthRule</b></p>	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>

## Description

`[Times,F] = date2time(Settle,Dates,Compounding,Basis,EndMonthRule)` computes time factors appropriate to compounded rate quotes beyond the settlement date.

Times is a vector of time factors.

F is a scalar of related compounding frequencies.

---

**Note** To obtain accurate results from this function, the **Basis** and **Dates** arguments must be consistent. If the **Dates** argument contains months that have 31 days, **Basis** must be one of the values that allow months to contain more than 30 days; for example, **Basis** = 0, 3, or 7.

---

date2time is the inverse of time2date.

## See Also

cftimes | disc2rate | rate2disc | time2date

## Topics

“Modeling the Interest-Rate Term Structure” on page 2-67

“Interest-Rate Term Conversions” on page 2-62

“Interest Rates Versus Discount Factors” on page 2-55

“Graphical Representation of Trees” on page 2-157

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## datedisp

Display date entries

### Syntax

```
datedisp(NumMat,DateForm)
CharMat = datedisp(NumMat,DateForm)
```

### Arguments

NumMat	Numeric matrix to display.
DateForm	(Optional) Date format. See <code>datestr</code> for available and default format flags.

### Description

`datedisp(NumMat,DateForm)` displays the matrix with the serial dates formatted as date character vectors, using a matrix with mixed numeric entries and serial date number entries. Integers between `datenum('01-Jan-1900')` and `datenum('01-Jan-2200')` are assumed to be serial date numbers, while all other values are treated as numeric entries.

`CharMat` is a character array representing `NumMat`. If no output variable is assigned, the function prints the array to the display (`CharMat = datedisp(NumMat,DateForm)`).

### Examples

```
NumMat = [ 730730, 0.03, 1200, 730100;
           730731, 0.05, 1000, NaN]
```

```
NumMat =
    1.0e+05 *
```



```
7.3073    0.0000    0.0120    7.3010
7.3073    0.0000    0.0100     NaN
```

`datedisp(NumMat)`

```
01-Sep-2000  0.03  1200  11-Dec-1998
02-Sep-2000  0.05  1000     NaN
```

## Tips

This function is identical to the `datedisp` function in Financial Toolbox software.

## See Also

`datenum` | `datestr`

## Topics

“Modeling the Interest-Rate Term Structure” on page 2-67

“Interest-Rate Term Conversions” on page 2-62

“Interest Rates Versus Discount Factors” on page 2-55

“Graphical Representation of Trees” on page 2-157

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## derivget

Get derivatives pricing options

### Syntax

```
Value = derivget(Options, 'Parameter')
```

### Arguments

Options	Existing options specification structure, probably created from previous call to <code>derivset</code> .
Parameter	Must be 'Diagnostics', 'Warnings', 'ConstRate', or 'BarrierMethod'. It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for parameter names.

### Description

`Value = derivget(Options, 'Parameter')` extracts the value of the named parameter from the derivative options structure `Options`. Parameter values can be 'off' or 'on', except for 'BarrierMethod', which can be 'unenhanced' or 'interp'. Specifying 'unenhanced' uses no correction calculation. Specifying 'interp' uses an enhanced valuation interpolating between nodes on barrier boundaries.

### Examples

**Example 1.** Create an Options structure with the value of `Diagnostics` set to 'on'.

```
Options = derivset('Diagnostics', 'on')
```

Use `derivget` to extract the value of `Diagnostics` from the `Options` structure.

```
Value = derivget(Options, 'Diagnostics')
```

```
Value =
```

```
on
```

**Example 2.** Use `derivget` to extract the value of `ConstRate`.

```
Value = derivget(Options, 'ConstRate')
```

```
Value =
```

```
on
```

Because the value of `'ConstRate'` was not previously set with `derivset`, the answer represents the default setting for `'ConstRate'`.

**Example 3.** Find the value of `'BarrierMethod'` in this structure.

```
derivget(Options, 'BarrierMethod')
```

```
ans =
```

```
unenhanced
```

## See Also

`barrierbycrr` | `barrierbyeqp` | `derivset`

## Topics

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

**Introduced before R2006a**

## derivset

Set or modify derivatives pricing options

### Syntax

```
Options = derivset(Options, 'Parameter1', Value1, ...
'Parameter4', Value4)
Options = derivset(OldOptions, NewOptions)
Options = derivset
derivset
```

### Arguments

Options	(Optional) Existing options specification structure, probably created from a previous call to <code>derivset</code> .
Parameter <i>n</i>	The parameter must be 'Diagnostics', 'Warnings', 'ConstRate', or 'BarrierMethod'. Parameters can be entered in any order.
Value <i>n</i>	<p>(BDT, BK, HJM, or HW pricing only) The parameter values for the following three options can be 'on' or 'off':</p> <ul style="list-style-type: none"> <li>'Diagnostics' 'on' generates diagnostic information. The default is 'Diagnostics' 'off'.</li> <li>'Warnings' 'on' (default) displays a warning message when executing a pricing function.</li> <li>'ConstRate' 'on' (default) assumes a constant rate between tree nodes.</li> </ul> <p>For pricing barrier options, the 'BarrierMethod' pricing option can be 'unenhanced' (default) or 'interp'. Specifying 'unenhanced' uses no correction calculation. Specifying 'interp' uses an enhanced valuation interpolating between nodes on barrier boundaries.</p>

<code>OldOptions</code>	Existing options specification structure.
<code>NewOptions</code>	New options specification structure.

## Description

`Options = derivset(Options, 'Parameter1', Value1, ... 'Parameter4', Value4)` creates a derivatives pricing options structure `Options` in which the named parameters have the specified values. Any unspecified value is set to the default value for that parameter when `Options` is passed to the pricing function. It is sufficient to type only the leading characters that uniquely identify the parameter name. Case is also ignored for parameter names.

If the optional input argument `Options` is specified, `derivset` modifies an existing pricing options structure by changing the named parameters to the specified values.

---

**Note** For parameter *values*, correct case and the complete character vector values are required; if an invalid character vector value is provided, the default is used.

---

`Options = derivset(OldOptions, NewOptions)` combines an existing options structure `OldOptions` with a new options structure `NewOptions`. Any parameters in `NewOptions` with nonempty values overwrite the corresponding old parameters in `OldOptions`.

`Options = derivset` creates an options structure `Options` whose fields are set to the default values.

`derivset` with no input or output arguments displays all parameter names and information about their possible values.

## Examples

```
Options = derivset('Diagnostics', 'on')
```

enables the display of additional diagnostic information that appears when executing pricing functions.

```
Options = derivset(Options, 'ConstRate', 'off')
```

changes the `ConstRate` parameter in the existing `Options` structure so that the assumption of constant rates between tree nodes no longer applies.

With no input or output arguments, `derivset` displays all parameter names and information about their possible values.

`derivset`

```
Diagnostics: [ on      | {off} ]
Warnings:   [ {on}   | off  ]
ConstRate:  [ {on}   | off  ]
BarrierMethod: [ {unenhanced} | interp  ]
```

## See Also

`barrierbycrr` | `barrierbyeqp` | `derivget`

## Topics

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

**Introduced before R2006a**

## disc2rate

Interest rates from cash flow discounting factors

### Syntax

**Usage 1:** Interval points are input as times in periodic units.

```
Rates = disc2rate(Compounding,Disc,EndTimes)
```

```
Rates = disc2rate(Compounding,Disc,EndTimes,StartTimes)
```

**Usage 2:** ValuationDate is passed and interval points are input as dates.

```
[Rates,EndTimes,StartTimes] = disc2rate(Compounding,Disc,EndDates,StartDates,ValuationDate)
```

```
[Rates,EndTimes,StartTimes] = disc2rate(Compounding,Disc,EndDates,StartDates,ValuationDate)
```

## Arguments

Compounding	<p>Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors (<b>Disc</b>):</p> <ul style="list-style-type: none"> <li>• <b>Compounding</b> = 0 for simple interest <ul style="list-style-type: none"> <li>• <math>Disc = 1 / (1 + Z * T)</math>, where T is time in years and simple interest assumes annual times <math>F = 1</math>.</li> </ul> </li> <li>• <b>Compounding</b> = 1, 2, 3, 4, 6, 12 <ul style="list-style-type: none"> <li>• <math>Disc = (1 + Z/F)^{-T}</math>, where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units, for example, <math>T = F</math> is one year.</li> </ul> </li> <li>• <b>Compounding</b> = 365 <ul style="list-style-type: none"> <li>• <math>Disc = (1 + Z/F)^{-T}</math>, where F is the number of days in the basis year and T is a number of days elapsed computed by basis.</li> </ul> </li> <li>• <b>Compounding</b> = -1 <ul style="list-style-type: none"> <li>• <math>Disc = \exp(-T*Z)</math>, where T is time in years.</li> </ul> </li> </ul>
Disc	<p>Number of points (<b>NPOINTS</b>) by number of curves (<b>NCURVES</b>) matrix of discounts. <b>Disc</b> are unit bond prices over investment intervals from <b>StartTimes</b>, when the cash flow is valued, to <b>EndTimes</b>, when the cash flow is received.</p>
EndTimes	<p><b>NPOINTS</b>-by-1 vector or scalar of times in periodic units ending the interval to discount over.</p> <hr/> <p><b>Note</b> When <b>ValuationDate</b> is not passed, the <b>EndTimes</b> and <b>StartTimes</b> arguments are interpreted as times.</p>
StartTimes	<p>(Optional) <b>NPOINTS</b>-by-1 vector or scalar of times in periodic units starting the interval to discount over. Default = 0.</p>



EndDates	<p>NPOINTS-by-1 vector or scalar of serial maturity dates ending the interval to discount over.</p> <hr/> <p><b>Note</b> When ValuationDate is passed, EndDates and StartDates arguments are interpreted as dates. The date ValuationDate is used as the zero point for computing the times.</p>
StartDates	<p>(Optional) NPOINTS-by-1 vector or scalar of serial dates starting the interval to discount over. Default = ValuationDate. StartDates must be earlier than EndDates.</p>
ValuationDate	<p>Scalar value in serial date number form representing the observation date of the investment horizons entered in StartDates and EndDates. Required in <b>Usage 2</b>. Omitted or passed as an empty matrix to invoke <b>Usage 1</b>.</p>
Basis	<p>(Optional) Day-count basis of the instrument when using <b>Usage 2</b>. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>

EndMonthRule	(Optional) End-of-month rule when using <b>Usage 2</b> . A vector. This rule applies only when <code>Maturity</code> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
--------------	--

## Description

**Usage 1:** `Rates = disc2rate(Compounding,Disc,EndTimes)` or `Rates = disc2rate(Compounding, Disc,EndTimes,StartTimes)` where interval points are input as times in periodic units.

**Usage 2:** `[Rates,EndTimes,StartTimes] = disc2rate(Compounding,Disc,EndDates,StartDates, ValuationDate)` or `[Rates,EndTimes,StartTimes] = disc2rate(Compounding,Disc,EndDates,StartDates,ValuationDate,Basis, EndMonthRule)` where `ValuationDate` is passed and interval points are input as dates.

`disc2rate` computes the yields over a series of `NPOINTS` time intervals given the cash flow discounts over those intervals. `NCURVES` different rate curves can be translated at once if they have the same time structure. The time intervals can represent a zero or a forward curve.

`Rates` is an `NPOINTS`-by-`NCURVES` column vector of yields in decimal form over the `NPOINTS` time intervals.

Specify the investment intervals with either input times (**Usage 1**) or input dates (**Usage 2**). Entering `ValuationDate` invokes the date interpretation; omitting `ValuationDate` invokes the default time interpretations.

For **Usage 1**:

- `StartTimes` is an `NPOINTS`-by-1 column vector of times starting the interval to discount over, measured in periodic units.
- `EndTimes` is an `NPOINTS`-by-1 column vector of times ending the interval to discount over, measured in periodic units.

For **Usage 2**:

- **StartDates** is an NPOINTS-by-1 column vector of serial dates starting the interval to discount over, measured in days.
- **EndDates** is an NPOINTS-by-1 column vector of serial date ending the interval to discount over, measured in days.

If **Compounding** = 365 (daily), **StartTimes** and **EndTimes** are measured in days for **Usage 2**. Otherwise, for **Usage 1**, the arguments contain values, **T**, computed from SIA semiannual time factors, **Tsemi**, by the formula  $T = Tsemi/2 * F$ , where **F** is the compounding frequency.

## See Also

rate2disc | ratetimes

## Topics

“Modeling the Interest-Rate Term Structure” on page 2-67

“Interest-Rate Term Conversions” on page 2-62

“Interest Rates Versus Discount Factors” on page 2-55

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## eqpprice

Instrument prices from Equal Probabilities binomial tree

### Syntax

```
[Price,PriceTree] = eqpprice(EQPTree,InstSet)
[Price,PriceTree] = eqpprice( ____,Options)
```

### Description

[Price,PriceTree] = eqpprice(EQPTree,InstSet) computes stock option prices using an EQP binomial tree created with `eqptree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`eqpprice` handles instrument types: 'Asian', 'Barrier', 'Compound', 'CBond', 'Lookback', 'OptStock'. See `instadd` to construct defined types.

[Price,PriceTree] = eqpprice( \_\_\_\_,Options) adds an optional input argument for `Options`.

### Examples

#### Price the Put Options Contained in the Instrument Set

Load the EQP tree and instruments from the data file `deriv.mat`. Price the put options contained in the instrument set.

```
load deriv.mat;
EQPSubSet = instselect(EQPInstSet, 'FieldName', 'OptSpec', ...
'Data', 'put')

EQPSubSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
```

```
Type: {5x1 cell}
FieldName: {5x1 cell}
FieldClass: {5x1 cell}
FieldData: {5x1 cell}
```

```
instdisp(EQPSubSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
1	OptStock	put	105	01-Jan-2003	01-Jan-2006	0	Put1	5
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPri
2	Asian	put	110	01-Jan-2003	01-Jan-2006	0	arithmetic	NaN
3	Asian	put	110	01-Jan-2003	01-Jan-2007	0	arithmetic	NaN

Price the put options.

```
[Price, PriceTree] = eqpprice(EQPtree, EQPSubSet)
```

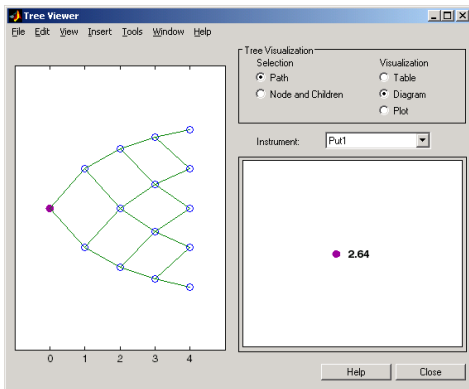
```
Price = 3x1
```

```
2.6375
4.7444
3.9178
```

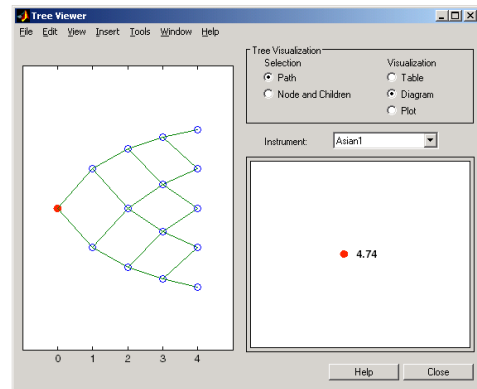
```
PriceTree = struct with fields:
```

```
FinObj: 'BinPriceTree'
Ptree: {1x5 cell}
tobs: [0 1 2 3 4]
dobs: [731582 731947 732313 732678 733043]
```

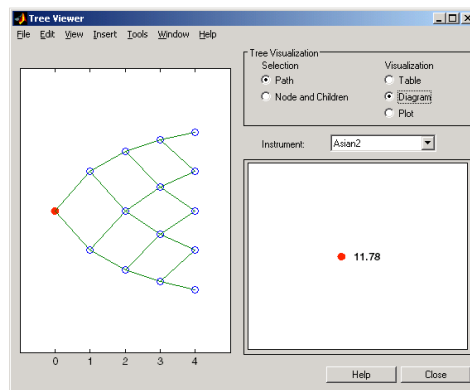
You can use `treeview` to see the prices of these three instruments along the price tree.



Put1



Asian11



Asian2

## Input Arguments

### EQPTree — Stock price tree structure

Stock price tree structure, specified by using `eqptree`.

Data Types: `struct`

**InstSet — Instrument variable**

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`**Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## Output Arguments

**Price — Price for each instrument**

vector

Price for each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned in that entry.

Related single-type pricing functions are:

- `asianbyeqp`: Price an Asian option from an EQP tree.
- `barrierbyeqp`: Price a barrier option from an EQP tree.
- `cbondbyeqp`: Price convertible bonds from an EQP tree.
- `compoundbyeqp`: Price a compound option from an EQP tree.
- `lookbackbyeqp`: Price a lookback option from an EQP tree.
- `optstockbyeqp`: Price an American, Bermuda, or European option from an EQP tree.

**PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.t0bs` contains the observation times.
- `PriceTree.d0bs` contains the observation dates.

### See Also

`cbondbyeqp` | `eqpsens` | `eqptimespec` | `eqptree` | `instadd` | `instcbond`

### Topics

“Computing Prices Using EQP” on page 3-133

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**



## eqpsens

Instrument prices and sensitivities from Equal Probabilities binomial tree

### Syntax

```
[Delta,Gamma,Vega,Price] = eqpsens(EQPTree,InstSet)
[Delta,Gamma,Vega,Price] = eqpsens( ____,Options)
```

### Description

[Delta,Gamma,Vega,Price] = eqpsens(EQPTree,InstSet) computes instrument sensitivities and prices for instruments using a binomial tree created with the `eqptree` function. All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

eqpsens handles instrument types: 'Asian', 'Barrier', 'Compound', 'CBond', 'Lookback', and 'OptStock'. See `instadd` for information on instrument types.

[Delta,Gamma,Vega,Price] = eqpsens( \_\_\_\_,Options) adds an optional input argument for Options.

### Examples

#### Compute Sensitivities for Instruments Using an eqptree

Load the EQP tree and instruments from the data file `deriv.mat`. Compute the Delta and Gamma sensitivities of the put options contained in the instrument set.

```
load deriv.mat;
```

```
EQPSubSet = instselect(EQPInstSet, 'FieldName', 'OptSpec', ...
    'Data', 'put')
```

```
EQPSubSet = struct with fields:
    FinObj: 'Instruments'
```

```
IndexTable: [1x1 struct]
    Type: {5x1 cell}
    FieldName: {5x1 cell}
    FieldClass: {5x1 cell}
    FieldData: {5x1 cell}
```

```
instdisp(EQPSubSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
1	OptStock	put	105	01-Jan-2003	01-Jan-2006	0	Put1	5

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPri
2	Asian	put	110	01-Jan-2003	01-Jan-2006	0	arithmetic	NaN
3	Asian	put	110	01-Jan-2003	01-Jan-2007	0	arithmetic	NaN

Obtain the Delta and Gamma for the put options contained in the instrument set.

```
[Delta, Gamma] = eqpsens(EQPtree, EQPSubSet)
```

```
Delta = 3x1
```

```
-0.2336
-0.5443
-0.4516
```

```
Gamma = 3x1
```

```
0.0218
0.0000
0.0000
```

## Input Arguments

### **EQPtree** — Stock tree structure

structure

Stock tree structure, specified by using `eqptree`.

Data Types: `struct`

### **InstSet — Instrument variable**

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Delta — Rate of change of instruments prices with respect to changes in the stock price**

vector

Rate of change of instruments prices with respect to changes in the stock price, returned as a NINST-by-1 vector of deltas.

For path-dependent options ('Lookback' and 'Asian'), Delta and Gamma are computed by finite differences in calls to `eqpprice`. For the rest of the options ('OptStock', 'Barrier', 'CBond', and 'Compound'), Delta and Gamma are computed from the EQPTree and the corresponding option price tree.

### **Gamma — Rate of change of instruments deltas with respect to changes in stock price**

vector

Rate of change of instruments deltas with respect to changes in the stock price, returned as a NINST-by-1 vector of gammas.

For path-dependent options ('Lookback' and 'Asian'), Delta and Gamma are computed by finite differences in calls to `eqpprice`. For the rest of the options

('OptStock', 'Barrier', 'CBond', and 'Compound'), Delta and Gamma are computed from the EQPTree and the corresponding option price tree.

### **Vega — Rate of change of instruments prices with respect to changes in volatility of the stock**

vector

Rate of change of instruments prices with respect to changes in the volatility of the stock, returned as a NINST-by-1 vector of vegas. Vega is computed by finite differences in calls to eqptree.

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned in that entry.

## **References**

[1] Chriss, Neil. *Black-Scholes and Beyond: Option Pricing Models*. McGraw-Hill, 1996, pp 308-312.

## **See Also**

cbondbyeqp | eqpprice | eqptree | instcbond

## **Topics**

“Computing Prices Using EQP” on page 3-133

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

# eqptimespec

Specify time structure for Equal Probabilities binomial tree

## Syntax

```
TimeSpec = eqptimespec(ValuationDate,Maturity,NumPeriods)
```

## Description

`TimeSpec = eqptimespec(ValuationDate,Maturity,NumPeriods)` sets the number of levels and node times for an equal probabilities tree (eqptree).

## Examples

### Set the Number of Levels and Node Times for an EQP Tree

This example shows how to set the number of levels and node times for an EQP tree by specifying a four-period tree with time steps of 1 year.

```
ValuationDate = '1-July-2002';
Maturity = '1-July-2006';
TimeSpec = eqptimespec(ValuationDate, Maturity, 4)

TimeSpec = struct with fields:
    FinObj: 'BinTimeSpec'
    ValuationDate: 731398
    Maturity: 732859
    NumPeriods: 4
    Basis: 0
    EndMonthRule: 1
    tObs: [0 1 2 3 4]
    dObs: [731398 731763 732128 732493 732859]
```

## Input Arguments

### **ValuationDate** — Pricing date and first observation in the tree

serial date number | character vector date

Pricing date and first observation in the `eqptree`, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### **Maturity** — Date marking the depth of the EQP stock tree

serial date number | date character vector

Date marking the depth of the `eqptree` binomial tree, specified as scalar serial date number or date character vector.

Data Types: double | char

### **NumPeriods** — Number of time steps in the EQP tree

integer

Number of time steps in the `eqptree` binomial tree, specified as scalar integer value.

Data Types: double

## Output Arguments

### **TimeSpec** — Specification for the time layout for `eqptree`

structure

Specification for the time layout for `eqptree`, returned as a structure.

## See Also

`eqptree` | `stockspec`

## Topics

“Computing Prices Using EQP” on page 3-133

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## eqptree

Build Equal Probabilities stock tree

### Syntax

```
EQPTree = eqptree(StockSpec,RateSpec,TimeSpec)
```

### Description

EQPTree = eqptree(StockSpec,RateSpec,TimeSpec) builds an Equal Probabilities stock tree.

### Examples

#### Create an EQP Tree

Using the data provided, create a stock specification (StockSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then use these specifications to create an EQP stock tree with eqptree.

```
Sigma = 0.20;  
AssetPrice = 50;  
DividendType = 'cash';  
DividendAmounts = [0.50; 0.50; 0.50; 0.50];  
ExDividendDates = {'03-Jan-2003'; '01-Apr-2003'; '05-July-2003';  
    '01-Oct-2003'};  
  
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...  
    DividendAmounts, ExDividendDates);  
  
RateSpec = intenvset('Rates', 0.05, 'StartDates', ...  
    '01-Jan-2003', 'EndDates', '31-Dec-2003');  
  
ValuationDate = '1-Jan-2003';  
Maturity = '31-Dec-2003';
```



```
TimeSpec = eqptimespec(ValuationDate, Maturity, 4);
```

```
EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)
```

Warning: RateSpec was not created with continuous compounding. Compounding will be set to continuous while leaving discount factors unaltered. This will result in the recalculation of the interest rates.

```
EQPTree =
```

```
struct with fields:
```

```
    FinObj: 'BinStockTree'
    Method: 'EQP'
    StockSpec: [1×1 struct]
    TimeSpec: [1×1 struct]
    RateSpec: [1×1 struct]
        tObs: [0 0.2493 0.4986 0.7479 0.9972]
        dObs: [731582 731673 731764 731855 731946]
    STree: {1×5 cell}
    UpProbs: [0.5000 0.5000 0.5000 0.5000]
```

Use `treeview` to observe the tree you have created.

## Input Arguments

### StockSpec — Stock specification

structure

Stock specification, specified by the `StockSpec` obtained from `stockspec`. See `stockspec` for information on creating a stock specification.

Data Types: `struct`

### RateSpec — Interest-rate specification for initial risk-free rate curve

structure

Interest-rate specification for initial risk-free rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

**Note** The standard equal probabilities tree assumes a constant interest rate, but `RateSpec` allows you to specify an interest-rate curve with varying rates. If you specify variable interest rates, the resulting tree is not a standard equal probabilities tree.

Data Types: `struct`

### **TimeSpec — Tree time layout specification**

structure

Tree time layout specification, specified by the `TimeSpec` obtained from `eqptimespec`. The `TimeSpec` defines the observation dates of the EQP stock tree. See `eqptimespec` for information on the tree structure.

Data Types: `struct`

## **Output Arguments**

### **EQPTree — EQP stock tree**

structure

EQP stock tree, returned as a structure specifying the time layout for the tree.

## **See Also**

`eqptimespec` | `intenvset` | `stockspec`

## **Topics**

“Computing Prices Using EQP” on page 3-133

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## fixedbybdt

Price fixed-rate note from Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = fixedbybdt(BDTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = fixedbybdt( ____,Name,Value)
```

### Description

[Price,PriceTree] = fixedbybdt(BDTree,CouponRate,Settle,Maturity) prices a fixed-rate note from a Black-Derman-Toy interest-rate tree.

[Price,PriceTree] = fixedbybdt( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a 10% Fixed-Rate Note Using a BDT Interest-Rate Tree

This example shows how to price a 10% fixed-rate note using a BDT interest-rate tree by loading the file `deriv.mat`, which provides `BDTree`. The `BDTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat
```

```
CouponRate = 0.10;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
FixedReset = 1;
```

```
Price = fixedbybdt(BDTree, CouponRate, Settle, Maturity, FixedReset)
```

```
Price = 92.9974
```

## Input Arguments

### **BDTTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `bdttree`

Data Types: `struct`

### **CouponRate** — Coupon annual rate

decimal

Coupon annual rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the BDT Tree. The fixed-rate note argument `Settle` is ignored.

Data Types: `char` | `double`

### **Maturity** — Maturity date

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each fixed-rate note.

Data Types: `char` | `double`

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle, Maturity, 'FixedReset', 4)`

**FixedReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FixedReset' and a NINST-by-1 vector.

Data Types: double

**Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

`Principal` accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: `cell` | `double`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using `derivset`.

Data Types: `struct`

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when `Maturity` is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: `logical`

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: `logical`

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHHolidays-by-1 vector.

Data Types: double

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

## **Output Arguments**

### **Price — Expected fixed-rate note prices at time 0**

vector

Expected fixed-rate note prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## See Also

`bdttree` | `bondbybdt` | `capbybdt` | `cfbybdt` | `floatbybdt` | `floorbybdt` | `swapbybdt`

## Topics

“Computing Instrument Prices” on page 2-99

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# fixedbybk

Price fixed-rate note from Black-Karasinski interest-rate tree

## Syntax

```
[Price,PriceTree] = fixedbybk(BKTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = fixedbybk( ____,Name,Value)
```

## Description

[Price,PriceTree] = fixedbybk(BKTree,CouponRate,Settle,Maturity) prices a fixed-rate note from a Black-Karasinski interest-rate tree.

[Price,PriceTree] = fixedbybk( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a 5% Fixed-Rate Note Using a Black-Karasinski Interest-Rate Tree

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
CouponRate = 0.05;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use `fixedbybk` to compute the price of the note.

```
Price = fixedbybk(BKTree, CouponRate, Settle, Maturity)
```

Warning: Fixed rate notes are valued at `Tree ValuationDate` rather than `Settle`  
Price = 103.5126

## Input Arguments

### **BKTree — Interest-rate structure**

structure

Interest-rate tree structure, created by `bktree`

Data Types: `struct`

### **CouponRate — Coupon annual rate**

decimal

Coupon annual rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the BK Tree. The fixed-rate note argument `Settle` is ignored.

Data Types: `char` | `double`

### **Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each fixed-rate note.

Data Types: `char` | `double`

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle, Maturity, 'FixedReset', 4)`

### **FixedReset – Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FixedReset' and a NINST-by-1 vector.

Data Types: double

### **Basis – Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

**Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using derivset.

Data Types: struct

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

### **BusinessDayConvention — Business day conventions**

`actual` (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

## **Output Arguments**

### **Price — Expected fixed-rate note prices at time 0**

vector

Expected fixed-rate note prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## **See Also**

`bktree` | `bondbybk` | `capbybk` | `cfbybk` | `floatbybk` | `floorbybk` | `swapbybk`

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# fixedbycir

Price fixed rate note from Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = fixedbycir(CIRTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = fixedbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = fixedbycir(CIRTree,CouponRate,Settle,Maturity) prices a fixed-rate note from a Cox-Ingersoll-Ross (CIR) interest-rate tree using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = fixedbycir( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Fixed-Rate Note Using a CIR Interest-Rate Tree

Define the CouponRate for a fixed-rate note.

```
CouponRate = 0.03;
```

Create a RateSpec using the intenvset function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```

NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2021';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [736696 737061 737426 737791]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

Price the 3% fixed-rate note.

```
[Price,PriceTree] = fixedbycir(CIRT,CouponRate,Settle,Maturity)
```

```
Price = 92.1422
```

```

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    tObs: [0 1 2 3 4]
    dObs: [736696 737061 737426 737791 738157]
    PTree: {1x5 cell}
    AITree: {[0] [0 0 0] [0 0 0 0 0] [0 0 0 0 0 0 0] [0 0 0 0 0 0 0 0]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}

```

## Input Arguments

### CIRTree — Interest-rate structure

structure



Interest-rate tree structure, created by `cirtree`

Data Types: `struct`

### **CouponRate — Coupon annual rate**

decimal

Coupon annual rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | character vector | string array | datetime

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the CIR tree. The fixed-rate note argument `Settle` is ignored.

Data Types: `char` | `double` | `string` | `datetime`

### **Maturity — Maturity date**

serial date number | character vector | string array | datetime

Maturity date, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays representing the maturity date for each fixed-rate note.

Data Types: `char` | `double` | `string` | `datetime`

## **Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = fixedbycir(CIRTree, CouponRate, Settle, Maturity, 'FixedReset', 4)`

### **FixedReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FixedReset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second

column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: double

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character

vectors of business day conventions. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

## Output Arguments

### **Price — Expected fixed-rate note prices at time 0**

vector

Expected fixed-rate note prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.tObs` contains the observation times.
- `PriceTree.dObs` contains the observation dates.
- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | cfbycir | floatbycir | floorbycir | instfixed |  
oasbycir | optbndbycir | optembndbycir | optemfloatbycir | optfloatbycir |  
rangefloatbycir | swapbycir | swaptionbycir

## Topics

"Computing Instrument Prices" on page 2-99  
"Pricing a Portfolio Using the Black-Derman-Toy Model" on page 1-13  
"Understanding Interest-Rate Tree Models" on page 2-79  
"Pricing Options Structure" on page B-2  
"Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

## fixedbyhjm

Price fixed-rate note from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = fixedbyhjm(HJMTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = fixedbyhjm( ____,Name,Value)
```

### Description

[Price,PriceTree] = fixedbyhjm(HJMTree,CouponRate,Settle,Maturity) prices a fixed-rate note from a Heath-Jarrow-Morton interest-rate tree.

[Price,PriceTree] = fixedbyhjm( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a 4% Fixed-Rate Note Using an HJM Forward-Rate Tree

This example shows how to price a 4% fixed-rate note using an HJM forward-rate tree by loading the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and forward-rate information needed to price the note.

```
load deriv.mat

CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';

Price = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity)

Price = 98.7159
```

## Input Arguments

### **HJMTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `hjm tree`

Data Types: `struct`

### **CouponRate** — Coupon annual rate

decimal

Coupon annual rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the HJM tree. The fixed-rate note argument `Settle` is ignored.

Data Types: `char` | `double`

### **Maturity** — Maturity date

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each fixed-rate note.

Data Types: `char` | `double`

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity, 'FixedReset', 4)`

**FixedReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FixedReset' and a NINST-by-1 vector.

Data Types: double

**Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.



`Principal` accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: `cell` | `double`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using `derivset`.

Data Types: `struct`

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when `Maturity` is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: `logical`

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: `logical`

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: `double`

### **BusinessDayConvention — Business day conventions**

`actual` (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

## **Output Arguments**

### **Price — Expected fixed-rate note prices at time 0**

vector

Expected fixed-rate note prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PBush` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## See Also

[bondbyhjm](#) | [capbyhjm](#) | [cfbyhjm](#) | [floatbyhjm](#) | [floorbyhjm](#) | [hjmtree](#) | [swapbyhjm](#)

## Topics

[“Computing Instrument Prices”](#) on page 2-99

[“Understanding Interest-Rate Tree Models”](#) on page 2-79

[“Pricing Options Structure”](#) on page B-2

[“Supported Interest-Rate Instruments”](#) on page 2-2

**Introduced before R2006a**

## fixedbyhw

Price fixed-rate note from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = fixedbyhw(HWTree,CouponRate,Settle,Maturity)
[Price,PriceTree] = fixedbyhw( ____,Name,Value)
```

### Description

[Price,PriceTree] = fixedbyhw(HWTree,CouponRate,Settle,Maturity) prices a fixed-rate note from a Hull-White interest-rate tree.

[Price,PriceTree] = fixedbyhw( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a 5% Fixed-Rate Note Using a Hull-White Interest-Rate Tree

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
CouponRate = 0.05;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use `fixedbyhw` to compute the price of the note.

```
Price = fixedbyhw(HWTree, CouponRate, Settle, Maturity)
```

Warning: Fixed rate notes are valued at `Tree ValuationDate` rather than `Settle`  
Price = 103.5126

## Input Arguments

### **HWTree — Interest-rate structure**

structure

Interest-rate tree structure, created by hwtree

Data Types: struct

### **CouponRate — Coupon annual rate**

decimal

Coupon annual rate, specified as a NINST-by-1 vector.

Data Types: double

### **Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the HW tree. The fixed-rate note argument `Settle` is ignored.

Data Types: char | double

### **Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each fixed-rate note.

Data Types: char | double

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle, Maturity, 'FixedReset', 4)`

### **FixedReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FixedReset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

**Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using derivset.

Data Types: struct

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

### **BusinessDayConvention — Business day conventions**

`actual` (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

## **Output Arguments**

### **Price — Expected fixed-rate note prices at time 0**

vector



Expected fixed-rate note prices at time 0, returned as a NINST-by-1 vector.

## **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## **See Also**

[bondbyhw](#) | [capbyhw](#) | [cfbyhw](#) | [floatbyhw](#) | [floorbyhw](#) | [hwtree](#) | [swapbyhw](#)

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## fixedbyzero

Price fixed-rate note from set of zero curves

### Syntax

```
[Price,DirtyPrice,CFlowAmounts,CFlowDates] = fixedbyzero(RateSpec,  
CouponRate,Settle,Maturity)  
[Price,DirtyPrice,CFlowAmounts,CFlowDates] = fixedbyzero( ____,  
Name,Value)
```

### Description

[Price,DirtyPrice,CFlowAmounts,CFlowDates] = fixedbyzero(RateSpec, CouponRate,Settle,Maturity) prices a fixed-rate note from a set of zero curves.

[Price,DirtyPrice,CFlowAmounts,CFlowDates] = fixedbyzero( \_\_\_\_, Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a 4% Fixed-Rate Note Using a Set of Zero Curves

This example shows how to price a 4% fixed-rate note using a set of zero curves by loading the file `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure needed to price the note.

```
load deriv.mat  
  
CouponRate = 0.04;  
Settle = '01-Jan-2000';  
Maturity = '01-Jan-2003';  
  
Price = fixedbyzero(ZeroRateSpec, CouponRate, Settle, Maturity)  
  
Price = 98.7159
```

## Pricing a Fixed-Fixed Cross Currency Swap

Assume that a financial institution has an existing swap with three years left to maturity where they are receiving 5% per year in yen and paying 8% per year in USD. The reset frequency for the swap is annual, the principals for the two legs are 1200 million yen and \$10 million USD, and both term structures are flat.

```
Settle = datenum('15-Aug-2015');
Maturity = datenum('15-Aug-2018');
Reset = 1;
```

```
r_d = .09;
r_f = .04;
```

```
FixedRate_d = .08;
FixedRate_f = .05;
```

```
Principal_d = 10000000;
Principal_f = 1200000000;
```

```
S0 = 1/110;
```

Construct term structures.

```
RateSpec_d = intenvset('StartDate',Settle,'EndDate',Maturity,'Rates',r_d,'Compounding');
RateSpec_f = intenvset('StartDate',Settle,'EndDate',Maturity,'Rates',r_f,'Compounding');
```

Use fixedbyzero:

```
B_d = fixedbyzero(RateSpec_d,FixedRate_d,Settle,Maturity,'Principal',Principal_d,'Reset');
B_f = fixedbyzero(RateSpec_f,FixedRate_f,Settle,Maturity,'Principal',Principal_f,'Reset');
```

Compute swap price. Based on Hull (see References), a cross currency swap can be valued with the following formula  $V_{\text{swap}} = S0*B_f - B_d$ .

```
V_swap = S0*B_f - B_d
```

```
V_swap = 1.5430e+06
```

## Input Arguments

### **RateSpec** — Annualized zero rate term structure

structure

Annualized zero rate term structure, specified using `intenvset` to create a `RateSpec`.

Data Types: `struct`

### **CouponRate** — Annual rate

decimal

Annual rate, specified as `NINST`-by-1 decimal annual rate or a `NINST`-by-1 cell array where each element is a `NumDates`-by-2 cell array and the first column is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or `NINST`-by-1 vector of serial date numbers or date character vectors.

`Settle` must be earlier than `Maturity`.

Data Types: `char` | `double`

### **Maturity** — Maturity date

serial date number | character vector

Maturity date, specified as a `NINST`-by-1 vector of serial date numbers or date character vectors representing the maturity date for each fixed-rate note.

Data Types: `char` | `double`

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price,DirtyPrice,CFlowAmounts,CFlowDates] = fixedbyzero(RateSpec,CouponRate,Settle,Maturity,'Principal',Principal)`

### **FixedReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FixedReset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: double

**BusinessDayConvention — Business day conventions**

`actual` (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

**Output Arguments****Price — Fixed-rate note prices**

matrix

Floating-rate note prices, returned as a (NINST) by number of curves (NUMCURVES) matrix. Each column arises from one of the zero curves.

**DirtyPrice — Dirty bond price**

matrix

Dirty bond price (clean + accrued interest), returned as a NINST- by-NUMCURVES matrix. Each column arises from one of the zero curves.

### **CFlowAmounts — Cash flow amounts**

matrix

Cash flow amounts, returned as a NINST- by-NUMCFS matrix of cash flows for each bond.

### **CFlowDates — Cash flow dates**

matrix

Cash flow dates, returned as a NINST- by-NUMCFS matrix of payment dates for each bond.

## **References**

[1] Hull, J. *Options, Futures, and Other Derivatives*. Prentice-Hall, 2011.

## **See Also**

bondbyzero | cfbyzero | floatbyzero | swapbyzero

## **Topics**

“Pricing Using Interest-Rate Term Structure” on page 2-72

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# floatbybdt

Price floating-rate note from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price,PriceTree] = floatbybdt(BDTree,Spread,Settle,Maturity)
[Price,PriceTree] = floatbybdt( ____,Name,Value)
```

## Description

[Price,PriceTree] = floatbybdt(BDTree,Spread,Settle,Maturity) prices a floating-rate note from a Black-Derman-Toy interest-rate tree.

floatbybdt computes prices of vanilla floating-rate notes, amortizing floating-rate notes, capped floating-rate notes, floored floating-rate notes and collared floating-rate notes.

[Price,PriceTree] = floatbybdt( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Floating-Rate Note Using a BDT Tree

Price a 20-basis point floating-rate note using a BDT interest-rate tree.

Load the file `deriv.mat`, which provides `BDTree`. The `BDTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use `floatbybdt` to compute the price of the note.

```
Price = floatbybdt(BDTree, Spread, Settle, Maturity)
```

```
Price = 100.4865
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014' ; '15-Nov-2015'; '15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
    ValuationDate: 734822
    Basis: 0
    EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle = '15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};
```

Build the BDT tree and assume volatility is 10%.

```
MatDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014'; '15-Nov-2015'; '15-Nov-2016'; '15-Nov-2017'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates)));
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbybdt(BDTT, Spread, Settle, Maturity, 'Principal', Principal)
```

```
Price = 100.3059
```

### Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the CapRate and FloorRate input argument to define the collar pricing.

Create the RateSpec.

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013'; '1-April-2014'; '1-April-2015'; ...
            '1-April-2016'; '1-April-2017'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intensvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
                    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

Build the BDT tree and assume volatility is 5%.

```
MatDates = {'1-April-2013'; '1-April-2014'; '1-April-2015'; '1-April-2016'; '1-April-2017'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.05;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates)))';
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Create the floating rate note instrument.

```
Settle = '1-April-2012';
Maturity = '1-April-2016';
Spread = 10;
Principal = 100;
```

Compute the price of a collared floating-rate note.

```
CapStrike = {'1-April-2013' 0.03; '1-April-2015' 0.055};
FloorStrike = {'1-April-2013' 0.025; '1-April-2015' 0.04};

Price = floatbybdt(BDTT, Spread, Settle, Maturity, 'CapRate', ...
    CapStrike, 'FloorRate', FloorStrike)

Price = 101.2414
```

### Pricing a Floating-Rate Note When the Reset Dates Are Not Tree Level Dates

When using `floatbybdt` to price floating-rate notes, there are cases where the dates specified in the BDT tree `TimeSpec` are not aligned with the cash flow dates.

Price floating-rate notes using the following data:

```
ValuationDate = '01-Sep-2013';
Rates = [0.0235; 0.0239; 0.0311; 0.0323];
EndDates = {'01-Sep-2014'; '01-Sep-2015'; '01-Sep-2016'; '01-Sep-2017'};
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates', ...
    ValuationDate,'EndDates',EndDates,'Rates',Rates,'Compounding', 1);
```

Build the BDT tree.

```
VolCurve = [.10; .11; .11; .12];
```

```
BDTVolatilitySpec = bdtvolspec(RateSpec.ValuationDate, EndDates, ...
                               VolCurve);
```

```
BDTTimeSpec = bdttimespec(RateSpec.ValuationDate, EndDates, 1);
```

```
BDTT = bdttree(BDTVolatilitySpec, RateSpec, BDTTimeSpec);
```

Compute the price of the floating-rate note using the following data:

```
Spread = 5;
```

```
Settle = '01-Sep-2013';
```

```
Maturity = '01-Dec-2015';
```

```
Reset = 2;
```

```
Price = floatbybdt(BDTT, Spread, Settle, Maturity, 'FloatReset', Reset)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

```
> In floatengbybdt at 204
```

```
  In floatbybdt at 123
```

```
Error using floatengbybdt (line 299)
```

```
Instrument '1' has cash flow dates that span across tree nodes.
```

```
Error in floatbybdt (line 123)
```

```
[Price, PriceTree, CFTree, TLPpal] = floatengbybdt(BDTTree, Spread, Settle, Maturity, 0Args{:});
```

This error indicates that it is not possible to determine the applicable rate used to calculate the payoff at the reset dates, given that the applicable rate needed cannot be calculated (the information was lost due to the recombination of the tree nodes). Note, if the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates. The simplest solution is to place the tree levels at the cash flow dates of the instrument, which is done by specifying `BDTTimeSpec`. It is also acceptable to have reset dates between tree levels, as long as there are reset dates on the tree levels.

To recover from this error, build a tree that lines up with the instrument.

```
Basis = intenvget(RateSpec, 'Basis');
```

```
EOM = intenvget(RateSpec, 'EndMonthRule');
```

```
resetDates = cfdates(ValuationDate, Maturity, Reset, Basis, EOM);
```

```
BDTTimeSpec = bdttimespec(RateSpec.ValuationDate, resetDates, Reset);
```

```
BDTT = bdttree(BDTVolatilitySpec, RateSpec, BDTTimeSpec);
```

```
Price = floatbybdt(BDTT, Spread, RateSpec.ValuationDate, ...
                  Maturity, 'FloatReset', Reset)
```

```
Price =  
    100.1087
```

## Input Arguments

### **BDTTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `bdttree`

Data Types: `struct`

### **Spread** — Number of basis points over the reference rate

vector

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the BDT tree. The floating-rate note argument `Settle` is ignored.

Data Types: `char` | `double`

### **Maturity** — Maturity date

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each floating-rate note.

Data Types: `char` | `double`

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = floatbybdt(BDTree, Spread, Settle, Maturity, 'Basis', 3)`

### **FloatReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

---

Data Types: double

### **Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)

- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal — Notional principal amounts or principal value schedules**

`100` (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

`Principal` accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: `cell` | `double`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using `derivset`.

Data Types: `struct`

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

`1` (in effect) (default) | nonnegative integer `[0, 1]`

End-of-month rule flag for generating dates when `Maturity` is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer `[0, 1]` using a NINST-by-1 vector.

- `0` = Ignore rule, meaning that a payment date is always the same numerical day of the month.



- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: double

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.

- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

### **CapRate — Annual cap rate**

decimal

Annual cap rate, specified as the comma-separated pair consisting of 'CapRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

Data Types: `double` | `cell`

### **FloorRate — Annual floor rate**

decimal

Annual floor rate, specified as the comma-separated pair consisting of 'FloorRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

Data Types: `double` | `cell`

## **Output Arguments**

### **Price — Expected floating-rate note prices at time 0**

vector

Expected floating-rate note prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.

- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.t0bs` contains the observation times.

## See Also

`bdttree` | `bondbybdt` | `capbybdt` | `cfbybdt` | `fixedbybdt` | `floorbybdt` | `swapbybdt`

## Topics

“Computing Instrument Prices” on page 2-99

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## floatbybk

Price floating-rate note from Black-Karasinski interest-rate tree

### Syntax

```
[Price,PriceTree] = floatbybk(BKTree,Spread,Settle,Maturity)
[Price,PriceTree] = floatbybk( ____,Name,Value)
```

### Description

[Price,PriceTree] = floatbybk(BKTree,Spread,Settle,Maturity) prices a floating-rate note from a Black-Karasinski interest-rate tree.

floatbybk computes prices of vanilla floating-rate notes, amortizing floating-rate notes, capped floating-rate notes, floored floating-rate notes and collared floating-rate notes.

[Price,PriceTree] = floatbybk( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a Floating-Rate Note Using a Black-Karasinski Tree

Price a 20-basis point floating-rate note using a Black-Karasinski interest-rate tree.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use floatbybk to compute the price of the note.

```
Price = floatbybk(BKTree, Spread, Settle, Maturity)
```

Warning: Floating range notes are valued at Tree ValuationDate rather than Settle.

```
Price = 100.3825
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014' ; '15-Nov-2015'; '15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
    ValuationDate: 734822
    Basis: 0
    EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle = '15-Nov-2011';  
Maturity = '15-Nov-2015';  
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};
```

Build the BK tree and assume the volatility is 10%.

```
VolDates = ['15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014'; '15-Nov-2015'; '15-Nov-2016'; '15-Nov-2017'];  
VolCurve = 0.1;  
AlphaDates = '15-Nov-2017';  
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...  
AlphaDates, AlphaCurve);  
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);  
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbybk(BKT, Spread, Settle, Maturity, 'Principal', Principal)  
  
Price = 100.3059
```

### **Price a Collar with a Floating-Rate Note**

Price a collar with a floating-rate note using the `CapRate` and `FloorRate` input argument to define the collar pricing.

Price a portfolio of collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];  
ValuationDate = '1-April-2012';  
StartDates = ValuationDate;  
EndDates = {'1-April-2013'; '1-April-2014'; '1-April-2015'; ...  
'1-April-2016'; '1-April-2017'};  
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

Build the BK tree and assume the volatility to be 5%.

```
VolDates = ['1-April-2013'; '1-April-2014'; '1-April-2015'; '1-April-2016'; ...
'1-April-2017'; '1-April-2018'];
VolCurve = 0.05;
AlphaDates = '15-Nov-2018';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Create the floating-rate note instrument.

```
Settle = '1-April-2012';
Maturity = '1-April-2016';
Spread = [15;10];
Principal = 100;
```

Compute the price of the two vanilla floaters.

```
Price = floatbybk(BKT, Spread, Settle, Maturity)
```

```
Price = 2×1
```

```
100.5519
100.3680
```

Compute the price of the collared floating-rate notes.

```
CapStrike = {{'1-April-2013' 0.045; '1-April-2014' 0.05; ...
'1-April-2015' 0.06}; 0.06};
```

```
FloorStrike = {{'1-April-2013' 0.035; '1-April-2014' 0.04; ...
'1-April-2015' 0.05}; 0.03};
```

```
PriceCollared = floatbybk(BKT, Spread, Settle, Maturity, ...
'CapRate', CapStrike, 'FloorRate', FloorStrike)
```

```
PriceCollared = 2×1
```

```
102.8537
```

100.4918

## Pricing a Floating-Rate Note When the Reset Dates Are Not Tree Level Dates

When using `floatbybk` to price floating-rate notes, there are cases where the dates specified in the BK tree Time Specs are not aligned with the cash flow dates.

Price floating-rate notes using the following data:

```
ValuationDate = '13-Sep-2013';
ForwardRatesVector = [ 0.0001; 0.0001; 0.0010; 0.0015];
EndDatesVector = ['13-Dec-2013'; '14-Mar-2014'; '13-Jun-2014'; '13-Sep-2014'];
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',...
ValuationDate,'EndDates',EndDatesVector,'Rates',ForwardRatesVector,'Compounding', 1);
```

Build the BK tree.

```
Volcurve = 0.1;
Alpha = 0.01;
BKVolatilitySpec = bkvolspec(RateSpec.ValuationDate, ...
EndDatesVector, Volcurve,...
EndDatesVector, Alpha);

BKTimeSpec = bktimespec(RateSpec.ValuationDate, EndDatesVector, 1);

BKT = bktree(BKVolatilitySpec, RateSpec, BKTimeSpec);
```

Create the floating-rate note instrument using the following data;

```
Spread = 0;
Maturity = '13-Jun-2014';
reset = 4;
```

Compute the price of the floating-rate note.

```
Price = floatbybk(BKT, Spread, RateSpec.ValuationDate,...
Maturity, 'FloatReset', reset)
```

```
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
> In floatengbytrintree at 214
In floatbybk at 136
Error using floatengbytrintree (line 319)
Instrument '1' has cash flow dates that span across tree nodes.
```

```
Error in floatbybk (line 136)
[Price, PriceTree, CFTree] = floatengbytrintree(BKTree, Spread, Settle, Maturity, OArgs{:});
```

This error indicates that it is not possible to determine the applicable rate used to calculate the payoff at the reset dates, given that the applicable rate needed cannot be



calculated (the information was lost due to the recombination of the tree nodes). Note, if the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates. The simplest solution is to place the tree levels at the cash flow dates of the instrument, which is done by specifying `BKTimeSpec`. It is also acceptable to have reset dates between tree levels, as long as there are reset dates on the tree levels.

To recover from this error, build a tree that lines up with the instrument.

```
Basis = intenvget(RateSpec, 'Basis');
EOM = intenvget(RateSpec, 'EndMonthRule');
resetDates = cfdates(ValuationDate, Maturity, reset, Basis, EOM);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, resetDates, reset);
BKT = bktree(BKVolatilitySpec, RateSpec, BKTimeSpec);

Price = floatbybk(BKT, Spread, RateSpec.ValuationDate, ...
    Maturity, 'FloatReset', reset)
```

```
Price =
```

```
100.0004
```

## Input Arguments

### **BKTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `bktree`

Data Types: `struct`

### **Spread** — Number of basis points over the reference rate

vector

Number of basis points over the reference rate, specified as a `NINST-by-1` vector.

Data Types: `double`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or `NINST-by-1` vector of serial date numbers or date character vectors.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the BK tree. The floating-rate note argument `Settle` is ignored.

Data Types: `char` | `double`

**Maturity — Maturity date**

`serial date number` | `character vector`

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each floating-rate note.

Data Types: `char` | `double`

**Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = floatbybk(BKTree, Spread, Settle, Maturity, 'Basis', 3)`

**FloatReset — Frequency of payments per year**

1 (default) | `vector`

Frequency of payments per year, specified as the comma-separated pair consisting of `'FloatReset'` and a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

---

Data Types: `double`

**Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using derivset.

Data Types: struct

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

**Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: double

**BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character

vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **CapRate — Annual cap rate**

decimal

Annual cap rate, specified as the comma-separated pair consisting of 'CapRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

Data Types: double | cell

### **FloorRate — Annual floor rate**

decimal

Annual floor rate, specified as the comma-separated pair consisting of 'FloorRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

Data Types: double | cell

## Output Arguments

### **Price — Expected floating-rate note prices at time 0**

vector

Expected floating-rate note prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:

- PriceTree.PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.
- PriceTree.Connect contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are NumNodes elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- PriceTree.Probs contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## See Also

bktree | bondbybk | capbybk | cfbybk | fixedbybk | floorbybk | swapbybk

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# floatbycir

Price floating-rate note from Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = floatbycir(CIRTree,Spread,Settle,Maturity)
[Price,PriceTree] = floatbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = floatbycir(CIRTree,Spread,Settle,Maturity) prices a floating-rate note from a Cox-Ingersoll-Ross (CIR) interest-rate tree.

floatbycir computes prices of vanilla floating-rate notes, amortizing floating-rate notes, capped floating-rate notes, floored floating-rate notes, and collared floating-rate notes using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = floatbycir( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Floating-Rate Note Using a CIR Interest-Rate Tree

Define a Spread of 20-basis points for a floating-rate note.

```
Spread = 20;
```

Create a RateSpec using the intenvset function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
```

```
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2021';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolSpec(Sigma, Alpha, Theta);
```

```
CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)
```

```
CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [736696 737061 737426 737791]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

Price the 20-basis point floating-rate note.

```
[Price,PriceTree] = floatbycir(CIRT,Spread,Settle,Maturity)
```

```
Price = 100.7143
```

```
PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    PTree: {1x5 cell}
    AITree: {[0] [0 0 0] [0 0 0 0 0] [0 0 0 0 0 0 0] [0 0 0 0 0 0 0 0]}
    tObs: [0 1 2 3 4]
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
```



## Input Arguments

### **CIRTree — Interest-rate tree structure**

structure

Interest-rate tree structure, created by `cirtree`

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

vector

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | character vector | string array | datetime

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the CIR tree. The floating-rate note argument `Settle` is ignored.

Data Types: `char` | `double` | `string` | `datetime`

### **Maturity — Maturity date**

serial date number | character vector | string array | datetime

Maturity date, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays representing the maturity date for each floating-rate note.

Data Types: `char` | `double` | `string` | `datetime`

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: [Price,PriceTree] =  
floatbycir(CIRTree,Spread,Settle,Maturity,'Basis',3)
```

**FloatReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)

- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array, and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

**Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

**BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a `N-by-1` cell array of character vectors of business day conventions. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

**CapRate — Annual cap rate**

decimal

Annual cap rate, specified as the comma-separated pair consisting of 'CapRate' and a `NINST-by-1` decimal annual rate or `NINST-by-1` cell array, where each element is a `NumDates-by-2` cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

Data Types: double | cell

### **FloorRate — Annual floor rate**

decimal

Annual floor rate, specified as the comma-separated pair consisting of 'FloorRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array.

For the NINST-by-1 cell array, each element is a NumDates-by-2 cell array, where the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

Data Types: double | cell

## **Output Arguments**

### **Price — Expected floating-rate note prices at time 0**

vector

Expected floating-rate note prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:

- PriceTree.PTree contains the clean prices.
- PriceTree.AITree contains the accrued interest.
- PriceTree.tObs contains the observation times.
- PriceTree.Connect contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are NumNodes elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- PriceTree.Probs contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | cfbycir | fixedbycir | floorbycir | instfloat |  
oasbycir | optbndbycir | optembndbycir | optemfloatbycir | optfloatbycir |  
rangefloatbycir | swapbycir | swaptionbycir

## Topics

- "Computing Instrument Prices" on page 2-99
- "Understanding Interest-Rate Tree Models" on page 2-79
- "Pricing Options Structure" on page B-2
- "Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

## floatbyhjm

Price floating-rate note from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = floatbyhjm(HJMTtree,Spread,Settle,Maturity)
[Price,PriceTree] = floatbyhjm( ____,Name,Value)
```

### Description

[Price,PriceTree] = floatbyhjm(HJMTtree,Spread,Settle,Maturity) prices a floating-rate note from a Heath-Jarrow-Morton interest-rate tree.

floatbyhjm computes prices of vanilla floating-rate notes, amortizing floating-rate notes, capped floating-rate notes, floored floating-rate notes and collared floating-rate notes.

[Price,PriceTree] = floatbyhjm( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a Floating-Rate Note Using an HJM Tree

Price a 20-basis point floating-rate note using an HJM forward-rate tree.

Load the file `deriv.mat`, which provides HJMTtree. The HJMTtree structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use `floatbyhjm` to compute the price of the note.

```
Price = floatbyhjm(HJMTree, Spread, Settle, Maturity)
```

```
Price = 100.5529
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014' ; '15-Nov-2015'; '15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
    ValuationDate: 734822
    Basis: 0
    EndMonthRule: 1
```

Create the floating-rate instrument using the following data:



```
Settle = '15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};
```

Build the HJM tree using the following data:

```
MatDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014'; '15-Nov-2015'; '15-Nov-2016'; '15-Nov-2017'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbyhjm(HJMT, Spread, Settle, Maturity, 'Principal', Principal)
Price = 100.3059
```

### Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the CapRate and FloorRate input argument to define the collar pricing.

Price a portfolio of collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013'; '1-April-2014'; '1-April-2015'; ...
'1-April-2016'; '1-April-2017'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

Build the HJM tree with the following data:

```
MatDates = {'1-April-2013'; '1-April-2014'; '1-April-2015'; ...  
            '1-April-2016'; '1-April-2017'; '1-April-2018'};  
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);  
Volatility = [.10; .08; .06; .04];  
CurveTerm = [ 1; 2; 3; 4];  
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);  
HJMT = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec);
```

Create the floating-rate note instrument.

```
Settle = '1-April-2012';  
Maturity = '1-April-2016';  
Spread = 10;  
Principal = 100;
```

Compute the price of two capped collared floating-rate notes.

```
CapStrike = [0.04; 0.055];  
PriceCapped = floatbyhjm(HJMT, Spread, Settle, Maturity, ...  
                        'CapRate', CapStrike)
```

```
PriceCapped = 2×1
```

```
    98.9986  
   100.2051
```

Compute the price of two collared floating-rate notes.

```
FloorStrike = [0.035; 0.040];  
PriceCollared = floatbyhjm(HJMT, Spread, Settle, Maturity, ...  
                          'CapRate', CapStrike, 'FloorRate', FloorStrike)
```

```
PriceCollared = 2×1
```

```
    99.9246  
   102.2321
```

## Input Arguments

**HJMTree** — Interest-rate structure  
structure

Interest-rate tree structure, created by `hjm tree`

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

vector

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the HJM tree. The floating-rate note argument `Settle` is ignored.

Data Types: `char` | `double`

### **Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each floating-rate note.

Data Types: `char` | `double`

## **Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = floatbyhjm(HJMTree, Spread, Settle, Maturity, 'Basis', 3)`

### **FloatReset — Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

---

Data Types: double

### **Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using derivset.

Data Types: struct

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: `logical`

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: `double`

### **BusinessDayConvention — Business day conventions**

`actual` (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- `modifiedfollow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

### **CapRate — Annual cap rate**

`decimal`

Annual cap rate, specified as the comma-separated pair consisting of 'CapRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

Data Types: double | cell

### **FloorRate — Annual floor rate**

decimal

Annual floor rate, specified as the comma-separated pair consisting of 'FloorRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

Data Types: double | cell

## **Output Arguments**

### **Price — Expected floating-rate note prices at time 0**

vector

Expected floating-rate note prices at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within PriceTree:

- PriceTree.PBush contains the clean prices.
- PriceTree.AIBush contains the accrued interest.
- PriceTree.tObs contains the observation times.

## **See Also**

bondbyhjm | capbyhjm | cfbyhjm | fixedbyhjm | floorbyhjm | hjmtree | swapbyhjm

**Topics**

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# floatbyhw

Price floating-rate note from Hull-White interest-rate tree

## Syntax

```
[Price,PriceTree] = floatbyhw(HWTree,Spread,Settle,Maturity)
[Price,PriceTree] = floatbyhw( ____,Name,Value)
```

## Description

[Price,PriceTree] = floatbyhw(HWTree,Spread,Settle,Maturity) prices a floating-rate note from a Hull-White interest-rate tree.

floatbyhw computes prices of vanilla floating-rate notes, amortizing floating-rate notes, capped floating-rate notes, floored floating-rate notes and collared floating-rate notes.

[Price,PriceTree] = floatbyhw( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Floating-Rate Note Using a Hull-White Tree

Price a 20-basis point floating-rate note using a Hull-White interest-rate tree.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;  
Settle = '01-Jan-2004';  
Maturity = '01-Jan-2007';
```

Use `floatbyhw` to compute the price of the note.

```
Price = floatbyhw(HWTree, Spread, Settle, Maturity)
```

```
Price = 100.5618
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];  
ValuationDate = '15-Nov-2011';  
StartDates = ValuationDate;  
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014' ; '15-Nov-2015'; '15-Nov-2016'};  
Compounding = 1;  
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: [5x1 double]  
    Rates: [5x1 double]  
    EndTimes: [5x1 double]  
    StartTimes: [5x1 double]  
    EndDates: [5x1 double]  
    StartDates: 734822  
    ValuationDate: 734822  
    Basis: 0  
    EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle = '15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};
```

Build the HW tree and assume the volatility is 10%.

```
VolDates = ['15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014'; '15-Nov-2015'; '15-Nov-2016'; '15-Nov-2017'];
VolCurve = 0.1;
AlphaDates = '15-Nov-2017';
AlphaCurve = 0.1;
```

```
HWVolSpec = hwwolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbyhw(HWT, Spread, Settle, Maturity, 'Principal', Principal)
Price = 100.3059
```

### Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the CapRate and FloorRate input argument to define the collar pricing.

Price two collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013'; '1-April-2014'; '1-April-2015'; ...
'1-April-2016'; '1-April-2017'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

Build the HW tree and assume the volatility to be 5%.

```
VolDates = ['1-April-2013'; '1-April-2014'; '1-April-2015'; ...  
'1-April-2016'; '1-April-2017'; '1-April-2018'];  
VolCurve = 0.05;  
AlphaDates = '15-Nov-2018';  
AlphaCurve = 0.1;
```

```
HWVolSpec = hwwolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...  
AlphaDates, AlphaCurve);  
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);  
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Create the floating-rate note instrument.

```
Settle = '1-April-2012';  
Maturity = '1-April-2016';  
Spread = 10;  
Principal = 100;
```

Compute the price of a vanilla floater.

```
Price = floatbyhw(HWT, Spread, Settle, Maturity)
```

```
Price = 100.3680
```

Compute the price of the collared floating-rate notes.

```
CapStrike = {{'1-April-2014' 0.045; '1-April-2015' 0.05; ...  
'1-April-2016' 0.06}; 0.06};  
FloorStrike = {{'1-April-2014' 0.035; '1-April-2015' 0.04; ...  
'1-April-2016' 0.05}; 0.03};  
PriceCollared = floatbyhw(HWT, Spread, Settle, Maturity, ...  
'CapRate', CapStrike, 'FloorRate', FloorStrike)
```

```
PriceCollared = 2×1
```

```
102.0458  
100.9299
```

## Pricing a Floating-Rate Note When the Reset Dates Are Not Tree Level Dates

When using `floatbyhw` to price floating-rate notes, there are cases where the dates specified in the HW tree `TimeSpec` are not aligned with the cash flow dates.

Price floating-rate notes using the following data:

```
ValuationDate = '01-Sep-2013';
Rates = [0.0001; 0.0001; 0.0010; 0.0015];
EndDates = ['01-Dec-2013'; '01-Mar-2014'; '01-Jun-2014'; '01-Sep-2014'];
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',...
ValuationDate,'EndDates',EndDates,'Rates',Rates,'Compounding', 1);
```

Build the HW tree.

```
Volcurve = 0.1;
Alpha = 0.01;
HWVolatilitySpec = hmwolspec(RateSpec.ValuationDate, ...
EndDates, Volcurve,...
EndDates, Alpha);

HWTimeSpec = hwtimespec(RateSpec.ValuationDate, EndDates, 1);

HWT = hwtree(HWVolatilitySpec, RateSpec, HWTimeSpec);
```

Compute the price of the floating-rate note using the following data.

```
Spread = 10;
Settle = '01-Sep-2013';
Maturity = '01-Jun-2014';
Reset = 2;

Price = floatbyhw(HWT, Spread, Settle, Maturity, 'FloatReset', Reset)

Error using floatengbytrintree (line 318)
Instrument '1' has cash flow dates that span across tree nodes.

Error in floatbyhw (line 136)
[Price, PriceTree, CFTree] = floatengbytrintree(HWTTree, Spread, Settle, Maturity, 0Args{:});
```

This error indicates that it is not possible to determine the applicable rate used to calculate the payoff at the reset dates, given that the applicable rate needed cannot be calculated (the information was lost due to the recombination of the tree nodes). Note, if the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates. The simplest

solution is to place the tree levels at the cash flow dates of the instrument, which is done by specifying `HWTimeSpec`. It is also acceptable to have reset dates between tree levels, as long as there are reset dates on the tree levels.

To recover from this error, build a tree that lines up with the instrument.

```
Basis = intenvget(RateSpec, 'Basis');
EOM = intenvget(RateSpec, 'EndMonthRule');
resetDates = cfdates(ValuationDate, Maturity, Reset, Basis, EOM);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, resetDates, Reset);
HWT = hwtree(HWVolatilitySpec, RateSpec, HWTimeSpec);

Price = floatbyhw(HWT, Spread, RateSpec.ValuationDate, ...
    Maturity, 'FloatReset', Reset)
```

```
Price =
```

```
100.0748
```

## Input Arguments

### **HWTree — Interest-rate structure**

structure

Interest-rate tree structure, created by `hwtree`

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

vector

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the HW tree. The floating-rate note argument `Settle` is ignored.

Data Types: `char` | `double`

**Maturity – Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each floating-rate note.

Data Types: char | double

**Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = floatbyhw(HWTree, Spread, Settle, Maturity, 'Basis', 3)`

**FloatReset – Frequency of payments per year**

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis – Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal** — Notional principal amounts or principal value schedules

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

`Principal` accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: `cell` | `double`

### **Options** — Derivatives pricing options structure

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure using `derivset`.

Data Types: `struct`



### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 vector of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: double

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **CapRate — Annual cap rate**

decimal

Annual cap rate, specified as the comma-separated pair consisting of 'CapRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

Data Types: double | cell

### **FloorRate — Annual floor rate**

decimal

Annual floor rate, specified as the comma-separated pair consisting of 'FloorRate' and a NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

Data Types: double | cell

## **Output Arguments**

### **Price — Expected floating-rate note prices at time 0**

vector

Expected floating-rate note prices at time 0, returned as a NINST-by-1 vector.

## **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## **See Also**

`bondbyhw` | `capbyhw` | `cfbyhw` | `fixedbyhw` | `floorbyhw` | `hwtree` | `swapbyhw`

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## floatbyzero

Price floating-rate note from set of zero curves

### Syntax

```
[Price,DirtyPrice,CFlowAmounts,CFlowDates] = floatbyzero(RateSpec,  
Spread,Settle,Maturity)  
[Price,DirtyPrice,CFlowAmounts,CFlowDates] = floatbyzero( ____,  
Name,Value)
```

### Description

[Price,DirtyPrice,CFlowAmounts,CFlowDates] = floatbyzero(RateSpec, Spread,Settle,Maturity) prices a floating-rate note from a set of zero curves.

floatbyzero computes prices of vanilla floating-rate notes and amortizing floating-rate notes.

[Price,DirtyPrice,CFlowAmounts,CFlowDates] = floatbyzero( \_\_\_\_, Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a Floating-Rate Note Using a Set of Zero Curves

Price a 20-basis point floating-rate note using a set of zero curves.

Load `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure, needed to price the bond.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use floatbyzero to compute the price of the note.

```
Price = floatbyzero(ZeroRateSpec, Spread, Settle, Maturity)
```

```
Price = 100.5529
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014'; '15-Nov-2015'; '15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
    ValuationDate: 734822
    Basis: 0
    EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle = '15-Nov-2011';  
Maturity = '15-Nov-2015';  
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbyzero(RateSpec, Spread, Settle, Maturity, 'Principal', Principal)
```

```
Price = 100.3059
```

### **Specify the Rate at the Instrument's Starting Date When It Cannot Be Obtained from the RateSpec**

If `Settle` is not on a reset date of a floating-rate note, `floatbyzero` attempts to obtain the latest floating rate before `Settle` from `RateSpec` or the `LatestFloatingRate` parameter. When the reset date for this rate is out of the range of `RateSpec` (and `LatestFloatingRate` is not specified), `floatbyzero` fails to obtain the rate for that date and generates an error. This example shows how to use the `LatestFloatingRate` input parameter to avoid the error.

Create the error condition when a floating-rate instrument's `StartDate` cannot be determined from the `RateSpec`.

```
load deriv.mat;
```

```
Spread = 20;  
Settle = '01-Jan-2000';  
Maturity = '01-Dec-2003';
```

```
Price = floatbyzero(ZeroRateSpec, Spread, Settle, Maturity)
```

```
Error using floatbyzero (line 256)
```

```
The rate at the instrument starting date cannot be obtained from RateSpec.
```

```
Its reset date (01-Dec-1999) is out of the range of dates contained in RateSpec.
```

```
This rate is required to calculate cash flows at the instrument starting date.
```

```
Consider specifying this rate with the 'LatestFloatingRate' input parameter.
```

Here, the reset date for the rate at `Settle` was 01-Dec-1999, which was earlier than the valuation date of `ZeroRateSpec` (01-Jan-2000). This error can be avoided by specifying the rate at the instrument's starting date using the `LatestFloatingRate` name-value pair argument.

Define LatestFloatingRate and calculate the floating-rate price.

```
Price = floatbyzero(ZeroRateSpec, Spread, Settle, Maturity, 'LatestFloatingRate', 0.03)
```

```
Price =
```

```
100.0285
```

### Price a Floating-Rate Note Using a Different Curve to Generate Floating Cash Flows

Define the OIS and Libor rates.

```
Settle = datenum('15-Mar-2013');
```

```
CurveDates = daysadd(Settle,360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10],1);
```

```
OISRates = [.0018 .0019 .0021 .0023 .0031 .006 .011 .017 .021 .026 .03]';
```

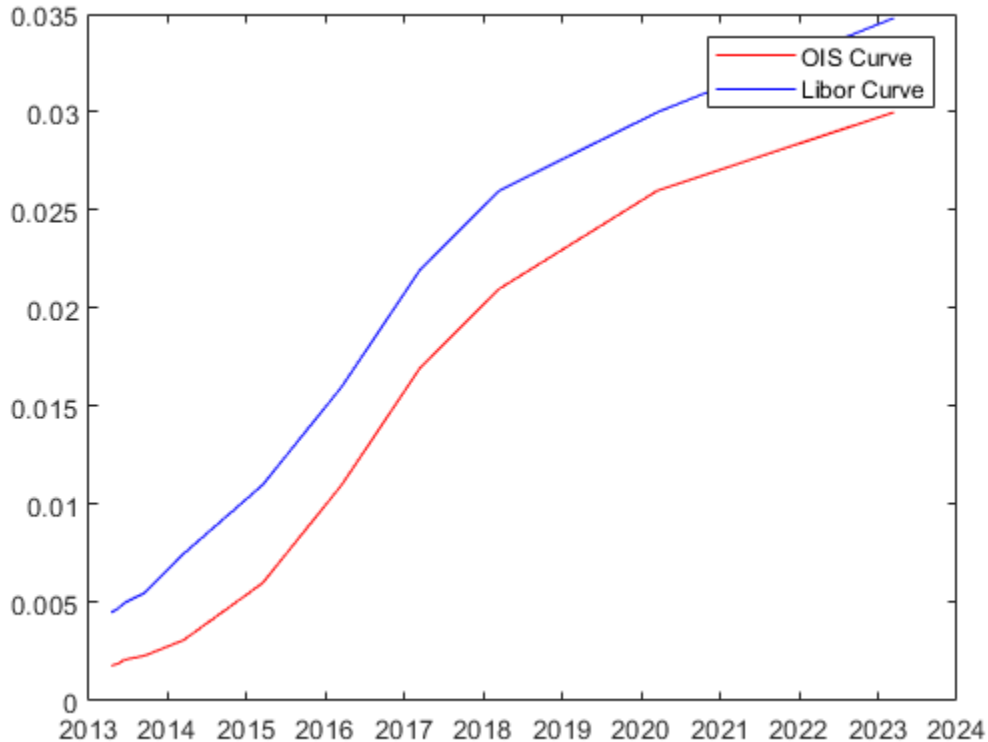
```
LiborRates = [.0045 .0047 .005 .0055 .0075 .011 .016 .022 .026 .030 .0348]';
```

Plot the dual curves.

```
figure,plot(CurveDates,OISRates,'r');hold on;plot(CurveDates,LiborRates,'b')
```

```
datetick
```

```
legend({'OIS Curve', 'Libor Curve'})
```



Create an associated RateSpec for the OIS and Libor curves.

```
OISCurve = intenvset('Rates',OISRates,'StartDate',Settle,'EndDates',CurveDates);
LiborCurve = intenvset('Rates',LiborRates,'StartDate',Settle,'EndDates',CurveDates);
```

Define the floating-rate note.

```
Maturity = datenum('15-Mar-2018');
```

Compute the price for the floating-rate note. The LiborCurve term structure will be used to generate the floating cash flows of the floater instrument. The OISCurve term structure will be used for discounting the cash flows.

```
Price = floatbyzero(OISCurve,0,Settle,Maturity,'ProjectionCurve',LiborCurve)
```



Price = 102.4214

Some instruments require using different interest-rate curves for generating the floating cash flows and discounting. This is when the `ProjectionCurve` parameter is useful. When you provide both `RateSpec` and `ProjectionCurve`, floatbyzero uses the `RateSpec` for the purpose of discounting and it uses the `ProjectionCurve` for generating the floating cash flows.

## Input Arguments

### **RateSpec — Annualized zero rate term structure**

structure

Annualized zero rate term structure, specified using `intenvset` to create a `RateSpec`.

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

vector

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

Settle must be earlier than `Maturity`.

Data Types: `char` | `double`

### **Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each floating-rate note.

Data Types: `char` | `double`

## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, DirtyPrice, CFlowAmounts, CFlowDates] = floatbyzero(RateSpec, Spread, Settle, Maturity, 'Principal', Principal)`

### **FloatReset** — Frequency of payments per year

1 (default) | vector

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and a NINST-by-1 vector.

Data Types: double

### **Basis** — Day count basis

0 (actual/actual) (default) | integer from 0 to 13

Day count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)

- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array, where each element of the cell array is a NumDates-by-2 cell array and the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **LatestFloatingRate — Rate for the next floating payment**

if not specified, the floating rate at the previous reset date is computed from RateSpec (default) | numeric

Rate for the next floating payment set at the last reset date, specified as the comma-separated pair consisting of 'LatestFloatingRate' and a NINST-by-1.

Data Types: double

**ProjectionCurve — Rate curve used in generating future forward rates**

if not specified, then `RateSpec` is used both for discounting cash flows and projecting future forward rates (default) | structure

The rate curve to be used in generating the future forward rates, specified as the comma-separated pair consisting of 'ProjectionCurve' and a structure created using `intenvset`. Use this optional input if the forward curve is different from the discount curve.

Data Types: `struct`

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

`false` (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a `NINST-by-1` vector of logicals with values of 0 (false) or 1 (true).

Data Types: `logical`

**Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: `double`

**BusinessDayConvention — Business day conventions**

`actual` (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a `N-by-1` cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.

- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

## Output Arguments

### **Price — Floating-rate note prices**

matrix

Floating-rate note prices, returned as a (NINST) by number of curves (NUMCURVES) matrix. Each column arises from one of the zero curves.

### **DirtyPrice — Dirty note price**

matrix

Dirty note price (clean + accrued interest), returned as a NINST- by-NUMCURVES matrix. Each column arises from one of the zero curves.

### **CFlowAmounts — Cash flow amounts**

matrix

Cash flow amounts, returned as a NINST- by-NUMCFS matrix of cash flows for each note. If there is more than one curve specified in the RateSpec input, then the first NCURVES rows correspond to the first note, the second NCURVES rows correspond to the second note, and so on.

### **CFlowDates — Cash flow dates**

matrix

Cash flow dates, returned as a NINST- by-NUMCFS matrix of payment dates for each note.

## **See Also**

bondbyzero | cfbyzero | fixedbyzero | intenvset | swapbyzero

## **Topics**

“Pricing Using Interest-Rate Term Structure” on page 2-72

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# floorbybdt

Price floor instrument from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price,PriceTree] = floorbybdt(BDTree,Strike,Settle,Maturity)
[Price,PriceTree] = floorbybdt( ____,FloorReset,Basis,Principal,
Options)
```

## Description

[Price,PriceTree] = floorbybdt(BDTree,Strike,Settle,Maturity) computes the price of a floor instrument from a Black-Derman-Toy interest-rate tree. floorbybdt computes prices of vanilla floors and amortizing floors.

[Price,PriceTree] = floorbybdt( \_\_\_\_,FloorReset,Basis,Principal, Options) adds optional arguments.

## Examples

### Price a 10% Floor Instrument Using a BDT Interest-Rate Tree

Load the file `deriv.mat`, which provides `BDTree`. `BDTree` contains the time and interest-rate information needed to price the floor instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.10;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use `floorbybdt` to compute the price of the floor instrument.

```
Price = floorbybdt(BDTree, Strike, Settle, Maturity)
Price = 0.2428
```

### Price a 10% Floor Instrument Using a Newly Created BDT Interest-Rate Tree

First set the required arguments for the three needed specifications.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
```

Create the specifications.

```
RateSpec = intenvset('Compounding', Compounding,...
'ValuationDate', ValuationDate,...
'StartDates', StartDate,...
'EndDates', EndDates,...
'Rates', Rates);
BDTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
```

Create the BDT tree from the specifications.

```
BDTree = bdttree(BDTVolSpec, RateSpec, BDTimeSpec)
```

```
BDTree = struct with fields:
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [730486 730852 731217 731582 731947]
    TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
    CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
    FwdTree: {1x5 cell}
```

Set the floor arguments. Remaining arguments will use defaults.



```
FloorStrike = 0.10;
Settlement = ValuationDate;
Maturity = '01-01-2002';
FloorReset = 1;
```

Use floorbybdt to find the price of the floor instrument.

```
Price= floorbybdt(BDTTree, FloorStrike, Settlement, Maturity,...
FloorReset)
```

```
Price = 0.0863
```

### Compute the Price of an Amortizing Floor Using the BDT Model

Define the RateSpec.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014' ; '15-Nov-2015'; '15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 734822
    ValuationDate: 734822
    Basis: 0
    EndMonthRule: 1
```

Define the floor instrument.

```
Settle = '15-Nov-2011';
Maturity = '15-Nov-2015';
Strike = 0.039;
```

```
Reset = 1;
Principal = {'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10};;
```

Build the BDT Tree.

```
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility*ones(1,length(EndDates)))
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
```

BDTTree = struct with fields:

```
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3 4]
        dObs: [734822 735188 735553 735918 736283]
        TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
        CFLOWT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
    FwdTree: {1x5 cell}
```

Price the amortizing floor.

```
Basis = 0;
Price = floorbybdt(BDTTree, Strike, Settle, Maturity, Reset, Basis, Principal)
```

```
Price = 0.3060
```

## Input Arguments

### **BDTTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdttree`.

Data Types: `struct`

### **Strike** — Rate at which floor is exercised

decimal

Rate at which the floor is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: double

**Settle — Settlement date for floor**

serial date number | date character vector | cell array of date character vectors

Settlement date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The **Settle** date for every floor is set to the **ValuationDate** of the BDT tree. The floor argument **Settle** is ignored.

Data Types: double | char | cell

**Maturity — Maturity date for floor**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**FloorReset — Reset frequency payment per year**

1 (default) | numeric

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)

- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal — Notional principal amount**

100 (default) | `numeric`

(Optional) Notional principal amount, specified as a NINST-by-1 of notional principal amounts, or a NINST-by-1 cell array, where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing floor.

Data Types: `double` | `cell`

### **Options — Derivatives pricing options structure**

`structure`

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected price of floor at time 0**

`vector`

Expected price of the floor at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure with values of floor at each node**

`vector`

Tree structure with values of the floor at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains floor prices.
- `PriceTree.tObs` contains the observation times.

## See Also

`bdttree` | `capbybdt` | `cfbybdt` | `floorbynormal` | `swapbybdt`

## Topics

“Computing Instrument Prices” on page 2-99

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## floorbybk

Price floor instrument from Black-Karasinski interest-rate tree

### Syntax

```
[Price,PriceTree] = floorbybk(BKTree,Strike,Settle,Maturity)
[Price,PriceTree] = floorbybk( ____,Reset,Basis,Principal,Options)
```

### Description

[Price,PriceTree] = floorbybk(BKTree,Strike,Settle,Maturity) computes the price of a floor instrument from a Black-Karasinski interest-rate tree. floorbybk computes prices of vanilla floors and amortizing floors.

[Price,PriceTree] = floorbybk( \_\_\_\_,Reset,Basis,Principal,Options) adds optional arguments.

### Examples

#### Price a 3% Floor Instrument Using a Black-Karasinski Interest-Rate Tree

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest rate information needed to price the floor instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2004';
Maturity = '01-Jan-2007';
```

Use `floorbybk` to compute the price of the floor instrument.

```
Price = floorbybk(BKTree, Strike, Settle, Maturity)
```

```
Price = 0.2061
```

## Compute the Price of an Amortizing and Vanilla Floors Using the BK Model

Load `deriv.mat` to specify the BKTtree and then define the floor instrument.

```
load deriv.mat;
Settle = '01-Jan-2004';
Maturity = '01-Jan-2008';
Strike = 0.045;
Reset = 1;
Principal ={'01-Jan-2005' 100;'01-Jan-2006' 60;'01-Jan-2007' 30;'01-Jan-2008' 30};...
          100};
```

Price the amortizing and vanilla floors.

```
Basis = 1;
Price = floorbybk(BKTtree, Strike, Settle, Maturity, Reset, Basis, Principal)
```

```
Price = 2×1
```

```
    2.2000
    2.5564
```

## Input Arguments

### BKTtree — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### Strike — Rate at which cap is exercised

decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

**Settle — Settlement date for floor**

serial date number | date character vector | cell array of date character vectors

Settlement date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The `Settle` date for every floor is set to the `ValuationDate` of the BK tree. The floor argument `Settle` is ignored.

Data Types: double | char | cell

**Maturity — Maturity date for floor**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**Reset — Reset frequency payment per year**

1 (default) | numeric

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)



- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

(Optional) Notional principal amount, specified as a NINST-by-1 of notional principal amounts, or a NINST-by-1 cell array, where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing floor.

Data Types: double | cell

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected price of floor at time 0**

vector

Expected price of the floor at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure with values of floor at each node**

vector

Tree structure with values of the floor at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains floor prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

### See Also

`bktree` | `capbybk` | `cfbybk` | `floorbynormal` | `swapbybk`

### Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# floorbycir

Price floor instrument from Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = floorbycir(CIRTree,Strike,Settle,Maturity)
[Price,PriceTree] = floorbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = floorbycir(CIRTree,Strike,Settle,Maturity) computes the price of a floor instrument from a Cox-Ingersoll-Ross (CIR) interest-rate tree. floorbycir computes prices of vanilla floors and amortizing floors using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = floorbycir( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price a Floor Using a CIR Interest-Rate Tree

Define the Strike for a floor.

```
Strike = 0.02;
```

Create a RateSpec using the intenvset function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```

NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2021';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tobs: [0 1 2 3]
    dobs: [736696 737061 737426 737791]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

Price the 2% floor.

```

[Price,PriceTree] = floorbycir(CIRT,Strike,Settle,Maturity)

Price = 1.4211e-14

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    tobs: [0 1 2 3 4]
    PTree: {1x5 cell}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

## Input Arguments

**CIRtree** — Interest-rate tree structure  
structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **Strike — Rate at which cap is exercised**

decimal

Rate at which cap is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

### **Settle — Settlement date for floor**

serial date number | date character vector | cell array of date character vectors | string array | datetime

Settlement date for the floor, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays. The `Settle` date for every floor is set to the `ValuationDate` of the CIR tree. The floor argument `Settle` is ignored.

Data Types: `double` | `char` | `cell` | `datetime`

### **Maturity — Maturity date for floor**

serial date number | date character vector | cell array of date character vectors | string array | datetime

Maturity date for the floor, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: `double` | `char` | `cell` | `datetime`

## **Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = floorbycir(CIRTree, CouponRate, Settle, Maturity, 'Basis', 3)`

### **FloorReset — Reset frequency payment per year**

1 (default) | numeric

Reset frequency payment per year, specified as the comma-separated pair consisting of 'FloorReset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 of notional principal amounts, or a NINST-by-1 cell array.

For the NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates, and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing floor.

Data Types: `double` | `cell`

## Output Arguments

### **Price** — Expected price of floor at time 0

vector

Expected price of the floor at time 0, returned as a NINST-by-1 vector.

### **PriceTree** — Tree structure with values of floor at each node

vector

Tree structure with values of the floor at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains floor prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

- [3] Hirsa, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | cfbycir | fixedbycir | floatbycir | instfloor |  
oasbycir | optbndbycir | optembndbycir | optemfloatbycir | optfloatbycir |  
rangefloatbycir | swapbycir | swaptionbycir

## Topics

- "Pricing Using Interest-Rate Tree Models" on page 2-99  
"Understanding Interest-Rate Tree Models" on page 2-79  
"Pricing Options Structure" on page B-2  
"Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a



# floorbyblk

Price floors using Black option pricing model

## Syntax

```
[FloorPrice,Floorlets] = floorbyblk(RateSpec,Strike,Settle,Maturity,
Volatility)
[FloorPrice,Floorlets] = floorbyblk( ____,Name,Value)
```

## Description

[FloorPrice,Floorlets] = floorbyblk(RateSpec,Strike,Settle,Maturity, Volatility) price floors using the Black option pricing model. floorbyblk computes prices of vanilla floors and amortizing floors.

[FloorPrice,Floorlets] = floorbyblk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a Floor Using the Black Option Pricing Model

This example shows how to price a floor using the Black option pricing model. Consider an investor who gets into a contract that floors the interest rate on a \$100,000 loan at 6% quarterly compounded for 3 months, starting on January 1, 2009. Assuming that on January 1, 2008 the zero rate is 6.9394% continuously compounded and the volatility is 20%, use this data to compute the floor price.

```
ValuationDate = 'Jan-01-2008';
EndDates = 'April-01-2010';
Rates = 0.069394;
Compounding = -1;
Basis = 1;
```

```

% calculate the RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

Settle = 'Jan-01-2009'; % floor starts in a year
Maturity = 'April-01-2009';
Volatility = 0.20;
FloorRate = 0.06;
FloorReset = 4;
Principal=100000;

FloorPrice = floorbyblk(RateSpec, FloorRate, Settle, Maturity, Volatility,...
'Reset', FloorReset, 'ValuationDate', ValuationDate, 'Principal', Principal,...
'Basis', Basis)

FloorPrice = 37.4864

```

### Price a Floor Using a Different Curve to Generate the Future Forward Rates

Define the OIS and Libor rates.

```

Settle = datenum('15-Mar-2013');
CurveDates = daysadd(Settle, 360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10], 1);
OISRates = [.0018 .0019 .0021 .0023 .0031 .006 .011 .017 .021 .026 .03]';
LiborRates = [.0045 .0047 .005 .0055 .0075 .0109 .0162 .0216 .0262 .0309 .0348]';

```

Create an associated RateSpec for the OIS and Libor curves.

```

OISCurve = intenvset('Rates', OISRates, 'StartDate', Settle, 'EndDates', CurveDates, 'Compounding', Compounding, 'Basis', Basis);
LiborCurve = intenvset('Rates', LiborRates, 'StartDate', Settle, 'EndDates', CurveDates, 'Compounding', Compounding, 'Basis', Basis);

```

Define the Floor instruments.

```

Maturity = {'15-Mar-2018'; '15-Mar-2020'};
Strike = [.04; .05];
BlackVol = .2;

```

Price the floor instruments using the term structure OISCurve both for discounting the cash flows and generating future forward rates.

```

[Price, Floorlets] = floorbyblk(OISCurve, Strike, Settle, Maturity, BlackVol)

```

```
Price = 2x1
```

```
  9.9808
 16.9057
```

```
Floorlets = 2x7
```

```
  3.6783  3.0706  1.8275  0.7280  0.6764  NaN  NaN
  4.6753  4.0587  2.7921  1.4763  1.3442  1.4130  1.1462
```

Price the floor instruments using the term structure `LiborCurve` to generate future forward rates. The term structure `OISCurve` is used for discounting the cash flows.

```
[PriceLC, FloorletsLC] = floorbyblk(OISCurve, Strike, Settle, Maturity, BlackVol, 'Proj
```

```
PriceLC = 2x1
```

```
  8.0524
 14.3184
```

```
FloorletsLC = 2x7
```

```
  3.2385  2.5338  1.2895  0.5889  0.4017  NaN  NaN
  4.2355  3.5219  2.2286  1.2751  0.9169  1.1698  0.9706
```

## Compute the Price of an Amortizing Floor Using the Black Model

Define the `RateSpec`.

```
Rates = [0.0358; 0.0421; 0.0473; 0.0527; 0.0543];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012'; '15-Nov-2013'; '15-Nov-2014'; '15-Nov-2015'; '15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
```

```
Compounding: 1
  Disc: [5x1 double]
  Rates: [5x1 double]
  EndTimes: [5x1 double]
  StartTimes: [5x1 double]
  EndDates: [5x1 double]
  StartDates: 734822
ValuationDate: 734822
  Basis: 0
EndMonthRule: 1
```

Define the floor instrument.

```
Settle = '15-Nov-2011';
Maturity = '15-Nov-2015';
Strike = 0.05;
Reset = 2;
Principal = {{'15-Nov-2012' 100; '15-Nov-2013' 70; '15-Nov-2014' 40; '15-Nov-2015' 10}};
```

Price the amortizing floor.

```
Volatility = 0.20;
Price = floorbyblk(RateSpec, Strike, Settle, Maturity, Volatility, ...
  'Reset', Reset, 'Principal', Principal)

Price = 1.9315
```

### Price a Floor Using the Shifted Black Model

Create the RateSpec.

```
ValuationDate = 'Mar-01-2016';
EndDates = {'Mar-01-2017'; 'Mar-01-2018'; 'Mar-01-2019'; 'Mar-01-2020'; 'Mar-01-2021'};
Rates = [-0.21; -0.12; 0.01; 0.10; 0.20]/100;
Compounding = 1;
Basis = 1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, ...
  'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec = struct with fields:
  FinObj: 'RateSpec'
```

```

Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 736390
ValuationDate: 736390
    Basis: 1
EndMonthRule: 1

```

Price the floor with a negative strike using the Shifted Black model.

```

Settle = 'Jun-01-2016'; % Floor starts in 3 months.
Maturity = 'Sep-01-2016';
ShiftedBlackVolatility = 0.31;
FloorRate = -0.001; % -0.1 percent strike.
FloorReset = 4;
Principal = 100000;
Shift = 0.01; % 1 percent shift.

```

```

FloorPrice = floorbyblk(RateSpec,FloorRate,Settle,Maturity,ShiftedBlackVolatility,...
'Reset',FloorReset,'ValuationDate',ValuationDate,'Principal',Principal,...
'Basis',Basis,'Shift',Shift)

```

```
FloorPrice = 31.2099
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### Strike — Rate at which floor is exercised

decimal

Rate at which floor is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: double

**Settle — Settlement date for floor**

serial date number | date character vector

Settlement date for the floor, specified as a serial date number or a date character vector.

Data Types: double | char

**Maturity — Maturity date for floor**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floor, specified as a serial date number or date character vector.

Data Types: double | char

**Volatility — Volatilities values**

numeric

Volatilities values, specified as a NINST-by-1 vector of numeric values.

The `Volatility` input is not intended for volatility surfaces or cubes. If you specify a matrix for the `Volatility` input, `floorbyblk` internally converts it into a vector. `floorbyblk` assumes that the volatilities specified in the `Volatility` input are flat volatilities, which are applied equally to each of the floorlets.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike, Settle, Maturity, Volatility, 'Reset', CapReset, 'Principal', 100000, 'Basis', 7)`

**Reset — Reset frequency payment per year**

1 (default) | numeric

Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as a NINST-by-1 of notional principal amounts, or a NINST-by-1 cell array. When **Principal** is a NINST-by-1 cell array, each element is a NumDates-by-2 cell array, where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use **Principal** to pass a schedule to compute the price for an amortizing floor.

Data Types: double | cell

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**ProjectionCurve — Rate curve used in generating future forward rates**

if `ProjectionCurve` is not specified, then `RateSpec` is used both for discounting cash flows and projecting future forward rates (default) | structure

The rate curve to be used in generating the future forward rates. This structure must be created using `intenvset`. Use this optional input if the forward curve is different from the discount curve.

Data Types: struct

**Shift — Shift in decimals for shifted Black model**

0 (no shift) (default) | positive decimal

Shift in decimals for the shifted Black model, specified using a scalar or NINST-by-1 vector of rate shifts in positive decimals. Set this parameter to a positive rate shift in decimals to add a positive shift to the forward rate and strike, which effectively sets a negative lower bound for the forward rate. For example, a `Shift` of 0.01 is equal to a 1% shift.

Data Types: double

## Output Arguments

**FloorPrice — Expected price of floor**

vector

Expected price of the floor, returned as a NINST-by-1 vector.

**Floorlets — Floorlets**

array

Floorlets, returned as a NINST-by-NCF array of floorlets, padded with NaNs.



## Definitions

### Shifted Black

The Shifted Black model is essentially the same as the Black's model, except that it models the movements of  $(F + Shift)$  as the underlying asset, instead of  $F$  (which is the forward rate in the case of floorlets).

This model allows negative rates, with a fixed negative lower bound defined by the amount of shift; that is, the zero lower bound of Black's model has been shifted.

## Algorithms

### Black Model

$$dF = \sigma_{Black} F dw$$

$$call = e^{-\nu T} [FN(d_1) - KN(d_2)]$$

$$put = e^{-\nu T} [KN(-d_2) - FN(-d_1)]$$

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \left(\frac{\sigma_B^2}{2}\right)T}{\sigma_B\sqrt{T}}, \quad d_2 = d_1 - \sigma_B\sqrt{T}$$

$$\sigma_B = \sigma_{Black}$$

Where  $F$  is the forward value and  $K$  is the strike.

### Shifted Black Model

$$dF = \sigma_{Shifted\_Black} (F + Shift) dw$$

$$call = e^{-\nu T} [(F + Shift)N(d_{s1}) - (K + Shift)N(d_{s2})]$$

$$put = e^{-\nu T} [(K + Shift)N(-d_{s2}) - (F + Shift)N(-d_{s1})]$$

$$d_{s1} = \frac{\ln\left(\frac{F + Shift}{K + Shift}\right) + \left(\frac{\sigma_{sB}^2}{2}\right)T}{\sigma_{sB}\sqrt{T}}, \quad d_{s2} = d_{s1} - \sigma_{sB}\sqrt{T}$$

$$\sigma_{sB} = \sigma_{Shifted\_Black}$$

Where  $F+Shift$  is the forward value and  $K+Shift$  is the strike for the shifted version.

## **See Also**

capbyblk | floorbynormal | intenvset

## **Topics**

“Work with Negative Interest Rates” on page 2-22

“Supported Interest-Rate Instruments” on page 2-2

## **External Websites**

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

## **Introduced in R2009a**

# floorbyhjm

Price floor instrument from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price,PriceTree] = floorbyhjm(HJMTree,Strike,Settle,Maturity)
[Price,PriceTree] = floorbyhjm( ____,FloorReset,Basis,Principal,
Options)
```

## Description

[Price,PriceTree] = floorbyhjm(HJMTree,Strike,Settle,Maturity) computes the price of a floor instrument from a Heath-Jarrow-Morton interest-rate tree. floorbyhjm computes prices of vanilla floors and amortizing floors.

[Price,PriceTree] = floorbyhjm( \_\_\_\_,FloorReset,Basis,Principal,Options) adds optional arguments.

## Examples

### Price a 3% Floor Instrument Using an HJM Forward-Rate Tree

This example shows how to price a 3% floor instrument using an HJM forward-rate tree by loading the file `deriv.mat`, which provides HJMTree. The HJMTree structure contains the time and forward-rate information needed to price the floor instrument.

```
load deriv.mat;
```

```
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

```
Price = floorbyhjm(HJMTree, Strike, Settle, Maturity)
```

```
Price = 0.0486
```

### Compute the Price of an Amortizing Floor Using the HJM Model

Load `deriv.mat` to specify the HJMTree and then define the floor instrument.

```
load deriv.mat;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
Strike = 0.05;
FloorReset = 1;
Principal ={'01-Jan-2001' 100;'01-Jan-2002' 80;'01-Jan-2003' 70;'01-Jan-2004' 30};
```

Price the amortizing floor.

```
Price = floorbyhjm(HJMTree, Strike, Settle, Maturity, FloorReset, Principal)
```

```
Price = 2.8215
```

## Input Arguments

### HJMTree — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hjm tree`.

Data Types: `struct`

### Strike — Rate at which floor is exercised

decimal

Rate at which the floor is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

### Settle — Settlement date for floor

serial date number | date character vector | cell array of date character vectors

Settlement date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The `Settle` date for every floor is set to the `ValuationDate` of the HJM tree. The floor argument `Settle` is ignored.

Data Types: double | char | cell

**Maturity — Maturity date for floor**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**FloorReset — Reset frequency payment per year**

1 (default) | numeric

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

**Principal — Notional principal amount**

`100` (default) | `numeric`

(Optional) Notional principal amount, specified as a `NINST`-by-1 of notional principal amounts, or a `NINST`-by-1 cell array, where each element is a `NumDates`-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing floor.

Data Types: `double` | `cell`

**Options — Derivatives pricing options structure**

`structure`

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## Output Arguments

**Price — Expected price of floor at time 0**

`vector`

Expected price of the floor at time 0, returned as a `NINST`-by-1 vector.

**PriceTree — Tree structure with values of floor at each node**

`vector`

Tree structure with values of the floor at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.tObs` contains the observation times.
- `PriceTree.PBush` contains the clean prices.

## See Also

`capbyhjm` | `cfbyhjm` | `floorbynormal` | `hjmtree` | `swapbyhjm`

## **Topics**

"Computing Instrument Prices" on page 2-99

"Pricing Options Structure" on page B-2

"Understanding Interest-Rate Tree Models" on page 2-79

"Supported Interest-Rate Instruments" on page 2-2

**Introduced before R2006a**

## floorbyhw

Price floor instrument from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = floorbyhw(HWTree,Strike,Settle,Maturity)
[Price,PriceTree] = floorbyhw( ____,FloorReset,Basis,Principal,
Options)
```

### Description

[Price,PriceTree] = floorbyhw(HWTree,Strike,Settle,Maturity) computes the price of a floor instrument from a Hull-White interest-rate tree. capbyhw computes prices of vanilla floors and amortizing floors.

[Price,PriceTree] = floorbyhw( \_\_\_\_,FloorReset,Basis,Principal,Options) adds optional arguments.

### Examples

#### Price a 3% Floor Instrument Using a Hull-White Interest-Rate Tree

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest rate information needed to price the floor instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2004';
Maturity = '01-Jan-2007';
```

Use `floorbyhw` to compute the price of the floor instrument.



```
Price = floorbyhw(HWTree, Strike, Settle, Maturity)
```

```
Price = 0.4186
```

### Compute the Price of an Amortizing and Vanilla Floors Using the HW Model

Define the RateSpec.

```
Rates = [0.035; 0.042; 0.047; 0.052; 0.054];
ValuationDate = '01-April-2014';
StartDates = ValuationDate;
EndDates = {'01-April-2019'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: 737516
    StartDates: 735690
    ValuationDate: 735690
    Basis: 0
    EndMonthRule: 1
```

Define the floor instruments.

```
Settle = '01-April-2014';
Maturity = '01-April-2018';
Strike = 0.05;
FloorReset = 1;
Principal = {'01-April-2015' 100; '01-April-2016' 60; '01-April-2017' 40; '01-April-2018'
    100};
```

Build the HW Tree.

```
VolDates = ['01-April-2015'; '01-April-2016'; '01-April-2017'; '01-April-2018'];
VolCurve = 0.05;
```

```
AlphaDates = '01-April-2018';
AlphaCurve = 0.10;

HWVolSpec = hwwolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
                    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWTTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)

HWTTree = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [735690 736055 736421 736786]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0350] [1.1300 1.0363 0.9503] [1x5 double] [1x7 double]}
```

Price the amortizing and vanilla floors.

```
Basis = 0;
Price = floorbyhw(HWTTree, Strike, Settle, Maturity, FloorReset, Basis, Principal)

Price = 2x1

    4.8675
   10.3881
```

## Input Arguments

**HWTTree — Interest-rate tree structure**  
structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: `struct`

**Strike — Rate at which floor is exercised**  
decimal

Rate at which the floor is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: double

### **Settle — Settlement date for floor**

serial date number | date character vector | cell array of date character vectors

Settlement date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The **Settle** date for every floor is set to the **ValuationDate** of the HW tree. The floor argument **Settle** is ignored.

Data Types: double | char | cell

### **Maturity — Maturity date for floor**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floor, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **FloorReset — Reset frequency payment per year**

1 (default) | numeric

(Optional) Reset frequency payment per year, specified as a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis representing the basis used when annualizing the input forward rate, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal — Notional principal amount**

100 (default) | `numeric`

(Optional) Notional principal amount, specified as a `NINST`-by-1 of notional principal amounts, or a `NINST`-by-1 cell array, where each element is a `NumDates`-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing floor.

Data Types: `double` | `cell`

### **Options — Derivatives pricing options structure**

`structure`

(Optional) Derivatives pricing options structure, specified using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected price of floor at time 0**

`vector`

Expected price of the floor at time 0, returned as a `NINST`-by-1 vector.

### **PriceTree — Tree structure with values of floor at each node**

`vector`

Tree structure with values of the floor at each node, returned as a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node:

- `PriceTree.PTree` contains floor prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## See Also

`capbyhw` | `cfbyhw` | `floorbynormal` | `hwtree` | `swapbyhw`

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## floorbylg2f

Price floor using Linear Gaussian two-factor model

### Syntax

```
FloorPrice = floorbylg2f(ZeroCurve,a,b,sigma,eta,rho,Strike,  
Maturity)  
FloorPrice = floorbylg2f(____,Name,Value)
```

### Description

FloorPrice = floorbylg2f(ZeroCurve,a,b,sigma,eta,rho,Strike,Maturity) returns the floor price for a two-factor additive Gaussian interest-rate model.

FloorPrice = floorbylg2f(\_\_\_\_,Name,Value) adds optional name-value pair arguments.

---

**Note** Use the optional name-value pair argument, `Notional`, to pass a schedule to compute the price for an amortizing floor.

---

### Examples

#### Price a Floor Using a Linear Gaussian Two-Factor Model

Define the ZeroCurve, a, b, sigma, eta, and rho parameters to compute the floor price.

```
Settle = datenum('15-Dec-2007');  
  
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';  
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';  
CurveDates = daysadd(Settle,360*ZeroTimes,1);  
  
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
```

```

a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;

FloorMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);

Strike = [0.035 0.037 0.038 0.039 0.040 0.042 0.044 0.046 0.047 0.047 0.047]';

Price = floorbylg2f(irdc,a,b,sigma,eta,rho,Strike,FloorMaturity)

Price = 11x1

    0
    0.4041
    0.8282
    1.3103
    1.8346
    3.0636
    4.9172
    7.7614
    9.7166
   11.4163
    :

```

### Price an Amortizing Floor Using a Linear Gaussian Two-Factor Model

Define the ZeroCurve, a, b, sigma, eta, rho, and Notional parameters for the amortizing floor.

```

Settle = datenum('15-Dec-2007');
% Define ZeroCurve
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
CurveDates = daysadd(Settle,360*ZeroTimes);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

% Define a, b, sigma, eta, and rho

```

```
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;

% Define the amortizing floors
FloorMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);
Strike = [0.025 0.036 0.037 0.038 0.039 0.041 0.043 0.045 0.046 0.046 0.046]';
Notional = {'15-Dec-2012' 100;'15-Dec-2017' 70;'15-Dec-2022' 40;'15-Dec-2037' 10};;

% Price the amortizing floors
Price = floorbylg2f(irdc,a,b,sigma,eta,rho,Strike,FloorMaturity,'Notional',Notional)

Price = 11x1

    0
    0.2633
    0.6438
    1.0815
    1.5637
    2.5196
    3.9061
    5.4326
    6.0416
    6.2033
    :
```

## Input Arguments

### **ZeroCurve** — Zero curve for Linear Gaussian two-factor model

structure

Zero curve for the Linear Gaussian two-factor model, specified using `IRDataCurve` or `RateSpec`.

Data Types: `struct`

### **a** — Mean reversion for first factor for Linear Gaussian two-factor model

scalar



Mean reversion for the first factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: `single` | `double`

**b — Mean reversion for second factor for Linear Gaussian two-factor model**

scalar

Mean reversion for the second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: `single` | `double`

**sigma — Volatility for first factor for Linear Gaussian two-factor model**

scalar

Volatility for the first factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: `single` | `double`

**eta — Volatility for second factor for Linear Gaussian two-factor model**

scalar

Volatility for the second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: `single` | `double`

**rho — Scalar correlation of factors**

scalar

Scalar correlation of the factors, specified as a scalar.

Data Types: `single` | `double`

**Strike — Floor strike price**

`nonnegative integer` | `vector of nonnegative integers`

Floor strike price specified, as a nonnegative integer using a `NumFloors-by-1` vector of floor strike prices.

Data Types: `single` | `double`

**Maturity — Floor maturity date**

`serial date number` | `vector of serial date numbers` | `date character vector`

Floor maturity date, specified using a `NumFloors-by-1` vector of serial date numbers or date character vectors.

Data Types: `single` | `double` | `char` | `cell`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price = floorbylg2f(irdc,a,b,sigma,eta,rho,Strike,FloorMaturity,'Reset',1,'Notional',100)`

### **Reset** — Frequency of floor payments per year

2 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1,2,3,4,6,12]

Frequency of floor payments per year, specified as the comma-separated pair consisting of 'Reset' and positive integers for the values [1,2,4,6,12] in a `NumFloors-by-1` vector.

Data Types: `single` | `double`

### **Notional** — Notional value of floor

100 (default) | nonnegative integer | vector of nonnegative integers

`NINST-by-1` of notional principal amounts or `NINST-by-1` cell array where each element is a `NumDates-by-2` cell array where the first column is dates and the second column is the associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: `single` | `double`

## Output Arguments

### **FloorPrice** — Floor price

scalar | vector

Floor price, returned as a scalar or a `NumFloors-by-1` vector.

## Algorithms

The following defines the two-factor additive Gaussian interest-rate model, given the ZeroCurve, a, b, sigma, eta, and rho parameters:

$$r(t) = x(t) + y(t) + \phi(t)$$

$$dx(t) = -a(x)(t)dt + \sigma(dW_1(t), x(0) = 0$$

$$dy(t) = -b(y)(t)dt + \eta(dW_2(t), y(0) = 0$$

where  $dW_1(t)dW_2(t) = \rho dt$  is a two-dimensional Brownian motion with correlation  $\rho$  and  $\phi$  is a function chosen to match the initial zero curve.

## References

- [1] Brigo, D. and F. Mercurio, *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

## See Also

LinearGaussian2F | capbylg2f | swaptionbylg2f

## Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

## floorbynormal

Price floors using Normal or Bachelier pricing model

### Syntax

```
[FloorPrice, Floorlets] = floorbynormal(RateSpec, Strike, Settle,
Maturity, Volatility)
[FloorPrice, Floorlets] = floorbynormal( ____, Name, Value)
```

### Description

[FloorPrice, Floorlets] = floorbynormal(RateSpec, Strike, Settle, Maturity, Volatility) prices floors using the Normal (Bachelier) pricing model for negative rates. floorbynormal computes prices of vanilla floors and amortizing floors.

[FloorPrice, Floorlets] = floorbynormal( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

### Examples

#### Price a Floor Using Normal Model for Negative Rates

Consider an investor who gets into a contract that floors the interest rate on a \$100,000 loan at  $-0.6\%$  quarterly compounded for 3 months, starting on January 1, 2009. Assuming that on January 1, 2008 the zero rate is  $0.69394\%$  continuously compounded and the volatility is  $20\%$ , use this data to compute the floor price. First, calculate the RateSpec, and then use floorbynormal to compute the FloorPrice.

```
ValuationDate = 'Jan-01-2008';
EndDates = 'April-01-2010';
Rates = 0.0069394;
Compounding = -1;
Basis = 1;
```

```

% calculate the RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

Settle = 'Jan-01-2009'; % floor starts in a year
Maturity = 'April-01-2009';
Volatility = 0.20;
FloorRate = -0.006;
FloorReset = 4;
Principal=100000;

FloorPrice = floorbynormal(RateSpec, FloorRate, Settle, Maturity, Volatility, ...
'Reset', FloorReset, 'ValuationDate', ValuationDate, 'Principal', Principal, ...
'Basis', Basis)

FloorPrice = 1.8212e+03

```

## Price a Floor Using floorbynormal and Compare to floorbyblk

Define the RateSpec.

```

Settle = datenum('20-Jan-2016');
ZeroTimes = [.5 1 2 3 4 5 7 10 20 30]';
ZeroRates = [0.0052 0.0055 0.0061 0.0073 0.0094 0.0119 0.0168 0.0222 0.0293 0.0307]';
ZeroDates = datemnth(Settle, 12*ZeroTimes);
RateSpec = intenvset('StartDate', Settle, 'EndDates', ZeroDates, 'Rates', ZeroRates)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
        Disc: [10x1 double]
        Rates: [10x1 double]
    EndTimes: [10x1 double]
    StartTimes: [10x1 double]
    EndDates: [10x1 double]
    StartDates: 736349
    ValuationDate: 736349
        Basis: 0
    EndMonthRule: 1

```

Define the floor instrument and price with floorbyblk.

```
ExerciseDate = datenum('20-Jan-2026');  
[~,ParSwapRate] = swapbyzero(RateSpec,[NaN 0],Settle,ExerciseDate)  
ParSwapRate = 0.0216  
  
Strike = .01;  
BlackVol = .3;  
NormalVol = BlackVol*ParSwapRate;  
  
Price = floorbyblk(RateSpec,Strike,Settle,ExerciseDate,BlackVol)  
Price = 1.2297
```

Price the floor instrument using `floorbynormal`.

```
Price_Normal = floorbynormal(RateSpec,Strike,Settle,ExerciseDate,NormalVol)  
Price_Normal = 1.9099
```

Price the floor instrument using `floorbynormal` for a negative strike.

```
Price_Normal = floorbynormal(RateSpec,-.005,Settle,ExerciseDate,NormalVol)  
Price_Normal = 0.0857
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **Strike — Rate at which floor is exercised**

decimal

Rate at which floor is exercised, specified as a NINST-by-1 vector of decimal values.

Data Types: double

### **Settle — Settlement date for floor**

serial date number | date character vector | datetime object | string object

Settlement date for the floor, specified as a NINST-by-1 vector of serial date numbers, date character vectors, datetime objects, or string objects.

Data Types: double | char | datetime | string

### **Maturity — Maturity date for floor**

serial date number | date character vector | datetime object | string object

Maturity date for the floor, specified as a NINST-by-1 vector of serial date numbers, date character vectors, datetime objects, or string objects.

Data Types: double | char | datetime | string

### **Volatility — Normal volatilities values**

numeric

Normal volatilities values, specified as a NINST-by-1 vector of numeric values.

For more information on the Normal model, see “Work with Negative Interest Rates” on page 2-22.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[FloorPrice, Floorlets] = floorbynormal(RateSpec, Strike, Settle, Maturity, Volatility, 'Reset', CapReset, 'Principal', 100000, 'Basis', 7)`

### **Reset — Reset frequency payment per year**

1 (default) | numeric

Reset frequency payment per year, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector.

Data Types: double

**Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector or a NINST-by-1 cell array. Each element in the NINST-by-1 cell array is a NumDates-by-2 cell array, where the first column is dates, and the second column is the associated principal amount. The date indicates the last day that the principal value is valid.

Use `Principal` to pass a schedule to compute the price for an amortizing cap.

Data Types: double | cell

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of instrument representing the basis used when annualizing the input forward rate, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers. Values are:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .



Data Types: double

**ValuationDate — Observation date of investment horizon**

if `ValuationDate` is not specified, then `Settle` is used (default) | serial date number | date character vector | datetime object | string object

Observation date of the investment horizon, specified as the comma-separated pair consisting of 'ValuationDate' and a serial date number, date character vector, datetime object, or string array.

Data Types: double | char | datetime | string

**ProjectionCurve — Rate curve used in generating future cash flows**

if `ProjectionCurve` is not specified, then `RateSpec` is used both for discounting cash flows and projecting future cash flows (default) | structure

The rate curve to be used in projecting the future cash flows, specified as the comma-separated pair consisting of 'ProjectionCurve' and a rate curve structure. This structure must be created using `intenvset`. Use this optional input if the forward curve is different from the discount curve.

Data Types: struct

## Output Arguments

**FloorPrice — Expected price of floor**

vector

Expected price of the floor, returned as a NINST-by-1 vector.

**Floorlets — Floorlets**

array

Floorlets, returned as a NINST-by-NCF array of caplets, padded with NaNs.

## See Also

`capbynormal` | `floorbyblk` | `intenvset` | `swaptionbynormal`

## Topics

“Calibrating Floorlets Using the Normal (Bachelier) Model”

“Work with Negative Interest Rates” on page 2-22  
“Supported Interest-Rate Instruments” on page 2-2

### **External Websites**

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

**Introduced in R2017a**

# floorvolstrip

Strip floorlet volatilities from flat floor volatilities

## Syntax

```
[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(
ZeroCurve, FloorSettle, FloorMaturity, FloorVolatility)
[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(____
, Name, Value)
```

## Description

[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(ZeroCurve, FloorSettle, FloorMaturity, FloorVolatility) strips floorlet volatilities from the flat floor volatilities by using the bootstrapping method. The function interpolates the cap volatilities on each floorlet payment date before stripping the floorlet volatilities.

[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(\_\_\_\_, Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Stripping Floorlet Volatilities from At-The-Money (ATM) Floors

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('10-Aug-2015');
ZeroRates = [0.12 0.24 0.40 0.73 1.09 1.62]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero', ValuationDate, CurveDates, ZeroRates)
```

```
ZeroCurve =
    Type: Zero
```

```

        Settle: 736186 (10-Aug-2015)
    Compounding: 2
        Basis: 0 (actual/actual)
    InterpMethod: linear
        Dates: [6x1 double]
        Data: [6x1 double]

```

Define the ATM floor volatility data.

```

FloorSettle = datenum('12-Aug-2015');
FloorMaturity = datenum({'12-Aug-2016'; '14-Aug-2017'; '13-Aug-2018'; ...
    '12-Aug-2019'; '12-Aug-2020'});
FloorVolatility = [0.31;0.39;0.43;0.42;0.40];

```

Strip floorlet volatilities from ATM floors.

```

[FloorletVols, FloorletPaymentDates, ATMFloorStrikes] = floorvolstrip(ZeroCurve,...
    FloorSettle, FloorMaturity, FloorVolatility);

```

```

PaymentDates = cellstr(datestr(FloorletPaymentDates));
format;
table(PaymentDates, FloorletVols, ATMFloorStrikes)

```

```

ans=9x3 table
    PaymentDates    FloorletVols    ATMFloorStrikes
    _____    _____    _____
    '12-Aug-2016'    0.31          0.0056551
    '13-Feb-2017'    0.3646       0.0073508
    '14-Aug-2017'    0.41948      0.0090028
    '12-Feb-2018'    0.43152      0.010827
    '13-Aug-2018'    0.46351      0.012617
    '12-Feb-2019'    0.40407      0.013862
    '12-Aug-2019'    0.39863      0.015105
    '12-Feb-2020'    0.3674       0.016369
    '12-Aug-2020'    0.35371      0.01762

```

### Stripping Floorlet Volatilities from Floors with the Same Strikes

Compute the zero curve for discounting and projecting forward rates.

```

ValuationDate = datenum('10-Jun-2015');
ZeroRates = [0.02 0.10 0.28 0.75 1.15 1.80]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero',ValuationDate,CurveDates,ZeroRates)

```

```

ZeroCurve =
    Type: Zero
    Settle: 736125 (10-Jun-2015)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [6x1 double]
    Data: [6x1 double]

```

Define the floor volatility data.

```

FloorSettle = datenum('12-Jun-2015');
FloorMaturity = datenum({'13-Jun-2016'; '12-Jun-2017'; '12-Jun-2018'; ...
    '12-Jun-2019'; '12-Jun-2020'});
FloorVolatility = [0.41;0.43;0.43;0.41;0.38];
FloorStrike = 0.015;

```

Strip floorlet volatilities from floors with the same strike.

```

[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(ZeroCurve, ...
    FloorSettle, FloorMaturity, FloorVolatility, 'Strike', FloorStrike);

```

```

PaymentDates = cellstr(datestr(FloorletPaymentDates));
format;
table(PaymentDates, FloorletVols, FloorStrikes)

```

```

ans=9x3 table
    PaymentDates    FloorletVols    FloorStrikes
    _____    _____    _____
    '13-Jun-2016'    0.41           0.015
    '12-Dec-2016'    0.42           0.015
    '12-Jun-2017'    0.43433        0.015
    '12-Dec-2017'    0.43001        0.015
    '12-Jun-2018'    0.43           0.015
    '12-Dec-2018'    0.39173        0.015
    '12-Jun-2019'    0.37244        0.015
    '12-Dec-2019'    0.32056        0.015
    '12-Jun-2020'    0.28308        0.015

```

### Stripping Floorlet Volatilities Using Manually Specified Floorlet Dates

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('19-May-2015');
ZeroRates = [0.02 0.07 0.23 0.63 1.01 1.60]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero',ValuationDate,CurveDates,ZeroRates)
```

```
ZeroCurve =
    Type: Zero
    Settle: 736103 (19-May-2015)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [6x1 double]
    Data: [6x1 double]
```

Define the floor volatility data.

```
FloorSettle = datenum('19-May-2015');
FloorMaturity = datenum({'19-May-2016'; '19-May-2017'; '21-May-2018'; ...
    '20-May-2019'; '19-May-2020'});
FloorVolatility = [0.39;0.42;0.43;0.42;0.40];
FloorStrike = 0.010;
```

Specify the quarterly and semiannual dates.

```
FloorletDates = [cfdates(FloorSettle, '19-May-2016', 4)...
    cfdates('19-May-2016', '19-May-2020', 2)];
FloorletDates(~isbusday(FloorletDates)) = ...
    busdate(FloorletDates(~isbusday(FloorletDates)), 'modifiedfollow');
```

Strip floorlet volatilities using specified FloorletDates.

```
[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(ZeroCurve, ...
    FloorSettle, FloorMaturity, FloorVolatility, 'Strike', FloorStrike, ...
    'FloorletDates', FloorletDates);

PaymentDates = cellstr(datestr(FloorletPaymentDates));
format;
table(PaymentDates, FloorletVols, FloorStrikes)
```

```
ans=11x3 table
  PaymentDates   FloorletVols   FloorStrikes
  _____   _____   _____
  '19-Nov-2015'   0.39          0.01
  '19-Feb-2016'   0.39          0.01
  '19-May-2016'   0.39          0.01
  '21-Nov-2016'   0.4058        0.01
  '19-May-2017'   0.4307        0.01
  '20-Nov-2017'   0.43317       0.01
  '21-May-2018'   0.44309       0.01
  '19-Nov-2018'   0.40831       0.01
  '20-May-2019'   0.39831       0.01
  '19-Nov-2019'   0.3524        0.01
  '19-May-2020'   0.32765       0.01
```

### Stripping Floorlet Volatilities from Floors Using the Shifted Black Model

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('3-May-2016');
ZeroRates = [-0.31 -0.21 -0.15 -0.10 0.009 0.19]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero',ValuationDate,CurveDates,ZeroRates)
```

```
ZeroCurve =
    Type: Zero
    Settle: 736453 (03-May-2016)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [6x1 double]
    Data: [6x1 double]
```

Define the floor volatility (Shifted Black) data.

```
FloorSettle = datenum('3-May-2016');
FloorMaturity = datenum({'3-May-2017';'3-May-2018';'3-May-2019'; ...
    '4-May-2020';'3-May-2021'});
FloorVolatility = [0.42;0.45;0.43;0.40;0.36]; % Shifted Black volatilities
```

```
Shift = 0.01; % 1 percent shift.
FloorStrike = -0.001; % -0.1 percent strike.
```

Strip floorlet volatilities from floors using the Shifted Black Model.

```
[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(ZeroCurve, ...
FloorSettle, FloorMaturity, FloorVolatility, 'Strike', FloorStrike, 'Shift', Shift);
```

```
PaymentDates = string(datestr(FloorletPaymentDates));
format;
table(PaymentDates, FloorletVols, FloorStrikes)
```

```
ans=9x3 table
    PaymentDates    FloorletVols    FloorStrikes
    _____    _____    _____
    "03-May-2017"      0.42          -0.001
    "03-Nov-2017"     0.44575        -0.001
    "03-May-2018"     0.47092        -0.001
    "05-Nov-2018"     0.41911        -0.001
    "03-May-2019"     0.40197        -0.001
    "04-Nov-2019"     0.36262        -0.001
    "04-May-2020"     0.33615        -0.001
    "03-Nov-2020"     0.27453        -0.001
    "03-May-2021"     0.23045        -0.001
```

### Stripping Floorlet Volatilities from Floors Using Normal Model

Compute the zero curve for discounting and projecting forward rates.

```
ValuationDate = datenum('1-May-2018');
ZeroRates = [-0.31 -0.27 -0.18 -0.05 0.015 0.22]/100;
CurveDates = datemnth(ValuationDate, [0.25 0.5 1 2 3 5]*12);
ZeroCurve = IRDataCurve('Zero', ValuationDate, CurveDates, ZeroRates)
```

```
ZeroCurve =
    Type: Zero
    Settle: 737181 (01-May-2018)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
```



```
Dates: [6x1 double]
Data: [6x1 double]
```

Define the normal floor volatility data.

```
FloorSettle = datenum('1-May-2018');
FloorMaturity = datenum({'1-May-2019'; '1-May-2020'; '3-May-2021'; ...
    '2-May-2022'; '1-May-2023'});
FloorVolatility = [0.0065; 0.0067; 0.0064; 0.0058; 0.0055]; % Normal volatilities
FloorStrike = -0.005; % -0.5 percent strike.
```

Strip floorlet volatilities from floors using the Normal (Bachelier) model.

```
[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(ZeroCurve, ...
    FloorSettle, FloorMaturity, FloorVolatility, 'Strike', FloorStrike, 'Model', 'normal');
```

```
PaymentDates = string(datestr(FloorletPaymentDates));
format;
table(PaymentDates, FloorletVols, FloorStrikes)
```

ans=9x3 table

PaymentDates	FloorletVols	FloorStrikes
"01-May-2019"	0.0065	-0.005
"01-Nov-2019"	0.0066644	-0.005
"01-May-2020"	0.0068354	-0.005
"02-Nov-2020"	0.006266	-0.005
"03-May-2021"	0.0060101	-0.005
"01-Nov-2021"	0.004942	-0.005
"02-May-2022"	0.0042668	-0.005
"01-Nov-2022"	0.0047986	-0.005
"01-May-2023"	0.0044738	-0.005

## Input Arguments

### ZeroCurve — Zero rate curve

RateSpec object | IRDataCurve object

Zero rate curve, specified using a RateSpec or IRDataCurve object containing the zero rate curve for discounting according to its day count convention. If you do not specify the

optional argument `ProjectionCurve`, the function uses `ZeroCurve` to compute the underlying forward rates as well. The observation date of the `ZeroCurve` specifies the valuation date. For more information on creating a `RateSpec`, see `intenvset`. For more information on creating an `IRDataCurve` object, see `IRDataCurve`.

Data Types: `struct`

### **FloorSettle — Common floor settle date**

serial date number | date character vector

Common floor settle date, specified as a scalar serial date number or date character vector. The `FloorSettle` date cannot be earlier than the `ZeroCurve` valuation date.

Data Types: `double` | `char`

### **FloorMaturity — Floor maturity dates**

serial date numbers | date character vectors

Floor maturity dates, specified using serial date numbers or cell array of date character vectors as an `NFloor-by-1` vector.

Data Types: `double` | `char` | `cell`

### **FloorVolatility — Flat floor volatilities**

vector of positive decimals

Flat floor volatilities, specified as an `NFloor-by-1` vector of positive decimals.

Data Types: `double`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[FloorletVols, FloorletPaymentDates, FloorStrikes] = floorvolstrip(ZeroCurve, FloorSettle, FloorMaturity, FloorVolatility, 'Strike', .2)`

### **Strike — Floor strike rate**

if not specified, all floors are at-the-money and the function computes the ATM strike for each floor maturing on each floorlet payment date (default) | scalar decimal | vector

Floor strike rate, specified as the comma-separated pair consisting of 'Strike' and a scalar decimal value or an `NFloorletVols-by-1` vector. Use `Strike` as a scalar to specify a single strike that applies equally to all floors. Or, specify an `NCapletVols-by-1` vector of strikes for the floors.

Data Types: double

### **FloorletDates — Floorlet reset and payment dates**

If not specified, the default is to automatically generate periodic floorlet dates (default) | serial date numbers | date character vectors

Floorlet reset and payment dates, specified as the comma-separated pair consisting of 'FloorletDates' and an `NFloorletDates-by-1` vector using serial date numbers or a cell array of date character vectors.

Use `FloorletDates` to manually specify all floorlet reset and payment dates. For example, some date intervals may be quarterly while others may be semiannual. All dates must be later than `FloorSettle` and cannot be later than the last `FloorMaturity` date. Dates are adjusted according to the `BusDayConvention` and `Holidays` inputs.

If `FloorletDates` is not specified, the default is to automatically generate periodic floorlet dates after `FloorSettle` based on the last `FloorMaturity` date as the reference date, using the following optional inputs: `Reset`, `EndMonthRule`, `BusDayConvention`, and `Holidays`.

Data Types: double | char | cell

### **Reset — Frequency of periodic payments per year within a floor**

2 (default) | positive scalar integer with values 1,2, 3, 4, 6, or 12

Frequency of periodic payments per year within a floor, specified as the comma-separated pair consisting of 'Reset' and a positive scalar integer with values 1,2, 3, 4, 6, or 12.

---

**Note** If you specify `FloorletDates`, the function ignores the input for `Reset`.

---

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating floorlet dates**

1 (in effect) (default) | scalar nonnegative integer [0, 1]

End-of-month rule flag for generating floorlet dates, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1].

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **BusDayConvention — Business day conventions**

'modifiedfollow' (default) | character vector with values 'actual', 'follow', 'modifiedfollow', 'previous', 'modifiedprevious'

Business day conventions, specified as the comma-separated pair consisting of 'BusDayConvention' and a character vector. Use this argument to specify how the function treats non-business days, which are days on which businesses are not open (such as weekends and statutory holidays).

- 'actual' — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- 'follow' — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- 'modifiedfollow' — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However, if the following business day is in a different month, the previous business day is adopted instead.
- 'previous' — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- 'modifiedprevious' — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However, if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | vector of MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and NHolidays-by-1 vector of MATLAB date numbers.

Data Types: double

**ProjectionCurve — Rate curve for computing underlying forward rates**

if not specified, the default is to use the ZeroCurve input for computing the underlying forward rates (default) | RateSpec object | IRDataCurve object

Rate curve for computing underlying forward rates, specified as the comma-separated pair consisting of 'ProjectionCurve' and a RateSpec object or IRDataCurve object. For more information on creating a RateSpec, see `intenvset`. For more information on creating an IRDataCurve object, see `IRDataCurve`.

Data Types: `struct`

**MaturityInterpMethod — Method for interpolating floor volatilities on each floorlet maturity date before stripping floorlet volatilities**

'linear' (default) | character vector with values: 'linear', 'nearest', 'next', 'previous', 'spline', 'pchip'

Method for interpolating the floor volatilities on each floorlet maturity date before stripping the floorlet volatilities, specified as the comma-separated pair consisting of 'MaturityInterpMethod' and a character vector with values: 'linear', 'nearest', 'next', 'previous', 'spline', or 'pchip'.

- 'linear' — Linear interpolation. The interpolated value at a query point is based on linear interpolation of the values at neighboring grid points in each respective dimension. This is the default interpolation method.
- 'nearest' — Nearest neighbor interpolation. The interpolated value at a query point is the value at the nearest sample grid point.
- 'next' — Next neighbor interpolation. The interpolated value at a query point is the value at the next sample grid point.
- 'previous' — Previous neighbor interpolation. The interpolated value at a query point is the value at the previous sample grid point.
- 'spline' — Spline interpolation using not-a-knot end conditions. The interpolated value at a query point is based on a cubic interpolation of the values at neighboring grid points in each respective dimension.
- 'pchip' — Shape-preserving piecewise cubic interpolation. The interpolated value at a query point is based on a shape-preserving piecewise cubic interpolation of the values at neighboring grid points.

For more information on interpolation methods, see `interp1`.

---

**Note** The function uses constant extrapolation to calculate volatilities falling outside the range of user-supplied data.

---

Data Types: `char`

**Limit — Upper bound of implied volatility search interval**

10 (or 1000% per annum) (default) | positive scalar decimal

Upper bound of implied volatility search interval, specified as the comma-separated pair consisting of 'Limit' and a positive scalar decimal.

Data Types: `double`

**Tolerance — Implied volatility search termination tolerance**

1e-5 (default) | positive scalar

Implied volatility search termination tolerance, specified as the comma-separated pair consisting of 'Tolerance' and a positive scalar.

Data Types: `double`

**OmitFirstFloorlet — Flag to omit the first floorlet payment in the floors**

true always omit the first floorlet (default) | logical

Flag to omit the first floorlet payment in the floors, specified as the comma-separated pair consisting of 'OmitFirstFloorlet' and a scalar logical.

If the floors are spot-starting, the first floorlet payment is omitted. If the floors are forward-starting, the first floorlet payment is included. Regardless of the status of the floors, if you set this logical to `false`, then the function includes the first floorlet payment.

In general, “spot lag” is the delay between the fixing date and the effective date for LIBOR-like indices. “Spot lag” determines whether a floor is spot-starting or forward-starting (Corb, 2012). Floors are considered to be spot-starting if they settle within “spot lag” business days after the valuation date. Those that settle later are considered to be forward-starting. The first floorlet is omitted if floors are spot-starting, while it is included if they are forward-starting (Tuckman, 2012).

Data Types: `logical`

**Shift — Shift in decimals for shifted SABR model**

0 (no shift) (default) | positive scalar decimal

Shift in decimals for the shifted SABR model (to be used with the Shifted Black model), specified as the comma-separated pair consisting of 'Shift' and a positive scalar decimal value. Set this parameter to a positive shift in decimals to add a positive shift to the forward rate and strike, which effectively sets a negative lower bound for the forward rate and strike. For example, a Shift value of 0.01 is equal to a 1% shift.

Data Types: double

### Model — Model used for implied volatility

'lognormal' (default) | character vector with value of 'lognormal' or 'normal' | string scalar with value of "lognormal" or "normal"

Model used for the implied volatility calculation, specified as the comma-separated pair consisting of 'Model' and a scalar character vector or string scalar with one of the following values:

- 'lognormal' - Implied Black (no shift) or Shifted Black volatility.
- 'normal' - Implied Normal (Bachelier) volatility. If you specify 'normal', Shift must be zero.

The floorvolstrip function supports three volatility types.

'Model' Value	'Shift' Value	Volatility Type
'lognormal'	Shift = 0	Black
'lognormal'	Shift > 0	Shifted Black
'normal'	Shift = 0	Normal (Bachelier)

Data Types: char | string

## Output Arguments

### FloorletVols — Stripped floorlet volatilities

vector of decimals

Stripped floorlet volatilities, returned as a NFloorletVols-by-1 vector of decimals.

---

**Note** floorvolstrip can output NaNs for some caplet volatilities. You might encounter this output if no volatility matches the caplet price implied by the user-supplied cap data.

---

**FloorletPaymentDates — Payment dates**

vector of date numbers

Payment dates (in date numbers), returned as an `NFloorletVols-by-1` vector of date numbers corresponding to `FloorletVols`.

**FloorStrikes — Floor strikes**

vector of decimals

Floor strikes, returned as a `NFloorletVols-by-1` vector of strikes in decimals for floors maturing on the corresponding `FloorletPaymentDates`.

## Limitations

When bootstrapping the floorlet volatilities from ATM floors, the function reuses the floorlet volatilities stripped from the shorter maturity floors in the longer maturity floors without adjusting for the difference in strike. `floorvolstrip` follows the simplified approach described in Gatarek, 2006.

## Definitions

### At-The-Money

A cap or floor is at-the-money (ATM) if its strike is equal to the forward swap rate.

The forward swap rate is the fixed rate of a swap that makes the present value of the floating leg equal to that of the fixed leg. In comparison, a caplet or floorlet is ATM if its strike is equal to the forward rate (not the forward swap rate). In general (except over a single period), the forward rate is not equal to the forward swap rate. So, to be precise, the individual caplets in an ATM cap have slightly different moneyness and are only approximately ATM (Alexander, 2003).

In addition, the swap rate changes with swap maturity. Similarly, the ATM cap strike also changes with cap maturity, so the ATM cap strikes are computed for each cap maturity before stripping the caplet volatilities. As a result, when stripping the caplet volatilities from the ATM caps with increasing maturities, the ATM strikes of consecutive caps are different.



## References

- [1] Alexander, C. "Common Correlation and Calibrating the Lognormal Forward Rate Model." *Wilmott Magazine*, 2003.
- [2] Corb, H. *Interest Rate Swaps and Other Derivatives*. Columbia Business School Publishing, 2012.
- [3] Gatarek, D., P. Bachert, and R. Maksymiuk. *The LIBOR Market Model in Practice*. Chichester, UK: Wiley, 2006.
- [4] Tuckman, B., and Serrat, A. *Fixed Income Securities: Tools for Today's Markets*. Hoboken, NJ: Wiley, 2012.

## See Also

[capvolstrip](#) | [floorbyblk](#) | [floorbynormal](#) | [intenvset](#) | [interp1](#)

## Topics

"Price Swaptions with Negative Strikes Using the Shifted SABR Model" on page 2-27  
"Work with Negative Interest Rates" on page 2-22

## External Websites

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

## Introduced in R2016a

## gapbyls

Determine price of gap digital options using Black-Scholes model

### Syntax

```
Price = gapbyls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold)
```

### Description

Price = gapbyls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold) calculates gap European digital option prices using the Black-Scholes option pricing model.

### Examples

#### Compute Gap Option Prices Using the Black-Scholes Option Pricing Model

This example shows how to compute gap option prices using the Black-Scholes option pricing model. Consider a gap call and put options on a nondividend paying stock with a strike of 57 and expiring on January 1, 2008. On July 1, 2008 the stock is trading at 50. Using this data, compute the price of the option if the risk-free rate is 9%, the strike threshold is 50, and the volatility is 20%.

```
Settle = 'Jan-1-2008';  
Maturity = 'Jul-1-2008';  
Compounding = -1;  
Rates = 0.09;  
% calculate the RateSpec  
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...  
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', 1);  
% define the StockSpec  
AssetPrice = 50;  
Sigma = .2;
```

```

StockSpec = stockspec(Sigma, AssetPrice);
% define the call and put options
OptSpec = {'call'; 'put'};
Strike = 57;
StrikeThreshold = 50;
% calculate the price
Pgap = gapbyls(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, StrikeThreshold)

Pgap = 2×1

    -0.0053
     4.4866

```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspect`.

`stockspect` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle** — Settlement or trade date

serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as an NINST-by-1 vector.

Data Types: char | cell

**Strike — Pay-off strike value**

vector

Pay-off strike value, specified as an NINST-by-1 vector.

Data Types: double

**StrikeThreshold — Strike values that determine if the option pays off**

vector

Strike values that determine if the option pays off, specified as an NINST-by-1 vector.

Data Types: double

## Output Arguments

**Price — Expected prices for gap option**

vector

Expected prices for gap option, returned as a NINST-by-1 vector.

## **See Also**

assetbys | cashbys | gapsensbys | supersharebys

## **Topics**

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**

## gapsensbyls

Determine price or sensitivities of gap digital options using Black-Scholes model

### Syntax

```
PriceSens = gapsensbyls(RateSpec,StockSpec,Settle,Maturity,OptSpec,  
Strike,StrikeThreshold)  
PriceSens = gapsensbyls( ____,Name,Value)
```

### Description

`PriceSens = gapsensbyls(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,StrikeThreshold)` calculates gap European digital option prices or sensitivities using the Black-Scholes option pricing model.

`PriceSens = gapsensbyls( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute Gap Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model

This example shows how to compute gap option prices and sensitivities using the Black-Scholes option pricing model. Consider a gap call and put options on a nondividend paying stock with a strike of 57 and expiring on January 1, 2008. On July 1, 2008 the stock is trading at 50. Using this data, compute the price and sensitivity of the option if the risk-free rate is 9%, the strike threshold is 50, and the volatility is 20%.

```
Settle = 'Jan-1-2008';  
Maturity = 'Jul-1-2008';  
Compounding = -1;  
Rates = 0.09;  
%create the RateSpec
```

```

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', 1);
% define the StockSpec
AssetPrice = 50;
Sigma = .2;
StockSpec = stockspec(Sigma, AssetPrice);
% define the call and put options
OptSpec = {'call'; 'put'};
Strike = 57;
StrikeThreshold = 50;
% compute the price
Pgap = gapbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, StrikeThreshold)

Pgap = 2×1

    -0.0053
     4.4866

% compute the gamma and delta
OutSpec = {'gamma'; 'delta'};
[Gamma ,Delta] = gapsensbybls(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, Strike, StrikeThreshold, 'OutSpec', OutSpec)

Gamma = 2×1

    0.0724
    0.0724

Delta = 2×1

    0.2852
   -0.7148

```

## Input Arguments

**RateSpec** — Interest-rate term structure  
structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec — Stock specification for underlying asset**  
structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**Settle — Settlement or trade date**  
serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**Maturity — Maturity date**  
serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**OptSpec — Definition of option**  
character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as an NINST-by-1 vector.

Data Types: `char` | `cell`

**Strike — Pay-off strike value**  
vector



Pay-off strike value, specified as an NINST-by-1 vector.

Data Types: double

**StrikeThreshold** — **Strike values that determine if the option pays off**  
vector

Strike values that determine if the option pays off, specified as an NINST-by-1 vector.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Gamma,Delta] = gapsensbyls(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,StrikeThreshold,'OutSpec',{'gamma'; 'delta'})`

### OutSpec — Define outputs

`{'Price'}` (default) | character vector with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | cell array of character vectors with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output is `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

Example: `OutSpec = {'delta','gamma','vega','lambda','rho','theta','price'}`

Data Types: char | cell

## Output Arguments

### **PriceSens — Expected prices or sensitivities for gap option**

vector

Expected prices or sensitivities (defined using `OutSpec`) for gap option, returned as a NINST-by-1 vector.

## See Also

`gapbybls`

## Topics

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**

# hedgeopt

Allocate optimal hedge for target costs or sensitivities

## Syntax

```
[PortSens,PortCost,PortHolds] =
hedgeopt(Sensitivities,Price,CurrentHolds,FixedInd,NumCosts,TargetCost,TargetSens,ConSet)
```

## Arguments

Sensitivities	Number of instruments (NINST) by number of sensitivities (NSENS) matrix of dollar sensitivities of each instrument. Each row represents a different instrument. Each column represents a different sensitivity.
Price	NINST-by-1 vector of portfolio instrument unit prices.
CurrentHolds	NINST-by-1 vector of contracts allocated to each instrument.
FixedInd	(Optional) Number of fixed instruments (NFIXED)-by-1 vector of indices of instruments to hold fixed. For example, to hold the first and third instruments of a 10 instrument portfolio unchanged, set FixedInd = [1 3]. Default = [], no instruments held fixed.
NumCosts	(Optional) Number of points generated along the cost frontier when a vector of target costs (TargetCost) is not specified. The default is 10 equally spaced points between the point of minimum cost and the point of minimum exposure. When specifying TargetCost, enter NumCosts as an empty matrix [].

TargetCost	(Optional) Vector of target cost values along the cost frontier. If TargetCost is empty, or not entered, hedgeopt evaluates NumCosts equally spaced target costs between the minimum cost and minimum exposure. When specified, the elements of TargetCost should be positive numbers that represent the maximum amount of money the owner is willing to spend to rebalance the portfolio.
TargetSens	(Optional) 1-by-NSENS vector containing the target sensitivity values of the portfolio. When specifying TargetSens, enter NumCosts and TargetCost as empty matrices [ ].
ConSet	(Optional) Number of constraints (NCONS) by number of instruments (NINST) matrix of additional conditions on the portfolio reallocations. An eligible NINST-by-1 vector of contract holdings, PortWts, satisfies all the inequalities $A * PortWts \leq b$ , where $A = ConSet(:, 1:end-1)$ and $b = ConSet(:, end)$ .

**Notes** The user-specified constraints included in ConSet may be created with the functions pcalims or portcons. However, the portcons default PortHolds positivity constraints are typically inappropriate for hedging problems since short-selling is usually required.

NPOINTS, the number of rows in PortSens and PortHolds and the length of PortCost, is inferred from the inputs. When the target sensitivities, TargetSens, is entered, NPOINTS = 1; otherwise NPOINTS = NumCosts, or is equal to the length of the TargetCost vector.

Not all problems are solvable (for example, the solution space may be infeasible or unbounded, or the solution may fail to converge). When a valid solution is not found, the corresponding rows of PortSens, PortHolds, and the elements of PortCost are padded with NaNs as placeholders.

## Description

[PortSens, PortCost, PortHolds] =  
hedgeopt(Sensitivities, Price, CurrentHolds, FixedInd, NumCosts, TargetCost, TargetSens, ConSet) allocates an optimal hedge by one of two criteria:

- Minimize portfolio sensitivities (exposure) for a given set of target costs.
- Minimize the cost of hedging a portfolio given a set of target sensitivities.

Hedging involves the fundamental tradeoff between portfolio insurance and the cost of insurance coverage. This function lets investors modify portfolio allocations among instruments to achieve either of the criteria. The chosen criterion is inferred from the input argument list. The problem is cast as a constrained linear least-squares problem.

PortSens is a number of points (NPOINTS-by-NSENS) matrix of portfolio sensitivities. When a perfect hedge exists, PortSens is zeros. Otherwise, the best hedge possible is chosen.

PortCost is a 1-by-NPOINTS vector of total portfolio costs.

PortHolds is an NPOINTS-by-NINST matrix of contracts allocated to each instrument. These are the reallocated portfolios.

## See Also

`hedgeslf` | `lsqlin` | `pcalims` | `portcons` | `portopt`

## Topics

“Portfolio Creation” on page 1-8

“Hedging with hedgeopt” on page 4-5

“Instrument Constructors” on page 1-20

“Hedging” on page 4-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## hedgeslf

Self-financing hedge

### Syntax

[PortSens,PortValue,PortHolds] =  
hedgeslf(Sensitivities,Price,CurrentHolds,FixedInd,ConSet)

### Arguments

Sensitivities	Number of instruments (NINST) by number of sensitivities (NSENS) matrix of dollar sensitivities of each instrument. Each row represents a different instrument. Each column represents a different sensitivity.
Price	NINST-by-1 vector of instrument unit prices.
CurrentHolds	NINST-by-1 vector of contracts allocated in each instrument.
FixedInd	(Optional) Empty or number of fixed instruments (NFIXED-by-1) vector of indices of instruments to hold fixed. The default is FixedInd = 1; the holdings in the first instrument are held fixed. If NFIXED instruments will not be changed, enter all their locations in the portfolio in a vector. If no instruments are to be held fixed, enter FixedInd = [].
ConSet	(Optional) Number of constraints (NCONS)-by-NINST matrix of additional conditions on the portfolio reallocations. An eligible NINST-by-1 vector of contract holdings, PortHolds, satisfies all the inequalities for $A * PortHolds \leq b$ , where $A = ConSet(:, 1:end-1)$ and $b = ConSet(:, end)$ .

### Description

[PortSens,PortValue,PortHolds] =  
hedgeslf(Sensitivities,Price,CurrentHolds,FixedInd,ConSet) allocates a

self-financing hedge among a collection of instruments. `hedgeslf` finds the reallocation in a portfolio of financial instruments that hedges the portfolio against market moves and that is closest to being self-financing (maintaining constant portfolio value). By default the first instrument entered is hedged with the other instruments.

`PortSens` is a 1-by-`NSENS` vector of portfolio dollar sensitivities. When a perfect hedge exists, `PortSens` is zeros. Otherwise, the best possible hedge is chosen.

`PortValue` is the total portfolio value (scalar). When a perfectly self-financing hedge exists, `PortValue` is equal to `dot(Price, CurrentWts)` of the initial portfolio.

`PortHolds` is an `NINST`-by-1 vector of contracts allocated to each instrument. This is the reallocated portfolio.

---

## Notes

- The constraints `PortHolds(FixedInd) = CurrentHolds(FixedInd)` are appended to any constraints passed in `ConSet`. Pass `FixedInd = []` to specify all constraints through `ConSet`.
  - The default constraints generated by `portcons` are inappropriate, since they require the sum of all holdings to be positive and equal to one.
  - `hedgeslf` first tries to find the allocations of the portfolio that make it closest to being self-financing, while reducing the sensitivities to 0. If no solution is found, it finds the allocations that minimize the sensitivities. If the resulting portfolio is self-financing, `PortValue` is equal to the value of the original portfolio.
- 

## Examples

**Example 1.** Perfect sensitivity cannot be reached.

```
Sens = [0.44  0.32; 1.0 0.0];
Price = [1.2; 1.0];
W0 = [1; 1];
[PortSens, PortValue, PortHolds]= hedgeslf(Sens, Price, W0)

PortSens =

    0.0000
    0.3200
```

```
PortValue =  
    0.7600  
  
PortHolds =  
    1.0000  
   -0.4400
```

**Example 2.** Constraints are in conflict.

```
Sens = [0.44 0.32; 1.0 0.0];  
Price = [1.2; 1.0];  
W0 = [1; 1];  
ConSet = pcalims([2 2])  
  
% O.K. if nothing fixed.  
  
[PortSens, PortValue, PortHolds]= hedgeslf(Sens, Price, W0,...  
[], ConSet)  
  
PortSens =  
    2.8800  
    0.6400  
  
PortValue =  
    4.4000  
  
PortHolds =  
    2  
    2  
  
% W0(1) is not greater than 2.  
  
[PortSens, PortValue, PortHolds] = hedgeslf(Sens, Price, W0,...  
1, ConSet)  
  
??? Error using ==> hedgeslf  
Overly restrictive allocation constraints implied by ConSet and  
by fixing the weight of instruments(s): 1
```

**Example 3.** Constraints are impossible to meet.

```
Sens = [0.44 0.32; 1.0 0.0];  
Price = [1.2; 1.0];
```



```
W0 = [1; 1];  
ConSet = pcalims([2 2],[1 1]);  
  
[PortSens, PortValue, PortHolds] = hedgeslf(Sens, Price, W0,...  
[],ConSet)  
  
??? Error using ==> hedgeslf  
Overly restrictive allocation constraints specified in ConSet
```

## See Also

hedgeopt | lsqlin | portcons

## Topics

“Portfolio Creation” on page 1-8  
“Self-Financing Hedges with hedgeslf” on page 4-12  
“Instrument Constructors” on page 1-20  
“Hedging” on page 4-2  
“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## hjmprice

Instrument prices from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = hjmprice(HJMTree,InstSet)
[Price,PriceTree] = hjmprice( ____,Options)
```

### Description

[Price,PriceTree] = hjmprice(HJMTree,InstSet) computes arbitrage-free prices for instruments using an interest-rate tree created with `hjmtree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`hjmprice` handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See `instadd` to construct defined types.

[Price,PriceTree] = hjmprice( \_\_\_\_,Options) adds an optional input argument for `Options`.

### Examples

#### Price the Cap and Bond Instruments Contained in an Instrument Set

Load the HJM tree and instruments from the data file `deriv.mat`.

```
load deriv.mat;
HJMSubSet = instselect(HJMInstSet,'Type',{'Bond','Cap'});
```

```
instdisp(HJMSubSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate	FirstCouponDate	LastCouponDate	Sta
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	NaN	NaN	NaN	NaN
2	Bond	0.04	01-Jan-2000	01-Jan-2004	2	NaN	NaN	NaN	NaN	NaN	NaN

Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal	Name	Quantity
3	Cap	0.03	01-Jan-2000	01-Jan-2004	1	NaN	NaN	3% Cap 30	

Use `hjmprice` to price the instruments.

```
[Price, PriceTree] = hjmprice(HJMTree, HJMSubSet)
```

```
Warning: Not all cash flows are aligned with the tree. Result will be approximated.  
> In checktree (line 289)  
  In hjmprice (line 85)
```

```
Price =
```

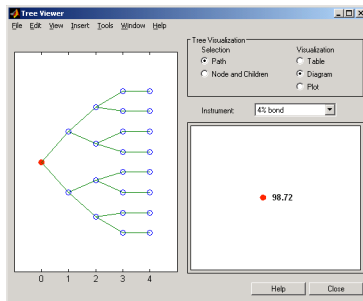
```
 98.7159  
 97.5280  
  6.2831
```

```
PriceTree =
```

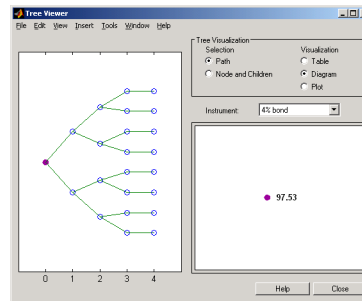
```
FinObj: 'HJMPriceTree'  
PBush: {[3x1 double] [3x1x2 double] [3x2x2 double] [3x4x2 double] [3x8 double]}  
AIBush: {[3x1 double] [3x1x2 double] [3x2x2 double] [3x4x2 double] [3x8 double]}  
tObs: [0 1 2 3 4]
```

You can use `treeview` to see the prices of these three instruments along the price tree.

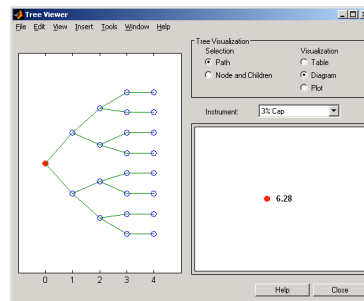
```
treeview
```



First 4% Bond (Maturity 2003)



Second 4% Bond (Maturity 2004)



3% Cap

## Price Multi-Stepped Coupon Bonds

The data for the interest-rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create a RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
```

```

        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: 734139
ValuationDate: 734139
        Basis: 0
        EndMonthRule: 1

```

Create a portfolio of stepped coupon bonds with different maturities.

```

Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07};

```

```

ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)

```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	[Cell]	01-Jan-2010	01-Jan-2011	1	0	1	NaN
2	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
3	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
4	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build the tree with the following data:

```

Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RS, HJMTimeSpec)

HJMT = struct with fields:
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734139 734504 734869 735235]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    FwdTree: {[4x1 double] [3x1x2 double] [2x2x2 double] [1x4x2 double]}

```

Compute the price of the stepped coupon bonds.

```
PHJM = hjmprice(HJMT, ISet)
```

```
PHJM = 4×1
```

```
100.6763
100.7368
100.9266
101.0115
```

### Price a Portfolio of Stepped Callable Bonds and Stepped Vanilla Bonds

The data for the interest-rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create a RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x1 double]
    Rates: [4x1 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734139
    ValuationDate: 734139
    Basis: 0
    EndMonthRule: 1
```

Create an instrument portfolio of three stepped callable bonds and three stepped vanilla bonds and display the instrument portfolio.

```

Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .06};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year

% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);

```

```

% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);

```

```

instdisp(ISet)

```

Index	Type	CouponRate	Settle	Maturity	OptSpec	Strike	ExerciseDates
1	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2012	call	100	01-Jan-2011
2	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2013	call	100	01-Jan-2011
3	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2014	call	100	01-Jan-2011

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
4	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
5	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
6	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build the tree with the following data:

```

Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RS,HJMTimeSpec)

```

```

HJMT = struct with fields:
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734139 734504 734869 735235]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}

```

```
FwdTree: {[4x1 double] [3x1x2 double] [2x2x2 double] [1x4x2 double]}
```

Price the instrument set using `hjmprice`.

```
PHJM = hjmprice(HJMT, ISet)
```

```
PHJM = 6x1
```

```
100.3682  
100.1557  
99.9232  
100.7368  
100.9266  
101.0115
```

The first three rows correspond to the price of the stepped callable bonds and the last three rows correspond to the price of the stepped vanilla bonds.

### Compute the Price of a Portfolio of Instruments

The data for the interest-rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];  
ValuationDate = 'Jan-1-2011';  
StartDates = ValuationDate;  
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};  
Compounding = 1;
```

Create a `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', ...  
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: [4x1 double]  
    Rates: [4x1 double]  
    EndTimes: [4x1 double]  
    StartTimes: [4x1 double]
```



```

        EndDates: [4x1 double]
        StartDates: 734504
        ValuationDate: 734504
        Basis: 0
        EndMonthRule: 1

```

Create an instrument portfolio with two range notes and a floating rate note with the following data and display the results:

```

Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

% First Range Note
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];

% Second Range Note
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates = [0.048 0.059; 0.055 0.068 ; 0.07 0.09];

% Create an InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);

% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);

% Display the portfolio instrument
instdisp(InstSet)

```

Index	Type	Spread	Settle	Maturity	RateSched	FloatReset	Basis	Princ
1	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100
2	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100
Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRu
3	Float	200	01-Jan-2011	01-Jan-2014	1	0	100	1

The data to build the tree is as follows:

```

Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
HJMTree = hjmtimespec(ValuationDate, MaTree);

```

```
HJMVS = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);  
HJMT = hjmtree(HJMVS, RS, HJMVS)
```

```
HJMT = struct with fields:  
    FinObj: 'HJMfwdTree'  
    VolSpec: [1x1 struct]  
    TimeSpec: [1x1 struct]  
    RateSpec: [1x1 struct]  
    tObs: [0 1 2 3]  
    dObs: [734504 734869 735235 735600]  
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}  
    CFLOWT: {[4x1 double] [3x1 double] [2x1 double] [4]}  
    FwdTree: {[4x1 double] [3x1x2 double] [2x2x2 double] [1x4x2 double]}
```

Price the portfolio.

```
Price = hjmprice(HJMT, InstSet)
```

```
Price = 3x1
```

```
    91.1555  
    90.6656  
   105.5147
```

### Create a Float-Float Swap and Price with `hjmprice`

Use `instswap` to create a float-float swap and price the swap with `hjmprice`.

```
RateSpec = intenvset('Rates', .05, 'StartDate', today, 'EndDate', datemnth(today, 60));  
IS = instswap([.02 .03], today, datemnth(today, 60), [], [], [], [1 1]);  
VolSpec = hjmvolspec('Constant', .2);  
TimeSpec = hjmtimespec(today, cfdates(today, datemnth(today, 60), 1));  
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec);  
hjmprice(HJMTree, IS)
```

```
ans = -4.3220
```

## Price Multiple Swaps with hjmprice

Use `instswap` to create multiple swaps and price the swaps with `hjmprice`.

```
RateSpec = intenvset('Rates', .05, 'StartDate', today, 'EndDate', datemnth(today,60));
IS = instswap([.03 .02], today, datemnth(today,60), [], [], [], [1 1]);
IS = instswap(IS, [200 300], today, datemnth(today,60), [], [], [], [0 0]);
IS = instswap(IS, [.08 300], today, datemnth(today,60), [], [], [], [1 0]);
VolSpec = hjmvolspec('Constant', .2);
TimeSpec = hjmtimespec(today, cfdates(today, datemnth(today,60), 1));
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec);
hjmprice(HJMTree, IS)
```

```
ans = 3×1

    4.3220
   -4.3220
   -0.2701
```

## Input Arguments

### HJMTree — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hjmtree`.

Data Types: `struct`

### InstSet — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### Options — Derivatives pricing options structure

structure

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## Output Arguments

### **Price — Price for each instrument**

`vector`

Price for each instrument, returned as a `NINST`-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a `NaN` is returned in that entry.

Related single-type pricing functions are:

- `bondbyhjm` — Price a bond from an HJM tree.
- `capbyhjm` — Price a cap from an HJM tree.
- `cfbyhjm` — Price an arbitrary set of cash flows from an HJM tree.
- `fixedbyhjm` — Price a fixed-rate note from an HJM tree.
- `floatbyhjm` — Price a floating-rate note from an HJM tree.
- `floorbyhjm` — Price a floor from an HJM tree.
- `optbndbyhjm` — Price a bond option from an HJM tree.
- `optembndbyhjm` — Price a bond with embedded option by an HJM tree.
- `optfloatbybdt` — Price a floating-rate note with an option from an HJM tree.
- `optemfloatbybdt` — Price a floating-rate note with an embedded option from an HJM tree.
- `rangefloatbyhjm` — Price range floating note using an HJM tree.
- `swapbyhjm` — Price a swap from an HJM tree.
- `swaptionbyhjm` — Price a swaption from an HJM tree.

### **PriceTree — Tree structure of instrument prices**

`structure`

Tree structure of instrument prices, returned as a `MATLAB` structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.

- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.t0bs` contains the observation times.

## See Also

`hjmsens` | `hjmtree` | `hjmvolspec` | `instadd` | `intenvprice` | `intenvsens`

## Topics

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## hjmsens

Instrument prices and sensitivities from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Delta,Gamma,Vega,Price] = hjmsens(HJMTree,InstSet)
[Delta,Gamma,Vega,Price] = hjmsens( ____,Options)
```

### Description

`[Delta,Gamma,Vega,Price] = hjmsens(HJMTree,InstSet)` computes instrument sensitivities and prices for instruments using an interest-rate tree created with the `hjmtree` function. All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

`hjmsens` handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See `instadd` for information on instrument types.

`[Delta,Gamma,Vega,Price] = hjmsens( ____,Options)` adds an optional input argument for `Options`.

### Examples

#### Compute Instrument Sensitivities Using an HJM Interest-Rate Tree

Load the tree and instruments from the `deriv.mat` data file. Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
HJMSubSet = instselect(HJMInstSet,'Type', {'Bond', 'Cap'});
instdisp(HJMSubSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	NaN

```

2      Bond 0.04      01-Jan-2000      01-Jan-2004      2      NaN      NaN      NaN
Index Type Strike Settle      Maturity      CapReset Basis Principal Name      Quanti
3      Cap 0.03      01-Jan-2000      01-Jan-2004      1      NaN      NaN      3% Cap 30

```

Compute the Delta and Gamma for the cap and bond instruments.

```
[Delta, Gamma] = hjmsens(HJMTtree, HJMSubSet)
```

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

```
Delta = 3×1
```

```

-272.6462
-347.4315
 294.9700

```

```
Gamma = 3×1
103 ×
```

```

 1.0299
 1.6227
 6.8526

```

## Input Arguments

### HJMTtree — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hjmtree`.

Data Types: `struct`

### InstSet — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Delta — Rate of change of instruments prices with respect to changes in interest rate**

vector

Rate of change of instruments prices with respect to changes in the interest rate, returned as a NINST-by-1 vector of deltas. `Delta` is computed by finite differences in calls to `hjmtree`.

---

**Note** `Delta` is calculated based on yield shifts of 100 basis points.

---

### **Gamma — Rate of change of instruments deltas with respect to changes in interest rate**

vector

Rate of change of instruments deltas with respect to changes in the interest rate, returned as a NINST-by-1 vector of gammas. `Gamma` is computed by finite differences in calls to `hjmtree`.

---

**Note** `Gamma` is calculated based on yield shifts of 100 basis points.

---

### **Vega — Rate of change of instruments prices with respect to changes in volatility**

vector

Rate of change of instruments prices with respect to changes in the volatility, returned as a NINST-by-1 vector of vegas. Volatility is  $\sigma(t, T)$  of the interest rate. `Vega` is computed by finite differences in calls to `hjmtree`. For information on the volatility process, see `hjmvolspec`.



---

**Note** Vega is calculated based on 1% shift in the volatility process.

---

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

## **See Also**

hjmprice | hjmtree | hjmvolspec | instadd

## **Topics**

“Computing Instrument Sensitivities” on page 2-108

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## hjmtimespec

Specify time structure for Heath-Jarrow-Morton interest-rate tree

### Syntax

```
TimeSpec = hjmtimespec(ValuationDate,Maturity)
TimeSpec = hjmtimespec( ____,Compounding)
```

### Description

`TimeSpec = hjmtimespec(ValuationDate,Maturity)` sets the number of levels and node times for a `hjmtree` and determines the mapping between dates and time for rate quoting.

`TimeSpec = hjmtimespec( ____,Compounding)` adds the optional argument `Compounding`.

### Examples

#### Set the Number of Levels and Node Times for an HJM Tree

This example shows how to specify an eight-period tree with semiannual nodes (every six months) and use exponential compounding to report rates.

```
Compounding = -1;
ValuationDate = '15-Jan-1999';
Maturity = datemnth(ValuationDate, 6*(1:8));
TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)
```

```
TimeSpec = struct with fields:
    FinObj: 'HJMTimeSpec'
    ValuationDate: 730135
    Maturity: [8x1 double]
    Compounding: -1
```

Basis: 0  
EndMonthRule: 1

## Input Arguments

### ValuationDate — Pricing date and first observation in the tree

serial date number | character vector date

Pricing date and first observation in the tree, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### Maturity — Dates marking the cash flow dates of the tree

serial date number | date character vector

Dates marking the cash flow dates of the tree, specified as NLEVELS-by-1 vector of serial date numbers or date character vectors. Cash flows with these maturities fall on tree nodes. **Maturity** should be in increasing order.

Data Types: double | char | cell

### Compounding — Rate at which the input zero rates were compounded when annualized

1 (default) | integer with value of 1, 2, 3, 4, 6, 12, 365, or -1

(Optional) Rate at which the input zero rates were compounded when annualized, specified as a scalar integer value.

- If Compounding = 1, 2, 3, 4, 6, 12:

$Disc = (1 + Z/F)^{-T}$ , where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units; for example,  $T = F$  is one year.

- If Compounding = 365:

$Disc = (1 + Z/F)^{-T}$ , where F is the number of days in the basis year and T is a number of days elapsed computed by basis.

- If Compounding = -1:

$Disc = \exp(-T*Z)$ , where T is time in years.

Data Types: double

## Output Arguments

**TimeSpec** — Specification for the time layout for `hjmtree` structure

Specification for the time layout for `hjmtree`, returned as a structure. The state observation dates are `[ValuationDate; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity(end)`.

## See Also

`hjmtree` | `hjmvolspec`

## Topics

“Specifying the Time Structure (TimeSpec)” on page 2-85

“Creating Trees” on page 2-87

“Examining Trees” on page 2-88

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# hjmtree

Build Heath-Jarrow-Morton interest-rate tree

## Syntax

```
HJMTree = hjmtree(VolSpec,RateSpec,TimeSpec)
```

## Description

HJMTree = hjmtree(VolSpec,RateSpec,TimeSpec) creates a structure containing time and forward-rate information on a bushy tree.

## Examples

### Create a HJMTree

Using the data provided, create a HJM volatility specification (using `hjmvolspec`), rate specification (using `intenvset`), and tree time layout specification (using `hjmtimespec`). Then use these specifications to create a HJM tree using `hjmtree`.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ['01-01-2000'; '01-01-2001'; '01-01-2002'; '01-01-2003'; '01-01-2004'];
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003'; '01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
CurveTerm = [1; 2; 3; 4; 5];

HJMVolSpec = hjmvolspec('Stationary', Volatility , CurveTerm);

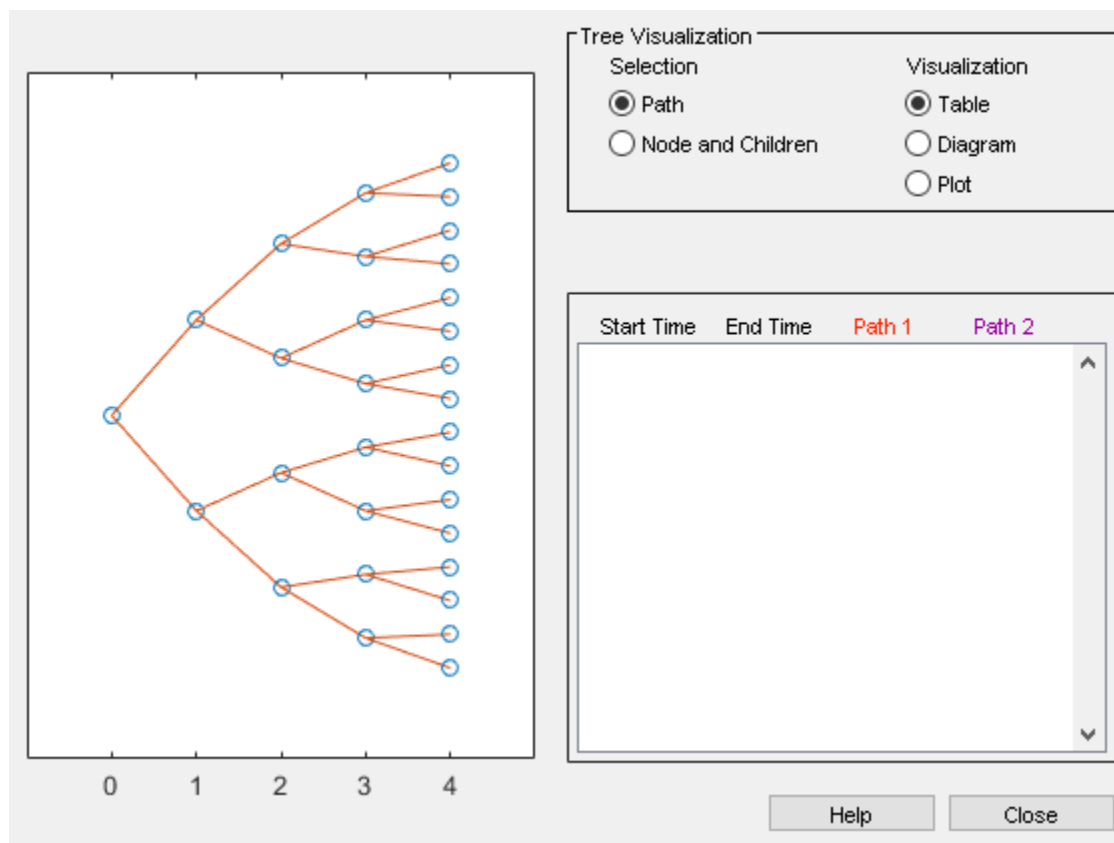
RateSpec = intenvset('Compounding', Compounding,...
    'ValuationDate', ValuationDate,...
    'StartDates', StartDate,...
    'EndDates', EndDates,...
    'Rates', Rates);
```

```
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);
HJMTree = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec)

HJMTree = struct with fields:
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [730486 730852 731217 731582 731947]
    TFwd: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [4]}
    CFlowT: {[5x1 double] [4x1 double] [3x1 double] [2x1 double] [5]}
    FwdTree: {1x5 cell}
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(HJMTree)
```



## Input Arguments

### **VolSpec – Volatility process specification**

structure

Volatility process specification, specified using the `VolSpec` output obtained from `hjmvolspec`. `VolSpec` sets the number of factors and the rules for computing the volatility  $\sigma(t, T)$  for each factor.

Data Types: `struct`

**RateSpec — Interest-rate specification for initial risk-free rate curve**

structure

Interest-rate specification for initial rate curve, specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`**TimeSpec — Time tree layout specification**

structure

Time tree layout specification, specified using the TimeSpec output obtained from `hjmtimespec`. The TimeSpec defines the observation dates of the HJM tree and the Compounding rule for date to time mapping and price-yield formulas.

Data Types: `struct`

## Output Arguments

**HJMTree — Time and interest-rate information of a bushy tree**

structure

Time and interest-rate information of a bushy tree, returned as a structure.

## See Also

`hjmprice` | `hjmtimespec` | `hjmvolspec` | `intenvset`

## Topics

“Creating Trees” on page 2-87

“Examining Trees” on page 2-88

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# hjmvolspec

Specify Heath-Jarrow-Morton interest-rate volatility process

## Syntax

```
VolSpec = hjmvolspec(Factor, Sigma_0)
VolSpec = hjmvolspec(Factor, CurveVol, CurveTerm)
VolSpec = hjmvolspec(Factor, Sigma_0, Lambda)
VolSpec = hjmvolspec(Factor, Sigma_0, CurveDecay, CurveTerm)
VolSpec = hjmvolspec(Factor, CurveProp, CurveTerm, MaxSpot)
```

## Description

`VolSpec = hjmvolspec(Factor, Sigma_0)` creates a Constant volatility (Ho-Lee) structure for `hjmtree` by specifying the `Factor` as `'Constant'`.

`VolSpec = hjmvolspec(Factor, CurveVol, CurveTerm)` creates a Stationary volatility structure for `hjmtree` by specifying the `Factor` as `'Stationary'`.

`VolSpec = hjmvolspec(Factor, Sigma_0, Lambda)` creates an Exponential volatility structure for `hjmtree` by specifying the `Factor` as `'Exponential'`.

`VolSpec = hjmvolspec(Factor, Sigma_0, CurveDecay, CurveTerm)` creates a Vasicek, Hull-White volatility structure for `hjmtree` by specifying the `Factor` as `'Vasicek'`.

`VolSpec = hjmvolspec(Factor, CurveProp, CurveTerm, MaxSpot)` creates a Nearly proportional stationary volatility structure for `hjmtree` by specifying the `Factor` as `'Proportional'`.

## Examples

### Compute the VolSpec Structure to Specify a Proportional Volatility Model for HJMTree

This example shows how to compute the VolSpec structure to specify the volatility model for `hjmtree` when volatility is single-factor proportional.

```
CurveProp = [0.11765; 0.08825; 0.06865];  
CurveTerm = [1; 2; 3];  
VolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm, 1e6)
```

```
VolSpec = struct with fields:  
    FinObj: 'HJMVolSpec'  
    FactorModels: {'Proportional'}  
    FactorArgs: {{1x3 cell}}  
    SigmaShift: 0  
    NumFactors: 1  
    NumBranch: 2  
    PBranch: [0.5000 0.5000]  
    Fact2Branch: [-1 1]
```

### Compute the VolSpec Structure to Specify an Exponential Volatility Model for HJMTree

This example shows how to compute the VolSpec structure to specify the volatility model for `hjmtree` when volatility is two-factor exponential and constant.

```
VolSpec = hjmvolspec('Exponential', 0.1, 1, 'Constant', 0.2)
```

```
VolSpec = struct with fields:  
    FinObj: 'HJMVolSpec'  
    FactorModels: {'Exponential' 'Constant'}  
    FactorArgs: {{1x2 cell} {1x1 cell}}  
    SigmaShift: 0  
    NumFactors: 2  
    NumBranch: 3  
    PBranch: [0.2500 0.2500 0.5000]  
    Fact2Branch: [2x3 double]
```

## Input Arguments

### Factor — Volatility factor

character vector with value of 'Constant', 'Stationary', 'Exponential', 'Vasicek', or 'Proportional'

Volatility factor, specified as a character vector with one of the following values:

- 'Constant'

$$\sigma(t, T) = \text{Sigma}_0$$

- 'Stationary'

$$\sigma(t, T) = \text{Vol}(T - t) = \text{Vol}(\text{Term})$$

- 'Exponential'

$$\sigma(t, T) = \text{Sigma}_0 * \exp(-\text{Lambda} * (T - t))$$

- 'Vasicek'

$$\sigma(t, T) = \text{Sigma}_0 * \exp(-\text{Decay} * (T - t))$$

- 'Proportional'

$$\sigma(t, T) = \text{Prop}(T - t) * \max(\text{SpotRate}(t), \text{MaxSpot})$$

---

**Note** You can specify more than one Factor by concatenating Factor names and their associated parameters.

---

Data Types: char

### Sigma\_0 — Base volatility over a unit time

numeric

Base volatility over a unit, specified as a scalar numeric value.

Data Types: double

### Lambda — Decay factor

numeric

Decay factor, specified as a scalar numeric value.

Data Types: double

**CurveVol — Number of curve Vol values at sample points**

numeric vector

Number of curve Vol values at sample points, specified as a NCURVES-by-1 vector.

Data Types: double

**CurveTerm — Number of curve Term values at sample points**

numeric vector

Number of curve Term values at sample points, specified as a NCURVES-by-1 vector.

Data Types: double

**CurveDecay — Number of curve Decay values at sample points**

numeric vector

Number of curve Decay values at sample points, specified as a NPOINTS-by-1 vector.

Data Types: double

**CurveProp — Number of curve Prop values at sample points**

numeric vector

Number of curve Prop values at sample points, specified as a NCURVES-by-1 vector.

Data Types: double

**MaxSpot — Maximum spot rate**

numeric

Maximum spot rate, specified as a scalar numeric value.

Data Types: double

## Output Arguments

**VolSpec — Specification for the volatility model for bktree**

structure

Structure specifying the volatility model for `bktree`. `hjmvolspec` defines an HJM forward-rate volatility process based on the specified input `Factor`.

## Definitions

### Volatility Process

The volatility process is  $\sigma(t, T)$ , where  $t$  is the observation time and  $T$  is the starting time of a forward rate.

In a stationary process, the volatility term is  $T-t$ . Multiple factors can be specified sequentially.

The time values  $T$ ,  $t$ , and Term are in coupon interval units specified by the Compounding input of `hjmtimespec`. For instance if `Compounding = 2`, `Term = 1` is a semiannual period (six months).

### See Also

`hjmtimespec` | `hjmtree`

### Topics

“Specifying the Volatility Model (VolSpec)” on page 2-82

“Creating Trees” on page 2-87

“Examining Trees” on page 2-88

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## HullWhite1F

Create Hull-White one-factor model

### Description

The Hull-White one-factor model is specified using the zero curve, alpha, and sigma parameters.

Specifically, the HullWhite1F model is defined using the following equations:

$$dr = [\theta(t) - a(t)r]dt + \sigma(t)dW$$

where:

$dr$  is the change in the short-term interest rate over a small interval.

$r$  is the short-term interest rate.

$\theta(t)$  is a function of time determining the average direction in which  $r$  moves, chosen such that movements in  $r$  are consistent with today's zero coupon yield curve.

$\alpha$  is the mean reversion rate.

$dt$  is a small change in time.

$\sigma$  is the annual standard deviation of the short rate.

$W$  is the Brownian motion.

### Creation

### Syntax

HW1F = HullWhite1F(ZeroCurve,Alpha,Sigma)

## Description

`HW1F = HullWhite1F(ZeroCurve,Alpha,Sigma)` creates a `HullWhite1F` (`HW1F`) object using the required arguments to set the Properties on page 11-823.

## Properties

### **ZeroCurve — Zero curve**

`IRDataCurve` object | `RateSpec`

Zero curve, specified as an output from `IRDataCurve` or a `RateSpec` that is obtained from `intenvset`. This is the zero curve used to evolve the path of future interest rates.

Data Types: `object` | `struct`

### **Alpha — Mean reversion**

numeric

Mean reversion, specified either as a scalar or function handle which takes time as input and returns a scalar mean reversion value.

Data Types: `double`

### **Sigma — Volatility**

numeric

Volatility, specified either as a scalar or function handle which takes time as input and returns a scalar mean volatility.

Data Types: `double`

## Object Functions

`simTermStructs` Simulate term structures for Hull-White one-factor model

## Examples

### Create a Hull-White One-Factor Model Using an IRDataCurve

Create a Hull-White one-factor model using an IRDataCurve.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

alpha = .1;
sigma = .01;

HW1F = HullWhite1F(irdc,alpha,sigma)
```

```
HW1F =
  HullWhite1F with properties:
    ZeroCurve: [1x1 IRDataCurve]
    Alpha: @(t,V)inAlpha
    Sigma: @(t,V)inSigma
```

Use the `simTermStructs` method with the `HullWhite1F` model to simulate term structures.

```
SimPaths = simTermStructs(HW1F, 10, 'nTrials',100);
```

### Create a Hull-White One-Factor Model Using a RateSpec

Create a Hull-White one-factor model using a RateSpec.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

RateSpec = intenvset('Rates',ZeroRates,'EndDates',CurveDates,'StartDate',Settle);

alpha = .1;
sigma = .01;
```



```
HW1F = HullWhite1F(RateSpec,alpha,sigma)
```

```
HW1F =
  HullWhite1F with properties:

    ZeroCurve: [1x1 IRDataCurve]
      Alpha: @(t,V)inAlpha
      Sigma: @(t,V)inSigma
```

Use the `simTermStructs` method with the `HullWhite1F` model to simulate term structures.

```
SimPaths = simTermStructs(HW1F, 10, 'nTrials',100);
```

### Simulate the Price of a Bond Using a Hull-White One-Factor Model Until the Bond's Maturity

Define the zero curve data.

```
Settle = datenum('4-Apr-2016');
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
ZeroDates = datemnth(Settle,ZeroTimes*12);
RateSpec = intenvset('StartDates', Settle,'EndDates', ZeroDates, 'Rates', ZeroRates)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
        Disc: [8x1 double]
        Rates: [8x1 double]
        EndTimes: [8x1 double]
        StartTimes: [8x1 double]
        EndDates: [8x1 double]
        StartDates: 736424
        ValuationDate: 736424
        Basis: 0
        EndMonthRule: 1
```

Define the bond parameters.

```
Maturity = datemnth(Settle,12*5);
CouponRate = 0;
```

Define the Hull-White parameters.

```
alpha = .1;
sigma = .01;
HW1F = HullWhite1F(RateSpec,alpha,sigma)
```

```
HW1F =
    HullWhite1F with properties:

    ZeroCurve: [1x1 IRDataCurve]
    Alpha: @(t,V)inAlpha
    Sigma: @(t,V)inSigma
```

Define the simulation parameters.

```
nTrials = 100;
nPeriods = 12*5;
deltaTime = 1/12;
SimZeroCurvePaths = simTermStructs(HW1F, nPeriods, 'nTrials',nTrials, 'deltaTime',deltaTime);
SimDates = datemnth(Settle,1:nPeriods);
```

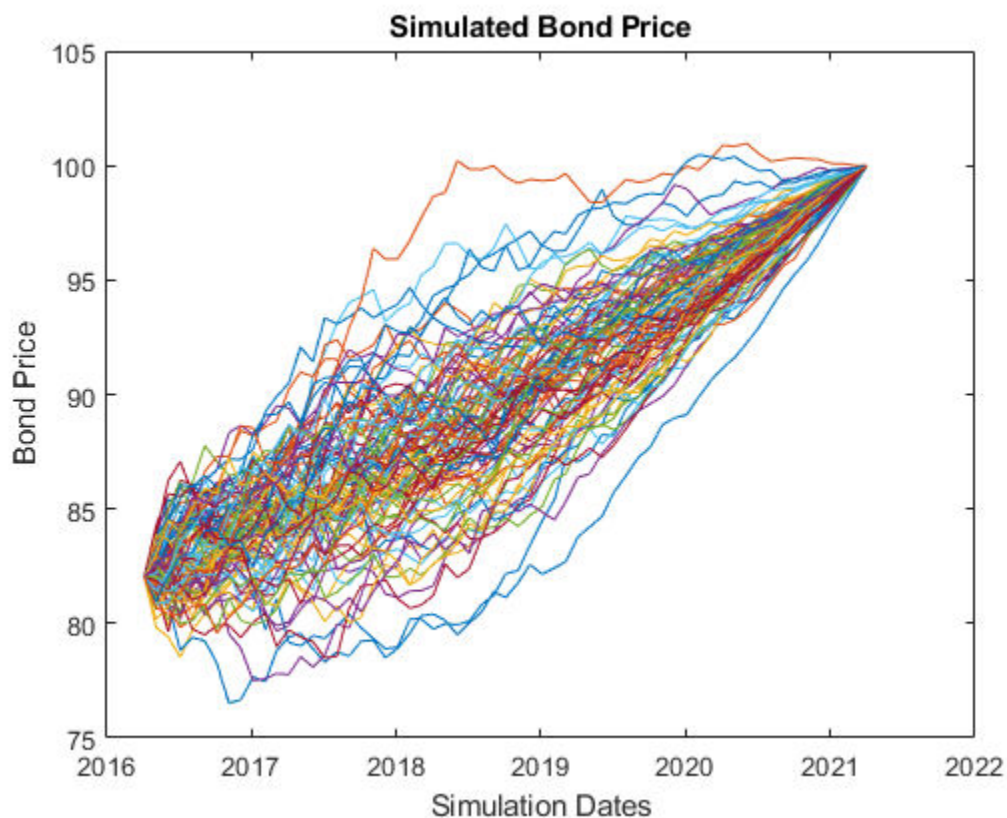
Preallocate and initialize for the simulation.

```
SimBondPrice = zeros(nPeriods+1,nTrials);
SimBondPrice(1,:,:) = bondbyzero(RateSpec,CouponRate,Settle,Maturity);
SimBondPrice(end,:,:) = 100;
```

Compute the bond values for each simulation date and path, note that you can vectorize over the trial dimension.

```
for periodidx=1:nPeriods-1
    simRateSpec = intenvset('StartDate',SimDates(periodidx),'EndDates',...
        datemnth(SimDates(periodidx),ZeroTimes*12),'Rates',squeeze(SimZeroCurvePaths(periodidx,:)))
    SimBondPrice(periodidx+1,:) = bondbyzero(simRateSpec,CouponRate,SimDates(periodidx),Maturity);
end

plot([Settle SimDates],SimBondPrice)
datetick
ylabel('Bond Price')
xlabel('Simulation Dates')
title('Simulated Bond Price')
```



### Simulate the Total Return of a Bond Portfolio Until Maturity

Define the zero curve data.

```
Settle = datenum('4-Apr-2016');
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [-0.01 -0.009 -0.0075 -0.003 -0.002 -0.001 0.002 0.0075]';
ZeroDates = datemnth(Settle,ZeroTimes*12);
RateSpec = intenvset('StartDates', Settle,'EndDates', ZeroDates, 'Rates', ZeroRates)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
```

```

Compounding: 2
    Disc: [8x1 double]
    Rates: [8x1 double]
    EndTimes: [8x1 double]
    StartTimes: [8x1 double]
    EndDates: [8x1 double]
    StartDates: 736424
ValuationDate: 736424
    Basis: 0
EndMonthRule: 1

```

Define the bond parameters for the five bonds in the portfolio.

```

Maturity = datemnth(Settle,12*5); % All bonds have the same maturity
CouponRate = [0.035;0.04;0.02;0.015;0.042]; % Different coupon rates for the bonds
nBonds = length(CouponRate);

```

Define the Hull-White parameters.

```

alpha = .1;
sigma = .01;
HW1F = HullWhite1F(RateSpec,alpha,sigma)

```

```

HW1F =
    HullWhite1F with properties:

    ZeroCurve: [1x1 IRDataCurve]
    Alpha: @(t,V)inAlpha
    Sigma: @(t,V)inSigma

```

Define the simulation parameters.

```

nTrials = 1000;
nPeriods = 12*5;
deltaTime = 1/12;
SimZeroCurvePaths = simTermStructs(HW1F, nPeriods,'nTrials',nTrials,'deltaTime',deltaTime);
SimDates = datemnth(Settle,1:nPeriods);

```

Preallocate and initialize for the simulation.

```

SimBondPrice = zeros(nPeriods+1,nBonds,nTrials);
SimBondPrice(1,:,:) = repmat(bondbyzero(RateSpec,CouponRate,Settle,Maturity)',[1 1 nTrials]);
SimBondPrice(end,:,:) = 100;

```

```
[BondCF,BondCFDates,~,CFlowFlags] = cfamounts(CouponRate,Settle,Maturity);
BondCF(CFlowFlags == 4) = BondCF(CFlowFlags == 4) - 100;
SimBondCF = zeros(nPeriods+1,nBonds,nTrials);
```

Compute bond values for each simulation date and path. Note that you can vectorize over the trial dimension.

```
for periodidx=1:nPeriods
    if periodidx < nPeriods
        simRateSpec = intenvset('StartDate',SimDates(periodidx),'EndDates',...
            datemnth(SimDates(periodidx),ZeroTimes*12),'Rates',squeeze(SimZeroCurvePaths(periodidx,1,:)))
        SimBondPrice(periodidx+1,,:) = bondbyzero(simRateSpec,CouponRate,SimDates(periodidx+1,1,:))
    end

    simidx = SimDates(periodidx) == BondCFDates;
    SimCF = zeros(1,nBonds);
    SimCF(any(simidx,2)) = BondCF(simidx);
    ReinvestRate = 1 + SimZeroCurvePaths(periodidx+1,1,:);
    SimBondCF(periodidx+1,,:) = bsxfun(@times,bsxfun(@plus,SimBondCF(periodidx,,:),S...
end
```

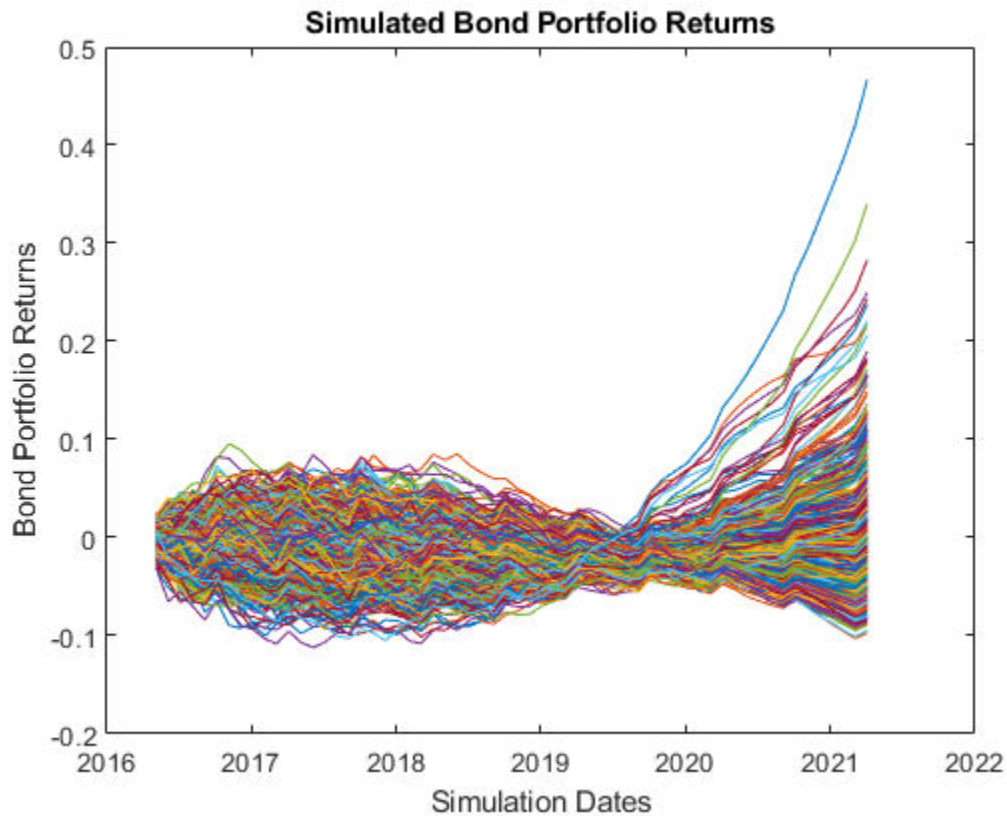
Compute the total return series.

```
TotalCF = SimBondPrice + SimBondCF;
```

Assume the bond portfolio is equally weighted and plot the simulated bond portfolio returns.

```
TotalCF = squeeze(sum(TotalCF,2));

TotRetSeries = bsxfun(@rdivide,TotCF(2:end,:),TotCF(1,:)) - 1;
plot(SimDates,TotRetSeries)
datetick
ylabel('Bond Portfolio Returns')
xlabel('Simulation Dates')
title('Simulated Bond Portfolio Returns')
```



## Definitions

### Hull-White One-Factor Model

The Hull-White model is a single-factor, no-arbitrage yield curve model in which the short-term rate of interest is the random factor or state variable.

No-arbitrage means that the model parameters are consistent with the bond prices implied in the zero coupon yield curve.

## References

- [1] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [2] Hull, J. *Options, Futures, and Other Derivatives*. Prentice-Hall, 2011.

## See Also

LiborMarketModel | LinearGaussian2F | hwcalbycap | hwcalbyfloor |  
simTermStructs

## Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123  
“Pricing Bermudan Swaptions with Monte Carlo Simulation”  
“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

## simTermStructs

Simulate term structures for Hull-White one-factor model

### Syntax

```
[ZeroRates,ForwardRates] = simTermStructs(HW1F,nPeriods)
[ZeroRates,ForwardRates] = simTermStructs( ____,Name,Value)
```

### Description

[ZeroRates,ForwardRates] = simTermStructs(HW1F,nPeriods) simulates future zero curve paths using a specified HullWhite1F object.

[ZeroRates,ForwardRates] = simTermStructs( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Simulate Term Structures for the HullWhite1F Model

Create a HW1F object.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

alpha = .1;
sigma = .01;

HW1F = HullWhite1F(irdc,alpha,sigma)

HW1F =
    HullWhite1F with properties:
```



```
ZeroCurve: [1x1 IRDataCurve]
Alpha: @(t,V)inAlpha
Sigma: @(t,V)inSigma
```

Simulate the term structures for the specified HW1F object.

```
SimPaths = simTermStructs(HW1F, 10, 'nTrials', 100);
```

## Input Arguments

### HW1F — HullWhite1F object

object

HullWhite1F object, specified using the HW1F object created using HullWhite1F.

Data Types: object

### nPeriods — Number of simulation periods

numeric

Number of simulation periods, specified as a numeric value. For example, to simulate 12 years with an annual spacing, specify 12 as the nPeriods input and 1 as the optional deltaTime input (note that the default value for deltaTime is 1).

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

```
Example: [ZeroRates, ForwardRates] =
simTermStructs(HW1F, NPeriods, 'nTrials', 100, 'deltaTime', dt)
```

### deltaTime — Time step between nPeriods

1 (default) | numeric

Time step between `nPeriods` measured in years, specified as the comma-separated pair consisting of `'deltaTime'` and a scalar numeric value. For example, to simulate 12 years with an annual spacing, specify 12 as the `nPeriods` input and 1 as the optional `deltaTime` input (note that the default value for `deltaTime` is 1).

Data Types: `double`

### **nTrials — Number of simulated trials**

1 (default) | positive integer

Number of simulated trials (sample paths), specified as the comma-separated pair consisting of `'nTrials'` and a positive scalar integer value of `nPeriods` observations each. If you do not specify a value for this argument, the default is 1, indicating a single path of correlated state variables.

Data Types: `double`

### **antithetic — Flag indicating whether antithetic sampling is used to generate Gaussian random variates**

false (default) | positive integer

Flag indicating whether antithetic sampling is used to generate the Gaussian random variates that drive the zero-drift, unit-variance rate Brownian vector  $dW(t)$ , specified as the comma-separated pair consisting of `'antithetic'` and a Boolean scalar flag. For details, see `simBySolution`.

Data Types: `logical`

### **Z — Direct specification of dependent random noise process**

Gaussian variates generated by `simBySolution` function (default) | numeric

Direct specification of the dependent random noise process, specified as the comma-separated pair consisting of `'Z'` and a numeric value. The `Z` value is used to generate the zero-drift, unit-variance rate Brownian vector  $dW(t)$  that drives the simulation. For details, see `simBySolution` for the HWV model. If you do not specify a value for `Z`, `simBySolution` generates Gaussian variates.

Data Types: `double`

### **Tenor — Maturities to compute at each time step**

tenor of `HullWhite1F` object zero curve (default) | numeric vector

Maturities to compute at each time step, specified as the comma-separated pair consisting of `'Tenor'` and a numeric vector.

Tenor enables you to choose a different set of rates to output than the underlying rates. For example, you may want to simulate quarterly data but only report annual rates; this can be done by specifying the optional input `Tenor`.

Data Types: `double`

## Output Arguments

### **ZeroRates** — Simulated zero-rate term structures

`matrix`

Simulated zero-rate term structures, returned as a `nPeriods+1-by-nTenors-by-nTrials` matrix.

### **ForwardRates** — Simulated forward-rate term structures

`matrix`

Simulated zero-rate term structures, returned as a `nPeriods+1-by-nTenors-by-nTrials` matrix. The `ForwardRates` output is computed using the simulated short rates and by using the model definition to recover the entire yield curve at each simulation date.

## See Also

`HullWhite1F`

## Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

## hwcalbycap

Calibrate Hull-White tree using caps

### Syntax

```
[Alpha,Sigma,OptimOut] = hwcalbycap(RateSpec,MarketStrike  
MarketMaturity,MarketVolatility)  
[Alpha,Sigma,OptimOut = hwcalbycap(RateSpec,MarketStrike  
MarketMaturity,MarketVolatility,Strike,Settle,Maturity)  
[Alpha,Sigma,OptimOut] = hwcalbycap( ____,Name,Value)
```

### Description

[Alpha,Sigma,OptimOut] = hwcalbycap(RateSpec,MarketStrike MarketMaturity,MarketVolatility) calibrates the Alpha (mean reversion) and Sigma (volatility) using cap market data and the Hull-White model using the entire cap surface.

The Hull-White calibration functions (hwcalbycap and hwcalbyfloor) support three models: Black (default), Bachelier or Normal, and Shifted Black. For more information, see the optional arguments for Shift and Model.

[Alpha,Sigma,OptimOut = hwcalbycap(RateSpec,MarketStrike MarketMaturity,MarketVolatility,Strike,Settle,Maturity) estimates the Alpha (mean reversion) and Sigma (volatility) using cap market data and the Hull-White model to price a cap at a particular maturity/volatility using the additional optional input arguments for Strike, Settle, and Maturity.

Strike, Settle, and Maturity arguments are specified to calibrate to a specific point on the market volatility surface. If omitted, the calibration is performed across all the market instruments

For an example of calibrating using the Hull-White model with Strike, Settle, and Maturity input arguments, see “Calibrating Hull-White Model Using Market Data” on page 2-111.

[Alpha, Sigma, OptimOut] = hwcalbycap( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

## Examples

### Calibrate Hull-White Model from Market Data Using the Entire Cap Volatility Surface

This example shows how to use hwcalbycap input arguments for MarketStrike, MarketMaturity, and MarketVolatility to calibrate the HW model using the entire cap volatility surface.

Cap market volatility data covering two strikes over 12 maturity dates.

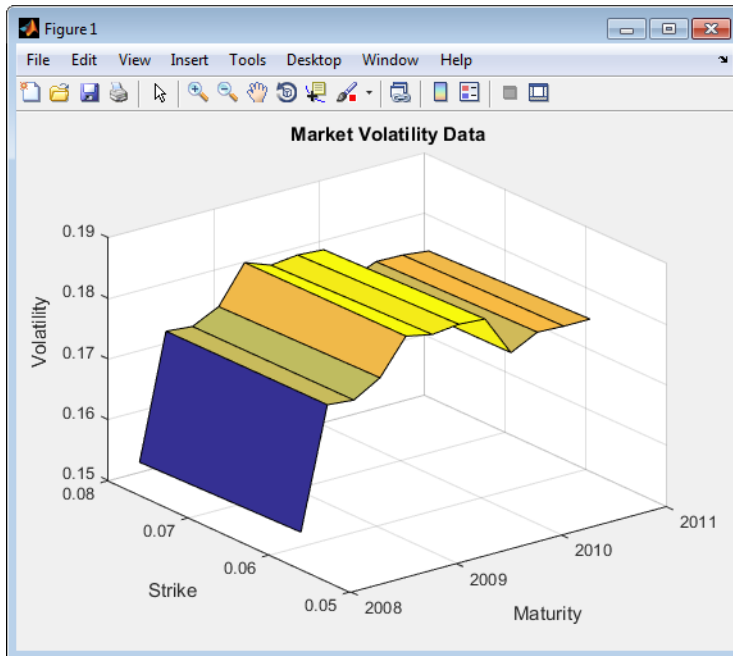
```
Reset = 4;
MarketStrike = [0.0590; 0.0790];

MarketMaturity = {'21-Mar-2008'; '21-Jun-2008'; '21-Sep-2008'; '21-Dec-2008';
                 '21-Mar-2009'; '21-Jun-2009'; '21-Sep-2009'; '21-Dec-2009';
                 '21-Mar-2010'; '21-Jun-2010'; '21-Sep-2010'; '21-Dec-2010'};
MarketMaturity = datenum(MarketMaturity);

MarketVolatility = [0.1533 0.1731 0.1727 0.1752 0.1809 0.1800 0.1805 0.1802...
                   0.1735 0.1757 0.1755 0.1755;
                   0.1526 0.1730 0.1726 0.1747 0.1808 0.1792 0.1797 0.1794...
                   0.1733 0.1751 0.1750 0.1745];
```

Plot market volatility surface.

```
[AllMaturities, AllStrikes] = meshgrid(MarketMaturity, MarketStrike);
figure;
surf(AllMaturities, AllStrikes, MarketVolatility)
datetick
xlabel('Maturity')
ylabel('Strike')
zlabel('Volatility')
title('Market Volatility Data')
```



Set interest rate term structure and create a RateSpec.

```
Settle = '21-Jan-2008';
Compounding = 4;
Basis = 0;
Rates = [0.0627; 0.0657; 0.0691; 0.0717; 0.0739; 0.0755; 0.0765; 0.0772;
         0.0779; 0.0783; 0.0786; 0.0789];
EndDates = {'21-Mar-2008'; '21-Jun-2008'; '21-Sep-2008'; '21-Dec-2008'; ...
            '21-Mar-2009'; '21-Jun-2009'; '21-Sep-2009'; '21-Dec-2009'; ...
            '21-Mar-2010'; '21-Jun-2010'; '21-Sep-2010'; '21-Dec-2010'};
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
                    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, ...
                    'Basis', Basis)
```

RateSpec =

```
FinObj: 'RateSpec'
Compounding: 4
Disc: [12x1 double]
Rates: [12x1 double]
EndTimes: [12x1 double]
StartTimes: [12x1 double]
EndDates: [12x1 double]
StartDates: 733428
```

```

ValuationDate: 733428
      Basis: 0
EndMonthRule: 1

```

### Calibrate Hull-White model from market data.

```

o = optimoptions('lsqnonlin','TolFun',1e-5,'Display','off');

[Alpha, Sigma] = hwcalbycap(RateSpec, MarketStrike, MarketMaturity,...
    MarketVolatility, 'Reset', Reset, 'Basis', Basis, 'OptimOptions', o)

Warning: LSQNONLIN did not converge to an optimal solution. It exited with exitflag = 3.

> In hwcalbycapfloor>optimizeOverCapSurface at 232
   In hwcalbycapfloor at 79
   In hwcalbycap at 81

```

```

Alpha =
    0.0943

```

```

Sigma =
    0.0146

```

### Compare with Black prices.

```

BlkPrices = capbyblk(RateSpec, AllStrikes(:), Settle, AllMaturities(:),...
    MarketVolatility(:), 'Reset', Reset, 'Basis', Basis);

```

```

BlkPrices =

    0.0604
         0
    0.2729
    0.0006
    0.6498
    0.0412
    1.1121
    0.1426
    1.6426
    0.3131
    2.1869
    0.4998
    2.7056
    0.6894
    3.2124
    0.8815
    3.7311
    1.0686
    4.2246

```

```
1.2790
4.7027
1.4810
5.1877
1.6919
```

Setup Hull-White tree using calibrated parameters, alpha, and sigma.

```
VolDates = EndDates;
VolCurve = Sigma*ones(numel(EndDates),1);
AlphaDates = EndDates;
AlphaCurve = Alpha*ones(numel(EndDates),1);
HWVolSpec = hmwolspec(Settle, VolDates, VolCurve, AlphaDates, AlphaCurve);

HWTimeSpec = hwtimespec(Settle, EndDates, Compounding);
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec, 'Method', 'HW2000')

HWTree =

    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
             tObs: [0 0.6593 1.6612 2.6593 3.6612 4.6593 5.6612 6.6593 7.6612 8.6593 9.6612 10.6593]
             dObs: [733428 733488 733580 733672 733763 733853 733945 734037 734128 734218 734310 734402]
    CFlowT: {1x12 cell}
    Probs: {1x11 cell}
    Connect: {1x11 cell}
    FwdTree: {1x12 cell}
```

Compute Hull-White prices based on the calibrated tree.

```
HWPPrices = capbyhw(HWTree, AllStrikes(:), Settle, AllMaturities(:), Reset, Basis)
```

```
HWPPrices =
```

```
0.0601
0
0.2788
0
0.6580
0.0518
1.1254
0.1485
1.6591
0.3123
2.2076
0.5022
2.7319
0.6883
3.2459
0.8774
```



```

3.7771
1.0900
4.2769
1.2875
4.7645
1.4845
5.2572
1.6921

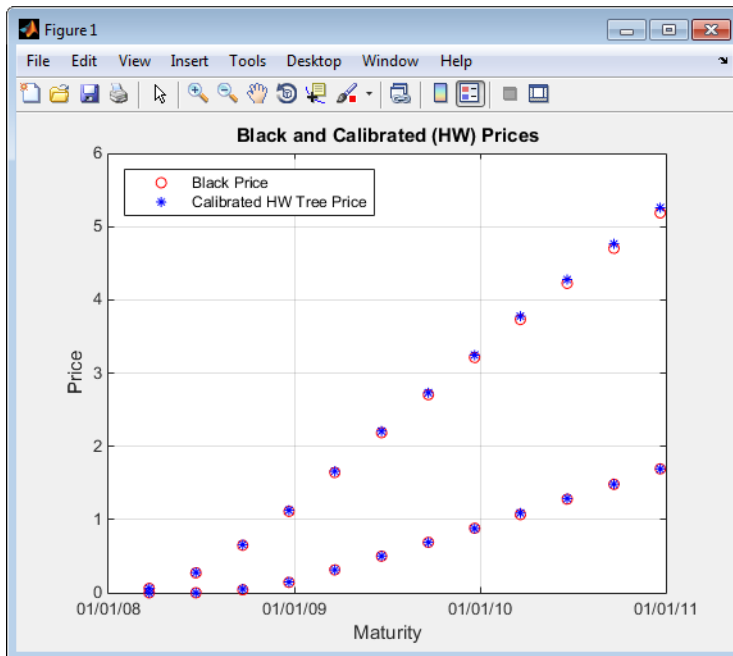
```

Plot Black prices against the calibrated Hull-White tree prices.

```

figure;
plot(AllMaturities(:), BlkPrices, 'or', AllMaturities(:), HWPrices, '*b');
datetick('x', 2)
xlabel('Maturity');
ylabel('Price');
title('Black and Calibrated (HW) Prices');
legend('Black Price', 'Calibrated HW Tree Price', 'Location', 'NorthWest');
grid on

```



## Calibrating Caplets Using the Normal (Bachelier) Model

This example shows how to use `hwcalbycap` to calibrate market data with the Normal (Bachelier) model to price caplets. Use the Normal (Bachelier) model to perform calibrations when working with negative interest rates, strikes, and normal implied volatilities.

Consider a cap with these parameters:

```
Settle = 'Dec-30-2016';  
Maturity = 'Dec-30-2019';  
Strike = -0.001075;  
Reset = 2;  
Principal = 100;  
Basis = 0;
```

The caplets and market data for this example are defined as:

```
capletDates = cfdates(Settle, Maturity, Reset, Basis);  
datestr(capletDates')
```

```
ans = 6x11 char array  
    '30-Jun-2017'  
    '30-Dec-2017'  
    '30-Jun-2018'  
    '30-Dec-2018'  
    '30-Jun-2019'  
    '30-Dec-2019'
```

```
% Market data information
```

```
MarketStrike = [-0.0013; 0];
```

```
MarketMat = {'30-Jun-2017'; '30-Dec-2017'; '30-Jun-2018'; '30-Dec-2018'; '30-Jun-2019';
```

```
MarketVol = [0.184 0.2329 0.2398 0.2467 0.2906 0.3348; % First row in table correspond  
            0.217 0.2707 0.2760 0.2814 0.3160 0.3508]; % Second row in table correspond
```

Define the `RateSpec`.

```
Rates = [-0.002210; -0.002020; -0.00182; -0.001343; -0.001075];
```

```
ValuationDate = 'Dec-30-2016';
```

```
EndDates = {'30-Jun-2017'; 'Dec-30-2017'; '30-Jun-2018'; 'Dec-30-2018'; 'Dec-30-2019'};
```

```
Compounding = 2;
```

```
Basis = 0;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
```

Use `hwcalbycap` to find values for the volatility parameters Alpha and Sigma using the Normal (Bachelier) model.

```
format short
o=optimoptions('lsqnonlin','TolFun',100*eps);
warning('off','fininst:hwcalbycapfloor:NoConverge')
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike, MarketMat,...
MarketVol, Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal,...
'Basis', Basis, 'OptimOptions', o, 'model', 'normal')
```

Local minimum possible.

`lsqnonlin` stopped because the size of the current step is less than the value of the step size tolerance.

Alpha = 1.0000e-06

Sigma = 0.3384

```
OptimOut = struct with fields:
    resnorm: 1.5181e-04
    residual: [5x1 double]
    exitflag: 2
    output: [1x1 struct]
    lambda: [1x1 struct]
    jacobian: [5x2 double]
```

The `OptimOut.residual` field of the `OptimOut` structure is the optimization residual. This value contains the difference between the Normal (Bachelier) caplets and those calculated during the optimization. Use the `OptimOut.residual` value to calculate the percentual difference (error) compared to Normal (Bachelier) caplet prices, and then decide whether the residual is acceptable. There is almost always some residual, so decide if it is acceptable to parameterize the market with a single value of Alpha and Sigma.

Price the caplets using the market data and Normal (Bachelier) model to obtain the reference caplet values. To determine the effectiveness of the optimization, calculate reference caplet values using the Normal (Bachelier) formula and the market data. Note, you must first interpolate the market data to obtain the caplets for calculation.

```

MarketMatNum = datenum(MarketMat);
[Mats, Strikes] = meshgrid(MarketMatNum, MarketStrike);
FlatVol = interp2(Mats, Strikes, MarketVol, datenum(Maturity), Strike, 'spline');

[CapPrice, Caplets] = capbynormal(RateSpec, Strike, Settle, Maturity, FlatVol,...
'Reset', Reset, 'Basis', Basis, 'Principal', Principal);
Caplets = Caplets(2:end)

Caplets = 5×1

    4.7392
    6.7799
    8.2609
    9.6136
   10.6455

```

Compare the optimized values and Normal (Bachelier) values, and display the results graphically. After calculating the reference values for the caplets, compare the values analytically and graphically to determine whether the calculated single values of Alpha and Sigma provide an adequate approximation.

```
OptimCaplets = Caplets+OptimOut.residual;
```

```
disp(' ');
```

```
disp(' Bachelier   Calibrated Caplets');
```

```

    Bachelier   Calibrated Caplets
disp([Caplets      OptimCaplets])

    4.7392    4.7453
    6.7799    6.7851
    8.2609    8.2657
    9.6136    9.6112
   10.6455   10.6379

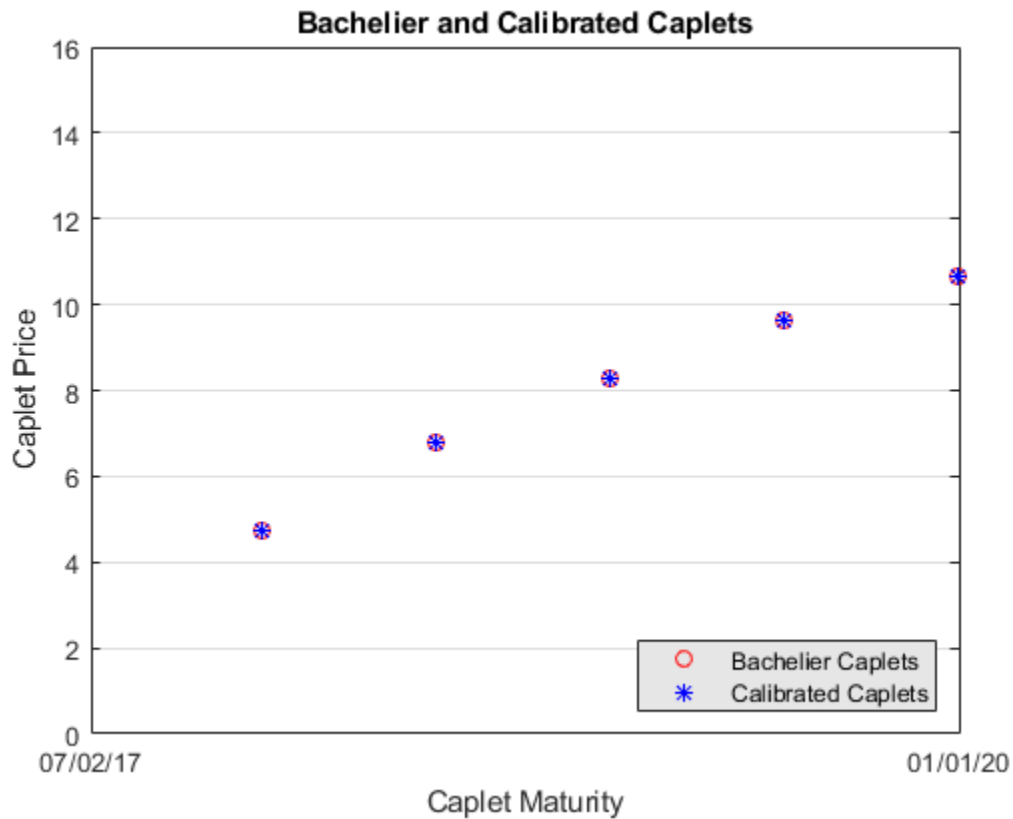
```

```

plot(MarketMatNum(2:end), Caplets, 'or', MarketMatNum(2:end), OptimCaplets, '*b');
datetick('x', 2)
xlabel('Caplet Maturity');
ylabel('Caplet Price');
ylim ([0 16]);

```

```
title('Bachelier and Calibrated Caplets');  
h = legend('Bachelier Caplets', 'Calibrated Caplets');  
set(h, 'color', [0.9 0.9 0.9]);  
set(h, 'Location', 'SouthEast');  
set(gcf, 'NumberTitle', 'off')  
grid on
```



## Input Arguments

**RateSpec** — Interest-rate specification for initial rate curve structure

Interest-rate specification for initial rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**MarketStrike — Market cap strike**

vector

Market cap strike, specified as a NINST-by-1 vector.

Data Types: `double`

**MarketMaturity — Market cap maturity date**

vector

Market cap maturity dates, specified as a NINST-by-1 vector.

Data Types: `double`

**MarketVolatility — Market flat volatilities**

matrix

Market flat volatilities, specified as a NSTRIKES-by-NMATS matrix of market flat volatilities, where NSTRIKES is the number of caplet strikes from `MarketStrike` and NMATS is the caplet maturity dates from `MarketMaturity`.

Data Types: `double`

**Strike — Rate at which cap is exercised**

decimal scalar

(Optional) Rate at which the cap is exercised, specified as a decimal scalar value.

Data Types: `single`

**Settle — Settlement date of the cap**

serial date number | date character vector

(Optional) Settlement date of the cap, specified as a scalar serial date number or date character.

Data Types: `single` | `char`

**Maturity — Maturity date of the cap**

serial date number | date character vector

(optional) Maturity date of the cap, specified as scalar serial date number or date character vector.

Data Types: `single` | `char`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `[Alpha,Sigma,OptimOut] = hwcalbycap(RateSpec,MarketStrike,MarketMaturity,MarketVolatility,'Reset',2,'Principal',100000,'Basis',3,'OptimOptions',o)`

### Reset — Frequency of payments per year

1 (default) | numeric

Frequency of payments per year, specified as the comma-separated pair consisting of `'Reset'` and a scalar numeric value.

Data Types: `double`

### Principal — Notional principal amount

100 (default) | nonnegative integer

Notional principal amount, specified as the comma-separated pair consisting of `'Principal'` and a scalar nonnegative integer.

Data Types: `single`

### Basis — Day-count basis used when annualizing the input forward rate

0 (actual/actual) (default) | integers of the set [0...13]

Day-count basis used when annualizing the input forward rate, specified as the comma-separated pair consisting of `'Basis'` and a scalar value. Values are:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365

- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: single

### **LB — Lower bounds**

[0;0] (default) | numeric vector

Lower bounds, specified as the comma-separated pair consisting of 'LB' and a 2-by-1 vector of the lower bounds, defined as [LBSigma; LBAalpha], used in the search algorithm function. For more information, see `lsqnonlin`.

Data Types: double

### **UB — Upper bounds**

[ ] (unbound) (default) | numeric vector

Upper bounds, specified as the comma-separated pair consisting of 'UB' and a 2-by-1 vector of the upper bounds, defined as [UBSigma; LBAalpha], used in the search algorithm function. For more information, see `lsqnonlin`.

Data Types: double

### **X0 — Initial values**

[0.5;0.5] (default) | numeric vector

Initial values, specified as the comma-separated pair consisting of 'X0' and a 2-by-1 vector of the initial values, defined as [Sigma0; Alpha0], used in the search algorithm function. For more information, see `lsqnonlin`.

Data Types: double



**OptimOptions — Optimization parameters**

structure

Optimization parameters, specified as the comma-separated pair consisting of 'OptimOptions' and a structure defined by using `optimoptions`.

Data Types: `struct`**Shift — Shift in decimals for shifted Black model**

0 (no shift) (default) | positive decimal

Shift in decimals for the shifted Black model, specified as the comma-separated pair consisting of 'Shift' and a scalar positive decimal value. Set this parameter to a positive shift in decimals to add a positive shift to forward rate and `Strike`, which effectively sets a negative lower bound for forward rate and `Strike`. For example, a `Shift` value of `0.01` is equal to a 1% shift.

Data Types: `single`**Model — Indicator for model used for calibration routine**`lognormal` (Black model) (default) | values are `normal` and `lognormal`

Indicator for model used for calibration routine, specified as the comma-separated pair consisting of 'Model' and a scalar character vector with a value of `normal` or `lognormal`.

Data Types: `char`

## Output Arguments

**Alpha — Mean reversion value obtained from calibrating the cap using market information**

scalar numeric

Mean reversion value obtained from calibrating the cap using market information, returned as a scalar value.

**Sigma — Volatility value obtained from calibrating cap using market information**

scalar numeric

Volatility value obtained from calibrating the cap using market information, returned as a scalar.

**OptimOut — Optimization results**

numeric structure

Optimization results, returned as a structure.

**See Also**

HullWhite1F | capbyblk | hwcalbyfloor | hwtree | lsqnonlin

**Topics**

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2009a**

# hwcalbyfloor

Calibrate Hull-White tree using floors

## Syntax

```
[Alpha,Sigma,OptimOut] = hwcalbyfloor(RateSpec,MarketStrike
MarketMaturity,MarketVolatility)
[Alpha,Sigma,OptimOut = hwcalbyfloor(RateSpec,MarketStrike
MarketMaturity,MarketVolatility,Strike,Settle,Maturity)
[Alpha,Sigma,OptimOut] = hwcalbyfloor( ____,Name,Value)
```

## Description

[Alpha,Sigma,OptimOut] = hwcalbyfloor(RateSpec,MarketStrike MarketMaturity,MarketVolatility) calibrates the Alpha (mean reversion) and Sigma (volatility) using floor market data and the Hull-White model using the entire floor surface.

The Hull-White calibration functions (hwcalbyfloor and hwcalbycap) support three models: Black (default), Bachelier or Normal, and Shifted Black. For more information, see the optional arguments for Shift and Model.

[Alpha,Sigma,OptimOut = hwcalbyfloor(RateSpec,MarketStrike MarketMaturity,MarketVolatility,Strike,Settle,Maturity) estimates the Alpha (mean reversion) and Sigma (volatility) using floor market data and the Hull-White model to price a floor at a particular maturity/volatility using the additional optional input arguments for Strike, Settle, and Maturity.

Strike, Settle, and Maturity arguments are specified to calibrate to a specific point on the market volatility surface. If omitted, the calibration is performed across all the market instruments

For an example of calibrating using the Hull-White model with Strike, Settle, and Maturity input arguments, see “Calibrating Hull-White Model Using Market Data” on page 2-111.

[Alpha, Sigma, OptimOut] = hwcalbyfloor( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

## Examples

### Calibrate Hull-White Model from Market Data Using the Entire Floor Volatility Surface

This example shows how to use hwcalbyfloor input arguments for MarketStrike, MarketMaturity, and MarketVolatility to calibrate the HW model using the entire floor volatility surface.

Floor market volatility data covering two strikes over 12 maturity dates.

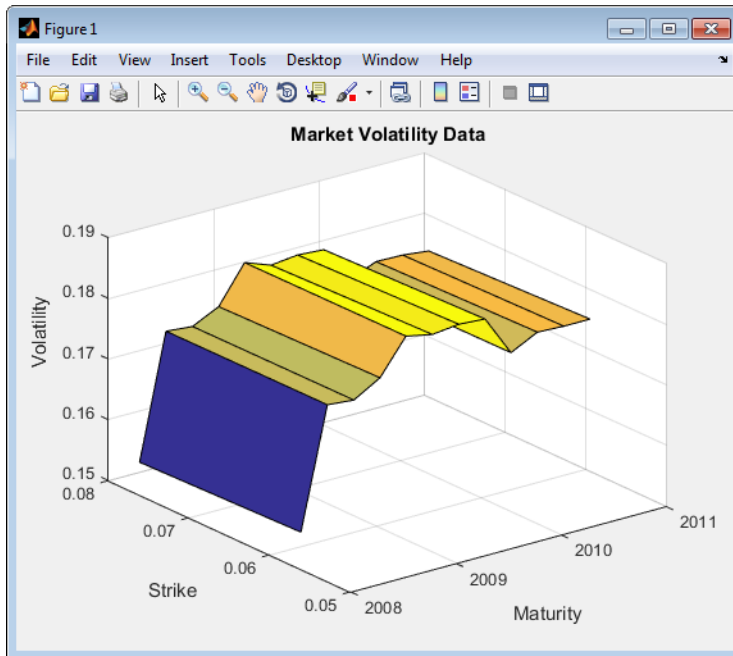
```
Reset = 4;
MarketStrike = [0.0590; 0.0790];

MarketMaturity = {'21-Mar-2008'; '21-Jun-2008'; '21-Sep-2008'; '21-Dec-2008';
                 '21-Mar-2009'; '21-Jun-2009'; '21-Sep-2009'; '21-Dec-2009';
                 '21-Mar-2010'; '21-Jun-2010'; '21-Sep-2010'; '21-Dec-2010'};
MarketMaturity = datenum(MarketMaturity);

MarketVolatility = [0.1533 0.1731 0.1727 0.1752 0.1809 0.1800 0.1805 0.1802...
                   0.1735 0.1757 0.1755 0.1755;
                   0.1526 0.1730 0.1726 0.1747 0.1808 0.1792 0.1797 0.1794...
                   0.1733 0.1751 0.1750 0.1745];
```

Plot market volatility surface.

```
[AllMaturities, AllStrikes] = meshgrid(MarketMaturity, MarketStrike);
figure;
surf(AllMaturities, AllStrikes, MarketVolatility)
datetick
xlabel('Maturity')
ylabel('Strike')
zlabel('Volatility')
title('Market Volatility Data')
```



Set interest rate term structure and create a RateSpec.

```
Settle = '21-Jan-2008';
Compounding = 4;
Basis = 0;
Rates = [0.0627; 0.0657; 0.0691; 0.0717; 0.0739; 0.0755; 0.0765; 0.0772;
         0.0779; 0.0783; 0.0786; 0.0789];
EndDates = {'21-Mar-2008'; '21-Jun-2008'; '21-Sep-2008'; '21-Dec-2008'; ...
            '21-Mar-2009'; '21-Jun-2009'; '21-Sep-2009'; '21-Dec-2009'; ...
            '21-Mar-2010'; '21-Jun-2010'; '21-Sep-2010'; '21-Dec-2010'};
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
                    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, ...
                    'Basis', Basis)
```

RateSpec =

```
FinObj: 'RateSpec'
Compounding: 4
Disc: [12x1 double]
Rates: [12x1 double]
EndTimes: [12x1 double]
StartTimes: [12x1 double]
EndDates: [12x1 double]
StartDates: 733428
```

```
ValuationDate: 733428
              Basis: 0
              EndMonthRule: 1
```

### Calibrate Hull-White model from market data.

```
o = optimoptions('lsqnonlin','TolFun',1e-5,'Display','off');
[Alpha, Sigma] = hwalbyfloor(RateSpec, MarketStrike, MarketMaturity,...
    MarketVolatility, 'Reset', Reset, 'Basis', Basis, 'OptimOptions', o)
Warning: LSQNONLIN did not converge to an optimal solution. It exited with exitflag = 3.
> In hwalbycapfloor>optimizeOverCapSurface at 232
   In hwalbycapfloor at 79
   In hwalbyfloor at 81
```

```
Alpha =
    0.0835
```

```
Sigma =
    0.0145
```

### Compare with Black prices.

```
BlkPrices = floorbyblk(RateSpec, AllStrikes(:), Settle, AllMaturities(:),...
    MarketVolatility(:), 'Reset', Reset, 'Basis', Basis)
```

```
BlkPrices =
    0
    0.2659
    0.0010
    0.5426
    0.0021
    0.6841
    0.0042
    0.7947
    0.0081
    0.8970
    0.0128
    0.9947
    0.0217
    1.1145
    0.0340
    1.2448
    0.0402
    1.3415
    0.0610
```

```

1.4947
0.0827
1.6458
0.1071
1.7951

```

Setup Hull-White tree using calibrated parameters, alpha, and sigma.

```

VolDates    = EndDates;
VolCurve    = Sigma*ones(numel(EndDates),1);
AlphaDates  = EndDates;
AlphaCurve  = Alpha*ones(numel(EndDates),1);
HWVolSpec   = hmwolspec(Settle, VolDates, VolCurve, AlphaDates, AlphaCurve);

HWTimeSpec  = hwtimespec(Settle, EndDates, Compounding);
HWTree      = hwtree(HWVolSpec, RateSpec, HWTimeSpec, 'Method', 'HW2000')

HWTree =
    FinObj: 'HwFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
      tObs: [0 0.6593 1.6612 2.6593 3.6612 4.6593 5.6612 6.6593 7.6612 8.6593 9.6612 10.6593]
      dObs: [733428 733488 733580 733672 733763 733853 733945 734037 734128 734218 734310 734402]
    CFlowT: {1x12 cell}
    Probs: {1x11 cell}
    Connect: {1x11 cell}
    FwdTree: {1x12 cell}

```

Compute Hull-White prices based on the calibrated tree.

```
HWPrices = floorbyhw(HWTree, AllStrikes(:), Settle, AllMaturities(:), Reset, Basis)
```

```

HWPrices =
    0
    0.2644
    0.0067
    0.5404
    0.0101
    0.6924
    0.0169
    0.7974
    0.0236
    0.8919
    0.0320
    0.9919
    0.0460
    1.1074
    0.0649

```

```

1.2340
0.0829
1.3558
0.1096
1.4957
0.1406
1.6418
0.1724
1.7877

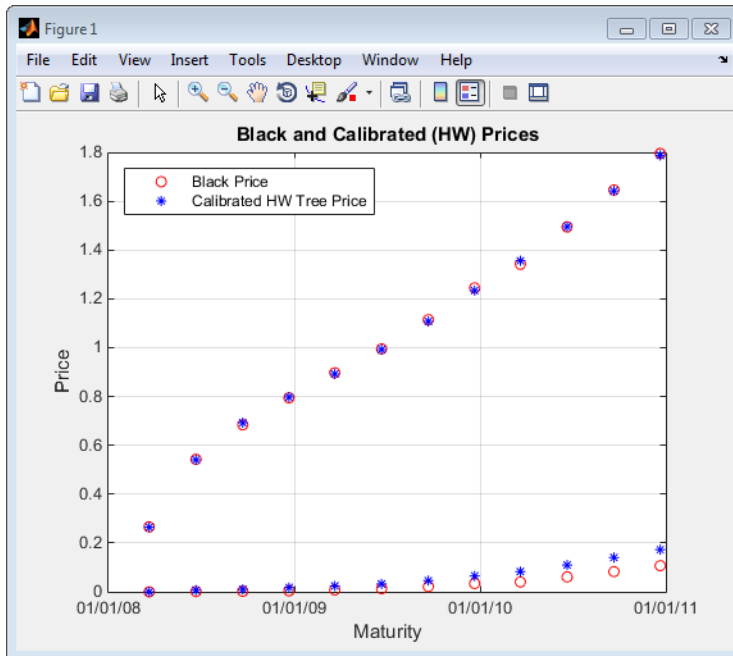
```

Plot Black prices against the calibrated Hull-White tree prices.

```

figure;
plot(AllMaturities(:), BlkPrices, 'or', AllMaturities(:), HWPrices, '*b');
datetick('x', 2)
xlabel('Maturity');
ylabel('Price');
title('Black and Calibrated (HW) Prices');
legend('Black Price', 'Calibrated HW Tree Price','Location', 'NorthWest');
grid on

```





## Calibrating Floorlets Using the Normal (Bachelier) Model

This example shows how to use `hwcalbyfloor` to calibrate market data with the Normal (Bachelier) model to price floorlets. Use the Normal (Bachelier) model to perform calibrations when working with negative interest rates, strikes, and normal implied volatilities.

Consider a floor with these parameters:

```
Settle = 'Dec-30-2016';
Maturity = 'Dec-30-2019';
Strike = -0.004075;
Reset = 2;
Principal = 100;
Basis = 0;
```

The floorlets and market data for this example are defined as:

```
floorletDates = cfdates(Settle, Maturity, Reset, Basis);
datestr(floorletDates')
```

```
ans = 6x11 char array
    '30-Jun-2017'
    '30-Dec-2017'
    '30-Jun-2018'
    '30-Dec-2018'
    '30-Jun-2019'
    '30-Dec-2019'
```

```
% Market data information
```

```
MarketStrike = [-0.00595; 0];
MarketMat = {'30-Jun-2017'; '30-Dec-2017'; '30-Jun-2018'; '30-Dec-2018'; '30-Jun-2019';
MarketVol = [0.184 0.2329 0.2398 0.2467 0.2906 0.3348; % First row in table correspon
             0.217 0.2707 0.2760 0.2814 0.3160 0.3508]; % Second row in table correspon
```

Define the `RateSpec`.

```
Rates= [-0.003210; -0.003020; -0.00182; -0.001343; -0.001075];
ValuationDate = 'Dec-30-2016';
EndDates = {'30-Jun-2017'; 'Dec-30-2017'; '30-Jun-2018'; 'Dec-30-2018'; 'Dec-30-2019'};
Compounding = 2;
Basis = 0;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, ...  
'StartDates', ValuationDate, 'EndDates', EndDates, ...  
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
```

Use `hwcalbyfloor` to find values for the volatility parameters Alpha and Sigma using the Normal (Bachelier) model.

```
format short  
o=optimoptions('lsqnonlin','TolFun',100*eps);  
warning('off','fininst:hwcalbycapfloor:NoConverge')  
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec, MarketStrike, MarketMat,...  
MarketVol, Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal,...  
'Basis', Basis, 'OptimOptions', o, 'model', 'normal')
```

Local minimum possible.

`lsqnonlin` stopped because the size of the current step is less than the value of the step size tolerance.

```
Alpha = 1.0000e-06
```

```
Sigma = 0.3410
```

```
OptimOut = struct with fields:  
    resnorm: 1.9233e-04  
    residual: [5x1 double]  
    exitflag: 2  
    output: [1x1 struct]  
    lambda: [1x1 struct]  
    jacobian: [5x2 double]
```

The `OptimOut.residual` field of the `OptimOut` structure is the optimization residual. This value contains the difference between the Normal (Bachelier) floorlets and those calculated during the optimization. Use the `OptimOut.residual` value to calculate the percentual difference (error) compared to Normal (Bachelier) floorlet prices, and then decide whether the residual is acceptable. There is almost always some residual, so decide if it is acceptable to parameterize the market with a single value of Alpha and Sigma.

Price the floorlets using the market data and Normal (Bachelier) model to obtain the reference floorlet values. To determine the effectiveness of the optimization, calculate reference floorlet values using the Normal (Bachelier) formula and the market data. Note, you must first interpolate the market data to obtain the floorlets for calculation.

```

MarketMatNum = datenum(MarketMat);
[Mats, Strikes] = meshgrid(MarketMatNum, MarketStrike);
FlatVol = interp2(Mats, Strikes, MarketVol, datenum(Maturity), Strike, 'spline');

[FloorPrice, Floorlets] = floorbynormal(RateSpec, Strike, Settle, Maturity, FlatVol,..
'Reset', Reset, 'Basis', Basis, 'Principal', Principal);
Floorlets = Floorlets(2:end)

```

```
Floorlets = 5×1
```

```

    4.7637
    6.7180
    8.1833
    9.5825
   10.6090

```

Compare the optimized values and Normal (Bachelier) values, and display the results graphically. After calculating the reference values for the floorlets, compare the values analytically and graphically to determine whether the calculated single values of Alpha and Sigma provide an adequate approximation.

```
OptimFloorlets = Floorlets+OptimOut.residual;
```

```
disp(' ');
```

```
disp(' Bachelier   Calibrated Floorlets');
```

```
    Bachelier   Calibrated Floorlets
```

```
disp([Floorlets      OptimFloorlets])
```

```

    4.7637    4.7685
    6.7180    6.7263
    8.1833    8.1878
    9.5825    9.5795
   10.6090   10.6007

```

```

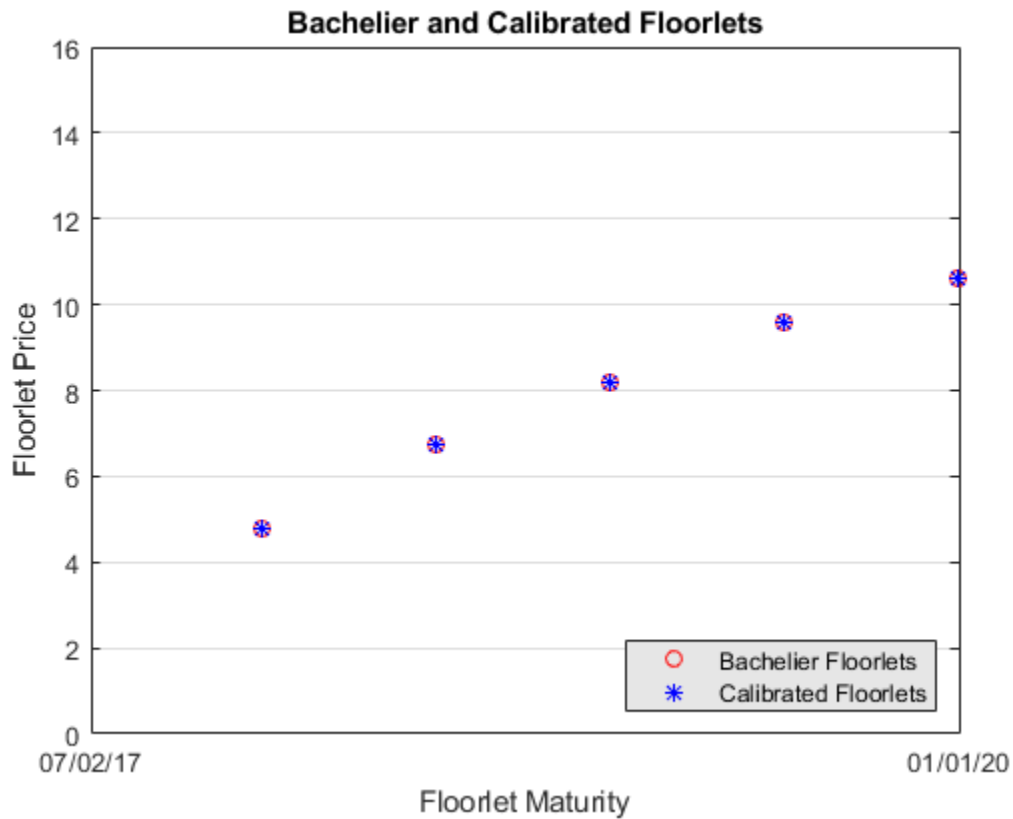
plot(MarketMatNum(2:end), Floorlets, 'or', MarketMatNum(2:end), OptimFloorlets, '*b');
datetick('x', 2)
xlabel('Floorlet Maturity');
ylabel('Floorlet Price');
ylim ([0 16]);

```

```

title('Bachelier and Calibrated Floorlets');
h = legend('Bachelier Floorlets', 'Calibrated Floorlets');
set(h, 'color', [0.9 0.9 0.9]);
set(h, 'Location', 'SouthEast');
set(gcf, 'NumberTitle', 'off')
grid on

```



## Input Arguments

**RateSpec** — Interest-rate specification for initial rate curve structure

Interest-rate specification for initial rate curve, specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**MarketStrike — Market floor strike**

vector

Market floor strike, specified as a NINST-by-1 vector.

Data Types: `double`

**MarketMaturity — Market floor maturity date**

vector

Market floor maturity dates, specified as a NINST-by-1 vector.

Data Types: `double`

**MarketVolatility — Market flat volatilities**

matrix

Market flat volatilities, specified as a NSTRIKES-by-NMATS matrix of market flat volatilities, where NSTRIKES is the number of caplet strikes from `MarketStrike` and NMATS is the caplet maturity dates from `MarketMaturity`.

Data Types: `double`

**Strike — Rate at which floor is exercised**

decimal scalar

(Optional) Rate at which the floor is exercised, specified as a decimal scalar value.

Data Types: `single`

**Settle — Settlement date of the floor**

serial date number | date character vector

(Optional) Settlement date of the floor, specified as a scalar serial date number or date character.

Data Types: `single` | `char`

**Maturity — Maturity date of the floor**

serial date number | date character vector

(optional) Maturity date of the floor, specified as scalar serial date number or date character vector.

Data Types: `single` | `char`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec, MarketStrike, MarketMaturity, MarketVolatility, 'Reset', 2, 'Principal', 100000, 'Basis', 3, 'OptimOptions', o)`

### **Reset** — Frequency of payments per year

1 (default) | numeric

Frequency of payments per year, specified as the comma-separated pair consisting of `'Reset'` and a scalar numeric value.

Data Types: `double`

### **Principal** — Notional principal amount

100 (default) | nonnegative integer

Notional principal amount, specified as the comma-separated pair consisting of `'Principal'` and a scalar nonnegative integer.

Data Types: `single`

### **Basis** — Day-count basis used when annualizing the input forward rate

0 (actual/actual) (default) | integers of the set [0 . . . 13]

Day-count basis used when annualizing the input forward rate, specified as the comma-separated pair consisting of `'Basis'` and a scalar value. Values are:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365

- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: single

### **LB — Lower bounds**

[0;0] (default) | numeric vector

Lower bounds, specified as the comma-separated pair consisting of 'LB' and a 2-by-1 vector of the lower bounds, defined as [LBSigma; LBAalpha], used in the search algorithm function. For more information, see `lsqnonlin`.

Data Types: double

### **UB — Upper bounds**

[ ] (unbound) (default) | numeric vector

Upper bounds, specified as the comma-separated pair consisting of 'UB' and a 2-by-1 vector of the upper bounds, defined as [UBSigma; LBAalpha], used in the search algorithm function. For more information, see `lsqnonlin`.

Data Types: double

### **X0 — Initial values**

[0.5;0.5] (default) | numeric vector

Initial values, specified as the comma-separated pair consisting of 'X0' and a 2-by-1 vector of the initial values, defined as [Sigma0; Alpha0], used in the search algorithm function. For more information, see `lsqnonlin`.

Data Types: double

**OptimOptions — Optimization parameters**

structure

Optimization parameters, specified as the comma-separated pair consisting of 'OptimOptions' and a structure defined by using `optimoptions`.

Data Types: `struct`**Shift — Shift in decimals for shifted Black model**

0 (no shift) (default) | positive decimal

Shift in decimals for the shifted Black model, specified as the comma-separated pair consisting of 'Shift' and a scalar positive decimal value. Set this parameter to a positive shift in decimals to add a positive shift to forward rate and `Strike`, which effectively sets a negative lower bound for forward rate and `Strike`. For example, a `Shift` value of `0.01` is equal to a 1% shift.

Data Types: `single`**Model — Indicator for model used for calibration routine**`lognormal` (Black model) (default) | values are `normal` and `lognormal`

Indicator for model used for calibration routine, specified as the comma-separated pair consisting of 'Model' and a scalar character vector with a value of `normal` or `lognormal`.

Data Types: `char`

## Output Arguments

**Alpha — Mean reversion value obtained from calibrating the floor using market information**

scalar numeric

Mean reversion value obtained from calibrating the floor using market information, returned as a scalar value.

**Sigma — Volatility value obtained from calibrating floor using market information**

scalar numeric

Volatility value obtained from calibrating the floor using market information, returned as a scalar.



**OptimOut — Optimization results**

numeric structure

Optimization results, returned as a structure.

**See Also**

HullWhite1F | floorbyblk | hwcalbycap | hwtree | lsqnonlin

**Topics**

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2009a**

## hwprice

Instrument prices from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = hwprice(HWTree,InstSet)
[Price,PriceTree] = hwprice(____,Options)
```

### Description

[Price,PriceTree] = hwprice(HWTree,InstSet) computes arbitrage-free prices for instruments using an interest-rate tree created with `hwtree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`hwprice` handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See `instadd` to construct defined types.

[Price,PriceTree] = hwprice(\_\_\_\_,Options) adds an optional input argument for `Options`.

### Examples

#### Price the Cap and Bond Instruments Contained in an Instrument Set

Load the HW tree and instruments from the data file `deriv.mat`.

```
load deriv.mat;
HWSubSet = instselect(HWInstSet,'Type', {'Bond', 'Cap'});
```

```
instdisp(HWSubSet)
```

```
instdisp(HWSubSet)
Index Type CouponRate Settle Maturity Period Basis EndMonthRule IssueDate FirstCouponDate LastCouponDate Sta
1 Bond 0.04 01-Jan-2004 01-Jan-2007 1 0 1 NaN NaN NaN NaN
```

---

2	Bond	0.04	01-Jan-2004	01-Jan-2008	1	0	1	NaN	NaN	NaN	NaN
Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal	Name	Quantity		
3	Cap	0.06	01-Jan-2004	01-Jan-2008	1	0	100	6% Cap	10		

Price the cap and bond instruments.

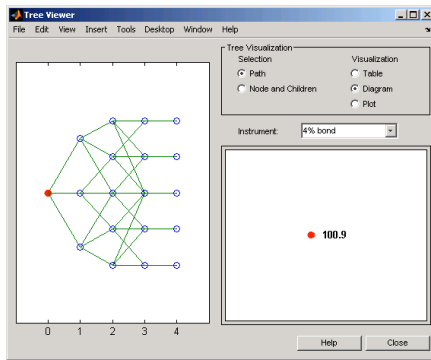
```
[Price, PriceTree] = hwprice(HWTree, HWSubSet);
```

```
100.9188
```

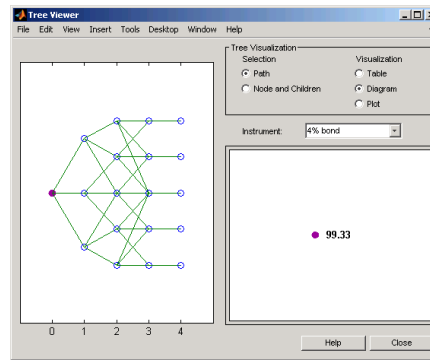
```
99.3296
```

```
0.5837
```

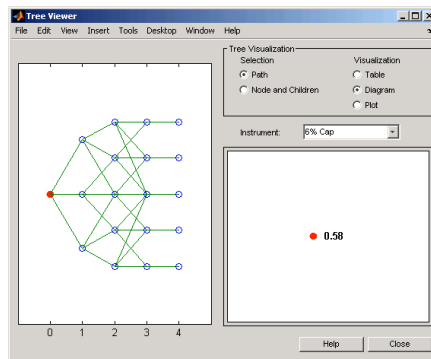
You can use `treeviewer` to see the prices of these three instruments along the price tree.



First 4% Bond (Maturity 2007)



Second 4% Bond (Maturity 2008)



6% Cap

## Price Multi-Stepped Coupon Bonds

The data for the interest-rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
    StartDates: 734139
    ValuationDate: 734139
        Basis: 0
    EndMonthRule: 1
```

Create a portfolio of stepped coupon bonds with different maturities.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}
```

```
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	[Cell]	01-Jan-2010	01-Jan-2011	1	0	1	NaN
2	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
3	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
4	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build the tree with the following data:

```
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```

```
HWVolSpec = hwwolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTTimeSpec)
```

```
HWT = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734139 734504 734869 735235]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0350] [1.0677 1.0494 1.0314] [1x5 double] [1x7 double]}
```

Compute the price of the stepped coupon bonds.

```
PHW = hwprice(HWT, ISet)
```

```
PHW = 4x1
```

```
100.6763
100.7368
100.9266
101.0115
```

### Price a Portfolio of Stepped Callable Bonds and Stepped Vanilla Bonds

The data for the interest-rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create a RateSpec.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
```

```

        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: 734139
ValuationDate: 734139
        Basis: 0
        EndMonthRule: 1

```

Create an instrument portfolio of three stepped callable bonds and three stepped vanilla bonds.

```

Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year

```

Bonds with embedded option.

```

ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike, ...
ExerciseDates, 'Period', 1);

```

Vanilla bonds.

```

ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);

```

Display the instrument portfolio.

```

instdisp(ISet)

```

Index	Type	CouponRate	Settle	Maturity	OptSpec	Strike	ExerciseDates
1	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2012	call	100	01-Jan-2011
2	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2013	call	100	01-Jan-2011
3	OptEmBond	[Cell]	01-Jan-2010	01-Jan-2014	call	100	01-Jan-2011

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
4	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
5	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
6	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build the tree with the following data:

```

VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;

HWVolSpec = hmwolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTTimeSpec)

HWT = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734139 734504 734869 735235]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0350] [1.0677 1.0494 1.0314] [1x5 double] [1x7 double]}

```

Compute the price of the stepped callable bonds and the stepped vanilla bonds.

```
PHW = hwprice(HWT, ISet)
```

```
PHW = 6x1
```

```

100.4089
100.2043
100.0197
100.7368
100.9266
101.0115

```

The first three rows correspond to the price of the stepped callable bonds and the last three rows correspond to the price of the stepped vanilla bonds.

### Compute the Price of a Portfolio of Instruments

The data for the interest-rate term structure is as follows:



```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

```

Create a RateSpec.

```

RS = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

```

```

RS = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x1 double]
    Rates: [4x1 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734504
    ValuationDate: 734504
    Basis: 0
    EndMonthRule: 1

```

Create an instrument portfolio with two range notes and a floating rate note with the following data:

```

Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

```

*% First Range Note*

```

RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];

```

*% Second Range Note*

```

RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates = [0.048 0.059; 0.055 0.068 ; 0.07 0.09];

```

Create InstSet, add a floating-rate note, and display the portfolio instruments.

```

InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);

```

*% Add a floating-rate note*

```
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);
```

```
% Display the portfolio instrument
instdisp(InstSet)
```

Index	Type	Spread	Settle	Maturity	RateSched	FloatReset	Basis	Princ
1	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100
2	RangeFloat	200	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRu
3	Float	200	01-Jan-2011	01-Jan-2014	1	0	100	1

The data to build the tree is as follows:

```
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;
```

```
HWVS = hwvolspec(RS.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
HWTS = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVS, RS, HWTS)
```

```
HWT = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734504 734869 735235 735600]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0350] [1.0677 1.0494 1.0314] [1x5 double] [1x7 double]}
```

Price the portfolio.

```
Price = hwprice(HWT, InstSet)
```

```
Price = 3x1
```

```
99.3327
```

```
98.1580
105.5147
```

## Create a Float-Float Swap and Price with hwprice

Use `instswap` to create a float-float swap and price the swap with `hwprice`.

```
RateSpec = intenvset('Rates',.05,'StartDate',today,'EndDate',datemnth(today,60));
IS = instswap([.02 .03],today,datemnth(today,60),[], [], [], [1 1]);
VolSpec = hwwolspec(today,datemnth(today,60),.01,datemnth(today,60),.1);
TimeSpec = hwtimespec(today,cfdates(today,datemnth(today,60),1));
HWTtree = hwtree(VolSpec,RateSpec,TimeSpec);
hwprice(HWTtree,IS)
```

```
ans =
```

```
-4.3220
```

## Price Multiple Swaps with hwprice

Use `instswap` to create multiple swaps and price the swaps with `hwprice`.

```
RateSpec = intenvset('Rates',.05,'StartDate',today,'EndDate',datemnth(today,60));
IS = instswap([.03 .02],today,datemnth(today,60),[], [], [], [1 1]);
IS = instswap(IS,[200 300],today,datemnth(today,60),[], [], [], [0 0]);
IS = instswap(IS,[.08 300],today,datemnth(today,60),[], [], [], [1 0]);
VolSpec = hwwolspec(today,datemnth(today,60),.01,datemnth(today,60),.1);
TimeSpec = hwtimespec(today,cfdates(today,datemnth(today,60),1));
HWTtree = hwtree(VolSpec,RateSpec,TimeSpec);
hwprice(HWTtree,IS)
```

```
ans =
```

```
4.3220
-4.3220
```

-0.2701

## Input Arguments

### **HWTree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: `struct`

### **InstSet — Instrument variable**

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options — Derivatives pricing options structure**

structure

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## Output Arguments

### **Price — Price for each instrument**

vector

Price for each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

Related single-type pricing functions are:

- `bondbyhw`: Price a bond from a Hull-White tree.

- `capbyhw`: Price a cap from a Hull-White tree.
- `cfbyhw`: Price an arbitrary set of cash flows from a Hull-White tree.
- `fixedbyhw`: Price a fixed-rate note from a Hull-White tree.
- `floatbyhw`: Price a floating-rate note from a Hull-White tree.
- `floorbyhw`: Price a floor from a Hull-White tree.
- `optbndbyhw`: Price a bond option from a Hull-White tree.
- `optembndbyhw`: Price a bond with embedded option by a Hull-White tree.
- `optfloatbybdt`: Price a floating-rate note with an option from a Hull-White tree.
- `optemfloatbybdt`: Price a floating-rate note with an embedded option from a Hull-White tree.
- `rangefloatbyhw`: Price range floating note using a Hull-White tree.
- `swapbyhw`: Price a swap from a Hull-White tree.
- `swaptionbyhw`: Price a swaption from a Hull-White tree.

### **PriceTree — Tree structure of instrument prices**

#### structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## **See Also**

`hwsens` | `hwtree` | `instadd` | `intenvprice` | `intenvsens`

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## **Introduced before R2006a**

# hwsens

Instrument prices and sensitivities from Hull-White interest-rate tree

## Syntax

```
[Delta,Gamma,Vega,Price] = hwsens(HWTree,InstSet)  
[Delta,Gamma,Vega,Price] = hwsens( ____,Options)
```

## Description

[Delta,Gamma,Vega,Price] = hwsens(HWTree,InstSet) computes instrument sensitivities and prices for instruments using an interest-rate tree created with the hwtree function. All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

hwsens handles instrument types: 'Bond', 'CashFlow', 'OptBond', 'OptEmBond', 'OptEmBond', 'OptFloat', 'OptEmFloat', 'Fixed', 'Float', 'Cap', 'Floor', 'RangeFloat', 'Swap'. See instadd for information on instrument types.

[Delta,Gamma,Vega,Price] = hwsens( \_\_\_\_,Options) adds an optional input argument for Options.

## Examples

### Compute Instrument Sensitivities Using an HW Interest-Rate Tree

Load the tree and instruments from the deriv.mat data file. Compute Delta and Gamma for the cap and bond instruments contained in the instrument set.

```
load deriv.mat;  
HWSubSet = instselect(HWInstSet,'Type', {'Bond', 'Cap'});  
  
instdisp(HWSubSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.04	01-Jan-2004	01-Jan-2007	1	0	1	NaN
2	Bond	0.04	01-Jan-2004	01-Jan-2008	1	0	1	NaN

Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal	Name	Quantity
3	Cap	0.06	01-Jan-2004	01-Jan-2008	1	0	100	6% Cap	10

Compute the Delta and Gamma for the cap and bond instruments.

```
[Delta, Gamma] = hwsens(HWTtree, HWSubSet)
```

```
Delta = 3x1
```

```
-291.2580
-374.6368
 60.9580
```

```
Gamma = 3x1
```

```
103 ×
```

```
0.8584
1.4609
5.5994
```

## Input Arguments

### **HWTtree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.



Data Types: `struct`

### **Options — Derivatives pricing options structure**

`structure`

Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Delta — Rate of change of instruments prices with respect to changes in interest rate**

`vector`

Rate of change of instruments prices with respect to changes in the interest rate, returned as a NINST-by-1 vector of deltas. `Delta` is computed by finite differences in calls to `hwtree`.

---

**Note** `Delta` is calculated based on yield shifts of 100 basis points.

---

### **Gamma — Rate of change of instruments deltas with respect to changes in interest rate**

`vector`

Rate of change of instruments deltas with respect to changes in the interest rate, returned as a NINST-by-1 vector of gammas. `Gamma` is computed by finite differences in calls to `hwtree`.

---

**Note** `Gamma` is calculated based on yield shifts of 100 basis points.

---

### **Vega — Rate of change of instruments prices with respect to changes in volatility**

`vector`

Rate of change of instruments prices with respect to changes in the volatility, returned as a NINST-by-1 vector of vegas. Volatility is  $\sigma(t, T)$  of the interest rate. `Vega` is computed by finite differences in calls to `hwtree`. For information on the volatility process, see `hwvolspec`.

---

**Note** Vega is calculated based on 1% shift in the volatility process.

---

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, a NaN is returned in that entry.

### **See Also**

hwprice | hwtree | hwvolspec | instadd

### **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# hwtimespec

Specify time structure for Hull-White interest-rate tree

## Syntax

```
TimeSpec = hwtimespec(ValuationDate,Maturity)
TimeSpec = hwtimespec( ____,Compounding)
```

## Description

`TimeSpec = hwtimespec(ValuationDate,Maturity)` sets the number of levels and node times for a hwtree and determines the mapping between dates and time for rate quoting.

`TimeSpec = hwtimespec( ____,Compounding)` adds the optional argument `Compounding`.

## Examples

### Set the Number of Levels and Node Times for a Hull-White Tree

This example shows how to specify a four-period tree with annual nodes and use annual compounding to report rates.

```
ValuationDate = 'Jan-1-2004';
Maturity = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
Compounding = 1;
TimeSpec = hwtimespec(ValuationDate, Maturity, Compounding)

TimeSpec = struct with fields:
    FinObj: 'HWTimeSpec'
    ValuationDate: 731947
    Maturity: [4x1 double]
```

```
Compounding: 1
           Basis: 0
EndMonthRule: 1
```

## Input Arguments

### **ValuationDate — Pricing date and first observation in the tree**

serial date number | character vector date

Pricing date and first observation in the tree, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### **Maturity — Dates marking the cash flow dates of the tree**

serial date number | date character vector

Dates marking the cash flow dates of the tree, specified as NLEVELS-by-1 vector of serial date numbers or date character vectors. Cash flows with these maturities fall on tree nodes. **Maturity** should be in increasing order.

Data Types: double | char | cell

### **Compounding — Rate at which the input zero rates were compounded when annualized**

1 (default) | integer with value of 1, 2, 3, 4, 6, 12, 365, or -1

(Optional) Rate at which the input zero rates were compounded when annualized, specified as a scalar integer value.

- If **Compounding** = 1, 2, 3, 4, 6, 12:

$Disc = (1 + Z/F)^{-T}$ , where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units; for example,  $T = F$  is one year.

- If **Compounding** = 365:

$Disc = (1 + Z/F)^{-T}$ , where F is the number of days in the basis year and T is a number of days elapsed computed by basis.

- If **Compounding** = -1:

$\text{Disc} = \exp(-T*Z)$ , where T is time in years.

Data Types: double

## Output Arguments

**TimeSpec** — Specification for the time layout for `hwtree` structure

Specification for the time layout for `hwtree`, returned as a structure. The state observation dates are `[ValuationDate; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity(end)`.

## See Also

`hwtree` | `hwvolspec`

## Topics

“Specifying the Time Structure (TimeSpec)” on page 2-85

“Creating Trees” on page 2-87

“Examining Trees” on page 2-88

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## hwtree

Build Hull-White interest-rate tree

### Syntax

```
HWTtree = hwtree(VolSpec,RateSpec,TimeSpec)  
HWTtree = hwtree( ____,Name,Value)
```

### Description

HWTtree = hwtree(VolSpec,RateSpec,TimeSpec) builds a Hull-White interest-rate tree.

HWTtree = hwtree( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Create an HWTtree

Using the data provided, create a Hull-White volatility specification (VolSpec), rate specification (RateSpec), and tree time layout specification (TimeSpec). Then, use these specifications to create a Hull-White tree using hwtree.

```
Compounding = -1;  
ValuationDate = '01-01-2004';  
StartDate = ValuationDate;  
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';  
            '12-31-2007'];  
VolCurve = 0.01;  
AlphaDates = '01-01-2008';  
AlphaCurve = 0.1;  
Rates = [0.0275; 0.0312; 0.0363; 0.0415];  
  
HwVolSpec = hwwolspec(ValuationDate, VolDates, VolCurve,...
```

```

AlphaDates, AlphaCurve);

RateSpec = intenvset('Compounding', Compounding,...
                    'ValuationDate', ValuationDate,...
                    'StartDates', ValuationDate,...
                    'EndDates', VolDates,...
                    'Rates', Rates);

HWTimeSpec = hwtimespec(ValuationDate, VolDates, Compounding);
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)

HWTree = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 0.9973 1.9973 2.9973]
    dObs: [731947 732312 732677 733042]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [3.9973]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0278] [1.0536 1.0356 1.0178] [1x5 double] [1x7 double]}

```

Use `treeview` to observe the tree you have created.

## Input Arguments

### **VolSpec — Volatility process specification**

structure

Volatility process specification, specified using the `VolSpec` obtained from `hvwolspec`. See `hvwolspec` for information on the volatility process.

Data Types: `struct`

### **RateSpec — Interest-rate specification for initial rate curve**

structure

Interest-rate specification for initial rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**TimeSpec — Time layout specification**

structure

Time layout specification, specified using the `TimeSpec` obtained from `hwtimespec`. The `TimeSpec` defines the observation dates of the HW tree and the compounding rule for date to time mapping and price-yield formulas. See `hwtimespec` for information on the tree structure.

Data Types: `struct`**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `HWTree = hwtree(VolSpec, RateSpec, TimeSpec, 'Method', 'HW1996')`**Method — Hull-White method upon which tree-node connectivity algorithm is based**`HW2000` (default) | character vector with values of `HW1996` or `HW2000`

Hull-White method upon which the tree-node connectivity algorithm is based, specified a character vector with a value of `HW1996` or `HW2000`.

---

**Note** `hwtree` supports two tree-node connectivity algorithms. `HW1996` is based on the original paper published in the *Journal of Derivatives*, and `HW2000` is the general version of the algorithm, as specified in the paper published in August 2000.

---

Data Types: `char`**Output Arguments****HWTree — Hull-White interest-rate tree**

structure

Hull-White interest-rate tree, returned as a structure containing time and interest rate information of a trinomial recombining tree.



The `HWTtree` structure returned contains all the information necessary to propagate back any cash flows occurring during the time span of the tree. The main fields of `HWTtree` are:

- `HWTtree.tObs` contains the time factor of each level of the tree.
- `HWTtree.dObs` contains the date of each level of the tree.
- `HWTtree.Probs` contains a cell array of 3-by-N numeric arrays with the up/mid/down probabilities of each node of the tree except for the last level. The cells in the cell array are ordered from root node. The arrays are 3-by-N with the first row corresponding to an up-move, the mid row to a mid-move and so on. Each column of the array represents a node starting from the top node of a given level.
- `HWTtree.Connect` contains a cell array with connectivity information for each node of the tree. The arrangement is similar to `HWTtree.Probs`, with the exception that it has only one row in each cell. The number represents the node in the next level to which the middle branch connects to. The top branch connects to the value above (minus one) and the lower branch connects to the value below (plus one).
- `HWTtree.FwdTree` contains the forward spot rate from one node to the next. The forward spot rate is defined as the inverse of the discount factor.

## References

[1] Hull, J., and A. White. "Using Hull-White Interest Rate Trees." *Journal of Derivatives*. 1996.

[2] Hull, J., and A. White. "*The General Hull-White Model and Super Calibration.*" August 2000.

## See Also

`hwcalbycap` | `hwcalbyfloor` | `hwprice` | `hwtimespec` | `hwvolspec` | `intenvset`

## Topics

"Creating Trees" on page 2-87

"Examining Trees" on page 2-88

"Calibrating Hull-White Model Using Market Data" on page 2-111

"Understanding Interest-Rate Tree Models" on page 2-79

"Pricing Options Structure" on page B-2

"Supported Interest-Rate Instruments" on page 2-2

**Introduced before R2006a**

## hvwolspec

Specify Hull-White interest-rate volatility process

### Syntax

```
VolSpec = hvwolspec(ValuationDate,VolDates,VolCurve,AlphaDates,
AlphaCurve)
VolSpec = hvwolspec(____,InterpMethod)
```

### Description

VolSpec = hvwolspec(ValuationDate,VolDates,VolCurve,AlphaDates,AlphaCurve) creates a structure specifying the volatility for hwtree.

The volatility process is such that the variance of  $r(t + dt) - r(t)$  is defined as follows:  $V = (\text{Volatility}.^2 .* (1 - \exp(-2*\text{Alpha} .* dt))) ./ (2 * \text{Alpha})$ . For more information on using Hull-White interest rate trees, see “Hull-White (HW) and Black-Karasinski (BK) Modeling” on page C-2.

VolSpec = hvwolspec(\_\_\_\_,InterpMethod) adds the optional argument InterpMethod.

### Examples

#### Create a Structure Specifying the Volatility for hwtree

This example shows how to create a Hull-White volatility specification (VolSpec) using the following data.

```
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
```

```
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;

HWVolSpec = hwvolSpec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve)

HWVolSpec = struct with fields:
    FinObj: 'HWVolSpec'
    ValuationDate: 731947
    VolDates: [4x1 double]
    VolCurve: [4x1 double]
    AlphaCurve: 0.1000
    AlphaDates: 733408
    VolInterpMethod: 'linear'
```

## Input Arguments

### **ValuationDate** — Observation date of the investment horizon

serial date number | character vector date

Observation date of the investment horizon, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### **VolDates** — Number of points of yield volatility end dates

serial date number | date character vector

Number of points of yield volatility end dates, specified as a NPOINTS-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **VolCurve** — Yield volatility values

decimal

Yield volatility values, specified as a NPOINTS-by-1 vector of decimal values. The term structure of `VolCurve` is the yield volatility represented by the value of the volatility of the yield from time  $t = 0$  to time  $t + i$ , where  $i$  is any point within the volatility curve.

---

**Note** The number of points in `VolCurve` and `AlphaCurve` do not have to be the same.

---

Data Types: double

**AlphaDates — Mean reversion end dates**

serial date number | date character vector

Mean reversion end dates, specified as a `NPOINTS`-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**AlphaCurve — Positive mean reversion values**

positive decimal

Positive mean reversion values, specified as a `NPOINTS`-by-1 vector of positive decimal values.

---

**Note** The number of points in `VolCurve` and `AlphaCurve` do not have to be the same.

---

Data Types: double

**InterpMethod — Interpolation method**

'linear' (default) | character vector with values supported by `interp1`

(Optional) Interpolation method, specified as a character vector with values supported by `interp1`.

Data Types: char

## Output Arguments

**VolSpec — Specification for the volatility model for `hwtree`**

structure

Structure specifying the volatility model for `hwtree`.

## See Also

hwcalbycap | hwcalbyfloor | interp1

## Topics

“Specifying the Volatility Model (VolSpec)” on page 2-82

“Creating Trees” on page 2-87

“Examining Trees” on page 2-88

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## Introduced before R2006a

# impvbybaw

Calculate implied volatility using Barone-Adesi and Whaley option pricing model

## Syntax

```
Volatility = impvbybaw(RateSpec,StockSpec,Settle,Maturity,OptSpec,  
Strike,OptPrice)  
Volatility = impvbybaw( ____,Name,Value)
```

## Description

`Volatility = impvbybaw(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,OptPrice)` calculates implied volatility using the Barone-Adesi and Whaley option pricing model.

`Volatility = impvbybaw( ____,Name,Value)` adds optional name-value pair arguments.

## Examples

### Compute the Implied Volatility for an American Option Using the Barone-Adesi and Whaley Option Pricing Model

This example shows how to compute implied volatility using the Barone-Adesi and Whaley option pricing model. Consider three American call options with exercise prices of \$100 that expire on July 1, 2017. The underlying stock is trading at \$100 on January 1, 2017 and pays a continuous dividend yield of 10%. The annualized continuously compounded risk-free rate is 10% per annum, and the option prices are \$4.063, \$6.77 and \$9.46. Using this data, calculate the implied volatility of the stock using the Barone-Adesi and Whaley option pricing model.

```
AssetPrice = 100;  
Settle = 'Jan-1-2017';  
Maturity = 'Jul-1-2017';
```

```
Strike = 100;  
DivAmount = 0.1;  
Rate = 0.05;
```

Define the RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...  
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 1)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9753  
    Rates: 0.0500  
    EndTimes: 0.5000  
    StartTimes: 0  
    EndDates: 736877  
    StartDates: 736696  
    ValuationDate: 736696  
    Basis: 1  
    EndMonthRule: 1
```

Define the StockSpec.

```
StockSpec = stockspec(NaN, AssetPrice, {'continuous'}, DivAmount)
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
    Sigma: NaN  
    AssetPrice: 100  
    DividendType: {'continuous'}  
    DividendAmounts: 0.1000  
    ExDividendDates: []
```

Define the American option.

```
OptSpec = {'call'};  
OptionPrice = [4.063;6.77;9.46];
```

Compute the implied volatility for the American option.

```
ImpVol = impvbybaw(RateSpec, StockSpec, Settle, Maturity, OptSpec,...  
Strike, OptionPrice)
```



```
ImpVol = 3×1
    0.1802
    0.2808
    0.3803
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle** — Settlement date

serial date number | date character vector | datetime object

Settlement date for the American option, specified as a NINST-by-1 matrix using a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

### **Maturity** — Maturity date

serial date number | date character vector | datetime object

Maturity date for the American option, specified as a NINST-by-1 matrix using a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors or string arrays with values 'call' or 'put'.

Data Types: `char` | `string`

### **Strike — American option strike price value**

nonnegative scalar | nonnegative vector

American option strike price value, specified as a nonnegative scalar or NINST-by-1 matrix of strike price values. Each row is the schedule for one option.

Data Types: `single` | `double`

### **OptPrice — American option price**

nonnegative scalar | nonnegative vector

American option prices from which the implied volatility of the underlying asset is derived, specified as a nonnegative scalar or NINST-by-1 matrix of strike price values.

Data Types: `single` | `double`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Volatility = impvbybaw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptionPrice)`

### **Limit — Lower and upper bound of implied volatility search interval**

[0.1 10] (or 10% to 1000% per annum) (default) | positive value

Lower and upper bound of implied volatility search interval, specified as the comma-separated pair consisting of 'Limit' and a 1-by-2 positive vector.

Data Types: double

### **Tolerance — Implied volatility search termination tolerance**

1e-6 (default) | positive scalar

Implied volatility search termination tolerance, specified as the comma-separated pair consisting of 'Tolerance' and a positive scalar.

Data Types: double

## **Output Arguments**

### **Volatility — Expected implied volatility values**

matrix

Expected implied volatility values, returned as a NINST-by-1 matrix. If no solution can be found, a NaN is returned.

## **References**

- [1] Barone-Adesi, G. and Robert E. Whaley. "Efficient Analytic Approximation of American Option Values." *The Journal of Finance*. Volume 42, Issue 2 (June 1987), 301-320.
- [2] Haug, E. *The Complete Guide to Option Pricing Formulas. Second Edition*. McGraw-Hill Education, January 2007.

## **See Also**

optstockbybaw | optstocksensbybaw

## **Topics**

"Supported Equity Derivatives" on page 3-24

**Introduced in R2017a**

## impvbybjs

Determine implied volatility using Bjerksund-Stensland 2002 option pricing model

### Syntax

```
Volatility = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec,  
Strike, OptPrice)  
Volatility = impvbybjs( ____, Name, Value)
```

### Description

`Volatility = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice)` computes implied volatility using the Bjerksund-Stensland 2002 pricing model.

---

**Note** `impvbybjs` computes implied volatility of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model.

---

`Volatility = impvbybjs( ____, Name, Value)` adds optional name-value pair arguments.

### Examples

#### Compute the Implied Volatility Using the Bjerksund-Stensland 2002 Option Pricing Model

This example shows how to compute implied volatility using the Bjerksund-Stensland 2002 option pricing model. Consider three American call options with exercise prices of \$100 that expire on July 1, 2008. The underlying stock is trading at \$100 on January 1, 2008 and pays a continuous dividend yield of 10%. The annualized continuously compounded risk-free rate is 10% per annum and the option prices are \$4.063, \$6.77 and

\$9.46. Using this data, calculate the implied volatility of the stock using the Bjerksund-Stensland 2002 option pricing model.

```

AssetPrice = 100;
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Strike = 100;
DivAmount = 0.1;
Rate = 0.1;

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 1);

StockSpec = stockspec(NaN, AssetPrice, {'continuous'}, DivAmount);

OptSpec = {'call'};
OptionPrice = [4.063;6.77;9.46];

ImpVol = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, OptionPrice)

ImpVol = 3×1

    0.1500
    0.2501
    0.3500

```

The implied volatility is 15% for the first call, and 25% and 35% for the second and third call options.

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`**Settle — Settlement date**

serial date number | date character vector

Settlement date, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: `double` | `char`**Maturity — Maturity date**

serial date number | date character vector

Maturity date for the American option, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: `double` | `char`**OptSpec — Definition of option**

character vector with values 'call' or 'put'

Definition of the option from which the implied volatility is derived, specified as a NINST-by-1 cell array of character vectors with a value of 'call' or 'put'.

Data Types: `char` | `cell`**Strike — Option strike price value**

nonnegative scalar | nonnegative vector

Option strike price value, specified as a nonnegative scalar or NINST-by-1 vector of strike price values. Each row is the schedule for one option.

Data Types: `double`

**OptPrice — American option price**

nonnegative scalar | nonnegative vector

American option prices from which the implied volatility of the underlying asset is derived, specified as a nonnegative scalar or NINST-by-1 vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Volatility = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice, 'Limit', [0.2 20], 'Tolerance', 1e-5)`

**Limit — Lower and upper bound of implied volatility search interval**

[0.1 10] (10% to 1000% per annum) (default) | positive vector

Lower and upper bound of implied volatility search interval, specified as the comma-separated pair consisting of `'Limit'` and a 1-by-2 positive vector.

Data Types: double

**Tolerance — Implied volatility search termination tolerance**

1e-6 (default) | positive scalar

Implied volatility search termination tolerance, specified as the comma-separated pair consisting of `'Tolerance'` and a positive scalar.

Data Types: double

**Output Arguments****Volatility — Expected implied volatility values**

vector

Expected implied volatility values, returned as a NINST-by-1 vector. If no solution can be found, a NaN is returned.

## References

- [1] Bjerksund, P. and G. Stensland. "Closed-Form Approximation of American Options." *Scandinavian Journal of Management*. Vol. 9, 1993, Suppl., pp. S88–S99.
- [2] Bjerksund, P. and G. Stensland. "Closed Form Valuation of American Options." Discussion paper 2002 (<https://www.scribd.com/doc/215619796/Closed-form-Valuation-of-American-Options-by-Bjerksund-and-Stensland#scribd>)

## See Also

optstockbybjs | optstocksensbybjs

## Topics

"Equity Derivatives Using Closed-Form Solutions" on page 3-150  
"Pricing Using the Bjerksund-Stensland Model" on page 3-158  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2008b**



# impvbyblk

Determine implied volatility using Black option pricing model

## Syntax

```
Volatility = impvbyblk(RateSpec,StockSpec,Settle,Maturity,OptSpec,  
Strike,OptPrice)  
Volatility = impvbyblk( ____,Name,Value)
```

## Description

`Volatility = impvbyblk(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,OptPrice)` computes implied volatility using the Black option pricing model.

`Volatility = impvbyblk( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute the Implied Volatility Using the Black Option Pricing Model

This example shows how to compute the implied volatility using the Black option pricing model. Consider a European call and put options on a futures contract with exercise prices of \$30 for the put and \$40 for the call that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at \$35. The annualized continuously compounded risk-free rate is 5% per annum. Find the implied volatilities of the stock, if on that date, the call price is \$1.14 and the put price is \$0.82.

```
AssetPrice = 35;  
Strike = [30; 40];  
Rates = 0.05;  
Settle = 'May-01-08';  
Maturity = 'Sep-01-08';
```

```
% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(NaN, AssetPrice);

% define the options
OptSpec = {'put';'call'};

Price = [1.14;0.82];
Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, Price, 'Method', 'jackel2016')

Volatility = 2×1

    0.4052
    0.3021
```

The implied volatility is 41% and 30%.

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: struct

**Settle — Settlement date**

serial date number | date character vector

Settlement date, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date for the American option, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: double | char

**OptSpec — Definition of option**

character vector with values 'call' or 'put'

Definition of the option from which the implied volatility is derived, specified as a NINST-by-1 cell array of character vectors with a value of 'call' or 'put'.

Data Types: char | cell

**Strike — Option strike price**

nonnegative scalar | nonnegative vector

Option strike price value, specified as a nonnegative scalar or NINST-by-1 vector of strike prices. Each row is the schedule for one option.

Data Types: double

**OptPrice — European option price**

nonnegative scalar | nonnegative vector

European option prices from which the implied volatility of the underlying asset is derived, specified as a nonnegative scalar or NINST-by-1 vector.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice, 'Limit', 5, 'Tolerance', 1e-5)`

### **Limit** — Upper bound of implied volatility search interval

10 (1000% per annum) (default) | positive value

Upper bound of implied volatility search interval, specified as the comma-separated pair consisting of `'Limit'` and a positive scalar.

---

**Note** If you are using `Method` with a value of `'jackel2016'`, the `Limit` argument is ignored.

---

Data Types: double

### **Tolerance** — Implied volatility search termination tolerance

1e-6 (default) | positive scalar

Implied volatility search termination tolerance, specified as the comma-separated pair consisting of `'Tolerance'` and a positive scalar.

---

**Note** If you are using `Method` with a value of `'jackel2016'`, the `Tolerance` argument is ignored.

---

Data Types: double

### **Method** — Method for computing implied volatility

`'jackel2016'` (default) | character vector with values `'search'` or `'jackel2016'` | string with values `"search"` or `"jackel2016"`

Method for computing implied volatility, specified as the comma-separated pair consisting of 'Method' and a character vector with a value of 'search' or 'jackel2016' or a string with a value of "search" or "jackel2016".

Data Types: char | string

## Output Arguments

### **Volatility** – Expected implied volatility values

vector

Expected implied volatility values, returned as a NINST-by-1 vector. If no solution can be found, a NaN is returned.

## References

[1] Jäckel, Peter. "Let's Be Rational." *Wilmott Magazine.*, January, 2015 (<https://onlinelibrary.wiley.com/doi/pdf/10.1002/wilm.10395>).

## See Also

optstockbyblk | optstocksensbyblk

## Topics

"Equity Derivatives Using Closed-Form Solutions" on page 3-150

"Pricing Using the Black Model" on page 3-156

"Black Model" on page 3-151

"Supported Equity Derivatives" on page 3-24

**Introduced in R2008b**

## impvbybls

Determine implied volatility using Black-Scholes option pricing model

### Syntax

```
Volatility = impvbybls(RateSpec,StockSpec,Settle,Maturity,OptSpec,  
Strike,OptPrice)  
Volatility = impvbybls( ____,Name,Value)
```

### Description

`Volatility = impvbybls(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,OptPrice)` computes implied volatility using the Black-Scholes option pricing model.

`Volatility = impvbybls( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Implied Volatility Using the Black-Scholes Option Pricing Model

This example shows how to compute the implied volatility using the Black-Scholes option pricing model. Consider a European call and put options with an exercise price of \$40 that expires on June 1, 2008. The underlying stock is trading at \$45 on January 1, 2008 and the risk-free rate is 5% per annum. The option price is \$7.10 for the call and \$2.85 for the put. Using this data, calculate the implied volatility of the European call and put using the Black-Scholes option pricing model.

```
AssetPrice = 45;  
Settlement = 'Jan-01-2008';  
Maturity = 'June-01-2008';  
Strike = 40;  
Rates = 0.05;
```

```

OptionPrice = [7.10; 2.85];
OptSpec = {'call'; 'put'};

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settlement, 'StartDates', Settlement, ...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);

StockSpec = stockspec(NaN, AssetPrice);

ImpvVol = impvbybls(RateSpec, StockSpec, Settlement, Maturity, OptSpec, ...
    Strike, OptionPrice, 'Method', 'jackel2016')

ImpvVol = 2×1

    0.3175
    0.4878

```

The implied volatility is 31.75% for the call and 48.78% for the put.

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: struct

### StockSpec — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: struct

**Settle — Settlement date**

serial date number | date character vector

Settlement date, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date for the American option, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: double | char

**OptSpec — Definition of option**

character vector with values 'call' or 'put'

Definition of the option from which the implied volatility is derived, specified as a NINST-by-1 cell array of character vectors with a value of 'call' or 'put'.

Data Types: char | cell

**Strike — Option strike price**

nonnegative scalar | nonnegative vector

Option strike price value, specified as a nonnegative scalar or NINST-by-1 vector of strike prices. Each row is the schedule for one option.

Data Types: double

**OptPrice — European option price**

nonnegative scalar | nonnegative vector

European option prices from which the implied volatility of the underlying asset is derived, specified as a nonnegative scalar or NINST-by-1 vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.



You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

```
Example: Volatility =  
impvbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice  
, 'Limit', 5, 'Tolerance', 1e-5)
```

**Limit — Upper bound of implied volatility search interval**

10 (1000% per annum) (default) | positive value

Upper bound of implied volatility search interval, specified as the comma-separated pair consisting of 'Limit' and a positive scalar.

---

**Note** If you are using Method with a value of 'jackel2016', the Limit argument is ignored.

---

Data Types: double

**Tolerance — Implied volatility search termination tolerance**

1e-6 (default) | positive scalar

Implied volatility search termination tolerance, specified as the comma-separated pair consisting of 'Tolerance' and a positive scalar.

---

**Note** If you are using Method with a value of 'jackel2016', the Tolerance argument is ignored.

---

Data Types: double

**Method — Method for computing implied volatility**

'jackel2016' (default) | character vector with values 'search' or 'jackel2016' | string with values "search" or "jackel2016"

Method for computing implied volatility, specified as the comma-separated pair consisting of 'Method' and a character vector with a value of 'search' or 'jackel2016' or a string with a value of "search" or "jackel2016".

Data Types: char | string

## Output Arguments

### **Volatility — Expected implied volatility values**

vector

Expected implied volatility values, returned as a NINST-by-1 vector. If no solution can be found, a NaN is returned.

## References

[1] Jäckel, Peter. "Let's Be Rational." *Wilmott Magazine.*, January, 2015 (<https://onlinelibrary.wiley.com/doi/pdf/10.1002/wilm.10395>).

## See Also

optstockbybls | optstocksensbybls

## Topics

"Equity Derivatives Using Closed-Form Solutions" on page 3-150

"Pricing Using the Black-Scholes Model" on page 3-154

"Pricing European Call Options Using Different Equity Models"

"Supported Equity Derivatives" on page 3-24

**Introduced in R2008b**

## impvbyrgw

Determine implied volatility using Roll-Geske-Whaley option pricing model for American call option

### Syntax

```
Volatility = impvbyrgw(RateSpec,StockSpec,Settle,Maturity,Strike,  
OptPrice)  
Volatility = impvbyrgw( ____,Name,Value)
```

### Description

Volatility = impvbyrgw(RateSpec,StockSpec,Settle,Maturity,Strike,OptPrice) computes implied volatility using Roll-Geske-Whaley option pricing model for American call option.

---

**Note** impvbyrgw computes implied volatility of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model.

---

Volatility = impvbyrgw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Implied Volatility Using the Roll-Geske-Whaley Option Pricing Model

This example shows how to compute the implied volatility using the Roll-Geske-Whaley option pricing model. Assume that on July 1, 2008 a stock is trading at \$13 and pays a single cash dividend of \$0.25 on November 1, 2008. The American call option with a strike price of \$15 expires on July 1, 2009 and is trading at \$1.346. The annualized continuously compounded risk-free rate is 5% per annum. Calculate the implied volatility of the stock using the Roll-Geske-Whaley option pricing model.

```
AssetPrice = 13;
Strike = 15;
Rates = 0.05;
Settle = 'July-01-08';
Maturity = 'July-01-09';

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(NaN, AssetPrice, {'cash'}, 0.25, {'Nov 1,2008'});

Price = [1.346];
Volatility = impvbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike, Price)

Volatility = 0.3539
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types: struct

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see stockspec.

stockspec handles several types of underlying assets. For example, for physical commodities the price is StockSpec.Asset, the volatility is StockSpec.Sigma, and the convenience yield is StockSpec.DividendAmounts.

Data Types: struct

### **Settle** — Settlement date

serial date number | date character vector

Settlement date, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: double | char

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date for the American option, specified as a NINST-by-1 vector of serial date numbers or a date character vectors.

Data Types: double | char

### **Strike — Option strike price value**

nonnegative scalar | nonnegative vector

Option strike price value, specified as a nonnegative scalar or NINST-by-1 vector of strike price values. Each row is the schedule for one option.

Data Types: double

### **OptPrice — American option price**

nonnegative scalar | nonnegative vector

American option prices from which the implied volatility of the underlying asset is derived, specified as a nonnegative scalar or NINST-by-1 vector.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: Volatility =
impvbyrgw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice
, 'Limit', 5, 'Tolerance', 1e-5)
```

### **Limit — Upper bound of implied volatility search interval**

10 (1000% per annum) (default) | positive value

Upper bound of implied volatility search interval, specified as the comma-separated pair consisting of 'Limit' and a positive scalar.

Data Types: double

### **Tolerance — Implied volatility search termination tolerance**

1e-6 (default) | positive scalar

Implied volatility search termination tolerance, specified as the comma-separated pair consisting of 'Tolerance' and a positive scalar.

Data Types: double

## **Output Arguments**

### **Volatility — Expected implied volatility values**

vector

Expected implied volatility values, returned as a NINST-by-1 vector. If no solution can be found, a NaN is returned.

## **See Also**

optstockbyrgw | optstocksensbyrgw

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing Using the Roll-Geske-Whaley Model” on page 3-157

“Roll-Geske-Whaley Model” on page 3-152

“Supported Equity Derivatives” on page 3-24

**Introduced in R2008b**

# instadd

Add types to instrument collection

## Syntax

```

InstSet = instadd(InstSetOld,TypeString,Data1,Data2, ...)
InstSet = instadd('CashFlow',CFlowAmounts,CFlowDates,Settle,Basis)
InstSet = instadd('CashFlow',CFlowAmounts,CFlowDates,Settle,Basis)
InstSet =
instadd('Barrier',OptSpec,Strike,Settle,ExerciseDates,AmericanOpt,BarrierType,Barrier,Rebate)
InstSet =
instadd('Bond',CouponRate,Settle,Maturity,Period,Basis,EndMonthRule,IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face)
InstSet =
instadd('CBond',CouponRate,Settle,Maturity,ConvRatio'CallStrike',CallStrike,'CallExDates',CallExDates,'AmericanCall',AmericanCall,'PutStrike',PutStrike,
'PutExDates',PutExDates,'AmericanPut',AmericanPut,'Period',Period,'Face',Face,'Spread',Spread,'IssueDate',IssueDate,'FirstCouponDate',FirstCouponDate,'LastCouponDate',LastCouponDate,'StartDate',StartDate)
InstSet =
instadd('OptEmBond',CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates,'AmericanOpt',AmericanOpt,'Period',Period,'Basis',Basis,'EndMonthRule',EndMonthRule,'Face',Face,
'IssueDate',IssueDate,'FirstCouponDate',FirstCouponDate,'LastCouponDate',LastCouponDate,'StartDate',StartDate)
InstSet =
instadd('OptBond',BondIndex,OptSpec,Strike,ExerciseDates,AmericanOpt)
InstSet =
instadd('Cap',Strike,Settle,Maturity,Reset,Basis,Principal)
InstSet =
instadd('Compound',UOptSpec,UStrike,USettle,UExerciseDates,UAmericanOpt,COptSpec,CStrike,CSettle,CExerciseDates,CAmericanOpt)

```

```
InstSet =  
instadd('Fixed', CouponRate, Settle, Maturity, Reset, Basis, Principal, End  
MonthRule)  
InstSet =  
instadd('Float', Spread, Settle, Maturity, Reset, Basis, Principal, EndMont  
hRule, CapRate, FloorRate)  
InstSet =  
instadd('Floor', Strike, Settle, Maturity, Reset, Basis, Principal)  
InstSet =  
instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)  
InstSet =  
instadd('OptFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)  
InstSet =  
instadd('OptEmFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt  
, Reset, Basis, EndMonthRule, Principal)  
InstSet =  
instadd('RangeFloat', Spread, Settle, Maturity, RateSched, Reset, Basis, Pr  
incipal, EndMonthRule)  
InstSet =  
instadd('OptStock', OptSpec, Strike, Settle, Maturity, AmericanOpt)  
InstSet =  
instadd('Swap', LegRate, Settle, Maturity, LegReset, Basis, Principal, LegT  
ype, EndMonthRule, StartDate)  
InstSet =  
instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, Settle, Maturi  
ty, AmericanOpt, SwapReset, Basis, Principal)
```

## Description

`InstSet = instadd(InstSetOld, TypeString, Data1, Data2, ...)` adds an instrument to an existing collection.

`InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)` adds an arbitrary cash flow instrument. (See also `instcf`.)

`InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)` adds an asian instrument. (See also `instasian`.)



InstSet =  
instadd('Barrier', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierType, Barrier, Rebate) adds a barrier instrument. (See also instbarrier.)

InstSet =  
instadd('Bond', CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) adds a bond instrument. (See also instbond.)

InstSet =  
instadd('CBond', CouponRate, Settle, Maturity, ConvRatio, CallStrike, CallStrike, 'CallExDates', CallExDates, 'AmericanCall', AmericanCall, 'PutStrike', PutStrike, 'PutExDates', PutExDates, 'AmericanPut', AmericanPut, 'Period', Period, 'Face', Face, 'Spread', Spread, 'IssueDate', IssueDate, 'FirstCouponDate', FirstCouponDate, 'LastCouponDate', LastCouponDate, 'StartDate', StartDate) adds a convertible bond instrument. (See also instcbond.)

InstSet =  
instadd('OptEmBond', CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period', Period, 'Basis', Basis, 'EndMonthRule', EndMonthRule, 'Face', Face, 'IssueDate', IssueDate, 'FirstCouponDate', FirstCouponDate, 'LastCouponDate', LastCouponDate, 'StartDate', StartDate) adds a bond with embedded option instrument. (See also instoptembnd.)

InstSet =  
instadd('OptBond', BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt) adds a bond option instrument. (See also instoptbnd.)

InstSet =  
instadd('Cap', Strike, Settle, Maturity, Reset, Basis, Principal) adds a cap instrument. (See also instcap.)

InstSet =  
instadd('Compound', UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt) adds a compound instrument. (See also instcompound.)

InstSet =  
instadd('Fixed', CouponRate, Settle, Maturity, Reset, Basis, Principal, EndMonthRule) adds a fixed-rate note instrument. (See also instfixed.)

InstSet =  
instadd('Float', Spread, Settle, Maturity, Reset, Basis, Principal, EndMonthRule, CapRate, FloorRate) adds a floating-rate note instrument. (See also instfloat.)

InstSet =  
instadd('Floor', Strike, Settle, Maturity, Reset, Basis, Principal) adds a floor instrument. (See also instfloor.)

InstSet =  
instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) adds a lookback instrument. (See also instlookback.)

InstSet =  
instadd('OptFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt) adds a floating-rate option instrument. (See also instoptfloat.)

InstSet =  
instadd('OptEmFloat', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, Reset, Basis, EndMonthRule, Principal) adds a floating-rate embedded option instrument. (See also instoptemfloat.)

InstSet =  
instadd('RangeFloat', Spread, Settle, Maturity, RateSched, Reset, Basis, Principal, EndMonthRule) adds a range floating note instrument. (See also instrangefloat.)

InstSet =  
instadd('OptStock', OptSpec, Strike, Settle, Maturity, AmericanOpt) adds a stock option instrument. (See also instoptstock.)

InstSet =  
instadd('Swap', LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule, StartDate) adds a swap instrument. (See also instswap.)

InstSet =  
instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal) adds a swaption instrument. (See also instswaption.)

instadd stores instruments of types 'Asian', 'Barrier', 'Bond', 'Cap', 'CashFlow', 'Compound', 'Fixed', 'Float', 'Floor', 'Lookback', 'OptBond',

'OptStock', 'Swap', or 'Swaption'. Financial Instruments Toolbox provides pricing and sensitivity routines for these instruments.

## Input Arguments

### InstSetOld

Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument. For more information on instrument data parameters, see the reference entries for individual instrument types. For example, see `instcap` for additional information on the cap instrument.

## Output Arguments

### InstSet

InstSet is an instrument set variable containing the new input data.

## Examples

### Create a Portfolio with Two Cap Instruments and a 4% Bond

Define the bond:

```
Strike = [0.06; 0.07];
CouponRate = 0.04;
Settle = '06-Feb-2000';
Maturity = '15-Jan-2003';
```

Create a portfolio with two cap instruments and a 4% bond and then display the portfolio:

```
InstSet = instadd('Cap', Strike, Settle, Maturity);
InstSet = instadd(InstSet, 'Bond', CouponRate, Settle, Maturity);
instdisp(InstSet)
```

Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal
1	Cap	0.06	06-Feb-2000	15-Jan-2003	1	0	100

2	Cap	0.07	06-Feb-2000	15-Jan-2003	1	0	100		
Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate	
3	Bond	0.04	06-Feb-2000	15-Jan-2003	2	0	1	NaN	

## See Also

`instaddfield` | `instasian` | `instbarrier` | `instbond` | `instcap` | `instcbond` | `instcf` | `instcompound` | `instdisp` | `instfixed` | `instfloat` | `instfloor` | `instlookback` | `instoptbnd` | `instoptembnd` | `instoptstock` | `instswap` | `instswaption`

## Topics

“Portfolio Creation” on page 1-8  
“Creating Instruments or Properties” on page 1-21  
“Instrument Constructors” on page 1-20

**Introduced before R2006a**

# instaddfield

Add new instruments to instrument collection

## Syntax

```

InstSet =
instaddfield('FieldName',FieldList,'Data',DataList,'Type',TypeString
)
InstSet =
instaddfield('FieldName',FieldList,'FieldClass',ClassList,'Data',DataList,'Type',TypeString)
InstSetNew =
instaddfield(InstSet,'FieldName',FieldList,'Data',DataList,'Type',TypeString)

```

## Arguments

<b>FieldList</b>	Number of fields, specified as a NFIELDS-by-1 cell array of character vectors listing the name of each data field. <b>FieldList</b> cannot be named with the reserved name <b>Type</b> or <b>Index</b> .
<b>DataList</b>	Number of instruments, specified as a NINST-by-M array or NFIELDS-by-1 cell array of data contents for each field. Each row in a data array corresponds to a separate instrument. Single rows are copied to apply to all instruments to be worked on. The number of columns is arbitrary, and data is padded along columns.
<b>ClassList</b>	(Optional) Character vector or NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how <b>DataList</b> is parsed. Valid character vectors are 'dble', 'date', and 'char'. The 'FieldClass', <b>ClassList</b> pair is always optional. <b>ClassList</b> is inferred from existing field names or from the data if not entered.

TypeString	Character vector specifying the type of instrument added. Instruments of different types can have different Fieldname collections.
InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different Data fields. The stored Data field is a row vector or character vector for each instrument.

## Description

InstSet = instadddfield('FieldName',FieldList,'Data',DataList,'Type',TypeString) to create your own types of instruments or to append new instruments to an existing collection. Argument value pairs can be entered in any order.

InstSet = instadddfield('FieldName',FieldList,'FieldClass',ClassList,'Data',DataList,'Type',TypeString) creates an instrument variable.

InstSetNew = instadddfield(InstSet,'FieldName',FieldList,'Data',DataList,'Type',TypeString) adds instruments to an existing instrument set, InstSet. The output InstSetNew is a new instrument set containing the input data.

## Examples

Build a portfolio around July options.

```
Strike  Call  Put
   95    12.2  2.9
  100     9.2  4.9
  105     6.8  7.4
```

```
Strike = (95:5:105)'
CallP = [12.2; 9.2; 6.8]
```

Enter three call options with data fields Strike, Price, and Opt.

```
InstSet = instadfield('Type','Option','FieldName',...
{'Strike','Price','Opt'}, 'Data',{ Strike, CallP, 'Call'});
instdisp(InstSet)
```

Index	Type	Strike	Price	Opt
1	Option	95	12.2	Call
2	Option	100	9.2	Call
3	Option	105	6.8	Call

Add a futures contract and set the input parsing class.

```
InstSet = instadfield(InstSet,'Type','Futures',...
'FieldName',{ 'Delivery','F'},'FieldClass',{ 'date','dblr'},...
'Data',{ '01-Jul-99',104.4 });
instdisp(InstSet)
```

Index	Type	Strike	Price	Opt
1	Option	95	12.2	Call
2	Option	100	9.2	Call
3	Option	105	6.8	Call

Index	Type	Delivery	F
4	Futures	01-Jul-1999	104.4

Add a put option.

```
FN = instfields(InstSet,'Type','Option')
InstSet = instadfield(InstSet,'Type','Option',...
'FieldName',FN, 'Data',{105, 7.4, 'Put'});
instdisp(InstSet)
```

Index	Type	Strike	Price	Opt
1	Option	95	12.2	Call
2	Option	100	9.2	Call
3	Option	105	6.8	Call

Index	Type	Delivery	F
4	Futures	01-Jul-1999	104.4

Index	Type	Strike	Price	Opt
5	Option	105	7.4	Put

Make a placeholder for another put.

```
InstSet = instadfield(InstSet,'Type','Option',...
'FieldName','Opt','Data','Put')
instdisp(InstSet)
```

Index	Type	Strike	Price	Opt
1	Option	95	12.2	Call
2	Option	100	9.2	Call
3	Option	105	6.8	Call

Index	Type	Delivery	F
4	Futures	01-Jul-1999	104.4

Index	Type	Strike	Price	Opt
5	Option	105	7.4	Put
6	Option	NaN	NaN	Put

Add a cash instrument.

```
InstSet = instaddfield(InstSet, 'Type', 'TBill', ...  
'FieldName', 'Price', 'Data', 99)  
instdisp(InstSet)
```

Index	Type	Strike	Price	Opt
1	Option	95	12.2	Call
2	Option	100	9.2	Call
3	Option	105	6.8	Call

Index	Type	Delivery	F
4	Futures	01-Jul-1999	104.4

Index	Type	Strike	Price	Opt
5	Option	105	7.4	Put
6	Option	NaN	NaN	Put

Index	Type	Price
7	TBill	99

## See Also

[instadd](#) | [instdisp](#) | [instget](#) | [instgetcell](#) | [instsetfield](#)

## Topics

“Portfolio Creation” on page 1-8

“Creating Instruments or Properties” on page 1-21

“Instrument Constructors” on page 1-20



**Introduced before R2006a**

## instasian

Construct Asian option

### Syntax

```
InstSet = instasian(InstSet,OptSpec,Strike,Settle,
ExerciseDates,AmericanOpt,AvgType,AvgPrice,AvgDate)
[FieldList,ClassList,TypeString] = instasian
```

### Arguments

InstSet	Instrument variable. This argument is specified only when adding asian instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
OptSpec	NINST-by-1 list of character vector values for 'Call' or 'Put'.
Strike	NINST-by-1 vector of strike price values. Each row is the schedule for one option.
Settle	NINST-by-1 vector of Settle dates.
ExerciseDates	For a European option ( <code>AmericanOpt = 0</code> ):  NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.  For an American option ( <code>AmericanOpt = 1</code> ):  NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if <code>ExerciseDates</code> is NINST-by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date.

AmericanOpt	(Optional) If AmericanOpt = 0, NaN, or is unspecified, the option is a European option. If AmericanOpt = 1, the option is an American option.
AvgType	(Optional) Character vector is 'arithmetic' for arithmetic average (default) or 'geometric' for geometric average.
AvgPrice	(Optional) Scalar representing the average price of the underlying asset at Settle. This argument is used when AvgDate < Settle. Default is the current stock price.
AvgDate	(Optional) Scalar representing the date on which the averaging period begins. Default = Settle.

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

## Description

InstSet = instasian(InstSet,OptSpec,Strike,Settle,ExerciseDates,AmericanOpt,AvgType,AvgPrice,AvgDate) specifies an Asian option.

[FieldList,ClassList,TypeString] = instasian displays the classes.

FieldList is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

TypeString is a character vector specifying the type of instrument added. For an Asian option instrument, TypeString = 'Asian'.

## Examples

## Create an Asian Option Instrument

Load the example instrument set, `deriv.mat`, and set the required values for an asian option instrument.

```
load deriv.mat
```

Create a subportfolio with barrier and lookback options.

```
CRRSubSet = instselect(CRRInstSet, 'Type', {'Barrier', 'Lookback'});
```

Define the asian instrument.

```
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';
```

Add a floating strike asian option to the instrument set.

```
InstSet = instasian(CRRSubSet, OptSpec, Strike, Settle, ExerciseDates);
instdisp(InstSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Barri
1	Barrier	call	105	01-Jan-2003	01-Jan-2006	1	ui	102

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quant
2	Lookback	call	115	01-Jan-2003	01-Jan-2006	0	Lookback1	7
3	Lookback	call	115	01-Jan-2003	01-Jan-2007	0	Lookback2	9

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPric
4	Asian	put	NaN	01-Jan-2003	01-Jan-2004	0	arithmetic	NaN

## See Also

`instadd` | `instdisp` | `instget` | `instgetcell` | `instsetfield`

## Topics

“Pricing Equity Derivatives Using Trees” on page 3-130

“Creating Instruments or Properties” on page 1-21

“Instrument Constructors” on page 1-20

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## instbarrier

Construct barrier option

### Syntax

```
InstSet = instbarrier(OptSpec,Strike,Settle,ExerciseDates,  
AmericanOpt,BarrierSpec,Barrier,Rebate)  
InstSet = instbarrier(InstSetOld,OptSpec,Strike,Settle,  
ExerciseDates,AmericanOpt,BarrierSpec,Barrier,Rebate)  
[FieldList,ClassList,TypeString] = instbarrier
```

### Description

`InstSet = instbarrier(OptSpec,Strike,Settle,ExerciseDates, AmericanOpt,BarrierSpec,Barrier,Rebate)` constructs a barrier instrument.

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with `NaN`. Only one data argument is required to create the instrument. The others can be omitted or passed as empty matrices `[]`.

`InstSet = instbarrier(InstSetOld,OptSpec,Strike,Settle, ExerciseDates,AmericanOpt,BarrierSpec,Barrier,Rebate)` adds barrier options to an existing instrument variable `InstSetOld`.

`[FieldList,ClassList,TypeString] = instbarrier` lists field metadata for the barrier instrument.

### Examples

#### Create Two Barrier Option Instruments

Create an instrument set of two barrier options with the following data:

```
OptSpec = {'put';'call'};  
Strike = 112;
```

```
Settle = '01-Jan-2012';
ExerciseDates = '01-Jan-2015';
BarrierSpec = {'do'; 'ui'};
Barrier = [101; 102];
AmericanOpt = 0;
```

Create the instrument set (InstSet) for the two barrier options.

```
InstSet = instbarrier(OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierSpec,
```

Display the instrument set.

```
instdisp(InstSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Bar
1	Barrier	put	112	01-Jan-2012	01-Jan-2015	0	do	101
2	Barrier	call	112	01-Jan-2012	01-Jan-2015	0	ui	102

## Input Arguments

### OptSpec — Definition of option

character vector with values 'call' or 'put'

Definition of an option as 'call' or 'put', specified as a NINST-by-1 list of character vector values.

Data Types: char

### Strike — Option strike price value

integer

Option strike price value, specified as an NINST-by-1 vector of strike values. Each row is the schedule for one option.

Data Types: double

### Settle — Settlement or trade date

serial date number | date character vector

Settlement date for the barrier option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**ExerciseDates — Option exercise dates**

date character vector | serial date number

Option exercise dates, specified as a date character vector or a serial date number:

- For a European option (`AmericanOpt = 0`), specified as a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- For an American option (`AmericanOpt = 1`), specified as a NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date.

Data Types: double | char

**AmericanOpt — Flag for American option**

integer with values 0 or 1

Flag for American option, specified as an integer with values 0 or 1. If `AmericanOpt = 0`, NaN, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option.

Data Types: logical

**BarrierSpec — Barrier option type**

character vector with values: 'UI', 'U0', 'DI', 'D0'

Barrier option type, specified as a character vector with the following values:

- 'UI' — Up Knock In

This option becomes effective when the price of the underlying asset passes above the barrier level. It gives the option holder the right, but not the obligation, to buy/sell (call/put) the underlying security at the strike price if the underlying asset goes above the barrier level during the life of the option.

- 'U0' — Up Knock Out

This option gives the option holder the right, but not the obligation, to buy/sell (call/put) the underlying security at the strike price as long as the underlying asset does not



go above the barrier level during the life of the option. This option terminates when the price of the underlying asset passes above the barrier level. Usually, with an up-and-out option, the rebate is paid if the spot price of the underlying reaches or exceeds the barrier level.

- 'DI' — Down Knock In

This option becomes effective when the price of the underlying stock passes below the barrier level. It gives the option holder the right, but not the obligation, to buy/sell (call/put) the underlying security at the strike price if the underlying security goes below the barrier level during the life of the option. With a down-and-in option, the rebate is paid if the spot price of the underlying does not reach the barrier level during the life of the option.

- 'DO' — Down Knock Up

This option gives the option holder the right, but not the obligation, to buy/sell (call/put) the underlying asset at the strike price as long as the underlying asset does not go below the barrier level during the life of the option. This option terminates when the price of the underlying security passes below the barrier level. Usually the option holder receives a rebate amount if the option expires worthless.

Option	Barrier Type	Payoff if Barrier Crossed	Payoff if Barrier not Crossed
Call/Put	Down Knock-out	Worthless	Standard Call/Put
Call/Put	Down Knock-in	Call/Put	Worthless
Call/Put	Up Knock-out	Worthless	Standard Call/Put
Call/Put	Up Knock-in	Standard Call/Put	Worthless

Data Types: char

#### **Barrier — Barrier value**

integer

Barrier value, specified as a vector of values.

Data Types: double

#### **Rebate — Rebate value**

integer

(Optional) Rebate value, specified as a vector of values.

Data Types: `double`

**InstSetOld** — Instrument variable

structure

(Optional) Instrument variable, this argument is specified only when adding barrier instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable.

Data Types: `struct`

## Output Arguments

**InstSet** — Instrument variable for barrier option

structure

Instrument variable for barrier option, returned as a structure. See `instget` for more information on the `InstSet` variable.

**FieldList** — Fields in `InstSet` instrument

cell array of character vectors

Fields in `InstSet` instrument are returned as a (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

**ClassList** — Data class of each field in `InstSet` instrument

cell array of character vectors

Data class of each field in `InstSet` instrument, returned as an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are `'dble'`, `'date'`, and `'char'`.

**TypeString** — Type of instrument added to `InstSet` instrument

character vector

Type of instrument added to `InstSet` instrument, returned as a character vector specifying the type of instrument added. For a barrier option instrument, `TypeString` = `'Barrier'`.

## Definitions

### Barrier Option

A barrier option has not only a strike price but also a barrier level and sometimes a rebate.

A rebate is a fixed amount that is paid if the option cannot be exercised because the barrier level has been reached or not reached. The payoff for this type of option depends on whether the underlying asset crosses the predetermined trigger value (barrier level), indicated by `Barrier`, during the life of the option.

### See Also

`barrierbybls` | `barrierbycrr` | `barrierbyeqp` | `barrierbyfd` | `barrierbyitt` |  
`barrierbyls` | `barrierbystt` | `instadd` | `instdisp` | `instget`

### Topics

“Pricing Equity Derivatives Using Trees” on page 3-130

“Creating Instruments or Properties” on page 1-21

“Instrument Constructors” on page 1-20

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## instbond

Construct bond instrument

### Syntax

```
InstSet =
instbond(CouponRate,Settle,Maturity,Period,Basis,EndMonthRule,IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face)
InstSet =
instbond(InstSet,CouponRate,Settle,Maturity,Period,Basis,EndMonthRule,IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face)
[FieldList,ClassList,TypeString] = instbond
```

### Arguments

InstSet	Instrument variable. This argument is specified only when adding bond instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
CouponRate	Decimal annual rate indicating the annual percentage rate used to determine the coupons payable on a bond. <code>CouponRate</code> is a NINST-by-1 vector or NINST-by-1 cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid.
Settle	Settlement date. A vector of serial date numbers or date character vectors. <code>Settle</code> must be earlier than <code>Maturity</code> .
Maturity	Maturity date. A vector of serial date numbers or date character vectors.

Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 1, 2, 3, 4, 6, and 12. Default = 2.
Basis	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
IssueDate	(Optional) Date when a bond was issued.

<b>FirstCouponDate</b>	(Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When <b>FirstCouponDate</b> and <b>LastCouponDate</b> are both specified, <b>FirstCouponDate</b> takes precedence in determining the coupon payment structure. If you do not specify a <b>FirstCouponDate</b> , the cash flow payment dates are determined from other inputs.
<b>LastCouponDate</b>	(Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified <b>FirstCouponDate</b> , a specified <b>LastCouponDate</b> determines the coupon structure of the bond. The coupon structure of a bond is truncated at the <b>LastCouponDate</b> , regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a <b>LastCouponDate</b> , the cash flow payment dates are determined from other inputs.
<b>StartDate</b>	(Optional) Date when a bond actually starts (that is, the date when a bond's cash flows can be considered). To make an instrument forward starting, specify this date as a future date. If <b>StartDate</b> is not explicitly specified, the effective start date is the <b>Settle</b> date.
<b>Face</b>	(Optional) Face or par value. <b>Face</b> is a <b>NINST</b> -by-1 vector or <b>NINST</b> -by-1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a <b>NumDates</b> -by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default = 100.

Data arguments are number of instruments (**NINST**)-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with **NaN**. Only one data argument is required to create the instrument. The others can be omitted or passed as empty matrices **[]**.

## Description

**InstSet** =  
**instbond**(**CouponRate**,**Settle**,**Maturity**,**Period**,**Basis**,**EndMonthRule**,**Issued**

ate, FirstCouponDate, LastCouponDate, StartDate, Face) creates a new instrument set containing bond instruments.

InstSet = instbond(InstSet, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) adds bond instruments to an existing instrument set.

[FieldList, ClassList, TypeString] = instbond displays the classes.

FieldList is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

TypeString is a character vector specifying the type of instrument added. For a bond instrument, TypeString = 'Bond'.

## Examples

### Create a Bond Instrument

Create a new instrument variable with the following information:

```
CouponRate= [0.035;0.04];
Settle= 'Nov-1-2013';
Maturity = 'Nov-1-2014';
Period =1;
```

```
InstSet = instbond(CouponRate, Settle, Maturity, ...
Period)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
    Type: {'Bond'}
    FieldName: {{11x1 cell}}
    FieldClass: {{11x1 cell}}
```

FieldData: {{11x1 cell}}

Display the instrument set.

`instdisp(InstSet)`

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.035	01-Nov-2013	01-Nov-2014	1	0	1	NaN
2	Bond	0.04	01-Nov-2013	01-Nov-2014	1	0	1	NaN

## See Also

`hjmpri` | `instaddfield` | `instdisp` | `instget` | `intenvprice`

## Topics

“Creating Instruments or Properties” on page 1-21

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



# instcap

Construct cap instrument

## Syntax

```
InstSet = instcap(Strike,Settle,Maturity,Reset,Basis,Principal)
InstSet =
instcap(InstSet,Strike,Settle,Maturity,CapReset,Basis,Principal)
[FieldList,ClassList,TypeString] = instcap
```

## Arguments

InstSet	Instrument variable. This argument is specified only when adding cap instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
Strike	Rate at which the cap is exercised, as a decimal number.
Settle	Settlement date. Serial date number representing the settlement date of the cap.
Maturity	Serial date number representing the maturity date of the cap.
CapReset	(Optional) NINST-by-1 vector representing the frequency of payments per year. Default = 1.

<p><b>Basis</b></p>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>Principal</b></p>	<p>(Optional) NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid. Default is 100.</p>

## Description

`InstSet = instcap(Strike,Settle,Maturity,Reset,Basis,Principal)`  
 creates a new instrument set containing cap instruments.

`InstSet = instcap(InstSet,Strike,Settle,Maturity,CapReset,Basis,Principal)` adds cap instruments to an existing instrument set.

`[FieldList,ClassList,TypeString] = instcap` displays the classes.

`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

`TypeString` is a character vector specifying the type of instrument added. For a cap instrument, `TypeString` = 'Cap'.

---

**Note** Use the optional argument, `Principal`, to pass a schedule for an amortizing cap.

---

## Examples

### Create Two Cap Instruments

Create a new instrument variable with the following information:

```
Strike = [0.035; 0.045];
Settle = 'Jan-1-2013';
Maturity = 'Jan-1-2014';
Reset = 1;
Basis = 1;
Principal = 1000;
```

Create the new cap instruments.

```
InstSet = instcap(Strike, Settle, Maturity, Reset, Basis, Principal)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
    Type: {'Cap'}
    FieldName: {{6x1 cell}}
    FieldClass: {{6x1 cell}}
    FieldData: {{6x1 cell}}
```

Display the cap instruments.

```
instdisp(InstSet)
```

Index	Type	Strike	Settle	Maturity	CapReset	Basis	Principal
1	Cap	0.035	01-Jan-2013	01-Jan-2014	1	1	1000
2	Cap	0.045	01-Jan-2013	01-Jan-2014	1	1	1000

### See Also

hjmprice | instaddfield | instbond | instdisp | instfloor | instswap |  
intenvprice

### Topics

“Creating Instruments or Properties” on page 1-21  
“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# instcbond

Construct CBond instrument for convertible bond

## Syntax

```
ISet = instcbond(CouponRate,Settle,Maturity,ConvRatio)  
ISet = instcbond( ____,Name,Value)
```

```
ISet = instcbond(ISet,CouponRate,Settle,Maturity,ConvRatio)  
ISet = instcbond( ____,Name,Value)
```

```
[FieldList,ClassList,TypeString] = instcbond
```

## Description

`ISet = instcbond(CouponRate,Settle,Maturity,ConvRatio)` creates a CBond instrument variable from data arrays.

`ISet = instcbond( ____,Name,Value)` creates a CBond instrument variable from data arrays using optional name-value pair arguments.

`ISet = instcbond(ISet,CouponRate,Settle,Maturity,ConvRatio)` adds a CBond to an existing instrument set.

`ISet = instcbond( ____,Name,Value)` adds a CBond instrument to an existing instrument set using optional name-value pair arguments.

`[FieldList,ClassList,TypeString] = instcbond` lists the field metadata for the CBond instrument.

## Examples

### Create a CBond Instrument

Create a CBond instrument.

```
CouponRate = 0.03;
Settle = 'Jan-1-2014';
Maturity = 'Jan-1-2016';
CallStrike = 125;
CallExDates = [datenum('Jan-1-2015') datenum('Jan-1-2016')];

ConvRatio = 1.5;
Spread = 0.045;

InstSet = instcbond(CouponRate,Settle,Maturity,ConvRatio,...
'Spread',Spread,'CallExDates',CallExDates,'CallStrike',CallStrike,...
'AmericanCall', 1);
```

Display the InstSet for the convertible bond.

```
instdisp(InstSet)
```

Index	Type	CouponRate	Settle	Maturity	ConvRatio	Period	IssueDate	FirstC
1	CBond	0.03	01-Jan-2014	01-Jan-2016	1.5	2	NaN	NaN

### Add a CBond Instrument to an Existing Portfolio Set

Create a bond instrument using instbond.

```
CouponRate= [0.035;0.04];
Settle= 'Nov-1-2013';
Maturity = 'Nov-1-2014';
Period =1;

InstSet = instbond(CouponRate,Settle,Maturity, ...
Period);
```

Add a CBond instrument to the existing portfolio set.

```
ConvRatio = 1.5;
InstSet = instadd(InstSet, 'CBond', CouponRate, Settle, Maturity, ConvRatio);
instdisp(InstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.035	01-Nov-2013	01-Nov-2014	1	0	1	NaN
2	Bond	0.04	01-Nov-2013	01-Nov-2014	1	0	1	NaN

Index	Type	CouponRate	Settle	Maturity	ConvRatio	Period	IssueDate	FirstC
3	CBond	0.035	01-Nov-2013	01-Nov-2014	1.5	2	NaN	NaN
4	CBond	0.04	01-Nov-2013	01-Nov-2014	1.5	2	NaN	NaN

[FieldList,ClassList,TypeString] = instcbond

FieldList = 20x1 cell array

```
{'CouponRate'      }
{'Settle'          }
{'Maturity'        }
{'ConvRatio'       }
{'Period'          }
{'IssueDate'       }
{'FirstCouponDate' }
{'LastCouponDate' }
{'StartDate'       }
{'Face'            }
{'Spread'          }
{'CallStrike'      }
{'CallExDates'    }
{'AmericanCall'   }
{'PutStrike'       }
{'PutExDates'     }
{'AmericanPut'    }
{'ConvDates'      }
{'DefaultProbability'}
{'RecoveryRate'   }
```

ClassList = 20x1 cell array

```
{'cell'}
{'date'}
{'date'}
{'dble'}
{'dble'}
{'date'}
{'date'}
{'date'}
{'date'}
{'date'}
{'cell'}
```

```
{'dbl' }  
{'dbl' }  
{'date' }  
{'dbl' }  
{'dbl' }  
{'date' }  
{'dbl' }  
{'date' }  
{'dbl' }  
{'dbl' }
```

```
TypeString =  
'CBond'
```

## Input Arguments

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an NINST-by-1 positive decimal annual rate or an NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

### **Settle** — Settlement date

scalar for serial nonnegative date number | scalar for date character vector

Settlement date, specified as an NINST-by-1 scalar using a serial nonnegative date number or date character vector.

---

**Note** The **Settle** date for every convertible bond is set to the **ValuationDate** of the stock tree. The bond argument, **Settle**, is ignored.

---

Data Types: double | char

### **Maturity** — Maturity date

scalar for serial nonnegative date number | scalar for date character vector



Maturity date, specified as an NINST-by-1 scalar using a serial nonnegative date number or date character vector.

Data Types: double | char

### **ConvRatio — Number of shares convertible to one bond**

nonnegative scalar

Number of shares convertible to one bond, specified as an NINST-by-1 nonnegative scalar.

Data Types: double

### **ISet — Variable containing a collection of instruments**

structure

Variable containing a collection of instruments, specified as a structure. Use this argument to add a CBond (convertible bond) to an existing instrument set (ISet). Instruments within ISet are broken down by type, and each type can have different data fields. For more information on the ISet variable, see `instget`.

Data Types: struct

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: `InstSet = instcbond(CouponRate, Settle, Maturity, ConvRatio, 'Spread', Spread, 'CallExDates', CallExDates, 'CallStrike', CallStrike, 'AmericanCall', 1)`

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### **IssueDate — Bond issue date**

scalar for serial date number | scalar for date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 scalar using a serial date number or date character vector.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

scalar for serial date number | scalar for date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 scalar using a serial date number or date character vector.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

scalar for serial date number | scalar for date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 scalar using a serial date number or date character vector.

Data Types: double | char

**Face — Face value**

100 (default) | scalar of nonnegative value | cell array of nonnegative values

Face value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 scalar of nonnegative face values or an NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is the associated face value. The date indicates the last day that the face value is valid.

Data Types: cell | double

**Spread — Number of basis points over the reference rate**

0 (default) | vector

Number of basis points over the reference rate, specified as the comma-separated pair consisting of 'Spread' and a NINST-by-1 vector.

Data Types: double

**CallStrike — Call strike price for European, Bermuda, or American option**

nonnegative integer | vector of nonnegative integers

Call strike price for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallStrike' and one of the following values:

- For a European call option — NINST-by-1 vector of nonnegative integers
- For a Bermuda call option — NINST-by-NSTRIKES matrix of strike price values, where each row is the schedule for one call option. If a call option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American call option — NINST-by-1 vector of strike price values for each call option.

Data Types: single | double

### **CallExDates — Call exercise date for European, Bermuda, or American option**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Call exercise date for European, Bermuda, or American option, specified as the comma-separated pair consisting of 'CallExDates' and one of the following values:

- For a European option — NINST-by-1 vector of serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates, where each row is the schedule for one call option. For a European option, there is only one CallExDate on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the call option can be exercised on any tree date between or including the pair of dates on that row. If CallExDates is NINST-by-1, the call option can be exercised between the ValuationDate of the stock tree and the single listed CallExDate.

Data Types: double | char | cell

### **AmericanCall — Call option type indicator**

0 if AmericanCall is NaN or not entered (default) | scalar | vector of positive integers[0,1]

Call option type, specified as the comma-separated pair consisting of 'AmericanCall' and a NINST-by-1 positive integer scalar flags with values 0 or 1.

- For a European or Bermuda option — AmericanCall is 0 for each European or Bermuda option.

- For an American option — `AmericanCall` is 1 for each American option. The `AmericanCall` argument is required to invoke American exercise rules.

Data Types: `single` | `double`

### **PutStrike — Put strike values for European, Bermuda, or American option**

scalar | vector of positive integers [0, 1]

Put strike values for a European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutStrike' and one of the following values:

- For a European put option — NINST-by-1 vector of nonnegative integers
- For a Bermuda put option — NINST-by-NSTRIKES matrix of strike price values, where each row is the schedule for one put option. If a put option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American put option — NINST-by-1 vector of strike price values for each put option.

Data Types: `single` | `double`

### **PutExDates — Put exercise date for European, Bermuda, or American option**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Put exercise date for a European, Bermuda, or American option, specified as the comma-separated pair consisting of 'PutExDates' and one of the following values:

- For a European option — NINST-by-1 vector serial date numbers or date character vectors.
- For a Bermuda option — NINST-by-NSTRIKES matrix of exercise dates, where each row is the schedule for one put option. For a European option, there is only one `PutExDate` on the option expiry date.
- For an American option — NINST-by-1 or NINST-by-2 matrix of exercise date boundaries. For each instrument, the put option can be exercised on any tree date between or including the pair of dates on that row. If `PutExDates` is NINST-by-1, the put option can be exercised between the `ValuationDate` of the stock tree and the single listed `PutExDate`.

Data Types: `double` | `char` | `cell`

**AmericanPut — Put option type indicator**

0 if AmericanPut is NaN or not entered (default) | scalar | vector of positive integers [0, 1]

Put option type, specified as the comma-separated pair consisting of 'AmericanPut' and a NINST-by-1 positive integer scalar flags with values 0 or 1.

- For a European or Bermuda option — AmericanPut is 0 for each European or Bermuda option.
- For an American option — AmericanPut is 1 for each American option. The AmericanPut argument is required to invoke American exercise rules.

Data Types: single | double

**ConvDates — Convertible dates**

MaturityDate (default) | scalar for serial date number | scalar for date character vector

Convertible dates, specified as the comma-separated pair consisting of 'ConvDates' and a NINST-by-1 or NINST-by-2 matrix of serial nonnegative date numbers or date character vectors. If ConvDates is not specified, the bond is always convertible until maturity.

For each instrument, the bond can be converted on any tree date between or including the pair of dates on that row.

If ConvDates is NINST-by-1, the bond can be converted between the ValuationDate of the stock tree and the single listed ConvDates.

Data Types: single | double | char

## Output Arguments

**ISet — Variable containing a collection of instruments**

character vector | row vector

Variable containing a collection of instruments, returned as a row vector or character vector for each instrument. Instruments are broken down by type and each type can have different data fields. For more information on the ISet variable, see `instget`.

**FieldList — Name of each data field for instrument type**

cell array of character vectors

Name of each data field for instrument type, returned as an NFIELDS-by-1 cell array of character vectors.

### **ClassList — Data class of each field**

cell array of character vectors with valid values of 'dble', 'date', and 'char'

Data class of each field, returned as an NFIELDS-by-1 cell array of character vectors with valid character vector values of 'dble', 'date', and 'char'.

### **TypeString — Type of instrument added**

character vector

Type of instrument added, returned as character vector. When adding a CBond, the TypeString = 'CBond'.

## **See Also**

cbondbycrr | cbondbyeqp | crrprice | crrsens | eqprice | eqpsens | instadd | instdisp

## **Topics**

“Convertible Bond” on page 2-3

## **Introduced in R2015a**

## instcf

Construct cash flow instrument

### Syntax

```
InstSet = instcf(CFlowAmounts,CFlowDates,Settle,Basis)
InstSet = instcf(InstSet,CFlowAmounts,CFlowDates,Settle,Basis)
[FieldList,ClassList,TypeString] = instcf
```

### Arguments

InstSet	Instrument variable. This argument is specified only when adding cash flow instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
CFlowAmounts	Number of instruments (NINST) by maximum number of cash flows (MOSTCFS) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.
CFlowDates	NINST-by-MOSTCFS matrix of cash flow dates. Each entry contains the date of the corresponding cash flow in <code>CFlowAmounts</code> .
Settle	Settlement date on which the cash flows are priced.

<b>Basis</b>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	--

Only one data argument is required to create an instrument. Other arguments can be omitted or passed as empty matrices [ ]. Dates can be input as serial date numbers or date character vectors.

## Description

`InstSet = instcf(CFlowAmounts,CFlowDates,Settle,Basis)` creates a new cash flow instrument set from data arrays.

`InstSet = instcf(InstSet,CFlowAmounts,CFlowDates,Settle,Basis)` adds instruments of type `CashFlow` to an instrument set.

`[FieldList,ClassList,TypeString] = instcf` lists field metadata for an instrument of type `CashFlow`.



`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

`TypeString` specifies the type of instrument added; for example,

```
TypeString = 'CashFlow'
```

.

## See Also

`instadd` | `instdisp` | `instget` | `intenvprice`

## Topics

“Creating Instruments or Properties” on page 1-21

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## instcompound

Construct compound option

### Syntax

```
InstSet =
instcompound(UOptSpec,UStrike,USettle,UExerciseDates,UAmericanOpt,COp
tSpec,CStrike,CSettle,CExerciseDates,CAmericanOpt)
InstSet =
instcompound(InstSet,UOptSpec,UStrike,USettle,UExerciseDates,UAmeric
anOpt,COptSpec,CStrike,CSettle,CExerciseDates,CAmericanOpt)
[FieldList,ClassList,TypeString] = instcompound
```

### Arguments

InstSet	Instrument variable. This argument is specified only when adding compound instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
UOptSpec	Character vector with value of 'Call' or 'Put'.
UStrike	1-by-1 vector of strike price values.
USettle	1-by-1 vector of <code>Settle</code> dates.
UExerciseDates	For a European option ( <code>UAmericanOpt = 0</code> ):  1-by-1 vector of exercise dates. For a European option, there is only one exercise date, the option expiry date.  For an American option ( <code>UAmericanOpt = 1</code> ):  1-by-2 vector of exercise date boundaries. The option can be exercised on any tree date. If only one non- <code>NaN</code> date is listed, or if <code>ExerciseDates</code> is 1-by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date.

UAmericanOpt	If UAmericanOpt = 0, NaN, or is unspecified, the option is a European option. If UAmericanOpt = 1, the option is an American option.
C0ptSpec	NINST-by-1 list of character vector values for 'Call' or 'Put' of the compound option.
CStrike	NINST-by-1 vector of strike price values. Each row is the schedule for one option.
CSettle	1-by-1 vector containing the settlement or trade date.
CExerciseDates	For a European option (CAmericanOpt = 0):  NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.  For an American option (CAmericanOpt = 1):  NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date.
CAmericanOpt	If CAmericanOpt = 0, NaN, or is unspecified, the option is a European option. If CAmericanOpt = 1, the option is an American option.

## Description

InstSet =  
instcompound(U0ptSpec,UStrike,USettle,UExerciseDates,UAmericanOpt,C0ptSpec,CStrike,CSettle,CExerciseDates,CAmericanOpt) creates a new instrument set for a compound option.

InstSet =  
instcompound(InstSet,U0ptSpec,UStrike,USettle,UExerciseDates,UAmericanOpt,C0ptSpec,CStrike,CSettle,CExerciseDates,CAmericanOpt) adds a compound option to an existing instrument set.

[FieldList,ClassList,TypeString] = instcompound displays the classes.

FieldList is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

TypeString is a character vector specifying the type of instrument added. For a compound option instrument, TypeString = 'Compound'.

## Examples

### Create a Compound Option Instrument

Define a compound option instrument with the following data:

```
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2012';
UExerciseDates = '01-Jan-2015';
UAmericanOpt = 0;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2012';
CExerciseDates = '01-Jan-2014';
CAmericanOpt = 0;

InstSet = instcompound(UOptSpec, UStrike, USettle,UExerciseDates, ...
UAmericanOpt, COptSpec, CStrike, CSettle,CExerciseDates, CAmericanOpt)

InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
    Type: {'Compound'}
    FieldName: {{10x1 cell}}
    FieldClass: {{10x1 cell}}
    FieldData: {{10x1 cell}}
```

```
InstSet = instcompound(UOptSpec, UStrike, USettle,UExerciseDates, ...
UAmericanOpt, COptSpec, CStrike, CSettle,CExerciseDates)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
    Type: {'Compound'}
    FieldName: {{10x1 cell}}
    FieldClass: {{10x1 cell}}
    FieldData: {{10x1 cell}}
```

Display the instrument set.

```
instdisp(InstSet)
```

Index	Type	UOptSpec	UStrike	USettle	UExerciseDates	UAmericanOpt	COptSpec	CS
1	Compound	Call	130	01-Jan-2012	01-Jan-2015	0	Put	5

## See Also

[instadd](#) | [instdisp](#) | [instget](#)

## Topics

“Creating Instruments or Properties” on page 1-21

“Pricing Equity Derivatives Using Trees” on page 3-130

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## instdelete

Complement of instrument set by matching conditions

### Syntax

```
ISubSet = instdelete(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet, 'TypeList', TypeList)
```

### Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
FieldList	Number of fields, specified as a NFIELDS-by-1 cell array of character vectors listing the name of each data field to match with data values.
DataList	Number of values, specified as a NVALUES-by-M array or NFIELDS-by-1 cell array of acceptable data values for each field. Each row lists a data row value to search for in the corresponding FieldList. The number of columns is arbitrary and matching ignores trailing NaNs or spaces.
IndexSet	(Optional) Number of instruments, specified as a NINST-by-1 vector restricting positions of instruments to check for matches. The default is all indices available in the instrument variable.
TypeList	(Optional) Number of types, specified as a NTYPES-by-1 cell array of character vectors restricting instruments to match one of TypeList types. The default is all types in the instrument variable.

---

**Note** Argument value pairs can be entered in any order. The InstSet variable must be the first argument. 'FieldName' and 'Data' arguments must appear together or not at all.

---

## Description

The output argument `ISubSet` contains instruments *not* matching the input criteria. Instruments are deleted from `ISubSet` if all the `Field`, `Index`, and `Type` conditions are met. An instrument meets an individual `Field` condition if the stored `FieldName` data matches any of the rows listed in the `DataList` for that `FieldName`. See `instfind` for more examples on matching criteria.

## Examples

Retrieve the instrument set variable `ExampleInst` from the data file `InstSetExamples.mat`. The variable contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)
```

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	100	9.2	Call	0
3	Option	105	6.8	Call	1000

Index	Type	Delivery	F	Contracts
4	Futures	01-Jul-1999	104.4	-1000

Index	Type	Strike	Price	Opt	Contracts
5	Option	105	7.4	Put	-1000
6	Option	95	2.9	Put	0

Index	Type	Price	Maturity	Contracts
7	TBill	99	01-Jul-1999	6

Create a new variable, `ISet`, with all options deleted.

```
ISet = instdelete(ExampleInst, 'Type','Option');
instdisp(ISet)
```

Index	Type	Delivery	F	Contracts
1	Futures	01-Jul-1999	104.4	-1000

Index	Type	Price	Maturity	Contracts
2	TBill	99	01-Jul-1999	6

## **See Also**

`instaddfield` | `instfind` | `instget` | `instselect`

## **Topics**

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**



# instdisp

Display instruments

## Syntax

```
CharTable = instdisp(InstSet)
```

## Arguments

InstSet	Variable containing a collection of instruments. See <code>instaddfield</code> for examples on constructing the variable.
---------	---

## Description

`CharTable = instdisp(InstSet)` creates a character array displaying the contents of an instrument collection, `InstSet`. If `instdisp` is called without output arguments, the table is displayed in the Command Window.

---

**Note** When using `instdisp`, a value of `NaN` in one of the columns for an instrument indicates that the default value for that parameter will be used in the instrument's pricing function.

---

`CharTable` is a character array with a table of instruments in `InstSet`. For each instrument row, the `Index` and `Type` are printed along with the field contents. Field headers are printed at the tops of the columns.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)
```

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	100	9.2	Call	0
3	Option	105	6.8	Call	1000

Index	Type	Delivery	F	Contracts
4	Futures	01-Jul-1999	104.4	-1000

Index	Type	Strike	Price	Opt	Contracts
5	Option	105	7.4	Put	-1000
6	Option	95	2.9	Put	0

Index	Type	Price	Maturity	Contracts
7	TBill	99	01-Jul-1999	6

Create a swap instrument and use `instdisp` to display the instrument. Notice that value of NaN in two columns for this instrument indicates that the default values for `LegReset` and `LegType` parameters will be used in the swap instrument's pricing function.

```
LegRate1 = [0.065, 0];
Settle1 = datenum('jan-1-2007');
Maturity1 = datenum('jan-1-2012');

ISet = instswap(LegRate1, Settle1, Maturity1);
instdisp(ISet)
```

Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	EndMonthRule
1	Swap	[0.065 0]	01-Jan-2007	01-Jan-2012	[NaN]	0	100	[NaN]	1

## See Also

`datestr` | `instaddfield` | `instcbond` | `instget` | `num2str`

## Topics

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**

# instfields

List field names

## Syntax

```
FieldList = instfields(InstSet, 'Type', TypeList)
```

## Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
TypeList	(Optional) Number of types, specified as a NTYPEES-by-1 cell array of character vectors listing the instrument types to query.

## Description

`FieldList = instfields(InstSet, 'Type', TypeList)` retrieves the list of fields stored in an instrument variable.

`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field corresponding to the listed types.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)
```

```
Index Type    Strike Price Opt Contracts
1      Option  95      12.2 Call      0
```

```
2   Option 100    9.2  Call    0
3   Option 105    6.8  Call  1000
```

```
Index Type    Delivery      F      Contracts
4     Futures 01-Jul-1999  104.4 -1000
```

```
Index Type  Strike Price Opt  Contracts
5     Option 105    7.4 Put -1000
6     Option 95     2.9 Put  0
```

```
Index Type Price Maturity      Contracts
7     TBill 99    01-Jul-1999  6
```

Get the fields listed for type 'Option'.

```
[FieldList, ClassList] = instfields(ExampleInst, 'Type', ...
'Option')
```

```
FieldList =
```

```
    'Strike'
    'Price'
    'Opt'
    'Contracts'
```

```
ClassList =
```

```
    'dble'
    'dble'
    'char'
    'dble'
```

Get the fields listed for types 'Option' and 'TBill'.

```
FieldList = instfields(ExampleInst, 'Type', {'Option', 'TBill'})
```

```
FieldList =
```

```
    'Strike'
    'Opt'
    'Price'
    'Maturity'
    'Contracts'
```

Get all the fields listed in any type in the variable.

```
FieldList = instfields(ExampleInst)
```

```
FieldList =  
    'Delivery'  
    'F'  
    'Strike'  
    'Opt'  
    'Price'  
    'Maturity'  
    'Contracts'
```

## See Also

instdisp | instlength | insttypes

## Topics

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**

## instfind

Search instruments for matching conditions

### Syntax

```
IndexMatch =  
instfind(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', Index  
Set, 'Type', TypeList)
```

### Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
FieldList	Number of fields, specified as a NFIELDS-by-1 cell array of character vectors listing the name of each data field to match with data values.
DataList	Number of values, specified as a NVALUES-by-M array or NFIELDS-by-1 cell array of acceptable data values for each field. Each row lists a data row value to search for in the corresponding FieldList. The number of columns is arbitrary, and matching ignores trailing NaNs or spaces.
IndexSet	(Optional) Number of instruments, specified as a NINST-by-1 vector restricting positions of instruments to check for matches. The default is all indices available in the instrument variable.
TypeList	(Optional) Number of types, specified as a NTYPES-by-1 cell array of character vectors restricting instruments to match one of TypeList types. The default is all types in the instrument variable.

Argument value pairs can be entered in any order. The InstSet variable must be the first argument. 'FieldName' and 'Data' arguments must appear together or not at all.

## Description

IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet, 'Type', TypeList) returns indices of instruments matching Type, Field, or Index values.

IndexMatch is an NINST-by-1 vector of positions of instruments matching the input criteria. Instruments are returned in IndexMatch if all the Field, Index, and Type conditions are met. An instrument meets an individual Field condition if the stored FieldName data matches any of the rows listed in the DataList for that FieldName.

## Examples

Retrieve the instrument set ExampleInst from the data file InstSetExamples.mat. ExampleInst contains three types of instruments: Option, Futures, and TBill.

```
load InstSetExamples;
instdisp(ExampleInst)
```

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	100	9.2	Call	0
3	Option	105	6.8	Call	1000

Index	Type	Delivery	F	Contracts
4	Futures	01-Jul-1999	104.4	-1000

Index	Type	Strike	Price	Opt	Contracts
5	Option	105	7.4	Put	-1000
6	Option	95	2.9	Put	0

Index	Type	Price	Maturity	Contracts
7	TBill	99	01-Jul-1999	6

Make a vector, Opt95, containing the indexes within ExampleInst of the options struck at 95.

```
Opt95 = instfind(ExampleInst, 'FieldName', 'Strike', 'Data', '95')
```

```
Opt95 =
```

1  
6

Locate the futures and Treasury bill instruments within ExampleInst.

```
Types = instfind(ExampleInst, 'Type', {'Futures'; 'TBill'})
```

```
Types =
```

4  
7

### See Also

`instaddfield` | `instget` | `instgetcell` | `instselect`

### Topics

“Portfolio Creation” on page 1-8

“Searching or Subsetting a Portfolio” on page 1-23

“Instrument Constructors” on page 1-20

**Introduced before R2006a**



# instfixed

Construct fixed-rate instrument

## Syntax

```
InstSet =
instfixed(CouponRate,Settle,Maturity,FixedReset,Basis,Principal,EndM
onthRule)
InstSet =
instfixed(InstSet,CouponRate,Settle,Maturity,FixedReset,Basis,Princi
pal,EndMonthRule)
[FieldList,ClassList,TypeString] = instfixed
```

## Arguments

InstSet	Instrument variable. This argument is specified only when adding fixed-rate note instruments to an existing instrument set. See <code>instget</code> for more information on the InstSet variable.
CouponRate	Decimal annual rate.
Settle	Settlement date. Date character vector or serial date number representing the settlement date of the fixed-rate note.
Maturity	Date character vector or serial date number representing the maturity date of the fixed-rate note.
FixedReset	(Optional) NINST-by-1 vector representing the frequency of payments per year. Default = 1.

<p><b>Basis</b></p>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>Principal</b></p>	<p>(Optional) NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid. Default is 100.</p>
<p><b>EndMonthRule</b></p>	<p>(Optional) NINST-by-1 vector representing the End-of-month rule. Default = 1.</p>

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [ ].

## Description

`InstSet = instfixed(CouponRate,Settle,Maturity,FixedReset,Basis,Principal,EndMonthRule)` creates a new instrument set containing fixed-rate instruments.

`InstSet = instfixed(InstSet,CouponRate,Settle,Maturity,FixedReset,Basis,Principal,EndMonthRule)` adds fixed-rate instruments to an existing instrument set.

`[FieldList,ClassList,TypeString] = instfixed` displays the classes.

`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

`TypeString` is a character vector specifying the type of instrument added. For a fixed-rate instrument, `TypeString = 'Fixed'`.

## Examples

### Create a Fixed-Rate Instrument

Define the characteristics of the fixed-rate instrument.

```
CouponRate = .03;
Settle = datenum('15-Mar-2013');
Maturity = datenum('15-Mar-2018');
FixedReset = 4;
Basis = 1;
Principal = 1000;
EndMonthRule = 1;
```

Create the new cap instrument.

```
ISet = instfixed(CouponRate, Settle, Maturity, FixedReset, Basis, Principal,EndMonthRule);
```

```
ISet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {'Fixed'}
    FieldName: {{7x1 cell}}
    FieldClass: {{7x1 cell}}
    FieldData: {{7x1 cell}}
```

Display the fixed-rate instrument.

```
instdisp(ISet)
```

Index	Type	CouponRate	Settle	Maturity	FixedReset	Basis	Principal	EndMonth
1	Fixed	0.03	15-Mar-2013	15-Mar-2018	4	1	1000	1

## See Also

[hjmprice](#) | [instaddfield](#) | [instbond](#) | [instcap](#) | [instdisp](#) | [instswap](#) | [intenvprice](#)

## Topics

“Creating Instruments or Properties” on page 1-21

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# instfloat

Construct floating-rate instrument

## Syntax

```
InstSet =
instfloat(Spread,Settle,Maturity,FloatReset,Basis,Principal,EndMonth
Rule)
InstSet =
instfloat(InstSet,Spread,Settle,Maturity,FloatReset,Basis,Principal,
EndMonthRule)
InstSet =
instfloat(Spread,Settle,Maturity,FloatReset,Basis,Principal,EndMonth
Rule,CapRate,FloorRate)
InstSet =
instfloat(InstSet,Spread,Settle,Maturity,FloatReset,Basis,Principal,
EndMonthRule,CapRate,FloorRate)
[FieldList,ClassList,TypeString] = instfloat
```

## Arguments

InstSet	Instrument variable. This argument is specified only when adding floating-rate note instruments to an existing instrument set. See <code>instget</code> for more information on the InstSet variable.
Spread	Number of basis points over the reference rate.
Settle	Settlement date. Date character vector or serial date number representing the settlement date of the floating-rate note.
Maturity	Date character vector or serial date number representing the maturity date of the floating-rate note.
FloatReset	(Optional) NINST-by-1 vector representing the frequency of payments per year. Default = 1.

<p><b>Basis</b></p>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>Principal</b></p>	<p>(Optional) NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid. Default is 100.</p>
<p><b>EndMonthRule</b></p>	<p>(Optional) NINST-by-1 vector representing the End-of-month rule. Default = 1.</p>
<p><b>CapRate</b></p>	<p>(Optional) NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.</p>
<p><b>FloorRate</b></p>	<p>(Optional) NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.</p>

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [ ].

## Description

InstSet =  
instfloat(Spread,Settle,Maturity,FloatReset,Basis,Principal,EndMonthRule) creates a new instrument set containing floating-rate instruments.

InstSet =  
instfloat(InstSet,Spread,Settle,Maturity,FloatReset,Basis,Principal,EndMonthRule) adds floating-rate instruments to an existing instrument set.

InstSet =  
instfloat(Spread,Settle,Maturity,FloatReset,Basis,Principal,EndMonthRule,CapRate,FloorRate) creates a new instrument set containing capped floating-rate instruments.

InstSet =  
instfloat(InstSet,Spread,Settle,Maturity,FloatReset,Basis,Principal,EndMonthRule,CapRate,FloorRate) adds capped floating-rate instruments to an existing instrument set.

[FieldList,ClassList,TypeString] = instfloat displays the classes.

FieldList is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

ClassList is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

TypeString is a character vector specifying the type of instrument added. For a floating-rate instrument, TypeString = 'Float'.

## Examples

## Create a Floating-Rate Instrument

Define the characteristics of the floating-rate instrument.

```
Spread = 2;
Settle = datenum('15-Mar-2013');
Maturity = datenum('15-Mar-2018');
FloatReset = 4;
Basis = 1;
Principal = 1000;
EndMonthRule = 1;
CapRate = 0.35;
FloorRate = 0.27;
```

Create the new floating-rate instrument.

```
ISet = instfloat(Spread, Settle, Maturity, FloatReset, Basis, Principal, ...
EndMonthRule, CapRate, FloorRate)
```

```
ISet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
    Type: {'Float'}
    FieldName: {{9x1 cell}}
    FieldClass: {{9x1 cell}}
    FieldData: {{9x1 cell}}
```

Display the floating-rate instrument.

```
instdisp(ISet)
```

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRule
1	Float	2	15-Mar-2013	15-Mar-2018	4	1	1000	1

## See Also

[hjmprice](#) | [instaddfield](#) | [instbond](#) | [instcap](#) | [instdisp](#) | [instswap](#) | [intenvprice](#)

## Topics

“Creating Instruments or Properties” on page 1-21



“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2012b**

## instfloor

Construct floor instrument

### Syntax

```
InstSet =
instfloor(Strike,Settle,Maturity,FloorReset,Basis,Principal)
InstSet =
instfloor(InstSet,Strike,Settle,Maturity,FloorReset,Basis,Principal)
[FieldList,ClassList,TypeString] = instfloor
```

### Arguments

InstSet	Instrument variable. This argument is specified only when adding floor instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
Strike	Rate at which the floor is exercised, as a decimal number.
Settle	Settlement date. A vector of serial date numbers or date character vectors. <code>Settle</code> must be earlier than <code>Maturity</code> .
Maturity	Maturity date. A vector of serial date numbers or date character vectors.
FloorReset	(Optional) NINST-by-1 vector representing the frequency of payments per year. Default = 1.

Basis	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
Principal	<p>(Optional) NINST-by-1 of notional principal amounts or NINST-by-1 cell array where each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid. Default is 100.</p>

## Description

InstSet =  
 instfloor(Strike,Settle,Maturity,FloorReset,Basis,Principal) creates a new instrument set containing floor instruments.

InstSet =  
 instfloor(InstSet,Strike,Settle,Maturity,FloorReset,Basis,Principal) adds floor instruments to an existing instrument set.

[FieldList,ClassList,TypeString] = instfloor displays the classes.

`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

`TypeString` is a character vector specifying the type of instrument added. For a floor instrument, `TypeString` = 'Floor'.

---

**Note** Use the optional argument, `Principal`, to pass a schedule for an amortizing floor.

---

## Examples

### Create a Floor Instrument

Define the characteristics of the floor instrument.

```
Strike = 0.22;  
Settle = datenum('15-Mar-2013');  
Maturity = datenum('15-Mar-2018');  
FloorReset = 4;  
Basis = 1;  
Principal = 1000;
```

Create the new floor instrument.

```
ISet = instfloor(Strike, Settle, Maturity, FloorReset, Basis, Principal)
```

```
ISet = struct with fields:  
    FinObj: 'Instruments'  
    IndexTable: [1x1 struct]  
    Type: {'Floor'}  
    FieldName: {{6x1 cell}}  
    FieldClass: {{6x1 cell}}  
    FieldData: {{6x1 cell}}
```

Display the floor instrument.

```
instdisp(ISet)
```

Index	Type	Strike	Settle	Maturity	FloorReset	Basis	Principal
1	Floor	0.22	15-Mar-2013	15-Mar-2018	4	1	1000

## See Also

hjmpri | instaddfield | instbond | instcap | instdisp | instswap |  
intenvprice

## Topics

“Creating Instruments or Properties” on page 1-21  
“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## instget

Data from instrument variable

### Syntax

```
[Data_1,Data_2,...,Data_n] =
instget(InstSet,'FieldName',FieldList,'Index',IndexSet,'Type',TypeList)
```

### Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
FieldList	(Optional) Number of fields, specified as a NFIELDS-by-1 cell array of character vectors listing the name of each data field to match with data values. FieldList entries can also be either 'Type' or 'Index'; these return type character vectors and index numbers respectively. The default is all fields available for the returned set of instruments.
IndexSet	(Optional) Number of instruments, specified as a NINST-by-1 vector of positions of instruments to work on. If TypeList is also entered, instruments referenced must be one of TypeList types and contained in IndexSet. The default is all indices available in the instrument variable.
TypeList	(Optional) Number of types, specified as a NTYPES-by-1 cell array of character vectors restricting instruments to match one of TypeList types. The default is all types in the instrument variable.

Argument value pairs can be entered in any order. The InstSet variable must be the first argument.

## Description

`[Data_1,Data_2,...,Data_n] = instget(InstSet,'FieldName',FieldList,'Index',IndexSet,'Type',TypeList)` retrieves data arrays from an instrument variable.

`Data_1` is an NINST-by-M array of data contents for the first field in `FieldList`. Each row corresponds to a separate instrument in `IndexSet`. Unavailable data is returned as NaN or as spaces.

`Data_n` is an NINST-by-M array of data contents for the last field in `FieldList`.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)
```

```
Index Type    Strike Price Opt  Contracts
1    Option  95    12.2 Call    0
2    Option 100    9.2  Call    0
3    Option 105    6.8  Call  1000
```

```
Index Type    Delivery      F    Contracts
4    Futures 01-Jul-1999  104.4 -1000
```

```
Index Type    Strike Price Opt  Contracts
5    Option 105    7.4 Put  -1000
6    Option  95    2.9 Put    0
```

```
Index Type Price Maturity      Contracts
7    TBill 99    01-Jul-1999  6
```

Extract the price from all instruments.

```
P = instget(ExampleInst,'FieldName','Price')
```

```
P =
```

```
12.2000
```

```

9.2000
6.8000
NaN
7.4000
2.9000
99.0000

```

Get all the prices and the number of contracts held.

```
[P,C] = instget(ExampleInst, 'FieldName', {'Price', 'Contracts'})
```

P =

```

12.2000
9.2000
6.8000
Nan
7.4000
2.9000
99.0000

```

C =

```

0
0
1000
-1000
-1000
0
6

```

Compute a value V. Create a new variable ISet that appends V to ExampleInst.

```

V = P.*C
ISet = instsetfield(ExampleInst, 'FieldName', 'Value', 'Data',...
V);
instdisp(ISet)

```

Index	Type	Strike	Price	Opt	Contracts	Value
1	Option	95	12.2	Call	0	0
2	Option	100	9.2	Call	0	0
3	Option	105	6.8	Call	1000	6800

Index	Type	Delivery	F	Contracts	Value
4	Futures	01-Jul-1999	104.4	-1000	NaN

Index	Type	Strike	Price	Opt	Contracts	Value
-------	------	--------	-------	-----	-----------	-------



```

5   Option 105    7.4 Put -1000    -7400
6   Option  95    2.9 Put   0         0

```

```

Index Type Price Maturity      Contracts Value
7     TBill 99    01-Jul-1999   6         594

```

Look at only the instruments that have nonzero `Contracts`.

```
Ind = find(C ~= 0)
```

```
Ind =
```

```

3
4
5
7

```

Get the `Type` and `Opt` parameters from those instruments. (Only options have a stored `'Opt'` field.)

```
[T,0] = instget(ExampleInst, 'Index', Ind, 'FieldName',...
{'Type', 'Opt'})
```

```
T =
```

```

Option
Futures
Option
TBill

```

```
0 =
```

```
Call
```

```
Put
```

Create a report of holdings `Type`, `Opt`, and `Value`.

```
rstring = [T, 0, num2str(V(Ind))]
```

```
rstring =
```

```

Option Call    6800
Futures      NaN
Option Put   -7400
TBill        594

```

## **See Also**

`instaddfield` | `instdisp` | `instgetcell` | `intenvprice`

## **Topics**

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**

# instgetcell

Data and context from instrument variable

## Syntax

```
[DataList,FieldList,ClassList] =
instgetcell(InstSet,'FieldName',FieldList,'Index',IndexSet,'Type',TypeList)
```

## Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
FieldList	(Optional) Number of fields, specified as a NFIELDS-by-1 cell array of character vectors listing the name of each data field to match with data values. FieldList should not be either Type or Index; these field names are reserved. The default is all fields available for the returned set of instruments.
IndexSet	(Optional) Number of instruments, specified as a NINST-by-1 vector of positions of instruments to work on. If TypeList is also entered, instruments referenced must be one of TypeList types and contained in IndexSet. The default is all indices available in the instrument variable.
TypeList	(Optional) Number of types, specified as a NTYPES-by-1 cell array of character vectors restricting instruments to match one of TypeList types. The default is all types in the instrument variable.

Argument value pairs can be entered in any order. The InstSet variable must be the first argument.

## Description

`[DataList,FieldList,ClassList] = instgetcell(InstSet,'FieldName',FieldList,'Index',IndexSet,'Type',TypeList)` retrieves data and context from an instrument variable.

`DataList` is an `NFIELDS-by-1` cell array of data contents for each field. Each cell is an `NINST-by-M` array, where each row corresponds to a separate instrument in `IndexSet`. Any data which is not available is returned as `NaN` or as spaces.

`FieldList` is an `NFIELDS-by-1` cell array of character vectors listing the name of each field in `DataList`.

`ClassList` is an `NFIELDS-by-1` cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are `'dble'`, `'date'`, and `'char'`.

`IndexSet` is an `NINST-by-1` vector of positions of instruments returned in `DataList`.

`TypeSet` is an `NINST-by-1` cell array of character vectors listing the type of each instrument row returned in `DataList`.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;  
instdisp(ExampleInst)
```

```
Index Type   Strike Price Opt  Contracts  
1      Option  95     12.2 Call   0  
2      Option 100     9.2  Call   0  
3      Option 105     6.8  Call 1000
```

```
Index Type   Delivery      F    Contracts  
4      Futures 01-Jul-1999 104.4 -1000
```

```
Index Type   Strike Price Opt  Contracts  
5      Option 105     7.4  Put  -1000  
6      Option  95     2.9  Put   0
```

Index	Type	Price	Maturity	Contracts
7	TBill	99	01-Jul-1999	6

Get the prices and contracts from all instruments.

```
FieldList = {'Price'; 'Contracts'}
DataList = instgetcell(ExampleInst, 'FieldName', FieldList )
P = DataList{1}
C = DataList{2}
```

P =

```
12.2000
 9.2000
 6.8000
   NaN
 7.4000
 2.9000
99.0000
```

C =

```
0
0
1000
-1000
-1000
0
6
```

Get all the option data: Strike, Price, Opt, Contracts.

```
[DataList, FieldList, ClassList] = instgetcell(ExampleInst,...
'Type','Option')
```

DataList =

```
[5x1 double]
[5x1 double]
[5x4 char ]
[5x1 double]
```

FieldList =

```
'Strike'
'Price'
```

```
'Opt'  
'Contracts'  
  
ClassList =  
  
    'dbl'  
    'dbl'  
    'char'  
    'dbl'
```

Look at the data as a comma-separated list. Type `help lists` for more information on cell array lists.

```
DataList{:}
```

```
ans =
```

```
    95  
   100  
   105  
   105  
    95
```

```
ans =
```

```
  12.2100  
   9.2000  
   6.8000  
   7.3900  
   2.9000
```

```
ans =
```

```
    Call  
    Call  
    Call  
    Put  
    Put
```

```
ans =
```

```
     0  
     0  
    100
```

-100  
0

## **See Also**

instaddfield | instdisp | instget

## **Topics**

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**

## instlength

Count instruments

### Syntax

```
NInst = instlength(InstSet)
```

### Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
---------	---

### Description

`NInst = instlength(InstSet)` computes `NInst`, the number of instruments contained in the variable, `InstSet`.

### See Also

`instdisp` | `instfields` | `insttypes`

### Topics

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**



# instlookback

Construct lookback option

## Syntax

```
InstSet =
instlookback(OptSpec,Strike,Settle,ExerciseDates,AmericanOpt)
InstSet =
instlookback(InstSet,OptSpec,Strike,Settle,ExerciseDates,AmericanOpt
)
[FieldList,ClassList,TypeString] = instlookback
```

## Arguments

InstSet	Instrument variable. This argument is specified only when adding lookback instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
OptSpec	NINST-by-1 list of character vector values for 'Call' or 'Put'.
Strike	NINST-by-1 vector of strike price values. Each row is the schedule for one option.
Settle	NINST-by-1 vector of Settle dates.

<p><b>ExerciseDates</b></p>	<p>For a European option (<code>AmericanOpt = 0</code>):</p> <p>NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.</p> <p>For an American option (<code>AmericanOpt = 1</code>):</p> <p>NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if <code>ExerciseDates</code> is NINST-by-1, the option can be exercised between the valuation date of the stock tree and the single listed exercise date.</p>
<p><b>AmericanOpt</b></p>	<p>(Optional) If <code>AmericanOpt = 0</code>, NaN, or is unspecified, the option is a European option. If <code>AmericanOpt = 1</code>, the option is an American option.</p>

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [ ].

## Description

`InstSet = instlookback(OptSpec,Strike,Settle,ExerciseDates,AmericanOpt)` creates an instrument set for lookback options.

`InstSet = instlookback(InstSet,OptSpec,Strike,Settle,ExerciseDates,AmericanOpt)` adds lookback options to an existing instrument set.

`[FieldList,ClassList,TypeString] = instlookback` displays the classes.

`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

TypeString is a character vector specifying the type of instrument added. For a lookback option instrument, TypeString = 'Lookback'.

## Examples

### Create a Lookback Option Instrument

Define a floating strike lookback instrument with the following data:

```
OptSpec = 'call';
Strike = NaN;
Settle = '01-Jan-2012';
ExerciseDates = '01-Jan-2015';
```

Create the instrument set.

```
InstSet = instlookback(OptSpec, Strike, Settle, ExerciseDates);
```

Display the lookback instrument.

```
instdisp(InstSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt
1	Lookback	call	NaN	01-Jan-2012	01-Jan-2015	0

## See Also

instadd | instdisp | instget

## Topics

“Pricing Equity Derivatives Using Trees” on page 3-130

“Creating Instruments or Properties” on page 1-21

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## instoptbnd

Construct bond option

### Syntax

```
InstSet = instoptbnd(BondIndex,OptSpec,Strike,ExerciseDates)
InstSet = instoptbnd(InstSet,BondIndex,OptSpec,Strike,ExerciseDates)
InstSet =
instoptbnd(InstSet,BondIndex,OptSpec,Strike,ExerciseDates,AmericanOpt)
[FieldList,ClassList,TypeString] = instoptbnd
```

### Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
BondIndex	Number of instruments (NINST)-by-1 vector of indices pointing to underlying instruments of Type 'Bond' which are also stored in InstSet. See instbond for information on specifying the bond data.
OptSpec	NINST-by-1 list of character vector values for 'Call' or 'Put'.
<b>Note</b> The interpretation of the Strike and ExerciseDates arguments depends upon the setting of the AmericanOpt argument. If AmericanOpt = 0, NaN, or is unspecified, the option is a European or Bermuda option. If AmericanOpt = 1, the option is an American option.	

Strike	<p>European option: NINST-by-1 vector of strike price values.</p> <p>Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.</p> <p>Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.</p> <p>For an American option:</p> <p>NINST-by-1 vector of strike price values for each option.</p>
ExerciseDates	<p>NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.</p> <p>For an American option:</p> <p>NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond <code>Settle</code> and the single listed exercise date.</p>

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [ ].

## Description

`InstSet = instoptbnd(BondIndex,OptSpec,Strike,ExerciseDates)` creates a bond option, specified as a European or Bermuda option.

`InstSet = instoptbnd(InstSet,BondIndex,OptSpec,Strike,ExerciseDates)` adds a bond option, specified as a European or Bermuda option, to an existing instrument set.

`InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)` specifies an American option if `AmericanOpt` is set to 1. If `AmericanOpt` is not set to 1, the function specifies a European or Bermuda option.

`[FieldList, ClassList, TypeString] = instoptbnd` displays the classes.

`FieldList` is a number of fields (`NFIELDS-by-1`) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS-by-1` cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

`TypeString` is a character vector specifying the type of instrument added. For a bond option instrument, `TypeString = 'OptBond'`.

## Examples

### Create a Bond Option Instrument

Create a new instrument variable with the following information:

```
BondIndex = 1;
OptSpec = 'call';
Strike= 85;
ExerciseDates = 'Nov-1-2014';
AmericanOpt = 1;
CouponRate= [0.035;0.04];
Settle= 'Nov-1-2013';
Maturity = 'Nov-1-2014';
Period =1;
```

Create the instrument portfolio with two bonds.

```
InstSet = instbond(CouponRate, Settle, Maturity, ...
Period)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
```

```
Type: {'Bond'}
FieldName: {{11x1 cell}}
FieldClass: {{11x1 cell}}
FieldData: {{11x1 cell}}
```

Create an option on the first bond

```
InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {2x1 cell}
    FieldName: {2x1 cell}
    FieldClass: {2x1 cell}
    FieldData: {2x1 cell}
```

Display the instrument set.

```
instdisp(InstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.035	01-Nov-2013	01-Nov-2014	1	0	1	NaN
2	Bond	0.04	01-Nov-2013	01-Nov-2014	1	0	1	NaN

Index	Type	UnderInd	OptSpec	Strike	ExerciseDates	AmericanOpt
3	OptBond	1	call	85	01-Nov-2014	1

## See Also

[hjmprice](#) | [instadd](#) | [instdisp](#) | [instget](#)

## Topics

“Creating Instruments or Properties” on page 1-21

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## instoptembnd

Construct bond with embedded option

### Syntax

```
InstSet = instoptembnd
(CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates,'AmericanOpt',
AmericanOpt,'Period',Period,'Basis',Basis,'EndMonthRule',EndMonth
Rule,'Face',Face,
'IssueDate',IssueDate,'FirstCouponDate',FirstCouponDate,'LastCouponD
ate',LastCouponDate,'StartDate',StartDate)
InstSet = instoptembnd(InstSetOld, CouponRate,...)
[FieldList,ClassList,TypeString] = instoptembnd
```

### Arguments

CouponRate	Decimal annual rate indicating the annual percentage rate used to determine the coupons payable on a bond. CouponRate is a NINST-by-1 vector or NINST-by-1 cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid.
Settle	NINST-by-1 vector of settlement dates.
Maturity	NINST-by-1 vector of maturity dates.
OptSpec	NINST-by-1 vector of character vector values for 'Call' or 'Put'.
For a European or Bermuda option	



Strike	NINST-by-NSTRIKES matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaN's.
ExerciseDates	NINST-by-NSTRIKES matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one ExerciseDate on the option expiry date.
AmericanOpt	(Optional) NINST-by-1 vector of flags. AmericanOpt is 0 for each European or Bermuda option. The default is 0 if AmericanOpt is NaN or not entered.
For an American option	
Strike	NINST-by-1 vector of strike price values for each option.
ExerciseDates	NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed ExerciseDate.
AmericanOpt	NINST-by-1 vector of flags. AmericanOpt is 1 for each American option. The AmericanOpt argument is required to invoke American exercise rules.
Period	(Optional) NINST-by-1 matrix for coupons per year. The default value is 2.

<p><b>Basis</b></p>	<p>(Optional) Day-count basis of the instrument. <b>Basis</b> is a vector of integers with the following possible values:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>EndMonthRule</b></p>	<p>(Optional) NINST-by-1 matrix for the end-of-month rule. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. When the value is 0, the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is 1, the end-of-month rule is set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
<p><b>IssueDate</b></p>	<p>(Optional) NINST-by-1 matrix for the bond issue date.</p>

<b>FirstCouponDate</b>	(Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When <b>FirstCouponDate</b> and <b>LastCouponDate</b> are both specified, <b>FirstCouponDate</b> takes precedence in determining the coupon payment structure. If you do not specify a <b>FirstCouponDate</b> , the cash flow payment dates are determined from other inputs.
<b>LastCouponDate</b>	(Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified <b>FirstCouponDate</b> , a specified <b>LastCouponDate</b> determines the coupon structure of the bond. The coupon structure of a bond is truncated at the <b>LastCouponDate</b> , regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a <b>LastCouponDate</b> , the cash flow payment dates are determined from other inputs.
<b>StartDate</b>	(Optional) NINST-by-1 matrix for date when a bond actually starts (that is, the date from which a bond's cash flows can be considered). To make an instrument forward starting, specify this date as a future date. If <b>StartDate</b> is not explicitly specified, the effective start date is the <b>Settle</b> date.
<b>Face</b>	(Optional) <b>Face</b> is a NINST-by-1 vector or NINST-by-1 cell array of face values, or face value schedules. For the latter case, each element of the cell array is a NumDates-by-2 cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100.

**Note** Data arguments are NINST-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [].

## Description

InstSet = instoptembnd  
 (CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates,'AmericanOp

`t',AmericanOpt','Period',Period,'Basis',Basis,'EndMonthRule',EndMonthRule,'Face',Face,'IssueDate',IssueDate,'FirstCouponDate',FirstCouponDate,'LastCouponDate',LastCouponDate,'StartDate',StartDate)` creates `InstSet`, a variable containing a collection of instruments.

---

**Note** `instopembnd` uses optional parameter name-value pairs such that, `'Name1', Value1, 'Name2', Value2`, and so on, are a variable length list of name-value pairs.

---

Instruments are broken down by type and each type can have different data fields. Each stored data field has a row vector or character vector for each instrument. See `instget` for more information on the `InstSet` variable.

`InstSet = instoptembnd(InstSetOld, CouponRate, ...)` adds `'OptEmBond'` instruments to an instrument variable.

`[FieldList,ClassList,TypeString] = instoptembnd` lists field metadata for the `'OptEmBond'` instrument.

`FieldList` is a number of fields (`NFIELDS-by-1`) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS-by-1` cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are `'dble', 'date', and 'char'`.

`TypeString` is a character vector specifying the type of instrument added. For a bond option instrument, `TypeString = 'OptEmBond'`.

## Examples

### Create a Bond With an Embedded Option

This example shows how to create a bond with an embedded option using the following data.

```
Settle = 'jan-1-2007';  
Maturity = 'jan-1-2010';
```

```

CouponRate = 0.07;
OptSpec = 'call';
Strike= 100;
ExerciseDates= {'jan-1-2008' '01-Jan-2010'};
AmericanOpt=1;
Period = 1;

InstSet = instoptembnd(CouponRate, ...
Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', AmericanOpt, ...
'Period', Period);

% display the instrument
instdisp(InstSet)

Index Type      CouponRate Settle      Maturity      OptSpec Strike ExerciseDates
1      OptEmBond 0.07         01-Jan-2007   01-Jan-2010   call    100    01-Jan-2008

```

## See Also

instadd | instdisp | instget

## Topics

“Creating Instruments or Properties” on page 1-21

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2008a**

## instoptfloat

Create option instrument on floating-rate note or add instrument to current portfolio

### Syntax

```
InstSet = instoptfloat(FloatIndex,OptSpec,Strike,ExerciseDates)
InstSet = instoptfloat(FloatIndex,OptSpec,Strike,ExerciseDates,
AmericanOpt)
```

```
InstSet = instoptfloat(InstSetOld, ___)
```

```
[FieldList,ClassList,TypeString] = instoptfloat
```

### Description

`InstSet = instoptfloat(FloatIndex,OptSpec,Strike,ExerciseDates)` to specify a European option for a floating-rate note.

`InstSet = instoptfloat(FloatIndex,OptSpec,Strike,ExerciseDates, AmericanOpt)` to specify an American or Bermuda option for a floating-rate note.

`InstSet = instoptfloat(InstSetOld, ___)` to add instruments to an existing portfolio.

`[FieldList,ClassList,TypeString] = instoptfloat` lists the field metadata for the 'OptFloat' instrument.

### Examples

#### Create an Instrument Portfolio with a Call Option for a Floating-Rate Note

Define the floating-rate note:

```
Settle = 'Nov-1-2012';
Maturity = 'Nov-1-2015';
```

```
Spread = 50;
Reset = 1;
```

Create InstSet:

```
InstSet = instfloat(Spread, Settle, Maturity, Reset)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {'Float'}
    FieldName: {{9x1 cell}}
    FieldClass: {{9x1 cell}}
    FieldData: {{9x1 cell}}
```

Display the instrument:

```
instdisp(InstSet)
```

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRu
1	Float	50	01-Nov-2012	01-Nov-2015	1	0	100	1

Add a European call option to the instrument portfolio:

```
OptSpec = 'call';
Strike = 100;
ExerciseDates = 'Nov-1-2015';
```

Create InstSet:

```
InstSet = instoptfloat(InstSet, 1, OptSpec, Strike, ExerciseDates)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {2x1 cell}
    FieldName: {2x1 cell}
    FieldClass: {2x1 cell}
    FieldData: {2x1 cell}
```

Display the instrument:

```
instdisp(InstSet)
```

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	EndMonthRu
1	Float	50	01-Nov-2012	01-Nov-2015	1	0	100	1

Index	Type	UnderInd	OptSpec	Strike	ExerciseDates	AmericanOpt
2	OptFloat	1	call	100	01-Nov-2015	0

## Input Arguments

### FloatIndex — Indices pointing to underlying instruments

vector of nonnegative integers

Indices pointing to underlying instruments of Type 'Float' specified by a NINST-by-1 vector. The instruments of Type 'Float' are also stored in the InstSet variable. For more information, see `instfloat`.

Data Types: double

### OptSpec — Definition of option

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: char | cell

### Strike — Option strike price values for European, Bermuda, or American option

nonnegative integer | vector of nonnegative integers

Option strike price values for option (European, Bermuda, or American) specified as nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

- For a European or Bermuda option — NINST-by-NSTRIKES matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American Option — NINST-by-1 vector of strike price values for each option.

Data Types: single | double



**ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American), specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates, depending on the option type.

- For a European or Bermuda option — NINST-by-NSTRIKES matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date
- For an American option — NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between the underlying bond `Settle` date and the single listed `ExerciseDate`.

Data Types: double | char | cell

**AmericanOpt — Option type**

0 if `AmericanOpt` is `NaN` or not entered (default) | scalar | vector of positive integers[0, 1]

Option type specified as NINST-by-1 positive integer scalar flags with values.

- For a European or Bermuda option — `AmericanOpt` is 0 for each European or Bermuda option.
- For an American option — `AmericanOpt` is 1 for each American option. The `AmericanOpt` argument is required to invoke American exercise rules.

Data Types: single | double

**InstSetOld — Variable containing an existing collection of instruments**

struct

Variable containing an existing collection of instruments, specified as a struct. For more information on the `InstSet` variable, see `instget`.

Data Types: struct

## Output Arguments

### **InstSet — Variable containing a collection of instruments**

scalar | vector

Variable containing a collection of instruments returned as a scalar or vector with the instruments broken down by type and each type can have different data fields. Each stored data field has a row vector or character vector for each instrument. For more information on the `InstSet` variable, see `instget`.

### **FieldList — Data field for instrument type**

character vector | cell array of character vectors

Data field for instrument type returned as a `NFIELDS`-by-1 cell array of character vectors listing the name of each data field for this instrument type.

### **ClassList — Data class of each field**

character vector with value: 'dble', 'date', 'char' | cell array of character vectors with values: 'dble', 'date', 'char'

Data class of each field returned as a `nFIELDS`-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed.

### **TypeString — Type of instrument added**

character vector with value 'OptFloat'

Type of instrument added returned as a character vector. The character vector for a floating-rate option instrument is `TypeString = 'OptFloat'`.

## See Also

`instadd` | `instoptemfloat`

## Topics

“Creating Instruments or Properties” on page 1-21

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# instoptemfloat

Create embedded option instrument on floating-rate note or add instrument to current portfolio

## Syntax

```
InstSet = instoptemfloat(Spread,Settle,Maturity,OptSpec,Strike,  
ExerciseDates)
```

```
InstSet = instoptemfloat( ____,Name,Value)
```

```
InstSet = instoptemfloat(InstSetOld,Spread,Settle,Maturity,OptSpec,  
Strike,ExerciseDates)
```

```
[FieldList,ClassList,TypeString] = instoptemfloat
```

## Description

`InstSet = instoptemfloat(Spread,Settle,Maturity,OptSpec,Strike,ExerciseDates)` creates an embedded option instrument for a floating-rate note.

`InstSet = instoptemfloat( ____,Name,Value)` adds optional name-value pair arguments.

`InstSet = instoptemfloat(InstSetOld,Spread,Settle,Maturity,OptSpec,Strike,ExerciseDates)` to add 'OptEmFloat' instruments to an instrument variable.

`[FieldList,ClassList,TypeString] = instoptemfloat` lists field metadata for the 'OptEmFloat' instrument.

## Examples

### Create an Instrument Portfolio with a Embedded Option Floating-Rate Note

Define the embedded call option:

```
Settle = 'Nov-1-2012';
Maturity = 'Nov-1-2015';
Spread = 25;
OptSpec = 'call';
Strike = 100;
ExerciseDates = 'Nov-1-2015';
Reset = 1;
```

Create InstSet:

```
InstSet = instoptemfloat(Spread, Settle, Maturity, OptSpec, ...
Strike, ExerciseDates, 'FloatReset', Reset)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {'OptEmFloat'}
    FieldName: {{13x1 cell}}
    FieldClass: {{13x1 cell}}
    FieldData: {{13x1 cell}}
```

Display the instrument:

```
instdisp(InstSet)
```

Index	Type	Spread	Settle	Maturity	OptSpec	Strike	ExerciseDates	FL
1	OptEmFloat	25	01-Nov-2012	01-Nov-2015	call	100	01-Nov-2015	1

## Input Arguments

### Spread — Number of basis points over the reference rate

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: single | double

### Settle — Settlement dates of floating-rate note

ValuationDate of HW Tree (default) | serial date number | character vector | cell array of character vectors

Settlement dates of floating-rate note, specified as serial date numbers or date character vectors using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

### **Maturity — Floating-rate note maturity date**

serial date number | character vector | cell array of character vectors

Floating-rate note maturity date, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

### **OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: char | cell

### **Strike — Embedded option strike price values**

nonnegative integer | vector of nonnegative integers

Embedded option strike price values for option specified as nonnegative integers using as NINST-by-NSTRIKES or NINST-by-1 vector of strike price values, depending on the type of option.

- For a European or Bermuda Option — NINST-by-NSTRIKES matrix of strike price values where each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American Option — NINST-by-1 vector of strike price values for each option.

Data Types: single | double

### **ExerciseDates — Exercise date for embedded option**

serial date nonnegative number | vector of serial date nonnegative numbers | date character vector | cell array of date character vectors

Exercise date for embedded option, specified as serial date nonnegative numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of the option exercise dates, depending on the type of option.

- For a European or Bermuda Option — NINST-by-NSTRIKES of exercise dates where each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American Option — NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between the underlying bond `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

### **InstSetOld — Variable containing an existing collection of instruments**

struct

Variable containing an existing collection of instruments, specified as a struct. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument. For more information on instrument data parameters, see the reference entries for individual instrument types. For example, see `instfloat` for additional information on the float instrument.

Data Types: `struct`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `InstSet = instoptemfloat(Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'FloatReset', Reset)`

### **AmericanOpt — Embedded option type**

0 if `AmericanOpt` is `NaN` or not entered (default) | scalar | vector of positive integers[0, 1]

Embedded option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and a NINST-by-1 positive integer scalar flags with values:

- For a European or Bermuda option — `AmericanOpt` is 0 for each European or Bermuda option. The default is 0 if `AmericanOpt` is `NaN` or not entered.

- For an American option — `AmericanOpt` is 1 for each American option. The `AmericanOpt` argument is required to invoke American exercise rules.

Data Types: `single` | `double`

### **FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 4, 6, 12] in a NINST-by-1 vector.

Data Types: `single` | `double`

### **Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The `Basis` value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `single` | `double`

### **Principal — Principal values**

100 (default) | nonnegative integer | vector of nonnegative integers | cell array of nonnegative integers

Principal values, specified as the comma-separated pair consisting of 'Principal' and a nonnegative integer using a NINST-by-1 vector of notional principal amounts.

Data Types: `single` | `double`

### **Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure using `derivset`.

Data Types: `struct`

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: `single` | `double`

## **Output Arguments**

### **InstSet — Variable containing a collection of instruments**

scalar | vector

Variable containing a collection of instruments returned as a scalar or vector with the instruments broken down by type and each type can have different data fields. Each



stored data field has a row vector or character vector for each instrument. For more information on the InstSet variable, see instget.

**FieldList — Name of each data field**

character vector | cell array of character vectors

NFIELDS-by-1 cell array of character vectors listing the name of each data field for this instrument type.

**ClassList — Determines how arguments are parsed**

character vector with value: 'dble', 'date', or 'char' | cell array of character vectors with values: 'dble', 'date', or 'char'

NFIELDS-by-1 cell array of character vectors listing the data class of each field.

**TypeString — Type of instrument added**

character vector with value 'OptEmFloat'

Character vector specifying the type of instrument added where TypeString = 'OptEmFloat'.

## See Also

instadd | instoptfloat

## Topics

“Creating Instruments or Properties” on page 1-21

**basis** on page Glossary-0

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

## instoptstock

Construct stock option

### Syntax

```
InstSet = instoptstock(OptSpec,Strike,Settle,ExerciseDates)
InstSet = instoptstock(InstSet,OptSpec,Strike,Settle,ExerciseDates)
InstSet =
instoptstock(InstSet,OptSpec,Strike,Settle,ExerciseDates,AmericanOpt
)
[FieldList,ClassList,TypeString] = instoptstock
```

### Arguments

InstSet	Instrument variable. This argument is specified only when adding stock option instruments to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
OptSpec	NINST-by-1 list of character vector values 'Call' or 'Put'.
<b>Note</b> The interpretation of the <code>Strike</code> and <code>ExerciseDates</code> arguments depends upon the setting of the <code>AmericanOpt</code> argument. If <code>AmericanOpt</code> = 0, NaN, or is unspecified, the option is a European or Bermuda option. If <code>AmericanOpt</code> = 1, the option is an American option.	
Strike	<p>European option: NINST-by-1 vector of strike price values.</p> <p>Bermuda option: NINST by number of strikes (NSTRIKES) matrix of strike price values.</p> <p>Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.</p> <p>American option: NINST-by-1 vector of strike price values for each option.</p>

Settle	NINST-by-1 vector of settlement dates.
ExerciseDates	<p>NINST-by-1 (European option) or NINST-by-NSTRIKES (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.</p> <p>For an American option:</p> <p>NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying bond Settle and the single listed exercise date.</p>

Data arguments are NINST-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices [ ].

## Description

`InstSet = instoptstock(OptSpec, Strike, Settle, ExerciseDates)` creates a stock option instrument, specified as a European or Bermuda option.

`InstSet = instoptstock(InstSet, OptSpec, Strike, Settle, ExerciseDates)` adds a stock option instrument, specified as a European or Bermuda option, to an existing instrument set.

`InstSet = instoptstock(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)` specifies an American option if `AmericanOpt` is set to 1. If `AmericanOpt` is not set to 1, the function specifies a European or Bermuda option.

`[FieldList, ClassList, TypeString] = instoptstock` displays the classes.

`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS-by-1` cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a character vector specifying the type of instrument added. For a stock option instrument, `TypeString = 'OptStock'`.

## Examples

### Create a Stock Option Instrument

Create an instrument set of two stock options with the following data:

```
OptSpec = {'put';'call'};  
Strike = [95;98];  
Settle = '01-May-2012';  
ExerciseDates = {'01-May-2014';'01-May-2015'};  
AmericanOpt = [0;1];
```

Create the stock option instruments.

```
InstSet = instoptstock(OptSpec, Strike,Settle, ExerciseDates, AmericanOpt)
```

```
InstSet = struct with fields:  
    FinObj: 'Instruments'  
    IndexTable: [1x1 struct]  
    Type: {'OptStock'}  
    FieldName: {{5x1 cell}}  
    FieldClass: {{5x1 cell}}  
    FieldData: {{5x1 cell}}
```

Display the instrument set.

```
instdisp(InstSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt
1	OptStock	put	95	01-May-2012	01-May-2014	0
2	OptStock	call	98	01-May-2012	01-May-2015	1

## **See Also**

instadd | instdisp | instget

## **Topics**

“Pricing Equity Derivatives Using Trees” on page 3-130

“Creating Instruments or Properties” on page 1-21

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## instrangefloat

Construct range note instrument

### Syntax

```
ISet =  
instrangefloat(Spread,Settle,Maturity,RateSched,Reset,Basis,Principa  
l,EndMonthRule)  
ISet =  
instrangefloat(ISet,Spread,Settle,Maturity,RateSched,Reset,Basis,Pri  
ncipal,EndMonthRule)
```

### Description

ISet =  
instrangefloat(Spread,Settle,Maturity,RateSched,Reset,Basis,Principa  
l,EndMonthRule) creates a range instrument from data arrays.

ISet =  
instrangefloat(ISet,Spread,Settle,Maturity,RateSched,Reset,Basis,Pri  
ncipal,EndMonthRule) adds a new range instrument to an existing instrument set.

### Input Arguments

#### Spread

Number of basis points over the reference rate.

#### Settle

NINST-by-1 vector of dates representing the settle date of the floating-rate note.

#### Maturity

NINST-by-1 vector of dates representing the maturity date of the floating-rate note.

**RateSched**

NINST-by-1 vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- `RateSched.Dates` — NDates-by-1 cell array of dates corresponding to the range schedule.
- `RateSched.Rates` — NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates(n)` is nonzero for rates in the range `RateSched.Rates(n,1) < Rate < RateSched.Rates(n,2)`.

**Reset**

(Optional) NINST-by-1 vector representing the frequency of payments per year.

**Default: 1****Basis**

(Optional) Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

**Default:** 0 (actual/actual)

### **Principal**

(Optional) NINST-by-1 vector of the notional principal amount.

**Default:** 100

### **EndMonthRule**

(Optional) NINST-by-1 vector for end-of-month rule. Values are 1 (in effect) and 0 (not in effect).

**Default:** 1 (in effect)

---

**Note** Data arguments are number of instruments NINST-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with NaN. Only one data argument is required to create the instrument. You can omit or pass the others as empty matrices [ ]. However, you cannot price the instrument when using the range note pricing function if you are missing any of the required input arguments.

---

## **Output Arguments**

### **ISet**

Variable containing a collection of instruments. Instruments are divided by type and each type can have different data fields. Each stored data field has a row vector or character vector for each instrument. Values are:

- **FieldList** — NFIELDS-by-1 cell array of character vectors listing the name of each data field for this instrument type.
- **ClassList** — NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.
- **TypeString** — Character vector specifying the type of instrument added. TypeString = 'RangeFloat'.

For more information, on ISet see `instget`.



## Examples

### Create a Range Note Instrument

Create an instrument portfolio with a range note.

```
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

RateSched.Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched.Rates = [0.045 0.055; 0.0525 0.0675; 0.06 0.08];

% Create InstSet
InstSet = instrangefloat(Spread, Settle, Maturity, RateSched);

% Display the portfolio instrument
instdisp(InstSet)
```

Index	Type	Spread	Settle	Maturity	RateSched	FloatReset	Basis	Princ
1	RangeFloat	100	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100

Add a second range note instrument to the portfolio. Second Range Note:

```
Spread2 = 200;
Settle2 = 'Jan-1-2011';
Maturity2 = 'Jan-1-2013';
RateSched2.Dates = {'Jan-1-2012'; 'Jan-1-2013'};
RateSched2.Rates = [0.048 0.059; 0.055 0.068];

InstSet = instrangefloat(InstSet, Spread2, Settle2, Maturity2, RateSched2);

% Display the portfolio instrument
instdisp(InstSet)
```

Index	Type	Spread	Settle	Maturity	RateSched	FloatReset	Basis	Princ
1	RangeFloat	100	01-Jan-2011	01-Jan-2014	[Struct]	1	0	100
2	RangeFloat	200	01-Jan-2011	01-Jan-2013	[Struct]	1	0	100

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range.

If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets direct exposure to the reference rate. In return for the drawback that no interest is paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## References

Jarrow, Robert. "Modelling Fixed Income Securities and Interest Rate Options." *Stanford Economics and Finance*. 2nd Edition. 2002.

## See Also

`instaddfield` | `instbond` | `instcap` | `instdisp` | `instswap` | `intenvprice` | `rangefloatbybdt` | `rangefloatbybk` | `rangefloatbyhjm` | `rangefloatbyhw`

## Topics

"Creating Instruments or Properties" on page 1-21

"Supported Interest-Rate Instruments" on page 2-2

**Introduced in R2012a**

# instselect

Create instrument subset by matching conditions

## Syntax

```
InstSubSet =
instselect(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet, 'Type', TypeList)
```

## Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
FieldList	Number of fields, specified as a NFIELDS-by-1 cell array of character vectors listing the name of each data field to match with data values.
DataList	Number of values (NVALUES-by-M) array or NFIELDS-by-1 cell array of acceptable data values for each field. Each row lists a data row value to search for in the corresponding FieldList. The number of columns is arbitrary and matching ignores trailing NaNs or spaces.
IndexSet	(Optional) Number of instruments, specified as a NINST-by-1 vector restricting positions of instruments to check for matches. The default is all indices available in the instrument variable.
TypeList	(Optional) Number of types, specified as a NTYPES-by-1 cell array of character vectors restricting instruments to match one of TypeList types. The default is all types in the instrument variable.

Argument value pairs can be entered in any order. The InstSet variable must be the first argument. 'FieldName' and 'Data' arguments must appear together or not at all. 'Index' and 'Type' arguments are each optional.

## Description

`InstSubSet = instselect(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet, 'Type', TypeList)` creates an instrument subset (`InstSubSet`) from an existing set of instruments (`InstSet`).

`InstSubSet` is a variable containing instruments matching the input criteria. Instruments are returned in `InstSubSet` if all the `Field`, `Index`, and `Type` conditions are met. An instrument meets an individual `Field` condition if the stored `FieldName` data matches any of the rows listed in the `DataList` for that `FieldName`. See `instfind` for examples on matching criteria.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. The variable contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples
instdisp(ExampleInst)
```

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	100	9.2	Call	0
3	Option	105	6.8	Call	1000

Index	Type	Delivery	F	Contracts
4	Futures	01-Jul-1999	104.4	-1000

Index	Type	Strike	Price	Opt	Contracts
5	Option	105	7.4	Put	-1000
6	Option	95	2.9	Put	0

Index	Type	Price	Maturity	Contracts
7	TBill	99	01-Jul-1999	6

Make a new portfolio containing only options struck at 95.

```
Opt95 = instselect(ExampleInst, 'FieldName', 'Strike', ...
'Data', '95')
instdisp(Opt95)
```

Opt95 =

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	95	2.9	Put	0

Make a new portfolio containing only futures and Treasury bills.

```
FutTBill = instselect(ExampleInst, 'Type', {'Futures'; 'TBill'})
```

instdisp(FutTBill) =

Index	Type	Delivery	F	Contracts
1	Futures	01-Jul-1999	104.4	-1000

Index	Type	Price	Maturity	Contracts
2	TBill	99	01-Jul-1999	6

## See Also

`instaddfield` | `instdelete` | `instfind` | `instget` | `instgetcell`

## Topics

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**

## instsetfield

Add or reset data for existing instruments

### Syntax

```
InstSet =  
instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList)  
InstSet =  
instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', I  
ndexSet, 'Type', TypeList)
```

### Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument. <code>InstSet</code> must be the first argument in the list.
FieldList	Number of fields, specified as a <code>NFIELDS-by-1</code> cell array of character vectors listing the name of each data field. <code>FieldList</code> cannot be named with the reserved names <code>Type</code> or <code>Index</code> .
DataList	Number of instruments, specified as a <code>NINST-by-M</code> array or <code>NFIELDS-by-1</code> cell array of data contents for each field. Each row in a data array corresponds to a separate instrument. Single rows are copied to apply to all instruments to be worked on. The number of columns is arbitrary, and data is padded along columns.
IndexSet	<code>NINST-by-1</code> vector of positions of instruments to work on. If <code>TypeList</code> is also entered, instruments referenced must be one of <code>TypeList</code> types and contained in <code>IndexSet</code> .
TypeList	Number of types, specified as a <code>NTYPES-by-1</code> cell array of character vectors restricting instruments worked on to match one of <code>TypeList</code> types.

Argument value pairs can be entered in any order.

## Description

`instsetfield` sets data for existing instruments in a collection variable.

```
InstSet =
instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList) resets or
adds fields to every instrument.
```

```
InstSet =
instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', I
ndexSet, 'Type', TypeList) resets or adds fields to a subset of instruments.
```

The output `InstSet` is a new instrument set variable containing the input data.

## Examples

Retrieve the instrument set `ExampleInstSF` from the data file `InstSetExamples.mat`. `ExampleInstSF` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
ISet = ExampleInstSF;
instdisp(ISet)
```

```
Index Type    Strike Price Opt
1    Option  95    12.2 Call
2    Option 100    9.2  Call
3    Option 105    6.8  Call
```

```
Index Type    Delivery      F
4    Futures 01-Jul-1999  104.4
```

```
Index Type    Strike Price Opt
5    Option 105    7.4  Put
6    Option NaN    NaN  Put
```

```
Index Type    Price
7    TBill 99
```

Enter data for the option in Index 6: Price 2.9 for a Strike of 95.

```
ISet = instsetfield(ISet, 'Index',6,...
'FieldName',{'Strike','Price'}, 'Data',{ 95 , 2.9 });
instdisp(ISet)
```

```
Index Type  Strike Price Opt
1      Option  95     12.2  Call
2      Option  100    9.2   Call
3      Option  105    6.8   Call
Index Type  Delivery      F
4      Futures 01-Jul-1999  104.4
Index Type  Strike Price Opt
5      Option  105    7.4   Put
6      Option  95     2.9   Put
```

```
Index Type  Price
7      TBill  99
```

Create a field Maturity for the cash instrument.

```
MDate = datenum('7/1/99');
ISet = instsetfield(ISet, 'Type', 'TBill', 'FieldName',...
'Maturity','FieldClass', 'date', 'Data', MDate);
instdisp(ISet)
```

```
Index Type  Price  Maturity
7      TBill  99     01-Jul-1999
```

Create a field Contracts for all instruments.

```
ISet = instsetfield(ISet, 'FieldName', 'Contracts', 'Data', 0);
instdisp(ISet)
```

```
Index Type  Strike Price Opt  Contracts
1      Option  95     12.2  Call  0
2      Option  100    9.2   Call  0
3      Option  105    6.8   Call  0

Index Type  Delivery      F      Contracts
4      Futures 01-Jul-1999  104.4  0

Index Type  Strike Price Opt  Contracts
5      Option  105    7.4   Put  0
6      Option  95     2.9   Put  0

Index Type  Price  Maturity  Contracts
7      TBill  99     01-Jul-1999  0
```



Set the Contracts fields for some instruments.

```
ISet = instsetfield(ISet, 'Index', [3; 5; 4; 7], ...
'FieldName', 'Contracts', 'Data', [1000; -1000; -1000; 6]);
```

```
instdisp(ISet)
```

Index	Type	Strike	Price	Opt	Contracts
1	Option	95	12.2	Call	0
2	Option	100	9.2	Call	0
3	Option	105	6.8	Call	1000

Index	Type	Delivery	F	Contracts
4	Futures	01-Jul-1999	104.4	-1000

Index	Type	Strike	Price	Opt	Contracts
5	Option	105	7.4	Put	-1000
6	Option	95	2.9	Put	0

Index	Type	Price	Maturity	Contracts
7	TBill	99	01-Jul-1999	6

## See Also

[instaddfield](#) | [instdisp](#) | [instget](#) | [instgetcell](#)

## Topics

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**

## instswap

Construct swap instrument

### Syntax

```

InstSet = instswap(LegRate,Settle,Maturity)
InstSet = instswap(InstSet,LegRate,Settle,Maturity)
InstSet =
instswap(InstSet,LegRate,Settle,Maturity,InstSet,LegReset,Basis,Prin
cipal,LegType,EndMonthRule)
InstSet =
instswap(InstSet,LegRate,Settle,Maturity,LegReset,Basis,Principal,Le
gType,EndMonthRule,StartDate)
[FieldList,ClassList,TypeString] = instswap
    
```

### Arguments

InstSet	Instrument variable. This argument is specified only when adding a swap to an existing instrument set. See <code>instget</code> for more information on the <code>InstSet</code> variable.
LegRate	Leg rate, specified as a NINST-by-2 matrix, with each row defined as:  [CouponRate Spread] (fixed-float), [Spread CouponRate] (float-fixed), [CouponRate CouponRate] (fixed-fixed), or [Spread Spread] (float-float).  CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.
Settle	Settlement date. NINST-by-1 vector of serial date numbers or date character vectors. Settle must be earlier than Maturity.
Maturity	Maturity date. NINST-by-1 vector of dates representing the maturity date for each swap.

<b>LegReset</b>	(Optional) NINST-by-2 matrix representing the reset frequency per year for each swap. Default = [1 1].
<b>Basis</b>	<p>(Optional) Day-count basis representing the basis for each leg. NINST-by-1 array (or NINST-by-2 if Basis is different for each leg).</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<b>Principal</b>	(Optional) Principal accepts a NINST-by-1 vector or NINST-by-1 cell array (or NINST-by-2 if Principal is different for each leg) of the notional principal amounts or principal value schedules. For schedules, each element of the cell array is a NumDates-by-2 matrix where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid. Default = 100.
<b>LegType</b>	(Optional) NINST-by-2 matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate. Default is [1, 0] for each instrument.
<b>EndMonthRule</b>	(Optional) NINST-by-1 (or NINST-by-2 if EndMonthRule is different for each leg). Default = 1.

<b>StartDate</b>	(Optional) NINST-by-1 vector of dates when the swaps actually start. Default is <code>Settle</code> .
------------------	---

Data arguments are number of instruments (NINST)-by-1 vectors, scalar, or empty. Fill in unspecified entries vectors with `NaN`. Only one data argument is required to create the instrument; the others may be omitted or passed as empty matrices [ ].

## Description

`InstSet = instswap(LegRate,Settle,Maturity)` creates a new instrument set containing swap instruments.

`InstSet = instswap(InstSet,LegRate,Settle,Maturity)` adds swap instruments to an existing instrument set.

`InstSet = instswap(InstSet,LegRate,Settle,Maturity,InstSet,LegReset,Basis,Principal,LegType,EndMonthRule)` uses optional input arguments to create a new instrument set containing swap instruments or adds swap instruments to an existing instrument set.

`InstSet = instswap(InstSet,LegRate,Settle,Maturity,LegReset,Basis,Principal,LegType,EndMonthRule,StartDate)` to create a new forward swap instrument or to add a forward swap instrument to an existing portfolio.

`[FieldList,ClassList,TypeString] = instswap` displays the classes.

`FieldList` is a number of fields (NFIELDS-by-1) cell array of character vectors listing the name of each data field for this instrument type.

`ClassList` is an NFIELDS-by-1 cell array of character vectors listing the data class of each field. The class determines how arguments are parsed. Valid character vectors are 'dble', 'date', and 'char'.

`TypeString` is a character vector specifying the type of instrument added. For a swap instrument, `TypeString = 'Swap'`.

## Examples

### Create a Vanilla Swap Instrument

Create a vanilla swap using market data.

Use the following market data to create a swap instrument.

```
LegRate = [0.065, 0]
```

```
LegRate = 1x2
```

```
    0.0650    0
```

```
Settle = 'jan-1-2007';
Maturity = 'jan-1-2012';
LegReset = [1, 1];
Basis = 0
```

```
Basis = 0
```

```
Principal = 100
```

```
Principal = 100
```

```
LegType = [1, 0]
```

```
LegType = 1x2
```

```
    1    0
```

```
InstSet = instswap(LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
InstSet = struct with fields:
    FinObj: 'Instruments'
    IndexTable: [1x1 struct]
        Type: {'Swap'}
    FieldName: {{9x1 cell}}
    FieldClass: {{9x1 cell}}
    FieldData: {{9x1 cell}}
```

View the swap instrument using `instdisp`.

```
instdisp(InstSet)
```

Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	End
1	Swap	[0.065 0]	01-Jan-2007	01-Jan-2012	[1 1]	0	100	[1 0]	1

### Create a Float-Float Swap and Price with `intenvprice`

Use `instswap` to create a float-float swap and price the swap with `intenvprice`.

```
RateSpec = intenvset('Rates',.05,'StartDate',today,'EndDate',datemnth(today,60));
IS = instswap([40 20],today,datemnth(today,60),[], [], [], [0 0]);
intenvprice(RateSpec,IS)
```

```
ans =
```

```
0.8644
```

### Create Float-Float, Fixed-Fixed, and Float-Fixed Swaps and Price with `intenvprice`

Use `instswap` to create swaps and price the swaps with `intenvprice`.

```
RateSpec = intenvset('Rates',.05,'StartDate',today,'EndDate',datemnth(today,60));
IS = instswap([.03 .02],today,datemnth(today,60),[], [], [], [1 1]);
IS = instswap(IS,[200 300],today,datemnth(today,60),[], [], [], [0 0]);
IS = instswap(IS,[300 .07],today,datemnth(today,60),[], [], [], [0 1]);
intenvprice(RateSpec,IS)
```

```
ans = 3×1
```

```
4.3220
-4.3220
4.5921
```

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

### See Also

[hjmprice](#) | [instaddfield](#) | [instbond](#) | [instcap](#) | [instdisp](#) | [instfloor](#) | [intenvprice](#)

### Topics

“Creating Instruments or Properties” on page 1-21  
“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## instswaption

Construct swaption instrument

### Syntax

```
InstSet = instswaption(OptSpec,Strike,ExerciseDates,Spread,Settle,  
Maturity)  
InstSet = instswaption( ____,AmericanOpt,SwapReset,Basis,Principal)  
InstSet = instswaption(InstSetOld, ____)  
[FieldList,ClassList,TypeString] = instswaption
```

### Description

InstSet = instswaption(OptSpec,Strike,ExerciseDates,Spread,Settle, Maturity) to specify a European swaption.

Fill in unspecified entries vectors with the value NaN. Only one data argument is required to create the instruments; the others may be omitted or passed as empty matrices [].

InstSet = instswaption( \_\_\_\_,AmericanOpt,SwapReset,Basis,Principal) to specify an American swaption.

InstSet = instswaption(InstSetOld, \_\_\_\_) to add swaption instruments to an instrument variable.

[FieldList,ClassList,TypeString] = instswaption to list field metadata for the swaption instrument.

### Examples

#### Create Two Swaption Instruments

This example shows how to create two European swaption instruments using the following data.



```

OptSpec = {'Call'; 'Put'};
Strike = .05;
ExerciseDates = 'jan-1-2011';
Spread=0;
Settle = 'jan-1-2007';
Maturity = 'jan-1-2012';
AmericanOpt = 0;

InstSet = instswaption(OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, ...
    AmericanOpt);

% view the European swaption instruments using instdisp
instdisp(InstSet)

```

Index	Type	OptSpec	Strike	ExerciseDates	Spread	Settle	Maturity	Amer
1	Swaption	Call	0.05	01-Jan-2011	0	01-Jan-2007	01-Jan-2012	0
2	Swaption	Put	0.05	01-Jan-2011	0	01-Jan-2007	01-Jan-2012	0

### Create Two European Swaption Instruments with Receiving and Paying Legs

This example shows how to create two European swaption instruments with receiving and paying legs using the following data.

```

OptSpec = {'Call'; 'Put'};
Strike = .05;
ExerciseDates = 'jan-1-2011';
Spread=0;
Settle = 'jan-1-2007';
Maturity = 'jan-1-2012';
AmericanOpt = 0;
SwapReset = [2 4]; % 1st column represents receiving leg, 2nd column represents paying
Basis = [1 3]; % 1st column represents receiving leg, 2nd column represents paying

InstSet = instswaption(OptSpec,Strike,ExerciseDates,AmericanOpt,Spread,Settle,Maturity,
    SwapReset,Basis);

```

View the European swaption instruments using instdisp.

```

instdisp(InstSet)

```

Index	Type	OptSpec	Strike	ExerciseDates	Spread	Settle	Maturity	AmericanOpt	S
1	Swaption	Call	0.05	01-Jan-2011	0	0	01-Jan-2007	NaN	2

2      Swaption Put      0.05      01-Jan-2011      0      0      01-Jan-2007      NaN

## Input Arguments

### OptSpec — Definition of option

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'. A 'call' swaption entitles the buyer to pay the fixed rate. A 'put' swaption entitles the buyer to receive the fixed rate.

Data Types: char | cell

### Strike — Strike swap rate values

vector

Strike swap rate values, specified as a NINST-by-1 vector.

Data Types: double

### ExerciseDates — Option exercise dates

serial date number | date character vector

Option exercise dates, specified as a vector of date character vectors or serial date numbers, where each row is the schedule for one option and the last element of each row must be the same as the maturity of the tree.

- For a European option, use a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one ExerciseDate on the option expiry date.
- For an American option, use a NINST-by-2 vector of exercise dates. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is NINST-by-1, the option can be exercised between the underlying swap Settle and the single listed ExerciseDate.

Data Types: double | char

### Spread — Number of basis points over the reference rate

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate, specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: single | double

### **Settle — Settle date for each swap**

serial date number | date character vector

Settle date for each swap, specified as a NINST-by-1 vector of date character vectors or serial date numbers.

Data Types: char | double

### **Maturity — Maturity date for each swap**

serial date number | date character vector

Maturity date for each swap, specified as a NINST-by-1 vector of date character vectors or serial date numbers.

Data Types: char | double

### **AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

(Optional) Option type, specified as NINST-by-1 integer flags with values:

- 0 — European
- 1 — American

The AmericanOpt argument is required to invoke American exercise rules.

Data Types: double

### **SwapReset — Reset frequency per year for each leg**

1 (default) | numeric

(Optional) Reset frequency per year for each leg, specified as a NINST-by-1 vector or NINST-by-2 matrix. If SwapReset is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis of the instrument, specified as a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If Basis is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

(Optional) Notional principal amount, specified as a NINST-by-1 vector.

Data Types: double

### **InstSetOld — Variable containing an existing collection of instruments**

struct

Variable containing an existing collection of instruments, specified as a struct. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument. The `InstSetOld` argument is specified only when adding swaption instruments to an existing instrument set. For more information on the `InstSet` variable, see `instget`.

Data Types: struct

## Output Arguments

### **InstSet** — Variable containing collection of instruments

vector

(Optional) Variable containing a collection of instruments. Instruments are broken down by type and each type can have different data fields. Each stored data field has a row vector or character vector for each instrument. For more information on the InstSet variable, see `instget`.

Data Types: double

### **FieldList** — Name of each data field for this instrument type

cell array of character vectors

Name of each data field for this instrument type, returned as a NFIELDS-by-1 cell array of character vectors.

Data Types: char | cell

### **ClassList** — Data class of each field

character vector with value: 'dble', 'date', or 'char'.

Data class of each field, returned as a NFIELDS-by-1 cell array of character vectors. Valid character vectors are 'dble', 'date', and 'char'.

Data Types: char | cell

### **TypeString** — Type of instrument added

character vector

Type of instrument added, returned as a character vector (for a swaption, `TypeString` = 'Swaption').

Data Types: char

## See Also

`instadd` | `instdisp` | `instget`

**Topics**

“Creating Instruments or Properties” on page 1-21

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# insttypes

List types

## Syntax

```
TypeList = insttypes(InstSet)
```

## Arguments

InstSet	Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.
---------	---

## Description

`TypeList = insttypes(InstSet)` retrieves a list of types stored in an instrument variable.

`TypeList` is a number of types (NTYPES)-by-1 cell array of character vectors listing the Type of instruments contained in the variable.

## Examples

Retrieve the instrument set variable `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: Option, Futures, and TBill.

```
load InstSetExamples;
instdisp(ExampleInst)
```

```
Index Type    Strike Price Opt  Contracts
1      Option   95    12.2 Call    0
```

```
2    Option 100    9.2  Call    0
3    Option 105    6.8  Call  1000
```

```
Index Type    Delivery      F    Contracts
4    Futures 01-Jul-1999  104.4 -1000
```

```
Index Type    Strike Price Opt  Contracts
5    Option 105    7.4  Put  -1000
6    Option 95    2.9  Put    0
```

```
Index Type Price Maturity      Contracts
7    TBill 99    01-Jul-1999  6
```

List all of the types included in `ExampleInst`.

```
TypeList = insttypes(ExampleInst)
```

```
TypeList =
    'Futures'
    'Option'
    'TBill'
```

## See Also

`instdisp` | `instfields` | `instlength`

## Topics

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**



# intenvget

Properties of interest-rate structure

## Syntax

```
ParameterValue = intenvget(RateSpec,ParameterName)
```

## Description

`ParameterValue = intenvget(RateSpec,ParameterName)` obtains the value of the named parameter `ParameterName` extracted from the `RateSpec`.

## Examples

### Extract Values from a RateSpec

Use `intenvset` to set the interest-rate structure.

```
RateSpec = intenvset('Rates',0.05,'StartDates',...  
'20-Jan-2000','EndDates','20-Jan-2001')
```

```
RateSpec = struct with fields:
```

```
    FinObj: 'RateSpec'  
    Compounding: 2  
        Disc: 0.9518  
        Rates: 0.0500  
    EndTimes: 2  
    StartTimes: 0  
    EndDates: 730871  
    StartDates: 730505  
    ValuationDate: 730505  
        Basis: 0  
    EndMonthRule: 1
```

Use `intenvget` to extract values from the `RateSpec`.

```
[R, RateSpec] = intenvget(RateSpec, 'Rates')
```

```
R = 0.0500
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
    Disc: 0.9518
    Rates: 0.0500
    EndTimes: 2
    StartTimes: 0
    EndDates: 730871
    StartDates: 730505
    ValuationDate: 730505
    Basis: 0
    EndMonthRule: 1
```

## Input Arguments

### **RateSpec** — Interest-rate specification

structure

Interest-rate specification, specified by the `RateSpec` obtained previously from `intenvset` or `toRateSpec` for an `IRDataCurve` or `toRateSpec` for an `IRFunctionCurve`.

Data Types: `struct`

### **ParameterName** — Parameter name to be accessed

character vector

Parameter name to be accessed, specified as a character vector. The value of the named parameter is extracted from the structure `RateSpec`. It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for parameter names.

Data Types: `char`

## Output Arguments

**ParameterValue** — Value of the named parameter **ParameterName** extracted from **RateSpec**

scalar

Value of the named parameter 'ParameterName' extracted from RateSpec, returned as a scalar value.

## See Also

intenvset

## Topics

“Modeling the Interest-Rate Term Structure” on page 2-67

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## intenvprice

Price instruments from set of zero curves

### Syntax

```
Price = intenvprice(RateSpecInstSet)
```

### Description

`Price = intenvprice(RateSpecInstSet)` computes arbitrage-free prices for instruments against a set of zero coupon bond rate curves.

`intenvprice` handles the following instrument types: 'Bond', 'CashFlow', 'Fixed', 'Float', 'Swap'. See `instadd` for information about constructing defined types.

### Examples

#### Load Zero Curves and Instruments from Data File

Load the zero curves and instruments.

```
load deriv.mat
instdisp(ZeroInstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	NaN
2	Bond	0.04	01-Jan-2000	01-Jan-2004	2	NaN	NaN	NaN

Index	Type	CouponRate	Settle	Maturity	FixedReset	Basis	Principal	Name
3	Fixed	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	4% Fixed

Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	Name
4	Float	20	01-Jan-2000	01-Jan-2003	1	NaN	NaN	20BP Float

Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	Name
-------	------	---------	--------	----------	----------	-------	-----------	---------	------

```
5      Swap [0.06 20] 01-Jan-2000    01-Jan-2003    [1 1]    NaN    NaN    [NaN]    69
```

Price the instruments.

```
Price = intenvprice(ZeroRateSpec, ZeroInstSet)
```

```
Price = 5x1
```

```
98.7159
97.5334
98.7159
100.5529
3.6923
```

### Create a Float-Float Swap and Price with `intenvprice`

Use `instswap` to create a float-float swap and price the swap with `intenvprice`.

```
RateSpec = intenvset('Rates', .05, 'StartDate', today, 'EndDate', datemnth(today,60));
IS = instswap([400 200], today, datemnth(today,60), [], [], [], [0 0]);
intenvprice(RateSpec, IS)
```

```
ans = 8.6440
```

### Create Float-Float, Fixed-Fixed, and Float-Fixed Swaps and Price with `intenvprice`

Use `instswap` to create swaps and price the swaps with `intenvprice`.

```
RateSpec = intenvset('Rates', .05, 'StartDate', today, 'EndDate', datemnth(today,60));
IS = instswap([.03 .02], today, datemnth(today,60), [], [], [], [1 1]);
IS = instswap(IS, [200 300], today, datemnth(today,60), [], [], [], [0 0]);
IS = instswap(IS, [300 .07], today, datemnth(today,60), [], [], [], [0 1]);
intenvprice(RateSpec, IS)
```

```
ans =
```

4.3220  
 -4.3220  
 4.5921

## Input Arguments

### **RateSpec — Interest-rate specification**

structure

(Optional) Interest-rate specification, specified by the RateSpec obtained previously from `intenvset` or `toRateSpec` for an `IRDataCurve` or `toRateSpec` for an `IRFunctionCurve`.

Data Types: `struct`

### **InstSet — Instrument variable containing a collection of instruments**

structure

Instrument variable containing a collection of instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

## Output Arguments

### **Price — Prices of each instrument**

matrix

Prices of each instrument, returned as a number of instruments (`NINST`) by number of curves (`NUMCURVES`) matrix. If an instrument cannot be priced, a NaN is returned in that entry.

For single-type pricing functions to retrieve pricing information, see the following:

<code>bondbyzero</code>	Price bonds from a set of zero curves.
<code>cfbyzero</code>	Price arbitrary cash flow instrument from a set of zero curves.

fixedbyzero	Fixed-rate note prices from a set of zero curves.
floatbyzero	Floating-rate note prices from a set of zero curves.
swapbyzero	Swap prices from a set of zero curves.

## See Also

bondbyzero | cfbyzero | fixedbyzero | floatbyzero | hjmprice | hjmsens | instadd | instswap | intenvsens | intenvset | swapbyzero

## Topics

“Pricing Using Interest-Rate Term Structure” on page 2-72

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## intenvsens

Instrument price and sensitivities from set of zero curves

### Syntax

```
[Delta,Gamma,Price] = intenvprice(RateSpecInstSet)
```

### Description

`[Delta,Gamma,Price] = intenvprice(RateSpecInstSet)` computes dollar prices and price sensitivities for instruments that use a zero coupon bond rate structure.

`intenvsens` handles the following instrument types: 'Bond', 'CashFlow', 'Fixed', 'Float', 'Swap'. See `instadd` for information about constructing defined types.

### Examples

#### Compute Prices Sensitivities for Instruments Using a Zero Coupon Bond Rate Structure

Load the tree and instruments from the `deriv.mat` data file and use `intenvprice` to compute dollar prices and sensitivities for instruments that use a zero coupon bond rate structure.

```
load deriv.mat
instdisp(ZeroInstSet)
```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	NaN
2	Bond	0.04	01-Jan-2000	01-Jan-2004	2	NaN	NaN	NaN

Index	Type	CouponRate	Settle	Maturity	FixedReset	Basis	Principal	Name
3	Fixed	0.04	01-Jan-2000	01-Jan-2003	1	NaN	NaN	4% Fixe



Index	Type	Spread	Settle	Maturity	FloatReset	Basis	Principal	Name
4	Float	20	01-Jan-2000	01-Jan-2003	1	NaN	NaN	20BP Float

Index	Type	LegRate	Settle	Maturity	LegReset	Basis	Principal	LegType	Na
5	Swap	[0.06 20]	01-Jan-2000	01-Jan-2003	[1 1]	NaN	NaN	[NaN]	6

```
[Delta, Gamma] = intenvsens(ZeroRateSpec, ZeroInstSet)
```

```
Delta = 5×1
```

```
-272.6403
-347.4386
-272.6403
-1.0445
-282.0405
```

```
Gamma = 5×1
103 ×
```

```
1.0298
1.6227
1.0298
0.0033
1.0596
```

## Input Arguments

### RateSpec — Interest-rate specification

structure

(Optional) Interest-rate specification, specified by the RateSpec obtained previously from intenvset or toRateSpec for an IRDataCurve or toRateSpec for an IRFunctionCurve.

Data Types: struct

### InstSet — Instrument variable containing a collection of instruments

structure

Instrument variable containing a collection of instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

## Output Arguments

### **Delta — Rate of change of instrument prices with respect to shifts in the observed zero curve**

vector

Rate of change of instrument prices with respect to shifts in the observed zero curve, returned as a number of instruments (NINST) by number of curves (NUMCURVES) matrix. Delta is computed by finite differences.

---

**Note** Delta sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

### **Gamma — Rate of change of instrument deltas with respect to shifts in the observed zero curve**

vector

Rate of change of instrument deltas with respect to shifts in the observed zero curve, returned as a number of instruments (NINST) by number of curves (NUMCURVES) matrix. Gamma is computed by finite differences.

---

**Note** Gamma sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

### **Price — Prices of each instrument**

vector

Prices of each instrument, returned as a number of instruments (NINST) by number of curves (NUMCURVES) matrix. If an instrument cannot be priced, a NaN is returned in that entry.

## **See Also**

hjmprice | hjmsens | instadd | intenvprice | intenvset

## **Topics**

“Pricing Using Interest-Rate Term Structure” on page 2-72

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## intenvset

Set properties of interest-rate structure

### Syntax

```
RateSpec = intenvset(Name,Value)  
[RateSpec,RateSpecOld] = intenvset(RateSpec,Name,Value)  
[RateSpec,RateSpecOld] = intenvset
```

### Description

`RateSpec = intenvset(Name,Value)` creates an interest-rate term structure (`RateSpec`) where the input argument list is specified as name-value pairs.

---

**Note** When creating a new `RateSpec`, the set of arguments passed to `intenvset` must include `StartDates`, `EndDates`, and either `Rates` or `Disc`.

---

`[RateSpec,RateSpecOld] = intenvset(RateSpec,Name,Value)` creates an interest-rate term structure (`RateSpec`) where the input argument list is specified as name-value pairs along with the optional argument `RateSpec`. If the optional argument `RateSpec` is specified, `intenvset` modifies the existing interest-rate term structure `RateSpec` by changing the named argument to the specified values and recalculating the arguments dependent on the new values.

`[RateSpec,RateSpecOld] = intenvset` creates an interest-rate term structure `RateSpec` with all fields set to [ ].

### Examples

#### Create a `RateSpec` for a Zero Curve

Use `intenvset` to create a `RateSpec` for a zero curve.

```
RateSpec = intenvset('Rates', 0.05, 'StartDates', ...
'20-Jan-2000', 'EndDates', '20-Jan-2001')
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
        Disc: 0.9518
        Rates: 0.0500
    EndTimes: 2
    StartTimes: 0
    EndDates: 730871
    StartDates: 730505
    ValuationDate: 730505
    Basis: 0
    EndMonthRule: 1
```

Now change the Compounding argument to 1 (annual).

```
RateSpec = intenvset(RateSpec, 'Compounding', 1)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: 0.9518
        Rates: 0.0506
    EndTimes: 1
    StartTimes: 0
    EndDates: 730871
    StartDates: 730505
    ValuationDate: 730505
    Basis: 0
    EndMonthRule: 1
```

Calling `intenvset` with no input or output arguments displays a list of argument names and possible values.

```
intenvset
```

```
Compounding: [ 0 | 1 | {2} | 3 | 4 | 6 | 12 | 365 | -1 ]
    Disc: [ scalar | vector (NPOINTS x 1) ]
    Rates: [ scalar | vector (NPOINTS x 1) ]
    EndDates: [ scalar | vector (NPOINTS x 1) ]
    StartDates: [ scalar | vector (NPOINTS x 1) ]
```

```
ValuationDate: [ scalar ]
      Basis: [ {0} | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 ]
      EndMonthRule: [ 0 | {1} ]
```

### Create a RateSpec for a Forward Curve

Use `intenvset` to create a `RateSpec` for a forward curve.

```
RateSpec = intenvset('Rates', 0.05, 'StartDates', ...
'20-Jan-2001', 'EndDates', '20-Jan-2002', 'ValuationDate', '20-Jan-2000')
```

```
RateSpec = struct with fields:
      FinObj: 'RateSpec'
      Compounding: 2
      Disc: 0.9518
      Rates: 0.0500
      EndTimes: 4
      StartTimes: 2
      EndDates: 731236
      StartDates: 730871
      ValuationDate: 730505
      Basis: 0
      EndMonthRule: 1
```

Now change the `Compounding` argument to 1 (annual).

```
RateSpec = intenvset(RateSpec, 'Compounding', 1)
```

```
RateSpec = struct with fields:
      FinObj: 'RateSpec'
      Compounding: 1
      Disc: 0.9518
      Rates: 0.0506
      EndTimes: 2
      StartTimes: 1
      EndDates: 731236
      StartDates: 730871
      ValuationDate: 730505
      Basis: 0
      EndMonthRule: 1
```

## Create a RateSpec Using Two Curves

Define data for the interest-rate term structure and use `intenvset` to create a `RateSpec`.

```
StartDates = '01-Oct-2011';
EndDates = ['01-Oct-2012'; '01-Oct-2013'; '01-Oct-2014'; '01-Oct-2015'];
Rates = [[0.0356;0.041185;0.04489;0.047741],[0.0325;0.0423;0.0437;0.0465]];
RateSpec = intenvset('Rates', Rates, 'StartDates',StartDates,...
'EndDates', EndDates, 'Compounding', 1)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x2 double]
    Rates: [4x2 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734777
    ValuationDate: 734777
    Basis: 0
    EndMonthRule: 1
```

To look at the `Rates` for the two interest-rate curves:

```
RateSpec.Rates
```

```
ans = 4x2
```

```
0.0356    0.0325
0.0412    0.0423
0.0449    0.0437
0.0477    0.0465
```

## Create a RateSpec to Price Multi-Stepped Coupon Bonds

Price the following multi-stepped coupon bonds using the following data:

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec using intenvset
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011'; '01-Jan-2012'; '01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2011' .042; '01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014'

% Display the instrument portfolio
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)

```

Index	Type	CouponRate	Settle	Maturity	Period	Basis	EndMonthRule	IssueDate
1	Bond	[Cell]	01-Jan-2010	01-Jan-2011	1	0	1	NaN
2	Bond	[Cell]	01-Jan-2010	01-Jan-2012	1	0	1	NaN
3	Bond	[Cell]	01-Jan-2010	01-Jan-2013	1	0	1	NaN
4	Bond	[Cell]	01-Jan-2010	01-Jan-2014	1	0	1	NaN

Build a BDTT tree to price the stepped coupon bonds. Assume the volatility to be 10%

```

Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);

```

```

% Compute the price of the stepped coupon bonds
PBDT = bdtprice(BDTT, ISet)

```

```
PBDT = 4×1
```

```

100.6763
100.7368
100.9266
101.0115

```



## Input Arguments

### RateSpec — Interest-rate specification for initial risk-free rate curve

structure

(Optional) Interest-rate specification for initial rate curve, specified by the `RateSpec` obtained previously from `intenvset` or `toRateSpec` for an `IRDataCurve` or `toRateSpec` for an `IRFunctionCurve`.

Data Types: `struct`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `RateSpec = intenvset('Rates',0.05,'StartDates','20-Jan-2001','EndDates','20-Jan-2002','ValuationDate','20-Jan-2000')`

### Compounding — Rate at which the input zero rates were compounded when annualized

2 (default) | integer with value of 0,1, 2, 3, 4, 6, 12, 365, or -1

Rate at which the input zero rates were compounded when annualized, specified as the comma-separated pair consisting of `'Compounding'` and a scalar integer value. The `Compounding` argument determines the formula for the discount factors (`Disc`):

- `Compounding = 0` for simple interest
  - $Disc = 1/(1 + Z * T)$ , where  $T$  is time in years and simple interest assumes annual times  $F = 1$ .
- `Compounding = 1, 2, 3, 4, 6, 12`
  - $Disc = (1 + Z/F)^{-T}$ , where  $F$  is the compounding frequency,  $Z$  is the zero rate, and  $T$  is the time in periodic units, for example,  $T = F$  is one year.
- `Compounding = 365`
  - $Disc = (1 + Z/F)^{-T}$ , where  $F$  is the number of days in the basis year and  $T$  is a number of days elapsed computed by basis.

- Compounding = -1
  - $\text{Disc} = \exp(-T*Z)$ , where T is time in years.

Data Types: double

### **Disc — Unit bond prices over investment intervals**

[ ] (default) | matrix

Unit bond prices over investment intervals from `StartDates` (when the cash flow is valued) to `EndDates` (when the cash flow is received), specified as the comma-separated pair consisting of 'Disc' and a number of points (NPOINTS) by number of curves (NCURVES) matrix.

Data Types: double

### **Rates — Interest rates**

matrix of decimal values

Interest rates, specified as the comma-separated pair consisting of 'Rates' and a number of points (NPOINTS) by number of curves (NCURVES) matrix of decimal values. Rates can only contain negative decimal values if the resulting `RateSpec` is used with a Normal (Bachelier) model, shifted Black model, or a shifted SABR model.

Data Types: double

### **EndDates — Maturity dates ending the interval to discount over**

serial date number | date character vector

Maturity dates ending the interval to discount over, specified as the comma-separated pair consisting of 'EndDates' and a scalar or a NPOINTS-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **StartDates — Dates starting the interval to discount over**

serial date number | date character vector

Dates starting the interval to discount over, specified as the comma-separated pair consisting of 'StartDates' and a scalar or a NPOINTS-by-1 vector of serial date numbers or date character vectors. `StartDates` must be earlier than `EndDates`.

Data Types: double | char | cell

**ValuationDate — Observation date of the investment horizons entered in StartDates and EndDates**

min(StartDates) (default) | serial date number | character vector date

observation date of the investment horizons entered in StartDates and EndDates, specified as the comma-separated pair consisting of 'ValuationDate' and a specified as a scalar serial date number or date character vector.

Data Types: double | char

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a scalar integer value.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar integer with a value of 0 or 1. This rule applies only when EndDates is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

## Output Arguments

### **RateSpec** — Interest-rate specification for initial rate curve

structure

Interest-rate specification for initial rate curve, returned as a structure.

### **RateSpecOld** — Properties of an interest-rate structure before the changes introduced by the call to `intenvset`

structure

Properties of an interest-rate structure before the changes introduced by the call to `intenvset`, returned as a structure.

## See Also

`intenvget` | `intenvprice`

## Topics

“Pricing Using Interest-Rate Term Structure” on page 2-72

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## isafin

True if input argument is financial structure type or financial object class

## Syntax

```
IsFinObj = isafin(Obj,ClassName)
```

## Arguments

Obj	Name of a financial structure.
ClassName	Character vector containing the name of a financial structure class.

## Description

`IsFinObj = isafin(Obj,ClassName)` returns `True` (1) if input argument is a financial structure type or financial object class, otherwise `False` (0) is returned.

## Examples

```
load deriv.mat
IsFinObj = isafin(HJMTree, 'HJMFwdTree')

IsFinObj =

     1
```

## See Also

`classfin`

## Topics

“Portfolio Creation” on page 1-8

“Instrument Constructors” on page 1-20

**Introduced before R2006a**

# ittprice

Price instruments using implied trinomial tree (ITT)

## Syntax

```
[Price,PriceTree] = ittprice(ITTree,InstSet)
[Price,PriceTree] = ittprice( ____,Options)
```

## Description

[Price,PriceTree] = ittprice(ITTree,InstSet) price instruments using an implied trinomial tree (ITT) created with `itttree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`ittprice` handles the following instrument types: `optstock`, `barrier`, `Asian`, `lookback`, and `compound`. Use `instadd` to construct the defined types.

[Price,PriceTree] = ittprice( \_\_\_\_,Options) adds an optional input argument for `Options`.

## Examples

### Price Instruments Using Implied Trinomial Tree (ITT)

Load the ITT tree and instruments from the data file `deriv.mat`.

```
load deriv.mat
```

Display the barrier and Asian options contained in the instrument set.

```
ITTSubSet = instselect(ITTInstSet,'Type', {'Barrier', 'Asian'});
instdisp(ITTSubSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Bar
1	Barrier	call	85	01-Jan-2006	31-Dec-2008	1	ui	115
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPri
2	Asian	call	55	01-Jan-2006	01-Jan-2008	0	arithmetic	NaN
3	Asian	call	55	01-Jan-2006	01-Jan-2010	0	arithmetic	NaN

Price the barrier and Asian options contained in the instrument set.

```
[Price, PriceTree] = ittprice(ITTree, ITTSubSet)
```

```
Price = 3×1
```

```
2.4074
3.2052
6.6074
```

```
PriceTree = struct with fields:
```

```
FinObj: 'TrinPriceTree'
PTree: {1x5 cell}
tObs: [0 1 2 3 4]
dObs: [732678 733043 733408 733773 734139]
```

## Input Arguments

### ITTree — Implied trinomial stock tree structure

structure

Implied trinomial stock tree structure, specified by using `itttree`.

Data Types: `struct`

### InstSet — Instrument variable

structure

Instrument variable containing a collection of NINST instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`



## Options — Derivatives pricing options structure

structure

(Optional) Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## Output Arguments

### Price — Price for each instrument

vector

Price for each instrument, returned as a `NINST-by-1` vector. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a `NaN` is returned in that entry.

For information on single-type pricing functions to retrieve state-by-state pricing tree information, see the following:

- `barrierbyitt` for pricing barrier options using an ITT tree
- `optstockbyitt` for pricing American, European, or Bermuda options using an ITT tree
- `asianbyitt` for pricing Asian options using an ITT tree
- `lookbackbyitt` for pricing lookback options using an ITT tree
- `compoundbyitt` for price compound options using an ITT tree
- `cbondbyitt` for pricing convertible bonds using an ITT tree

### PriceTree — Tree structure of instrument prices

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.dObs` contains the observation dates.

## **See Also**

ittsens | itttree

## **Topics**

“Computing Prices Using ITT” on page 3-135

“Examining Output from the Pricing Functions” on page 3-139

“Computing Equity Instrument Sensitivities” on page 3-144

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced in R2007a**

# ittsens

Instrument sensitivities and prices using implied trinomial tree (ITT)

## Syntax

```
[Delta,Gamma,Vega,Price] = ittsens(ITTree,InstSet)
[Delta,Gamma,Vega,Price] = ittsens( ____,Options)
```

## Description

`[Delta,Gamma,Vega,Price] = ittsens(ITTree,InstSet)` calculates instrument sensitivities and prices using an implied trinomial tree (ITT) that is created with the `itttree` function. All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

`ittsens` handles the following instrument types: `optstock`, `barrier`, `Asian`, `lookback`, and `compound`. Use `instadd` to construct the defined types.

`[Delta,Gamma,Vega,Price] = ittsens( ____,Options)` adds an optional input argument for `Options`.

## Examples

### Compute Instrument Sensitivities Using an Implied Trinomial Tree (ITT)

Load the ITT tree and instruments from the data file `deriv.mat` and display the vanilla options and barrier option instruments.

```
load deriv.mat
ITTSubSet = instselect(ITTInstSet,'Type', {'OptStock', 'Barrier'});
```

```
instdisp(ITTSubSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
1	OptStock	call	95	01-Jan-2006	31-Dec-2008	1	Call1	10

```

2      OptStock put      80      01-Jan-2006      01-Jan-2010      0              Put1      4
Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt BarrierSpec Bar
3      Barrier call      85      01-Jan-2006      31-Dec-2008      1              ui              115

```

Compute the **Delta** and **Gamma** sensitivities of vanilla options and barrier option contained in the instrument set.

```
[Delta, Gamma] = ittSENS(ITTree, ITTSubSet)
```

Warning: The option set specified in StockOptSpec was too narrow for the generated tree.

```
Delta = 3x1
```

```

0.2387
-0.4283
0.3482

```

```
Gamma = 3x1
```

```

0.0260
0.0188
0.0380

```

## Input Arguments

### **ITTree** — Stock tree structure

structure

Stock tree structure, specified by using `itttree`.

Data Types: `struct`

### **InstSet** — Instrument variable

structure

Instrument variable containing a collection of `NINST` instruments, specified using `instadd`. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or character vector for each instrument.

Data Types: `struct`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, created using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Delta — Rate of change of instruments prices with respect to changes in the stock price**

vector

Rate of change of instruments prices with respect to changes in the stock price, returned as a NINST-by-1 vector of deltas.

For path-dependent options ('Lookback' and 'Asian'), Delta and Gamma are computed by finite differences in calls to `ittprice`. For the rest of the options ('OptStock', 'Barrier', 'CBond', and 'Compound'), Delta and Gamma are computed from the ITTtree and the corresponding option price tree.

### **Gamma — Rate of change of instruments deltas with respect to changes in stock price**

vector

Rate of change of instruments deltas with respect to changes in the stock price, returned as a NINST-by-1 vector of gammas.

For path-dependent options ('Lookback' and 'Asian'), Delta and Gamma are computed by finite differences in calls to `ittprice`. For the rest of the options ('OptStock', 'Barrier', 'CBond', and 'Compound'), Delta and Gamma are computed from the ITTtree and the corresponding option price tree.

### **Vega — Rate of change of instruments prices with respect to changes in volatility of the stock**

vector

Rate of change of instruments prices with respect to changes in the volatility of the stock, returned as a NINST-by-1 vector of vegas. Vega is computed by finite differences in calls to `itttree`.

### **Price — Price of each instrument**

vector

Price of each instrument, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned in that entry.

## **References**

[1] Chriss, Neil. *Black-Scholes and Beyond: Option Pricing Models*. McGraw-Hill, 1996, pp 308-312.

## **See Also**

`ittprice` | `itttree`

## **Topics**

“Computing Prices Using ITT” on page 3-135

“Examining Output from the Pricing Functions” on page 3-139

“Computing Equity Instrument Sensitivities” on page 3-144

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced in R2007a**

# itttimespec

Specify time structure using implied trinomial tree (ITT)

## Syntax

```
TimeSpec = itttimespec(ValuationDate,Maturity,NumPeriods)
```

## Description

`TimeSpec = itttimespec(ValuationDate,Maturity,NumPeriods)` creates the structure specifying the time layout for an ITT tree (`itttree`).

## Examples

### Creates the Structure Specifying the Time Layout for an ITT Tree

This example shows how to specify a four-period tree with time steps of 1 year.

```
ValuationDate = '1-July-2006';
Maturity = '1-July-2010';
TimeSpec = itttimespec(ValuationDate, Maturity, 4)

TimeSpec = struct with fields:
    FinObj: 'ITTTIMEspec'
    ValuationDate: 732859
    Maturity: 734320
    NumPeriods: 4
    Basis: 0
    EndMonthRule: 1
    tObs: [0 1 2 3 4]
    dObs: [732859 733224 733589 733954 734320]
```

## Input Arguments

### **ValuationDate — Pricing date and first observation in the tree**

serial date number | character vector date

Pricing date and first observation in the `itttree`, specified as a scalar date using a serial date number or date character vector.

Data Types: `double` | `char`

### **Maturity — Date marking the depth of the EQP stock tree**

serial date number | date character vector

Date marking the depth of the `itttree` trinomial tree, specified as scalar serial date number or date character vector.

Data Types: `double` | `char`

### **NumPeriods — Number of time steps in the ITT tree**

integer

Number of time steps in the `itttree` trinomial tree, specified as scalar integer value.

Data Types: `double`

## Output Arguments

### **TimeSpec — Specification for the time layout for `itttree`**

structure

Specification for the time layout for `itttree`, returned as a structure.

## See Also

`ittprice` | `itttree` | `stockspec`

## Topics

“Building Implied Trinomial Trees” on page 3-8

“Examining Equity Trees” on page 3-18

“Graphical Representation of Equity Derivative Trees” on page 3-142



"Understanding Equity Trees" on page 3-2  
"Pricing Options Structure" on page B-2  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2007a**

## itttree

Build implied trinomial stock tree

### Syntax

```
ITTTree = itttree(StockSpec,RateSpec,TimeSpec,StockOptSpec)
```

### Description

`ITTTree = itttree(StockSpec,RateSpec,TimeSpec,StockOptSpec)` builds an implied trinomial (ITT) stock tree.

### Examples

#### Create an ITT Tree

Assume that the interest rate is fixed at 8% annually between the valuation date of the tree (January 1, 2006) until its maturity.

```
Rate = 0.08;  
ValuationDate = '01-01-2006';  
EndDate = '01-01-2008';
```

```
RateSpec = intenvset('StartDates', ValuationDate, 'EndDates', EndDate, ...  
    'ValuationDate', ValuationDate, 'Rates', Rate, 'Compounding', -1);
```

To build an `ITTTree`, create the `StockSpec`, `TimeSpec`, and `StockOptSpec` structures.

```
Sigma = 0.20;  
AssetPrice = 50;  
DividendType = 'cash';  
DividendAmounts = [0.50; 0.50; 0.50; 0.50];  
ExDividendDates = {'03-Jan-2007'; '01-Apr-2007'; '05-July-2007'; '01-Oct-2007'};
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
```

```
DividendAmounts, ExDividendDates);

ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
NumPeriods = 4;

TimeSpec = itttimespec(ValuationDate, EndDate, NumPeriods);

Build a StockOptSpec structure.

Settle = '01/01/06';

Maturity = ['07/01/06';
            '07/01/06';
            '07/01/06';
            '01/01/07';
            '01/01/07';
            '01/01/07';
            '01/01/07';
            '07/01/07';
            '07/01/07';
            '07/01/07';
            '07/01/07';
            '01/01/08';
            '01/01/08';
            '01/01/08';
            '01/01/08'];

Strike = [113;
          101;
          100;
          88;
          128;
          112;
          100;
          78;
          144;
          112;
          100;
          69;
          162;
          112;
          100;
          61];
```

```
OptPrice = [                                0;
  4.807905472659144;
  1.306321897011867;
  0.048039195057173;
  0;
  2.310953054191461;
  1.421950392866235;
  0.020414826276740;
  0;
  5.091986935627730;
  1.346534812295291;
  0.005101325584140;
  0;
  8.047628153217246;
  1.219653432150932;
  0.001041436654748];
```

```
OptSpec = { 'call';
  'call';
  'put';
  'put';
  'call';
  'call';
  'put';
  'put';
  'put';
  'call';
  'call';
  'put';
  'put';
  'call';
  'call';
  'put';
  'put'};
```

```
StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec);
```

Use `itttree` to build the `ITTree` structure. Note, in this example, the extrapolation warnings are turned on. These warnings are a consequence of having to extrapolate to find the option price of the tree nodes. In this example, the set of inputs options was too narrow for the shift in the tree nodes introduced by the disturbance used to calculate the sensitivities. As a consequence extrapolation for some of the nodes was needed.

```
warning('on', 'fininst:itttree:Extrapolation');
ITTree = itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)
```

Warning: The option set specified in StockOptSpec was too narrow for the generated tree. This made extrapolation necessary. Below is a list of the options that were outside of the range of those specified in StockOptSpec.

```
Option Type: 'call'    Maturity: 02-Jul-2006  Strike=60.7466
Option Type: 'put'    Maturity: 02-Jul-2006  Strike=50.0731
Option Type: 'put'    Maturity: 02-Jul-2006  Strike=41.3344
Option Type: 'call'   Maturity: 01-Jan-2007  Strike=73.8592
Option Type: 'call'   Maturity: 01-Jan-2007  Strike=60.8227
Option Type: 'put'    Maturity: 01-Jan-2007  Strike=50.1492
Option Type: 'put'    Maturity: 01-Jan-2007  Strike=41.4105
Option Type: 'put'    Maturity: 01-Jan-2007  Strike=34.2559
Option Type: 'call'   Maturity: 02-Jul-2007  Strike=88.8310
Option Type: 'call'   Maturity: 02-Jul-2007  Strike=72.9081
Option Type: 'call'   Maturity: 02-Jul-2007  Strike=59.8715
Option Type: 'put'    Maturity: 02-Jul-2007  Strike=49.1980
Option Type: 'put'    Maturity: 02-Jul-2007  Strike=40.4594
Option Type: 'put'    Maturity: 02-Jul-2007  Strike=33.3047
Option Type: 'put'    Maturity: 02-Jul-2007  Strike=27.4470
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=107.2895
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=87.8412
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=71.9183
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=58.8817
Option Type: 'put'    Maturity: 01-Jan-2008  Strike=48.2083
Option Type: 'put'    Maturity: 01-Jan-2008  Strike=39.4696
Option Type: 'put'    Maturity: 01-Jan-2008  Strike=32.3150
Option Type: 'put'    Maturity: 01-Jan-2008  Strike=26.4573
Option Type: 'put'    Maturity: 01-Jan-2008  Strike=21.6614
```

```
> In itttree>InterpOptPrices at 675
  In itttree at 277
```

```
ITTree =
```

```
    FinObj: 'ITStockTree'
    StockSpec: [1x1 struct]
    StockOptSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 0.5000000000000000 1 1.5000000000000000 2]
    dObs: [732678 732860 733043 733225 733408]
```

```
STree: {1x5 cell}  
Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
```

## Input Arguments

### **StockSpec — Stock specification**

structure

Stock specification, specified by the `StockSpec` obtained from `stockspec`. See `stockspec` for information on creating a stock specification.

Data Types: `struct`

### **RateSpec — Interest-rate specification for initial risk-free rate curve**

structure

Interest-rate specification for initial risk-free rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **TimeSpec — Tree time layout specification**

structure

Tree time layout specification, specified by the `TimeSpec` obtained from `itttimespec`. The `TimeSpec` defines the observation dates of the ITT tree. See `itttimespec` for information on the tree structure.

Data Types: `struct`

### **StockOptSpec — Option stock specification**

structure

Option stock specification, specified by the `StockOptSpec` obtained from `stockoptspec`. See `stockoptspec` for information on creating a stock specification.

Data Types: `struct`

## Output Arguments

### **ITTree** – ITT trinomial tree

structure

ITT trinomial tree, returned as a structure specifying the time layout for the tree.

## See Also

`intenvset` | `ittprice` | `itttimespec` | `itttree` | `stockoptspec` | `stockspec`

## Topics

“Building Implied Trinomial Trees” on page 3-8

“Examining Equity Trees” on page 3-18

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Understanding Equity Trees” on page 3-2

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced in R2007a**

## LiborMarketModel

Create LIBOR Market Model

### Description

The LIBOR Market Model (LMM) is an interest-rate model that differs from short rate models in that it evolves a set of discrete forward rates.

Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate

$$\frac{dF_i(t)}{F_i} = -\mu_i dt + \sigma_i(t) dW_i$$

where:

$W$  is an  $N$ -dimensional geometric Brownian motion with

$$dW_i(t)dW_j(t) = \rho_{ij}$$

The LMM relates drifts of the forward rates based on no-arbitrage arguments. Specifically, under the Spot LIBOR measure, drifts are expressed as

$$\mu_i(t) = -\sigma_i(t) \sum_{j=q(t)}^i \frac{\tau_j \rho_{i,j} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)}$$

where:

$\rho_{i,j}$  represents the input argument Correlation.

$\sigma_j(t)$  represents the input argument VolFunc.

$F_j(t)$  represents the computation of the input argument for ZeroCurve.

$\tau_i$  is the time fraction associated with the  $i$  th forward rate

$q(t)$  is an index defined by the relation



$$T_{q(t)-1} < t < T_{q(t)}$$

and the Spot LIBOR numeraire is defined as

$$B(t) = P(t, T_{q(t)}) \prod_{n=0}^{q(t)-1} (1 + \tau_n F_n(T_n))$$

## Creation

## Syntax

```
LMM = LiborMarketModel(ZeroCurve,VolFunc,Correlation)
LMM = LiborMarketModel( ____,Name,Value)
```

## Description

`LMM = LiborMarketModel(ZeroCurve,VolFunc,Correlation)` creates a `LiborMarketModel` (LMM) object using the required arguments for `ZeroCurve`, `VolFunc`, `Correlation`.

`LMM = LiborMarketModel( ____,Name,Value)` sets Properties on page 11-1099 using name-value pairs. For example, `LMM = LiborMarketModel(irdc,VolFunc,Correlation,'Period',1)`. You can specify multiple name-value pairs. Enclose each property name in single quotes.

## Input Arguments

### **ZeroCurve** — Zero curve used to evolve path of future interest rates

IRDataCurve object | RateSpec

Zero curve used to evolve the path of future interest rates, specified as an output from `IRDataCurve` or a `RateSpec` that is obtained from `intenvset`. The `ZeroCurve` input sets the `ZeroCurve` on page 11-0 property.

Data Types: object | struct

### **VolFunc** — Volatility function

cell array of function handles

Volatility function, specified using a NumRates-by-1 cell array of function handles and sets the VolFunc on page 11-0 property. Each function handle must take time as an input and, return a scalar volatility.

---

**Note** The number of rates to simulate using the simTermStructs function is determined by the size of the VolFunc and Correlation inputs which must be consistent. These can be any value and, together with the Period property, determines the kinds and number of rates being simulated. For example, if the Period is set to 4 (quarterly) and VolFunc has length of 120 and Correlation has size 120-by-120, then 120 quarterly rates are simulated. In other words, 30 years of the yield curve are simulated (0-3mos, 3mos-6mos, 6mos-9mos, and so on, all the way up to 30 years). Therefore, if VolFunc and Correlation have size 120, the output of a call to simTermStructs is (nPeriods+1)-by-121-by-nTrials.

---

Data Types: cell

### **Correlation — Correlation matrix** matrix

Correlation matrix, specified using a NumRates-by-NumRates correlation matrix and sets the Correlation on page 11-0 property.

---

**Note** The number of rates to simulate using the simTermStructs function is determined by the size of the VolFunc and Correlation inputs which must be consistent. These can be any value and, together with the Period property, determines the kinds and number of rates being simulated. For example, if the Period is set to 4 (quarterly) and VolFunc has length of 120 and Correlation has size 120-by-120, then 120 quarterly rates are simulated. In other words, 30 years of the yield curve are simulated (0-3mos, 3mos-6mos, 6mos-9mos, and so on, all the way up to 30 years). Therefore, if VolFunc and Correlation have size 120, the output of a call to simTermStructs is (nPeriods+1)-by-121-by-nTrials.

---

Data Types: double

## Properties

### **ZeroCurve — Zero curve**

IRDataCurve object | RateSpec

Zero curve, specified as an output from IRDataCurve or a RateSpec that is obtained from `intenvset`.

Data Types: object | struct

### **VolFunc — Volatility function**

cell array of function handles

Volatility function, specified using a `NumRates`-by-1 cell array of function handles. Each function handle must take time as an input and, return a scalar volatility.

Data Types: cell

### **Correlation — Correlation matrix**

matrix

Correlation matrix, specified using a `NumRates`-by-`NumRates` correlation matrix.

Data Types: double

### **NumFactors — Number of Brownian factors**

NaN (default) | numeric

Number of Brownian factors, specified as a numeric value. The default is NaN, where the number of factors is equal to the number of rates.

Data Types: double

### **Period — Period of forward rates, specifically number of rates per year**

2 (default) | numeric with value 1, 2, 4, or 12

Period of the forward rates, specifically the number of rates per year, specified as a numeric value of 1, 2, 4, or 12. The default value is 2, meaning forward rates are spaced at 0, .5, 1, 1.5, and so on.

Data Types: double

## Object Functions

`simTermStructs` Simulate term structures for LIBOR Market Model

## Examples

### Create a LIBOR Market Model Using an IRDataCurve

Create a LMM object using an IRDataCurve.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -.02 .7 .14];

numRates = 20;
VolFunc(1:numRates,1) = {@(t) LMMVolFunc(LMMVolParams,t)};

Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
Correlation = CorrFunc(meshgrid(1:numRates)',meshgrid(1:numRates),Beta);

LMM = LiborMarketModel(irdc,VolFunc,Correlation,'Period',1)

LMM =
  LiborMarketModel with properties:
    ZeroCurve: [1x1 IRDataCurve]
  VolFunctions: {20x1 cell}
  Correlation: [20x20 double]
  NumFactors: NaN
  Period: 1
```

Simulate the term structures for the specified LMM object.

```
[ZeroRates, ForwardRates] = simTermStructs(LMM, 10,'nTrials',100);
```

## Create a LIBOR Market Model Using a RateSpec

Create a LMM object using a RateSpec.

```

Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

RateSpec = intenvset('Rates',ZeroRates,'EndDates',CurveDates,'StartDate',Settle);

LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -.02 .7 .14];

numRates = 20;
VolFunc(1:numRates,1) = {@(t) LMMVolFunc(LMMVolParams,t)};

Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
Correlation = CorrFunc(meshgrid(1:numRates)',meshgrid(1:numRates),Beta);

LMM = LiborMarketModel(RateSpec,VolFunc,Correlation,'Period',1)

LMM =
    LiborMarketModel with properties:
        ZeroCurve: [1x1 IRDataCurve]
        VolFunctions: {20x1 cell}
        Correlation: [20x20 double]
        NumFactors: NaN
        Period: 1

```

Simulate the term structures for the specified LMM object.

```
[ZeroRates, ForwardRates] = simTermStructs(LMM, 10,'nTrials',100);
```

## Definitions

### LIBOR Market Model

The LIBOR Market Model, also called the BGM Model (Brace, Gatarek, Musiela Model) is a financial model of interest rates.

The quantities that are modeled are a set of forward rates (also called forward LIBORs) which have the advantage of being directly observable in the market, and whose volatilities are naturally linked to traded contracts.

### References

[1] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

### See Also

HullWhite1F | IRDataCurve | LinearGaussian2F | intenvset | simTermStructs

### Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# simTermStructs

Simulate term structures for LIBOR Market Model

## Syntax

```
[ZeroRates,ForwardRates] = simTermStructs(LMM,nPeriods)
[ZeroRates,ForwardRates] = simTermStructs( ____,Name,Value)
```

## Description

[ZeroRates,ForwardRates] = simTermStructs(LMM,nPeriods) simulates future zero curve paths using a specified `LiborMarketModel` object.

[ZeroRates,ForwardRates] = simTermStructs( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Simulate Term Structures for a LIBOR Market Model

Create a LMM object.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -.02 .7 .14];

numRates = 20;
VolFunc(1:numRates-1) = {@(t) LMMVolFunc(LMMVolParams,t)};
```

```
Beta = .08;
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
Correlation = CorrFunc(meshgrid(1:numRates-1)',meshgrid(1:numRates-1),Beta);

LMM = LiborMarketModel(irdc,VolFunc,Correlation,'Period',1)

LMM =
  LiborMarketModel with properties:

    ZeroCurve: [1x1 IRDataCurve]
  VolFunctions: {1x19 cell}
  Correlation: [19x19 double]
  NumFactors: NaN
  Period: 1
```

Simulate the term structures for the specified LMM object.

```
[ZeroRates, ForwardRates] = simTermStructs(LMM, 20, 'nTrials',100);
```

## Input Arguments

### **LMM — LiborMarketModel object**

object

LiborMarketModel object, specified using the LMM object created using LiborMarketModel.

Data Types: object

### **nPeriods — Number of simulation periods**

numeric

Number of simulation periods, specified as a numeric value. The nPeriods value is determined by the swaption expiry and the periodicity of the rates of the model. For example, if you were to price a swaption expiring in 5 years with a semiannual LIBOR Market Model (LMM), then nPeriods would be 10.

Data Types: double



## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[ZeroRates, ForwardRates] = simTermStructs(LMM, 20, 'nTrials', 100)`

### **nTrials** — Number of simulated trials

1 (default) | positive integer

Number of simulated trials (sample paths), specified as the comma-separated pair consisting of `'nTrials'` and a positive scalar integer value of `nPeriods` observations each. If you do not specify a value for this argument, the default is 1, indicating a single path of correlated state variables.

Data Types: `double`

### **antithetic** — Flag indicating whether antithetic sampling is used to generate Gaussian random variates

false (default) | positive integer

Flag indicating whether antithetic sampling is used to generate the Gaussian random variates that drive the zero-drift, unit-variance rate Brownian vector  $dW(t)$ , specified as the comma-separated pair consisting of `'antithetic'` and a Boolean scalar flag. For details on the Brownian vector, see `simBySolution`.

Data Types: `logical`

### **Z** — Direct specification of dependent random noise process

generated by `simBySolution` function (default) | numeric

Direct specification of the dependent random noise process, specified as the comma-separated pair consisting of `'Z'` and a numeric value. The `Z` value is used to generate the zero-drift, unit-variance rate Brownian vector  $dW(t)$  that drives the simulation. For details, see `simBySolution` for the GBM model.

Data Types: `double`

### **Tenor** — Maturities to compute at each time step

number of rates in `LiborMarketModel` object (default) | numeric vector

Maturities to compute at each time step, specified as the comma-separated pair consisting of 'Tenor' and a numeric vector.

Tenor enables you to choose a different set of rates to output than the underlying rates. For example, you may want to simulate quarterly data but only report annual rates; this can be done by specifying the optional input Tenor.

The default for tenor is the number of rates in the LiborMarketModel object as specified by the Correlation and VolFunc input arguments for the LiborMarketModel object.

Data Types: double

## Output Arguments

### **ZeroRates — Simulated zero-rate term structures**

matrix

Simulated zero-rate term structures, returned as a nPeriods+1-by-nTenors-by-nTrials matrix.

### **ForwardRates — Simulated forward-rate term structures**

matrix

Simulated zero-rate term structures, returned as a nPeriods+1-by-nTenors-by-nTrials matrix.

## See Also

LiborMarketModel | blackvolbyrebonato

## Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# LinearGaussian2F

Create two-factor additive Gaussian interest-rate model

## Description

The two-factor additive Gaussian interest rate-model is specified using the zero curve,  $a$ ,  $b$ ,  $\sigma$ ,  $\eta$ , and  $\rho$  parameters.

Specifically, the LinearGaussian2F model is defined using the following equations:

$$r(t) = x(t) + y(t) + \phi(t)$$

$$dx(t) = -a(t)x(t)dt + \sigma(t)dW_1(t), x(0) = 0$$

$$dy(t) = -b(t)y(t)dt + \eta(t)dW_2(t), y(0) = 0$$

where  $dW_1(t)dW_2(t) = \rho dt$  is a two-dimensional Brownian motion with correlation  $\rho$ , and  $\phi$  is a function chosen to match the initial zero curve.

## Creation

## Syntax

```
G2PP = LinearGaussian2F(ZeroCurve,a,b,sigma,eta,rho)
```

## Description

`G2PP = LinearGaussian2F(ZeroCurve,a,b,sigma,eta,rho)` creates a LinearGaussian2F (G2PP) object using the required arguments to set the Properties on page 11-1108.

## Properties

### **ZeroCurve — Zero curve**

IRDataCurve object | RateSpec

Zero curve, specified as an output from IRDataCurve or a RateSpec that is obtained from `intenvset`. This is the zero curve used to evolve the path of future interest rates.

Data Types: `object` | `struct`

### **a — Mean reversion for first factor**

numeric

Mean reversion for the first factor, specified either as a scalar or function handle which takes time as input and returns a scalar mean reversion value.

Data Types: `double`

### **b — Mean reversion for second factor**

numeric

Mean reversion for the second factor, specified either as a scalar or as a function handle which takes time as input and returns a scalar mean reversion value.

Data Types: `double`

### **sigma — Volatility for first factor**

numeric

Volatility for the first factor, specified either as a scalar or function handle which takes time as input and returns a scalar mean volatility.

Data Types: `double`

### **eta — Volatility for second factor**

numeric

Volatility for the second factor, specified either as a scalar or function handle which takes time as input and returns a scalar mean volatility.

Data Types: `double`

### **rho — Scalar correlation of factors**

numeric

Scalar correlation of the factors, specified as a numeric value.

Data Types: double

## Object Functions

`simTermStructs` Simulate term structures for two-factor additive Gaussian interest-rate model

## Examples

### Create a Two-Factor Additive Gaussian Interest-Rate Model Using an IRDataCurve

Create a two-factor additive Gaussian interest-rate model using an IRdataCurve.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;

G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho)

G2PP =
    LinearGaussian2F with properties:
        ZeroCurve: [1x1 IRDataCurve]
                a: @(t,V)ina
                b: @(t,V)inb
        sigma: @(t,V)insigma
        eta: @(t,V)ineta
        rho: -0.7000
```

Use the `simTermStructs` method to simulate term structures based on the `LinearGaussian2F` model.

```
SimPaths = simTermStructs(G2PP, 10, 'nTrials', 100);
```

### Create a Two-Factor Additive Gaussian Interest-Rate Model Using a RateSpec

Create a two-factor additive Gaussian interest-rate model using a `RateSpec`.

```
Settle = datenum('15-Dec-2007');  
CurveTimes = [1:5 7 10 20]';  
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';  
CurveDates = daysadd(Settle, 360*CurveTimes, 1);  
  
RateSpec = intenvset('Rates', ZeroRates, 'EndDates', CurveDates, 'StartDate', Settle);  
  
a = .07;  
b = .5;  
sigma = .01;  
eta = .006;  
rho = -.7;  
  
G2PP = LinearGaussian2F(RateSpec, a, b, sigma, eta, rho)  
  
G2PP =  
LinearGaussian2F with properties:  
  
ZeroCurve: [1x1 IRDataCurve]  
    a: @(t,V)ina  
    b: @(t,V)inb  
sigma: @(t,V)insigma  
eta: @(t,V)ineta  
rho: -0.7000
```

Use the `simTermStructs` method to simulate term structures based on the `LinearGaussian2F` model.

```
SimPaths = simTermStructs(G2PP, 10, 'nTrials', 100);
```

## Definitions

### Two-Factor Additive Gaussian Interest-Rate Model

Short-rate model based on two factors where the short rate is the sum of the two factors and a deterministic function.

In this case  $\phi(t)$ , which is chosen to match the initial term structure.

### References

[1] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

### See Also

HullWhite1F | LiborMarketModel | capbylg2f | floorbylg2f | simTermStructs | swaptionbylg2f

### Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

## simTermStructs

Simulate term structures for two-factor additive Gaussian interest-rate model

### Syntax

```
[ZeroRates,ForwardRates] = simTermStructs(G2PP,nPeriods)
[ZeroRates,ForwardRates] = simTermStructs( ____,Name,Value)
```

### Description

[ZeroRates,ForwardRates] = simTermStructs(G2PP,nPeriods) simulates future zero curve paths using a specified LinearGaussian2F object.

[ZeroRates,ForwardRates] = simTermStructs( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Simulate Term Structures for the LinearGaussian2F Model

Create a two-factor additive Gaussian interest-rate model.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);

a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
```



```
G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho)
```

```
G2PP =
  LinearGaussian2F with properties:
```

```
  ZeroCurve: [1x1 IRDataCurve]
             a: @(t,V)ina
             b: @(t,V)inb
             sigma: @(t,V)insigma
             eta: @(t,V)ineta
             rho: -0.7000
```

Use the `simTermStructs` method to simulate term structures based on the `LinearGaussian2F` model.

```
SimPaths = simTermStructs(G2PP, 10, 'nTrials', 100);
```

### Simulate Term Structures for the LinearGaussian2F Model Using a Vector for `deltaTime`

Create a two-factor additive Gaussian interest-rate model.

```
Settle = datenum('15-Dec-2007');
CurveTimes = [1:5 7 10 20]';
ZeroRates = [.01 .018 .024 .029 .033 .034 .035 .034]';
CurveDates = daysadd(Settle,360*CurveTimes,1);

irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);
a = .07;
b = .5;
sigma = .01;
eta = .006;
rho = -.7;
G2PP = LinearGaussian2F(irdc,a,b,sigma,eta,rho)
```

```
G2PP =
  LinearGaussian2F with properties:
```

```
  ZeroCurve: [1x1 IRDataCurve]
             a: @(t,V)ina
```

```
b: @(t,V)inb
sigma: @(t,V)insigma
eta: @(t,V)ineta
rho: -0.7000
```

Use the `simTermStructs` method to simulate term structures based on the `LinearGaussian2F` object, where uneven simulation tenors are specified using the optional name-value argument `deltaTime` as a vector of length `NPeriods`.

```
NPeriods = 10;
dt = rand(NPeriods,1);
SimPaths = G2PP.simTermStructs(NPeriods,'nTrials',100,'DeltaTime',dt);
```

## Input Arguments

### **G2PP** — `LinearGaussian2F` object

object

`LinearGaussian2F` object, specified using the `G2PP` object created using `LinearGaussian2F`.

Data Types: `object`

### **nPeriods** — Number of simulation periods

numeric

Number of simulation periods, specified as a numeric value. For example, to simulate 12 years with an annual spacing, specify 12 as the `nPeriods` input and 1 as the optional `deltaTime` input (note that the default value for `deltaTime` is 1).

Data Types: `double`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name, Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[ZeroRates,ForwardRates] = simTermStructs(G2PP,NPeriods,'nTrials',100,'deltaTime',dt)`

### **deltaTime — Time step between nPeriods**

1 (default) | numeric

Time step between `nPeriods`, specified as the comma-separated pair consisting of `'deltaTime'` and a numeric scalar or vector. For example, to simulate 12 years with an annual spacing, specify 12 as the `nPeriods` input and 1 as the optional `deltaTime` input (note that the default value for `deltaTime` is 1).

Data Types: double

### **nTrials — Number of simulated trials**

1 (default) | positive integer

Number of simulated trials (sample paths), specified as the comma-separated pair consisting of `'nTrials'` and a positive scalar integer value of `nPeriods` observations each. If you do not specify a value for this argument, the default is 1, indicating a single path of correlated state variables.

Data Types: double

### **antithetic — Flag indicating whether antithetic sampling is used to generate Gaussian random variates**

false (default) | positive integer

Flag indicating whether antithetic sampling is used to generate the Gaussian random variates that drive the zero-drift, unit-variance rate Brownian vector  $dW(t)$ , specified as the comma-separated pair consisting of `'antithetic'` and a Boolean scalar flag. For details, see `simBySolution` for the HWV model.

Data Types: logical

### **Z — Direct specification of dependent random noise process**

Gaussian variates generated by `simBySolution` function (default) | numeric

Direct specification of the dependent random noise process, specified as the comma-separated pair consisting of `'Z'` and a numeric value. The `Z` value is used to generate the zero-drift, unit-variance rate Brownian vector  $dW(t)$  that drives the simulation. For details, see `simBySolution` for the HWV model. If you do not specify a value for `Z`, `simBySolution` generates Gaussian variates.

Data Types: double

**Tenor — Maturities to compute at each time step**

tenor of `LinearGaussian2F` object zero curve (default) | numeric vector

Maturities to compute at each time step, specified as the comma-separated pair consisting of 'Tenor' and a numeric vector.

Tenor enables you to choose a different set of rates to output than the underlying rates. For example, you may want to simulate quarterly data but only report annual rates; this can be done by specifying the optional input Tenor.

Data Types: double

## Output Arguments

**ZeroRates — Simulated zero-rate term structures**

matrix

Simulated zero-rate term structures, returned as a `nPeriods+1-by-nTenors-by-nTrials` matrix.

**ForwardRates — Simulated forward-rate term structures**

matrix

Simulated zero-rate term structures, returned as a `nPeriods+1-by-nTenors-by-nTrials` matrix. The `ForwardRates` output is computed using the simulated short rates and by using the model definition to recover the entire yield curve at each simulation date.

## See Also

`LinearGaussian2F`

## Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Pricing Bermudan Swaptions with Monte Carlo Simulation”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# lookbackbycrr

Price lookback option from Cox-Ross-Rubinstein binomial tree

## Syntax

```
Price = lookbackbycrr(CRRTree,OptSpec,Strike,Settle,ExerciseDates)
Price = lookbackbycrr( ____,AmericanOpt)
```

## Description

Price = lookbackbycrr(CRRTree,OptSpec,Strike,Settle,ExerciseDates) prices lookback options using a Cox-Ross-Rubinstein binomial tree.

Price = lookbackbycrr( \_\_\_\_,AmericanOpt) adds an optional argument for AmericanOpt.

## Examples

### Price a Lookback Option Using a CRR Binomial Tree

This example shows how to price a lookback option using a CRR binomial tree by loading the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;

OptSpec = 'Call';
Strike = 115;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';

Price = lookbackbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price = 7.6015
```

## Input Arguments

### **CRRTree** — Stock tree structure

structure

Stock tree structure for a Cox-Ross-Rubinstein binomial tree, specified by using `crرتree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `char` | `cell`

### **Strike** — Option strike price value

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values. Each row is the schedule for one option.

To compute the value of a floating-strike lookback option, `Strike` must be specified as NaN. Floating-strike lookback options are also known as average strike options.

Data Types: `double`

### **Settle** — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the lookback option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every lookback option is set to the `ValuationDate` of the stock tree. The lookback argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a `NINST`-by-1 matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST`-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a `NINST`-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

**AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

(Optional) Option type, specified as `NINST`-by-1 integer flags with values:

- 0 — European
- 1 — American

Data Types: `single` | `double`

## Output Arguments

**Price — Expected prices for lookback options at time 0**

vector

Expected prices for lookback options at time 0, returned as a `NINST`-by-1 vector. Pricing of lookback options is done using Hull-White (1993). Therefore, for these options there are no unique prices on the tree nodes except for the root node.

## References

- [1] Hull J. and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Fall 1993, pp. 21-31.

## See Also

`crmtree` | `instlookback`

## Topics

“Computing Prices Using CRR” on page 3-131

“Examining Output from the Pricing Functions” on page 3-139

“Computing Equity Instrument Sensitivities” on page 3-144

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing European Call Options Using Different Equity Models”

“Lookback Option” on page 3-51

“Pricing Options Structure” on page B-2

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**



# lookbackbycvgs

Calculate prices of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models

## Syntax

```
Price = lookbackbycvgs(RateSpec, StockSpec, OptSpec, Strike, Settle,
ExerciseDates)
Price = lookbackbycvgs( ____, Name, Value)
```

## Description

`Price = lookbackbycvgs(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates)` returns prices of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models. `lookbackbycvgs` calculates prices of European fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, `Strike` must be specified as `NaN`. The Goldman-Sosin-Gatto model is used for floating-strike lookback options. The Conze-Viswanathan model is used for fixed-strike lookback options.

`Price = lookbackbycvgs( ____, Name, Value)` adds optional name-value pair arguments.

## Examples

### Compute the Price of a Floating Lookback Option Using the Goldman-Sosin-Gatto Model

Define the `RateSpec`.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.042;
Compounding = -1;
```

```
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...  
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
        Disc: 0.9589  
        Rates: 0.0420  
    EndTimes: 1  
    StartTimes: 0  
    EndDates: 735600  
    StartDates: 735235  
    ValuationDate: 735235  
        Basis: 0  
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 50;  
Sigma = 0.36;  
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
        Sigma: 0.3600  
    AssetPrice: 50  
    DividendType: []  
    DividendAmounts: 0  
    ExDividendDates: []
```

Define the floating lookback options.

```
Settle = 'Jan-1-2013';  
Maturity = 'April-1-2013';  
OptSpec = {'put'; 'call'};  
Strike = NaN;
```

Compute the price of the European floating lookback options.

```
Price = lookbackbycvsg(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity)  
  
Price = 2×1  
  
    7.2581
```

6.9777

### Compute the Price of a Fixed Lookback Option Using the Conze-Viswanathan Model

Define the RateSpec.

```

StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.045;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9560
    Rates: 0.0450
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1

```

Define the StockSpec.

```

AssetPrice = 102;
Sigma = 0.45;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.4500
    AssetPrice: 102
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []

```

Define the fixed lookback options.

```
Settle = 'Jan-1-2013';  
Maturity = 'July-1-2013';  
OptSpec = {'put'; 'call'};  
Strike = [98; 101];
```

Price the European fixed lookback options.

```
Price = lookbackbycvsg(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity)
```

```
Price = 2×1
```

```
18.3130  
30.4021
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types: single | double

**Settle — Settlement or trade date**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement or trade date for the lookback option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

**ExerciseDates — European option expiry date**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

European option expiry date, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
lookbackbycvgsg(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'AssetMinMax', AssetMinMax)
```

### **AssetMinMax — Maximum or minimum underlying asset price**

if unspecified, the lookback option is newly issued, and `AssetMinMax = StockSpec.AssetPrice` (default) | nonnegative integer

Maximum or minimum underlying asset price, specified as the comma-separated pair consisting of 'AssetMinMax' and a NINST-by-1 vector.

Data Types: `single` | `double`

## Output Arguments

### **Price — Expected prices of lookback option**

vector

Expected prices of the lookback option, returned as a NINST-by-1 vector.

## References

[1] Hull, J. C. *Options, Futures, and Other Derivatives* 5th Edition. Englewood Cliffs, NJ: Prentice Hall, 2002.

## See Also

`intenvset` | `lookbackbyls` | `lookbacksensbycvgsg` | `lookbacksensbyls` | `stockspec`

## Topics

“Lookback Option” on page 3-51

“Supported Equity Derivatives” on page 3-24

**Introduced in R2014a**

# lookbacksensbycvgsg

Calculate prices or sensitivities of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models

## Syntax

```
PriceSens = lookbacksensbycvgsg(RateSpec,StockSpec,OptSpec,Strike,  
Settle,ExerciseDates)  
PriceSens = lookbacksensbycvgsg( ____,Name,Value)
```

## Description

`PriceSens = lookbacksensbycvgsg(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)` returns prices or sensitivities of European lookback options using Conze-Viswanathan and Goldman-Sosin-Gatto models. `lookbacksensbycvgsg` calculates prices of European fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, `Strike` must be specified as `NaN`. The Goldman-Sosin-Gatto model is used for floating-strike lookback options. The Conze-Viswanathan model is used for fixed-strike lookback options.

`PriceSens = lookbacksensbycvgsg( ____,Name,Value)` adds optional name-value pair arguments.

## Examples

### Compute the Price and Delta of a Floating Lookback Option Using the Goldman-Sosin-Gatto Model

Define the `RateSpec`.

```
StartDates = 'Jan-1-2013';  
EndDates = 'Jan-1-2014';  
Rates = 0.41;  
Compounding = -1;
```

```
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.6637
    Rates: 0.4100
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec with continuous dividend yield.

```
AssetPrice = 120;
Sigma = 0.3;
Yield = 0.045;
StockSpec = stockspec(Sigma, AssetPrice, 'Continuous', Yield)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
    AssetPrice: 120
    DividendType: {'continuous'}
    DividendAmounts: 0.0450
    ExDividendDates: []
```

Define the floating lookback option.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = NaN;
SMinMax = 100;
```

Compute the price and delta of the European floating lookback option.

```
OutSpec = {'price', 'delta'};
[Price, Delta] = lookbacksensbycvgs(RateSpec, StockSpec, OptSpec, Strike,...
    Settle, Maturity, 'AssetMinMax', SMinMax, 'OutSpec', OutSpec)
```



Price = 36.9926

Delta = 0.8659

### Compute the Price and Delta of a Fixed Lookback Option Using the Conze-Viswanathan Model

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.8187
    Rates: 0.1000
    EndTimes: 2
    StartTimes: 0
    EndDates: 735965
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 103;
Sigma = 0.30;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
    AssetPrice: 103
    DividendType: []
    DividendAmounts: 0
```

```
ExDividendDates: []
```

Define the fixed lookback option.

```
Settle    = 'Jan-1-2013';  
Maturity  = 'July-1-2013';  
OptSpec   = 'call';  
Strike    = 99;
```

Price and delta for the European fixed lookback option.

```
OutSpec   = {'price', 'delta'};  
[Price, Delta] = lookbacksensbyls(RateSpec, StockSpec, OptSpec, ...  
Strike, Settle, Maturity, 'OutSpec', OutSpec)
```

```
Price = 22.7227
```

```
Delta = 1.1349
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types: single | double

**Settle — Settlement or trade date**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement or trade date for the lookback option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

**ExerciseDates — European option expiry date**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

European option expiry date, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of dates.

Data Types: double | char | cell

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `PriceSens =`

```
lookbacksensbycvgsg(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'AssetMinMax', AssetMinMax, 'OutSpec', {'All'})
```

**AssetMinMax — Maximum or minimum underlying asset price**

if unspecified, the lookback option is newly issued, and `AssetMinMax = StockSpec.AssetPrice` (default) | nonnegative integer

Maximum or minimum underlying asset price, specified as a NINST-by-1 vector.

Data Types: `single` | `double`

**OutSpec — Define outputs**

`{'Price'}` (default) | character vector with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | cell array of character vectors with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

Example: `OutSpec = {'delta','gamma','vega','lambda','rho','theta','price'}`

Data Types: `char` | `cell`

## Output Arguments

**PriceSens — Expected prices or sensitivities of lookback option**

vector

Expected prices or sensitivities (defined by `OutSpec`) of the lookback option, returned as a NINST-by-1 vector.

## References

- [1] Hull, J. C. *Options, Futures, and Other Derivatives* 5th Edition. Englewood Cliffs, NJ, Prentice Hall, 2002.

## **See Also**

[intenvset](#) | [lookbackbycvgsg](#) | [lookbackbyls](#) | [lookbacksensbyls](#) | [stockspec](#)

## **Topics**

“Lookback Option” on page 3-51

“Supported Equity Derivatives” on page 3-24

**Introduced in R2014a**

## lookbackbyeqp

Price lookback option from Equal Probabilities binomial tree

### Syntax

```
Price = lookbackbyeqp(EQPTree,OptSpec,Strike,Settle,ExerciseDates)
Price = lookbackbyeqp( ____,AmericanOpt)
```

### Description

Price = lookbackbyeqp(EQPTree,OptSpec,Strike,Settle,ExerciseDates) prices lookback options using an Equal Probabilities binomial tree.

Price = lookbackbyeqp( \_\_\_\_,AmericanOpt) adds an optional argument for AmericanOpt.

### Examples

#### Price a Lookback Option Using an EQP Equity Tree

This example shows how to price a lookback option using an EQP equity tree by loading the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat

OptSpec = 'Call';
Strike = 115;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';

Price = lookbackbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price = 8.7941
```

## Input Arguments

### **EQPTree** — Stock tree structure

structure

Stock tree structure for an Equal Probabilities binomial tree, specified by using `eqptree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `char` | `cell`

### **Strike** — Option strike price value

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values. Each row is the schedule for one option.

To compute the value of a floating-strike lookback option, `Strike` must be specified as NaN. Floating-strike lookback options are also known as average strike options.

Data Types: `double`

### **Settle** — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the lookback option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every lookback option is set to the `ValuationDate` of the stock tree. The lookback argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a `NINST`-by-1 matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST`-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a `NINST`-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

**AmericanOpt — Option type**

0 European (default) | integer with values 0 or 1

(Optional) Option type, specified as `NINST`-by-1 integer flags with values:

- 0 — European
- 1 — American

Data Types: single | double

## Output Arguments

**Price — Expected prices for lookback options at time 0**

vector

Expected prices for lookback options at time 0, returned as a `NINST`-by-1 vector. Pricing of lookback options is done using Hull-White (1993). Therefore, for these options there are no unique prices on the tree nodes except for the root node.

## References

- [1] Hull J. and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Fall 1993, pp. 21-31.



## See Also

eqptree | instlookback

## Topics

“Computing Prices Using EQP” on page 3-133

“Examining Output from the Pricing Functions” on page 3-139

“Computing Equity Instrument Sensitivities” on page 3-144

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing European Call Options Using Different Equity Models”

“Lookback Option” on page 3-51

“Computing Instrument Prices” on page 3-130

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## lookbackbyitt

Price lookback option using implied trinomial tree (ITT)

### Syntax

```
Price = lookbackbyitt(ITTTree,OptSpec,Strike,Settle,ExerciseDates)
Price = lookbackbyitt( ____,AmericanOpt)
```

### Description

`Price = lookbackbyitt(ITTTree,OptSpec,Strike,Settle,ExerciseDates)` prices lookback options using an implied trinomial tree (ITT).

`Price = lookbackbyitt( ____,AmericanOpt)` adds an optional argument for `AmericanOpt`.

### Examples

#### Price a Lookback Option Using an ITT Equity Tree

This example shows how to price a lookback option using an ITT equity tree by loading the file `deriv.mat`, which provides the `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat

OptSpec = 'Call';
Strike = 85;
Settle = '01-Jan-2006';
ExerciseDates = '01-Jan-2008';

Price = lookbackbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates)

Price = 0.5426
```

## Input Arguments

### ITTree — Stock tree structure

structure

Stock tree structure for an implied trinomial tree (ITT), specified by using `itttree`.

Data Types: `struct`

### OptSpec — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `char` | `cell`

### Strike — Option strike price value

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values. Each row is the schedule for one option.

To compute the value of a floating-strike lookback option, `Strike` must be specified as NaN. Floating-strike lookback options are also known as average strike options.

Data Types: `double`

### Settle — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the lookback option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every lookback option is set to the `ValuationDate` of the stock tree. The lookback argument, `Settle`, is ignored.

---

Data Types: `double` | `char`

### ExerciseDates — Option exercise dates

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a `NINST-by-1` matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a `NINST-by-2` vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a `NINST-by-1` vector of serial date numbers or cell array of character vectors, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

### **AmericanOpt — Option type**

`0` European (default) | integer with values `0` or `1`

(Optional) Option type, specified as `NINST-by-1` integer flags with values:

- `0` — European
- `1` — American

Data Types: `single` | `double`

## **Output Arguments**

### **Price — Expected prices for lookback options at time 0**

vector

Expected prices for lookback options at time 0, returned as a `NINST-by-1` vector. Pricing of lookback options is done using Hull-White (1993). Therefore, for these options there are no unique prices on the tree nodes except for the root node.

## **References**

- [1] Hull J. and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Fall 1993, pp. 21-31.

## **See Also**

instlookback | itttree

## **Topics**

“Computing Prices Using ITT” on page 3-135

“Examining Output from the Pricing Functions” on page 3-139

“Computing Equity Instrument Sensitivities” on page 3-144

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Pricing European Call Options Using Different Equity Models”

“Lookback Option” on page 3-51

“Computing Instrument Prices” on page 3-130

“Supported Equity Derivatives” on page 3-24

## **Introduced in R2007a**

## lookbackbyls

Price European or American lookback options using Monte Carlo simulations

### Syntax

```
[Price,Paths,Times,Z] = lookbackbyls(RateSpec,StockSpec,OptSpec,  
Strike,Settle,ExerciseDates)  
[Price,Paths,Times,Z] = lookbackbyls( ____,Name,Value)
```

### Description

[Price,Paths,Times,Z] = lookbackbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates) returns prices of lookback options using the Longstaff-Schwartz model for Monte Carlo simulations. lookbackbyls computes prices of European and American lookback options.

For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium.

lookbackbyls calculates values of fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, Strike must be specified as NaN.

[Price,Paths,Times,Z] = lookbackbyls( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Price for a Floating Lookback Option Using Monte Carlo Simulation

Define the RateSpec.

```
StartDates = 'Jan-1-2013';  
EndDates = 'Jan-1-2014';  
Rates = 0.042;
```

```

Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9589
    Rates: 0.0420
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1

```

Define the StockSpec.

```

AssetPrice = 50;
Sigma = 0.36;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3600
    AssetPrice: 50
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []

```

Define the floating lookback option.

```

Settle = 'Jan-1-2013';
Maturity = 'April-1-2013';
OptSpec = 'put';
Strike = NaN;

```

Compute the price of the European floating lookback option.

```

Price = lookbackbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity)

Price = 6.6471

```

**Compute the Price of a Fixed Lookback Option Using Monte Carlo Simulation**

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2014';
Rates = 0.045;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9560
    Rates: 0.0450
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 102;
Sigma = 0.45;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.4500
    AssetPrice: 102
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the fixed lookback option.

```
Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
```



```
OptSpec = 'call';  
Strike = 98;
```

Compute the price of the European fixed lookback option.

```
Price = lookbackbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity)
```

```
Price = 30.2368
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: `char` | `cell`

**Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types: single | double

**Settle — Settlement or trade date**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement or trade date for the lookback option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

**ExerciseDates — Matrix of exercise callable or puttable dates for European or American options**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Matrix of exercise callable or puttable dates for European or American options, specified as date character vectors or as serial date numbers as follows:

- European option — NINST-by-1 vector of exercise dates. For a European option, there is only one exercise date which is the option expiry date.
- American option — NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector of serial date numbers or cell array of character vectors, the option is exercised between Settle and the single listed exercise date.

Data Types: double | char | cell

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: `Price = lookbackbyls(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, 'AmericanOpt', 1)`

### **AmericanOpt — Option type**

0 European (default) | scalar with value [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and an integer scalar flag with these values:

- 0 — European
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: single | double

### **NumTrials — Scalar number of independent sample paths**

1000 (default) | nonnegative scalar integer

Scalar number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of 'NumTrials' and a nonnegative integer.

Data Types: single | double

### **NumPeriods — Scalar number of simulation periods per trial**

100 (default) | nonnegative scalar integer

Scalar number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a nonnegative integer. NumPeriods is considered only when pricing European lookback options. For American lookback options, NumPeriods is equal to the number of exercise days during the life of the option.

Data Types: single | double

### **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-1-by-NumTrials 3-D array. The Z value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: single | double

**Antithetic — Indicator for antithetic sampling**

false (default) | scalar logical flag with value of true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

## Output Arguments

**Price — Expected price of lookback option**

scalar

Expected price of the lookback option, returned as a 1-by-1 scalar.

**Paths — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a NumPeriods + 1-by-1-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

**Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a NumPeriods + 1-by-1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths.

**Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, returned as a NumPeriods-by-1-by-NumTrials 3-D array when Z is specified as an input argument. If the Z input argument

is not specified, then the Z output argument contains the random variates generated internally.

## References

[1] Hull, J. C. *Options, Futures, and Other Derivatives* 5th Edition. Englewood Cliffs, NJ: Prentice Hall, 2002.

## See Also

intenvset | lookbackbycvgs | lookbacksensbycvgs | lookbacksensbyls | stockspect

## Topics

“Lookback Option” on page 3-51

“Supported Equity Derivatives” on page 3-24

**Introduced in R2014a**

## lookbacksensbyls

Calculate price and sensitivities for European or American lookback options using Monte Carlo simulations

### Syntax

```
[PriceSens,Paths,Times,Z] = lookbacksensbyls(RateSpec,StockSpec,  
OptSpec,Strike,Settle,ExerciseDates)  
[PriceSens,Paths,Times,Z] = lookbacksensbyls( ____,Name,Value)
```

### Description

[PriceSens,Paths,Times,Z] = lookbacksensbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates) returns prices or sensitivities of lookback options using the Longstaff-Schwartz model for Monte Carlo simulations. lookbacksensbyls computes prices of European and American lookback options.

For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium.

lookbacksensbyls calculates values of fixed- and floating-strike lookback options. To compute the value of a floating-strike lookback option, `Strike` must be specified as `NaN`.

[PriceSens,Paths,Times,Z] = lookbacksensbyls( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Price and Delta of a European Floating Lookback Option Using Monte Carlo Simulation

Define the RateSpec.

```
StartDates = 'Jan-1-2013';  
EndDates = 'Jan-1-2014';
```

```

Rates = 0.41;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.6637
    Rates: 0.4100
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1

```

Define the StockSpec with continuous dividend yield.

```

AssetPrice = 120;
Sigma = 0.3;
Yield = 0.045;
StockSpec = stockspec(Sigma, AssetPrice, 'Continuous', Yield)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
    AssetPrice: 120
    DividendType: {'continuous'}
    DividendAmounts: 0.0450
    ExDividendDates: []

```

Define the floating lookback option.

```

Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = NaN;

```

Compute the price and delta of the European floating lookback option.

```
OutSpec = {'price', 'delta'};
[Price, Delta] = lookbacksensbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity,
'OutSpec', OutSpec)

Price = 27.0768

Delta = 0.2256
```

### **Compute the Price and Delta of a European Fixed Lookback Option Using Monte Carlo Simulation**

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.8187
    Rates: 0.1000
    EndTimes: 2
    StartTimes: 0
    EndDates: 735965
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 103;
Sigma = 0.30;
StockSpec = stockspec(Sigma, AssetPrice)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
```



```

    AssetPrice: 103
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []

```

Define the fixed lookback option.

```

Settle = 'Jan-1-2013';
Maturity = 'July-1-2013';
OptSpec = 'call';
Strike = 99;

```

Compute the price and delta of the European fixed lookback option.

```

OutSpec = {'price', 'delta'};
[Price, Delta] = lookbacksensbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, Maturity,
'OutSpec', OutSpec)

```

```
Price = 22.7227
```

```
Delta = 1.1349
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### StockSpec — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` can handle several types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented

by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types: `single` | `double`

### **Settle — Settlement or trade date**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement or trade date for the lookback option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: `double` | `char` | `cell`

### **ExerciseDates — Matrix of exercise callable or puttable dates for European or American options**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Matrix of exercise callable or puttable dates for European or American options, specified as date character vectors or as serial date numbers as follows:

- European option — NINST-by-1 vector of exercise dates. For a European option, there is only one exercise date which is the option expiry date.
- American option — NINST-by-2 vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a

NINST-by-1 vector of serial date numbers or cell array of character vectors, the option is exercised between `Settle` and the single listed exercise date.

Data Types: `double` | `char` | `cell`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `PriceSens = lookbacksensbyls(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, 'AmericanOpt', 1, 'OutSpec', {'All'})`

### AmericanOpt — Option type

0 European (default) | scalar with value [0, 1]

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and an integer scalar flag with these values:

- 0 — European
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: `single` | `double`

### NumTrials — Scalar number of independent sample paths

1000 (default) | nonnegative scalar integer

Scalar number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of `'NumTrials'` and a nonnegative integer.

Data Types: `single` | `double`

**NumPeriods — Scalar number of simulation periods per trial**

100 (default) | nonnegative scalar integer

Scalar number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a nonnegative integer. NumPeriods is considered only when pricing European lookback options. For American lookback options, NumPeriod is equal to the number of exercise days during the life of the option.

Data Types: single | double

**Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-1-by-NumTrials 3-D array. The Z value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: single | double

**Antithetic — Indicator for antithetic sampling**

false (default) | scalar logical flag with value true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.

Example: OutSpec =

```
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}
```

Data Types: char | cell

## Output Arguments

### **PriceSens — Expected price or sensitivities of lookback option**

scalar

Expected price or sensitivities (defined by `OutSpec`) of the lookback option, returned as a 1-by-1 array.

### **Paths — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a `NumPeriods + 1`-by-1-by-`NumTrials` 3-D time series array. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

### **Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a `NumPeriods + 1`-by-1 column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

### **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, returned as a `NumPeriods`-by-1-by-`NumTrials` 3-D array when `Z` is specified as an input argument. If the `Z` input argument is not specified, then the `Z` output argument contains the random variates generated internally.

## References

- [1] Hull, J. C. *Options, Futures, and Other Derivatives* 5th Edition. Englewood Cliffs, NJ: Prentice Hall, 2002.

## **See Also**

intenvset | lookbackbycvgs | lookbackbyls | lookbacksensbycvgs |  
stockspec

## **Topics**

“Lookback Option” on page 3-51

“Supported Equity Derivatives” on page 3-24

**Introduced in R2014a**

# lookbackbystt

Price lookback options using standard trinomial tree

## Syntax

```
Price = lookbackbystt(STTTree,OptSpec,Strike,Settle,ExerciseDates)
Price = lookbackbystt(____,AmericanOpt)
```

## Description

Price = lookbackbystt(STTTree,OptSpec,Strike,Settle,ExerciseDates) prices lookback options using a standard trinomial (STT) tree.

Price = lookbackbystt(\_\_\_\_,AmericanOpt) prices lookback options using a standard trinomial (STT) tree with an optional argument for AmericanOpt.

## Examples

### Price a Lookback Option Using the Standard Trinomial Tree Model

Create a RateSpec.

```
StartDates = 'Jan-1-2009';
EndDates = 'Jan-1-2013';
Rates = 0.035;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.8694
    Rates: 0.0350
```

```
EndTimes: 4
StartTimes: 0
EndDates: 735235
StartDates: 733774
ValuationDate: 733774
Basis: 1
EndMonthRule: 1
```

Create a `StockSpec`.

```
AssetPrice = 85;
Sigma = 0.15;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1500
    AssetPrice: 85
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Create an `STTTree`.

```
NumPeriods = 4;
TimeSpec = stttimespec(StartDates, EndDates, 4);
STTTree = stttree(StockSpec, RateSpec, TimeSpec)
```

```
STTTree = struct with fields:
    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [733774 734139 734504 734869 735235]
    STree: {1x5 cell}
    Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
```

Define the lookback option and compute the price.

```
Settle = '1/1/09';
ExerciseDates = [datenum('1/1/12'); datenum('1/1/13')];
```



```

OptSpec = 'call';
Strike = [90;95];

Price= lookbackbstt(STTtree, OptSpec, Strike, Settle, ExerciseDates)

Price = 2×1

    11.7296
    12.9120

```

## Input Arguments

**STTtree** — Stock tree structure for standard trinomial tree structure

Stock tree structure for a standard trinomial tree, specified by using `stttree`.

Data Types: `struct`

**OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector or a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `char` | `cell`

**Strike** — Option strike price value

matrix of nonnegative integers

Option strike price value, specified with a nonnegative integer using a NINST-by-1 matrix of strike price values. Each row is the schedule for one option. To compute the value of a floating-strike lookback option, `Strike` should be specified as `NaN`. Floating-strike lookback options are also known as average strike options.

Data Types: `double`

**Settle** — Settlement date or trade date

serial date number | date character vector

Settlement date or trade date for the lookback option, specified as a NINST-by-1 matrix of settlement or trade dates using serial date numbers or date character vectors.

---

**Note** The `Settle` date for every lookback option is set to the `ValuationDate` of the stock tree. The lookback argument, `Settle`, is ignored.

---

Data Types: double | char

### **ExerciseDates** — Option exercise dates

serial date number | date character vector

Option exercise dates, specified as a serial date number or date character vector:

- For a European option, use a NINST-by-1 matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a NINST-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

### **AmericanOpt** — Option type

0 European (default) | scalar with values [0, 1]

Option type, specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

Data Types: single | double

## **Output Arguments**

### **Price** — Expected prices for lookback options at time 0

matrix

Expected prices for lookback options at time 0, returned as a NINST-by-1 matrix. Pricing of lookback options is done using Hull-White (1993). Consequently, for these options there are no unique prices on the tree nodes with the exception of the root node.

## References

- [1] Hull J. and A. White. "Efficient Procedures for Valuing European and American Path-Dependent Options." *Journal of Derivatives*. Fall 1993, pp. 21-31.

## See Also

sttprice | sttsens | stttimespec | stttree

## Topics

"Lookback Option" on page 3-51

"Supported Equity Derivatives" on page 3-24

**Introduced in R2015b**

## **lrtimespec**

Specify time structure for Leisen-Reimer binomial tree

### **Syntax**

```
TimeSpec = lrtimespec(ValuationDate,Maturity,NumPeriods)
```

### **Description**

`TimeSpec = lrtimespec(ValuationDate,Maturity,NumPeriods)` specifies a time structure for a Leisen-Reimer stock tree (`lrtree`).

### **Examples**

#### **Specify the Time Structure for Leisen-Reimer Binomial Tree**

This example shows how to specify a 5-period tree with time steps of 1 year.

```
ValuationDate = '1-July-2010';  
Maturity = '1-July-2015';  
TimeSpec = lrtimespec(ValuationDate, Maturity, 5)  
  
TimeSpec = struct with fields:  
    FinObj: 'BinTimeSpec'  
    ValuationDate: 734320  
    Maturity: 736146  
    NumPeriods: 5  
    Basis: 0  
    EndMonthRule: 1  
    tObs: [0 1 2 3 4 5]  
    dObs: [734320 734685 735050 735415 735780 736146]
```

## Input Arguments

### **ValuationDate** — Pricing date and first observation in Leisen-Reimer stock tree

serial date number | character vector date

Pricing date and first observation in the `lrtree`, specified as a scalar date using a serial date number or date character vector.

Data Types: double | char

### **Maturity** — Date marking the depth of the Leisen-Reimer stock tree

serial date number | date character vector

Date marking the depth of the Leisen-Reimer stock tree, specified as scalar serial date number or date character vector.

Data Types: double | char

### **NumPeriods** — Number of time steps in the Leisen-Reimer stock tree

odd integer value

Number of time steps in the Leisen-Reimer stock tree, specified as scalar odd integer value.

---

**Note** Leisen-Reimer requires the number of steps to be an odd number.

---

Data Types: double

## Output Arguments

### **TimeSpec** — Specification for the time layout for `lrtree`

structure

Specification for the time layout for `lrtree`, returned as a structure.

## References

- [1] Leisen D.P., M. Reimer. "Binomial Models for Option Valuation - Examining and Improving Convergence." *Applied Mathematical Finance*. Number 3, 1996, pp. 319-346.

## See Also

lrtree | stockspec

## Topics

- "Building Equity Binary Trees" on page 3-3  
"Understanding Equity Trees" on page 3-2  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2010b**

# lrtree

Build Leisen-Reimer stock tree

## Syntax

```
LRTree = lrtree(StockSpec,RateSpec,TimeSpec,Strike)
LRTree = lrtree( ____,Name,Value)
```

## Description

`LRTree = lrtree(StockSpec,RateSpec,TimeSpec,Strike)` builds a Leisen-Reimer stock tree.

`LRTree = lrtree( ____,Name,Value)` adds a name-value pair argument.

## Examples

### Build a Leisen-Reimer Stock Tree

This example shows how to build Leisen-Reimer stock tree. Consider a European put option with an exercise price of \$30 that expires on June 1, 2010. The underlying stock is trading at \$30 on January 1, 2010 and has a volatility of 30% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, create a Leisen-Reimer tree with 101 steps using the PP1 method.

```
AssetPrice = 30;
Strike = 30;

ValuationDate = 'Jan-1-2010';
Maturity = 'June-1-2010';

% define StockSpec
Sigma = 0.3;
StockSpec = stockspect(Sigma, AssetPrice);
```

```
% define RateSpec
Rates = 0.05;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

% build the Leisen-Reimer (LR) tree with 101 steps
LRTimeSpec = lrtimespec(ValuationDate, Maturity, 101);

% use the PP1 method
LRMethod = 'PP1';

LRTree = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike, ...
'method', LRMethod)

LRTree = struct with fields:
    FinObj: 'BinStockTree'
    Method: 'LR'
    Submethod: 'PP1'
    Strike: 30
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [1x102 double]
        dObs: [1x102 double]
    STree: {1x102 cell}
    UpProbs: [101x1 double]
```

## Input Arguments

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for underlying asset, specified using StockSpec obtained from stockspeg. For information on the stock specification, see stockspeg.



`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **RateSpec — Interest-rate specification for initial risk-free rate curve**

structure

Interest-rate specification for initial rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **TimeSpec — Time tree layout specification**

structure

Time tree layout specification, specified using the `TimeSpec` output obtained from `lrtimespec`.

Data Types: `struct`

### **Strike — Option strike price value**

nonnegative integer

Option strike price value, specified as a scalar nonnegative integer.

Data Types: `double`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `LRTree =`

```
lrtree(StockSpec, RateSpec, LRTimeSpec, Strike, 'Method', 'PP2')
```

### **Method — Computation method**

'PP1' (default) | character vector with value 'PP1' or 'PP2'

Computation method, specified as the comma-separated pair consisting of 'Method' and a character vector with a value of 'PP1' or 'PP2'. 'PP1' is for Peizer-Pratt method 1

inversion and 'PP2' is for Peizer-Pratt method 2 inversion. For more information on 'PP1' and 'PP2' methods, see “Leisen-Reimer Tree (LR) Modeling” on page C-3.

Data Types: char

## Output Arguments

### **LRTree — Stock and time information for a Leisen-Reimer tree**

structure

Stock and time information for a Leisen-Reimer tree, returned as a structure.

## References

- [1] Leisen D.P., M. Reimer. “Binomial Models for Option Valuation – Examining and Improving Convergence.” *Applied Mathematical Finance*. Number 3, 1996, pp. 319-346.

## See Also

`intenvset` | `lrtimespec` | `optstockbylr` | `optstocksensbylr` | `stockspec`

## Topics

“Building Equity Binary Trees” on page 3-3

“Understanding Equity Trees” on page 3-2

“Supported Equity Derivatives” on page 3-24

**Introduced in R2010b**

# maxassetbystulz

Determine European rainbow option price on maximum of two risky assets using Stulz option pricing model

## Syntax

Price =  
 maxassetbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)

## Arguments

RateSpec	The annualized, continuously compounded rate term structure. For information on the interest rate specification, see <code>intenvset</code> .
StockSpec1	Stock specification for asset 1. See <code>stockspec</code> .
StockSpec2	Stock specification for asset 2. See <code>stockspec</code> .
Settle	NINST-by-1 vector of settlement or trade dates.
Maturity	NINST-by-1 vector of maturity dates.
OptSpec	NINST-by-1 cell array of character vectors 'call' or 'put'.
Strike	NINST-by-1 vector of strike price values.
Corr	NINST-by-1 vector of correlation between the underlying asset prices.

## Description

Price =  
 maxassetbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr) computes rainbow option prices using the Stulz option pricing model.

Price is a NINST-by-1 vector of expected option prices.

## Examples

### Compute Rainbow Option Prices Using the Stulz Option Pricing Model

Consider a European rainbow option that gives the holder the right to buy either \$100,000 worth of an equity index at a strike price of 1000 (asset 1) or \$100,000 of a government bond (asset 2) with a strike price of 100% of face value, whichever is worth more at the end of 12 months. On January 15, 2008, the equity index is trading at 950, pays a dividend of 2% annually and has a return volatility of 22%. Also on January 15, 2008, the government bond is trading at 98, pays a coupon yield of 6%, and has a return volatility of 15%. The risk-free rate is 5%. Using this data, if the correlation between the rates of return is -0.5, 0, and 0.5, calculate the price of the European rainbow option.

Since the asset prices in this example are in different units, it is necessary to work in either index points (asset 1) or in dollars (asset 2). The European rainbow option allows the holder to buy the following: 100 units of the equity index at \$1000 each (for a total of \$100,000) or 1000 units of the government bonds at \$100 each (for a total of \$100,000). To convert the bond price (asset 2) to index units (asset 1), you must make the following adjustments:

- Multiply the strike price and current price of the government bond by 10 (1000/100).
- Multiply the option price by 100, considering that there are 100 equity index units in the option.

Once these adjustments are introduced, the strike price is the same for both assets (\$1000). First, create the `RateSpec`:

```
Settle = 'Jan-15-2008';
Maturity = 'Jan-15-2009';
Rates = 0.05;
Basis = 1;

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
```

```

    StartTimes: 0
      EndDates: 733788
    StartDates: 733422
  ValuationDate: 733422
      Basis: 1
  EndMonthRule: 1

```

Create the two StockSpec definitions.

```

AssetPrice1 = 950; % Asset 1 => Equity index
AssetPrice2 = 980; % Asset 2 => Government bond
Sigma1 = 0.22;
Sigma2 = 0.15;
Div1 = 0.02;
Div2 = 0.06;

```

```

StockSpec1 = stockspec(Sigma1, AssetPrice1, 'continuous', Div1)

```

```

StockSpec1 = struct with fields:
    FinObj: 'StockSpec'
      Sigma: 0.2200
    AssetPrice: 950
    DividendType: {'continuous'}
  DividendAmounts: 0.0200
  ExDividendDates: []

```

```

StockSpec2 = stockspec(Sigma2, AssetPrice2, 'continuous', Div2)

```

```

StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
      Sigma: 0.1500
    AssetPrice: 980
    DividendType: {'continuous'}
  DividendAmounts: 0.0600
  ExDividendDates: []

```

Calculate the price of the options for different correlation levels.

```

Strike = 1000 ;
Corr = [-0.5; 0; 0.5];
OptSpec = 'call';

```

```
Price = maxassetbystulz(RateSpec, StockSpec1, StockSpec2, ...  
Settle, Maturity, OptSpec, Strike, Corr)
```

```
Price = 3×1
```

```
111.6683  
103.7715  
92.4412
```

These are the prices of one unit. This means that the premium is 11166.83, 10377.15, and 9244.12 (for 100 units).

### See Also

[intenvset](#) | [maxassetsensbystulz](#) | [minassetbystulz](#) | [stockspec](#)

### Topics

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Rainbow Option” on page 3-35

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**

# maxassetsensbystulz

Determine European rainbow option prices or sensitivities on maximum of two risky assets using Stulz pricing model

## Syntax

```
PriceSens =
maxassetsensbystulz(RateSpec,StockSpec1,StockSpec2,Settle,Maturity,0
ptSpec,Strike,Corr)
PriceSens =
maxassetsensbystulz(RateSpec,StockSpec1,StockSpec2,Settle,Maturity,0
ptSpec,Strike,Corr,OutSpec)
```

## Arguments

RateSpec	The annualized, continuously compounded rate term structure. For information on the interest rate specification, see <code>intenvset</code> .
StockSpec1	Stock specification for asset 1. See <code>stockspec</code> .
StockSpec2	Stock specification for asset 2. See <code>stockspec</code> .
Settle	NINST-by-1 vector of settlement or trade dates.
Maturity	NINST-by-1 vector of maturity dates.
OptSpec	NINST-by-1 cell array of character vectors 'call' or 'put'.
Strike	NINST-by-1 vector of strike price values.
Corr	NINST-by-1 vector of correlation between the underlying asset prices.

<p><b>OutSpec</b></p>	<p>(Optional) All optional inputs are specified as matching parameter name-value pairs. The parameter name is specified as a character vector, followed by the corresponding parameter value. You can specify parameter name-value pairs in any order. Names are case-insensitive and partial matches are allowed provided no ambiguities exist. Valid parameter names are:</p> <ul style="list-style-type: none"> <li>• NOUT-by-1 or 1-by-NOUT cell array of character vectors indicating the nature and order of the outputs for the function. Possible values are 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', or 'All'.</li> </ul> <p>For example, <code>OutSpec = {'Price'; 'Lambda'; 'Rho'}</code> specifies that the output should be Price, Lambda, and Rho, in that order.</p> <p>To invoke from a function: <code>[Price, Lambda, Rho] = maxassetsensbystulz(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})</code></p> <p><code>OutSpec = {'All'}</code> specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying <code>OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'}</code>;</p> <ul style="list-style-type: none"> <li>• Default is <code>OutSpec = {'Price'}</code>.</li> </ul>
-----------------------	---

## Description

`PriceSens = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, 0, ptSpec, Strike, Corr)` computes rainbow option prices using the Stulz option pricing model.

`PriceSens = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, 0, ptSpec, Strike, Corr, OutSpec)` computes rainbow option prices or sensitivities using the Stulz option pricing model.



PriceSens is a NINST-by-1 or NINST-by-2 vector of expected prices or sensitivities values.

## Examples

### Compute Rainbow Option Prices and Sensitivities Using the Stulz Option Pricing Model

Consider a European rainbow option that gives the holder the right to buy either \$100,000 of an equity index at a strike price of 1000 (asset 1) or \$100,000 of a government bond (asset 2) with a strike price of 100% of face value, whichever is worth more at the end of 12 months. On January 15, 2008, the equity index is trading at 950, pays a dividend of 2% annually, and has a return volatility of 22%. Also on January 15, 2008, the government bond is trading at 98, pays a coupon yield of 6%, and has a return volatility of 15%. The risk-free rate is 5%. Using this data, calculate the price and sensitivity of the European rainbow option if the correlation between the rates of return is -0.5, 0, and 0.5.

Since the asset prices in this example are in different units, it is necessary to work in either index points (for asset 1) or in dollars (for asset 2). The European rainbow option allows the holder to buy the following: 100 units of the equity index at \$1000 each (for a total of \$100,000) or 1000 units of the government bonds at \$100 each (for a total of \$100,000). To convert the bond price (asset 2) to index units (asset 1), you must make the following adjustments:

- Multiply the strike price and current price of the government bond by 10 (1000/100).
- Multiply the option price by 100, considering that there are 100 equity index units in the option.

Once these adjustments are introduced, the strike price is the same for both assets (\$1000). First, create the RateSpec:

```
Settle = 'Jan-15-2008';
Maturity = 'Jan-15-2009';
Rates = 0.05;
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 733788
    StartDates: 733422
    ValuationDate: 733422
    Basis: 1
    EndMonthRule: 1
```

Create the two StockSpec definitions.

```
AssetPrice1 = 950; % Asset 1 => Equity index
AssetPrice2 = 980; % Asset 2 => Government bond
Sigma1 = 0.22;
Sigma2 = 0.15;
Div1 = 0.02;
Div2 = 0.06;
```

```
StockSpec1 = stockspec(Sigma1, AssetPrice1, 'continuous', Div1)
```

```
StockSpec1 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2200
    AssetPrice: 950
    DividendType: {'continuous'}
    DividendAmounts: 0.0200
    ExDividendDates: []
```

```
StockSpec2 = stockspec(Sigma2, AssetPrice2, 'continuous', Div2)
```

```
StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1500
    AssetPrice: 980
    DividendType: {'continuous'}
    DividendAmounts: 0.0600
    ExDividendDates: []
```

Calculate the price and delta for different correlation levels.

```
Strike = 1000 ;
Corr = [-0.5; 0; 0.5];
OutSpec = {'price'; 'delta'};
OptSpec = 'call';
[Price, Delta] = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2,...
Settle, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)
```

```
Price = 3×1
```

```
111.6683
103.7715
92.4412
```

```
Delta = 3×2
```

```
0.4594    0.3698
0.4292    0.3166
0.4053    0.2512
```

The output `Delta` has two columns: the first column represents the `Delta` with respect to the equity index (asset 1), and the second column represents the `Delta` with respect to the government bond (asset 2). The value 0.4595 represents `Delta` with respect to one unit of the equity index. Since there are 100 units of the equity index, the overall `Delta` would be 45.94 ( $100 * 0.4594$ ) for a correlation level of -0.5. To calculate the `Delta` with respect to the government bond, remember that an adjusted price of 980 was used instead of 98. Therefore, for example, the `Delta` with respect to government bond, for a correlation of 0.5 would be 251.2 ( $0.2512 * 100 * 10$ ).

## See Also

[intenvset](#) | [maxassetsensbystulz](#) | [stockspec](#)

## Topics

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Rainbow Option” on page 3-35

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**

## minassetbystulz

Determine European rainbow option prices on minimum of two risky assets using Stulz option pricing model

### Syntax

```
Price = minassetbystulz(RateSpec, StockSpec1, StockSpec2, Settle,  
Maturity, OptSpec, Strike, Corr)
```

### Description

Price = minassetbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr) computes option prices using the Stulz option pricing model.

### Examples

#### Compute Rainbow Option Prices Using the Stulz Option Pricing Model

Consider a European rainbow put option that gives the holder the right to sell either stock A or stock B at a strike of 50.25, whichever has the lower value on the expiration date May 15, 2009. On November 15, 2008, stock A is trading at 49.75 with a continuous annual dividend yield of 4.5% and has a return volatility of 11%. Stock B is trading at 51 with a continuous dividend yield of 5% and has a return volatility of 16%. The risk-free rate is 4.5%. Using this data, if the correlation between the rates of return is -0.5, 0, and 0.5, calculate the price of the minimum of two assets that are European rainbow put options. First, create the RateSpec:

```
Settle = 'Nov-15-2008';  
Maturity = 'May-15-2009';  
Rates = 0.045;  
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9778
    Rates: 0.0450
    EndTimes: 0.5000
    StartTimes: 0
    EndDates: 733908
    StartDates: 733727
    ValuationDate: 733727
    Basis: 1
    EndMonthRule: 1
```

Create the two StockSpec definitions.

```
AssetPriceA = 49.75;
AssetPriceB = 51;
SigmaA = 0.11;
SigmaB = 0.16;
DivA = 0.045;
DivB = 0.05;
```

```
StockSpecA = stockspec(SigmaA, AssetPriceA, 'continuous', DivA)
```

```
StockSpecA = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1100
    AssetPrice: 49.7500
    DividendType: {'continuous'}
    DividendAmounts: 0.0450
    ExDividendDates: []
```

```
StockSpecB = stockspec(SigmaB, AssetPriceB, 'continuous', DivB)
```

```
StockSpecB = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1600
    AssetPrice: 51
    DividendType: {'continuous'}
    DividendAmounts: 0.0500
```

```
ExDividendDates: []
```

Compute the price of the options for different correlation levels.

```
Strike = 50.25;  
Corr = [-0.5;0;0.5];  
OptSpec = 'put';  
Price = minassetbystulz(RateSpec, StockSpecA, StockSpecB, Settle,...  
Maturity, OptSpec, Strike, Corr)
```

```
Price = 3×1
```

```
3.4320  
3.1384  
2.7694
```

The values 3.43, 3.14, and 2.77 are the price of the European rainbow put options with a correlation level of -0.5, 0, and 0.5 respectively.

## Input Arguments

### **RateSpec — Annualized, continuously compounded rate term structure**

structure

Annualized, continuously compounded rate term structure, specified using `intenvset`.

Data Types: structure

### **StockSpec1 — Stock specification for asset 1**

structure

Stock specification for asset 1, specified using `stockspec`.

Data Types: structure

### **StockSpec2 — Stock specification for asset 2**

structure

Stock specification for asset 2, specified using `stockspec`.

Data Types: structure

**Settle — Settlement or trade dates**

vector

Settlement or trade dates, specified as an NINST-by-1 vector of numeric dates.

Data Types: double

**Maturity — Maturity dates**

vector

Maturity dates, specified as an NINST-by-1 vector.

Data Types: double

**OptSpec — Option type**

cell array of character vectors with a value of 'call' or 'put'

Option type, specified as an NINST-by-1 cell array of character vectors with a value of 'call' or 'put'.

Data Types: cell

**Strike — Strike prices**

vector

Strike prices, specified as an NINST-by-1 vector.

Data Types: double

**Corr — Correlation between the underlying asset prices**

vector

Correlation between the underlying asset prices, specified as an NINST-by-1 vector.

Data Types: double

**Output Arguments****Price — Expected option prices**

vector

Expected option prices, returned as an NINST-by-1 vector.

## **See Also**

intenvset | maxassetsensbystulz | minassetsensbystulz | stockspec

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Rainbow Option” on page 3-35

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**



# minassetsensbystulz

Determine European rainbow option prices or sensitivities on minimum of two risky assets using Stulz option pricing model

## Syntax

```
PriceSens = minassetsensbystulz(RateSpec,StockSpec1,StockSpec2,
Settle,Maturity,OptSpec,Strike,Corr)
PriceSens = minassetsensbystulz( ____,Name,Value)
```

## Description

`PriceSens = minassetsensbystulz(RateSpec,StockSpec1,StockSpec2,Settle,Maturity,OptSpec,Strike,Corr)` computes option prices using the Stulz option pricing model.

`PriceSens = minassetsensbystulz( ____,Name,Value)` specifies options using one or more optional name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute Rainbow Option Prices and Sensitivities Using the Stulz Option Pricing Model

Consider a European rainbow put option that gives the holder the right to sell either stock A or stock B at a strike of 50.25, whichever has the lower value on the expiration date May 15, 2009. On November 15, 2008, stock A is trading at 49.75 with a continuous annual dividend yield of 4.5% and has a return volatility of 11%. Stock B is trading at 51 with a continuous dividend yield of 5% and has a return volatility of 16%. The risk-free rate is 4.5%. Using this data, if the correlation between the rates of return is -0.5, 0, and 0.5, calculate the price and sensitivity of the minimum of two assets that are European rainbow put options. First, create the `RateSpec`:

```
Settle = 'Nov-15-2008';
Maturity = 'May-15-2009';
Rates = 0.045;
Basis = 1;

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9778
    Rates: 0.0450
    EndTimes: 0.5000
    StartTimes: 0
    EndDates: 733908
    StartDates: 733727
    ValuationDate: 733727
    Basis: 1
    EndMonthRule: 1
```

Create the two StockSpec definitions.

```
AssetPriceA = 49.75;
AssetPriceB = 51;
SigmaA = 0.11;
SigmaB = 0.16;
DivA = 0.045;
DivB = 0.05;

StockSpecA = stockspec(SigmaA, AssetPriceA, 'continuous', DivA)

StockSpecA = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1100
    AssetPrice: 49.7500
    DividendType: {'continuous'}
    DividendAmounts: 0.0450
    ExDividendDates: []

StockSpecB = stockspec(SigmaB, AssetPriceB, 'continuous', DivB)

StockSpecB = struct with fields:
    FinObj: 'StockSpec'
```

```

        Sigma: 0.1600
    AssetPrice: 51
    DividendType: {'continuous'}
    DividendAmounts: 0.0500
    ExDividendDates: []

```

Calculate price and delta for different correlation levels.

```

Strike = 50.25;
Corr = [-0.5;0;0.5];
OptSpec = 'put';
OutSpec = {'Price'; 'delta'};
[P, D] = minassetsensbystulz(RateSpec, StockSpecA, StockSpecB,...
    Settle, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)

```

P = 3×1

```

    3.4320
    3.1384
    2.7694

```

D = 3×2

```

-0.4183    -0.3496
-0.3746    -0.3189
-0.3304    -0.2905

```

The output `Delta` has two columns: the first column represents the `Delta` with respect to the stock A (asset 1), and the second column represents the `Delta` with respect to the stock B (asset 2). The value 0.4183 represents `Delta` with respect to the stock A for a correlation level of -0.5. The `Delta` with respect to stock B, for a correlation of zero is -0.3189.

## Input Arguments

**RateSpec** — Annualized, continuously compounded rate term structure  
structure

Annualized, continuously compounded rate term structure, specified using `intenvset`.

Data Types: structure

**StockSpec1 — Stock specification for asset 1**

structure

Stock specification for asset 1, specified using `stockspec`.

Data Types: structure

**StockSpec2 — Stock specification for asset 2**

structure

Stock specification for asset 2, specified using `stockspec`.

Data Types: structure

**Settle — Settlement or trade dates**

vector

Settlement or trade dates, specified as an NINST-by-1 vector of numeric dates.

Data Types: double

**Maturity — Maturity dates**

vector

Maturity dates, specified as an NINST-by-1 vector.

Data Types: double

**OptSpec — Option type**

cell array of character vectors with a value of 'call' or 'put'

Option type, specified as an NINST-by-1 cell array of character vectors with a value of 'call' or 'put'.

Data Types: cell

**Strike — Strike prices**

vector

Strike prices, specified as an NINST-by-1 vector.

Data Types: double

**Corr — Correlation between the underlying asset prices**

vector

Correlation between the underlying asset prices, specified as an NINST-by-1 vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[PriceSens] = minassetsensbystulz(RateSpec, StockSpecA, StockSpecB, Settle, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)`

**OutSpec — Define outputs**

`{'Price'}` (default) | cell array of character vectors with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a NOUT-by-1 or 1-by-NOUT cell array of character vectors or string array with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output is `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` to include each sensitivity:

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: cell

**Output Arguments****PriceSens — Expected prices or sensitivities**

vector

Expected prices or sensitivities, returned as an NINST-by-1 or NINST-by-2 vector.

## **See Also**

intenvset | minassetsensbystulz | stockspect

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Rainbow Option” on page 3-35

“Supported Equity Derivatives” on page 3-24

**Introduced in R2009a**

# mkbush

Create bushy tree

## Syntax

```
[Tree, NumStates] = mkbush(NumLevels, NumChild, NumPos, Trim, NodeVal)
```

## Arguments

NumLevels	Number of time levels of the tree.
NumChild	1-by- number of levels (NUMLEVELS) vector with number of branches (children) of the nodes in each level.
NumPos	1-by-NUMLEVELS vector containing the length of the state vectors in each time level.
Trim	(Optional) Scalar 0 or 1. If Trim = 1, NumPos decreases by 1 when moving from one time level to the next. Otherwise, if Trim = 0 (Default), NumPos does not decrease.
NodeVal	(Optional) Initial value at each node of the tree. Default = NaN.

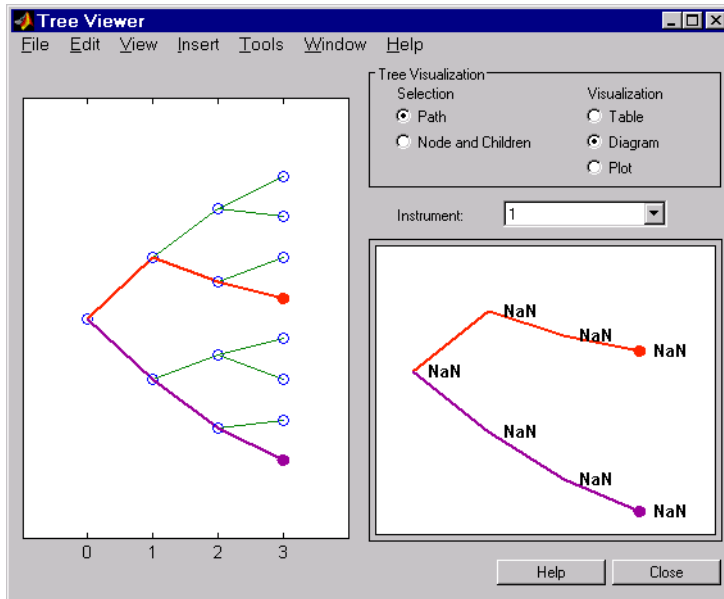
## Description

[Tree, NumStates] = mkbush(NumLevels, NumChild, NumPos, Trim, NodeVal) creates a bushy tree Tree with initial values NodeVal at each node. NumStates is a 1-by-NUMLEVELS vector containing the number of state vectors in each level.

## Examples

Create a tree with four time levels, two branches per node, and a vector of three elements in each node with each element initialized to NaN.

```
Tree = mkbush(4, 2, 3);  
treeviewer(Tree)
```



## See Also

bushpath | bushshape

## Topics

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50

**Introduced before R2006a**



# mktree

Create recombining binomial tree

## Syntax

```
Tree = mktree(NumLevels, NumPos, NodeVal, IsPriceTree)
```

## Arguments

NumLevels	Number of time levels of the tree.
NumPos	1-by-NUMLEVELS vector containing the length of the state vectors in each time level.
NodeVal	(Optional) Initial value at each node of the tree. Default = NaN.
IsPriceTree	(Optional) Boolean determining if a final horizontal branch is added to the tree. Default = 0.

## Description

`Tree = mktree(NumLevels, NumPos, NodeVal, IsPriceTree)` creates a recombining tree `Tree` with initial values `NodeVal` at each node.

## Examples

Create a recombining tree of four time levels with a vector of two elements in each node and each element initialized to NaN.

```
Tree = mktree(4, 2);
```

## See Also

`treepath` | `treeshape`

**Topics**

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50

**Introduced before R2006a**

# mktrintree

Create recombining trinomial tree

## Syntax

```
TrinTree = mktrintree(NumLevels, NumPos, NumStates, NodeVal)
```

## Arguments

NumLevels	Number of time levels of the tree.
NumPos	1-by- <code>NUMLEVELS</code> vector containing the length of the state vectors in each time level.
NumStates	1-by- <code>NUMLEVELS</code> vector containing the number of state vectors in each time level.
NodeVal	(Optional) Initial value at each node of the tree. Default = <code>NaN</code> .

## Description

`TrinTree = mktrintree(NumLevels, NumPos, NumStates, NodeVal)` creates a recombining tree `Tree` with initial values `NodeVal` at each node.

## Examples

Create a recombining trinomial tree of four time levels with a vector of two elements in each node and each element initialized to `NaN`.

```
TrinTree = mktrintree(4, [2 2 2 2], [1 3 5 7]);
```

## See Also

`trintreepath` | `trintreeshape`

**Topics**

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50

**Introduced before R2006a**

# mmktbybdt

Create money-market tree from Black-Derman-Toy interest-rate tree

## Syntax

```
MMktTree = mmktbybdt(BDTree)
```

## Arguments

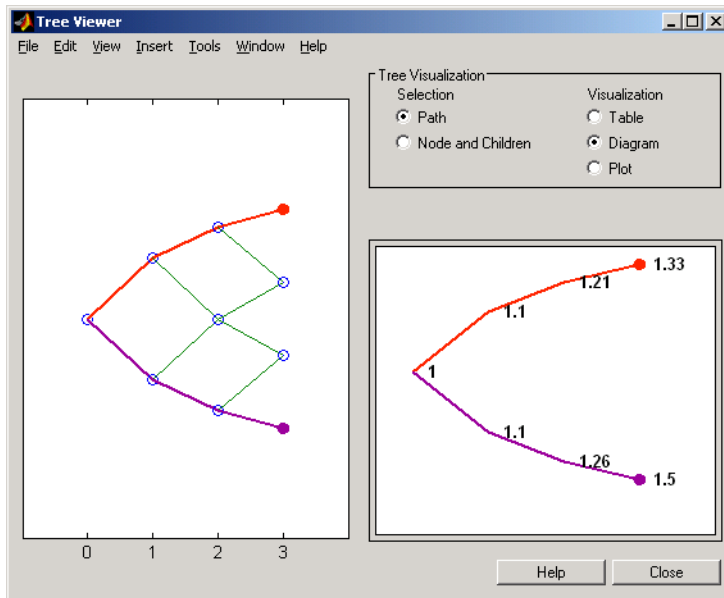
BDTree	Interest-rate tree structure created by <code>bdt tree</code> .
--------	---

## Description

`MMktTree = mmktbybdt(BDTree)` creates a money-market tree from an interest-rate tree structure created by `bdt tree`.

## Examples

```
load deriv.mat;  
MMktTree = mmktbybdt(BDTree);  
treeviewer(MMktTree)
```



## See Also

bdttree

## Topics

"Pricing a Portfolio Using the Black-Derman-Toy Model" on page 1-13

"Understanding Interest-Rate Tree Models" on page 2-79

"Supported Interest-Rate Instruments" on page 2-2

**Introduced before R2006a**

# mmktbyhjm

Create money-market tree from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
MMktTree = mmktbyhjm(HJMTree)
```

## Arguments

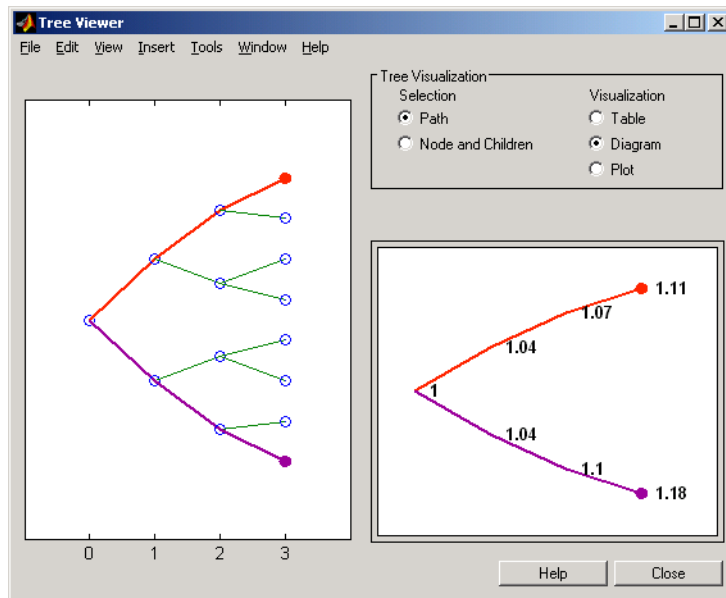
HJMTree	Forward-rate tree structure created by hjmtree.
---------	---

## Description

`MMktTree = mmktbyhjm(HJMTree)` creates a money-market tree from a forward-rate tree structure created by `hjmtree`.

## Examples

```
load deriv.mat;  
MMktTree = mmktbyhjm(HJMTree);  
treeviewer(MMktTree)
```



## See Also

hjmtree

## Topics

"Computing Instrument Prices" on page 2-99

"Understanding Interest-Rate Tree Models" on page 2-79

"Supported Interest-Rate Instruments" on page 2-2

**Introduced before R2006a**



# normalvolbysabr

Implied Normal (Bachelier) volatility by SABR model

## Syntax

```
outVol = normalvolbysabr(Alpha,Beta,Rho,Nu,Settle,ExerciseDate,  
ForwardValue,Strike)  
outVol = normalvolbysabr( ____,Name,Value)
```

## Description

`outVol = normalvolbysabr(Alpha,Beta,Rho,Nu,Settle,ExerciseDate,ForwardValue,Strike)` calculates the implied Normal (Bachelier) volatility by using the SABR stochastic volatility model.

`outVol = normalvolbysabr( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute the Implied Normal (Bachelier) Volatility Using the SABR Model

Define the model parameters and option data.

```
ForwardValue = 0.0209;  
Strike = 0.02;  
Alpha = 0.041;  
Beta = 0.5;  
Rho = -0.2;  
Nu = 0.33;
```

```
Settle = datenum('15-Feb-2018');  
ExerciseDate = datenum('15-Feb-2020');
```

Compute the Normal (Bachelier) volatility using the SABR model.

```
ComputedVols = normalvolbysabr(Alpha,Beta,Rho,Nu,Settle,ExerciseDate,ForwardValue,Strike)
ComputedVols = 0.0059
```

### **Compute the Implied Normal (Bachelier) Volatility Using the Normal SABR Model**

To use the Normal SABR model, set the **Beta** parameter to zero. Negative interest rates are allowed when the Normal SABR model is used in combination with Normal (Bachelier) implied volatility.

Define the model parameters and option data.

```
ForwardValue = -0.00383;
Strike = -0.003;
Alpha = 0.007;
Beta = 0; % Set the Beta parameter to zero to use the Normal SABR model
Rho = -0.18;
Nu = 0.29;

Settle = datenum('17-Jan-2018');
ExerciseDate = datenum('17-Apr-2018');
```

Compute the Normal (Bachelier) volatility using the Normal SABR model.

```
ComputedVols = normalvolbysabr(Alpha,Beta,Rho,Nu,Settle,ExerciseDate,ForwardValue,Strike)
ComputedVols = 0.0070
```

## **Input Arguments**

### **Alpha — Current SABR volatility**

scalar numeric

Current SABR volatility, specified as a scalar numeric.

Data Types: double

### **Beta — SABR constant elasticity of variance (CEV) exponent**

scalar numeric

SABR CEV exponent, specified as a scalar numeric.

---

**Note** Set the Beta parameter to 0 to allow a negative ForwardValue and Strike.

---

Data Types: double

**Rho — Correlation between forward value and volatility**

scalar numeric

Correlation between forward value and volatility, specified as a scalar numeric.

Data Types: double

**Nu — Volatility of volatility**

scalar numeric

Volatility of volatility, specified as a scalar numeric.

Data Types: double

**Settle — Settlement date**

scalar for serial nonnegative date number | scalar for date character vector

Settlement date, specified as a scalar using a serial nonnegative date number or date character vector.

Data Types: double | char

**ExerciseDate — Option exercise date**

scalar for serial nonnegative date number | scalar for date character vector

Option exercise date, specified as a scalar using a serial nonnegative date number or date character vector.

Data Types: double | char

**ForwardValue — Current forward value of underlying asset**

scalar numeric | vector

Current forward value of the underlying asset, specified as a scalar numeric or vector of size NumVols-by-1.

Data Types: double

**Strike — Option strike price value**

scalar numeric | vector

Option strike price value, specified as a scalar numeric or a vector of size `NumVols-by-1`.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `outVol = normalvolbysabr(Alpha, Beta, Rho, Nu, Settle, ExerciseDate, ForwardValue, Strike, 'Basis', 2)`

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | positive integers of the set [1...13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of `'Basis'` and a positive integer from the set [1...13].

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)

- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

## Output Arguments

**outVol** — Normal (Bachelier) volatility computed by SABR model

scalar numeric | vector

Normal (Bachelier) volatility computed by the SABR model, returned as a scalar numeric or vector of size NumVols-by-1.

## Algorithms

The two general case algorithms for normalvolbysabr are not At-The-Money (ATM) and ATM.

For not ATM ( $F \neq K$ ):

$$\begin{aligned}\sigma_N(\alpha, \beta, \rho, v, F, K, T) &= \alpha \frac{(F - K)(1 - \beta)}{F^{(1 - \beta)} - K^{(1 - \beta)}} \left( \frac{z}{x(z)} \right) \left\{ 1 + \left[ \frac{\beta(\beta - 2)\alpha^2}{24F_{mid}^2 - 2\beta} + \frac{\rho\beta v\alpha}{4F_{mid}^{1 - \beta}} + \frac{2 - 3\rho^2}{24}v^2 \right] T \right\} \\ &= v \frac{(F - K)}{x(z)} \left\{ 1 + \left[ \frac{\beta(\beta - 2)\alpha^2}{24F_{mid}^2 - 2\beta} + \frac{\rho\beta v\alpha}{4F_{mid}^{1 - \beta}} + \frac{2 - 3\rho^2}{24}v^2 \right] T \right\}\end{aligned}$$

$$F_{mid} = \frac{F + K}{2}$$

$$z = \frac{v}{\alpha} \left( \frac{F^{1 - \beta} - K^{1 - \beta}}{1 - \beta} \right) \quad x(z) = \ln \left( \frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho} \right)$$

For ATM ( $F = K$ ):

$$\sigma_{N, ATM}(\alpha, \beta, \rho, v, F, T) = \alpha F^\beta \left\{ 1 + \left[ \frac{\beta(\beta - 2)\alpha^2}{24F^2 - 2\beta} + \frac{\rho\beta v\alpha}{4F^{1 - \beta}} + \frac{2 - 3\rho^2}{24}v^2 \right] T \right\}$$

The special case for normalvolbysabr where  $\beta = 0$  for not ATM ( $F \neq K$ ) is:

$$\sigma_N(\alpha, \rho, v, F, K, T) = v \frac{(F - K)}{\hat{\lambda}(\zeta)} \left( 1 + \frac{2 - 3\rho^2}{24} v^2 T \right)$$

$$\zeta = \frac{v}{\alpha} (F - K)$$

$$\hat{\lambda}(\zeta) = \ln \left( \frac{\sqrt{1 - 2\rho\zeta + \zeta^2} + \zeta - \rho}{1 - \rho} \right)$$

For ATM ( $F = K$ ):

$$\sigma_{N, ATM}(\alpha, \rho, v, T) = \alpha \left( 1 + \frac{2 - 3\rho^2}{24} v^2 T \right)$$

The special case for normalvolbysabr where  $\beta = 1$  for not ATM ( $F \neq K$ ) is:

$$\sigma_N(\alpha, \rho, v, F, K, T) = v \frac{(F - K)}{\hat{\lambda}(\zeta)} \left\{ 1 + \left[ \frac{-\alpha^2}{24} + \frac{\rho v \alpha}{4} + \frac{2 - 3\rho^2}{24} v^2 \right] T \right\}$$

$$\zeta = \frac{v}{\alpha} \ln \frac{F}{K}$$

$$\hat{\lambda}(\zeta) = \ln \left( \frac{\sqrt{1 - 2\rho\zeta + \zeta^2} + \zeta - \rho}{1 - \rho} \right)$$

For ATM ( $F = K$ ):

$$\sigma_{N, ATM}(\alpha, \rho, v, F, T) = \alpha F \left\{ 1 + \left[ \frac{-\alpha^2}{24} + \frac{\rho v \alpha}{4} + \frac{2 - 3\rho^2}{24} v^2 \right] T \right\}$$

## References

- [1] Hagan, P. S., D. Kumar, A.S. Lesniewski, and D.E. Woodward. "Managing Smile Risk." *Wilmott Magazine*. September 2002, pp. 84-108.

## See Also

optsensbysabr | swaptionbyblk | swaptionbynormal

## **Topics**

“Calibrate the SABR Model using Normal (Bachelier) Volatilities with Negative Strikes”

“Price Swaptions with Negative Strikes Using the Shifted SABR Model” on page 2-27

“Calibrate the SABR Model” on page 2-36

“Price a Swaption Using the SABR Model” on page 2-42

“Work with Negative Interest Rates” on page 2-22

“Supported Interest-Rate Instruments” on page 2-2

## **External Websites**

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

## **Introduced in R2018b**

## numerix class

Create numerix object to set up Numerix CAIL environment

### Description

The `numerix` object makes the Numerix engine directly accessible from MATLAB. To use the capabilities of Numerix CAIL, you must have CAIL client software installed on your local desktop.

In addition, you must add the Numerix library file to MATLAB path to use the documentation examples:

- Add `<Numerix software package installation root>/lib` to `<matlabroot>/toolbox/local/librarypath.txt`
- or
- Place `<Numerix software package installation root>/lib/NxProjava.dll` in the folder `<matlabroot>/bin/win64`

### Construction

`N = numerix(DATADIRECTORYPATH)` constructs the `numerix` object and sets up the Numerix CrossAsset Integration Layer (CAIL) environment given the path, `DATADIRECTORYPATH`.

### Properties

The following properties are from the `numerix` class.

#### Path

Defines the path for `DATADIRECTORYPATH`. This path is the location of the templates and is created by the client installation of CAIL. A template defines the interface; it



encapsulates the instructions for performing calculations, the calculation's required and optional input parameters, and the calculation's outputs.

**Attributes:**

SetAccess	public
GetAccess	public

**RepositoryPath**

RepositoryPath defines the path location for the repository folder in a file system.

**Attributes:**

SetAccess	public
GetAccess	public

**Repository**

Repositories are collections of templates and are defined as a folder in a file system.

**Attributes:**

SetAccess	public
GetAccess	public

**Context**

The calculation context manages all the CAIL information. Context contains the location of the template repository and is responsible for creating a CAIL application context in which to perform the calculations.

**Attributes:**

SetAccess	public
GetAccess	public

**LookupsPath**

Defines the path for the numeric instruments data types.

**Attributes:**

SetAccess	public
GetAccess	public

**MarketsPath**

Defines a path for the logical schema for naming all the market data. **MarketsPath** enables you to provide a data dictionary to map business market data to CAIL to reduce the task of inputting market data into CAIL objects directly.

**Attributes:**

SetAccess	public
GetAccess	public

**FixingsPath**

Defines the path for the schema for naming historical fixing data for rates and prices.

**Attributes:**

SetAccess	public
GetAccess	public

**TradesPath**

Defines the path to the trade instrument definitions.

**Attributes:**

SetAccess	public
GetAccess	public

**Parameters**

Defines the calculation parameters and market data, if available.

**Attributes:**

```

SetAccess      public
GetAccess      public

```

## Methods

parseResults      Converts Numerix CAIL data to MATLAB data types

## Copy Semantics

Value. To learn how value classes affect copy operations, see Copying Objects (MATLAB).

## Examples

### Construct a Numerix Object

Initialize Numerix CAIL environment.

```

import com.numerix.integration.*;
import com.numerix.integration.implementation.*;

```

Construct a numerix object.

```

n = numerix('i:\NumeriX_java_10_3_0\data')

```

n =

```

      Path: 'i:\NumeriX_java_10_3_0\data'
RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
  Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
    Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings'

```

```
TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'  
Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

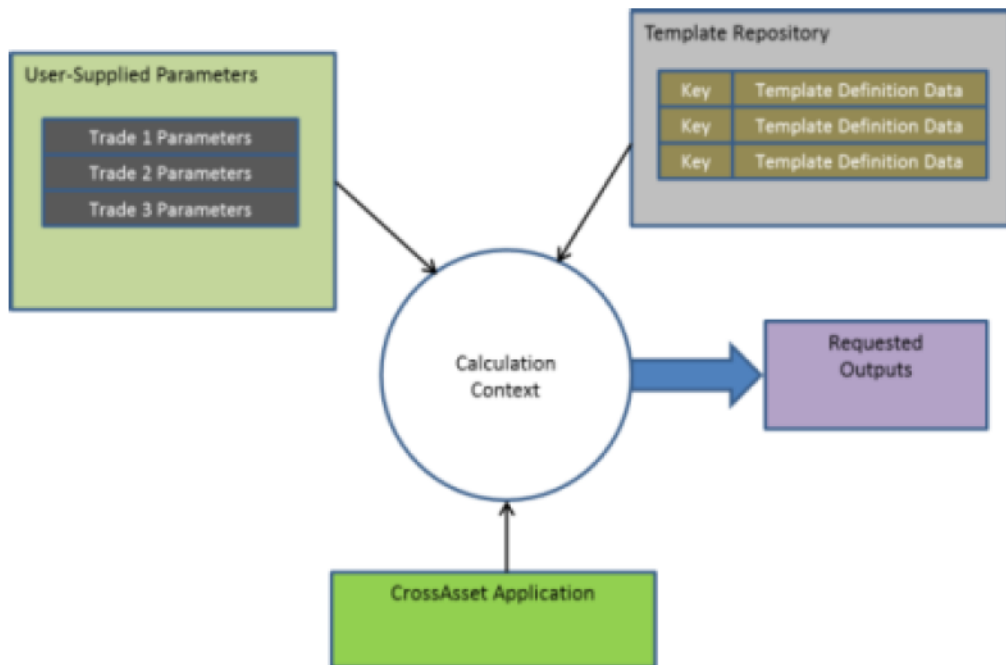
## Definitions

### CrossAsset Integration Layer (CAIL)

The CrossAsset Integration Layer (CAIL) is an application programming interface (API), which extends the data-driven approach of Numerix.

The calculation workflow of CAIL is:

- 1 Select a trade template for a specified deal or trade from the repository. The trade template specifies a set of inputs, a set of outputs, and the dependencies on other information (model, market data, calendar, and so on).
- 2 Provide the input parameters to the trade template and call the calculation context. The calculation context follows the dependency path to collect the needed information and produces the output specified by the template.



## See Also

`numerixCrossAsset | parseResults`

## Topics

“Working with Simple Numerix Trades” on page 10-2

“Working with Advanced Numerix Trades” on page 10-5

“Use Numerix to Price Cash Deposits” on page 10-10

“Use Numerix for Interest-Rate Risk Assessment” on page 10-13

Class Attributes (MATLAB)

Property Attributes (MATLAB)

## External Websites

<https://www.numerix.com/cail>

**Introduced in R2013b**

# numerixCrossAsset class

Create numerixCrossAsset object to set up Numerix CROSSASSET environment

## Description

Creating a numerixCrossAsset object initializes a Numerix CrossAsset object based on the Numerix data-driven CROSSASSET API. To use the Numerix engine directly from MATLAB, you must have the CROSSASSET client installed on your local desktop.

In addition, you must add the Numerix library file to MATLAB path to use the documentation examples:

- Add *<Numerix software package installation root>/lib* to *<matlabroot>/toolbox/local/librarypath.txt*
- or
- Place *<Numerix software package installation root>/lib/NxProjava.dll* in the folder *<matlabroot>/bin/win64*

## Construction

`c = numerixCrossAsset` constructs the numerixCrossAsset object and sets up the Numerix CROSSASSET environment.

## Properties

### Application — Application object

object

Application object, created when numerixCrossAsset object is initialized.

Example: `app = Application;`

### Attributes:

SetAccess	private
GetAccess	public

### **ApplicationWarning — ApplicationWarning object** object

ApplicationWarning object, created when numerixCrossAsset object is initialized.

Example: `appWarnings = ApplicationWarning;`

#### **Attributes:**

SetAccess	private
GetAccess	public

## **Methods**

applicationCall	Create and register Numerix CROSSASSET Call object
applicationData	Create and register data with Numerix CROSSASSET Application Data object
applicationMatrix	Create and register Numerix CROSSASSET Application Matrix object
close	Close numerixCrossAsset object
getdata	Convert Numerix CROSSASSET Application object to MATLAB structure

## **Examples**

### **Construct a numerixCrossAsset Object**

Construct a numerixCrossAsset object.

```
c = numerixCrossAsset
```

```
c =
```

```
numerixCrossAsset with properties:
```



Application: [1x1 com.numerix.pro.Application]  
ApplicationWarning: [1x1 com.numerix.pro.ApplicationWarning]

## Definitions

### **CROSSASSET**

The CROSSASSET API is an application programming interface (API), which extends the data-driven approach of Numerix.

### **See Also**

`applicationCall` | `applicationData` | `applicationMatrix` | `close` | `getdata`

### **Topics**

“Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects” on page 10-16  
Class Attributes (MATLAB)  
Property Attributes (MATLAB)

### **External Websites**

<https://www.numerix.com/CrossAsset>

**Introduced in R2016b**

## applicationCall

**Class:** numerixCrossAsset

Create and register Numerix CROSSASSET Call object

### Syntax

```
applicationCall(C,Headers,Name,Value)
```

### Description

`applicationCall(C,Headers,Name,Value)` creates and registers the Numerix CROSSASSET Call object with additional options specified by one or more `Name,Value` pair arguments. The name-value parameters conform to the Numerix Cross Asset Integration Layer interface and are defined by `N1, N2, ..., NN` to the values given in `V1, V2, ..., VN`.

Creating and registering the Call object calculates values in the Numerix Cross Asset Integration Layer and returns the data in MATLAB.

### Input Arguments

**C — Connection object to Numerix CROSSASSET**  
object

Connection object to Numerix CROSSASSET, specified using the `numerixCrossAsset` constructor.

**Headers — Output names of the returned values from numerixCrossAsset connection object**  
cell array of character vectors

Output names of the returned values from `numerixCrossAsset` connection object, specified as a cell array of character vectors.

Data Types: cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

### N1 — Numerix name-value parameter

name-value parameter defined by Numerix

Numerix parameters, specified as a `Name`, `Value` argument pair.

```
Example: applicationCall(c,Headers,'ID','RATESPEC','OBJECT','MARKET
DATA','TYPE','YIELD','COMMENT','Comments
here','SKIP',false,'INTERPMETHOD','LogLinear','INTERPVARIABLE','DF',
'CURRENCY','USD','DATA','USD_3MLIBOR_CURVE','BASIS','ACT/360');
```

Data Types: char | double | logical

### N2 — Numerix name-value parameters

name-value parameter defined by Numerix

Numerix parameter, specified as a `Name`, `Value` argument pair.

```
Example: applicationCall(c,Headers,'ID','RATESPEC','OBJECT','MARKET
DATA','TYPE','YIELD','COMMENT','Comments
here','SKIP',false,'INTERPMETHOD','LogLinear','INTERPVARIABLE','DF',
'CURRENCY','USD','DATA','USD_3MLIBOR_CURVE','BASIS','ACT/360');
```

Data Types: char | double | logical

## Attributes

Access	public
Static	false
Hidden	false

To learn about attributes of methods, see [Method Attributes \(MATLAB\)](#).

## Examples

### Create and Register a Numerix CROSSASSET Call Object

Create a datetime object.

```
dates = datetime({'18-Feb-2014'; '20-May-2014'; '18-Jun-2014'; '16-Jul-2014';  
                '20-Aug-2014'; '17-Sep-2014'; '15-Oct-2014'; '19-Nov-2014';  
                '17-Dec-2014'; '18-Mar-2015'; '17-Jun-2015'; '16-Sep-2015';  
                '16-Dec-2015'; '16-Mar-2016'; '15-Jun-2016'; '21-Sep-2016';  
                '21-Dec-2016'; '15-Mar-2017'; '20-Feb-2018'; '20-Feb-2019';  
                '20-Feb-2020'; '22-Feb-2021'; '22-Feb-2022'; '21-Feb-2023';  
                '20-Feb-2024'; '20-Feb-2025'; '20-Feb-2026'; '20-Feb-2029';  
                '21-Feb-2034'; '22-Feb-2039'; '22-Feb-2044'; '20-Feb-2054';  
                '20-Feb-2064'});
```

Create the corresponding vector of discount factors for a 3-month LIBOR curve.

```
discountFactors = [1;0.99942;0.999231;0.999037;0.998797;0.998616;0.998385;...  
                  0.998122;0.997941;0.997159;0.996157;0.994825;0.993065;...  
                  0.99078;0.987889;0.984092;0.979913;0.975459;0.952707;...  
                  0.922223;0.888128;0.852291;0.816462;0.781228;0.746677;...  
                  0.712892;0.680462;0.592285;0.474003;0.383493;0.312617;...  
                  0.213809;0.152345];
```

Create a Numerix CROSSASSET object.

```
c = numerixCrossAsset;
```

Add data to the Numerix Cross Asset Application Data object.

```
applicationData(c, 'USD_3MLIBOR_CURVE', 'DATE', dates, 'DISCOUNTFACTOR', discountFactors)
```

Define the Headers input and add the RateSpec Call object to the Numerix CROSSASSET Application object using name-value pairs, where USD\_3MLIBOR\_CURVE denotes the yield curve data object created previously.

```
headers = {'ID', 'LOCAL ID', 'TIMER', 'TIMER CPU', 'UPDATED'};  
applicationCall(c, headers, 'ID', 'RATESPEC', 'OBJECT', 'MARKET DATA', 'TYPE', 'YIELD', 'COMMENT', 'Comments here', ...  
                'SKIP', false, 'INTERPMETHOD', 'LogLinear', 'INTERVARIABLE', 'DF', ...  
                'CURRENCY', 'USD', 'DATA', 'USD_3MLIBOR_CURVE', 'BASIS', 'ACT/360');
```

## See Also

[applicationData](#) | [applicationMatrix](#) | [close](#) | [getdata](#) | [numerixCrossAsset](#)

## **Topics**

“Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects” on page 10-16

## **External Websites**

<https://www.numerix.com/CrossAsset>

**Introduced in R2016b**

## applicationData

**Class:** numerixCrossAsset

Create and register data with Numerix CROSSASSET Application Data object

### Syntax

```
applicationData(C,Desc,Name,Value)
applicationData(C,Desc,T)
applicationData(C,Desc,S)
```

### Description

`applicationData(C,Desc,Name,Value)` `applicationData` creates and registers the data for Numerix CROSSASSET Application Data object with additional options specified by one or more `Name, Value` pair arguments. The name-value parameters conform to the Numerix Cross Asset Integration Layer interface and are defined by `N1, N2, ..., NN` to the values given in `V1, V2, ..., VN`.

`applicationData(C,Desc,T)` creates and registers the data for Numerix CROSSASSET Application Data object in table `T`.

`applicationData(C,Desc,S)` creates and registers the data in the structure, `S`. The structure `fieldnames` represents the property names for the values in each field.

### Input Arguments

**C — Connection object to Numerix CROSSASSET**  
object

Connection object to Numerix CROSSASSET, specified using the `numerixCrossAsset` constructor.

**Desc — Description of data**  
character vector or cell array of character vectors

Description of data, specified as a character vector or cell array of character vectors.

Data Types: `char` | `cell`

### **T — Table input**

table `VariableNames` represents the property names for values in the corresponding column

Table input for data to register for the Numerix CROSSASSET Application Data object, specified using `VariableNames`.

Data Types: `table`

### **S — Structure input**

structure `fieldnames` represents the property names for the values in each field

Structure input for data to register for the Numerix CROSSASSET Application Data object, specified using the `fieldnames`.

Data Types: `struct`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

### **N1 — Numerix name-value parameter**

name-value parameter defined by Numerix

Numerix parameter, specified as `Name`, `Value` argument pair.

Example:

```
applicationData(c, 'USD_3MLIBOR_CURVE', 'DATE', dates, 'DISCOUNTFACTOR', discountFactors)
```

Data Types: `char` | `double` | `logical`

### **N2 — Numerix name-value parameters**

name-value parameter defined by Numerix

Numerix parameter, specified as `Name`, `Value` argument pair.

Example:

```
applicationData(c,'USD_3MLIBOR_CURVE','DATE',dates,'DISCOUNTFACTOR',  
discountFactors)
```

Data Types: char | double | logical

## Attributes

Access	public
Static	false
Hidden	false

To learn about attributes of methods, see Method Attributes (MATLAB).

## Examples

### Create a Numerix CROSSASSET Application Data Object

Create a datetime object.

```
dates = datetime({'18-Feb-2014';'20-May-2014';'18-Jun-2014';'16-Jul-2014';  
'20-Aug-2014';'17-Sep-2014';'15-Oct-2014';'19-Nov-2014';  
'17-Dec-2014';'18-Mar-2015';'17-Jun-2015';'16-Sep-2015';  
'16-Dec-2015';'16-Mar-2016';'15-Jun-2016';'21-Sep-2016';  
'21-Dec-2016';'15-Mar-2017';'20-Feb-2018';'20-Feb-2019';  
'20-Feb-2020';'22-Feb-2021';'22-Feb-2022';'21-Feb-2023';  
'20-Feb-2024';'20-Feb-2025';'20-Feb-2026';'20-Feb-2029';  
'21-Feb-2034';'22-Feb-2039';'22-Feb-2044';'20-Feb-2054';  
'20-Feb-2064'});
```

Create the corresponding vector of discount factors for a 3-month LIBOR curve.

```
discountFactors = [1;0.99942;0.999231;0.999037;0.998797;0.998616;0.998385;...  
0.998122;0.997941;0.997159;0.996157;0.994825;0.993065;...  
0.99078;0.987889;0.984092;0.979913;0.975459;0.952707;...  
0.922223;0.888128;0.852291;0.816462;0.781228;0.746677;...  
0.712892;0.680462;0.592285;0.474003;0.383493;0.312617;...  
0.213809;0.152345];
```

Create a Numerix CROSSASSET object.

```
c = numerixCrossAsset;
```

Create and register the data with the Numerix CROSSASSET Application Data object.



```
applicationData(c, 'USD_3MLIBOR_CURVE', 'DATE', dates, 'DISCOUNTFACTOR', discountFactors)
```

## See Also

applicationCall | applicationMatrix | close | getdata | numerixCrossAsset

## Topics

“Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects” on page 10-16

## External Websites

<https://www.numerix.com/CrossAsset>

**Introduced in R2016b**

# applicationMatrix

**Class:** numerixCrossAsset

Create and register Numerix CROSSASSET Application Matrix object

## Syntax

```
applicationMatrix(C,Desc,Rdata,Cdata,Mdata)
```

## Description

`applicationMatrix(C,Desc,Rdata,Cdata,Mdata)` creates the CROSSASSET Application Matrix object from the row information (Rdata), column information (Cdata), and matrix (Mdata).

## Input Arguments

**C — Connection object to Numerix CROSSASSET**  
object

Connection object to Numerix CROSSASSET, specified using the `numerixCrossAsset` constructor.

**Desc — Description of data**  
character vector or cell array of character vectors

Description of data, specified as a character vector or cell array of character vectors.

Data Types: `char` | `cell`

**Rdata — Row information for Application Matrix object**  
numeric values

Row information for Application Matrix object, specified using numeric values.

Example: `Rdata = [41992,42020,42449,42905,43115];`

Data Types: double

### **Cdata — Column information for Application Matrix object**

numeric values

Column information for Application Matrix object, specified as numeric values.

Example: `Cdata = [390,395,400,405];`

Data Types: double

### **Mdata — Volatility information for Application Matrix object**

numeric values

Volatility information for Application Matrix object, specified as numeric values.

Example: `Mdata = [0.35778, 0.35132, 0.34394, 0.33582; ...  
0.33405, 0.32819, 0.32669, 0.31904; ...  
0.31576, 0.31235, 0.30371, 0.30261; ...  
0.29391, 0.29366, 0.28962, 0.28932; ...  
0.28787, NaN, 0.28347, NaN];`

Data Types: double

## **Attributes**

Access	public
Static	false
Hidden	false

To learn about attributes of methods, see Method Attributes (MATLAB).

## **Examples**

### **Create a Numerix CROSSASSET Application Matrix Object**

Create a volatility matrix with dates describing the rows and strike prices describing the columns with the description `BYSTRIKEVOLDATA`. Missing values in the matrix input are denoted as `NaN`.

Create a Numerix CROSSASSET object.

```
c = numerixCrossAsset;
```

Define the matrix data.

```
rowData = [41992, 42020, 42449, 42905, 43115];  
colData = [390, 395, 400, 405];  
volData = [0.35778, 0.35132, 0.34394, 0.33582;...  
           0.33405, 0.32819, 0.32669, 0.31904;...  
           0.31576, 0.31235, 0.30371, 0.30261;...  
           0.29391, 0.29366, 0.28962, 0.28932;...  
           0.28787, NaN, 0.28347, NaN  ];
```

Create and register a Numerix CROSSASSET Application Matrix object.

```
applicationMatrix(c, 'BYSTRIKEVOLDATA', rowData, colData, volData);
```

## See Also

[applicationCall](#) | [applicationData](#) | [close](#) | [getdata](#) | [numerixCrossAsset](#)

## Topics

“Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects” on page 10-16

## External Websites

<https://www.numerix.com/CrossAsset>

## Introduced in R2016b

## close

**Class:** numerixCrossAsset

Close numerixCrossAsset object

## Syntax

```
AppData = close(C)
```

## Description

AppData = close(C) closes the numerixCrossAsset object (C).

## Input Arguments

**C — Connection object to Numerix CROSSASSET**

object

Connection object to Numerix CROSSASSET, specified using the numerixCrossAsset constructor.

## Attributes

Access	public
Static	false
Hidden	false

To learn about attributes of methods, see Method Attributes (MATLAB).

## Examples

### Close the Numerix CROSSASSET Object

Construct a `numerixCrossAsset` object.

```
c = numerixCrossAsset
```

```
c =
```

```
numerixCrossAsset with properties:
```

```
Application: [1x1 com.numerix.pro.Application]
```

```
ApplicationWarning: [1x1 com.numerix.pro.ApplicationWarning]
```

Close the `numerixCrossAsset` object.

```
close(c)
```

### See Also

`applicationCall` | `applicationData` | `applicationMatrix` | `getdata` | `numerixCrossAsset`

### Topics

“Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects” on page 10-16

### External Websites

<https://www.numerix.com/CrossAsset>

### Introduced in R2016b

# getdata

**Class:** numerixCrossAsset

Convert Numerix CROSSASSET Application object to MATLAB structure

## Syntax

```
AppData = getdata(C)
```

## Description

`AppData = getdata(C)` converts a Numerix CROSSASSET Application object to a MATLAB structure.

## Input Arguments

**C — Connection object to Numerix CROSSASSET**  
object

Connection object to Numerix CROSSASSET, specified using the `numerixCrossAsset` constructor.

## Output Arguments

**AppData — Converted Numerix CROSSASSET Application object**  
structure

Converted Numerix CROSSASSET Application object, returned as a MATLAB structure

## Attributes

Access	public
Static	false
Hidden	false

To learn about attributes of methods, see Method Attributes (MATLAB).

## Examples

### Convert Numerix CROSSASSET Application Object to a MATLAB Structure

Create a datetime object.

```
dates = datetime({'18-Feb-2014'; '20-May-2014'; '18-Jun-2014'; '16-Jul-2014';  
                '20-Aug-2014'; '17-Sep-2014'; '15-Oct-2014'; '19-Nov-2014';  
                '17-Dec-2014'; '18-Mar-2015'; '17-Jun-2015'; '16-Sep-2015';  
                '16-Dec-2015'; '16-Mar-2016'; '15-Jun-2016'; '21-Sep-2016';  
                '21-Dec-2016'; '15-Mar-2017'; '20-Feb-2018'; '20-Feb-2019';  
                '20-Feb-2020'; '22-Feb-2021'; '22-Feb-2022'; '21-Feb-2023';  
                '20-Feb-2024'; '20-Feb-2025'; '20-Feb-2026'; '20-Feb-2029';  
                '21-Feb-2034'; '22-Feb-2039'; '22-Feb-2044'; '20-Feb-2054';  
                '20-Feb-2064'});
```

Create the corresponding vector of discount factors for a 3-month LIBOR curve.

```
discountFactors = [1;0.99942;0.999231;0.999037;0.998797;0.998616;0.998385;...  
                  0.998122;0.997941;0.997159;0.996157;0.994825;0.993065;...  
                  0.99078;0.987889;0.984092;0.979913;0.975459;0.952707;...  
                  0.922223;0.888128;0.852291;0.816462;0.781228;0.746677;...  
                  0.712892;0.680462;0.592285;0.474003;0.383493;0.312617;...  
                  0.213809;0.152345];
```

Create a Numerix CROSSASSET object.

```
c = numerixCrossAsset;
```

Add data to the Numerix CROSSASSET Application Data object.

```
applicationData(c, 'USD_3MLIBOR_CURVE', 'DATE', dates, 'DISCOUNTFACTOR', discountFactors)
```

Add the RATESPEC Call object to the Numerix CROSSASSET Application object using name-value pairs, where USD\_3MLIBOR\_CURVE denotes yield curve data object created previously.

```
headers = {'ID', 'LOCAL ID', 'TIMER', 'TIMER CPU', 'UPDATED'};  
applicationCall(c, headers, 'ID', 'RATESPEC', 'OBJECT', 'MARKET DATA', 'TYPE', 'YIELD', 'COMMENT', 'Comments here', ...
```



```
'SKIP',false,'INTERPMETHOD','LogLinear','INTERPVARIABLE','DF',...  
'CURRENCY','USD','DATA','USD_3MLIBOR_CURVE','BASIS','ACT/360');
```

Use `getdata` to convert the Numerix CROSSASSET Application object to a MATLAB structure.

```
APPDATA = getdata(C)
```

## See Also

`applicationCall` | `applicationData` | `applicationMatrix` | `close` | `numerixCrossAsset`

## Topics

“Numerix CROSSASSET Interface Workflow Example Using Matrix, Data, and Call Objects” on page 10-16

## External Websites

<https://www.numerix.com/CrossAsset>

## Introduced in R2016b

## **parseResults**

**Class:** numerix

Converts Numerix CAIL data to MATLAB data types

### **Syntax**

`R = parseResults(N,Results)`

### **Description**

`R = parseResults(N,Results)` returns Numerix CAIL data in native MATLAB data types.

### **Input Arguments**

**N**

Numerix object constructed using `numerix`.

**Results**

Result instances for each trade instance.

### **Output Arguments**

**R**

Results from Numerix table output represented as MATLAB data types.

## Attributes

Access	public
Static	false
Hidden	false

To learn about attributes of methods, see Method Attributes (MATLAB).

## Examples

### Return Results for Numerix CAIL API to Price a Callable Reverse Floater

Initialize Numerix environment.

```
import com.numerix.integration.*;
import com.numerix.integration.implementation.*;

n = numerix('i:\NumeriX_java_10_3_0\data')

n =
    Path: 'i:\NumeriX_java_10_3_0\data'
    RepositoryPath: 'i:\NumeriX_java_10_3_0\data\Repository'
    Repository: [1x1 com.numerix.integration.implementation.FileSystemRepository]
    Context: [1x1 com.numerix.integration.implementation.LocalCalculationContext]
    LookupsPath: 'i:\NumeriX_java_10_3_0\data\Data\LookupRules'
    MarketsPath: 'i:\NumeriX_java_10_3_0\data\Data\Markets'
    FixingsPath: 'i:\NumeriX_java_10_3_0\data\Data\Fixings'
    TradesPath: 'i:\NumeriX_java_10_3_0\data\Data\Trades'
    Parameters: [1x1 com.numerix.integration.implementation.CalculationParameters]
```

Create a market.

```
quotes = java.util.HashMap;
quotes.put('IR.USD-LIBOR-3M.SWAP-1Y.MID', 0.0066056);
quotes.put('IR.USD-LIBOR-3M.SWAP-10Y.MID', 0.022465005);
quotes.put('IR.USD-LIBOR-3M.SWAP-20Y.MID', 0.027544995);
market = Market('EOD_14-NOV-2011', DateExtensions.date('14-Nov-2011'), quotes.entrySet());
```

Define a trade instance based on instrument template found in the Repository.

```
tradeDescriptor = 'TRADE.IR.CALLBLEREVERSEFLOATER';
tradeParameters = java.util.HashMap;
tradeParameters.put('Trade ID', '1001');
tradeParameters.put('Quote Type', 'MID');
tradeParameters.put('Currency', 'USD');
```

```

tradeParameters.put('Notional', 1000000.0);
tradeParameters.put('Effective Date', DateExtensions.date('1-Dec-2011'));
tradeParameters.put('Termination Date', DateExtensions.date('1-Dec-2021'));
tradeParameters.put('IR Index', 'LIBOR');
tradeParameters.put('IR Index Tenor', '3M');
tradeParameters.put('Structured Freq', '3M');
tradeParameters.put('Structured Side', 'Receive');
tradeParameters.put('Structured Coupon Floor', 0.0);
tradeParameters.put('Structured Coupon UpBd', 0.08);
tradeParameters.put('StructuredCoupon Multiplier', 1.4);
tradeParameters.put('Structured Coupon Cap', 0.05);
tradeParameters.put('Structured Basis', 'ACT/360');
tradeParameters.put('Funding Freq', '3M');
tradeParameters.put('Funding Side', 'Pay');
tradeParameters.put('Funding Spread', 0.003);
tradeParameters.put('Funding Basis', 'ACT/360');
tradeParameters.put('Call Start Date', DateExtensions.date('1-Dec-2013'));
tradeParameters.put('Call End Date', DateExtensions.date('1-Dec-2020'));
tradeParameters.put('Option Side', 'Short');
tradeParameters.put('Option Type', 'Right to Terminate');
tradeParameters.put('Call Frequency', '3M');
tradeParameters.put('Model', 'IR.USD-LIBOR-3M.MID.DET');
tradeParameters.put('Method', 'BackwardAnalytic');

```

Create a trade instance.

```
trade = RepositoryExtensions.createTradeInstance(n.Repository, tradeDescriptor, tradeParameters);
```

Price the trades.

```
results = CalculationContextExtensions.calculate(n.Context, trade, market, Request.getAll);
```

Parse the results for MATLAB and display.

```
r = n.parseResults(results)
disp([r.Name r.Category r.Currency r.Data])
```

r =

```

Category: {13x1 cell}
Currency: {13x1 cell}
Name: {13x1 cell}
Data: {13x1 cell}

'Reporting Currency'      'Price'      ''      'USD'
'Structured Cashflow Log' 'Cashflow'   ''      {41x20 cell}
'Structured Leg PV Accrued' 'Price'      'USD'   [ 0]
'PV'                     'Price'      'USD'   [ 6.4133e+04]
'Structured Leg PV Clean' 'Price'      'USD'   [ 4.2637e+05]
'Option PV'              'Price'      'USD'   [-1.3220e+05]
'Funding Cashflow Log'  'Cashflow'   ''      {41x20 cell}
'Structured Leg PV'     'Price'      'USD'   [ 4.2637e+05]
'Funding Leg PV'        'Price'      'USD'   [-2.3004e+05]
'Funding Leg PV Accrued' 'Price'      'USD'   [ 0]
'Funding Leg PV Clean'  'Price'      'USD'   [-2.3004e+05]

```

```
'Yield Risk Report'      ''      ''      { 4x30 cell}  
'Messages'              ''      ''      { 4x1  cell}
```

## See Also

numerix

## Topics

“Working with Simple Numerix Trades” on page 10-2

“Working with Advanced Numerix Trades” on page 10-5

“Use Numerix to Price Cash Deposits” on page 10-10

“Use Numerix for Interest-Rate Risk Assessment” on page 10-13

## External Websites

<https://www.numerix.com/cail>

## Introduced in R2013b

## oasbybdt

Determine option adjusted spread using Black-Derman-Toy model

### Syntax

```
[OAS,OAD,OAC] = oasbybdt(BDTree,Price,CouponRate,Settle,Maturity,  
OptSpec,Strike,ExerciseDates)  
[OAS,OAD,OAC] = oasbybdt( ____,Name,Value)
```

### Description

[OAS,OAD,OAC] = oasbybdt(BDTree,Price,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates) calculates option adjusted spread using a Black-Derman-Toy model.

[OAS,OAD,OAC] = oasbybdt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute OAS Using the Black-Derman-Toy (BDT) Model

This example shows how to compute OAS using the Black-Derman-Toy (BDT) model with the following data.

```
ValuationDate = 'Oct-1-2010';  
Rates = [0.035; 0.042; 0.047; 0.052];  
StartDates = ValuationDate;  
EndDates = datemnth(ValuationDate, 12:12:48)';  
Compounding = 1;  
% define RateSpec  
RateSpec = intenvset('ValuationDate', ValuationDate,...  
'StartDates', StartDates, 'EndDates', EndDates, ...  
'Rates', Rates, 'Compounding', Compounding);
```

```

% specify VolSpec and TimeSpec
Sigma = 0.20;
VS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)));
TS = bdttimespec(ValuationDate, EndDates, Compounding);

% build the BDT tree
BDTTree = bdttree(VS, RateSpec, TS);
BDTTreeNew = cvtree(BDTTree);

% instrument information
CouponRate = 0.065;
Settle = ValuationDate;
Maturity = '01-Oct-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates = '01-Oct-2011';
Period = 1;
Price = 101.58;

% compute the OAS
OAS = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period)

OAS = 32.7688

```

## Input Arguments

### **BDTTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdttree`.

Data Types: `struct`

### **Price** — Market prices of bonds with embedded options

numeric

Market prices of bonds with embedded options, specified as an NINST-by-1 vector.

Data Types: `double`

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond with an embedded option is set to the `ValuationDate` of the BDT tree. The bond argument `Settle` is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.



- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES of serial date numbers or character vectors depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates. Each row is the schedule for one option.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option is exercised between the underlying bond `Settle` date and the single listed exercise date.

Data Types: double | char

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `OAS =`

```
oasbybdt(BDTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 4)
```

### **AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated

at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: `char` | `double`

**StartDate — Forward starting date of payments**

`serial date number` | `date character vector`

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of `'StartDate'` and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: `char` | `double`

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of `'Face'` and a NINST-by-1 vector.

Data Types: `double`

**Options — Derivatives pricing options**

`structure`

Derivatives pricing options, specified as the comma-separated pair consisting of `'Options'` and a structure that is created with `derivset`.

Data Types: `struct`

## Output Arguments

**OAS — Option adjusted spread**

`vector`

Option adjusted spread, returned as a NINST-by-1 vector.

**OAD — Option adjusted duration**

`vector`

Option adjusted duration, returned as a NINST-by-1 vector.

**OAC — Option adjusted convexity**  
vector

Option adjusted convexity, returned as a NINST-by-1 vector.

## Definitions

### Bond with Embedded Options

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates.

Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds. The pricing for a bond with embedded options is as follows:

- Callable bond — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

$$\text{Price callable bond} = \text{Price Option free bond} - \text{Price call option}$$

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

$$\text{Price puttable bond} = \text{Price Option free bond} + \text{Price put option}$$

## References

[1] Fabozzi, F. *Handbook of Fixed Income Securities*. 7th Edition. McGraw-Hill, 2005.

[2] Windas, T. *Introduction to Option-Adjusted Spread Analysis*. 3rd Edition. Bloomberg Press, 2007.

## See Also

bdtpprice | bdttree | instoptembnd | oasbybk | oasbyhjm | oasbyhw |  
optembndbybdt

## Topics

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## Introduced in R2011a

# oasbybk

Determine option adjusted spread using Black-Karasinski model

## Syntax

```
[OAS,OAD,OAC] = oasbybk(BKTree,Price,CouponRate,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
[OAS,OAD,OAC] = oasbybk( ____,Name,Value)
```

## Description

[OAS,OAD,OAC] = oasbybk(BKTree,Price,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates) calculates option adjusted spread using a Black-Karasinski model.

[OAS,OAD,OAC] = oasbybk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Compute OAS and OAD Using the Black-Karasinski (BK) Model

This example shows how to compute OAS and OAD using the Black-Karasinski (BK) model using the following data.

```
ValuationDate = 'Aug-2-2010';
Rates = [0.0355; 0.0382; 0.0427; 0.0489];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;

% define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates,'EndDates', EndDates, ...
```

```

'Rates', Rates, 'Compounding', Compounding);

% specify VolSpec and TimeSpec
Sigma = 0.10;
Alpha = 0.01;
VS = bkvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)),...
EndDates, Alpha*ones(size(EndDates)));
TS = bktimespec(ValuationDate, EndDates, Compounding);

% build the BK tree
BKTree = bktree(VS, RateSpec, TS);

% instrument information
CouponRate = 0.045;
Settle = ValuationDate;
Maturity = '02-Aug-2014';
OptSpec = 'put';
Strike = 100;
ExerciseDates = '02-Aug-2013';
Period = 1;
AmericanOpt = 1;
Price = 101;

% compute OAS and OAD
[OAS, OAD] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'AmericanOpt', AmericanOpt)

OAS = 21.8655
OAD = 1.8654

```

## Input Arguments

### **BKTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **Price** — Market prices of bonds with embedded options

numeric



Market prices of bonds with embedded options, specified as an NINST-by-1 vector.

Data Types: double

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate.

Data Types: double

### **Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond with an embedded option is set to the `ValuationDate` of the BK tree. The bond argument `Settle` is ignored.

---

Data Types: double | char

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

### **OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

### **Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates. Each row is the schedule for one option.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option is exercised between the underlying bond `Settle` date and the single listed exercise date.

Data Types: double | char

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `OAS =`

```
oasbybk(BDTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 4)
```

### **AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers date or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: char | double

### **StartDate — Forward starting date of payments**

serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: char | double

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: double

### **Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: struct

## **Output Arguments**

### **OAS — Option adjusted spread**

vector

Option adjusted spread, returned as a NINST-by-1 vector.

**OAD — Option adjusted duration**

vector

Option adjusted duration, returned as a NINST-by-1 vector.

**OAC — Option adjusted convexity**

vector

Option adjusted convexity, returned as a NINST-by-1 vector.

## Definitions

### Bond with Embedded Options

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates.

Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds. The pricing for a bond with embedded options is as follows:

- Callable bond — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

$$\text{Price callable bond} = \text{Price Option free bond} - \text{Price call option}$$

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

$$\text{Price puttable bond} = \text{Price Option free bond} + \text{Price put option}$$

### References

- [1] Fabozzi, F. *Handbook of Fixed Income Securities*. 7th Edition. McGraw-Hill, 2005.
- [2] Windas, T. *Introduction to Option-Adjusted Spread Analysis*. 3rd Edition. Bloomberg Press, 2007.

## See Also

bkprice | bktree | instoptembnd | oasbybdt | oasbyhjm | oasbyhw | optembndbybk

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2011a**

## oasbycir

Determine option adjusted spread using Cox-Ingersoll-Ross model

### Syntax

```
[OAS,OAD,OAC] = oasbycir(CIRTree,Price,CouponRate,Settle,Maturity,  
OptSpec,Strike,ExerciseDates)  
[OAS,OAD,OAC] = oasbycir( ____,Name,Value)
```

### Description

[OAS,OAD,OAC] = oasbycir(CIRTree,Price,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates) calculates the option adjusted spread from a Cox-Ingersoll-Ross (CIR) interest-rate tree. oasbycir computes the price of an option adjusted spread for bonds with embedded options using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[OAS,OAD,OAC] = oasbycir( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute OAS Using a CIR Interest-Rate Tree

Create a RateSpec using the intenvset function.

```
ValuationDate = 'October-25-2018';  
Rates = [0.0355; 0.0382; 0.0427; 0.0489];  
StartDates = ValuationDate;  
EndDates = datemnth(ValuationDate, 12:12:48)';  
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.



```

NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Maturity = '01-Jan-2023';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1.0462 2.0924 3.1386]
        dObs: [737358 737740 738122 738504]
    FwdTree: {[1.0373] [1.0750 1.0443 1.0229] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

Define the OAS instrument.

```

CouponRate = 0.045;
Settle = ValuationDate;
Maturity = '25-October-2019';
OptSpec = 'call';
Strike = 100;
ExerciseDates = {'25-October-2018', '25-October-2019'};
Period = 1;
AmericanOpt = 0;
Price = 97;

```

Compute the OAS.

```

[OAS, OAD] = oasbycir(CIRT, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)

OAS = 416.9457
OAD = 0.9282

```

## Input Arguments

### **CIRtree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **Price — Market prices of bonds with embedded options**

numeric

Market prices of bonds with embedded options, specified as an NINST-by-1 vector.

Data Types: `double`

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate.

Data Types: `double`

### **Settle — Settlement date**

serial date number | date character vector | string array | datetime

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

---

**Note** The `Settle` date for every bond with an embedded option is set to the `ValuationDate` of the CIR tree. The bond argument `Settle` is ignored.

---

Data Types: `double` | `char` | `string` | `datetime`

### **Maturity — Maturity date**

serial date number | date character vector | string array | datetime

Maturity date, specified as an NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: `double` | `char` | `string` | `datetime`

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of option, specified as a NINST-by-1 cell array of character vectors or string arrays with values 'call' or 'put'.

Data Types: char | cell | string

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

**ExerciseDates — Option exercise dates**

serial date number | date character vector | string array | datetime

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers, date character vectors, string arrays, or datetime arrays depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

.

Data Types: double | char | string | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `OAS = oasbycir(CIRTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 4)`

### **AmericanOpt** — Option type

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and a NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **Period** — Coupons per year

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of `'Period'` and a NINST-by-1 vector.

Data Types: double

### **Basis** — Day-count basis

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of `'Basis'` and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)

- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector | string array | datetime

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector | string array | datetime

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | string | datetime

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector | string array | datetime

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: char | double | string | datetime

### **StartDate — Forward starting date of payments**

serial date number | date character vector | string array | datetime

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime array.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double | string | datetime

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: double

## Output Arguments

### **OAS — Option adjusted spread**

vector

Option adjusted spread, returned as a NINST-by-1 vector.

### **OAD — Option adjusted duration**

vector

Option adjusted duration, returned as a NINST-by-1 vector.

### **OAC — Option adjusted convexity**

vector

Option adjusted convexity, returned as a NINST-by-1 vector.

## Definitions

### **Bond with Embedded Options**

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates.

Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds. The pricing for a bond with embedded options is as follows:

- **Callable bond** — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

$$\text{Price callable bond} = \text{Price Option free bond} - \text{Price call option}$$

- **Puttable bond** — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

$$\text{Price puttable bond} = \text{Price Option free bond} + \text{Price put option}$$

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

[bondbycir](#) | [capbycir](#) | [cfbycir](#) | [fixedbycir](#) | [floatbycir](#) | [floorbycir](#) | [optbndbycir](#) | [optembndbycir](#) | [optemfloatbycir](#) | [optfloatbycir](#) | [rangefloatbycir](#) | [swapbycir](#) | [swaptionbycir](#)

## Topics

"Pricing Using Interest-Rate Tree Models" on page 2-99  
"Understanding Interest-Rate Tree Models" on page 2-79  
"Pricing Options Structure" on page B-2  
"Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a



# oasbyhjm

Determine option adjusted spread using Heath-Jarrow-Morton model

## Syntax

```
[OAS,OAD,OAC] = oasbyhjm(HJMTtree,Price,CouponRate,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
[OAS,OAD,OAC] = oasbyhjm( ____,Name,Value)
```

## Description

[OAS,OAD,OAC] = oasbyhjm(HJMTtree,Price,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates) calculates option adjusted spread using a Heath-Jarrow-Morton model.

[OAS,OAD,OAC] = oasbyhjm( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Compute OAS Using the Heath-Jarrow-Morton (HJM) Model

This example shows how to compute OAS using the Heath-Jarrow-Morton (HJM) model using the following data.

```
ValuationDate = 'Nov-1-2010';
Rates = [0.0356; 0.0427; 0.0478; 0.0529];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;

% define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates,'EndDates', EndDates, ...
```

```
'Rates', Rates, 'Compounding', Compounding);

% specify VolSpec and TimeSpec
Sigma = 0.02;
VS = hjmvolspec('Constant', Sigma);
TS = hjmtimespec(ValuationDate, EndDates, Compounding);

% build the HJM tree
HJMTree = hjmtree(VS, RateSpec, TS);
HJMTreeNew = cvtree(HJMTree);

% instrument information
CouponRate = 0.05;
Settle = ValuationDate;
Maturity = '01-Nov-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates = '01-Nov-2011';
Period = 1;
Price = 97.5;

% compute the OAS
OAS = oasbyhjm(HJMTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ...
ExerciseDates, 'Period', Period)

OAS = 5.0057
```

## Input Arguments

### **HJMTree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `hjmtree`.

Data Types: `struct`

### **Price — Market prices of bonds with embedded options**

numeric

Market prices of bonds with embedded options, specified as an NINST-by-1 vector.

Data Types: `double`

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond with an embedded option is set to the `ValuationDate` of the HJM tree. The bond argument `Settle` is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.

- Bermuda option — `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with NaNs.
- American option — `NINST-by-1` vector of strike price values for each option.

Data Types: `double`

### **ExerciseDates — Option exercise dates**

`serial date number` | `date character vector`

Option exercise dates, specified as a `NINST-by-1`, `NINST-by-2`, or `NINST-by-NSTRIKES` using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a `NINST-by-1` vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a `NINST-by-NSTRIKES` vector of dates. Each row is the schedule for one option.
- For an American option, use a `NINST-by-2` vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a `NINST-by-1` vector, the option is exercised between the underlying bond `Settle` date and the single listed exercise date.

Data Types: `double` | `char`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `OAS =`

```
oasbyk(BDTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 4)
```

### **AmericanOpt — Option type**

`0` European/Bermuda (default) | integer with values `0` or `1`

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and `NINST-by-1` positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers date or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated

at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: `char` | `double`

### **StartDate — Forward starting date of payments**

`serial date number` | `date character vector`

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of `'StartDate'` and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: `char` | `double`

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of `'Face'` and a NINST-by-1 vector.

Data Types: `double`

### **Options — Derivatives pricing options**

`structure`

Derivatives pricing options, specified as the comma-separated pair consisting of `'Options'` and a structure that is created with `derivset`.

Data Types: `struct`

## **Output Arguments**

### **OAS — Option adjusted spread**

`vector`

Option adjusted spread, returned as a NINST-by-1 vector.

### **OAD — Option adjusted duration**

`vector`

Option adjusted duration, returned as a NINST-by-1 vector.

**OAC — Option adjusted convexity**

vector

Option adjusted convexity, returned as a NINST-by-1 vector.

## Definitions

### Bond with Embedded Options

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates.

Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds. The pricing for a bond with embedded options is as follows:

- Callable bond — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

$$\text{Price callable bond} = \text{Price Option free bond} - \text{Price call option}$$

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

$$\text{Price puttable bond} = \text{Price Option free bond} + \text{Price put option}$$

## References

[1] Fabozzi, F. *Handbook of Fixed Income Securities*. 7th Edition. McGraw-Hill, 2005.

[2] Windas, T. *Introduction to Option-Adjusted Spread Analysis*. 3rd Edition. Bloomberg Press, 2007.



## **See Also**

hjmprice | hjmtree | instoptembnd | oasbybdt | oasbybk | oasbyhw |  
optembndbyhjm

## **Topics**

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## **Introduced in R2011a**

## **oasbyhw**

Determine option adjusted spread using Hull-White model

### **Syntax**

```
[OAS,OAD,OAC] = oasbyhw(HWTree,Price,CouponRate,Settle,Maturity,  
OptSpec,Strike,ExerciseDates)  
[OAS,OAD,OAC] = oasbyhw( ____,Name,Value)
```

### **Description**

[OAS,OAD,OAC] = oasbyhw(HWTree,Price,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates) calculates option adjusted spread using a Hull-White model.

[OAS,OAD,OAC] = oasbyhw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### **Examples**

#### **Compute OAS and OAD Using the Hull-White (HW) Model**

This example shows how to compute OAS and OAD using the Hull-White (HW) model using the following data.

```
ValuationDate = 'October-25-2010';  
Rates = [0.0355; 0.0382; 0.0427; 0.0489];  
StartDates = ValuationDate;  
EndDates = datemnth(ValuationDate, 12:12:48)';  
Compounding = 1;  
  
% define RateSpec  
RateSpec = intenvset('ValuationDate', ValuationDate,...  
'StartDates', StartDates, 'EndDates', EndDates, ...
```

```

'Rates', Rates, 'Compounding', Compounding);

% specify VolsSpec and TimeSpec
Sigma = 0.05;
Alpha = 0.01;
VS = hwvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)), ...
EndDates, Alpha*ones(size(EndDates)));
TS = hwtimespec(ValuationDate, EndDates, Compounding);

% build the HW tree
HWTree = hwtree(VS, RateSpec, TS);

% instrument information
CouponRate = 0.045;
Settle = ValuationDate;
Maturity = '25-October-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates = {'25-October-2010', '25-October-2013'};
Period = 1;
AmericanOpt = 0;
Price = 97;

% compute the OAS
[OAS, OAD] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, ...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'AmericanOpt', AmericanOpt)

OAS = -12.8538
OAD = 3.2910

```

### Compute the OAS to Measure Cost of an Embedded Option Relative to a Risk-Free Curve

This example shows how to compute the price of a callable bond using a Hull-White tree.

Use the following bond data:

```

Settle = datenum('20-Aug-2014');

% Bond Properties
Maturity = datenum('01-Apr-2034');

```

```
CouponRate = .0625;
CallDates = datemnth('01-Oct-2014',6*(0:19));
CallStrikes = [102.85 102.7 102.55 102.4 102.25 102.1 101.95 101.8 ...
    101.65 101.5 101.35 101.2 101.05 100.9 100.75 100.6 100.45 100.3 ...
    100.15 100];
```

Use the following zero-curve data:

```
CurveDates = datemnth(Settle,12*[1 2 3 5 7 10 20 30]');
ZeroRates = [.11 0.30 0.64 1.44 2.07 2.61 3.29 3.55]'/100;
```

Define the RateSpec and build the HW tree.

```
RateSpec = intenvset('StartDate',Settle,'EndDates',CurveDates,'Rates',ZeroRates);
```

```
% HW Model Properties
```

```
alpha = .1;
sigma = .01;
```

```
TimeSpec = hwtimespec(Settle,cfdates(Settle,Maturity,12),2);
VolSpec = hwvolspec(Settle,Maturity,sigma,Maturity,alpha);
```

```
HWTree = hwtree(VolSpec,RateSpec,TimeSpec,'method','HW2000')
```

```
HWTree = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [1x236 double]
    dObs: [1x236 double]
    CFlowT: {1x236 cell}
    Probs: {1x235 cell}
    Connect: {1x235 cell}
    FwdTree: {1x236 cell}
```

Compute the OAS for the bond.

```
Price = 103.25;
OAS = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, 'call', CallStrikes, CallDates);
OAS = 234.8209
```

If you want to compute an OAS that only measures the option cost, you can pass in an issuer-specific curve instead of a risk-free curve (this would be done in the `RateSpec` argument).

## Input Arguments

### **HWTree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: `struct`

### **Price — Market prices of bonds with embedded options**

numeric

Market prices of bonds with embedded options, specified as an NINST-by-1 vector.

Data Types: `double`

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate.

Data Types: `double`

### **Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond with an embedded option is set to the `ValuationDate` of the HW tree. The bond argument `Settle` is ignored.

---

Data Types: `double` | `char`

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

### **OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

### **Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates. Each row is the schedule for one option.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option is exercised between the underlying bond Settle date and the single listed exercise date.

Data Types: double | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: OAS =

```
oasbyk(BDTree,Price,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates,'Period',4)
```

### AmericanOpt — Option type

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### Period — Coupons per year

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### Basis — Day-count basis

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360

- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector



Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: char | double

### **StartDate — Forward starting date of payments**

serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: double

**Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: `struct`

**Output Arguments****OAS — Option adjusted spread**

vector

Option adjusted spread, returned as a NINST-by-1 vector.

**OAD — Option adjusted duration**

vector

Option adjusted duration, returned as a NINST-by-1 vector.

**OAC — Option adjusted convexity**

vector

Option adjusted convexity, returned as a NINST-by-1 vector.

**Definitions****Bond with Embedded Options**

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates.

Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds. The pricing for a bond with embedded options is as follows:

- **Callable bond** — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

Price callable bond = Price Option free bond – Price call option

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

Price puttable bond = Price Option free bond + Price put option

## References

[1] Fabozzi, F. *Handbook of Fixed Income Securities*. 7th Edition. McGraw-Hill, 2005.

[2] Windas, T. *Introduction to Option-Adjusted Spread Analysis*. 3rd Edition. Bloomberg Press, 2007.

## See Also

[hwprice](#) | [hwtree](#) | [instoptembnd](#) | [oasbybdt](#) | [oasbybk](#) | [oasbyhjm](#) | [optembndbyhw](#)

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## Introduced in R2011a

## optbndbybdt

Price bond option from Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = optbndbybdt(BDTree,OptSpec,Strike,
ExerciseDates,AmericanOpt,CouponRate,Settle,Maturity)
[Price,PriceTree] = optbndbybdt( ____,Period,Basis,EndMonthRule,
IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options)
```

### Description

`[Price,PriceTree] = optbndbybdt(BDTree,OptSpec,Strike, ExerciseDates,AmericanOpt,CouponRate,Settle,Maturity)` calculates the price for a bond option from a Black-Derman-Toy interest-rate tree.

`[Price,PriceTree] = optbndbybdt( ____,Period,Basis,EndMonthRule, IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options)` adds optional arguments.

### Examples

#### Price a European Call and Put Option on a Bond

Using the BDT interest-rate tree in the `deriv.mat` file, price a European call and put option on a 10% bond with a strike of 95. The exercise date for the option is Jan. 01, 2002. The settle date for the bond is Jan. 01, 2000, and the maturity date is Jan. 01, 2003.

Load the file `deriv.mat`, which provides `BDTree`. The `BDTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbndbybdt` to compute the price of the 'Call' option.

```
[Price,PriceTree] = optbndbybdt(BDTree, 'Call', 95, '01-Jan-2002', ...
0,0.10, '01-Jan-2000', '01-Jan-2003', 1)
```

```
Price = 1.7657
```

```
PriceTree = struct with fields:
  FinObj: 'BDTPriceTree'
  tObs: [0 1 2 3 4]
  PTree: {1x5 cell}
```

Now use `optbndbybdt` to compute the price of a 'Put' option on the same bond.

```
[Price,PriceTree] = optbndbybdt(BDTree, 'Put', 95, '01-Jan-2002', ...
0,0.10, '01-Jan-2000', '01-Jan-2003', 1)
```

```
Price = 0.5740
```

```
PriceTree = struct with fields:
  FinObj: 'BDTPriceTree'
  tObs: [0 1 2 3 4]
  PTree: {[0.5740] [0 1.2628] [0 0 2.8871] [0 0 0 0] [0 0 0 0]}
```

## Input Arguments

### **BDTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdtree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: `char`

### **Strike** — Option strike price values

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

Data Types: double | char

**AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

(Optional) Option type, specified as NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

### **Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The Settle date for every bond is set to the ValuationDate of the BDT tree. The bond argument Settle is ignored.

---

Data Types: double | char

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

### **Period — Coupons per year**

2 per year (default) | vector

(Optional) Coupons per year, specified as an NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360

- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

(Optional) End-of-month rule flag is specified as a nonnegative integer using a NINST-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector

(Optional) Bond issue date, specified as an NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector



(Optional) Irregular first coupon date, specified as an NINST-by-1 vector using serial date numbers date or date character vectors.

When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector

(Optional) Irregular last coupon date, specified as a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: char | double

### **StartDate — Forward starting date of payments**

serial date number | date character vector

(Optional) Forward starting date of payments (the date from which a bond cash flow is considered), specified as a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: char | double

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

(Optional) Face or par value, specified as an NINST-by-1 vector.

Data Types: double

### **Options — Derivatives pricing options**

structure

(Optional) Derivatives pricing options, specified as structure that is created with `derivset`.

Data Types: `struct`

## Output Arguments

### **Price — Expected prices of bond option at time 0**

`matrix`

Expected price of the bond option at time 0, returned as a NINST-by-1 matrix.

### **PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

`structure`

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## See Also

`bdtprice` | `bdttree` | `instoptbnd`

## Topics

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# optbndbybk

Price bond option from Black-Karasinski interest-rate tree

## Syntax

```
[Price,PriceTree] = optbndbybk(BKTree,OptSpec,Strike,ExerciseDates,
AmericanOpt,CouponRate,Settle,Maturity)
[Price,PriceTree] = optbndbybk( ____,Period,Basis,EndMonthRule,
IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options)
```

## Description

[Price,PriceTree] = optbndbybk(BKTree,OptSpec,Strike,ExerciseDates, AmericanOpt,CouponRate,Settle,Maturity) calculates the price for a bond option from a Black-Karasinski interest-rate tree.

[Price,PriceTree] = optbndbybk( \_\_\_\_,Period,Basis,EndMonthRule, IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options) adds optional arguments.

## Examples

### Price a European Call and Put Option on a Bond

Using the BK interest rate tree in the `deriv.mat` file, price a European call and put option on a 4% bond with a strike of 96. The exercise date for the option is Jan. 01, 2006. The settle date for the bond is Jan. 01, 2005, and the maturity date is Jan. 01, 2009.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbndbybk` to compute the price of the 'call' option.

```
[Price,PriceTree] = optbndbybk(BKTree, 'Call', 96, '01-Jan-2006', ...  
0,0.04, '01-Jan-2005', '01-Jan-2009')
```

Warning: OptBonds are valued at Tree ValuationDate rather than Settle

> In optbndbytrintree (line 41)

In optbndbybk (line 89)

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

> In optbndbytrintree (line 152)

In optbndbybk (line 89)

Price =

0.1512

PriceTree =

struct with fields:

FinObj: 'BKPriceTree'

PTree: {[0.1512] [0.0281 0.1481 0.3119] [0 0 0.1329 0.3886 0.3086] [0 0 0 0 0]}

tObs: [0 1 2 3 4]

Connect: {[2] [2 3 4] [2 2 3 4 4]}

Probs: {[3×1 double] [3×3 double] [3×5 double]}

Now use `optbndbybdt` to compute the price of a 'put' option on the same bond.

```
[Price,PriceTree] = optbndbybk(BKTree, 'Put', 96, '01-Jan-2006', ...  
0,0.04, '01-Jan-2005', '01-Jan-2009')
```

Warning: OptBonds are valued at Tree ValuationDate rather than Settle

> In optbndbytrintree (line 41)

In optbndbybk (line 89)

Warning: Not all cash flows are aligned with the tree. Result will be approximated.

> In optbndbytrintree (line 152)

In optbndbybk (line 89)

Price =

0.0272

PriceTree =

struct with fields:

```

FinObj: 'BKPriceTree'
PTree: {[0.0272] [0.0860 0.0204 0] [0.0474 0.1266 0 0 0] [0 0 0 0 0] [0 0 0 0 0]}
tObs: [0 1 2 3 4]
Connect: {[2] [2 3 4] [2 2 3 4 4]}
Probs: {[3×1 double] [3×3 double] [3×5 double]}

```

## Input Arguments

### **BKTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: `char`

### **Strike** — Option strike price values

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: `double`

### **ExerciseDates** — Option exercise dates

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

### **AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

(Optional) Option type, specified as NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: `double`

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond is set to the `ValuationDate` of the BK tree. The bond argument `Settle` is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**Period — Coupons per year**

2 per year (default) | vector

(Optional) Coupons per year, specified as an NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

(Optional) End-of-month rule flag is specified as a nonnegative integer using a NINST-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

(Optional) Bond issue date, specified as an NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

(Optional) Irregular first coupon date, specified as an NINST-by-1 vector using serial date numbers date or date character vectors.

When **FirstCouponDate** and **LastCouponDate** are both specified, **FirstCouponDate** takes precedence in determining the coupon payment structure. If you do not specify a **FirstCouponDate**, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector



(Optional) Irregular last coupon date, specified as a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: char | double

### **StartDate — Forward starting date of payments**

serial date number | date character vector

(Optional) Forward starting date of payments (the date from which a bond cash flow is considered), specified as a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: char | double

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

(Optional) Face or par value, specified as an NINST-by-1 vector.

Data Types: double

### **Options — Derivatives pricing options**

structure

(Optional) Derivatives pricing options, specified as structure that is created with `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices of bond option at time $\theta$**

matrix

Expected price of the bond option at time  $\theta$ , returned as a NINST-by-1 matrix.

**PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

**See Also**

`bkprice` | `bktree` | `instoptbnd`

**Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# optbndbycir

Price bond option from Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = optbndbycir(CIRTree,OptSpec,Strike,
ExerciseDates,AmericanOpt,CouponRate,Settle,Maturity)
[Price,PriceTree] = optbndbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = optbndbycir(CIRTree,OptSpec,Strike, ExerciseDates,AmericanOpt,CouponRate,Settle,Maturity) calculates the price for a bond option from a Cox-Ingersoll-Ross (CIR) interest-rate tree using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = optbndbycir( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a European Call and Put Option on a Bond Using a CIR Interest-Rate Tree

Compute the price for a European call option on a 4% bond with a strike of 96. The exercise date for the option is Jan. 01, 2018. The settle date for the bond is Jan. 01, 2017, and the maturity date is Jan. 01, 2020.

Create a RateSpec using the `intenvset` function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
```

```
Compounding = 1;  
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'EndDates', EndDates);
```

Create a CIR tree.

```
NumPeriods = length(EndDates);  
Alpha = 0.03;  
Theta = 0.02;  
Sigma = 0.1;  
Settle = '01-Jan-2017';  
Maturity = '01-Jan-2019';  
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);  
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);
```

```
CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)
```

```
CIRT = struct with fields:  
    FinObj: 'CIRFwdTree'  
    VolSpec: [1x1 struct]  
    TimeSpec: [1x1 struct]  
    RateSpec: [1x1 struct]  
        tObs: [0 0.5000 1 1.5000]  
        dObs: [736696 736878 737061 737243]  
    FwdTree: {[1.0173] [1.0276 1.0175 1.0097] [1x5 double] [1x7 double]}  
    Connect: {[3x1 double] [3x3 double] [3x5 double]}  
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

Price the call option.

```
[Price,PriceTree] = optbndbycir(CIRT,'Call',96,'01-Jan-2018',...  
0,0.04,'01-Jan-2017','01-Jan-2020')
```

```
Price = 2.6827
```

```
PriceTree = struct with fields:  
    FinObj: 'CIRPriceTree'  
    tObs: [0 0.5000 1 1.5000 2]  
    PTree: {1x5 cell}  
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
```

Price the put option.

```
[Price,PriceTree] = optbndbycir(CIRT,'Put',96,'01-Jan-2018',...  
0,0.04,'01-Jan-2017','01-Jan-2020')
```

```
Price = 0.6835

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    tObs: [0 0.5000 1 1.5000 2]
    PTree: {1x5 cell}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
```

## Input Arguments

### **CIRTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string arrays with values "call" or "put"

Definition of option, specified as a NINST-by-1 cell array of character vectors or string arrays.

Data Types: `char` | `cell` | `string`

### **Strike** — Option strike price values

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: `double`

**ExerciseDates — Option exercise dates**

serial date number | date character vector | string array | datetime

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers, data character vectors, string arrays, or datetime arrays depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

Data Types: double | char | string | datetime

**AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

**Settle — Settlement date**

serial date number | date character vector | string array | datetime

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

---

**Note** The `Settle` date for every bond is set to the `ValuationDate` of the CIR tree. The bond argument `Settle` is ignored.

---

Data Types: `double` | `char` | `string`

### **Maturity – Maturity date**

serial date number | date character vector | string array | datetime

Maturity date, specified as an NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: `double` | `char` | `string` | `datetime`

## **Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = optbndbycir(CIRTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, 'Period' 6, 'Basis', 7, 'Face', 1000)`

### **Period – Coupons per year**

2 per year (default) | possible values include: 0, 1, 2, 3, 4, 6, 12.

Coupons per year, specified as the comma-separated pair consisting of `'Period'` and a NINST-by-1 vector.

Data Types: `double`

### **Basis – Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of `'Basis'` and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector | string array | datetime



Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector | string array | datetime

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers date, date character vectors, string arrays, or datetime arrays.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | string | datetime

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector | string array | datetime

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: char | double | string | datetime

### **StartDate — Forward starting date of payments**

serial date number | date character vector | string array | datetime

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double | string | datetime

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: double

## Output Arguments

**Price — Expected prices of bond option at time 0**

matrix

Expected price of the bond option at time 0, returned as a NINST-by-1 matrix.

**PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- PriceTree.t0bs contains the observation times.
- PriceTree.PTree contains the clean prices.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.

- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | cfbycir | fixedbycir | floatbycir | floorbycir | instoptbnd | oasbycir | optembndbycir | optemfloatbycir | optfloatbycir | rangefloatbycir | swapbycir | swaptionbycir

## Topics

- "Pricing Using Interest-Rate Tree Models" on page 2-99
- "Calibrating Hull-White Model Using Market Data" on page 2-111
- "Understanding Interest-Rate Tree Models" on page 2-79
- "Pricing Options Structure" on page B-2
- "Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

## optbndbyhjm

Price bond option from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = optbndbyhjm(HJMTree,OptSpec,Strike,  
ExerciseDates,AmericanOpt,CouponRate,Settle,Maturity)  
[Price,PriceTree] = optbndbyhjm( ____,Period,Basis,EndMonthRule,  
IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options)
```

### Description

[Price,PriceTree] = optbndbyhjm(HJMTree,OptSpec,Strike, ExerciseDates,AmericanOpt,CouponRate,Settle,Maturity) calculates the price for a bond option from a Black-Karasinski interest-rate tree.

[Price,PriceTree] = optbndbyhjm( \_\_\_\_,Period,Basis,EndMonthRule, IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options) adds optional arguments.

### Examples

#### Price a European Call and Put Option on a Bond

Using the HJM forward-rate tree in the `deriv.mat` file, price a European call and put option on a 4% bond with a strike of 96. The exercise date for the option is Jan. 01, 2003. The settle date for the bond is Jan. 01, 2000, and the maturity date is Jan. 01, 2004.

Load the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbndbyhjm` to compute the price of the 'call' option.

```
[Price,PriceTree] = optbndbyhjm(HJMTree,'Call',96,'01-Jan-2003',...
0,0.04,'01-Jan-2000','01-Jan-2004')
```

```
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
> In optbndbyhjm (line 217)
```

```
Price =
```

```
2.2410
```

```
PriceTree =
```

```
struct with fields:
```

```
FinObj: 'HJMPriceTree'
```

```
tObs: [0 1 2 3 4]
```

```
PBush: {[2.2410] [1x1x2 double] [1x2x2 double] [1x4x2 double] [0 0 0 0 0 0 0 0]}
```

Now use `optbndbyhjm` to compute the price of a 'put' option on the same bond.

```
[Price,PriceTree] = optbndbyhjm(HJMTree,'Put',96,'01-Jan-2003',...
0,0.04,'01-Jan-2000','01-Jan-2004')
```

```
Warning: Not all cash flows are aligned with the tree. Result will be approximated.
> In optbndbyhjm (line 217)
```

```
Price =
```

```
0.0446
```

```
PriceTree =
```

```
struct with fields:
```

```
FinObj: 'HJMPriceTree'
```

```
tObs: [0 1 2 3 4]
```

```
PBush: {[0.0446] [1x1x2 double] [1x2x2 double] [1x4x2 double] [0 0 0 0 0 0 0 0]}
```

## Input Arguments

**HJMTree** — Interest-rate tree structure  
structure

Interest-rate tree structure, specified by using `hjm tree`.

Data Types: `struct`

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a `NINST-by-1` cell array of character vectors.

Data Types: `char`

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a `NINST-by-1` or `NINST-by-NSTRIKES` depending on the type of option:

- European option — `NINST-by-1` vector of strike price values.
- Bermuda option — `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with NaNs.
- American option — `NINST-by-1` vector of strike price values for each option.

Data Types: `double`

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a `NINST-by-1`, `NINST-by-2`, or `NINST-by-NSTRIKES` using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a `NINST-by-1` vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a `NINST-by-NSTRIKES` vector of dates.
- For an American option, use a `NINST-by-2` vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a `NINST-by-1` vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

**AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

(Optional) Option type, specified as NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

**Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The Settle date for every bond is set to the ValuationDate of the HJM tree. The bond argument Settle is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**Period — Coupons per year**

2 per year (default) | vector

(Optional) Coupons per year, specified as an NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

(Optional) End-of-month rule flag is specified as a nonnegative integer using a NINST-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.



- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector

(Optional) Bond issue date, specified as an NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

(Optional) Irregular first coupon date, specified as an NINST-by-1 vector using serial date numbers date or date character vectors.

When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector

(Optional) Irregular last coupon date, specified as a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: char | double

### **StartDate — Forward starting date of payments**

serial date number | date character vector

(Optional) Forward starting date of payments (the date from which a bond cash flow is considered), specified as a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: `char` | `double`

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

(Optional) Face or par value, specified as an NINST-by-1 vector.

Data Types: `double`

**Options — Derivatives pricing options**

structure

(Optional) Derivatives pricing options, specified as structure that is created with `derivset`.

Data Types: `struct`

## Output Arguments

**Price — Expected prices of bond option at time 0**

matrix

Expected price of the bond option at time 0, returned as a NINST-by-1 matrix.

**PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PBush` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## **See Also**

hjmprice | hjmtree | instoptbnd

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## optbndbyhw

Price bond option from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = optbndbyhw(HWTree,OptSpec,Strike,ExerciseDates,  
AmericanOpt,CouponRate,Settle,Maturity)  
[Price,PriceTree] = optbndbyhw( ____,Period,Basis,EndMonthRule,  
IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options)
```

### Description

[Price,PriceTree] = optbndbyhw(HWTree,OptSpec,Strike,ExerciseDates, AmericanOpt,CouponRate,Settle,Maturity) calculates the price for a bond option from a Hull-White interest-rate tree.

[Price,PriceTree] = optbndbyhw( \_\_\_\_,Period,Basis,EndMonthRule, IssueDate,FirstCouponDate,LastCouponDate,StartDate,Face,Options) adds optional arguments.

### Examples

#### Price a European Call and Put Option on a Bond

Using the HW interest rate tree in the `deriv.mat` file, price a European call option on a 4% bond with a strike of 96. The exercise date for the option is Jan. 01, 2006. The settle date for the bond is Jan. 01, 2005, and the maturity date is Jan. 01, 2009.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbndbyhw` to compute the price of the 'call' option.

```
[Price,PriceTree] = optbndbyhw(HWTree, 'Call', 96, '01-Jan-2006', ...
0,0.04, '01-Jan-2005', '01-Jan-2009')
```

```
Price =
```

```
1.1556
```

```
PriceTree =
```

```
struct with fields:
```

```
FinObj: 'HWPriceTree'
PTree: {[1.1556] [0.0150 0.8509 3.7085] [0 0 0.0722 4.9980 3.8429] [0 0 0 0 0]}
tObs: [0 1 2 3 4]
Connect: {[2] [2 3 4] [2 2 3 4 4]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

Now use `optbndbyhw` to compute the price of a 'put' option on the same bond.

```
[Price,PriceTree] = optbndbyhw(HWTree, 'Put', 96, '01-Jan-2006', ...
0,0.04, '01-Jan-2005', '01-Jan-2009')
```

```
Price =
```

```
1.0150
```

```
PriceTree =
```

```
struct with fields:
```

```
FinObj: 'HWPriceTree'
PTree: {[1.0150] [3.2945 0.7413 0] [3.5551 4.6060 0 0 0] [0 0 0 0 0] [0 0 0]}
tObs: [0 1 2 3 4]
Connect: {[2] [2 3 4] [2 2 3 4 4]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

## Input Arguments

### **HWTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: struct

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or data character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

Data Types: double | char

**AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

**Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The Settle date for every bond is set to the ValuationDate of the HW tree. The bond argument Settle is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**Period — Coupons per year**

2 per year (default) | vector

(Optional) Coupons per year, specified as an NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

(Optional) Day-count basis, specified as a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

(Optional) End-of-month rule flag is specified as a nonnegative integer using a NINST-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.



- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector

(Optional) Bond issue date, specified as an NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

(Optional) Irregular first coupon date, specified as an NINST-by-1 vector using serial date numbers date or date character vectors.

When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector

(Optional) Irregular last coupon date, specified as a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: char | double

### **StartDate — Forward starting date of payments**

serial date number | date character vector

(Optional) Forward starting date of payments (the date from which a bond cash flow is considered), specified as a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: `char` | `double`

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

(Optional) Face or par value, specified as an NINST-by-1 vector.

Data Types: `double`

### **Options — Derivatives pricing options**

structure

(Optional) Derivatives pricing options, specified as structure that is created with `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices of bond option at time 0**

matrix

Expected price of the bond option at time 0, returned as a NINST-by-1 matrix.

### **PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level,

there are NumNodes elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.

- PriceTree.Probs contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## See Also

hwprice | hwtree | instoptbnd

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## optByBatesFD

Option price by Bates model using finite differences

### Syntax

```
[Price,PriceGrid,AssetPrices,Variances,Times] = optByBatesFD(Rate,  
AssetPrice,Settle,ExerciseDates,OptSpec,Strike,V0,ThetaV,Kappa,  
SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq)  
[Price,PriceGrid,AssetPrices,Variances,Times] = optByBatesFD( ____,  
Name,Value)
```

### Description

[Price,PriceGrid,AssetPrices,Variances,Times] = optByBatesFD(Rate, AssetPrice,Settle,ExerciseDates,OptSpec,Strike,V0,ThetaV,Kappa, SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq) computes a vanilla European or American option price by the Bates model, using the alternating direction implicit (ADI) method.

[Price,PriceGrid,AssetPrices,Variances,Times] = optByBatesFD( \_\_\_\_, Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Price an American Option Using the Bates Model

Define the option variables and Bates model parameters.

```
AssetPrice = 90;  
Strike = 100;  
Rate = 0.03;  
Settle = '01-Jan-2018';  
ExerciseDates = '02-Jul-2018';
```

```

V0 = 0.04;
ThetaV = 0.04;
Kappa = 2;
SigmaV = 0.25;
RhoSV = -0.5;
JumpVol = 0.4;
MeanJ = exp(-0.5+JumpVol.^2/2)-1;
JumpFreq = 0.2;

```

Compute the American put option price using the finite differences method.

```

OptSpec = 'Put';
Price = optByBatesFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, ...
V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'AmericanOpt', 1)

Price = 11.6116

```

## Input Arguments

### Rate — Continuously compounded risk-free interest rate

scalar decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal.

Data Types: double

### AssetPrice — Current underlying asset price

scalar numeric

Current underlying asset price, specified as a scalar numeric.

Data Types: double

### Settle — Option settlement date

serial date number | date character vector | datetime | string array

Option settlement date, specified as a scalar using a serial date number, date character vector, datetime object, or string scalar.

Data Types: double | char | datetime | string

### ExerciseDates — Option exercise dates

serial date number | date character vector | datetime array | string array

Option exercise dates, specified as a serial date number, date character vector, datetime array, or string array:

- For a European option, use a scalar serial date number, date character vector, datetime object, or string scalar. For a European option, `ExerciseDates` contains only one value: the option expiry date.
- For an American option, use a 1-by-2 vector of serial date numbers, date character vectors, datetime arrays, or string arrays to specify the exercise date boundaries. An American option can be exercised on any date between or including the pair of dates. If only one non-NaN date is listed, then the option can be exercised between `Settle` date and the single listed value in `ExerciseDates`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

character vector with value of 'call' or 'put' | string with value of "call" or "put"

Definition of the option, specified as a scalar using a character vector or string with a value of 'call' or 'put'.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

### **V0 — Initial variance of underlying asset**

scalar numeric

Initial variance of the underling asset, specified as a scalar numeric.

Data Types: `double`

### **ThetaV — Long-term variance of underlying asset**

scalar numeric

Long-term variance of the underling asset, specified as a scalar numeric.

Data Types: `double`

**Kappa — Mean revision speed for the variance of underlying asset**

scalar numeric

Mean revision speed for the underlying asset, specified as a scalar numeric.

Data Types: double

**SigmaV — Volatility of the variance of underlying asset**

scalar numeric

Volatility of the variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**RhoSV — Correlation between Weiner processes for underlying asset and its variance**

scalar numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric.

Data Types: double

**MeanJ — Mean of random percentage jump size**

scalar decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal where  $\log(1+J)$  is normally distributed with the mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

**JumpVol — Standard deviation of  $\log(1+J)$** 

scalar decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal.

Data Types: double

**JumpFreq — Annual frequency of Poisson jump process**

scalar numeric

Annual frequency of the Poisson jump process, specified as a scalar numeric.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceGrid] = optByBatesFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

### **Basis** — Day-count basis of instrument

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double



**DividendYield — Continuously compounded underlying asset yield**

0 (default) | scalar numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates` = [ ] or do not enter them. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

**DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and an NDIV-by-1 vector.

---

**Note** Each dividend amount must have a corresponding ex-dividend date. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

**ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | string array | datetime array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and an NDIV-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, value is calculated based on asset price distribution at maturity (default) | positive scalar numeric

Maximum price for the price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar numeric.

Data Types: double

### **VarianceMax — Maximum variance for variance grid boundary**

1.0 (default) | scalar numeric

Maximum variance for the variance grid boundary, specified as the comma-separated pair consisting of 'VarianceMax' as a scalar numeric.

Data Types: double

### **AssetGridSize — Size of asset grid for finite difference grid**

400 (default) | scalar numeric

Size of the asset grid for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar numeric.

Data Types: double

### **VarianceGridSize — Number of nodes of variance grid for finite difference grid**

200 (default) | scalar numeric

Number of nodes of the variance grid for the finite difference grid, specified as the comma-separated pair consisting of 'VarianceGridSize' and a scalar numeric.

Data Types: double

### **TimeGridSize — Number of nodes of time grid for finite difference grid**

100 (default) | positive numeric scalar

Number of nodes of the time grid for the finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive numeric scalar.

Data Types: double

### **AmericanOpt — Option type**

0 (European) (default) | scalar with value of [0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

## Output Arguments

### Price — Option price

scalar numeric

Option price, returned as a scalar numeric.

### PriceGrid — Grid containing prices calculated by the finite difference method

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with size `AssetGridSize`  $\times$  `TimeGridSize`. The number of columns is not necessarily equal to the `TimeGridSize` because exercise the function adds exercise and ex-dividend dates to the time grid. `PriceGrid(:, :, end)` contains the price for  $t = 0$ .

### AssetPrices — Prices of the asset

vector

Prices of the asset corresponding to the first dimension of `PriceGrid`, returned as a vector.

### Variiances — Variiances

vector

Variiances corresponding to the second dimension of `PriceGrid`, returned as a vector.

### Times — Times

vector

Times corresponding to the third dimension of `PriceGrid`, returned as a vector.

## Definitions

### Bates Stochastic Volatility Jump Diffusion Model

The Bates model [1] extends the Heston model by including stochastic volatility and (similar to Merton) jump diffusion parameters in the modeling of sudden asset price movements.

The stochastic differential equation is:

$$\begin{aligned}
 dS_t &= (r - q - \lambda_p \mu_J) S_t dt + \sqrt{v_t} S_t dW_t + JS_t dP_t \\
 dv_t &= \kappa(\theta - v_t) dt + \sigma_v \sqrt{v_t} dW_t \\
 E[dW_t dW_t^v] &= \rho dt \\
 \text{prob}(dP_t = 1) &= \lambda_p dt
 \end{aligned}$$

where:

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$ .

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1 + J)\delta\sqrt{2\pi}} \exp \left\{ - \frac{\left[ \ln(1 + J) - \left( \ln(1 + \mu_J) - \frac{\delta^2}{2} \right) \right]^2}{2\delta^2} \right\}$$

where:

$v_0$  is the initial variance of the asset price at  $t = 0$  ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

## References

- [1] Bates, D. S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*. Vol. 9, Number 1, 1996.

## See Also

[optByHestonFD](#) | [optByLocalVolFD](#) | [optByMertonFD](#) | [optBySensMertonFD](#) | [optSensByBatesFD](#) | [optSensByHestonFD](#) | [optSensByLocalVolFD](#)

**Introduced in R2019a**

## optSensByBatesFD

Option price and sensitivities by Bates model using finite differences

### Syntax

```
[PriceSens,PriceGrid,AssetPrices,Variances,Times] =  
optSensByBatesFD(Rate,AssetPrice,Settle,ExerciseDates,OptSpec,  
Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq)  
[PriceSens,PriceGrid,AssetPrices,Variances,Times] =  
optSensByBatesFD( ____,Name,Value)
```

### Description

[PriceSens,PriceGrid,AssetPrices,Variances,Times] = optSensByBatesFD(Rate,AssetPrice,Settle,ExerciseDates,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq) computes a vanilla European or American option price and sensitivities by the Bates model, using the alternating direction implicit (ADI) method.

[PriceSens,PriceGrid,AssetPrices,Variances,Times] = optSensByBatesFD( \_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute Price and Sensitivities for an American Option Using the Bates Model

Define the option variables and Bates model parameters.

```
AssetPrice = 90;  
Strike = 100;  
Rate = 0.03;  
Settle = '01-Jan-2018';  
ExerciseDates = '02-Jul-2018';
```

```

V0 = 0.04;
ThetaV = 0.04;
Kappa = 2;
SigmaV = 0.25;
RhoSV = -0.5;
JumpVol = 0.4;
MeanJ = exp(-0.5+JumpVol.^2/2)-1;
JumpFreq = 0.2;

```

Compute the American put option price and sensitivities using the finite differences method.

```

OptSpec = 'Put';
[Price, Delta, Gamma, Rho, Theta, Vega, VegaLT] = optSensByBatesFD(Rate, AssetPrice, S
OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'American
'OutSpec', ["Price" "Delta" "Gamma" "Rho" "Theta" "Vega" "VegaLT"])

```

```
Price = 11.6116
```

```
Delta = -0.6578
```

```
Gamma = 0.0366
```

```
Rho = -20.1643
```

```
Theta = -4.8425
```

```
Vega = 14.3216
```

```
VegaLT = 8.1849
```

## Input Arguments

### Rate — Continuously compounded risk-free interest rate

scalar decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal.

Data Types: double

### AssetPrice — Current underlying asset price

scalar numeric

Current underlying asset price, specified as a scalar numeric.

Data Types: `double`

**Settle — Option settlement date**

serial date number | date character vector | datetime | string scalar

Option settlement date, specified as a scalar using a serial date number, date character vector, datetime object, or string scalar.

Data Types: `double` | `char` | `datetime` | `string`

**ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime array | string array

Option exercise dates, specified as a serial date number, date character vector, datetime object or string scalar:

- For a European option, use a scalar serial date number, date character vector, datetime array, or string array. For a European option, `ExerciseDates` contains only one value: the option expiry date.
- For an American option, use a 1-by-2 vector of serial date numbers, date character vectors, datetime arrays, or string arrays to specify the exercise date boundaries. An American option can be exercised on any date between or including the pair of dates. If only one non-NaN date is listed, then the option can be exercised between `Settle` date and the single listed value in `ExerciseDates`.

Data Types: `double` | `char` | `datetime` | `string`

**OptSpec — Definition of option**

character vector with value of 'call' or 'put' | string with value of "call" or "put"

Definition of the option, specified as a scalar using a character vector or string with a value of 'call' or 'put'.

Data Types: `cell` | `string`

**Strike — Option strike price value**

numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`



**V0 — Initial variance of underlying asset**

scalar numeric

Initial variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**ThetaV — Long-term variance of underlying asset**

scalar numeric

Long-term variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**Kappa — Mean revision speed for the variance of underlying asset**

scalar numeric

Mean revision speed for the underlying asset, specified as a scalar numeric.

Data Types: double

**SigmaV — Volatility of the variance of underlying asset**

scalar numeric

Volatility of the variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**RhoSV — Correlation between Weiner processes for underlying asset and its variance**

scalar numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric.

Data Types: double

**MeanJ — Mean of random percentage jump size**

scalar decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal where  $\log(1+J)$  is normally distributed with the mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

**JumpVol — Standard deviation of  $\log(1+J)$** 

scalar decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal.

Data Types: double

**JumpFreq — Annual frequency of Poisson jump process**

scalar numeric

Annual frequency of the Poisson jump process, specified as a scalar numeric.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

```
Example: [PriceSens, PriceGrid] =  
optSensByBatesFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike  
, V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'Basis', 7, 'OutS  
pec', 'delta')
```

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | scalar numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates` = [ ] or do not enter them. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

### **DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and an `NDIV-by-1` vector.

---

**Note** Each dividend amount must have a corresponding ex-dividend date. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

### **ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | string array | datetime array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and an NDIV-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, value is calculated based on asset price distribution at maturity (default) | positive scalar numeric

Maximum price for the price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar numeric.

Data Types: double

**VarianceMax — Maximum variance for variance grid boundary**

1.0 (default) | scalar numeric

Maximum variance for the variance grid boundary, specified as the comma-separated pair consisting of 'VarianceMax' as a scalar numeric.

Data Types: double

**AssetGridSize — Size of asset grid for finite difference grid**

400 (default) | scalar numeric

Size of the asset grid for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar numeric.

Data Types: double

**VarianceGridSize — Number of nodes of variance grid for finite difference grid**

200 (default) | scalar numeric

Number of nodes of the variance grid for the finite difference grid, specified as the comma-separated pair consisting of 'VarianceGridSize' and a scalar numeric.

Data Types: double

**TimeGridSize — Number of nodes of time grid for finite difference grid**

100 (default) | positive numeric scalar

Number of nodes of the time grid for the finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive numeric scalar.

Data Types: double

### **AmericanOpt — Option type**

0 (European) (default) | scalar with value of [0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

### **OutSpec — Define outputs**

['price'] (default) | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegalt' | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegalt"

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

---

**Note** 'vega' is the sensitivity with respect to the initial volatility  $\sqrt{V_0}$ . In contrast, 'vegalt' is the sensitivity with respect to the long-term volatility  $\sqrt{\text{Theta}V}$ .

---

Example: OutSpec =

```
['price','delta','gamma','vega','rho','theta','vegalt']
```

Data Types: string | cell

## **Output Arguments**

### **PriceSens — Option price or sensitivities**

numeric

Option price or sensitivities, returned as a numeric. The name-value pair argument OutSpec determines the types and order of the outputs.

### **PriceGrid — Grid containing prices calculated by the finite difference method**

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with size `AssetGridSize` × `TimeGridSize`. The number of columns is not necessarily equal to the `TimeGridSize` because the function adds exercise and ex-dividend dates to the time grid. `PriceGrid(:, :, end)` contains the price for  $t = 0$ .

**AssetPrices — Prices of the asset**

vector

Prices of the asset corresponding to the first dimension of `PriceGrid`, returned as a vector.

**Variiances — Variiances**

vector

Variiances corresponding to the second dimension of `PriceGrid`, returned as a vector.

**Times — Times**

vector

Times corresponding to the third dimension of `PriceGrid`, returned as a vector.

## Definitions

### Bates Stochastic Volatility Jump Diffusion Model

The Bates model [1] extends the Heston model by including stochastic volatility and (similar to Merton) jump diffusion parameters in the modeling of sudden asset price movements.

The stochastic differential equation is

$$dS_t = (r - q - \lambda_p \mu_J) S_t dt + \sqrt{v_t} S_t dW_t + JS_t dP_t$$

$$dv_t = \kappa(\theta - v_t) dt + \sigma_v \sqrt{v_t} dW_t$$

$$E[dW_t dW_t^y] = \rho dt$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where:

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$ .

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp \left\{ - \frac{\left[ \ln(1+J) - \left( \ln(1 + \mu_J) - \frac{\delta^2}{2} \right) \right]^2}{2\delta^2} \right\}$$

where:

$v_0$  is the initial variance of the asset price at  $t = 0$  ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

## References

- [1] Bates, D. S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*. Vol. 9, Number 1, 1996.

## **See Also**

[optByBatesFD](#) | [optByHestonFD](#) | [optByLocalVolFD](#) | [optByMertonFD](#) | [optBySensMertonFD](#) | [optSensByHestonFD](#) | [optSensByLocalVolFD](#)

**Introduced in R2019a**



# optByBatesFFT

Option price by Bates model using FFT and FRFT

## Syntax

```
[Price,StrikeOut] = optByBatesFFT(Rate,AssetPrice,Settle,Maturity,  
OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq)  
[Price,StrikeOut] = optByBatesFFT( ____,Name,Value)
```

## Description

[Price,StrikeOut] = optByBatesFFT(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq) computes vanilla European option price by Bates model, using Carr-Madan FFT and Chourdakis FRFT methods.

[Price,StrikeOut] = optByBatesFFT( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Workflow for Plotting an Option Price Surface Using the Bates Model

Use `optByBatesFFT` to calibrate the FFT strike grid, compute option prices, and plot an option price surface.

### Define Option Variables and Bates Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
V0 = 0.04;
```

```
ThetaV = 0.05;  
Kappa = 1.0;  
SigmaV = 0.2;  
RhoSV = -0.7;  
MeanJ = 0.02;  
JumpVol = 0.08;  
JumpFreq = 2;
```

### Compute Option Prices for the Entire FFT (or FRFT) Strike Grid, Without Specifying Strike

Compute option prices and also output the corresponding strikes. If the `Strike` input is empty (`[]`), option prices will be computed on the entire FFT (or FRFT) strike grid. The FFT (or FRFT) strike grid is determined as `exp(log-strike grid)`, where each column of the log-strike grid has `NumFFT` points with `LogStrikeStep` spacing that are roughly centered around each element of `log(AssetPrice)`. The default value for `NumFFT` is  $2^{12}$ . In addition to the prices in the first output, the optional last output contains the corresponding strikes.

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = []; % Strike is not specified (will use the entire FFT strike grid)  
  
% Compute option prices for the entire FFT strike grid  
[Call, Kout] = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...  
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...  
    'DividendYield', DividendYield);  
  
% Show the lowest and highest strike values on the FFT strike grid  
format  
MinStrike = Kout(1) % Lowest possible strike in the current FFT strike grid  
  
MinStrike = 2.9205e-135  
  
MaxStrike = Kout(end) % Highest possible strike in the current FFT strike grid  
  
MaxStrike = 1.8798e+138  
  
% Show a subset of the strikes and corresponding option prices  
Range = (2046:2052);  
[Kout(Range) Call(Range)]  
  
ans = 7x2
```

```

50.4929    29.4990
58.8640    21.4545
68.6231    12.8544
80.0000     5.3484
93.2631     1.2404
108.7251    0.1648
126.7505    0.0152

```

### Change the Number of FFT (or FRFT) Points and Compare with optByBatesNI

Try a different number of FFT (or FRFT) points, and compare the results with direct numerical integration. Unlike `optByBatesFFT`, which uses FFT (or FRFT) techniques for fast computation across the whole range of strikes, the `optByBatesNI` function uses direct numerical integration and it is typically slower, especially for multiple strikes. However, the values computed by `optByBatesNI` can serve as a benchmark for adjusting the settings for `optByBatesFFT`.

```

% Try a smaller number of FFT (or FRFT) points
% (e.g. for faster performance or smaller memory footprint)
NumFFT = 2^10; % Smaller than the default value of 2^12
Strike = []; % Strike is not specified (will use the entire FFT strike grid)
[Call, Kout] = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT);

% Compare with numerical integration method
Range = (510:516);
Strike = Kout(Range);
CallFFT = Call(Range);
CallNI = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield);
Error = abs(CallFFT-CallNI);
table(Strike, CallFFT, CallNI, Error)

```

```
ans=7x4 table
```

Strike	CallFFT	CallNI	Error
12.696	66.237	66.696	0.45912
23.449	55.86	56.103	0.24239
43.312	36.418	36.541	0.12246
80	5.4029	5.3484	0.054469
147.76	0.044921	0.0010864	0.043835

```
272.93    0.0094655    -7.8249e-08    0.0094656
504.11    0.0024986    -3.3873e-07    0.0024989
```

### Make Further Adjustments to FFT (or FRFT)

If the values in the output CallFFT are significantly different from those in CallNI, try making adjustments to optByBatesFFT settings, such as CharacteristicFcnStep, LogStrikeStep, NumFFT, DampingFactor, and so on. Note that if (LogStrikeStep \* CharacteristicFcnStep) is  $2\pi / \text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

```
Strike = []; % Strike is not specified (will use the entire FFT or FRFT strike grid)
[Call, Kout] = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001);
```

```
% Compare with numerical integration method
```

```
Strike = Kout(Range);
CallFFT = Call(Range);
CallNI = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'LogStrikeStep', 0.001);
Error = abs(CallFFT-CallNI);
table(Strike, CallFFT, CallNI, Error)
```

```
ans=7x4 table
```

Strike	CallFFT	CallNI	Error
79.76	5.4682	5.4682	1.5355e-08
79.84	5.4281	5.4281	1.4833e-08
79.92	5.3882	5.3882	1.4244e-08
80	5.3484	5.3484	1.359e-08
80.08	5.3088	5.3088	1.2875e-08
80.16	5.2693	5.2693	1.2101e-08
80.24	5.23	5.23	1.1272e-08

```
% Save the final FFT (or FRFT) strike grid for future reference. For
% example, it provides information about the range of |Strike| inputs for
% which the FFT (or FRFT) operation is valid.
```

```
FFTStrikeGrid = Kout;
MinStrike = FFTStrikeGrid(1) % Strike cannot be less than MinStrike
```

```
MinStrike = 47.9437
```

```
MaxStrike = FFTStrikeGrid(end) % Strike cannot be greater than MaxStrike
```

```
MaxStrike = 133.3566
```

### Compute the Option Price for a Single Strike

Once the desired FFT (or FRFT) settings are determined, use the `Strike` input to specify the strikes (rather than providing an empty array). If the specified strikes do not match a value on the FFT (or FRFT) strike grid, the outputs are interpolated on the specified strikes.

```
Settle = datenum('29-Jun-2017');
```

```
Maturity = datemnth(Settle, 6);
```

```
Strike = 80;
```

```
Call = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001)
```

```
Call = 5.3484
```

### Compute the Option Prices for a Vector of Strikes

Use the `Strike` input to specify the strikes.

```
Settle = datenum('29-Jun-2017');
```

```
Maturity = datemnth(Settle, 6);
```

```
Strike = (76:2:84)';
```

```
Call = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001)
```

```
Call = 5×1
```

```
7.5765
```

```
6.4020
```

```
5.3484
```

```
4.4173
```

```
3.6073
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the `Strike` input to specify the strikes. Also, the `Maturity` input can be a vector, but it must match the length of the `Strike` vector if the `ExpandOutput` name-value pair argument is not set to `"true"`.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes

Call = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001) % Five values in vector output

Call = 5×1

    9.7516
   10.3931
   10.8865
   11.2990
   11.6491
```

### Expand the Outputs for a Surface

Set the `ExpandOutput` name-value pair argument to `"true"` to expand the outputs into `NStrikes-by-NMaturities` matrices. In this case, they are square matrices.

```
[Call, Kout] = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 5) matrix output

Call = 5×5

    9.7516    11.4387    12.8395    14.0588    15.1361
    8.6554    10.3931    11.8344    13.0890    14.1980
    7.6432     9.4149    10.8865    12.1693    13.3046
    6.7153     8.5035     9.9952    11.2990    12.4553
    5.8705     7.6581     9.1594    10.4771    11.6491

Kout = 5×5
```

```

76    76    76    76    76
78    78    78    78    78
80    80    80    80    80
82    82    82    82    82
84    84    84    84    84

```

### Compute Option Prices for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is "true", `NStrikes` do not have to match `NMaturities` (that is, the output `NStrikes-by-NMaturities` matrix can be rectangular).

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes

Call = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 6) matrix output

```

Call = 5×6

```

7.5765    9.7516   11.4387   12.8395   14.0588   15.1361
6.4020    8.6554   10.3931   11.8344   13.0890   14.1980
5.3484    7.6432    9.4149   10.8865   12.1693   13.3046
4.4173    6.7153    8.5035    9.9952   11.2990   12.4553
3.6073    5.8705    7.6581    9.1594   10.4771   11.6491

```

### Compute the Option Prices for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

Call = optByBatesFFT(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 4) matrix output

```

```
Call = 5×4
```

4.2033	6.6918	9.7516	13.2808
3.5558	5.8112	8.6554	11.9993
2.9906	5.0181	7.6432	10.7934
2.5018	4.3096	6.7153	9.6652
2.0825	3.6818	5.8705	8.6158

### Plot an Option Price Surface

Use the `Strike` input to specify the strikes. Increase the value for `NumFFT` to support a wider range of strikes. Also, the `Maturity` input can be a vector. Set `ExpandOutput` to "true" to output the surface as a `NStrikes-by-NMaturities` matrix.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

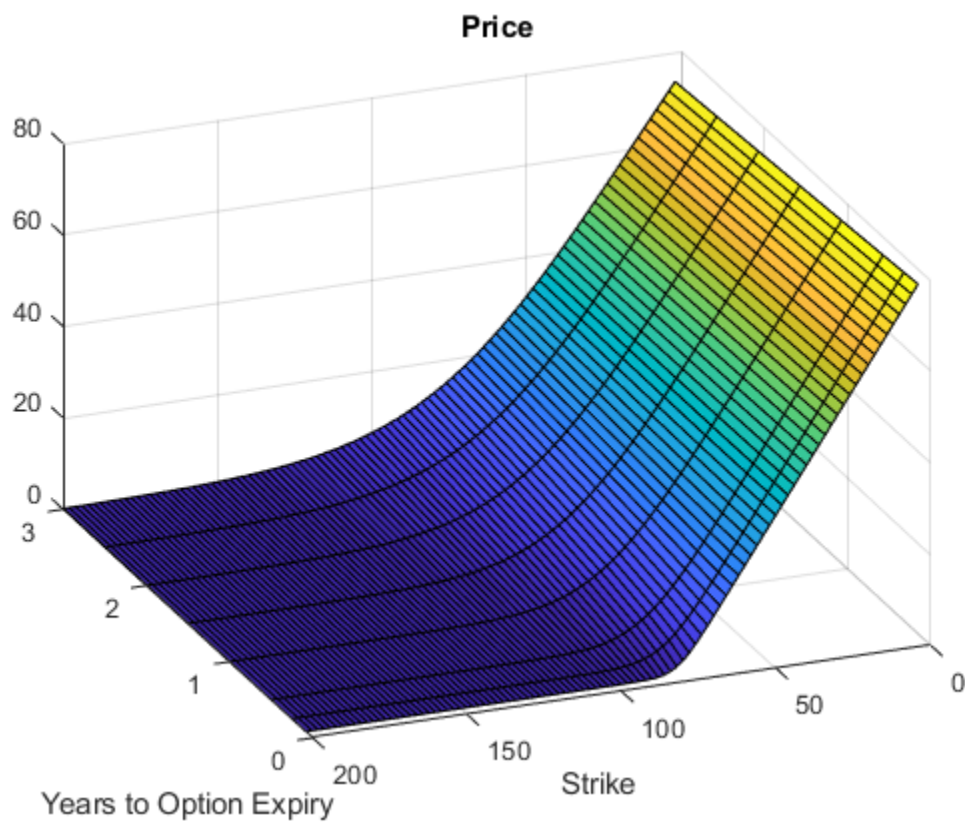
% Increase |NumFFT| to support a wider range of strikes
NumFFT = 2^13;

Call = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065,
    'LogStrikeStep', 0.001, 'ExpandOutput', true);

[X,Y] = meshgrid(Times,Strike);

figure;
surf(X,Y,Call);
title('Price');
xlabel('Years to Option Expiry');
ylabel('Strike');
view(-112,34);
xlim([0 Times(end)]);
zlim([0 80]);
```





## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

If this input is an empty array ( [] ), option prices are computed on the entire FFT (or FRFT) strike grid, which is determined as  $\exp(\log\text{-strike grid})$ . Each column of the log-strike grid has 'NumFFT' points with 'LogStrikeStep' spacing that are roughly centered around each element of  $\log(\text{AssetPrice})$ .

For more information on the proper dimensions for Strike, see the name-value pair argument ExpandOutput.

Data Types: double

### **V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

### **SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

**MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

**JumpVol — Standard deviation of  $\log(1+J)$** 

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

**JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, StrikeOut] = optByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

### **VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double

**LittleTrap — Flag indicating Little Heston Trap formulation**

true (default) | logical with values true or false

Flag indicating Little Heston Trap formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- true — Use the Albrecher *et al* formulation.
- false — Use the original Heston formation.

Data Types: logical

**NumFFT — Number of grid points in the characteristic function variable**

4096 (default) | numeric

Number of grid points in the characteristic function variable and in each column of the log-strike grid, specified as the comma-separated pair consisting of 'NumFFT' and a scalar numeric value.

Data Types: double

**CharacteristicFcnStep — Characteristic function variable grid spacing**

0.01 (default) | numeric

Characteristic function variable grid spacing, specified as the comma-separated pair consisting of 'CharacteristicFcnStep' and a scalar numeric value.

Data Types: double

**LogStrikeStep — Log-strike grid spacing**

$2\pi/\text{NumFFT}/\text{CharacteristicFcnStep}$  (default) | numeric

Log-strike grid spacing, specified as the comma-separated pair consisting of 'LogStrikeStep' and a scalar numeric value.

---

**Note** If  $(\text{LogStrikeStep} \times \text{CharacteristicFcnStep})$  is  $2\pi/\text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

---

Data Types: double

**DampingFactor — Damping factor for Carr-Madan formulation**

1.5 (default) | numeric

Damping factor for Carr-Madan formulation, specified as the comma-separated pair consisting of 'DampingFactor' and a scalar numeric value.

Data Types: double

### **Quadrature — Type of quadrature**

"simpson" (default) | character vector with values: 'simpson' or 'trapezoidal' | string array with values: "simpson" or "trapezoidal"

Type of quadrature, specified as the comma-separated pair consisting of 'Quadrature' and a single character vector or string array with a value of 'simpson' or 'trapezoidal'.

Data Types: char | string

### **ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- **true** — If true, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. If Strike is empty, NRows is equal to NumFFT. NColumns is determined by the sizes of AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.
- **false** — If false, the outputs are NINST-by-1 vectors. Also, the inputs Strike, AssetPrice, Settle, Maturity, and OptSpec must all be either scalar or NINST-by-1 vectors.

Data Types: logical

## **Output Arguments**

### **Price — Option prices**

numeric

Option prices, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput.

## StrikeOut — Strikes corresponding to Price

numeric

Strikes corresponding to Price, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput.

## Definitions

### Bates Stochastic Volatility Jump Diffusion Model

The Bates model (Bates (1996)) is an extension of the Heston model, where, in addition to stochastic volatility, the jump diffusion parameters similar to Merton (1976) were also added to model sudden asset price movements.

The stochastic differential equation is:

$$dS_t = (r - q - \lambda_p \mu_J) S_t dt + \sqrt{v_t} S_t dW_t + JS_t dP_t$$

$$dv_t = \kappa(\theta - v_t) dt + \sigma_v \sqrt{v_t} dW_t$$

$$E[dW_t^x dW_t^y] = \rho dt$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$ .

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:



$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp\left\{-\frac{\left[\ln(1+J) - (\ln(1+\mu_J) - \frac{\delta^2}{2})\right]^2}{2\delta^2}\right\}$$

$v_0$  is the initial variance of the asset price at  $t = 0$  ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

The characteristic function  $f_{Bates_j(\phi)}$  for  $j = 1$  (asset price mean measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Bates(\phi)} = \exp(C_j + D_j v_0 + i\phi \ln S_t) \exp(\lambda_p \tau (1 + \mu_j)^{m_j + \frac{1}{2}} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 (m_j i\phi + \frac{(i\phi)^2}{2})} - 1 \right] - \lambda_p \tau \mu_j i\phi)$$

$$m_j = \begin{cases} m_1 = \frac{1}{2} \\ m_2 = -\frac{1}{2} \end{cases}$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi + d_j)\tau - 2\ln\left(\frac{1 - g_j e^{d_j \tau}}{1 - g_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j \tau}}{1 - g_j e^{d_j \tau}} \right)$$

$$g_j = \frac{b_j - p\sigma_v i\phi + d_j}{b_j - p\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - p\sigma_v i\phi)^2 - \sigma_v^2 (2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - p\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity for ( $\tau = T - t$ ).

$i$  is the unit imaginary number for ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under “The Little Heston Trap” by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln\left(\frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Carr-Madan Formulation

The Carr and Madan (1999) formulation is a popular modified implementation of Heston (1993) framework.

Rather than computing the probabilities  $P_1$  and  $P_2$  as intermediate steps, Carr and Madan developed an alternative expression so that taking its inverse Fourier transform gives the option price itself directly.

$$Call(k) = \frac{e^{-\alpha k}}{\pi} \int_0^{\infty} \text{Re}\left[e^{-iuk} \psi(u)\right] du$$

$$\psi(u) = \frac{e^{-r\tau} f_2(\phi = (u - (\alpha + 1)i))}{\alpha^2 + \alpha - u^2 + iu(2\alpha + 1)}$$

$$Put(K) = Call(K) + Ke^{-r\tau} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$

$i$  is a unit imaginary number ( $i^2 = -1$ )

$\phi$  is the characteristic function variable.

$\alpha$  is the damping factor.

$u$  is the characteristic function variable for integration, where  $\phi = (u - (\alpha+1)i)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

To apply FFT or FRFT to this formulation, the characteristic function variable for integration,  $u$ , is discretized into NumFFT( $N$ ) points with the step size CharacteristicFcnStep ( $\Delta u$ ), and the log-strike  $k$  is discretized into  $N$  points with the step size LogStrikeStep( $\Delta k$ ).

The discretized characteristic function variable for integration,  $u_j$ (for  $j = 1, 2, 3, \dots, N$ ), has a minimum value of 0 and a maximum value of  $(N-1) (\Delta u)$ , and it approximates the continuous integration range from 0 to infinity.

The discretized log-strike grid,  $k_n$ (for  $n = 1, 2, 3, N$ ) is approximately centered around  $\ln(S_t)$ , with a minimum value of

$$\ln(S_t) - \frac{N}{2} \Delta k$$

and a maximum value of

$$\ln(S_t) + \left(\frac{N}{2} - 1\right) \Delta k$$

Where the minimum allowable strike is

$$S_t \exp\left(-\frac{N}{2} \Delta k\right)$$

and the maximum allowable strike is

$$S_t \exp\left[\left(\frac{N}{2} - 1\right) \Delta k\right]$$

As a result of the discretization, the expression for the call option becomes

$$Call(k_n) = \Delta u \frac{e^{-\alpha k_n}}{\pi} \sum_{j=1}^N \operatorname{Re} \left[ e^{-i \Delta k \Delta u (j-1)(n-1)} e^{iu_j \left[ \frac{N \Delta k}{2} - \ln(S_t) \right]} \psi(u_j) \right] w_j$$

where

$\Delta u$  is the step size of discretized characteristic function variable for integration.

$\Delta k$  is the step size of discretized log-strike.

$N$  is the number of FFT/FRFT points

$w_j$  is the weights for quadrature used for approximating the integral.

FFT is used to evaluate the above expression if  $\Delta k$  and  $\Delta u$  are subject to the following constraint:

$$\Delta k \Delta u = \left( \frac{2\pi}{N} \right)$$

otherwise, the functions use the FRFT method described in Chourdakis (2005).

## References

- [1] Albrecher, H., Mayer, P., Schoutens, W., and Tistaert, J. "The Little Heston Trap." Working Paper, Linz and Graz University of Technology, K.U. Leuven, ING Financial Markets, 2006.
- [2] Bates, D. S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [3] Carr, P. and D.B. Madan. "Option Valuation Using the Fast Fourier Transform." *Journal of Computational Finance*. Vol 2. No. 4. 1999.
- [4] Chourdakis, K. "Option Pricing Using Fractional FFT." *Journal of Computational Finance*. 2005.
- [5] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6. No. 2. 1993.

## See Also

optByBatesNI | optByHestonFFT | optByHestonNI | optByMertonFFT |  
 optByMertonNI | optSensByBatesFFT | optSensByBatesNI | optSensByHestonFFT  
 | optSensByHestonNI | optSensByMertonFFT | optSensByMertonNI

**Introduced in R2018a**

# optSensByBatesFFT

Option price and sensitivities by Bates model using FFT and FRFT

## Syntax

```
[PriceSens,StrikeOut] = optSensByBatesFFT(Rate,AssetPrice,Settle,
Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,
JumpFreq)
[PriceSens,StrikeOut] = optSensByBatesFFT( ____,Name,Value)
```

## Description

[PriceSens,StrikeOut] = optSensByBatesFFT(Rate,AssetPrice,Settle, Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol, JumpFreq) computes vanilla European option price and sensitivities by Bates model, using Carr-Madan FFT and Chourdakis FRFT methods.

[PriceSens,StrikeOut] = optSensByBatesFFT( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Workflow for Plotting an Option Sensitivity Surface Using the Bates Model

Use optSensByBatesFFT to calibrate the FFT strike grid for sensitivities, compute option sensitivities, and plot option sensitivity surfaces.

#### Define Option Variables and Bates Model Parameters

```
AssetPrice = 80;
Rate = 0.03;
DividendYield = 0.02;
OptSpec = 'call';
```

```
V0 = 0.04;  
ThetaV = 0.05;  
Kappa = 1.0;  
SigmaV = 0.2;  
RhoSV = -0.7;  
MeanJ = 0.02;  
JumpVol = 0.08;  
JumpFreq = 2;
```

### Compute the Option Sensitivities for the Entire FFT (or FRFT) Strike Grid, Without Specifying "Strike"

Compute option sensitivities and also output the corresponding strikes. If the Strike input is empty ( [] ), option sensitivities will be computed on the entire FFT (or FRFT) strike grid. The FFT (or FRFT) strike grid is determined as  $\exp(\log\text{-strike grid})$ , where each column of the log-strike grid has NumFFT points with LogStrikeStep spacing that are roughly centered around each element of  $\log(\text{AssetPrice})$ . The default value for NumFFT is  $2^{12}$ . In addition to the sensitivities in the first output, the optional last output contains the corresponding strikes.

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = []; % Strike is not specified (will use the entire FFT strike grid)  
  
% Compute option sensitivities for the entire FFT strike grid  
[Delta, Kout] = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,  
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...  
    'DividendYield', DividendYield, 'OutSpec', "delta");  
  
% Show the lowest and highest strike values on the FFT strike grid  
format  
MinStrike = Kout(1) % Lowest possible strike in the current FFT strike grid  
MinStrike = 2.9205e-135  
  
MaxStrike = Kout(end) % Highest possible strike in the current FFT strike grid  
MaxStrike = 1.8798e+138  
  
% Show a subset of the strikes and corresponding option sensitivities  
Range = (2046:2052);  
[Kout(Range) Delta(Range)]  
  
ans = 7x2
```



```

50.4929    0.9846
58.8640    0.9585
68.6231    0.8498
80.0000    0.5630
93.2631    0.1955
108.7251   0.0319
126.7505   0.0033

```

### Change the Number of FFT (or FRFT) Points and Compare with optSensByBatesNI

Try a different number of FFT (or FRFT) points, and compare the results with numerical integration. Unlike `optSensByBatesFFT`, which uses FFT (or FRFT) techniques for fast computation across the whole range of strikes, the `optSensByBatesNI` function uses direct numerical integration and it is typically slower, especially for multiple strikes. However, the values computed by `optSensByBatesNI` can serve as a benchmark for adjusting the settings for `optSensByBatesFFT`.

```

% Try a smaller number of FFT points
% (e.g. for faster performance or smaller memory footprint)
NumFFT = 2^10; % Smaller than the default value of 2^12
Strike = []; % Strike is not specified (will use the entire FFT strike grid)
[Delta, Kout] = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT);

% Compare with numerical integration method
Range = (510:516);
Strike = Kout(Range);
DeltaFFT = Delta(Range);
DeltaNI = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ..
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta");
Error = abs(DeltaFFT-DeltaNI);
table(Strike, DeltaFFT, DeltaNI, Error)

```

ans=7×4 table

Strike	DeltaFFT	DeltaNI	Error
12.696	0.9265	0.99002	0.063524
23.449	0.95153	0.99002	0.038497
43.312	0.95928	0.98928	0.029994

80	0.5355	0.56303	0.027531
147.76	0.0016267	0.00025691	0.0013698
272.93	0.00058267	1.8942e-09	0.00058267
504.11	0.00017752	8.7099e-10	0.00017752

### Make Further Adjustments to FFT (or FRFT)

If the values in the output DeltaFFT are significantly different from those in DeltaNI, try making adjustments to optSensByBatesFFT settings, such as CharacteristicFcnStep, LogStrikeStep, NumFFT, DampingFactor, and so on. Note that if (LogStrikeStep \* CharacteristicFcnStep) is  $2\pi / \text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

```
Strike = []; % Strike is not specified (will use the entire FFT or FRFT strike grid)
[Delta, Kout] = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001);
```

% Compare with numerical integration method

```
Strike = Kout(Range);
DeltaFFT = Delta(Range);
DeltaNI = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ..
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta");
Error = abs(DeltaFFT-DeltaNI);
table(Strike, DeltaFFT, DeltaNI, Error)
```

ans=7x4 table

Strike	DeltaFFT	DeltaNI	Error
79.76	0.57037	0.57037	6.3042e-09
79.84	0.56793	0.56793	7.156e-09
79.92	0.56548	0.56548	7.975e-09
80	0.56303	0.56303	8.7573e-09
80.08	0.56057	0.56057	9.4992e-09
80.16	0.55811	0.55811	1.0197e-08
80.24	0.55564	0.55564	1.0847e-08

% Save the final FFT (or FRFT) strike grid for future reference. For  
 % example, it provides information about the range of Strike inputs for

```

% which the FFT (or FRFT) operation is valid.
FFTStrikeGrid = Kout;
MinStrike = FFTStrikeGrid(1) % Strike cannot be less than MinStrike

MinStrike = 47.9437

MaxStrike = FFTStrikeGrid(end) % Strike cannot be greater than MaxStrike

MaxStrike = 133.3566

```

### Compute the Option Sensitivity for a Single Strike

Once the desired FFT (or FRFT) settings are determined, use the `Strike` input to specify the strikes rather than providing an empty array. If the specified strikes do not match a value on the FFT (or FRFT) strike grid, the outputs are interpolated on the specified strikes.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;

Delta = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)

Delta = 0.5630

```

### Compute the Option Sensitivities for a Vector of Strikes

Use the `Strike` input to specify the strikes.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';

Delta = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)

Delta = 5x1

    0.6807
    0.6234
    0.5630

```

```
0.5011
0.4392
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the `Strike` input to specify the strikes. Also, the `Maturity` input can be a vector, but it must match the length of the `Strike` vector if the `ExpandOutput` name-value pair argument is not set to `"true"`.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes
```

```
Delta = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001) % Five values in vector output
```

```
Delta = 5x1
```

```
0.6625
0.6232
0.5958
0.5748
0.5577
```

### Expand the Outputs for a Surface

Set the `ExpandOutput` name-value pair argument to `"true"` to expand the outputs into `NStrikes-by-NMaturities` matrices. In this case, they are square matrices.

```
[Delta, Kout] = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 5) matrix output
```

```
Delta = 5x5
```

```
0.6625    0.6556    0.6515    0.6483    0.6455
0.6222    0.6232    0.6239    0.6241    0.6238
0.5805    0.5900    0.5958    0.5996    0.6019
```

```

0.5381    0.5564    0.5674    0.5748    0.5798
0.4954    0.5225    0.5389    0.5499    0.5577

```

```
Kout = 5x5
```

```

76    76    76    76    76
78    78    78    78    78
80    80    80    80    80
82    82    82    82    82
84    84    84    84    84

```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of Different Lengths

When ExpandOutput is "true", NStrikes do not have to match NMaturities. That is, the output NStrikes-by-NMaturities matrix can be rectangular.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes

```

```

Delta = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 6) matrix output

```

```
Delta = 5x6
```

```

0.6807    0.6625    0.6556    0.6515    0.6483    0.6455
0.6234    0.6222    0.6232    0.6239    0.6241    0.6238
0.5630    0.5805    0.5900    0.5958    0.5996    0.6019
0.5011    0.5381    0.5564    0.5674    0.5748    0.5798
0.4392    0.4954    0.5225    0.5389    0.5499    0.5577

```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Asset Prices

When ExpandOutput is "true", the output can also be a NStrikes-by-NAssetPrices rectangular matrix by accepting a vector of asset prices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity

```

```

ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

Delta = optSensByBatesFFT(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, ...
    Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 4) matrix output

Delta = 5x4

    0.4350    0.5579    0.6625    0.7457
    0.3881    0.5124    0.6222    0.7120
    0.3432    0.4670    0.5805    0.6763
    0.3010    0.4223    0.5381    0.6390
    0.2619    0.3789    0.4954    0.6002

```

### Plot Option Sensitivity Surfaces

Use the `Strike` input to specify the strikes. Increase the value for `NumFFT` to support a wider range of strikes. Also, the `Maturity` input can be a vector. Set `ExpandOutput` to "true" to output the surfaces as `NStrikes-by-NMaturities` matrices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

% Increase 'NumFFT' to support a wider range of strikes
NumFFT = 2^13;

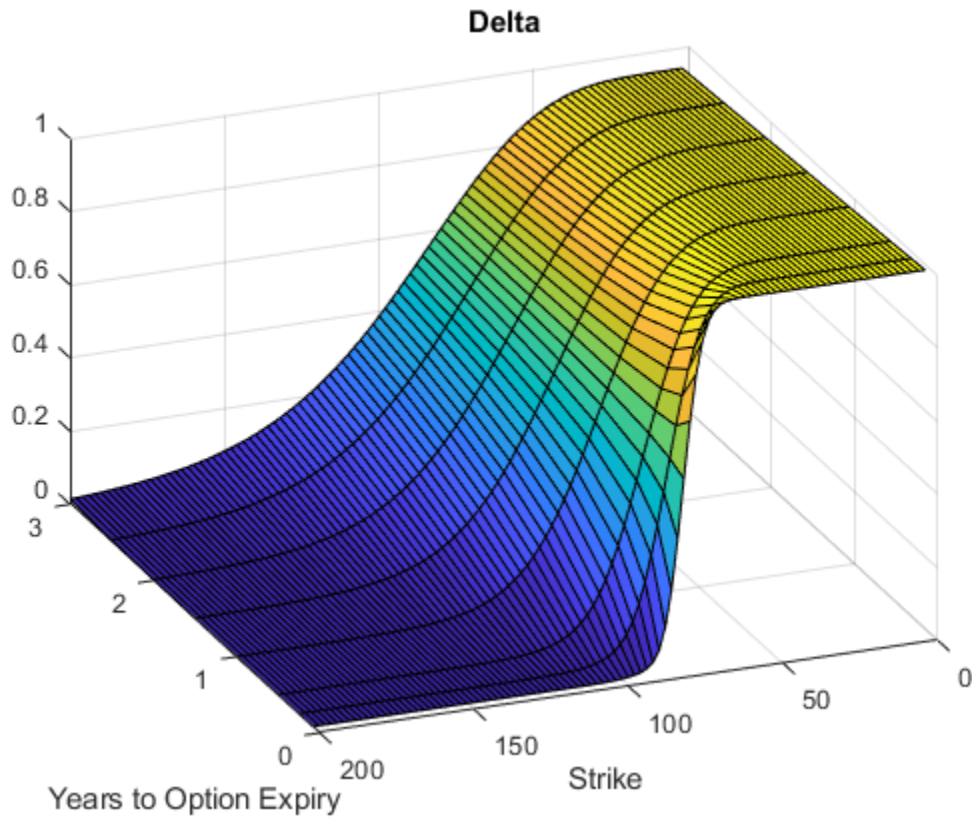
[Delta, Gamma, Rho, Theta, Vega, VegaLT] = optSensByBatesFFT(...
    Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'OutSpec', ["delta", "gamma", "rho", "theta", "vega", "vegalt"], ...
    'ExpandOutput', true);

[X,Y] = meshgrid(Times,Strike);

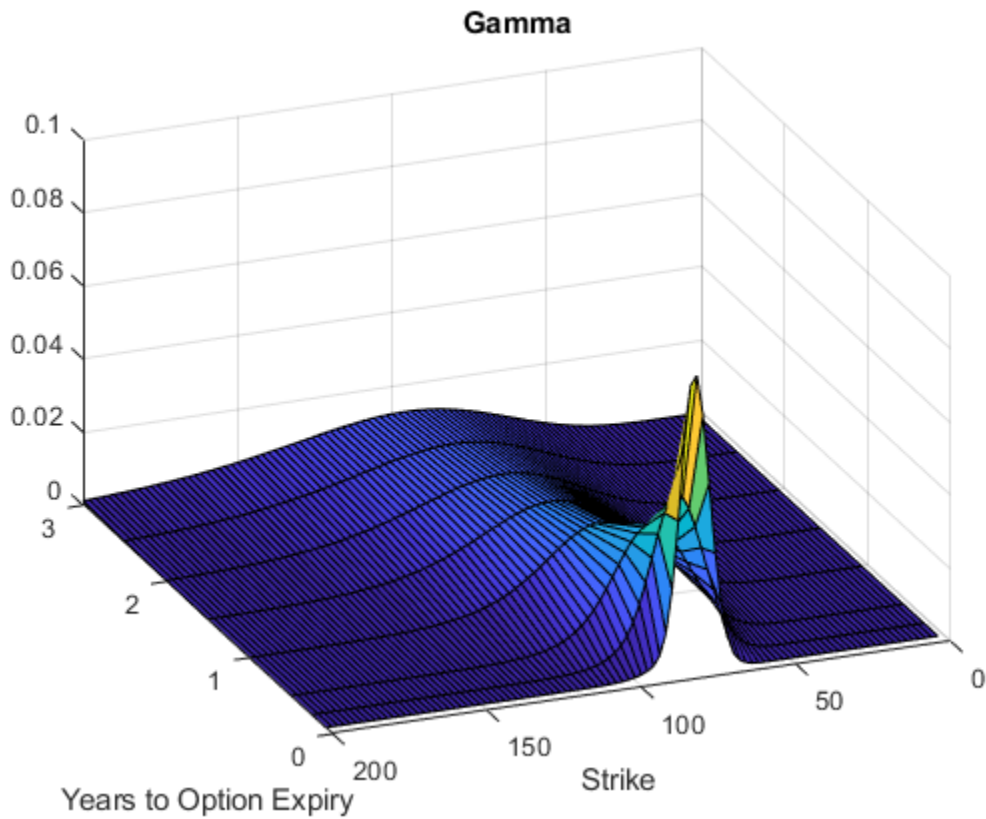
figure;
surf(X,Y,Delta);
title('Delta');

```

```
xlabel('Years to Option Expiry');  
ylabel('Strike');  
view(-112,34);  
xlim([0 Times(end)]);
```

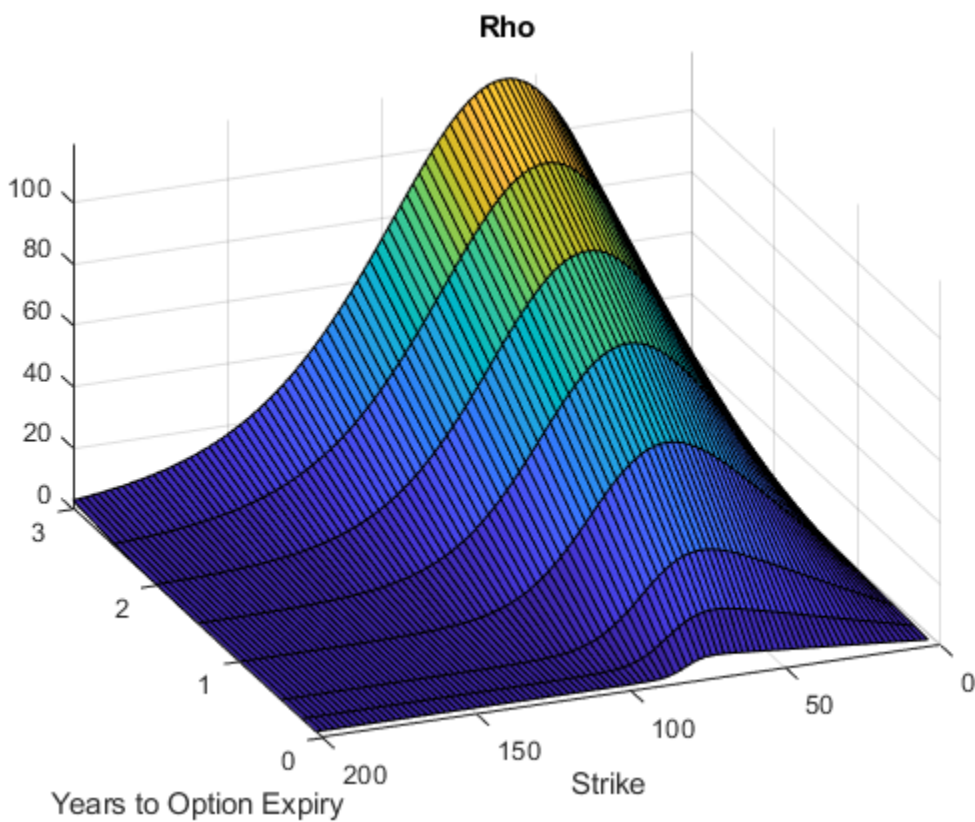


```
figure;  
surf(X,Y,Gamma)  
title('Gamma')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```

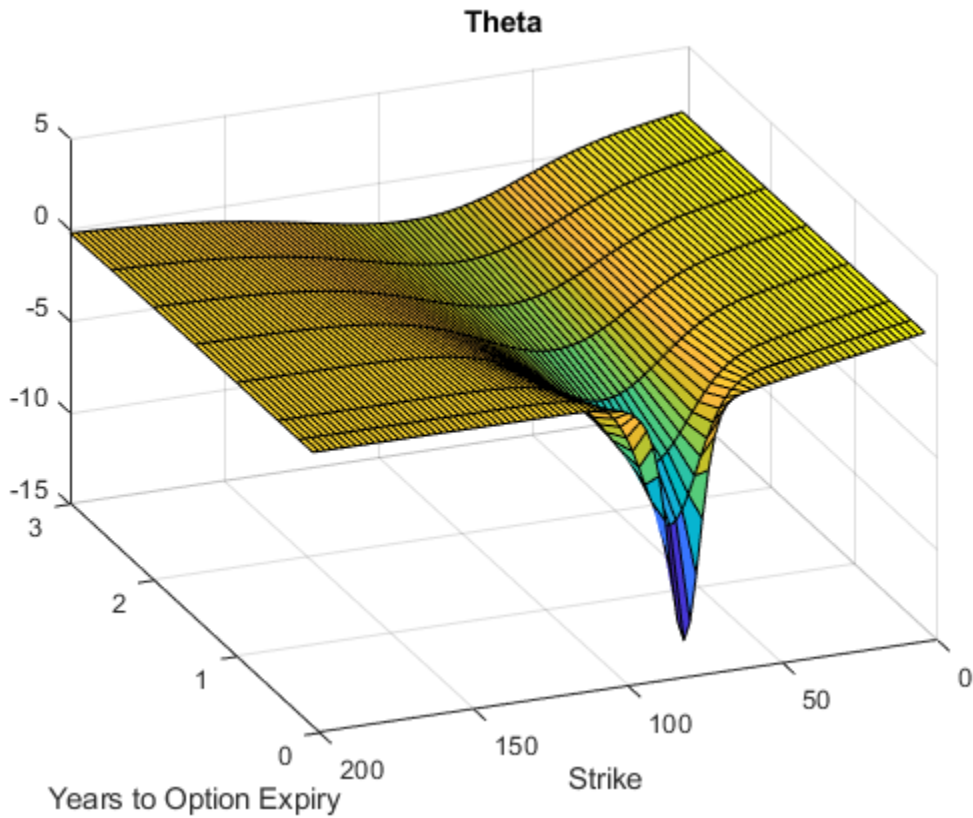


```
figure;
surf(X,Y,Rho)
title('Rho')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```

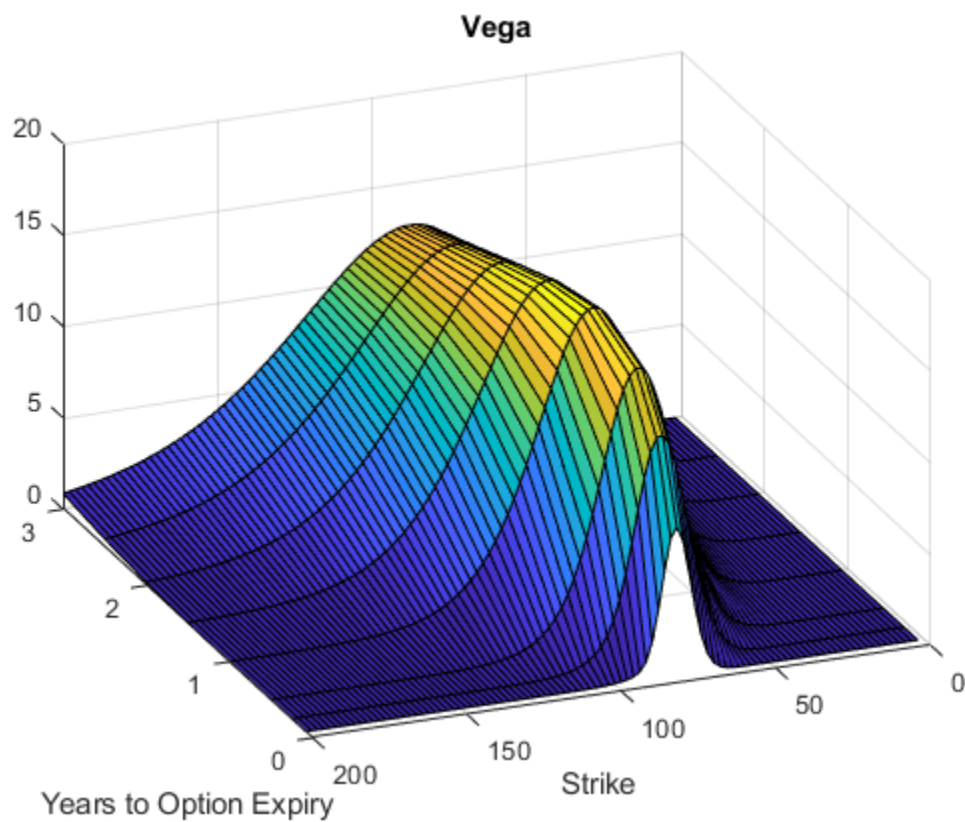




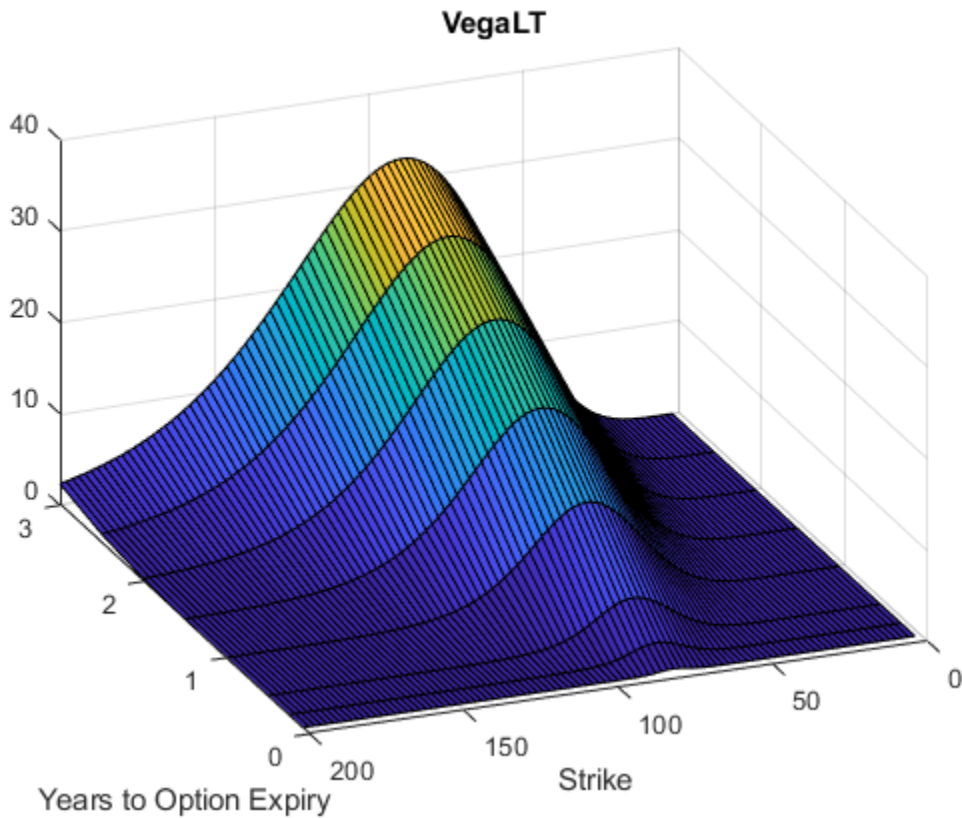
```
figure;  
surf(X,Y,Theta)  
title('Theta')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



```
figure;
surf(X,Y,Vega)
title('Vega')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



```
figure;  
surf(X,Y,VegaLT)  
title('VegaLT')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



## Input Arguments

**Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

**AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

If this input is an empty array (`[]`), option prices are computed on the entire FFT (or FRFT) strike grid, which is determined as `exp(log-strike grid)`. Each column of the log-strike grid has 'NumFFT' points with 'LogStrikeStep' spacing that are roughly centered around each element of `log(AssetPrice)`.

For more information on the proper dimensions for `Strike`, see the name-value pair argument `ExpandOutput`.

Data Types: double

### **V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

### **SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

### **MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

### **JumpVol — Standard deviation of $\log(1+J)$**

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

### **JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[PriceSens, StrikeOut] = optSensByBatesFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

### **Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

### **VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double



**LittleTrap — Flag indicating Little Heston Trap formulation**

true (default) | logical with values true or false

Flag indicating Little Heston Trap formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- true — Use the Albrecher *et al* formulation.
- false — Use the original Heston formation.

Data Types: logical

**OutSpec — Define outputs**

["price"] (default) | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegalt" | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegalt'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

---

**Note** "vega" is the sensitivity with respect to the initial volatility  $\sqrt{V_0}$ . In contrast, "vegalt" is the sensitivity with respect to the long-term volatility  $\sqrt{\text{Theta}V}$ .

---

Example: OutSpec =

```
["price","delta","gamma","vega","rho","theta","vegalt"]
```

Data Types: string | cell

**NumFFT — Number of grid points in the characteristic function variable**

4096 (default) | numeric

Number of grid points in the characteristic function variable and in each column of the log-strike grid, specified as the comma-separated pair consisting of 'NumFFT' and a scalar numeric value.

Data Types: double

**CharacteristicFcnStep — Characteristic function variable grid spacing**

0.01 (default) | numeric

Characteristic function variable grid spacing, specified as the comma-separated pair consisting of 'CharacteristicFcnStep' and a scalar numeric value.

Data Types: double

**LogStrikeStep — Log-strike grid spacing**

$2\pi/\text{NumFFT}/\text{CharacteristicFcnStep}$  (default) | numeric

Log-strike grid spacing, specified as the comma-separated pair consisting of 'LogStrikeStep' and a scalar numeric value.

---

**Note** If  $(\text{LogStrikeStep} \times \text{CharacteristicFcnStep})$  is  $2\pi/\text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

---

Data Types: double

**DampingFactor — Damping factor for Carr-Madan formulation**

1.5 (default) | numeric

Damping factor for Carr-Madan formulation, specified as the comma-separated pair consisting of 'DampingFactor' and a scalar numeric value.

Data Types: double

**Quadrature — Type of quadrature**

"simpson" (default) | character vector with values: 'simpson' or 'trapezoidal' | string array with values: "simpson" or "trapezoidal"

Type of quadrature, specified as the comma-separated pair consisting of 'Quadrature' and a single character vector or string array with a value of 'simpson' or 'trapezoidal'.

Data Types: char | string

**ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- **true** — If true, the outputs are NRows-by-NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. If Strike is empty, NRows is equal to NumFFT. NColumns is determined by the sizes of

AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.

- `false` — If `false`, the outputs are NINST-by-1 vectors. Also, the inputs `Strike`, `AssetPrice`, `Settle`, `Maturity`, and `OptSpec` must all be either scalar or NINST-by-1 vectors.

Data Types: `logical`

## Output Arguments

### PriceSens — Option prices or sensitivities

`numeric`

Option prices or sensitivities, returned as a NINST-by-1, or NRows-by-NColumns, depending on `ExpandOutput`. The name-value pair argument `OutSpec` determines the types and order of the outputs.

### StrikeOut — Strikes corresponding to Price

`numeric`

Strikes corresponding to `Price`, returned as a NINST-by-1, or NRows-by-NColumns, depending on `ExpandOutput`.

## Definitions

### Bates Stochastic Volatility Jump Diffusion Model

The Bates model (Bates (1996)) is an extension of the Heston model, where, in addition to stochastic volatility, the jump diffusion parameters similar to Merton (1976) were also added to model sudden asset price movements.

The stochastic differential equation is:

$$dS_t = (r - q - \lambda_p \mu_J) S_t dt + \sqrt{v_t} S_t dW_t + JS_t dP_t$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t$$

$$E[dW_t^x dW_t^y] = \rho dt$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$ .

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1 + J)\delta\sqrt{2\pi}} \exp\left\{-\frac{\left[\ln(1 + J) - \left(\ln(1 + \mu_J) - \frac{\delta^2}{2}\right)\right]^2}{2\delta^2}\right\}$$

$v_0$  is the initial variance of the asset price at  $t = 0$  ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of variance for ( $\sigma_v > 0$ ).

$p$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq p \leq 1$ ).

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

The characteristic function  $f_{Bates_j(\phi)}$  for  $j = 1$  (asset price mean measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Bates(\phi)} = \exp(C_j + D_j v_0 + i\phi \ln S_t) \exp(\lambda_p \tau (1 + \mu_j)^{m_j + \frac{1}{2}} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 (m_j i\phi + \frac{(i\phi)^2}{2})} - 1 \right] - \lambda_p \tau \mu_j i\phi)$$

$$m_j = \begin{cases} m_1 = \frac{1}{2} \\ m_2 = -\frac{1}{2} \end{cases}$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi + d_j)\tau - 2\ln\left(\frac{1 - g_j e^{d_j\tau}}{1 - g_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right)$$

$$g_j = \frac{b_j - p\sigma_v i\phi + d_j}{b_j - p\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - p\sigma_v i\phi)^2 - \sigma_v^2 (2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - p\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity for ( $\tau = T - t$ ).

$i$  is the unit imaginary number for ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under “The Little Heston Trap” by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln\left(\frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Carr-Madan Formulation

The Carr and Madan (1999) formulation is a popular modified implementation of Heston (1993) framework.

Rather than computing the probabilities  $P_1$  and  $P_2$  as intermediate steps, Carr and Madan developed an alternative expression so that taking its inverse Fourier transform gives the option price itself directly.

$$Call(k) = \frac{e^{-\alpha k}}{\pi} \int_0^{\infty} \text{Re}\left[e^{-iuk} \psi(u)\right] du$$

$$\psi(u) = \frac{e^{-r\tau} f_2(\phi = (u - (\alpha + 1)i))}{\alpha^2 + \alpha - u^2 + iu(2\alpha + 1)}$$

$$Put(K) = Call(K) + Ke^{-r\tau} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$\alpha$  is the damping factor.

$u$  is the characteristic function variable for integration, where  $\phi = (u - (\alpha+1)i)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

To apply FFT or FRFT to this formulation, the characteristic function variable for integration,  $u$ , is discretized into NumFFT( $N$ ) points with the step size CharacteristicFcnStep ( $\Delta u$ ), and the log-strike  $k$  is discretized into  $N$  points with the step size LogStrikeStep( $\Delta k$ ).

The discretized characteristic function variable for integration,  $u_j$  (for  $j = 1, 2, 3, \dots, N$ ), has a minimum value of 0 and a maximum value of  $(N-1) (\Delta u)$ , and it approximates the continuous integration range from 0 to infinity.

The discretized log-strike grid,  $k_n$  (for  $n = 1, 2, 3, N$ ) is approximately centered around  $\ln(S_t)$ , with a minimum value of

$$\ln(S_t) - \frac{N}{2} \Delta k$$

and a maximum value of

$$\ln(S_t) + \left(\frac{N}{2} - 1\right) \Delta k$$

Where the minimum allowable strike is

$$S_t \exp\left(-\frac{N}{2} \Delta k\right)$$

and the maximum allowable strike is

$$S_t \exp\left[\left(\frac{N}{2} - 1\right) \Delta k\right]$$

As a result of the discretization, the expression for the call option becomes

$$Call(k_n) = \Delta u \frac{e^{-\alpha k_n}}{\pi} \sum_{j=1}^N \operatorname{Re} \left[ e^{-i \Delta k \Delta u (j-1)(n-1)} e^{iu_j \left[ \frac{N \Delta k}{2} - \ln(S_t) \right]} \psi(u_j) \right] w_j$$

where

$\Delta u$  is the step size of discretized characteristic function variable for integration.

$\Delta k$  is the step size of discretized log-strike.

$N$  is the number of FFT/FRFT points

$w_j$  is the weights for quadrature used for approximating the integral.

FFT is used to evaluate the above expression if  $\Delta k$  and  $\Delta u$  are subject to the following constraint:

$$\Delta k \Delta u = \left( \frac{2\pi}{N} \right)$$

otherwise, the functions use the FRFT method described in Chourdakis (2005).

## References

- [1] Albrecher, H., Mayer, P., Schoutens, W., and Tistaert, J. "The Little Heston Trap." Working Paper, Linz and Graz University of Technology, K.U. Leuven, ING Financial Markets, 2006.
- [2] Bates, D. S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [3] Carr, P. and D.B. Madan. "Option Valuation Using the Fast Fourier Transform." *Journal of Computational Finance*. Vol 2. No. 4. 1999.
- [4] Chourdakis, K. "Option Pricing Using Fractional FFT." *Journal of Computational Finance*. 2005.
- [5] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6. No. 2. 1993.

## See Also

[optByBatesFFT](#) | [optByBatesNI](#) | [optByHestonFFT](#) | [optByHestonNI](#) | [optByMertonFFT](#) | [optByMertonNI](#) | [optSensByBatesNI](#) | [optSensByHestonFFT](#) | [optSensByHestonNI](#) | [optSensByMertonFFT](#) | [optSensByMertonNI](#)



**Introduced in R2018a**

## optByBatesNI

Option price by Bates model using numerical integration

### Syntax

```
Price = optByBatesNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,  
V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq)  
Price = optByBatesNI( ____,Name,Value)
```

### Description

Price = optByBatesNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike, V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq) computes vanilla European option price by Bates model, using numerical integration methods.

Price = optByBatesNI( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Workflow for Plotting an Option Price Surface Using the Bates Model

optByBatesNI uses numerical integration to compute option prices and then plot an option price surface.

#### Define Option Variables and Bates Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
V0 = 0.04;  
ThetaV = 0.05;
```

```
Kappa = 1.0;
SigmaV = 0.2;
RhoSV = -0.7;
MeanJ = 0.02;
JumpVol = 0.08;
JumpFreq = 2;
```

### Compute the Option Price for a Single Strike

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;
```

```
Call = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield)
```

```
Call = 5.3484
```

### Compute the Option Prices for a Vector of Strikes

The Strike input can be a vector.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';
```

```
Call = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield)
```

```
Call = 5×1
```

```
7.5765
6.4020
5.3484
4.4173
3.6073
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the Strike input to specify the strikes. Also, the Maturity input can be a vector, but it must match the length of the Strike vector if the ExpandOutput name-value pair argument is not set to "true".

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes

Call = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield) % Five values in vector output

Call = 5×1

    9.7516
   10.3931
   10.8865
   11.2990
   11.6491
```

### Expand the Output for a Surface

Set the `ExpandOutput` name-value pair argument to "true" to expand the output into a `NStrikes-by-NMaturities` matrix. In this case, it is a square matrix.

```
Call = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'ExpandOutput', true) % (5 x 5) matrix output

Call = 5×5

    9.7516    11.4387    12.8395    14.0588    15.1361
    8.6554    10.3931    11.8344    13.0890    14.1980
    7.6432     9.4149    10.8865    12.1693    13.3046
    6.7153     8.5035     9.9951    11.2990    12.4553
    5.8705     7.6581     9.1594    10.4771    11.6491
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is "true", `NStrikes` do not have to match `NMaturities` (that is, the output `NStrikes-by-NMaturities` matrix can be rectangular).

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes
```

```
Call = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'ExpandOutput', true) % (5 x 6) matrix output
```

```
Call = 5x6
```

```
    7.5765    9.7516   11.4387   12.8395   14.0588   15.1361
    6.4020    8.6554   10.3931   11.8344   13.0890   14.1980
    5.3484    7.6432    9.4149   10.8865   12.1693   13.3046
    4.4173    6.7153    8.5035    9.9951   11.2990   12.4553
    3.6073    5.8705    7.6581    9.1594   10.4771   11.6491
```

### Compute Option Prices for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes
```

```
Call = optByBatesNI(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'ExpandOutput', true) % (5 x 4) matrix output
```

```
Call = 5x4
```

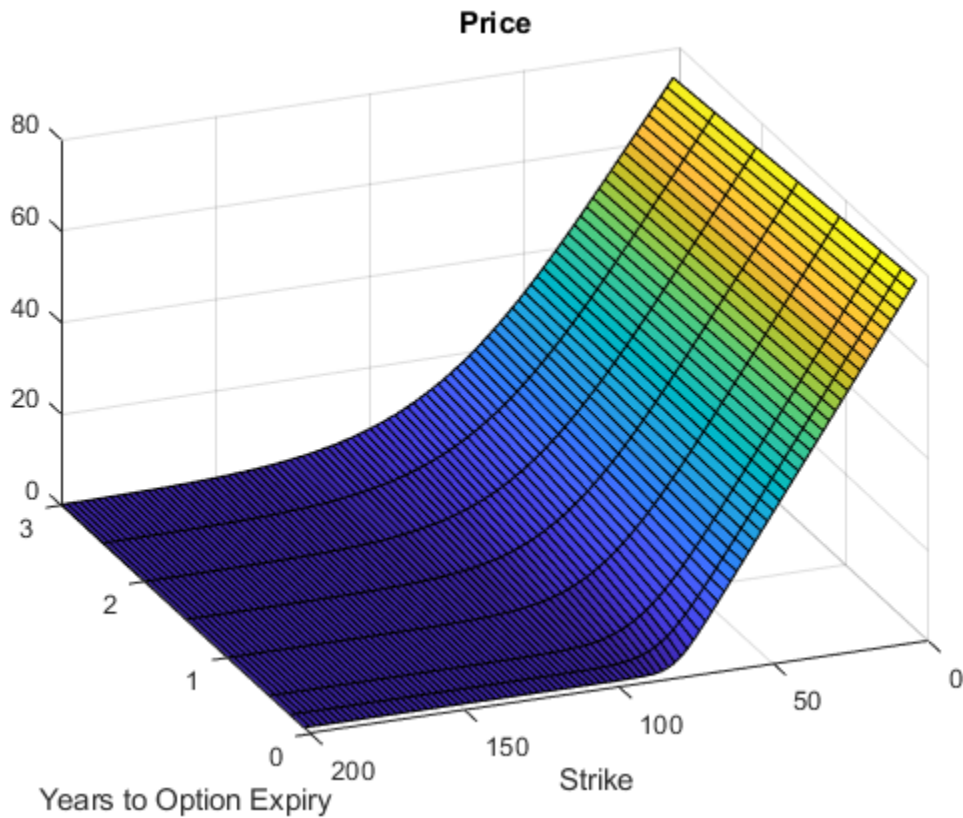
```
    4.2033    6.6918    9.7516   13.2808
    3.5558    5.8111    8.6554   11.9993
    2.9906    5.0181    7.6432   10.7934
    2.5018    4.3096    6.7153    9.6651
    2.0825    3.6818    5.8705    8.6158
```

### Plot an Option Price Surface

The `Strike` and `Maturity` inputs can be vectors. Set `ExpandOutput` to "true" to output the surface as a `NStrikes-by-NMaturities` matrix.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';
```

```
Call = optByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...  
    ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...  
    'DividendYield', DividendYield, 'ExpandOutput', true);  
  
[X,Y] = meshgrid(Times,Strike);  
  
figure;  
surf(X,Y,Call);  
title('Price');  
xlabel('Years to Option Expiry');  
ylabel('Strike');  
view(-112,34);  
xlim([0 Times(end)]);  
zlim([0 80]);
```



## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric



Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

For more information on the proper dimensions for `Strike`, see the name-value pair argument `ExpandOutput`.

Data Types: double

### **V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

### **SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

**MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

**JumpVol — Standard deviation of  $\log(1+J)$** 

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

**JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
optByBatesNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,V0,Theta  
V,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq,'Basis',7)
```

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0,1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual

- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

### **VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double

### **LittleTrap — Flag indicating "Little Heston Trap" formulation**

true (default) | logical with values true or false

Flag indicating "Little Heston Trap" formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- `true` — Use the Albrecher *et al* formulation.
- `false` — Use the original Heston formation.

Data Types: `logical`

**AbsTol — Absolute error tolerance for numerical integration**

`1e-10` (default) | `numeric`

Absolute error tolerance for numerical integration, specified as the comma-separated pair consisting of 'AbsTol' and a scalar numeric value.

Data Types: `double`

**RelTol — Relative error tolerance for numerical integration**

`1e-6` (default) | `numeric`

Relative error tolerance for numerical integration, specified as the comma-separated pair consisting of 'RelTol' and a scalar numeric value.

Data Types: `double`

**IntegrationRange — Numerical integration range used to approximate continuous integral over [0 Inf]**

`[1e-9 Inf]` (default) | `vector`

Numerical integration range used to approximate the continuous integral over `[0 Inf]`, specified as the comma-separated pair consisting of 'IntegrationRange' and a 1-by-2 vector representing `[LowerLimit UpperLimit]`.

Data Types: `double`

**Framework — Framework for computing option prices and sensitivities using numerical integration of models**

`"heston1993"` (default) | string with values `"heston1993"` or `"lewis2001"` | character vector with values `'heston1993'` or `'lewis2001'`

Framework for computing option prices and sensitivities using numerical integration of models, specified as the comma-separated pair consisting of 'Framework' and a scalar string or character vector with the following values:

- `"heston1993"` or `'heston1993'` — Method used in Heston (1993)
- `"lewis2001"` or `'lewis2001'` — Method used in Lewis (2001)

Data Types: `char` | `string`

**ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- **true** — If true, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. NColumns is determined by the sizes of AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.
- **false** — If false, the outputs are NINST-by-1 vectors. Also, the inputs Strike, AssetPrice, Settle, Maturity, and OptSpec must all be either scalar or NINST-by-1 vectors.

Data Types: logical

## Output Arguments

**Price — Option prices**

numeric

Option prices, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput.

## Definitions

**Bates Stochastic Volatility Jump Diffusion Model**

The Bates model (Bates (1996)) is an extension of the Heston model, where, in addition to stochastic volatility, the jump diffusion parameters similar to Merton (1976) were also added to model sudden asset price movements.

The stochastic differential equation is:

$$\begin{aligned}
 dS_t &= (r - q - \lambda_p \mu_J) S_t dt + \sqrt{v_t} S_t dW_t + JS_t dP_t \\
 dv_t &= \kappa(\theta - v_t) dt + \sigma_v \sqrt{v_t} dW_t \\
 E[dW_t dW_t^v] &= \rho dt \\
 \text{prob}(dP_t = 1) &= \lambda_p dt
 \end{aligned}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$ .

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp \left\{ - \frac{\left[ \ln(1+J) - \left( \ln(1 + \mu_J) - \frac{\delta^2}{2} \right) \right]^2}{2\delta^2} \right\}$$

$v_0$  is the initial variance of the asset price at  $t = 0$  ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

The characteristic function  $f_{Bates_j(\phi)}$  for  $j = 1$  (asset price mean measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Bates(\phi)} = \exp(C_j + D_j v_0 + i\phi \ln S_t) \exp(\lambda_p \tau (1 + \mu_j)^{m_j + \frac{1}{2}} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 (m_j i\phi + \frac{(i\phi)^2}{2})} - 1 \right] - \lambda_p \tau \mu_j i\phi)$$

$$m_j = \begin{cases} m_1 = \frac{1}{2} \\ m_2 = -\frac{1}{2} \end{cases}$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi + d_j)\tau - 2\ln\left(\frac{1 - g_j e^{d_j\tau}}{1 - g_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right)$$

$$g_j = \frac{b_j - p\sigma_v i\phi + d_j}{b_j - p\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - p\sigma_v i\phi)^2 - \sigma_v^2 (2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - p\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity for ( $\tau = T - t$ ).

$i$  is the unit imaginary number for ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under "The Little Heston Trap" by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln\left(\frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Numerical Integration Method Under Heston (1993) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Heston (1993) framework is based on the following expressions:

$$Call(K) = S_t e^{-q\tau} P_1 - K e^{-r\tau} P_2$$

$$Put(K) = Call(K) + K e^{-r\tau} - S_t e^{-q\tau}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \text{Re} \left[ \frac{e^{-i\phi \ln(K)} f_j(\phi)}{i\phi} \right] d\phi$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .



$i$  is a unit imaginary number ( $i^2 = -1$ )

$\phi$  is the characteristic function variable.

$f_j(\phi)$  is the characteristic function for  $P_j$  ( $j = 1, 2$ ).

$P_1$  is the probability of  $S_t > K$  under the asset price measure for the model.

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

Where  $j = 1, 2$  so that  $f_1(\phi)$  and  $f_2(\phi)$  are the characteristic functions for probabilities  $P_1$  and  $P_2$ , respectively.

This framework is chosen with the default value "Heston1993" for the Framework name-value pair argument.

## Numerical Integration Method Under Lewis (2001) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Lewis (2001) framework is based on the following expressions:

$$Call(k) = S_t e^{-qt} - \frac{\sqrt{K} e^{-\tau t}}{\pi} \int_0^{\infty} \text{Re} \left[ K^{-iu} f_2 \left( \phi = \left( u - \frac{i}{2} \right) \frac{1}{u^2 + \frac{1}{4}} \right) \right] du$$

$$Put(K) = Call(K) = K e^{-\tau t} - S_t e^{-qt}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T - t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$

$i$  is a unit imaginary number ( $i^2 = -1$ )

$\phi$  is the characteristic function variable.

$u$  is the characteristic function variable for integration, where  $\phi = \left(u - \frac{i}{2}\right)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

This framework is chosen with the value “Lewis2001” for the Framework name-value pair argument.

## References

- [1] Bates, D. S. “Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options.” *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [2] Heston, S. L. “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options.” *The Review of Financial Studies*. Vol 6. No. 2. 1993.
- [3] Lewis, A. L. “A Simple Option Formula for General Jump-Diffusion and Other Exponential Levy Processes.” Envision Financial Systems and OptionCity.net, 2001.

## See Also

optByBatesFFT | optByHestonFFT | optByHestonNI | optByMertonFFT |  
optByMertonNI | optSensByBatesFFT | optSensByBatesNI | optSensByHestonFFT  
| optSensByHestonNI | optSensByMertonFFT | optSensByMertonNI

**Introduced in R2018a**

# optSensByBatesNI

Option price or sensitivities by Bates model using numerical integration

## Syntax

```
PriceSens = optSensByBatesNI(Rate,AssetPrice,Settle,Maturity,
OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq)
PriceSens = optSensByBatesNI( ____,Name,Value)
```

## Description

`PriceSens = optSensByBatesNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,MeanJ,JumpVol,JumpFreq)` computes vanilla European option price and sensitivities by Bates model, using numerical integration methods.

`PriceSens = optSensByBatesNI( ____,Name,Value)` adds optional name-value pair arguments.

## Examples

### Workflow for Plotting an Option Sensitivity Surface Using the Bates Model

`optSensByBatesNI` uses numerical integration to compute option sensitivities and then to plot option sensitivity surfaces.

#### Define Option Variables and Bates Model Parameters

```
AssetPrice = 80;
Rate = 0.03;
DividendYield = 0.02;
OptSpec = 'call';

V0 = 0.04;
```

```
ThetaV = 0.05;  
Kappa = 1.0;  
SigmaV = 0.2;  
RhoSV = -0.7;  
MeanJ = 0.02;  
JumpVol = 0.08;  
JumpFreq = 2;
```

### Compute the Option Sensitivity for a Single Strike

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = 80;
```

```
Delta = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...  
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...  
    'DividendYield', DividendYield, 'OutSpec', "delta")
```

```
Delta = 0.5630
```

### Compute the Option Sensitivities for a Vector of Strikes

The Strike input can be a vector.

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = (76:2:84)';
```

```
Delta = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...  
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...  
    'DividendYield', DividendYield, 'OutSpec', "delta")
```

```
Delta = 5×1
```

```
0.6807  
0.6234  
0.5630  
0.5011  
0.4392
```

## Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the `Strike` input to specify the strikes. Also, the `Maturity` input can be a vector, but it must match the length of the `Strike` vector if the `ExpandOutput` name-value pair argument is not set to `"true"`.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes

Delta = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta") % Five values in vector output

Delta = 5x1

    0.6625
    0.6232
    0.5958
    0.5748
    0.5577
```

## Expand the Output for a Surface

Set the `ExpandOutput` name-value pair argument to `"true"` to expand the output into a `NStrikes-by-NMaturities` matrix. In this case, it is a square matrix.

```
Delta = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", ...
    'ExpandOutput', true) % (5 x 5) matrix output

Delta = 5x5

    0.6625    0.6556    0.6515    0.6483    0.6455
    0.6222    0.6232    0.6239    0.6241    0.6238
    0.5805    0.5900    0.5958    0.5996    0.6019
    0.5381    0.5564    0.5674    0.5748    0.5798
    0.4954    0.5225    0.5389    0.5499    0.5577
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is "true", `NStrikes` do not have to match `NMaturities`. That is, the output `NStrikes`-by-`NMaturities` matrix can be rectangular.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes

Delta = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", ...
    'ExpandOutput', true) % (5 x 6) matrix output
```

Delta = 5×6

0.6807	0.6625	0.6556	0.6515	0.6483	0.6455
0.6234	0.6222	0.6232	0.6239	0.6241	0.6238
0.5630	0.5805	0.5900	0.5958	0.5996	0.6019
0.5011	0.5381	0.5564	0.5674	0.5748	0.5798
0.4392	0.4954	0.5225	0.5389	0.5499	0.5577

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes`-by-`NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

Delta = optSensByBatesNI(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, ...
    'DividendYield', DividendYield, 'OutSpec', "delta", ...
    'ExpandOutput', true) % (5 x 4) matrix output
```

Delta = 5×4

0.4350	0.5579	0.6625	0.7457
0.3881	0.5124	0.6222	0.7120
0.3432	0.4670	0.5805	0.6763

```

0.3010    0.4223    0.5381    0.6390
0.2619    0.3789    0.4954    0.6002

```

## Plot Option Sensitivity Surfaces

The Strike and Maturity inputs can be vectors. Set ExpandOutput to "true" to output the surfaces as NStrikes-by-NMaturities matrices.

```

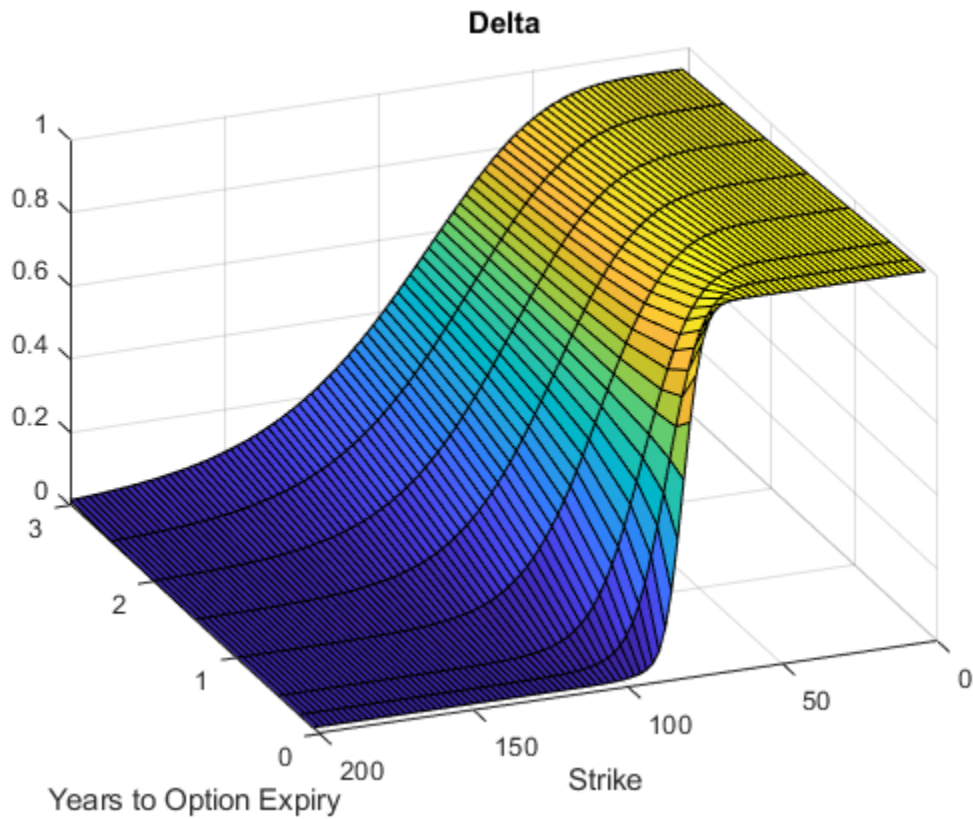
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

[Delta, Gamma, Rho, Theta, Vega, VegaLT] = optSensByBatesNI(...
    Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ThetaV, Kappa, ...
    SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', ["delta", "gamma", "rho", "theta", "vega", "vegalt"], ...
    'ExpandOutput', true);

[X,Y] = meshgrid(Times,Strike);

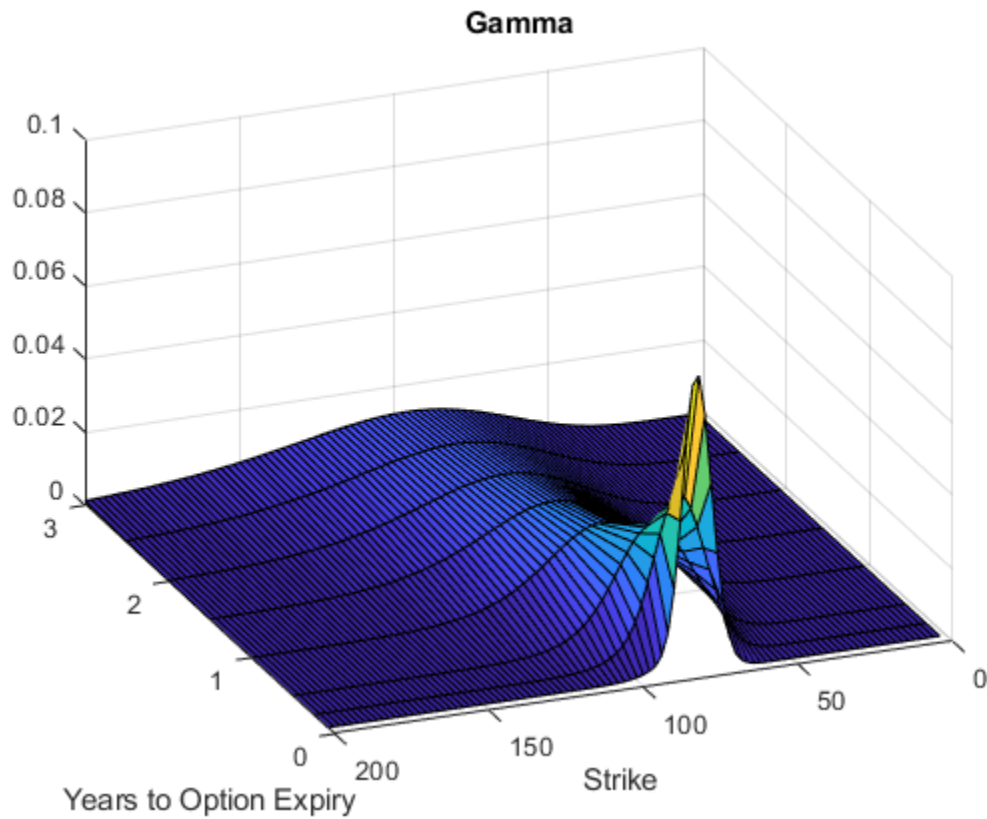
figure;
surf(X,Y,Delta);
title('Delta');
xlabel('Years to Option Expiry');
ylabel('Strike');
view(-112,34);
xlim([0 Times(end)]);

```

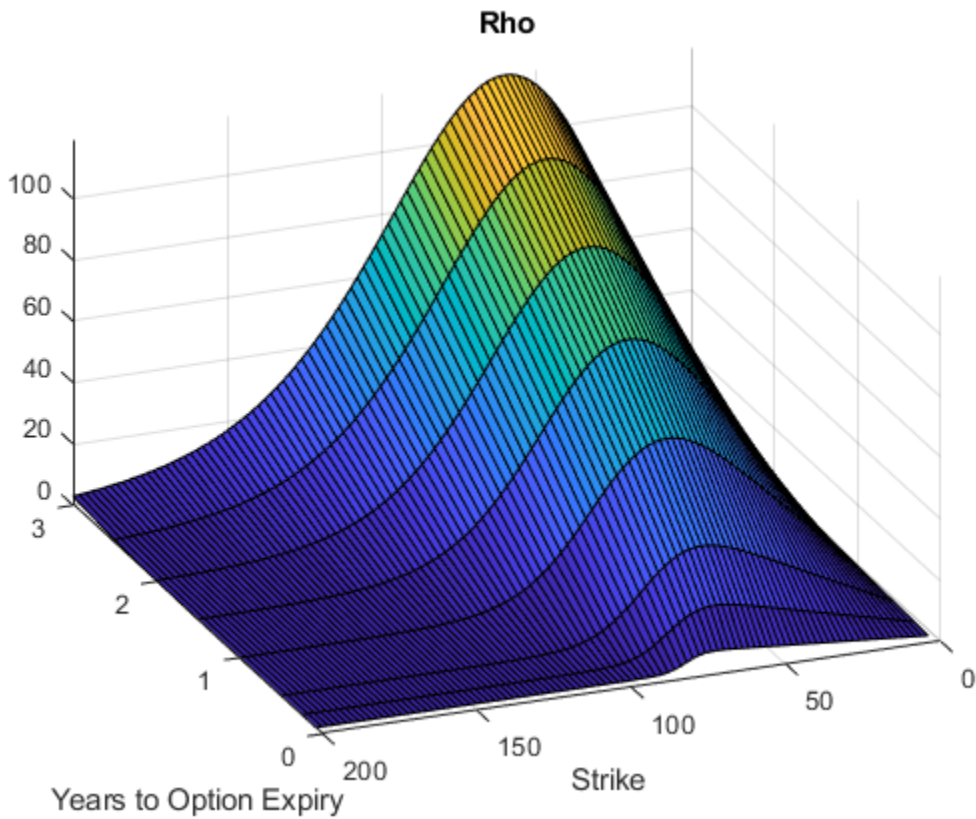


```
figure;
surf(X,Y,Gamma)
title('Gamma')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```

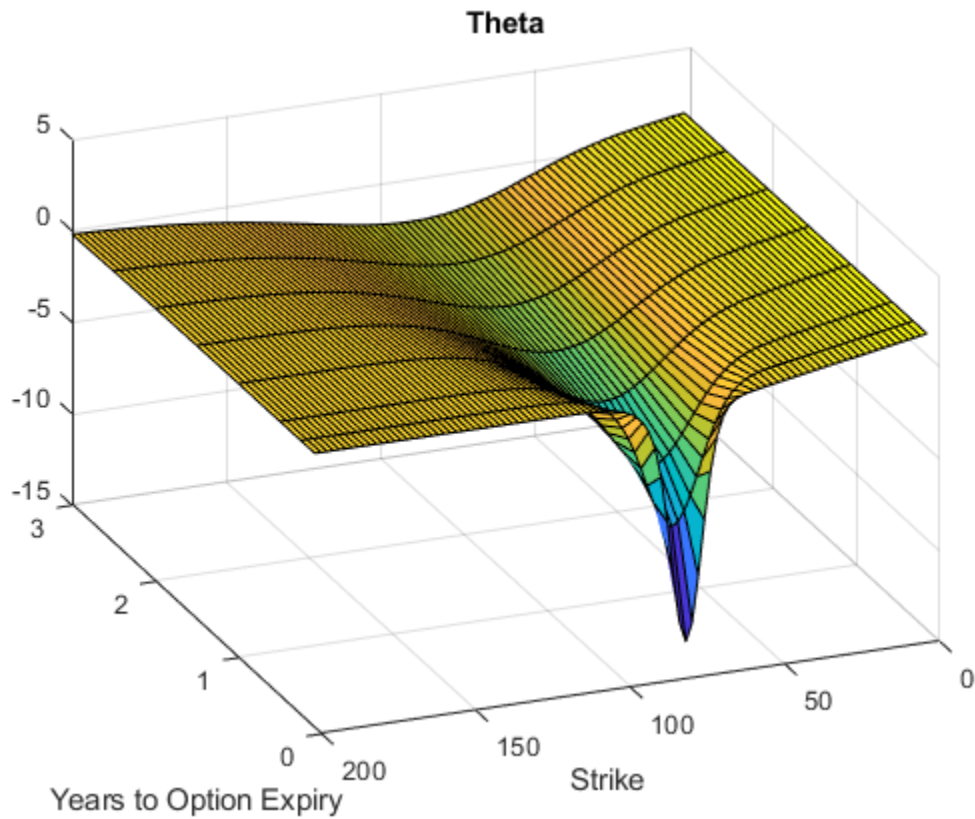




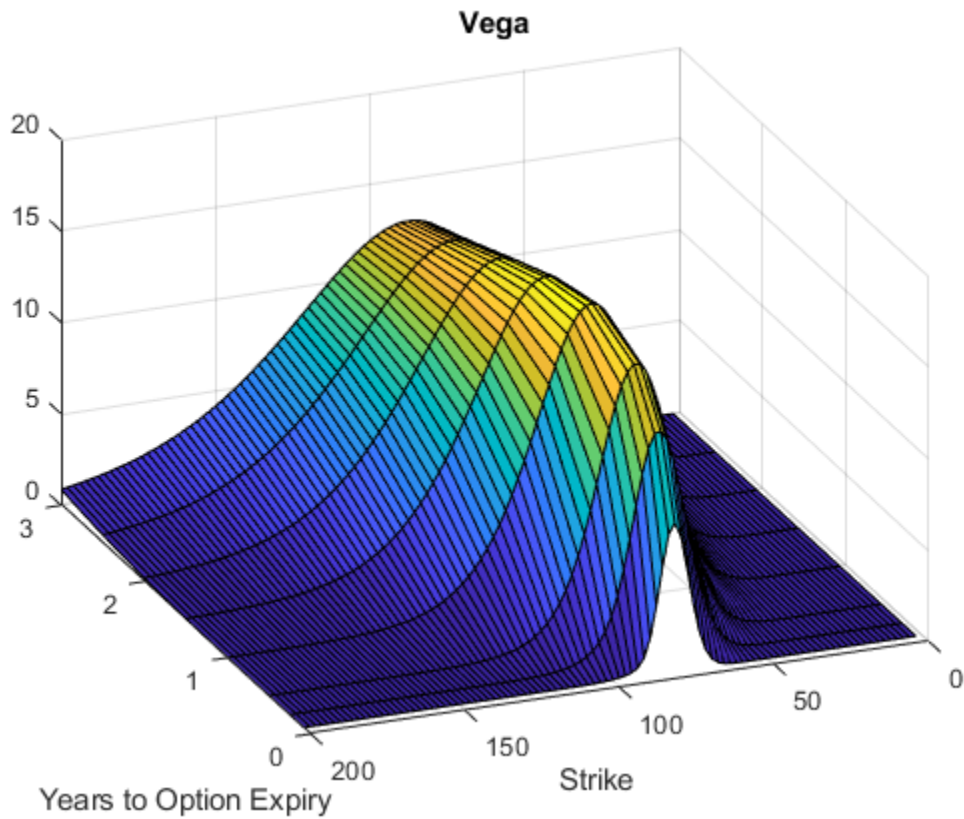
```
figure;  
surf(X,Y,Rho)  
title('Rho')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



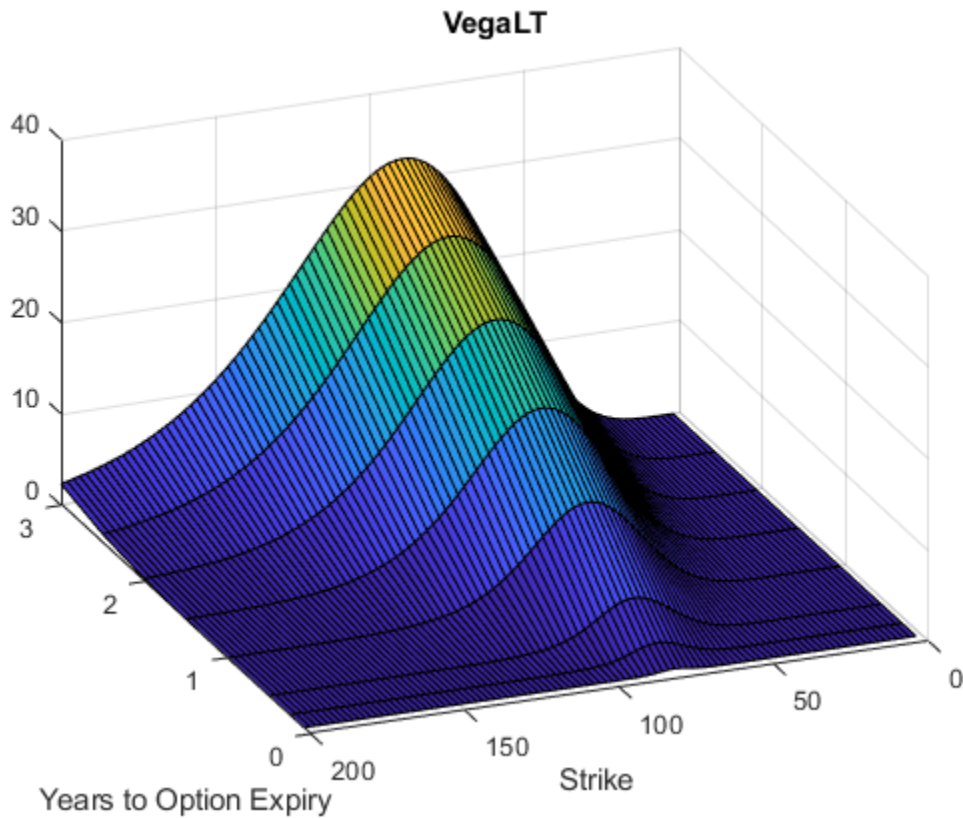
```
figure;
surf(X,Y,Theta)
title('Theta')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



```
figure;  
surf(X,Y,Vega)  
title('Vega')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



```
figure;
surf(X,Y,VegaLT)
title('VegaLT')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

For more information on the proper dimensions for `Strike`, see the name-value pair argument `ExpandOutput`.

Data Types: double

### **V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

### **SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

**MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

**JumpVol — Standard deviation of  $\log(1+J)$** 

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

**JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `PriceSens = optSensByBatesNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual



- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

### **VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double

### **LittleTrap — Flag indicating Little Heston Trap formulation**

true (default) | logical with values true or false

Flag indicating Little Heston Trap formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- `true` — Use the Albrecher *et al* formulation.
- `false` — Use the original Heston formation.

Data Types: `logical`

### **OutSpec — Define outputs**

`["price"]` (default) | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegalt" | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegalt'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

---

**Note** "vega" is the sensitivity with respect the initial volatility  $\sqrt{V_0}$ . In contrast, "vegalt" is the sensitivity with respect to the long-term volatility  $\sqrt{\text{Theta}V}$ .

---

Example: `OutSpec =`  
`["price","delta","gamma","vega","rho","theta","vegalt"]`

Data Types: `string` | `cell`

### **AbsTol — Absolute error tolerance for numerical integration**

`1e-10` (default) | `numeric`

Absolute error tolerance for numerical integration, specified as the comma-separated pair consisting of 'AbsTol' and a scalar numeric value.

Data Types: `double`

### **RelTol — Relative error tolerance for numerical integration**

`1e-6` (default) | `numeric`

Relative error tolerance for numerical integration, specified as the comma-separated pair consisting of 'RelTol' and a scalar numeric value.

Data Types: `double`

### **IntegrationRange — Numerical integration range used to approximate continuous integral over [0 Inf]**

`[1e-9 Inf]` (default) | `vector`

Numerical integration range used to approximate the continuous integral over  $[0 \text{ Inf}]$ , specified as the comma-separated pair consisting of 'IntegrationRange' and a 1-by-2 vector representing [LowerLimit UpperLimit].

Data Types: double

### **Framework — Framework for computing option prices and sensitivities using numerical integration of models**

"heston1993" (default) | string with values "heston1993" or "lewis2001" | character vector with values 'heston1993' or 'lewis2001'

Framework for computing option prices and sensitivities using numerical integration of models, specified as the comma-separated pair consisting of 'Framework' and a scalar string or character vector with the following values:

- "heston1993" or 'heston1993' — Method used in Heston (1993)
- "lewis2001" or 'lewis2001' — Method used in Lewis (2001)

Data Types: char | string

### **ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- true — If true, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. NColumns is determined by the sizes of AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.
- false — If false, the outputs are NINST-by-1 vectors. Also, the inputs Strike, AssetPrice, Settle, Maturity, and OptSpec must all be either scalar or NINST-by-1 vectors.

Data Types: logical

## **Output Arguments**

### **PriceSens — Option prices**

numeric

Option prices or sensitivities, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput. The name-value pair argument OutSpec determines the types and order of the outputs.

## Definitions

### Bates Stochastic Volatility Jump Diffusion Model

The Bates model (Bates (1996)) is an extension of the Heston model, where, in addition to stochastic volatility, the jump diffusion parameters similar to Merton (1976) were also added to model sudden asset price movements.

The stochastic differential equation is:

$$dS_t = (r - q - \lambda_p \mu_J) S_t dt + \sqrt{v_t} S_t dW_t + JS_t dP_t$$

$$dv_t = \kappa(\theta - v_t) dt + \sigma_v \sqrt{v_t} dW_t$$

$$E[dW_t dW_t^Y] = \rho dt$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$ .

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp \left\{ - \frac{\left[ \ln(1+J) - \left( \ln(1 + \mu_J) - \frac{\delta^2}{2} \right) \right]^2}{2\delta^2} \right\}$$

$v_0$  is the initial variance of the asset price at  $t = 0$  ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

$\mu_j$  is the mean of  $J$  for ( $\mu_j > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

The characteristic function  $f_{Bates_j(\phi)}$  for  $j = 1$  (asset price mean measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Bates(\phi)} = \exp(C_j + D_j v_0 + i\phi \ln S_t) \exp(\lambda_p \tau (1 + \mu_j)^{m_j + \frac{1}{2}} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 (m_j i\phi + \frac{(i\phi)^2}{2})} - 1 \right] - \lambda_p \tau \mu_j i\phi)$$

$$m_j = \begin{cases} m_1 = \frac{1}{2} \\ m_2 = -\frac{1}{2} \end{cases}$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi + d_j)\tau - 2\ln\left(\frac{1 - g_j e^{d_j\tau}}{1 - g_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right)$$

$$g_j = \frac{b_j - p\sigma_v i\phi + d_j}{b_j - p\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - p\sigma_v i\phi)^2 - \sigma_v^2 (2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - p\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity for ( $\tau = T - t$ ).

$i$  is the unit imaginary number for ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under “The Little Heston Trap” by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln\left(\frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Numerical Integration method Under Heston (1993) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Heston (1993) framework is based on the following expressions:

$$Call(K) = S_t e^{-q\tau} P_1 - K e^{-r\tau} P_2$$

$$Put(K) = Call(K) + K e^{-r\tau} - S_t e^{-q\tau}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \text{Re} \left[ \frac{e^{-i\phi \ln(K)} f_j(\phi)}{i\phi} \right] d\phi$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$f_j(\phi)$  is the characteristic function for  $P_j$  ( $j = 1, 2$ ).

$P_1$  is the probability of  $S_t > K$  under the asset price measure for the model.

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

Where  $j = 1, 2$  so that  $f_1(\phi)$  and  $f_2(\phi)$  are the characteristic functions for probabilities  $P_1$  and  $P_2$ , respectively.

This framework is chosen with the default value "Heston1993" for the Framework name-value pair argument.

## Numerical Integration Method Under Lewis (2001) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Lewis (2001) framework is based on the following expressions:

$$Call(k) = S_t e^{-q\tau} - \frac{\sqrt{K} e^{-\tau t}}{\pi} \int_0^{\infty} \operatorname{Re} \left[ K^{-iu} f_2 \left( \phi = \left( u - \frac{i}{2} \right) \right) \frac{1}{u^2 + \frac{1}{4}} \right] du$$

$$Put(K) = Call(K) = K e^{-\tau t} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$u$  is the characteristic function variable for integration, where  $\phi = \left(u - \frac{i}{2}\right)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

This framework is chosen with the value “Lewis2001” for the Framework name-value pair argument.

## References

- [1] Bates, D. S. “Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options.” *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [2] Heston, S. L. “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options.” *The Review of Financial Studies*. Vol 6. No. 2. 1993.
- [3] Lewis, A. L. “A Simple Option Formula for General Jump-Diffusion and Other Exponential Levy Processes.” Envision Financial Systems and OptionCity.net, 2001.



## See Also

[optByBatesFFT](#) | [optByBatesNI](#) | [optByHestonFFT](#) | [optByHestonNI](#) |  
[optByMertonFFT](#) | [optByMertonNI](#) | [optSensByBatesFFT](#) | [optSensByHestonFFT](#) |  
[optSensByHestonNI](#) | [optSensByMertonFFT](#) | [optSensByMertonNI](#)

**Introduced in R2018a**

## optByHestonFD

Option price by Heston model using finite differences

### Syntax

```
[Price,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD(Rate,  
AssetPrice,Settle,ExerciseDates,OptSpec,Strike,V0,ThetaV,Kappa,  
SigmaV,RhoSV)  
[Price,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD( ____,  
Name,Value)
```

### Description

[Price,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD(Rate, AssetPrice,Settle,ExerciseDates,OptSpec,Strike,V0,ThetaV,Kappa, SigmaV,RhoSV) computes a vanilla European or American option price by the Heston model, using the alternating direction implicit (ADI) method.

[Price,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD( \_\_\_\_, Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Price an American Option Using the Heston Model

Define the option variables and Heston model parameters.

```
AssetPrice = 10;  
Strike = 10;  
Rate = 0.1;  
Settle = '01-Jan-2017';  
ExerciseDates = '02-Apr-2017';
```

```
V0 = 0.0625;  
ThetaV = 0.16;  
Kappa = 5.0;  
SigmaV = 0.9;  
RhoSV = 0.1;
```

Compute the American put option price.

```
OptSpec = 'Put';  
Price = optByHestonFD(Rate, AssetPrice, Settle, ...  
ExerciseDates, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, 'AmericanOpt', 1)  
Price = 0.5188
```

## Input Arguments

### **Rate — Continuously compounded risk-free interest rate**

scalar decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal.

Data Types: double

### **AssetPrice — Current underlying asset price**

scalar numeric

Current underlying asset price, specified as numeric value using a scalar numeric.

Data Types: double

### **Settle — Option settlement date**

serial date number | date character vector | datetime array | string array

Option settlement date, specified as a scalar using serial date numbers, date character vectors, datetime arrays, or string arrays.

Data Types: double | char | datetime | string

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | string array | datetime array

Option exercise dates, specified as a serial date number, date character vector, string array, or datetime array:

- For a European option, there is only one `ExerciseDates` value and this is the option expiry date.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, the option can be exercised between the `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `string` | `datetime`

### **OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a scalar using a cell array of character vectors or string arrays with values 'call' or 'put'.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

### **V0 — Initial variance of underlying asset**

scalar numeric

Initial variance of the underlying asset, specified as a scalar numeric.

Data Types: `double`

### **ThetaV — Long-term variance of underlying asset**

scalar numeric

Long-term variance of the underlying asset, specified as a scalar numeric.

Data Types: `double`

### **Kappa — Mean revision speed for variance of underlying asset**

scalar numeric

Mean revision speed for the variance of the underlying asset, specified as a scalar numeric.

Data Types: double

### **SigmaV — Volatility of variance of underlying asset**

scalar numeric

Volatility of the variance of the underlying asset, specified as a scalar numeric.

Data Types: double

### **RhoSV — Correlation between Weiner processes for underlying asset and its variance**

scalar numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceGrid, AssetPrices, Variances, Times] = optByHestonD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, 'Basis', 7)`

### **Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of `'Basis'` and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)

- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | scalar numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates` = [ ] or do not enter them. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

### **DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and a NDIV-by-1 vector.

---

**Note** Each dividend amount must have a corresponding ex-dividend date. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

**ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | string array | datetime array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and an NDIV-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, value is calculated based on asset price distribution at maturity (default) | positive scalar

Maximum price for the price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar.

Data Types: single | double

**VarianceMax — Maximum variance to use for variance grid boundary**

1.0 (default) | scalar numeric

Maximum variance to use for the variance grid boundary, specified as the comma-separated pair consisting of 'VarianceMax' as a scalar numeric.

Data Types: double

**AssetGridSize — Size of asset grid for finite difference grid**

400 (default) | scalar numeric

Size of the asset grid for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar numeric.

Data Types: double

**VarianceGridSize — Number of nodes for variance grid for finite difference grid**

200 (default) | scalar numeric

Number of nodes for the variance grid for finite difference grid, specified as the comma-separated pair consisting of 'VarianceGridSize' and a scalar numeric.

Data Types: double

**TimeGridSize — Number of nodes of time grid for finite difference grid**

100 (default) | positive numeric scalar

Number of nodes of the time grid for finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive numeric scalar.

Data Types: double

**AmericanOpt — Option type**

0 (European) (default) | scalar with value of [0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

## Output Arguments

**Price — Option price**

numeric

Option price, returned as a scalar numeric.

**PriceGrid — Grid containing prices calculated by the finite difference method**

numeric

Grid containing prices calculated by the finite difference method, returned as a three-dimensional grid with size `AssetGridSize` × `VarianceGridSize` × `TimeGridSize`. The depth is not necessarily equal to the `TimeGridSize`, because exercise and ex-dividend dates are added to the time grid. `PriceGrid(:, :, end)` contains the price for  $t = 0$ .

**AssetPrices — Prices of the asset**

vector

Prices of the asset corresponding to the first dimension of `PriceGrid`, returned as a vector.

**Variiances — Variiances**

vector

Variiances corresponding to the second dimension of `PriceGrid`, returned as a vector.



**Times — Times**

vector

Times corresponding to the third dimension of PriceGrid, returned as a vector.

**Definitions****Heston Stochastic Volatility Model**

The Heston model is an extension of the Black-Scholes model, where the volatility (square root of variance) is no longer assumed to be constant, and the variance now follows a stochastic (CIR) process. This allows modeling the implied volatility smiles observed in the market.

The stochastic differential equation is:

$$dS_t = (r - q)S_t dt + \sqrt{v_t}S_t dW_t$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v$$

$$E[dW_t dW_t^v] = \rho dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$

$v_0$  is the initial variance of the asset price at  $t = 0$  for ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for the variance for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of the variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

## References

- [1] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6, Number 2, 1993.

## See Also

`optByBatesFD` | `optByLocalVolFD` | `optByMertonFD` | `optBySensMertonFD` | `optSensByBatesFD` | `optSensByHestonFD` | `optSensByLocalVolFD` | `optstockbyfd` | `optstocksensbyfd`

**Introduced in R2018b**

# optSensByHestonFD

Option price and sensitivities by Heston model using finite differences

## Syntax

```
[PriceSens,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD(
Rate,AssetPrice,Settle,ExerciseDates,OptSpec,Strike,V0,ThetaV,Kappa,
SigmaV,RhoSV)
```

```
[PriceSens,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD(
___,Name,Value)
```

## Description

[PriceSens,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD(Rate,AssetPrice,Settle,ExerciseDates,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV) computes a vanilla European or American option price and sensitivities by the Heston model, using the alternating direction implicit (ADI) method.

[PriceSens,PriceGrid,AssetPrices,Variances,Times] = optByHestonFD(\_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute an American Option Price and Sensitivities Using the Heston Model

Define the option variables and Heston model parameters.

```
AssetPrice = 10;
Strike = 10;
Rate = 0.1;
Settle = '01-Jan-2017';
ExerciseDates = '02-Apr-2017';
```

```
V0 = 0.0625;  
ThetaV = 0.16;  
Kappa = 5.0;  
SigmaV = 0.9;  
RhoSV = 0.1;
```

Compute the American put option price and sensitivities.

```
OptSpec = 'Put';  
[Price,Delta,Gamma,Rho,Theta,Vega,VegaLT] = optSensByHestonFD(Rate, AssetPrice, Settle  
OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, 'AmericanOpt', 1, ...  
'OutSpec', ["Price" "Delta" "Gamma" "Rho" "Theta" "Vega" "VegaLT"])
```

```
Price = 0.5188
```

```
Delta = -0.4472
```

```
Gamma = 0.2822
```

```
Rho = -0.9234
```

```
Theta = -1.1614
```

```
Vega = 0.8998
```

```
VegaLT = 1.0921
```

## Input Arguments

### **Rate — Continuously compounded risk-free interest rate**

scalar decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal.

Data Types: double

### **AssetPrice — Current underlying asset price**

scalar numeric

Current underlying asset price, specified as a scalar numeric.

Data Types: double

**Settle — Option settlement date**

serial date number | date character vector | datetime array | string array

Option settlement date, specified as a scalar using serial date numbers, date character vectors, datetime arrays, or string arrays.

Data Types: double | char | datetime | string

**ExerciseDates — Option exercise dates**

serial date number | date character vector | string array | datetime array

Option exercise dates, specified as a serial date number, date character vector, string array, or datetime array:

- For a European option, there is only one `ExerciseDates` value and this is the option expiry date.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, the option can be exercised between the `Settle` date and the single listed `ExerciseDate`.

Data Types: double | char | string | datetime

**OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a scalar using a cell array of character vectors or string arrays with values 'call' or 'put'.

Data Types: cell | string

**Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: double

**V0 — Initial variance of underlying asset**

scalar numeric

Initial variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**ThetaV — Long-term variance of underlying asset**

scalar numeric

Long-term variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**Kappa — Mean revision speed for the variance of underlying asset**

scalar numeric

Mean revision speed for the variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**SigmaV — Volatility of the variance of underlying asset**

scalar numeric

Volatility of the variance of the underlying asset, specified as a scalar numeric.

Data Types: double

**RhoSV — Correlation between Weiner processes for underlying asset and its variance**

scalar numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[PriceSens, PriceGrid, AssetPrices, Variances, Times] = optSensByHestonFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, 'Basis', 7)`

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0,1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**DividendYield — Continuously compounded underlying asset yield**

0 (default) | scalar numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for DividendYield, then set DividendAmounts and ExDividendDates = [ ] or do not enter them. If you enter values for DividendAmounts and ExDividendDates, then set DividendYield = 0.

---

Data Types: double

**DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and a NDIV-by-1 vector.

---

**Note** Each dividend amount must have a corresponding ex-dividend date. If you enter values for DividendAmounts and ExDividendDates, then set DividendYield = 0.

---

Data Types: double

**ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | string array | datetime array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and a NDIV-by-1 vector of serial date numbers, character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, value is calculated based on asset price distribution at maturity (default) | positive scalar

Maximum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar.

Data Types: single | double

**VarianceMax — Maximum variance to use for variance grid boundary**

1.0 (default) | scalar numeric

Maximum variance to use for variance grid boundary, specified as the comma-separated pair consisting of 'VarianceMax' as a scalar numeric.

Data Types: double

**AssetGridSize — Size of asset grid for finite difference grid**

400 (default) | scalar numeric

Size of the asset grid for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar numeric.



Data Types: double

**VarianceGridSize — Number of nodes for variance grid for finite difference grid**

200 (default) | scalar numeric

Number of nodes for the variance grid for finite difference grid, specified as the comma-separated pair consisting of 'VarianceGridSize' and a scalar numeric.

Data Types: double

**TimeGridSize — Number of nodes of time grid for finite difference grid**

100 (default) | positive numeric scalar

Number of nodes of the time grid for finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive numeric scalar.

Data Types: double

**AmericanOpt — Option type**

0 (European) (default) | scalar with values [0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

**OutSpec — Define outputs**

["price"] (default) | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegalt' | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegalt"

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and an NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with the supported values.

---

**Note** 'vega' is the sensitivity with respect to the initial volatility  $\sqrt{V_0}$ . In contrast, 'vegalt' is the sensitivity with respect to the long-term volatility  $\sqrt{\Theta\sigma}$ .

---

```
Example: OutSpec =  
{'price', 'delta', 'gamma', 'vega', 'rho', 'theta', 'vegalt'}
```

Data Types: string | cell

## Output Arguments

### **PriceSens — Option price and sensitivities**

scalar numeric

Option price and sensitivities, returned as a scalar numeric. OutSpec determines the types and order of the outputs.

### **PriceGrid — Grid containing prices calculated by the finite difference method**

numeric

Grid containing prices calculated by the finite difference method, returned as a three-dimensional grid with size AssetGridSize  $\times$  VarianceGridSize  $\times$  TimeGridSize. The depth is not necessarily equal to the TimeGridSize, because exercise and ex-dividend dates are added to the time grid. PriceGrid(:, :, end) contains the price for  $t = 0$ .

### **AssetPrices — Prices of the asset**

vector

Prices of the asset corresponding to the first dimension of PriceGrid, returned as a vector.

### **Variiances — Variiances**

vector

Variiances corresponding to the second dimension of PriceGrid, returned as a vector.

### **Times — Times**

vector

Times corresponding to the third dimension of PriceGrid, returned as a vector.

## Definitions

### Heston Stochastic Volatility Model

The Heston model is an extension of the Black-Scholes model, where the volatility (square root of variance) is no longer assumed to be constant, and the variance now follows a stochastic (CIR) process. This allows modeling the implied volatility smiles observed in the market.

The stochastic differential equation is:

$$\begin{aligned}dS_t &= (r - q)S_t dt + \sqrt{v_t}S_t dW_t \\dv_t &= \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v \\E[dW_t dW_t^v] &= \rho dt\end{aligned}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$

$v_0$  is the initial variance of the asset price at  $t = 0$  for ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for the variance for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of the variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

### References

- [1] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6, Number 2, 1993.

## **See Also**

`optByBatesFD` | `optByHestonFD` | `optByLocalVolFD` | `optByMertonFD` |  
`optBySensMertonFD` | `optSensByBatesFD` | `optSensByLocalVolFD` | `optstockbyfd`  
| `optstocksensbyfd`

**Introduced in R2018b**

# optByHestonFFT

Option price by Heston model using FFT and FRFT

## Syntax

```
[Price,StrikeOut] = optByHestonFFT(Rate,AssetPrice,Settle,Maturity,  
OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV)  
[Price,StrikeOut] = optByHestonFFT( ____,Name,Value)
```

## Description

[Price,StrikeOut] = optByHestonFFT(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV) computes vanilla European option price by Heston model, using Carr-Madan FFT and Chourdakis FRFT methods.

[Price,StrikeOut] = optByHestonFFT( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Workflow for Plotting an Option Price Surface Using the Heston Model

Use `optByHestonFFT` to calibrate a FFT strike grid and then compute option prices and plot an option price surface.

#### Define Option Variables and Heston Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
V0 = 0.04;  
ThetaV = 0.05;
```

```
Kappa = 1.0;  
SigmaV = 0.2;  
RhoSV = -0.7;
```

### Compute the Option Prices for the Entire FFT (or FRFT) Strike Grid, Without Specifying "Strike"

Compute option prices and also output the corresponding strikes. If the `Strike` input is empty ( `[]` ), option prices will be computed on the entire FFT (or FRFT) strike grid. The FFT (or FRFT) strike grid is determined as `exp(log-strike grid)`, where each column of the log-strike grid has `NumFFT` points with `LogStrikeStep` spacing that are roughly centered around each element of `log(AssetPrice)`. The default value for `NumFFT` is  $2^{12}$ . In addition to the prices in the first output, the optional last output contains the corresponding strikes.

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = []; % Strike is not specified (will use the entire FFT strike grid)  
  
% Compute option prices for the entire FFT strike grid  
[Call, Kout] = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ..  
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield);  
  
% Show the lowest and highest strike values on the FFT strike grid  
format  
MinStrike = Kout(1) % Lowest possible strike in the current FFT strike grid  
  
MinStrike = 2.9205e-135  
  
MaxStrike = Kout(end) % Highest possible strike in the current FFT strike grid  
  
MaxStrike = 1.8798e+138  
  
% Show a subset of the strikes and corresponding option prices  
Range = (2046:2052);  
[Kout(Range) Call(Range)]  
  
ans = 7x2  
  
    50.4929    29.4843  
    58.8640    21.3767  
    68.6231    12.5614  
    80.0000     4.7008  
    93.2631     0.6496
```

```
108.7251    0.0144
126.7505    0.0001
```

### Change the Number of FFT (or FRFT) Points, and Compare with optByHestonNI

Try a different number of FFT (or FRFT) points, and compare the results with direct numerical integration. Unlike `optByHestonFFT`, which uses FFT (or FRFT) techniques for fast computation across the whole range of strikes, the `optByHestonNI` function uses direct numerical integration and it is typically slower, especially for multiple strikes. However, the values computed by `optByHestonNI` can serve as a benchmark for adjusting the settings for `optByHestonFFT`.

```
% Try a smaller number of FFT (or FRFT) points
% (e.g. for faster performance or smaller memory footprint)
NumFFT = 2^10; % Smaller than the default value of 2^12
Strike = []; % Strike is not specified (will use the entire FFT strike grid)
[Call, Kout] = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT);

% Compare with numerical integration method
Range = (510:516);
Strike = Kout(Range);
CallFFT = Call(Range);
CallNI = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield);
Error = abs(CallFFT-CallNI);
table(Strike, CallFFT, CallNI, Error)
```

```
ans=7x4 table
```

Strike	CallFFT	CallNI	Error
12.696	66.066	66.696	0.62964
23.449	55.766	56.103	0.33672
43.312	36.359	36.539	0.17974
80	4.7727	4.7007	0.071928
147.76	0.066156	2.3472e-08	0.066156
272.93	0.013271	-2.5036e-09	0.013271
504.11	0.0034504	-3.0876e-07	0.0034508

**Make Further Adjustments to FFT (or FRFT)**

If the values in the output CallFFT are significantly different from those in CallNI, try making adjustments to optByHestonFFT settings, such as CharacteristicFcnStep, LogStrikeStep, NumFFT, DampingFactor, and so on. Note that if (LogStrikeStep \* CharacteristicFcnStep) is  $2\pi/\text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

```
Strike = []; % Strike is not specified (will use the entire FFT or FRFT strike grid)
[Call, Kout] = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001);
```

```
% Compare with numerical integration method
```

```
Strike = Kout(Range);
CallFFT = Call(Range);
CallNI = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ...
    ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield);
Error = abs(CallFFT-CallNI);
table(Strike, CallFFT, CallNI, Error)
```

```
ans=7x4 table
```

Strike	CallFFT	CallNI	Error
79.76	4.826	4.826	2.7708e-08
79.84	4.7841	4.7841	3.0111e-08
79.92	4.7423	4.7423	3.2376e-08
80	4.7007	4.7007	3.4496e-08
80.08	4.6593	4.6593	3.6457e-08
80.16	4.6181	4.6181	3.8253e-08
80.24	4.577	4.577	3.9872e-08

```
% Save the final FFT (or FRFT) strike grid for future reference. For
% example, it provides information about the range of Strike inputs for
% which the FFT (or FRFT) operation is valid.
```

```
FFTStrikeGrid = Kout;
MinStrike = FFTStrikeGrid(1) % Strike cannot be less than MinStrike
```

```
MinStrike = 47.9437
```

```
MaxStrike = FFTStrikeGrid(end) % Strike cannot be greater than MaxStrike
```

```
MaxStrike = 133.3566
```



### Compute the Option Price for a Single Strike

Once the desired FFT (or FRFT) settings are determined, use the `Strike` input to specify the strikes rather than providing an empty array. If the specified strikes do not match a value on the FFT (or FRFT) strike grid, the outputs are interpolated on the specified strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;
```

```
Call = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)
```

```
Call = 4.7007
```

### Compute the Option Prices for a Vector of Strikes

Use the `Strike` input to specify the strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';
```

```
Call = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)
```

```
Call = 5×1
```

```
7.0401
5.8053
4.7007
3.7316
2.8991
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the `Strike` input to specify the strikes. Also, the `Maturity` input can be a vector, but it must match the length of the `Strike` vector if the `ExpandOutput` name-value pair argument is not set to "true".

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes

Call = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001) % Five values in vector out

Call = 5×1

    8.9560
    9.3419
    9.6240
    9.8560
   10.0500

```

### Expand the Outputs for a Surface

Set the `ExpandOutput` name-value pair argument to "true" to expand the outputs into `NStrikes-by-NMaturities` matrices. In this case, they are square matrices.

```

[Call, Kout] = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 5) matrix outputs

Call = 5×5

    8.9560    10.4543    11.7058    12.8009    13.7728
    7.7946     9.3419    10.6337    11.7644    12.7685
    6.7244     8.3028     9.6240    10.7828    11.8134
    5.7475     7.3379     8.6771     9.8560    10.9074
    4.8645     6.4474     7.7930     8.9840    10.0500

Kout = 5×5

    76     76     76     76     76
    78     78     78     78     78
    80     80     80     80     80
    82     82     82     82     82
    84     84     84     84     84

```

## Compute the Option Prices for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is "true", `NStrikes` do not have to match `NMaturities` (that is, the output `NStrikes-by-NMaturities` matrix can be rectangular).

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes

Call = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 6) matrix output
```

```
Call = 5x6
```

7.0401	8.9560	10.4543	11.7058	12.8009	13.7728
5.8053	7.7946	9.3419	10.6337	11.7644	12.7685
4.7007	6.7244	8.3028	9.6240	10.7828	11.8134
3.7316	5.7475	7.3379	8.6771	9.8560	10.9074
2.8991	4.8645	6.4474	7.7930	8.9840	10.0500

## Compute the Option Prices for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

Call = optByHestonFFT(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT, ...
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 4) matrix output
```

```
Call = 5x4
```

3.2944	5.8047	8.9560	12.6052
2.6413	4.8810	7.7946	11.2507
2.0864	4.0575	6.7244	9.9738
1.6230	3.3325	5.7475	8.7783

1.2429    2.7028    4.8645    7.6676

### Plot an Option Price Surface

Use the `Strike` input to specify the strikes. Increase the value for `NumFFT` to support a wider range of strikes. Also, the `Maturity` input can be a vector. Set `ExpandOutput` to `"true"` to output the surface as a `NStrikes-by-NMaturities` matrix.

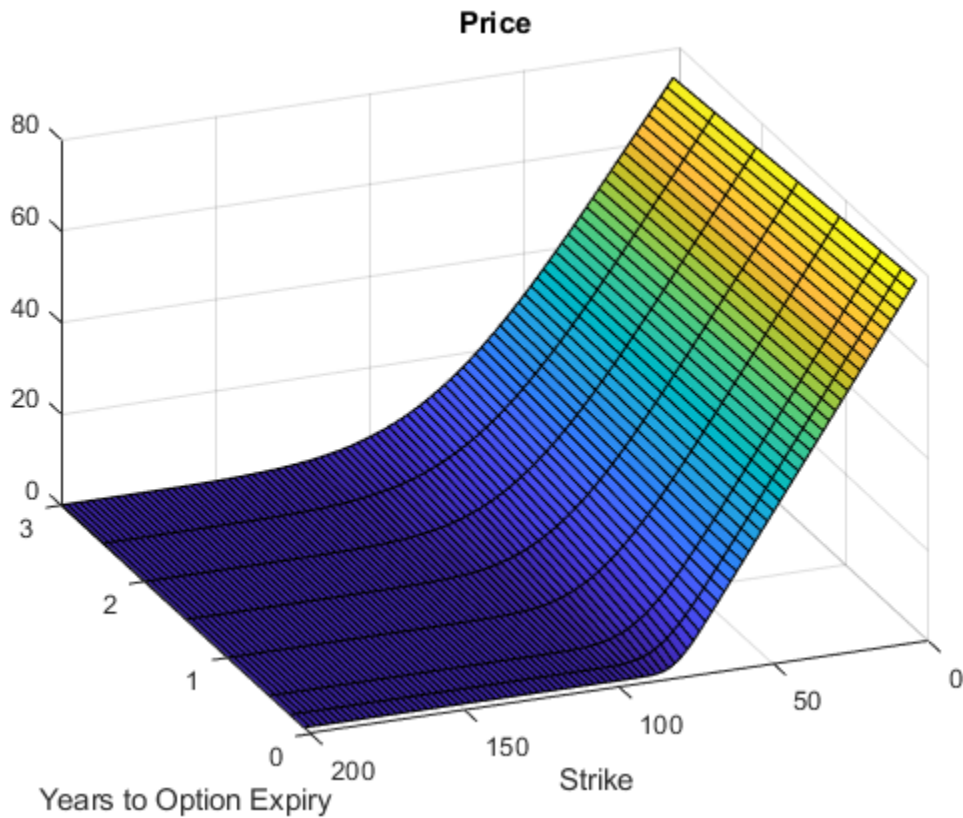
```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]);
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';
```

```
% Increase 'NumFFT' to support a wider range of strikes
NumFFT = 2^13;
```

```
Call = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, 'ExpandOutput', true);
```

```
[X,Y] = meshgrid(Times,Strike);
```

```
figure;
surf(X,Y,Call);
title('Price');
xlabel('Years to Option Expiry');
ylabel('Strike');
view(-112,34);
xlim([0 Times(end)]);
zlim([0 80]);
```



## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a `NINST-by-1` or `NColumns-by-1` vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle** — Option settlement date

`serial date number` | `date character vector` | `datetime` | `string array`

Option settlement date, specified as a `NINST-by-1` or `NColumns-by-1` vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity** — Option maturity date

`serial date number` | `date character vector` | `datetime` | `string array`

Option maturity date, specified as a `NINST-by-1` or `NColumns-by-1` vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec** — Definition of option

`cell array of character vector with values 'call' or 'put'` | `string array with values "call" or "put"`

Definition of the option as `'call'` or `'put'`, specified as a `NINST-by-1` or `NColumns-by-1` vector using a cell array of character vectors or string arrays with values `"call"` or `"put"`.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

**Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

If this input is an empty array ( [] ), option prices are computed on the entire FFT (or FRFT) strike grid, which is determined as  $\exp(\log\text{-strike grid})$ . Each column of the log-strike grid has 'NumFFT' points with 'LogStrikeStep' spacing that are roughly centered around each element of  $\log(\text{AssetPrice})$ .

For more information on the proper dimensions for Strike, see the name-value pair argument ExpandOutput.

Data Types: double

**V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

**ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

**Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

**SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

**RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, StrikeOut] = optByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, The taV, Kappa, SigmaV, RhoSV, 'Basis', 7)`

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)



- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

### **VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double

### **LittleTrap — Flag indicating Little Heston Trap formulation**

true (default) | logical with values true or false

Flag indicating Little Heston Trap formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- true — Use the Albrecher *et al* formulation.
- false — Use the original Heston formation.

Data Types: logical

### **NumFFT — Number of grid points in the characteristic function variable**

4096 (default) | numeric

Number of grid points in the characteristic function variable and in each column of the log-strike grid, specified as the comma-separated pair consisting of 'NumFFT' and a scalar numeric value.

Data Types: double

**CharacteristicFcnStep — Characteristic function variable grid spacing**

0.01 (default) | numeric

Characteristic function variable grid spacing, specified as the comma-separated pair consisting of 'CharacteristicFcnStep' and a scalar numeric value.

Data Types: double

**LogStrikeStep — Log-strike grid spacing**

$2\pi/\text{NumFFT}/\text{CharacteristicFcnStep}$  (default) | numeric

Log-strike grid spacing, specified as the comma-separated pair consisting of 'LogStrikeStep' and a scalar numeric value.

---

**Note** If  $(\text{LogStrikeStep} \times \text{CharacteristicFcnStep})$  is  $2\pi/\text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

---

Data Types: double

**DampingFactor — Damping factor for Carr-Madan formulation**

1.5 (default) | numeric

Damping factor for Carr-Madan formulation, specified as the comma-separated pair consisting of 'DampingFactor' and a scalar numeric value.

Data Types: double

**Quadrature — Type of quadrature**

"simpson" (default) | character vector with values: 'simpson' or 'trapezoidal' | string array with values: "simpson" or "trapezoidal"

Type of quadrature, specified as the comma-separated pair consisting of 'Quadrature' and a single character vector or string array with a value of 'simpson' or 'trapezoidal'.

Data Types: char | string

**ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- **true** — If **true**, the outputs are **NRows-by-NColumns** matrices. **NRows** is the number of strikes for each column and it is determined by the **Strike** input. For example, **Strike** can be a **NRows-by-1** vector, or a **NRows-by-NColumns** matrix. If **Strike** is empty, **NRows** is equal to **NumFFT**. **NColumns** is determined by the sizes of **AssetPrice**, **Settle**, **Maturity**, and **OptSpec**, which must all be either scalar or **NColumns-by-1** vectors.
- **false** — If **false**, the outputs are **NINST-by-1** vectors. Also, the inputs **Strike**, **AssetPrice**, **Settle**, **Maturity**, and **OptSpec** must all be either scalar or **NINST-by-1** vectors.

Data Types: `logical`

## Output Arguments

### **Price — Option prices**

`numeric`

Option prices, returned as a **NINST-by-1**, or **NRows-by-NColumns**, depending on **ExpandOutput**.

### **StrikeOut — Strikes corresponding to Price**

`numeric`

Strikes corresponding to **Price**, returned as a **NINST-by-1**, or **NRows-by-NColumns**, depending on **ExpandOutput**.

## Definitions

### **Heston Stochastic Volatility Model**

The Heston model is an extension of the Black-Scholes model, where the volatility (square root of variance) is no longer assumed to be constant, and the variance now follows a stochastic (CIR) process. This process allows modeling the implied volatility smiles observed in the market.

The stochastic differential equation is:

$$dS_t = (r - q)S_t dt + \sqrt{v_t}S_t dW_t$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v$$

$$\mathbb{E}[dW_t dW_t^v] = \rho dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$

$v_0$  is the initial variance of the asset price at  $t = 0$  for ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for the variance for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of the variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

The characteristic function  $f_{Heston_j}(\phi)$  for  $j = 1$  (asset price measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Heston_j}(\phi) = \exp(C_j + D_j v_0 + i\phi \ln S_t)$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi + d_j)\tau - 2\ln \left( \frac{1 - g_j e^{d_j\tau}}{1 - g_j} \right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right)$$

$$g_j = \frac{b_j - p\sigma_v i\phi + d_j}{b_j - p\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - p\sigma_v i\phi)^2 - \sigma_v^2 (2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - p\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under “The Little Heston Trap” by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln \left( \frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j} \right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Carr-Madan Formulation

The Carr and Madan (1999) formulation is a popular modified implementation of Heston (1993) framework.

Rather than computing the probabilities  $P_1$  and  $P_2$  as intermediate steps, Carr and Madan developed an alternative expression so that taking its inverse Fourier transform gives the option price itself directly.

$$Call(k) = \frac{e^{-\alpha k}}{\pi} \int_0^{\infty} \text{Re}[e^{-iuk} \psi(u)] du$$

$$\psi(u) = \frac{e^{-r\tau} f_2(\phi = (u - (\alpha + 1)i))}{\alpha^2 + \alpha - u^2 + iu(2\alpha + 1)}$$

$$Put(K) = Call(K) + Ke^{-r\tau} - S_t e^{-q\tau}$$

where

$\tau$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$

$i$  is a unit imaginary number ( $i^2 = -1$ )

$\phi$  is the characteristic function variable.

$\alpha$  is the damping factor.

$u$  is the characteristic function variable for integration, where  $\phi = (u - (\alpha+1)i)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

To apply FFT or FRFT to this formulation, the characteristic function variable for integration,  $u$ , is discretized into  $\text{NumFFT}(N)$  points with the step size  $\text{CharacteristicFcnStep}(\Delta u)$ , and the log-strike  $k$  is discretized into  $N$  points with the step size  $\text{LogStrikeStep}(\Delta k)$ .

The discretized characteristic function variable for integration,  $u_j$  (for  $j = 1, 2, 3, \dots, N$ ), has a minimum value of 0 and a maximum value of  $(N-1)(\Delta u)$ , and it approximates the continuous integration range from 0 to infinity.

The discretized log-strike grid,  $k_n$  (for  $n = 1, 2, 3, N$ ) is approximately centered around  $\ln(S_t)$ , with a minimum value of

$$\ln(S_t) - \frac{N}{2}\Delta k$$

and a maximum value of

$$\ln(S_t) + \left(\frac{N}{2} - 1\right)\Delta k$$

Where the minimum allowable strike is

$$S_t \exp\left(-\frac{N}{2}\Delta k\right)$$

and the maximum allowable strike is

$$S_t \exp\left[\left(\frac{N}{2} - 1\right)\Delta k\right]$$

As a result of the discretization, the expression for the call option becomes

$$\text{Call}(k_n) = \Delta u \frac{e^{-\alpha k_n}}{\pi} \sum_{j=1}^N \text{Re} \left[ e^{-i\Delta k \Delta u (j-1)(n-1)} e^{iu_j \left[ \frac{N\Delta k}{2} - \ln(S_t) \right]} \psi(u_j) \right] w_j$$

where

$\Delta u$  is the step size of discretized characteristic function variable for integration.

$\Delta k$  is the step size of discretized log-strike.

$N$  is the number of FFT or FRFT points.

$w_j$  is the weights for quadrature used for approximating the integral.

FFT is used to evaluate the above expression if  $\Delta k$  and  $\Delta u$  are subject to the following constraint:

$$\Delta k \Delta u = \left( \frac{2\pi}{N} \right)$$

otherwise, the functions use the FRFT method described in Chourdakis (2005).

## References

- [1] Albrecher, H., Mayer, P., Schoutens, W., and Tistaert, J. "The Little Heston Trap." Working Paper, Linz and Graz University of Technology, K.U. Leuven, ING Financial Markets, 2006.
- [2] Carr, P. and D.B. Madan. "Option Valuation using the Fast Fourier Transform." *Journal of Computational Finance*. Vol 2. No. 4. 1999.
- [3] Chourdakis, K. "Option Pricing Using Fractional FFT." *Journal of Computational Finance*. 2005.
- [4] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6. No. 2. 1993.

## See Also

[optByBatesFFT](#) | [optByBatesNI](#) | [optByHestonNI](#) | [optByMertonFFT](#) | [optByMertonNI](#) | [optSensByBatesFFT](#) | [optSensByBatesNI](#) | [optSensByHestonFFT](#) | [optSensByHestonNI](#) | [optSensByMertonFFT](#) | [optSensByMertonNI](#)

**Introduced in R2018a**



# optSensByHestonFFT

Option price and sensitivities by Heston model using FFT and FRFT

## Syntax

```
[PriceSens,StrikeOut] = optSensByHestonFFT(Rate,AssetPrice,Settle,  
Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV)  
[PriceSens,StrikeOut] = optSensByHestonFFT( ____,Name,Value)
```

## Description

[PriceSens,StrikeOut] = optSensByHestonFFT(Rate,AssetPrice,Settle, Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV) computes vanilla European option price and sensitivities by Heston model, using Carr-Madan FFT and Chourdakis FRFT methods.

[PriceSens,StrikeOut] = optSensByHestonFFT( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Workflow for Plotting an Option Sensitivity Surface Using the Heston Model

Use `optSensByHestonFFT` to calibrate the FFT strike grid for sensitivities, compute option sensitivities, and plot option sensitivity surfaces.

### Define Option Variables and Heston Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
V0 = 0.04;
```

```
ThetaV = 0.05;
Kappa = 1.0;
SigmaV = 0.2;
RhoSV = -0.7;
```

### Compute the Option Sensitivities for the Entire FFT (or FRFT) Strike Grid, Without Specifying "Strike"

Compute option sensitivities and also output the corresponding strikes. If the Strike input is empty ( [] ), option sensitivities will be computed on the entire FFT (or FRFT) strike grid. The FFT (or FRFT) strike grid is determined as  $\exp(\log\text{-strike grid})$ , where each column of the log-strike grid has NumFFT points with LogStrikeStep spacing that are roughly centered around each element of  $\log(\text{AssetPrice})$ . The default value for NumFFT is  $2^{12}$ . In addition to the sensitivities in the first output, the optional last output contains the corresponding strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = []; % Strike is not specified (will use the entire FFT strike grid)

% Compute option sensitivities for the entire FFT strike grid
[Delta, Kout] = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'OutSpec', "delta");

% Show the lowest and highest strike values on the FFT strike grid
format
MinStrike = Kout(1) % Lowest possible strike in the current FFT strike grid

MinStrike = 2.9205e-135

MaxStrike = Kout(end) % Highest possible strike in the current FFT strike grid

MaxStrike = 1.8798e+138

% Show a subset of the strikes and corresponding option sensitivities
Range = (2046:2052);
[Kout(Range) Delta(Range)]

ans = 7x2

    50.4929    0.9866
    58.8640    0.9671
    68.6231    0.8724
    80.0000    0.5775
```

```

93.2631    0.1545
108.7251   0.0059
126.7505   0.0000

```

### Change the Number of FFT (or FRFT) Points, and Compare With optSensByHestonNI

Try a different number of FFT (or FRFT) points, and compare the results with numerical integration. Unlike `optSensByHestonFFT`, which uses FFT (or FRFT) techniques for fast computation across the whole range of strikes, the `optSensByHestonNI` function uses direct numerical integration and it is typically slower, especially for multiple strikes. However, the values computed by `optSensByHestonNI` can serve as a benchmark for adjusting the settings for `optSensByHestonFFT`.

```

% Try a smaller number of FFT points
% (e.g. for faster performance or smaller memory footprint)
NumFFT = 2^10; % Smaller than the default value of 2^12
Strike = []; % Strike is not specified (will use the entire FFT strike grid)
[Delta, Kout] = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT);

% Compare with numerical integration method
Range = (510:516);
Strike = Kout(Range);
DeltaFFT = Delta(Range);
DeltaNI = optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'OutSpec', "delta");
Error = abs(DeltaFFT-DeltaNI);
table(Strike, DeltaFFT, DeltaNI, Error)

```

ans=7×4 table

Strike	DeltaFFT	DeltaNI	Error
12.696	0.90066	0.99002	0.08936
23.449	0.93635	0.99002	0.053677
43.312	0.94796	0.9896	0.041645
80	0.53274	0.57747	0.044733
147.76	0.0032769	2.45e-08	0.0032769
272.93	0.00098029	-1.399e-10	0.00098029
504.11	0.00028151	5.2868e-10	0.00028151

**Make Further Adjustments to FFT (or FRFT)**

If the values in the output `DeltaFFT` are significantly different from those in `DeltaNI`, try making adjustments to `optSensByHestonFFT` settings, such as `CharacteristicFcnStep`, `LogStrikeStep`, `NumFFT`, `DampingFactor`, and so on. Note that if  $(\text{LogStrikeStep} * \text{CharacteristicFcnStep})$  is  $2 * \pi / \text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

```
Strike = []; % Strike is not specified (will use the entire FFT or FRFT strike grid)
[Delta, Kout] = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, 'LogStrikeStep', LogStrikeStep);
```

```
% Compare with numerical integration method
```

```
Strike = Kout(Range);
DeltaFFT = Delta(Range);
DeltaNI = optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, 'LogStrikeStep', LogStrikeStep);
Error = abs(DeltaFFT-DeltaNI);
table(Strike, DeltaFFT, DeltaNI, Error)
```

```
ans=7x4 table
```

Strike	DeltaFFT	DeltaNI	Error
79.76	0.58558	0.58558	3.0538e-08
79.84	0.58289	0.58289	2.8865e-08
79.92	0.58018	0.58018	2.7053e-08
80	0.57747	0.57747	2.5111e-08
80.08	0.57476	0.57476	2.3049e-08
80.16	0.57203	0.57203	2.0875e-08
80.24	0.5693	0.5693	1.8601e-08

```
% Save the final FFT (or FRFT) strike grid for future reference. For
% example, it provides information about the range of Strike inputs for
% which the FFT (or FRFT) operation is valid.
```

```
FFTStrikeGrid = Kout;
MinStrike = FFTStrikeGrid(1) % Strike cannot be less than MinStrike
```

```
MinStrike = 47.9437
```

```
MaxStrike = FFTStrikeGrid(end) % Strike cannot be greater than MaxStrike
```

```
MaxStrike = 133.3566
```

### Compute the Option Sensitivity for a Single Strike

Once you determine FFT (or FRFT) settings, use the `Strike` input to specify the strikes rather than providing an empty array. If the specified strikes do not match a value on the FFT (or FRFT) strike grid, the outputs are interpolated on the specified strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;

Delta = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001)

Delta = 0.5775
```

### Compute the Option Sensitivities for a Vector of Strikes

Use the `Strike` input to specify the strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';

Delta = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001)

Delta = 5×1

    0.7043
    0.6433
    0.5775
    0.5083
    0.4377
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the `Strike` input to specify the strikes. Also, the `Maturity` input can be a vector, but it must match the length of the `Strike` vector if the `ExpandOutput` name-value pair argument is not set to `"true"`.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes

Delta = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001) % Five values in vector output

Delta = 5x1

    0.6848
    0.6413
    0.6095
    0.5841
    0.5631

```

### Expand the Outputs for a Surface

Set the `ExpandOutput` name-value pair argument to "true" to expand the outputs into `NStrikes-by-NMaturities` matrices. In this case, they are square matrices.

```

[Delta, Kout] = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 5) matrix output

Delta = 5x5

    0.6848    0.6762    0.6703    0.6654    0.6609
    0.6416    0.6413    0.6404    0.6390    0.6372
    0.5960    0.6048    0.6095    0.6119    0.6129
    0.5485    0.5671    0.5776    0.5841    0.5882
    0.4997    0.5286    0.5452    0.5559    0.5631

Kout = 5x5

    76    76    76    76    76
    78    78    78    78    78
    80    80    80    80    80
    82    82    82    82    82
    84    84    84    84    84

```

## Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is "true", `NStrikes` do not have to match `NMaturities` (that is, the output `NStrikes-by-NMaturities` matrix can be rectangular).

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes

Delta = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 6) matrix output

Delta = 5x6

    0.7043    0.6848    0.6762    0.6703    0.6654    0.6609
    0.6433    0.6416    0.6413    0.6404    0.6390    0.6372
    0.5775    0.5960    0.6048    0.6095    0.6119    0.6129
    0.5083    0.5485    0.5671    0.5776    0.5841    0.5882
    0.4377    0.4997    0.5286    0.5452    0.5559    0.5631
```

## Compute the Option Sensitivities for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

Delta = optSensByHestonFFT(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 4) matrix output

Delta = 5x4

    0.4293    0.5708    0.6848    0.7705
    0.3737    0.5193    0.6416    0.7364
    0.3200    0.4668    0.5960    0.6994
```

```
0.2693    0.4143    0.5485    0.6597
0.2226    0.3628    0.4997    0.6177
```

### Plot Option Sensitivity Surfaces

Use the `Strike` input to specify the strikes. Increase the value for `NumFFT` to support a wider range of strikes. Also, the `Maturity` input can be a vector. Set `ExpandOutput` to `"true"` to output the surfaces as `NStrikes-by-NMaturities` matrices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';
```

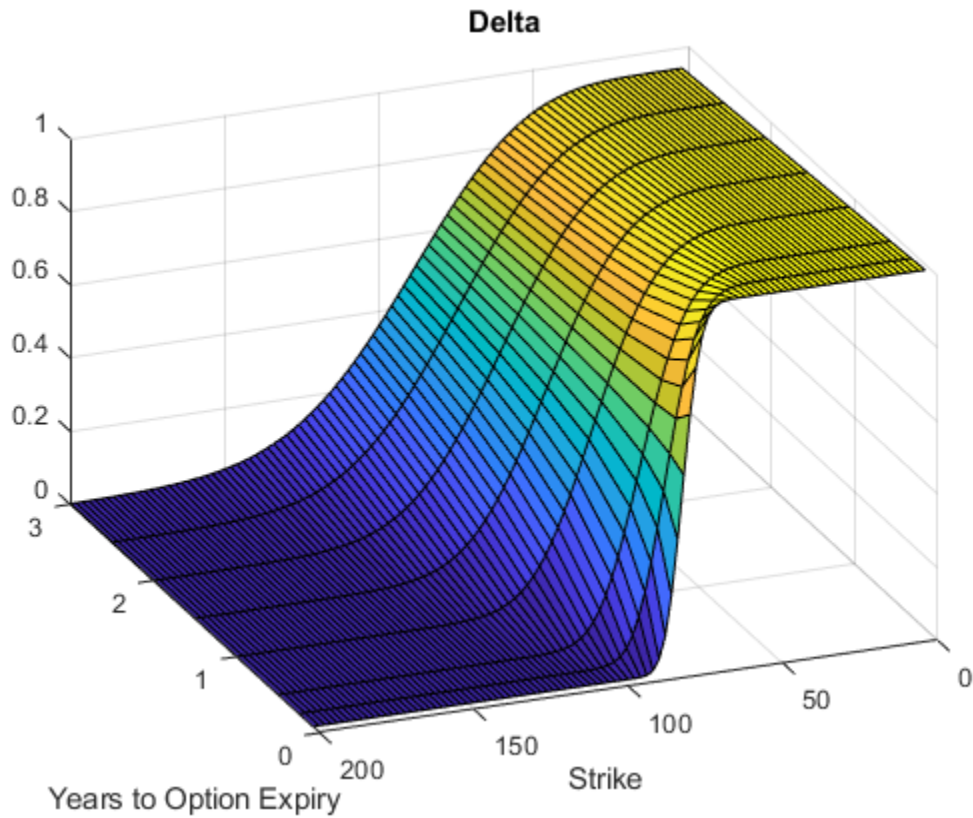
```
% Increase 'NumFFT' to support a wider range of strikes
NumFFT = 2^13;
```

```
[Delta, Gamma, Rho, Theta, Vega, VegaLT] = ...
    optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, 'ExpandOutput', true, ...
    'OutSpec', ["delta", "gamma", "rho", "theta", "vega", "vegalt"]);
```

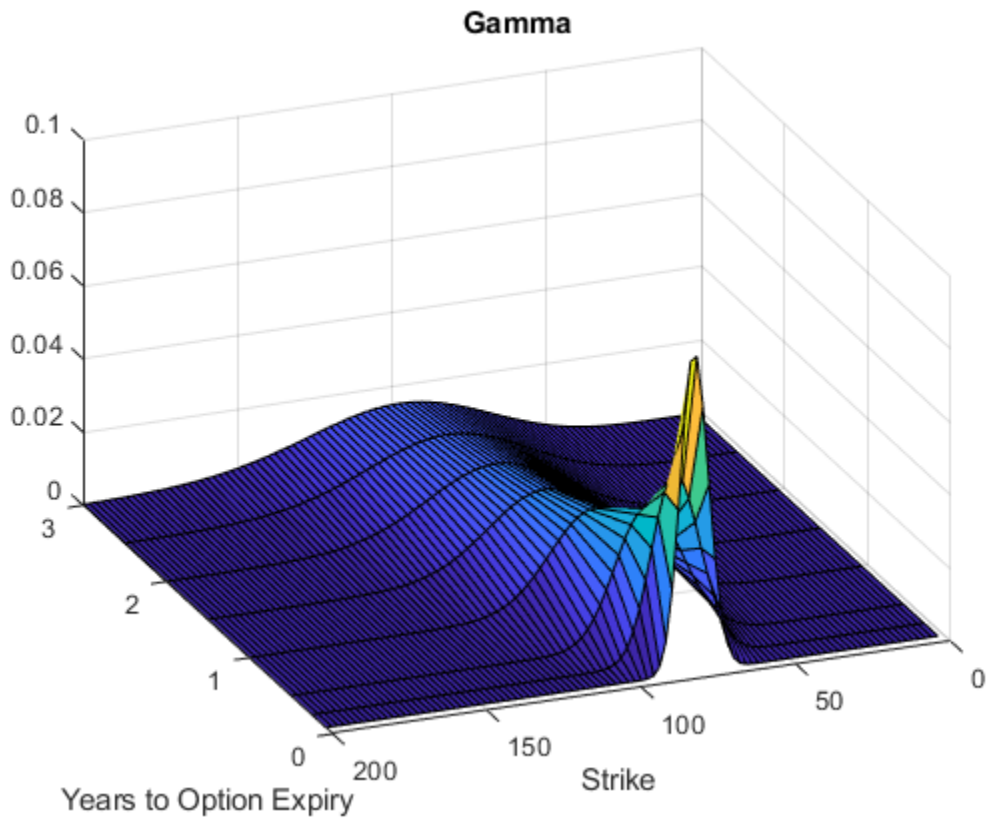
```
[X,Y] = meshgrid(Times,Strike);
```

```
figure;
surf(X,Y,Delta);
title('Delta');
xlabel('Years to Option Expiry');
ylabel('Strike');
view(-112,34);
xlim([0 Times(end)]);
```

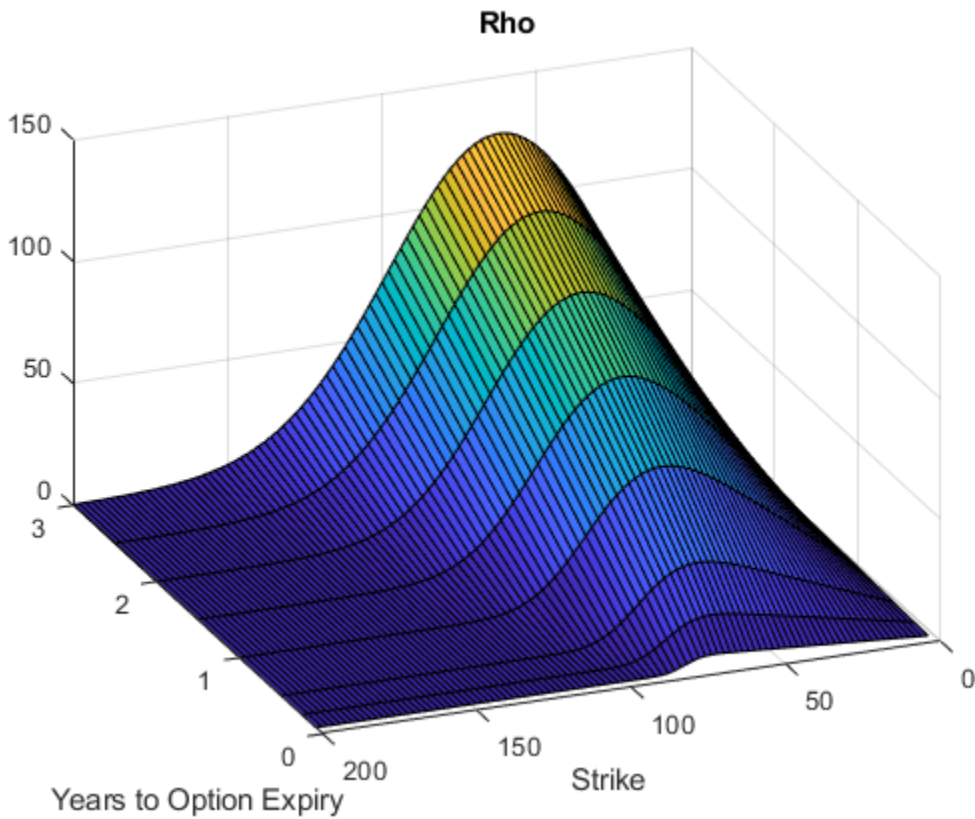




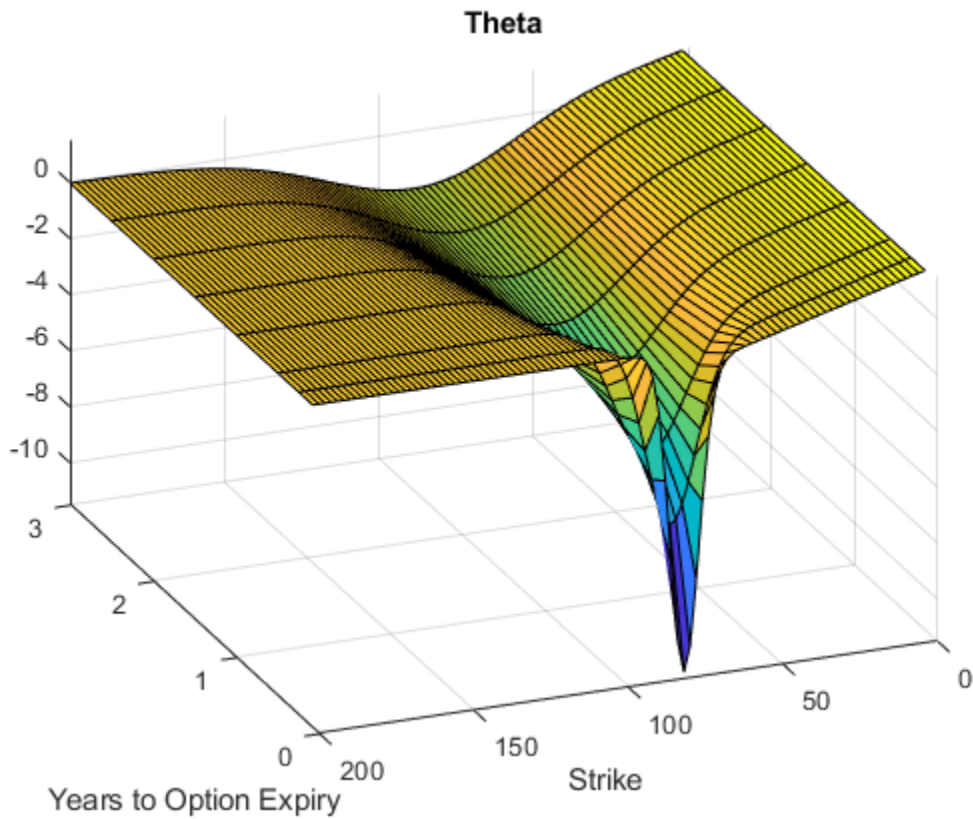
```
figure;  
surf(X,Y,Gamma)  
title('Gamma')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



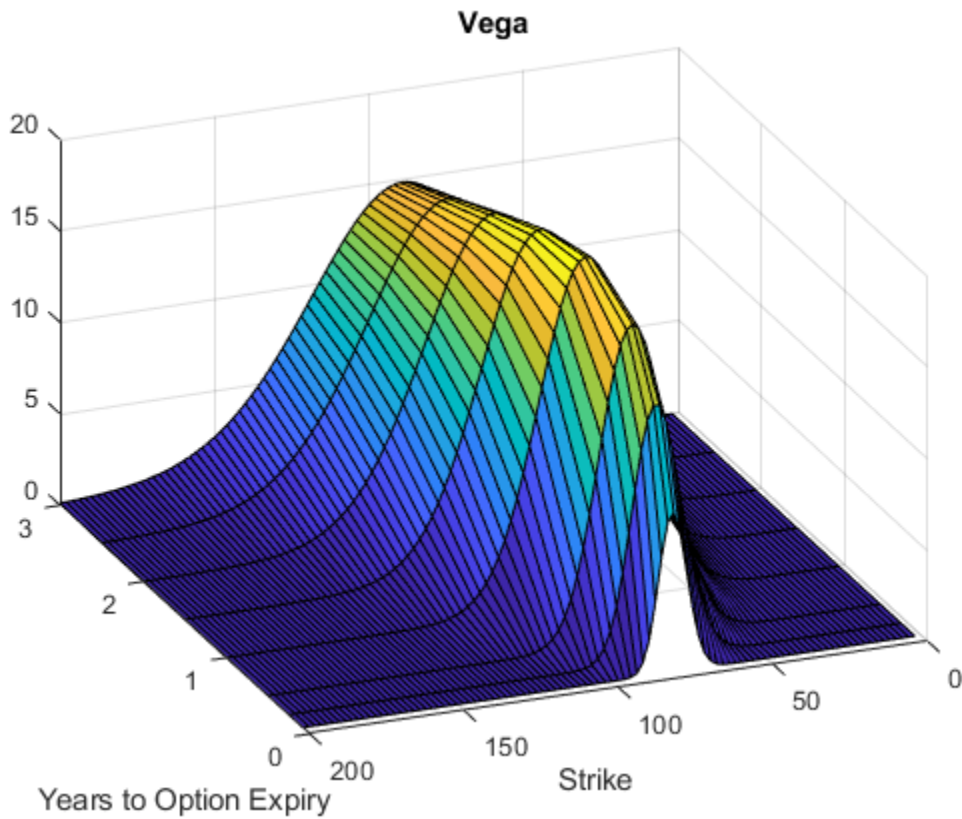
```
figure;
surf(X,Y,Rho)
title('Rho')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



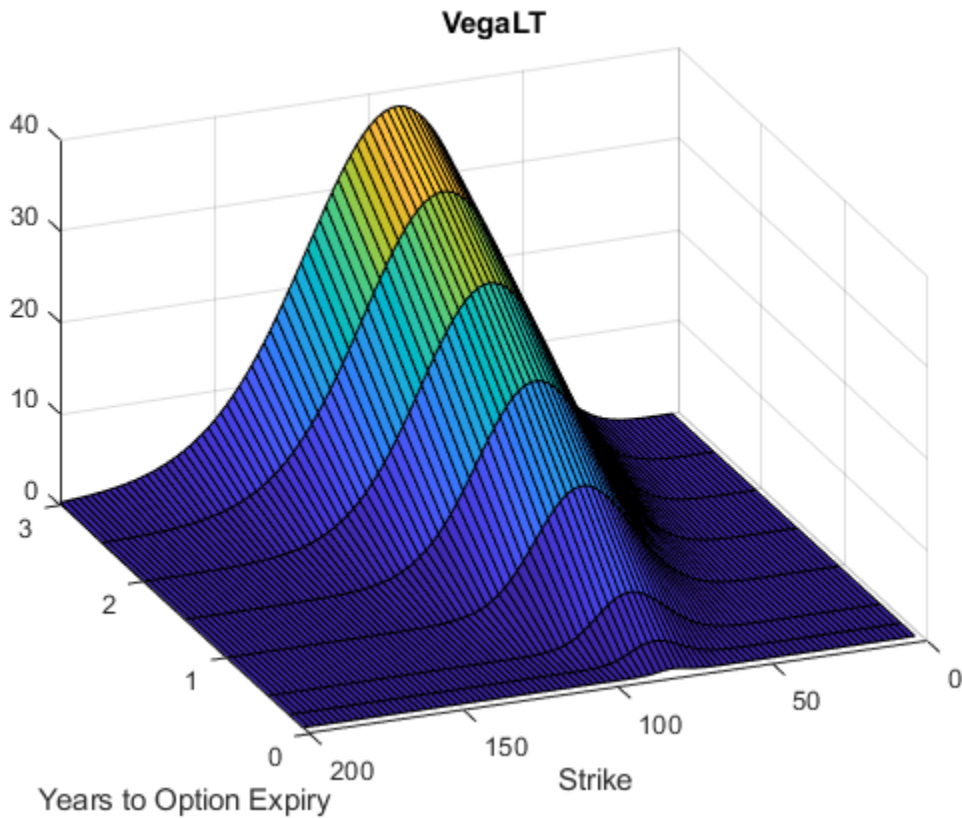
```
figure;  
surf(X,Y,Theta)  
title('Theta')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



```
figure;
surf(X,Y,Vega)
title('Vega')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



```
figure;  
surf(X,Y,VegaLT)  
title('VegaLT')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



## Input Arguments

**Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

**AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

If this input is an empty array (`[]`), option prices are computed on the entire FFT (or FRFT) strike grid, which is determined as `exp(log-strike grid)`. Each column of the log-strike grid has 'NumFFT' points with 'LogStrikeStep' spacing that are roughly centered around each element of `log(AssetPrice)`.

For more information on the proper dimensions for `Strike`, see the name-value pair argument `ExpandOutput`.

Data Types: double

### **V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

### **SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric



Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `[PriceSens,StrikeOut] = optSensByHestonFFT(Rate, AssetPrice,Settle,Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,Rho SV,'Basis',7,'OptSpec',"vega")`

### Basis — Day-count basis of instrument

0 (default) | numeric values: 0,1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of `'Basis'` and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

**VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double

**LittleTrap — Flag indicating Little Heston Trap formulation**

true (default) | logical with values true or false

Flag indicating Little Heston Trap formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- true — Use the Albrecher *et al* formulation.
- false — Use the original Heston formation.

Data Types: logical

**OutSpec — Define outputs**

["price"] (default) | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegal t" | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegal t'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

---

**Note** "vega" is the sensitivity with respect the initial volatility  $\sqrt{V_0}$ . In contrast, "vegal t" is the sensitivity with respect to the long-term volatility  $\sqrt{\text{Theta}V}$ .

---

Example: OutSpec =  
["price", "delta", "gamma", "vega", "rho", "theta", "vegaltd"]

Data Types: string | cell

### **NumFFT — Number of grid points in the characteristic function variable**

4096 (default) | numeric

Number of grid points in the characteristic function variable and in each column of the log-strike grid, specified as the comma-separated pair consisting of 'NumFFT' and a scalar numeric value.

Data Types: double

### **CharacteristicFcnStep — Characteristic function variable grid spacing**

0.01 (default) | numeric

Characteristic function variable grid spacing, specified as the comma-separated pair consisting of 'CharacteristicFcnStep' and a scalar numeric value.

Data Types: double

### **LogStrikeStep — Log-strike grid spacing**

$2\pi/\text{NumFFT}/\text{CharacteristicFcnStep}$  (default) | numeric

Log-strike grid spacing, specified as the comma-separated pair consisting of 'LogStrikeStep' and a scalar numeric value.

---

**Note** If  $(\text{LogStrikeStep} \times \text{CharacteristicFcnStep})$  is  $2\pi/\text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

---

Data Types: double

### **DampingFactor — Damping factor for Carr-Madan formulation**

1.5 (default) | numeric

Damping factor for Carr-Madan formulation, specified as the comma-separated pair consisting of 'DampingFactor' and a scalar numeric value.

Data Types: double

**Quadrature — Type of quadrature**

`simpson` (default) | character vector with values: `simpson` or `trapezoidal` | string array with values: `simpson` or `trapezoidal`

Type of quadrature, specified as the comma-separated pair consisting of 'Quadrature' and a single character vector or string array with a value of `simpson` or `trapezoidal`.

Data Types: `char` | `string`

**ExpandOutput — Flag to expand the outputs**

`false` (outputs are NINST-by-1 vectors) (default) | logical with value of `true` or `false`

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- `true` — If `true`, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the `Strike` input. For example, `Strike` can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. If `Strike` is empty, NRows is equal to NumFFT. NColumns is determined by the sizes of `AssetPrice`, `Settle`, `Maturity`, and `OptSpec`, which must all be either scalar or NColumns-by-1 vectors.
- `false` — If `false`, the outputs are NINST-by-1 vectors. Also, the inputs `Strike`, `AssetPrice`, `Settle`, `Maturity`, and `OptSpec` must all be either scalar or NINST-by-1 vectors.

Data Types: `logical`

## Output Arguments

**PriceSens — Option prices or sensitivities**

numeric

Option prices or sensitivities, returned as a NINST-by-1, or NRows-by-NColumns, depending on `ExpandOutput`. The name-value pair argument `OutSpec` determines the types and order of the outputs.

**StrikeOut — Strikes corresponding to Price**

numeric

Strikes corresponding to `Price`, returned as a NINST-by-1, or NRows-by-NColumns, depending on `ExpandOutput`.

## Definitions

### Heston Stochastic Volatility Model

The Heston model is an extension of the Black-Scholes model, where the volatility (square root of variance) is no longer assumed to be constant, and the variance now follows a stochastic (CIR) process. This process allows modeling the implied volatility smiles observed in the market.

The stochastic differential equation is:

$$\begin{aligned}dS_t &= (r - q)S_t dt + \sqrt{v_t}S_t dW_t \\dv_t &= \kappa(\theta - v_t)dt + \sigma_v\sqrt{v_t}dW_t^v \\E[dW_t dW_t^v] &= \rho dt\end{aligned}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$

$v_0$  is the initial variance of the asset price at  $t = 0$  for ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for the variance for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of the variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

The characteristic function  $f_{Heston_j}(\phi)$  for  $j = 1$  (asset price measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Heston_j}(\phi) = \exp(C_j + D_j v_0 + i\phi \ln S_t)$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi + d_j)\tau - 2\ln \left( \frac{1 - g_j e^{d_j\tau}}{1 - g_j} \right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right)$$

$$g_j = \frac{b_j - p\sigma_v i\phi + d_j}{b_j - p\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - p\sigma_v i\phi)^2 - \sigma_v^2(2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - p\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under “The Little Heston Trap” by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln \left( \frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j} \right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Carr-Madan Formulation

The Carr and Madan (1999) formulation is a popular modified implementation of Heston (1993) framework.

Rather than computing the probabilities  $P_1$  and  $P_2$  as intermediate steps, Carr and Madan developed an alternative expression so that taking its inverse Fourier transform gives the option price itself directly.

$$Call(k) = \frac{e^{-\alpha k}}{\pi} \int_0^{\infty} \text{Re}[e^{-iuk} \psi(u)] du$$

$$\psi(u) = \frac{e^{-r\tau} f_2(\phi = (u - (\alpha + 1)i))}{\alpha^2 + \alpha - u^2 + iu(2\alpha + 1)}$$

$$Put(K) = Call(K) + Ke^{-r\tau} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$\alpha$  is the damping factor.

$u$  is the characteristic function variable for integration, where  $\phi = (u - (\alpha+1)i)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

To apply FFT or FRFT to this formulation, the characteristic function variable for integration,  $u$ , is discretized into NumFFT( $N$ ) points with the step size CharacteristicFcnStep ( $\Delta u$ ), and the log-strike  $k$  is discretized into  $N$  points with the step size LogStrikeStep( $\Delta k$ ).

The discretized characteristic function variable for integration,  $u_j$ (for  $j = 1, 2, 3, \dots, N$ ), has a minimum value of 0 and a maximum value of  $(N-1) (\Delta u)$ , and it approximates the continuous integration range from 0 to infinity.

The discretized log-strike grid,  $k_n$ (for  $n = 1, 2, 3, N$ ) is approximately centered around  $\ln(S_t)$ , with a minimum value of

$$\ln(S_t) - \frac{N}{2}\Delta k$$

and a maximum value of

$$\ln(S_t) + \left(\frac{N}{2} - 1\right)\Delta k$$

Where the minimum allowable strike is

$$S_t \exp\left(-\frac{N}{2}\Delta k\right)$$

and the maximum allowable strike is

$$S_t \exp\left[\left(\frac{N}{2} - 1\right)\Delta k\right]$$

As a result of the discretization, the expression for the call option becomes

$$Call(k_n) = \Delta u \frac{e^{-\alpha k_n}}{\pi} \sum_{j=1}^N \operatorname{Re} \left[ e^{-i\Delta k \Delta u (j-1)(n-1)} e^{iu_j \left[ \frac{N\Delta k}{2} - \ln(S_t) \right]} \psi(u_j) \right] w_j$$

where

$\Delta u$  is the step size of discretized characteristic function variable for integration.

$\Delta k$  is the step size of discretized log-strike.

$N$  is the number of FFT or FRFT points.



$w_j$  is the weights for quadrature used for approximating the integral.

FFT is used to evaluate the above expression if  $\Delta k$  and  $\Delta u$  are subject to the following constraint:

$$\Delta k \Delta u = \left( \frac{2\pi}{N} \right)$$

otherwise, the functions use the FRFT method described in Chourdakis (2005).

## References

- [1] Albrecher, H., Mayer, P., Schoutens, W., and Tistaert, J. "The Little Heston Trap." Working Paper, Linz and Graz University of Technology, K.U. Leuven, ING Financial Markets, 2006.
- [2] Carr, P. and D.B. Madan. "Option Valuation using the Fast Fourier Transform." *Journal of Computational Finance*. Vol 2. No. 4. 1999.
- [3] Chourdakis, K. "Option Pricing Using Fractional FFT." *Journal of Computational Finance*. 2005.
- [4] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6. No. 2. 1993.

## See Also

optByBatesFFT | optByBatesNI | optByHestonFFT | optByHestonNI |  
 optByMertonFFT | optByMertonNI | optSensByBatesFFT | optSensByBatesNI |  
 optSensByHestonNI | optSensByMertonFFT | optSensByMertonNI

**Introduced in R2018a**

## optByHestonNI

Option price by Heston model using numerical integration

### Syntax

```
Price = optByHestonNI(Rate,AssetPrice,Settle,Maturity,OptSpec,  
Strike,V0,ThetaV,Kappa,SigmaV,RhoSV)  
Price = optByHestonNI( ____,Name,Value)
```

### Description

Price = optByHestonNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV) computes vanilla European option price by Heston model, using numerical integration methods.

Price = optByHestonNI( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Workflow for Plotting an Option Price Surface Using the Heston Model

optByHestonNI uses numerical integration to compute option prices and then to plot an option price surface.

#### Define Option Variables and Heston Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
V0 = 0.04;  
ThetaV = 0.05;
```

```
Kappa = 1.0;
SigmaV = 0.2;
RhoSV = -0.7;
```

### Compute the Option Price for a Single Strike

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;
```

```
Call = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield)
```

```
Call = 4.7007
```

### Compute the Option Prices for a Vector of Strikes

The Strike input can be a vector.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';
```

```
Call = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield)
```

```
Call = 5×1
```

```
7.0401
5.8053
4.7007
3.7316
2.8991
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the Strike input to specify the strikes. Also, the Maturity input can be a vector, but it must match the length of the Strike vector if the ExpandOutput name-value pair argument is not set to "true".

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes
```

```
Call = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield)
```

```
Call = 5×1
```

```
    8.9560
    9.3419
    9.6240
    9.8560
   10.0500
```

```
% Five values in vector output
```

### Expand the Output for a Surface

Set the ExpandOutput name-value pair argument to "true" to expand the output into a NStrikes-by-NMaturities matrix. In this case, it is a square matrix.

```
Call = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'ExpandOutput', true) % (5 x 5) matrix output
```

```
Call = 5×5
```

```
    8.9560    10.4543    11.7058    12.8009    13.7728
    7.7946     9.3419    10.6337    11.7644    12.7685
    6.7244     8.3028     9.6240    10.7828    11.8134
    5.7474     7.3378     8.6771     9.8560    10.9074
    4.8645     6.4474     7.7930     8.9840    10.0500
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of Different Lengths

When ExpandOutput is "true", NStrikes do not have to match NMaturities (that is, the output NStrikes-by-NMaturities matrix can be rectangular).

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes
```

```
Call = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'ExpandOutput', true) % (5 x 6) matrix output
```

```
Call = 5×6
```

```

7.0401    8.9560   10.4543   11.7058   12.8009   13.7728
5.8053    7.7946    9.3419   10.6337   11.7644   12.7685
4.7007    6.7244    8.3028    9.6240   10.7828   11.8134
3.7316    5.7474    7.3378    8.6771    9.8560   10.9074
2.8991    4.8645    6.4474    7.7930    8.9840   10.0500

```

### Compute the Option Prices for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

```

```

Call = optByHestonNI(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'ExpandOutput', true) % (5 x 4) matrix output

```

```
Call = 5×4
```

```

3.2944    5.8047    8.9560   12.6052
2.6413    4.8810    7.7946   11.2507
2.0864    4.0575    6.7244    9.9738
1.6230    3.3325    5.7474    8.7783
1.2429    2.7028    4.8645    7.6676

```

### Plot an Option Price Surface

The `Strike` and `Maturity` inputs can be vectors. Set `ExpandOutput` to "true" to output the surface as a `NStrikes-by-NMaturities` matrix.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

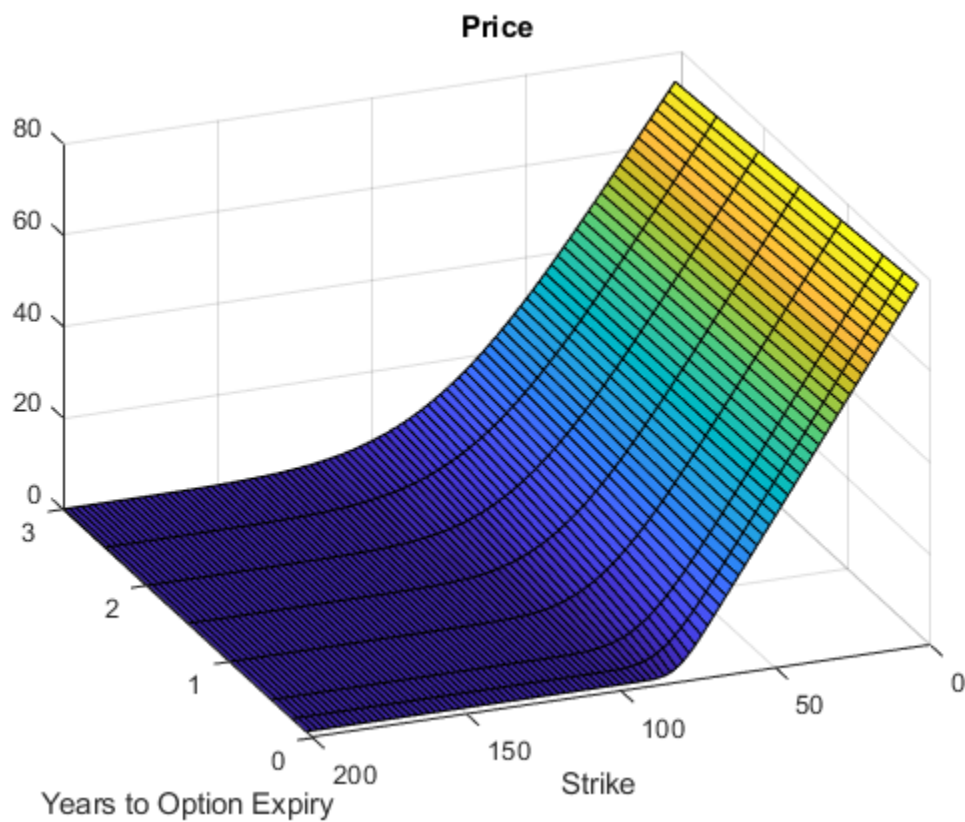
```

```

Call = optByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'ExpandOutput', true);

```

```
[X,Y] = meshgrid(Times,Strike);  
  
figure;  
surf(X,Y,Call);  
title('Price');  
xlabel('Years to Option Expiry');  
ylabel('Strike');  
view(-112,34);  
xlim([0 Times(end)]);  
zlim([0 80]);
```



## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a `NINST-by-1` or `NColumns-by-1` vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle** — Option settlement date

`serial date number` | `date character vector` | `datetime` | `string array`

Option settlement date, specified as a `NINST-by-1` or `NColumns-by-1` vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity** — Option maturity date

`serial date number` | `date character vector` | `datetime` | `string array`

Option maturity date, specified as a `NINST-by-1` or `NColumns-by-1` vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec** — Definition of option

`cell array of character vector with values 'call' or 'put'` | `string array with values 'call' or 'put'`

Definition of the option as `'call'` or `'put'`, specified as a `NINST-by-1` or `NColumns-by-1` vector using a cell array of character vectors or string arrays with values `"call"` or `"put"`.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`



**Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

For more information on the proper dimensions for `Strike`, see the name-value pair argument `ExpandOutput`.

Data Types: double

**V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

**ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

**Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

**SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

**RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `Price = optByHestonNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV,'Basis',7,'Framework','lewis2001')`

### **Basis** — Day-count basis of instrument

0 (default) | numeric values: 0,1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of `'Basis'` and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

**VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double

**LittleTrap — Flag indicating Little Heston Trap formulation**

true (default) | logical with values true or false

Flag indicating Little Heston Trap formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- true — Use the Albrecher *et al* formulation.
- false — Use the original Heston formation.

Data Types: logical

**AbsTol — Absolute error tolerance for numerical integration**

1e-10 (default) | numeric

Absolute error tolerance for numerical integration, specified as the comma-separated pair consisting of 'AbsTol' and a scalar numeric value.

Data Types: double

**RelTol — Relative error tolerance for numerical integration**

1e-6 (default) | numeric

Relative error tolerance for numerical integration, specified as the comma-separated pair consisting of 'RelTol' and a scalar numeric value.

Data Types: double

**IntegrationRange — Numerical integration range used to approximate continuous integral over [0 Inf]**

[1e-9 Inf] (default) | vector

Numerical integration range used to approximate the continuous integral over [0 Inf], specified as the comma-separated pair consisting of 'IntegrationRange' and a 1-by-2 vector representing [LowerLimit UpperLimit].

Data Types: double

**Framework — Framework for computing option prices and sensitivities using numerical integration of models**

"heston1993" (default) | string with values "heston1993" or "lewis2001" | character vector with values 'heston1993' or 'lewis2001'

Framework for computing option prices and sensitivities using numerical integration of models, specified as the comma-separated pair consisting of 'Framework' and a scalar string or character vector with the following values:

- "heston1993" or 'heston1993' — Method used in Heston (1993)
- "lewis2001" or 'lewis2001' — Method used in Lewis (2001)

Data Types: char | string

**ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- true — If true, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. NColumns is determined by the sizes of AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.
- false — If false, the outputs are NINST-by-1 vectors. Also, the inputs Strike, AssetPrice, Settle, Maturity, and OptSpec must all be either scalar or NINST-by-1 vectors.

Data Types: logical

## Output Arguments

### Price — Option prices

numeric

Option prices, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput.

## Definitions

### Heston Stochastic Volatility Model

The Heston model is an extension of the Black-Scholes model, where the volatility (square root of variance) is no longer assumed to be constant, and the variance now follows a stochastic (CIR) process. This allows modeling the implied volatility smiles observed in the market.

The stochastic differential equation is:

$$dS_t = (r - q)S_t dt + \sqrt{v_t}S_t dW_t$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v$$

$$E[dW_t dW_t^v] = \rho dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$

$v_0$  is the initial variance of the asset price at  $t = 0$  for ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for the variance for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of the variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

The characteristic function  $f_{Heston_j}(\phi)$  for  $j = 1$  (asset price measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Heston_j}(\phi) = \exp(C_j + D_j v_0 + i\phi \ln S_t)$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - \rho\sigma_v i\phi + d_j)\tau - 2\ln \left( \frac{1 - g_j e^{d_j\tau}}{1 - g_j} \right) \right]$$

$$D_j = \frac{b_j - \rho\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right)$$

$$g_j = \frac{b_j - \rho\sigma_v i\phi + d_j}{b_j - \rho\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - \rho\sigma_v i\phi)^2 - \sigma_v^2(2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - \rho\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under “The Little Heston Trap” by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln\left(\frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Numerical Integration Method Under Heston (1993) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Heston (1993) framework is based on the following expressions:

$$Call(K) = S_t e^{-q\tau} P_1 - K e^{-r\tau} P_2$$

$$Put(K) = Call(K) + K e^{-r\tau} - S_t e^{-q\tau}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \text{Re} \left[ \frac{e^{-i\phi \ln(K)} f_j(\phi)}{i\phi} \right] d\phi$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$f_j(\phi)$  is the characteristic function for  $P_j$  ( $j = 1, 2$ ).

$P_1$  is the probability of  $S_t > K$  under the asset price measure for the model.

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

Where  $j = 1, 2$  so that  $f_1(\phi)$  and  $f_2(\phi)$  are the characteristic functions for probabilities  $P_1$  and  $P_2$ , respectively.

This framework is chosen with the default value “Heston1993” for the Framework name-value pair argument.

## Numerical Integration Method Under Lewis (2001) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Lewis (2001) framework is based on the following expressions:

$$Call(k) = S_t e^{-q\tau} - \frac{\sqrt{K} e^{-\tau t}}{\pi} \int_0^{\infty} \operatorname{Re} \left[ K^{-iu} f_2 \left( \phi = \left( u - \frac{i}{2} \right) \frac{1}{u^2 + \frac{1}{4}} \right) \right] du$$

$$Put(K) = Call(K) = K e^{-\tau t} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T - t$ ).

$Call(K)$  is the call price at strike  $K$ .



$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$u$  is the characteristic function variable for integration, where  $\phi = \left(u - \frac{i}{2}\right)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

This framework is chosen with the value "Lewis2001" for the Framework name-value pair argument.

## References

- [1] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6. No. 2. 1993.
- [2] Lewis, A. L. "A Simple Option Formula for General Jump-Diffusion and Other Exponential Levy Processes." *Envision Financial Systems and OptionCity.net*, 2001.

## See Also

[optByBatesFFT](#) | [optByBatesNI](#) | [optByHestonFFT](#) | [optByMertonFFT](#) | [optByMertonNI](#) | [optSensByBatesFFT](#) | [optSensByBatesNI](#) | [optSensByHestonFFT](#) | [optSensByHestonNI](#) | [optSensByMertonFFT](#) | [optSensByMertonNI](#)

**Introduced in R2018a**

## optSensByHestonNI

Option price and sensitivities by Heston model using numerical integration

### Syntax

```
PriceSens = optSensByHestonNI(Rate,AssetPrice,Settle,Maturity,  
OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV)  
PriceSens = optSensByHestonNI( ____,Name,Value)
```

### Description

`PriceSens = optSensByHestonNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,V0,ThetaV,Kappa,SigmaV,RhoSV)` computes vanilla European option price and sensitivities by Heston model, using numerical integration methods.

`PriceSens = optSensByHestonNI( ____,Name,Value)` adds optional name-value pair arguments.

### Examples

#### Workflow for Plotting an Option Sensitivity Surface Using the Heston Model

`optSensByHestonNI` uses numerical integration to compute option sensitivities and then to plot option sensitivity surfaces.

#### Define Option Variables and Heston Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
V0 = 0.04;  
ThetaV = 0.05;
```

```
Kappa = 1.0;
SigmaV = 0.2;
RhoSV = -0.7;
```

### Compute the Option Sensitivity for a Single Strike

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;
```

```
Delta = optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'OutSpec', "delta");
```

```
Delta = 0.5775
```

### Compute the Option Sensitivities for a Vector of Strikes

The Strike input can be a vector.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';
```

```
Delta = optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, 'OutSpec', "delta");
```

```
Delta = 5×1
```

```
0.7043
0.6433
0.5775
0.5083
0.4377
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the Strike input to specify the strikes. Also, the Maturity input can be a vector, but it must match the length of the Strike vector if the ExpandOutput name-value pair argument is not set to "true".

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes
```

```
Delta = optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta") % Five values in vector output
```

```
Delta = 5×1
```

```
0.6848
0.6413
0.6095
0.5841
0.5631
```

### Expand the Output for a Surface

Set the ExpandOutput name-value pair argument to "true" to expand the output into a NStrikes-by-NMaturities matrix. In this case, it is a square matrix.

```
Delta = optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'ExpandOutput', true) % (5 x 5) matrix output
```

```
Delta = 5×5
```

```
0.6848    0.6762    0.6703    0.6654    0.6609
0.6416    0.6413    0.6404    0.6390    0.6372
0.5960    0.6048    0.6095    0.6119    0.6129
0.5485    0.5671    0.5776    0.5841    0.5882
0.4997    0.5286    0.5452    0.5559    0.5631
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of Different Lengths

When ExpandOutput is "true", NStrikes do not have to match NMaturities (that is, the output NStrikes-by-NMaturities matrix can be rectangular).

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84); % Five strikes
```

```
Delta = optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'ExpandOutput', true) % (5 x 6) matrix output
```

Delta = 5×6

```

0.7043    0.6848    0.6762    0.6703    0.6654    0.6609
0.6433    0.6416    0.6413    0.6404    0.6390    0.6372
0.5775    0.5960    0.6048    0.6095    0.6119    0.6129
0.5083    0.5485    0.5671    0.5776    0.5841    0.5882
0.4377    0.4997    0.5286    0.5452    0.5559    0.5631

```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Asset Prices

When ExpandOutput is "true", the output can also be a NStrikes-by-NAssetPrices rectangular matrix by accepting a vector of asset prices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

```

```

Delta = optSensByHestonNI(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ..
    V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'ExpandOutput', true) % (5 x 4) matrix output

```

Delta = 5×4

```

0.4293    0.5708    0.6848    0.7705
0.3737    0.5193    0.6416    0.7364
0.3200    0.4668    0.5960    0.6994
0.2693    0.4143    0.5485    0.6597
0.2226    0.3628    0.4997    0.6177

```

### Plot Option Sensitivity Surfaces

The Strike and Maturity inputs can be vectors. Set ExpandOutput to "true" to output the surfaces as NStrikes-by-NMaturities matrices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

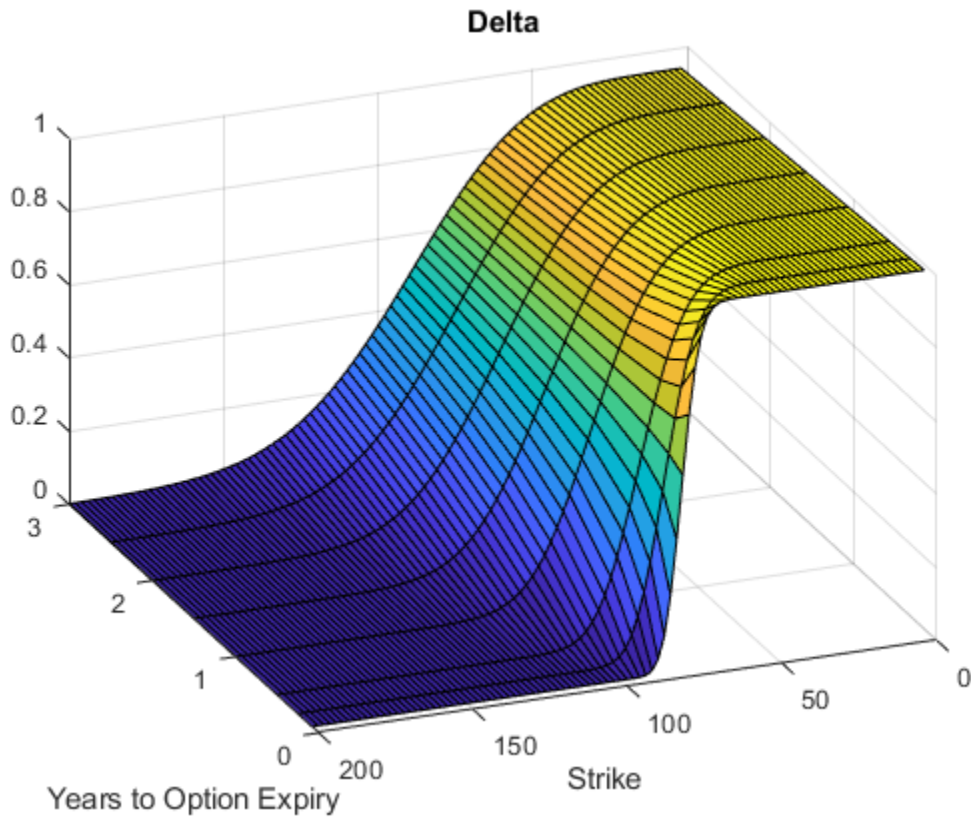
```

```

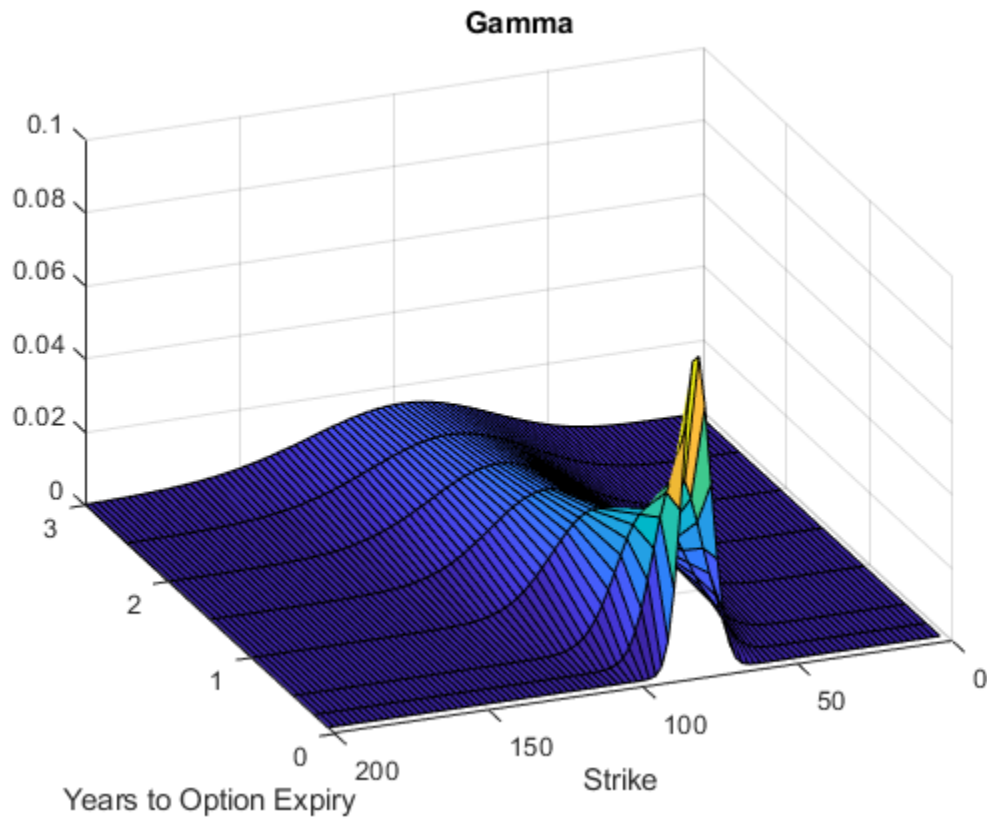
[Delta, Gamma, Rho, Theta, Vega, VegaLT] = ...
    optSensByHestonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...

```

```
V0, ThetaV, Kappa, SigmaV, RhoSV, 'DividendYield', DividendYield, ...  
'OutSpec', ["delta", "gamma", "rho", "theta", "vega", "vegalt"], ...  
'ExpandOutput', true);  
  
[X,Y] = meshgrid(Times,Strike);  
  
figure;  
surf(X,Y,Delta);  
title('Delta');  
xlabel('Years to Option Expiry');  
ylabel('Strike');  
view(-112,34);  
xlim([0 Times(end)]);
```

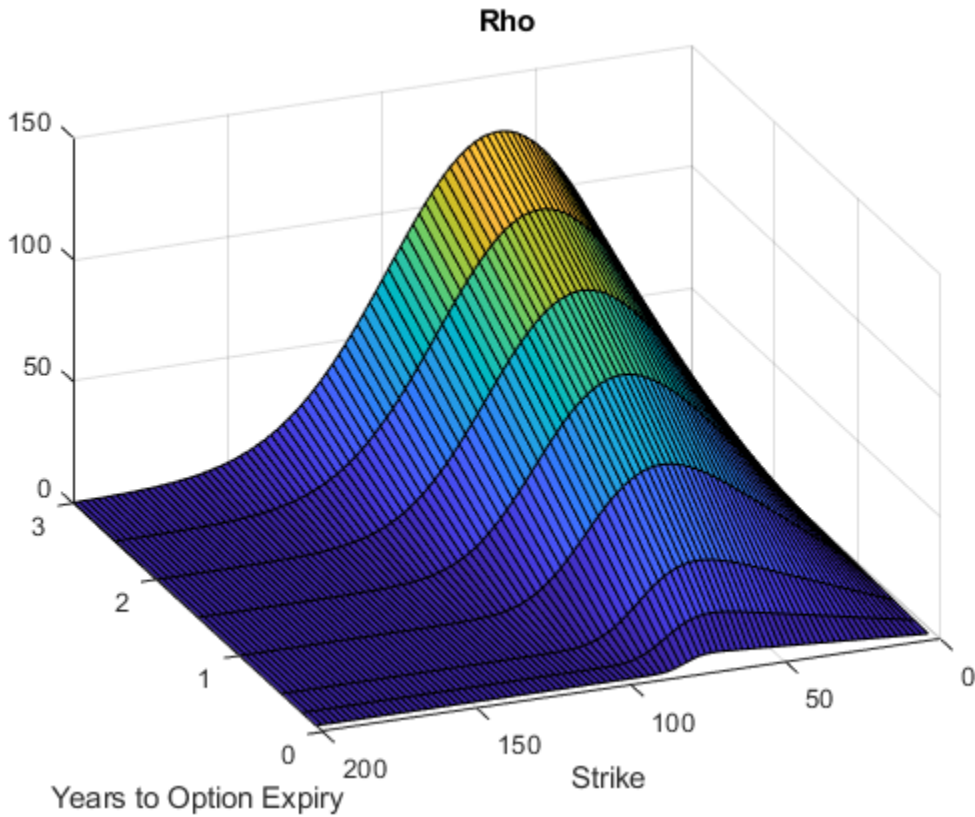


```
figure;  
surf(X,Y,Gamma)  
title('Gamma')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



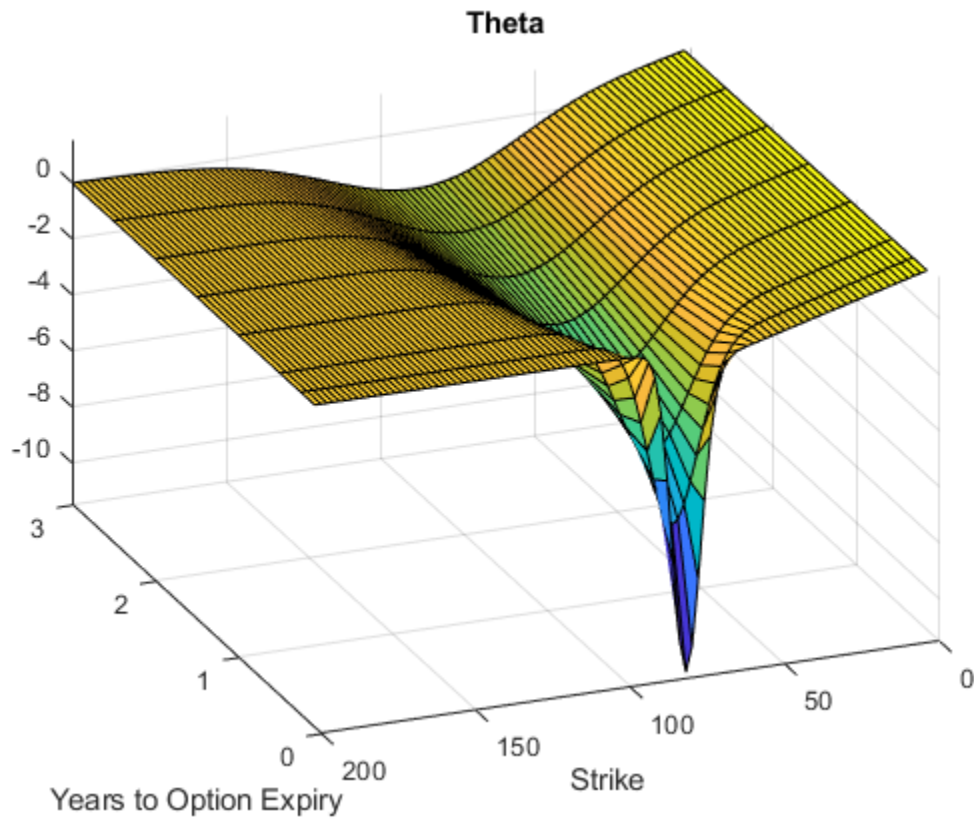
```
figure;  
surf(X,Y,Rho)  
title('Rho')  
xlabel('Years to Option Expiry')
```

```
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```

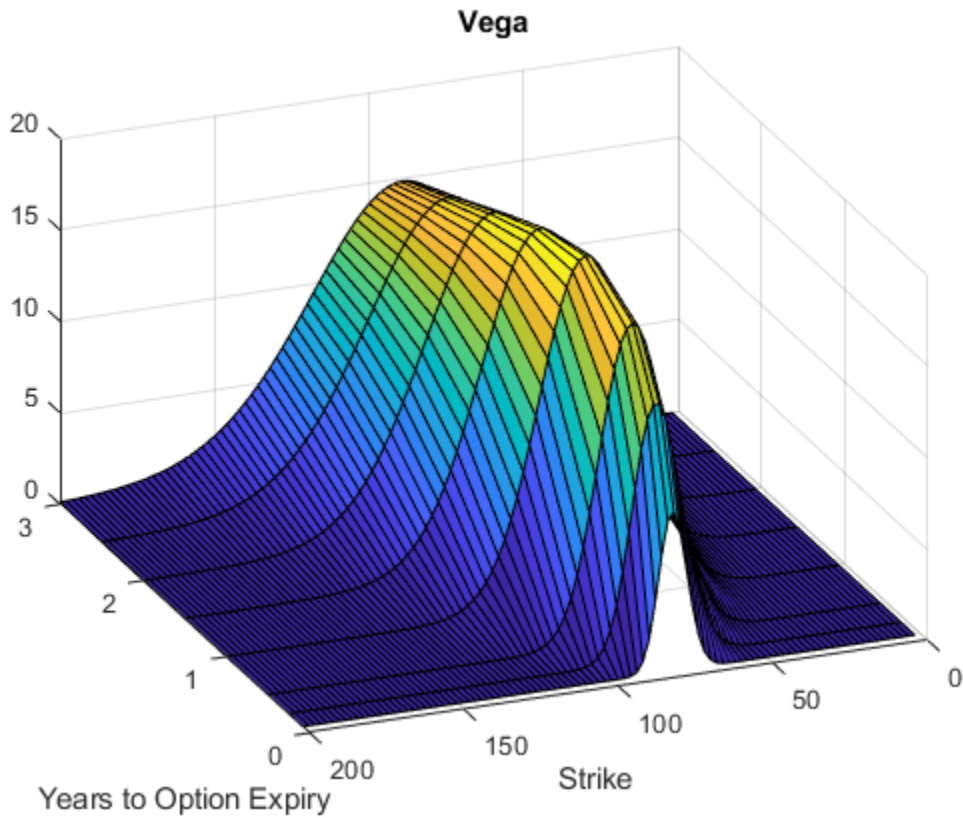


```
figure;
surf(X,Y,Theta)
title('Theta')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```

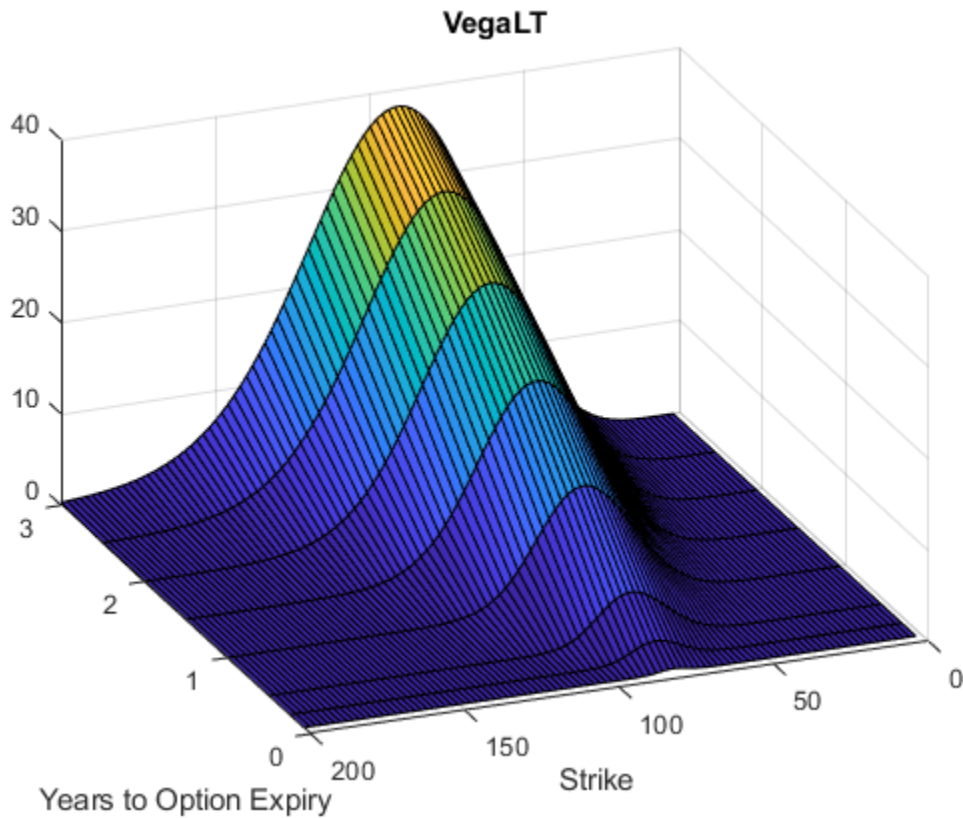




```
figure;  
surf(X,Y,Vega)  
title('Vega')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



```
figure;
surf(X,Y,VegaLT)
title('VegaLT')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

For more information on the proper dimensions for Strike, see the name-value pair argument ExpandOutput.

Data Types: double

### **V0 — Initial variance of underlying asset**

numeric

Initial variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **ThetaV — Long-term variance of underlying asset**

numeric

Long-term variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **Kappa — Mean revision speed for the variance of underlying asset**

numeric

Mean revision speed for the underling asset, specified as a scalar numeric value.

Data Types: double

### **SigmaV — Volatility of the variance of underlying asset**

numeric

Volatility of the variance of the underling asset, specified as a scalar numeric value.

Data Types: double

### **RhoSV — Correlation between Weiner processes for underlying asset and its variance**

numeric

Correlation between the Weiner processes for the underlying asset and its variance, specified as a scalar numeric value.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `PriceSens = optSensByHestonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, V0, ThetaV, Kappa, SigmaV, RhoSV, 'Basis', 7)`

### **Basis** — Day-count basis of instrument

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield** — Continuously compounded underlying asset yield

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

### **VolRiskPremium — Volatility risk premium**

0 (default) | numeric

Volatility risk premium, specified as the comma-separated pair consisting of 'VolRiskPremium' and a scalar numeric value.

Data Types: double

### **LittleTrap — Flag indicating Little Heston Trap formulation**

true (default) | logical with values true or false

Flag indicating Little Heston Trap formulation by Albrecher *et al*, specified as the comma-separated pair consisting of 'LittleTrap' and a logical:

- true — Use the Albrecher *et al* formulation.
- false — Use the original Heston formation.

Data Types: logical

### **OutSpec — Define outputs**

["price"] (default) | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegalt" | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegalt'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

---

**Note** "vega" is the sensitivity with respect the initial volatility  $\sqrt{V_0}$ . In contrast, "vegalt" is the sensitivity with respect to the long-term volatility  $\sqrt{\text{Theta}V}$ .

---

Example: OutSpec =

```
["price","delta","gamma","vega","rho","theta","vegalt"]
```

Data Types: string | cell

**AbsTol — Absolute error tolerance for numerical integration**`1e-10 (default) | numeric`

Absolute error tolerance for numerical integration, specified as the comma-separated pair consisting of 'AbsTol' and a scalar numeric value.

Data Types: double

**RelTol — Relative error tolerance for numerical integration**`1e-6 (default) | numeric`

Relative error tolerance for numerical integration, specified as the comma-separated pair consisting of 'RelTol' and a scalar numeric value.

Data Types: double

**IntegrationRange — Numerical integration range used to approximate continuous integral over [0 Inf]**`[1e-9 Inf] (default) | vector`

Numerical integration range used to approximate the continuous integral over [0 Inf], specified as the comma-separated pair consisting of 'IntegrationRange' and a 1-by-2 vector representing [LowerLimit UpperLimit].

Data Types: double

**Framework — Framework for computing option prices and sensitivities using numerical integration of models**`"heston1993" (default) | string with values "heston1993" or "lewis2001" | character vector with values 'heston1993' or 'lewis2001'`

Framework for computing option prices and sensitivities using numerical integration of models, specified as the comma-separated pair consisting of 'Framework' and a scalar string or character vector with the following values:

- "heston1993" or 'heston1993' — Method used in Heston (1993)
- "lewis2001" or 'lewis2001' — Method used in Lewis (2001)

Data Types: char | string

**ExpandOutput — Flag to expand the outputs**`false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false`



Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- `true` — If `true`, the outputs are `NRows`-by-`NColumns` matrices. `NRows` is the number of strikes for each column and it is determined by the `Strike` input. For example, `Strike` can be a `NRows`-by-1 vector, or a `NRows`-by-`NColumns` matrix. `NColumns` is determined by the sizes of `AssetPrice`, `Settle`, `Maturity`, and `OptSpec`, which must all be either scalar or `NColumns`-by-1 vectors.
- `false` — If `false`, the outputs are `NINST`-by-1 vectors. Also, the inputs `Strike`, `AssetPrice`, `Settle`, `Maturity`, and `OptSpec` must all be either scalar or `NINST`-by-1 vectors.

Data Types: `logical`

## Output Arguments

### PriceSens — Option prices or sensitivities

`numeric`

Option prices or sensitivities, returned as a `NINST`-by-1, or `NRows`-by-`NColumns`, depending on `ExpandOutput`. The name-value pair argument `OutSpec` determines the types and order of the outputs.

## Definitions

### Heston Stochastic Volatility Model

The Heston model is an extension of the Black-Scholes model, where the volatility (square root of variance) is no longer assumed to be constant, and the variance now follows a stochastic (CIR) process. This allows modeling the implied volatility smiles observed in the market.

The stochastic differential equation is:

$$dS_t = (r - q)S_t dt + \sqrt{v_t}S_t dW_t$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v\sqrt{v_t}dW_t^v$$

$$E[dW_t dW_t^v] = \rho dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$v_t$  is the asset price variance at time  $t$ .

$v_0$  is the initial variance of the asset price at  $t = 0$  for ( $v_0 > 0$ ).

$\theta$  is the long-term variance level for ( $\theta > 0$ ).

$\kappa$  is the mean reversion speed for the variance for ( $\kappa > 0$ ).

$\sigma_v$  is the volatility of the variance for ( $\sigma_v > 0$ ).

$\rho$  is the correlation between the Weiner processes  $W_t$  and  $W_t^v$  for ( $-1 \leq \rho \leq 1$ ).

The characteristic function  $f_{Heston_j}(\phi)$  for  $j = 1$  (asset price measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Heston_j}(\phi) = \exp(C_j + D_j v_0 + i\phi \ln S_t)$$

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - \rho\sigma_v i\phi + d_j)\tau - 2\ln \left( \frac{1 - g_j e^{d_j\tau}}{1 - g_j} \right) \right]$$

$$D_j = \frac{b_j - \rho\sigma_v i\phi + d_j}{\sigma_v^2} \left( \frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right)$$

$$g_j = \frac{b_j - \rho\sigma_v i\phi + d_j}{b_j - \rho\sigma_v i\phi - d_j}$$

$$d_j = \sqrt{(b_j - \rho\sigma_v i\phi)^2 - \sigma_v^2(2u_j i\phi - \phi^2)}$$

where for  $j = 1, 2$ :

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, b_1 = \kappa + \lambda_{VolRisk} - \rho\sigma_v, b_2 = \kappa + \lambda_{VolRisk}$$

where

$\phi$  is the characteristic function variable.

$\lambda_{VolRisk}$  is the volatility risk premium.

$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

The definitions for  $C_j$  and  $D_j$  under “The Little Heston Trap” by Albrecher et al. (2007) are:

$$C_j = (r - q)i\phi\tau + \frac{\kappa\theta}{\sigma_v^2} \left[ (b_j - p\sigma_v i\phi - d_j)\tau - 2\ln\left(\frac{1 - \varepsilon_j e^{-d_j\tau}}{1 - \varepsilon_j}\right) \right]$$

$$D_j = \frac{b_j - p\sigma_v i\phi - d_j}{\sigma_v^2} \left( \frac{1 - e^{-d_j\tau}}{1 - \varepsilon_j e^{-d_j\tau}} \right)$$

$$\varepsilon_j = \frac{b_j - p\sigma_v i\phi - d_j}{b_j - p\sigma_v i\phi + d_j}$$

## Numerical Integration Method Under Heston (1993) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Heston (1993) framework is based on the following expressions:

$$Call(K) = S_t e^{-q\tau} P_1 - K e^{-r\tau} P_2$$

$$Put(K) = Call(K) + K e^{-r\tau} - S_t e^{-q\tau}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \text{Re} \left[ \frac{e^{-i\phi \ln(K)} f_j(\phi)}{i\phi} \right] d\phi$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$f_j(\phi)$  is the characteristic function for  $P_j$  ( $j = 1, 2$ ).

$P_1$  is the probability of  $S_t > K$  under the asset price measure for the model.

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

Where  $j = 1, 2$  so that  $f_1(\phi)$  and  $f_2(\phi)$  are the characteristic functions for probabilities  $P_1$  and  $P_2$ , respectively.

This framework is chosen with the default value “Heston1993” for the Framework name-value pair argument.

## Numerical Integration Method Under Lewis (2001) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Lewis (2001) framework is based on the following expressions:

$$Call(k) = S_t e^{-q\tau} - \frac{\sqrt{K} e^{-\tau t}}{\pi} \int_0^{\infty} \operatorname{Re} \left[ K^{-iu} f_2 \left( \phi = \left( u - \frac{i}{2} \right) \frac{1}{u^2 + \frac{1}{4}} \right) \right] du$$

$$Put(K) = Call(K) = K e^{-\tau t} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$u$  is the characteristic function variable for integration, where  $\phi = \left(u - \frac{i}{2}\right)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

This framework is chosen with the value “Lewis2001” for the Framework name-value pair argument.

## References

- [1] Heston, S. L. “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options.” *The Review of Financial Studies*. Vol 6. No. 2. 1993.
- [2] Lewis, A. L. “A Simple Option Formula for General Jump-Diffusion and Other Exponential Levy Processes.” Envision Financial Systems and OptionCity.net, 2001.

## See Also

optByBatesFFT | optByBatesNI | optByHestonFFT | optByHestonNI |  
 optByMertonFFT | optByMertonNI | optSensByBatesFFT | optSensByBatesNI |  
 optSensByHestonFFT | optSensByMertonFFT | optSensByMertonNI

**Introduced in R2018a**

# optByLocalVolFD

Option price by local volatility model, using finite differences

## Syntax

```
[Price,PriceGrid,AssetPrices,Times] = optByLocalVolFD(Rate,  
AssetPrice,Settle,ExerciseDates,OptSpec,Strike,ImpliedVolData)  
[Price,PriceGrid,AssetPrices,Times] = optByLocalVolFD( ____,  
Name,Value)
```

## Description

[Price,PriceGrid,AssetPrices,Times] = optByLocalVolFD(Rate, AssetPrice,Settle,ExerciseDates,OptSpec,Strike,ImpliedVolData) compute a Vanilla European or American option price by the local volatility model, using the Crank-Nicolson method.

[Price,PriceGrid,AssetPrices,Times] = optByLocalVolFD( \_\_\_\_, Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Price a European Option Using the Local Volatility Model

Define the option variables.

```
AssetPrice = 590;  
Strike = 590;  
Rate = 0.06;  
DividendYield = 0.0262;  
Settle = '01-Jan-2018';  
ExerciseDates = '01-Jan-2020';
```

Define the implied volatility surface data.

```
Maturity = ["06-Mar-2018" "05-Jun-2018" "12-Sep-2018" "10-Dec-2018" "01-Jan-2019" ...  
"02-Jul-2019" "01-Jan-2020" "01-Jan-2021" "01-Jan-2022" "01-Jan-2023"];  
Maturity = repmat(Maturity,10,1);  
Maturity = Maturity(:);  
  
ExercisePrice = AssetPrice.*[0.85 0.90 0.95 1.00 1.05 1.10 1.15 1.20 1.30 1.40];  
ExercisePrice = repmat(ExercisePrice,1,10)';  
  
ImpliedVol = [...  
    0.190; 0.168; 0.133; 0.113; 0.102; 0.097; 0.120; 0.142; 0.169; 0.200; ...  
    0.177; 0.155; 0.138; 0.125; 0.109; 0.103; 0.100; 0.114; 0.130; 0.150; ...  
    0.172; 0.157; 0.144; 0.133; 0.118; 0.104; 0.100; 0.101; 0.108; 0.124; ...  
    0.171; 0.159; 0.149; 0.137; 0.127; 0.113; 0.106; 0.103; 0.100; 0.110; ...  
    0.171; 0.159; 0.150; 0.138; 0.128; 0.115; 0.107; 0.103; 0.099; 0.108; ...  
    0.169; 0.160; 0.151; 0.142; 0.133; 0.124; 0.119; 0.113; 0.107; 0.102; ...  
    0.169; 0.161; 0.153; 0.145; 0.137; 0.130; 0.126; 0.119; 0.115; 0.111; ...  
    0.168; 0.161; 0.155; 0.149; 0.143; 0.137; 0.133; 0.128; 0.124; 0.123; ...  
    0.168; 0.162; 0.157; 0.152; 0.148; 0.143; 0.139; 0.135; 0.130; 0.128; ...  
    0.168; 0.164; 0.159; 0.154; 0.151; 0.147; 0.144; 0.140; 0.136; 0.132];  
  
ImpliedVolData = table(Maturity, ExercisePrice, ImpliedVol);  
  
Compute the European call option price.  
  
OptSpec = 'Call';  
Price = optByLocalVolFD(Rate, AssetPrice, ...  
Settle, ExerciseDates, OptSpec, Strike, ImpliedVolData, 'DividendYield',DividendYield)  
  
Price = 65.5302
```

## Input Arguments

### Rate — Continuously compounded risk-free interest rate

scalar numeric

Continuously compounded risk-free interest rate, specified by a scalar numeric.

Data Types: double

### AssetPrice — Current underlying asset price

scalar numeric



Current underlying asset price, specified as a scalar numeric.

Data Types: double

### **Settle — Settlement date**

serial date number | date character vector | datetime array | string array

Settlement date, specified as a scalar serial date number, date character vector, datetime array, or string array.

Data Types: double | char | datetime | string

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime array | string array

Option exercise dates, specified as a serial date number, a date character vector, a datetime array, or a string array:

- For a European option, there is only one `ExerciseDates` value and this is the option expiry date.
- For an American option, use a 1-by-2 vector of serial date numbers, date character vectors, datetime arrays, or string arrays. The American option can be exercised on any date between or including the pair of dates. If only one non-`NaN` date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: double | char | cell | datetime | string

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a character vector or string array with values 'call' or 'put'.

Data Types: char | string

### **Strike — Option strike price value**

nonnegative scalar

Option strike price value, specified as a nonnegative scalar.

Data Types: double

**ImpliedVolData — Table of maturity dates, strike or exercise prices, and corresponding implied volatilities**

table

Table of maturity dates, strike or exercise prices, and their corresponding implied volatilities, specified as a NVOL-by-3 table.

Data Types: table

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Price = optByLocalVolFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, ImpliedVolData, 'AssetGridSize', 1000)

**Basis — Day-count basis**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a scalar using one of the supported values:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates` = [ ] or do not enter them. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

### **DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and a NDIV-by-1 vector.

For each dividend amount, there must be a corresponding `ExDividendDates` date. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates` = [ ] or do not enter them.

---

Data Types: double

### **ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | datetime array | string array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and a NDIV-by-1 vector.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, AssetPriceMax values are calculated using asset distributions at maturity (default) | positive scalar

Maximum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar.

Data Types: double

**AssetGridSize — Size of asset grid for finite difference grid**

400 (default) | positive scalar

Size of the asset grid for a finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a positive scalar.

Data Types: double

**TimeGridSize — Size of time grid for finite difference grid**

100 (default) | positive scalar

Size of the time grid for a finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive scalar.

Data Types: double

**AmericanOpt — Option type**

0 (European) (default) | scalar with values [0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a positive integer scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

**InterpMethod — Method of interpolation for estimating the implied volatility surface from ImpliedVolData**

'linear' (default) | character vector with values 'linear', 'makima', 'spline', or 'tpaps' | string with values "linear", "makima", "spline", or "tpaps"

Method of interpolation for estimating the implied volatility surface from `ImpliedVolData`, specified as the comma-separated pair consisting of `'InterpMethod'` and a character vector or string array with one of the following values:

- `'linear'` — Linear interpolation
- `'makima'` — Modified Akima cubic Hermite interpolation
- `'spline'` — Cubic spline interpolation
- `'tpaps'` — Thin-plate smoothing spline interpolation

---

**Note** The `'tpaps'` method uses the thin-plate smoothing spline functionality from Curve Fitting Toolbox.

The `'makima'` and `'spline'` methods work only for gridded data. For scattered data, use the `'linear'` or `'tpaps'` methods.

---

For more information on gridded or scattered data and details on interpolation methods, see “Gridded and Scattered Sample Data” (MATLAB) and “Interpolating Gridded Data” (MATLAB).

Data Types: `char` | `string`

## Output Arguments

### **Price — Option price**

scalar numeric

Option price, returned as a scalar numeric.

### **PriceGrid — Grid containing prices calculated by finite difference method**

grid

Grid containing prices calculated by the finite difference method, returned as a grid that is two-dimensional with size `AssetGridSize` × `TimeGridSize`. The number of columns does not have to be equal to the `TimeGridSize`, because `ExerciseDates` and `ExDividendDates` are added to the time grid. `PriceGrid(:, :, end)` contains the price for  $t = 0$ .

### **AssetPrices — Prices of asset**

vector

Prices of the asset corresponding to the first dimension of PriceGrid, returned as a vector.

**Times — Times**

vector

Times corresponding to second dimension of the PriceGrid, returned as a vector.

## Definitions

### Local Volatility Model

A local volatility model treats volatility as a function both of the current asset level and of time.

The local volatility can be estimated by using the Dupire formula [2]:

$$\sigma_{loc}^2(K, \tau) = \frac{\sigma_{imp}^2 + 2\tau\sigma_{imp}\frac{\partial\sigma_{imp}}{\partial\tau} + 2(\tau - d)K\tau\sigma_{imp}\frac{\partial\sigma_{imp}}{\partial K}}{\left(1 + Kd_1\sqrt{\tau}\frac{\partial\sigma_{imp}}{\partial K}\right)^2 + K^2\tau\sigma_{imp}\left(\frac{\partial^2\sigma_{imp}}{\partial K^2} - d_1\sqrt{\tau}\left(\frac{\partial\sigma_{imp}}{\partial K}\right)^2\right)}$$

$$d_1 = \frac{\ln(S_0/K) + ((\tau - d) + \sigma_{imp}^2/2)\tau}{\sigma_{imp}\sqrt{\tau}}$$

### References

- [1] Andersen, L. B., and R. Brotherton-Ratcliffe. "The Equity Option Volatility Smile: An Implicit Finite-Difference Approach." *Journal of Computational Finance*. Vol. 1, Number 2, 1997, pp. 5-37.
- [2] Dupire, B. "Pricing with a Smile." *Risk*. Vol. 7, Number 1, 1994, pp. 18-20.

### See Also

optByBatesFD | optByHestonFD | optByMertonFD | optBySensMertonFD |  
 optSensByBatesFD | optSensByHestonFD | optSensByLocalVolFD | optstockbyfd  
 | optstocksensbyfd

## **Topics**

“Supported Equity Derivatives” on page 3-24

**Introduced in R2018b**

## optSensByLocalVolFD

Option price and sensitivities by local volatility model, using finite differences

### Syntax

```
[PriceSens,PriceGrid,AssetPrices,Times] = optSensByLocalVolFD(Rate,  
AssetPrice,Settle,ExerciseDates,OptSpec,Strike,ImpliedVolData)  
[PriceSens,PriceGrid,AssetPrices,Times] = optSensByLocalVolFD( ____,  
Name,Value)
```

### Description

```
[PriceSens,PriceGrid,AssetPrices,Times] = optSensByLocalVolFD(Rate,  
AssetPrice,Settle,ExerciseDates,OptSpec,Strike,ImpliedVolData)  
compute option price and sensitivities by the local volatility model, using the Crank-  
Nicolson method.
```

```
[PriceSens,PriceGrid,AssetPrices,Times] = optSensByLocalVolFD( ____,  
Name,Value) specifies options using one or more name-value pair arguments in addition  
to the input arguments in the previous syntax.
```

### Examples

#### Compute European Option Price and Sensitivities Using the Local Volatility Model

Define the option variables.

```
AssetPrice = 590;  
Strike = 590;  
Rate = 0.06;  
DividendYield = 0.0262;  
Settle = '01-Jan-2018';  
ExerciseDates = '01-Jan-2020';
```



Define the implied volatility surface data.

```
Maturity = ["06-Mar-2018" "05-Jun-2018" "12-Sep-2018" "10-Dec-2018" "01-Jan-2019" ...
"02-Jul-2019" "01-Jan-2020" "01-Jan-2021" "01-Jan-2022" "01-Jan-2023"];
Maturity = repmat(Maturity,10,1);
Maturity = Maturity(:);
```

```
ExercisePrice = AssetPrice.*[0.85 0.90 0.95 1.00 1.05 1.10 1.15 1.20 1.30 1.40];
ExercisePrice = repmat(ExercisePrice,1,10)';
```

```
ImpliedVol = [...
0.190; 0.168; 0.133; 0.113; 0.102; 0.097; 0.120; 0.142; 0.169; 0.200; ...
0.177; 0.155; 0.138; 0.125; 0.109; 0.103; 0.100; 0.114; 0.130; 0.150; ...
0.172; 0.157; 0.144; 0.133; 0.118; 0.104; 0.100; 0.101; 0.108; 0.124; ...
0.171; 0.159; 0.149; 0.137; 0.127; 0.113; 0.106; 0.103; 0.100; 0.110; ...
0.171; 0.159; 0.150; 0.138; 0.128; 0.115; 0.107; 0.103; 0.099; 0.108; ...
0.169; 0.160; 0.151; 0.142; 0.133; 0.124; 0.119; 0.113; 0.107; 0.102; ...
0.169; 0.161; 0.153; 0.145; 0.137; 0.130; 0.126; 0.119; 0.115; 0.111; ...
0.168; 0.161; 0.155; 0.149; 0.143; 0.137; 0.133; 0.128; 0.124; 0.123; ...
0.168; 0.162; 0.157; 0.152; 0.148; 0.143; 0.139; 0.135; 0.130; 0.128; ...
0.168; 0.164; 0.159; 0.154; 0.151; 0.147; 0.144; 0.140; 0.136; 0.132];
```

```
ImpliedVolData = table(Maturity, ExercisePrice, ImpliedVol);
```

Compute the European call option price and sensitivities.

```
OptSpec = 'Call';
[Delta,Gamma,Lambda,Theta,Price] = optSensByLocalVolFD(Rate, AssetPrice, ...
Settle, ExerciseDates, OptSpec, Strike, ImpliedVolData, 'DividendYield',DividendYield,
'OutSpec',["Delta" "Gamma" "Lambda" "Theta" "Price"])
```

```
Delta = 0.5462
```

```
Gamma = 0.0082
```

```
Lambda = 4.9173
```

```
Theta = -20.8350
```

```
Price = 65.5302
```

## Input Arguments

### **Rate — Continuously compounded risk-free interest rate**

scalar numeric

Continuously compounded risk-free interest rate, specified by a scalar numeric.

Data Types: double

### **AssetPrice — Current underlying asset price**

scalar numeric

Current underlying asset price, specified as a scalar numeric.

Data Types: double

### **Settle — Settlement date**

serial date number | date character vector | datetime array | string

Settlement date, specified as a scalar serial date number, date character vector, datetime object, or string array

Data Types: double | char | datetime | string

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime array | string array

Option exercise dates, specified as a serial date number, a date character vector, a datetime array, or a string array:

- For a European option, there is only one `ExerciseDates` value and this is the option expiry date.
- For an American option, use a 1-by-2 vector of serial date numbers, date character vectors, datetimes, or strings. The American option can be exercised on any date between or including the pair of dates. If only one non-NaN date is listed, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: double | char | cell | datetime | string

### **OptSpec — Definition of option**

character vector with value of 'call' or 'put' | string array with value of "call" or "put"

Definition of the option, specified as a character vector or string array with a value of 'call' or 'put'.

Data Types: char | string

### **Strike — Option strike price value**

nonnegative scalar

Option strike price value, specified as a nonnegative scalar.

Data Types: double

### **ImpliedVolData — Table of maturity dates, strike or exercise prices, and corresponding implied volatilities**

table

A table of maturity dates, strike or exercise prices, and their corresponding implied volatilities, specified as a NVOL-by-3 table.

Data Types: table

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `PriceSens = Price = optByLocalVolFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, ImpliedVolData, 'AssetGridSize', 1000, 'OutSpec', {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'})`

### **Basis — Day-count basis**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a scalar using one of these supported values:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365

- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | scalar numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for DividendYield, then set DividendAmounts and ExDividendDates = [ ] or do not enter them. If you enter values for DividendAmounts and ExDividendDates, then set DividendYield = 0.

---

Data Types: double

### **DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and a NDIV-by-1 vector.

For each dividend amount, there must be a corresponding ExDividendDates date. If you enter values for DividendAmounts and ExDividendDates, then set DividendYield = 0.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates = [ ]` or do not enter them.

---

Data Types: double

**ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | datetime array | string array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and a NDIV-by-1 vector.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, `AssetPriceMax` values are calculated using asset distributions at maturity (default) | positive scalar

Maximum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar.

Data Types: double

**AssetGridSize — Size of asset grid for finite difference grid**

400 (default) | positive scalar

Size of the asset grid for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a positive scalar.

Data Types: double

**TimeGridSize — Size of time grid for finite difference grid**

100 (default) | positive scalar

Size of the time grid for finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive scalar.

Data Types: double

**AmericanOpt — Option type**

0 (European) (default) | scalar with values [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a positive integer scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

**InterpMethod — Method of interpolation for estimating the implied volatility surface from ImpliedVolData**

'linear' (default) | character vector with values 'linear', 'makima', 'spline', or 'tpaps' | string with values "linear", "makima", "spline", or "tpaps"

Method of interpolation for estimating the implied volatility surface from `ImpliedVolData`, specified as the comma-separated pair consisting of 'InterpMethod' and a character vector or string with one of the following values:

- 'linear' — Linear interpolation
- 'makima' — Modified Akima cubic Hermite interpolation
- 'spline' — Cubic spline interpolation
- 'tpaps' — Thin-plate smoothing spline interpolation

---

**Note** The 'tpaps' method uses the thin-plate smoothing spline functionality from Curve Fitting Toolbox.

The 'makima' and 'spline' methods work only for gridded data. For scattered data, use the 'linear' or 'tpaps' methods.

---

For more information on gridded or scattered data and details on interpolation methods, see “Gridded and Scattered Sample Data” (MATLAB) and “Interpolating Gridded Data” (MATLAB).

Data Types: char | string

**OutSpec — Define outputs**

{'price'} (default) | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta' | string array with values "price", "delta", "gamma", "vega", "rho", "theta"

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and an NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'price', 'delta', 'gamma', 'vega', 'lambda', 'rho', and 'theta'.

```
Example: OutSpec =
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}
```

Data Types: cell | string

## Output Arguments

### PriceSens — Option price and sensitivities

scalar numeric

Option price and sensitivities, returned as a scalar numeric. OutSpec determines the types and order of the output.

### PriceGrid — Grid containing prices calculated by finite difference method

grid

Grid containing prices calculated by the finite difference method, returned as a grid that is two-dimensional with size AssetGridSize  $\times$  TimeGridSize. The number of columns does not have to be equal to the TimeGridSize, because ExerciseDates and ExDividendDates are added to the time grid. PriceGrid(:, :, end) contains the price for  $t = 0$ .

### AssetPrices — Prices of asset

vector

Prices of the asset corresponding to the first dimension of PriceGrid, returned as a vector.

### Times — Times

vector

Times corresponding to second dimension of the PriceGrid, returned as a vector.

## Definitions

### Local Volatility Model

A local volatility model treats volatility as a function both of the current asset level and of time.

The local volatility can be estimated by using the Dupire formula [2]:

$$\sigma_{loc}^2(K, \tau) = \frac{\sigma_{imp}^2 + 2\tau\sigma_{imp}\frac{\partial\sigma_{imp}}{\partial\tau} + 2(\tau - d)K\tau\sigma_{imp}\frac{\partial\sigma_{imp}}{\partial K}}{\left(1 + Kd_1\sqrt{\tau}\frac{\partial\sigma_{imp}}{\partial K}\right)^2 + K^2\tau\sigma_{imp}\left(\frac{\partial^2\sigma_{imp}}{\partial K^2} - d_1\sqrt{\tau}\left(\frac{\partial\sigma_{imp}}{\partial K}\right)^2\right)}$$

$$d_1 = \frac{\ln(S_0/K) + ((\tau - d) + \sigma_{imp}^2/2)\tau}{\sigma_{imp}\sqrt{\tau}}$$

## References

- [1] Andersen, L. B., and R. Brotherton-Ratcliffe. "The Equity Option Volatility Smile: An Implicit Finite-Difference Approach." *Journal of Computational Finance*. Vol. 1, Number 2, 1997, pp. 5-37.
- [2] Dupire, B. "Pricing with a Smile." *Risk*. Vol. 7, Number 1, 1994, pp. 18-20.

## See Also

optByBatesFD | optByHestonFD | optByLocalVolFD | optByMertonFD |  
 optBySensMertonFD | optSensByBatesFD | optSensByHestonFD | optstockbyfd |  
 optstocksensbyfd

## Topics

"Supported Equity Derivatives" on page 3-24

**Introduced in R2018b**



# optByMertonFD

Option price by Merton76 model using finite differences

## Syntax

```
[Price,PriceGrid,AssetPrices,Times] = optByMertonFD(Rate,AssetPrice,Settle,ExerciseDates,OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq)
[Price,PriceGrid,AssetPrices,Times] = optByMertonFD( ____,Name,Value)
```

## Description

```
[Price,PriceGrid,AssetPrices,Times] = optByMertonFD(Rate,AssetPrice,Settle,ExerciseDates,OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq)
```

computes a vanilla European or American option price by the Merton76 model, using the Crank-Nicolson Adams-Bashforth (CNAB) IMEX method.

```
[Price,PriceGrid,AssetPrices,Times] = optByMertonFD( ____,Name,Value)
```

specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute Price for an American Option Using the Merton Model

Define the option variables and Merton model parameters.

```
AssetPrice = 90;
Strike = 100;
Rate = 0.06;
DividendYield = 0.1;
Settle = '01-Jan-2018';
ExerciseDates = '02-Apr-2018';

Sigma = 0.40;
```

```
MeanJ = -0.10;  
JumpVol = 0.01;  
JumpFreq = 1.00;
```

Compute the American call option price using the finite differences method.

```
OptSpec = 'Call';
```

```
Price = optSensByMertonFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike,...  
Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'AmericanOpt', 1)
```

```
Price = 3.4551
```

## Input Arguments

### **Rate — Continuously compounded risk-free interest rate**

scalar decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice — Current underlying asset price**

scalar numeric

Current underlying asset price, specified as a scalar numeric.

Data Types: double

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string scalar

Option settlement date, specified as a scalar using a serial date number, date character vector, datetime object, or string scalar.

Data Types: double | char | datetime | string

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime array | string array

Option exercise dates, specified as a serial date number, date character vector, datetime array, or string array:

- For a European option, use a scalar serial date number, date character vector, datetime object, or string scalar. For a European option, `ExerciseDates` contains only one value: the option expiry date.
- For an American option, use a 1-by-2 vector of serial date numbers, date character vectors, datetime arrays, or string arrays to specify the exercise date boundaries. An American option can be exercised on any date between or including the pair of dates. If only one non-NaN date is listed, then the option can be exercised between `Settle` date and the single listed value in `ExerciseDates`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

character vector with value of 'call' or 'put' | string array with value of "call" or "put"

Definition of the option, specified as a scalar using a character vector or string array with a value of 'call' or 'put'.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: `double`

### **Sigma — Volatility of underlying asset**

scalar numeric

Volatility of the underling asset, specified as a scalar numeric.

Data Types: `double`

### **MeanJ — Mean of the random percentage jump size**

scalar decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with the mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation `JumpVol`.

Data Types: `double`

**JumpVol — Standard deviation of  $\log(1+J)$** 

scalar decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal.

Data Types: double

**JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of the Poisson jump process, specified as a scalar numeric.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceGrid] = optByMertonFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of `'Basis'` and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)

- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | scalar numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates` = [ ] or do not enter them. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

### **DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and an NDIV-by-1 vector.

---

**Note** Each dividend amount must have a corresponding ex-dividend date. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

### **ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | string array | datetime array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and an NDIV-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, value is calculated based on asset price distribution at maturity (default) | positive scalar numeric

Maximum price for the price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar numeric.

Data Types: double

**AssetGridSize — Size of asset grid for finite difference grid**

400 (default) | scalar numeric

Size of the asset grid for the finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar numeric.

Data Types: double

**TimeGridSize — Number of nodes of time grid for finite difference grid**

100 (default) | positive numeric scalar

Number of nodes of the time grid for the finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive numeric scalar.

Data Types: double

**AmericanOpt — Option type**

0 (European) (default) | scalar with value of [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

## Output Arguments

### Price — Option price

scalar numeric

Option price, returned as a scalar numeric.

### PriceGrid — Grid containing prices calculated by the finite difference method

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with size `AssetGridSize` × `TimeGridSize`. The number of columns is not necessarily equal to the `TimeGridSize` because exercise and ex-dividend dates are added to the time grid. `PriceGrid(:, :, end)` contains the price for  $t = 0$ .

### AssetPrices — Prices of the asset

vector

Prices of the asset corresponding to the first dimension of `PriceGrid`, returned as a vector.

### Times — Times

vector

Times corresponding to the second dimension of `PriceGrid`, returned as a vector.

## Definitions

### Merton Jump Diffusion Model

The Merton jump diffusion model [2] extends the Black-Scholes model by using the Poisson process to include jump diffusion parameters in the modeling of sudden asset price movements (both up and down).

The stochastic differential equation is

$$dS_t = (r - q - \lambda_p \mu_j) S_t dt + \sigma S_t dW_t + JS_t dP_t$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where:

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$W_t$  is the Weiner process.

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1 + J)\delta\sqrt{2\pi}} \exp\left\{-\frac{\left[\ln(1 + J) - \left(\ln(1 + \mu_J) - \frac{\delta^2}{2}\right)\right]^2}{2\delta^2}\right\}$$

where:

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

$\sigma$  is the volatility of the asset price for ( $\sigma > 0$ ).

## References

- [1] Cont, R., and E. Voltchkova. "A Finite Difference Scheme for Option Pricing in Jump Diffusion and Exponential Lévy Models." *SIAM Journal on Numerical Analysis*. Vol. 43, Number 4, 2005, pp. 1596-1626.
- [2] Merton, R. "Option Pricing When Underlying Stock Returns Are Discontinuous." *The Journal of Financial Economics*. Vol 3. 1976, pp. 125-144.

## See Also

optByBatesFD | optByHestonFD | optByLocalVolFD | optBySensMertonFD |  
optSensByBatesFD | optSensByHestonFD | optSensByLocalVolFD



**Introduced in R2019a**

## optSensByMertonFD

Option price and sensitivities by Merton76 model using finite differences

### Syntax

```
[PriceSens,PriceGrid,AssetPrices,Times] = optSensByMertonFD(Rate,  
AssetPrice,Settle,ExerciseDates,OptSpec,Strike,Sigma,MeanJ,JumpVol,  
JumpFreq)  
[PriceSens,PriceGrid,AssetPrices,Times] = optSensByMertonFD( ____,  
Name,Value)
```

### Description

[PriceSens,PriceGrid,AssetPrices,Times] = optSensByMertonFD(Rate, AssetPrice,Settle,ExerciseDates,OptSpec,Strike,Sigma,MeanJ,JumpVol, JumpFreq) computes a vanilla European or American option price and sensitivities by the Merton76 model, using the Crank-Nicolson Adams-Bashforth (CNAB) IMEX method.

[PriceSens,PriceGrid,AssetPrices,Times] = optSensByMertonFD( \_\_\_\_, Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute Price and Sensitivities for an American Option Using the Merton Model

Define the option variables and Merton model parameters.

```
AssetPrice = 90;  
Strike = 100;  
Rate = 0.06;  
DividendYield = 0.1;  
Settle = '01-Jan-2018';  
ExerciseDates = '02-Apr-2018';
```

```
Sigma = 0.40;  
MeanJ = -0.10;  
JumpVol = 0.01;  
JumpFreq = 1.00;
```

Compute the American call option price and sensitivities using the finite differences method.

```
OptSpec = 'Call';
```

```
[Price, Delta, Gamma, Rho, Theta, Vega] = optSensByMertonFD(Rate, AssetPrice, Settle, Maturity, Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'AmericanOpt', 1, ...  
'OutSpec', ["Price" "Delta" "Gamma" "Rho" "Theta" "Vega"])
```

```
Price = 3.4551
```

```
Delta = 0.3211
```

```
Gamma = 0.0195
```

```
Rho = 5.6610
```

```
Theta = -11.9877
```

```
Vega = 15.5156
```

## Input Arguments

### **Rate — Continuously compounded risk-free interest rate**

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice — Current underlying asset price**

numeric

Current underlying asset price, specified as a scalar numeric.

Data Types: double

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a scalar using a serial date number, date character vector, datetime array, or string array.

Data Types: double | char | datetime | string

### **ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime array | string array

Option exercise dates, specified as a serial date number, date character vector, datetime array, or string array:

- For a European option, use a scalar serial date number, date character vector, datetime array, or string array. For a European option, `ExerciseDates` contains only one value: the option expiry date.
- For an American option, use a 1-by-2 vector of serial date numbers, date character vectors, datetime arrays, or string arrays to specify the exercise date boundaries. An American option can be exercised on any date between or including the pair of dates. If only one non-NaN date is listed, then the option can be exercised between `Settle` date and the single listed value in `ExerciseDates`.

Data Types: double | char | datetime | string

### **OptSpec — Definition of option**

character vector with value of 'call' or 'put' | string array with value of "call" or "put"

Definition of the option, specified as a scalar using a character vector or string array with a value of 'call' or 'put'.

Data Types: cell | string

### **Strike — Option strike price value**

scalar numeric

Option strike price value, specified as a scalar numeric.

Data Types: double

### **Sigma — Volatility of underlying asset**

scalar numeric

Volatility of the underlying asset, specified as a scalar numeric.

Data Types: double

### **MeanJ — Mean of the random percentage jump size**

scalar decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with the mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

### **JumpVol — Standard deviation of $\log(1+J)$**

scalar decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal.

Data Types: double

### **JumpFreq — Annual frequency of Poisson jump process**

scalar numeric

Annual frequency of the Poisson jump process, specified as a scalar numeric.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceGrid] = optByMertonFD(Rate, AssetPrice, Settle, ExerciseDates, OptSpec, Strike, MeanJ, JumpVol, JumpFreq, 'Basis', 7, 'OutSpec', 'delta')`

### **Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of `'Basis'` and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | scalar numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric.

---

**Note** If you enter a value for `DividendYield`, then set `DividendAmounts` and `ExDividendDates` = [ ] or do not enter them. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield` = 0.

---

Data Types: double

### **DividendAmounts — Cash dividend amounts**

[ ] (default) | vector

Cash dividend amounts, specified as the comma-separated pair consisting of 'DividendAmounts' and an `NDIV-by-1` vector.

---

**Note** Each dividend amount must have a corresponding ex-dividend date. If you enter values for `DividendAmounts` and `ExDividendDates`, then set `DividendYield = 0`.

---

Data Types: double

**ExDividendDates — Ex-dividend dates**

[ ] (default) | serial date number | date character vector | string array | datetime array

Ex-dividend dates, specified as the comma-separated pair consisting of 'ExDividendDates' and an NDIV-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

**AssetPriceMax — Maximum price for price grid boundary**

if unspecified, value is calculated based on asset price distribution at maturity (default) | positive scalar numeric

Maximum price for the price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar numeric.

Data Types: double

**AssetGridSize — Size of asset grid for the finite difference grid**

400 (default) | scalar numeric

Size of the asset grid for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a scalar numeric.

Data Types: double

**TimeGridSize — Number of nodes of time grid for the finite difference grid**

100 (default) | positive numeric scalar

Number of nodes of the time grid for finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive numeric scalar.

Data Types: double

**AmericanOpt — Option type**

0 (European) (default) | scalar with value of [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar flag with one of these values:

- 0 — European
- 1 — American

Data Types: double

### **OutSpec — Define outputs**

`['price']` (default) | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', and 'theta' | string array with values "price", "delta", "gamma", "vega", "rho", and "theta"

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

Example: `OutSpec = ['price','delta','gamma','vega','rho','theta']`

Data Types: string | cell

## **Output Arguments**

### **PriceSens — Option price or sensitivities**

numeric

Option price or sensitivities, returned as a numeric. The name-value pair argument `OutSpec` determines the types and order of the outputs.

### **PriceGrid — Grid containing prices calculated by the finite difference method**

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with size `AssetGridSize` × `TimeGridSize`. The number of columns is not necessarily equal to the `TimeGridSize` because exercise and ex-dividend dates are added to the time grid. `PriceGrid(:, :, end)` contains the price for  $t = 0$ .

### **AssetPrices — Prices of the asset**

vector

Prices of the asset corresponding to the first dimension of `PriceGrid`, returned as a vector.

### **Times — Times**

vector



Times corresponding to the second dimension of PriceGrid, returned as a vector.

## Definitions

### Merton Jump Diffusion Model

The Merton jump diffusion model [2] extends the Black-Scholes model by using the Poisson process to include jump diffusion parameters in the modeling of sudden asset price movements (both up and down).

The stochastic differential equation is

$$\begin{aligned} dS_t &= (r - q - \lambda_p \mu_J) S_t dt + \sigma S_t dW_t + JS_t dP_t \\ \text{prob}(dP_t = 1) &= \lambda_p dt \end{aligned}$$

where:

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$W_t$  is the Weiner process.

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp \left\{ - \frac{\left[ \ln(1+J) - \left( \ln(1 + \mu_J) - \frac{\delta^2}{2} \right) \right]^2}{2\delta^2} \right\}$$

where:

$\mu_J$  is the mean of  $J$  for  $(\mu_J > -1)$ .

$\delta$  is the standard deviation of  $\ln(1+J)$  for  $(\delta \geq 0)$ .

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

$\sigma$  is the volatility of the asset price for ( $\sigma > 0$ ).

## References

- [1] Cont, R., and E. Voltchkova. "A Finite Difference Scheme for Option Pricing in Jump Diffusion and Exponential Lévy Models." *SIAM Journal on Numerical Analysis*. Vol. 43, Number 4, 2005, pp. 1596-1626.
- [2] Merton, R. "Option Pricing When Underlying Stock Returns Are Discontinuous." *The Journal of Financial Economics*. Vol 3. 1976, pp. 125-144.

## See Also

[optByBatesFD](#) | [optByHestonFD](#) | [optByLocalVolFD](#) | [optByMertonFD](#) | [optSensByBatesFD](#) | [optSensByHestonFD](#) | [optSensByLocalVolFD](#)

**Introduced in R2019a**

# optByMertonFFT

Option price by Merton76 model using FFT and FRFT

## Syntax

```
[Price,StrikeOut] = optByMertonFFT(Rate,AssetPrice,Settle,Maturity,
OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq)
[Price,StrikeOut] = optByMertonFFT( ____,Name,Value)
```

## Description

[Price,StrikeOut] = optByMertonFFT(Rate,AssetPrice,Settle,Maturity, OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq) computes vanilla European option price by Merton76 model, using Carr-Madan FFT and Chourdakis FRFT methods.

[Price,StrikeOut] = optByMertonFFT( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Workflow for Plotting an Option Price Surface Using the Merton76 Model

Use optByMertonFFT to calibrate the FFT strike grid, compute option prices, and plot an option price surface.

### Define Option Variables and Merton76 Model Parameters

```
AssetPrice = 80;
Rate = 0.03;
DividendYield = 0.02;
OptSpec = 'call';
```

```
Sigma = 0.16;
MeanJ = 0.02;
```

```
JumpVol = 0.08;  
JumpFreq = 2;
```

### Compute the Option Prices for the Entire FFT (or FRFT) Strike Grid, Without Specifying "Strike"

Compute option prices and also output the corresponding strikes. If the `Strike` input is empty ( `[]` ), option prices will be computed on the entire FFT (or FRFT) strike grid. The FFT (or FRFT) strike grid is determined as `exp(log-strike grid)`, where each column of the log-strike grid has `NumFFT` points with `LogStrikeStep` spacing that are roughly centered around each element of `log(AssetPrice)`. The default value for `NumFFT` is  $2^{12}$ . In addition to the prices in the first output, the optional last output contains the corresponding strikes.

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = []; % Strike is not specified (will use the entire FFT strike grid)  
  
% Compute option prices for the entire FFT strike grid  
[Call, Kout] = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...  
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield);  
  
% Show the lowest and highest strike values on the FFT strike grid  
format  
MinStrike = Kout(1) % Lowest possible strike in the current FFT strike grid  
  
MinStrike = 2.9205e-135  
  
MaxStrike = Kout(end) % Highest possible strike in the current FFT strike grid  
  
MaxStrike = 1.8798e+138  
  
% Show a subset of the strikes and corresponding option prices  
Range = (2046:2052);  
[Kout(Range) Call(Range)]  
  
ans = 7x2  
  
    50.4929    29.4645  
    58.8640    21.2601  
    68.6231    12.2218  
    80.0000     4.5600  
    93.2631     0.9579  
   108.7251     0.1236
```

126.7505      0.0113

### Change the Number of FFT (or FRFT) Points and Compare with optByMertonNI

Try a different number of FFT(or FRFT) points, and compare the results with direct numerical integration. Unlike optByMertonFFT, which uses FFT (or FRFT) techniques for fast computation across the whole range of strikes, the optByMertonNI function uses direct numerical integration and it is typically slower, especially for multiple strikes. However, the values computed by optByMertonNI can serve as a benchmark for adjusting the settings for optByMertonFFT.

```
% Try a smaller number of FFT (or FRFT) points
% (e.g. for faster performance or smaller memory footprint)
NumFFT = 2^10; % Smaller than the default value of 2^12
Strike = []; % Strike is not specified (will use the entire FFT strike grid)
[Call, Kout] = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT)

% Compare with numerical integration method
Range = (510:516);
Strike = Kout(Range);
CallFFT = Call(Range);
CallNI = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield);
Error = abs(CallFFT-CallNI);
table(Strike, CallFFT, CallNI, Error)
```

ans=7×4 table

Strike	CallFFT	CallNI	Error
12.696	66.328	66.696	0.36786
23.449	55.922	56.103	0.18071
43.312	36.481	36.536	0.055233
80	4.7387	4.56	0.17867
147.76	0.046602	0.0008089	0.045793
272.93	0.0092842	-7.0709e-08	0.0092842
504.11	0.0024041	-2.4515e-07	0.0024044

### Make Further Adjustments to FFT (or FRFT)

If the values in the output CallFFT are significantly different from those in CallNI, try making adjustments to optByMertonFFT settings, such as CharacteristicFcnStep,

LogStrikeStep, NumFFT, DampingFactor, and so on. Note that if (LogStrikeStep \* CharacteristicFcnStep) is  $2\pi / \text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

```
Strike = []; % Strike is not specified (will use the entire FFT or FRFT strike grid)
[Call, Kout] = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001);
```

*% Compare with numerical integration method*

```
Strike = Kout(Range);
CallFFT = Call(Range);
CallNI = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield);
Error = abs(CallFFT-CallNI);
table(Strike, CallFFT, CallNI, Error)
```

ans=7×4 table

Strike	CallFFT	CallNI	Error
79.76	4.674	4.674	4.9664e-10
79.84	4.6358	4.6358	4.9651e-10
79.92	4.5978	4.5978	4.9642e-10
80	4.56	4.56	4.9641e-10
80.08	4.5224	4.5224	4.9642e-10
80.16	4.485	4.485	4.965e-10
80.24	4.4478	4.4478	4.966e-10

*% Save the final FFT (or FRFT) strike grid for future reference. For % example, it provides information about the range of Strike inputs for % which the FFT (or FRFT) operation is valid.*

```
FFTStrikeGrid = Kout;
MinStrike = FFTStrikeGrid(1) % Strike cannot be less than MinStrike
```

```
MinStrike = 47.9437
```

```
MaxStrike = FFTStrikeGrid(end) % Strike cannot be greater than MaxStrike
```

```
MaxStrike = 133.3566
```

### Compute Option Price for a Single Strike

Once the desired FFT (or FRFT) settings are determined, use the `Strike` input to specify the strikes rather than providing an empty array. If the specified strikes do not match a

value on the FFT (or FRFT) strike grid, the outputs are interpolated on the specified strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;

Call = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)

Call = 4.5600
```

### Compute the Option Prices for a Vector of Strikes

Use the `Strike` input to specify the strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';

Call = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)

Call = 5×1

    6.7411
    5.5762
    4.5600
    3.6891
    2.9551
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the `Strike` input to specify the strikes. Also, the `Maturity` input can be a vector, but it must match the length of the `Strike` vector if the `ExpandOutput` name-value pair argument is not set to `"true"`.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes
```

```
Call = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001) % Five values in vector out
Call = 5×1

    8.5589
    8.9439
    9.2316
    9.4653
    9.6565
```

### Expand the Outputs for a Surface

Set the `ExpandOutput` name-value pair argument to "true" to expand the outputs into `NStrikes-by-NMaturities` matrices. In this case, they are square matrices.

```
[Call, Kout] = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 5) matrix output
Call = 5×5

    8.5589    9.9675    11.1343    12.1492    13.0464
    7.4844    8.9439    10.1481    11.1939    12.1181
    6.5125    8.0023     9.2316    10.2999    11.2449
    5.6401    7.1402     8.3827     9.4653    10.4249
    4.8630    6.3545     7.5990     8.6881     9.6565
```

```
Kout = 5×5

    76    76    76    76    76
    78    78    78    78    78
    80    80    80    80    80
    82    82    82    82    82
    84    84    84    84    84
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is "true", `NStrikes` do not have to match `NMaturities`. That is, the output `NStrikes-by-NMaturities` matrix can be rectangular.



```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes

Call = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 6) matrix output

Call = 5x6

    6.7411    8.5589    9.9675    11.1343    12.1492    13.0464
    5.5762    7.4844    8.9439    10.1481    11.1939    12.1181
    4.5600    6.5125    8.0023    9.2316    10.2999    11.2449
    3.6891    5.6401    7.1402    8.3827    9.4653    10.4249
    2.9551    4.8630    6.3545    7.5990    8.6881    9.6565

```

### Compute the Option Prices for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

Call = optByMertonFFT(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'ExpandOutput', true) % (5 x 4) matrix output

Call = 5x4

    3.4187    5.6579    8.5589    12.0417
    2.8538    4.8401    7.4844    10.7343
    2.3718    4.1205    6.5125    9.5230
    1.9635    3.4922    5.6401    8.4090
    1.6198    2.9476    4.8630    7.3921

```

### Plot an Option Price Surface

Use the `Strike` input to specify the strikes. Increase the value for `NumFFT` to support a wider range of strikes. Also, the `Maturity` input can be a vector. Set `ExpandOutput` to "true" to output the surface as a `NStrikes-by-NMaturities` matrix.

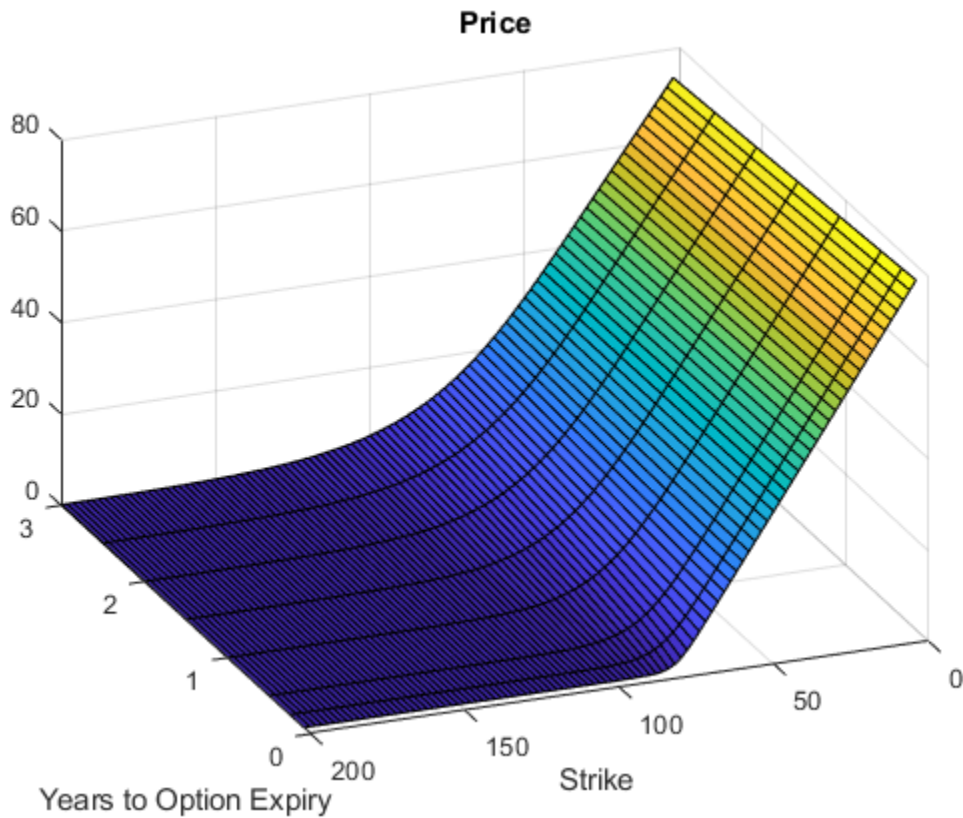
```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

% Increase 'NumFFT' to support a wider range of strikes
NumFFT = 2^13;

Call = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'NumFFT', NumFFT,
    'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, 'ExpandOutput', true);

[X,Y] = meshgrid(Times,Strike);

figure;
surf(X,Y,Call);
title('Price');
xlabel('Years to Option Expiry');
ylabel('Strike');
view(-112,34);
xlim([0 Times(end)]);
zlim([0 80]);
```



## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

If this input is an empty array ( [] ), option prices are computed on the entire FFT (or FRFT) strike grid, which is determined as  $\exp(\log\text{-strike grid})$ . Each column of the log-strike grid has 'NumFFT' points with 'LogStrikeStep' spacing that are roughly centered around each element of  $\log(\text{AssetPrice})$ .

For more information on the proper dimensions for Strike, see the name-value pair argument ExpandOutput.

Data Types: double

### **Sigma — Volatility of underlying asset**

numeric

Volatility of the underling asset, specified as a scalar numeric value.

Data Types: double

### **MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

### **JumpVol — Standard deviation of $\log(1+J)$**

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

### **JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, StrikeOut] = optByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, Sigma, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

### **Basis** — Day-count basis of instrument

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield** — Continuously compounded underlying asset yield

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

**NumFFT — Number of grid points in the characteristic function variable**

4096 (default) | numeric

Number of grid points in the characteristic function variable and in each column of the log-strike grid, specified as the comma-separated pair consisting of 'NumFFT' and a scalar numeric value.

Data Types: double

**CharacteristicFcnStep — Characteristic function variable grid spacing**

0.01 (default) | numeric

Characteristic function variable grid spacing, specified as the comma-separated pair consisting of 'CharacteristicFcnStep' and a scalar numeric value.

Data Types: double

**LogStrikeStep — Log-strike grid spacing**

$2\pi/\text{NumFFT}/\text{CharacteristicFcnStep}$  (default) | numeric

Log-strike grid spacing, specified as the comma-separated pair consisting of 'LogStrikeStep' and a scalar numeric value.

---

**Note** If  $(\text{LogStrikeStep} \times \text{CharacteristicFcnStep})$  is  $2\pi/\text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

---

Data Types: double

**DampingFactor — Damping factor for Carr-Madan formulation**

1.5 (default) | numeric

Damping factor for Carr-Madan formulation, specified as the comma-separated pair consisting of 'DampingFactor' and a scalar numeric value.

Data Types: double

**Quadrature — Type of quadrature**

"simpson" (default) | character vector with values: 'simpson' or 'trapezoidal' | string array with values: "simpson" or "trapezoidal"

Type of quadrature, specified as the comma-separated pair consisting of 'Quadrature' and a single character vector or string array with a value of 'simpson' or 'trapezoidal'.

Data Types: char | string

**ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- **true** — If true, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. If Strike is empty, NRows is equal to NumFFT. NColumns is determined by the sizes of AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.
- **false** — If false, the outputs are NINST-by-1 vectors. Also, the inputs Strike, AssetPrice, Settle, Maturity, and OptSpec must all be either scalar or NINST-by-1 vectors.

Data Types: logical

## Output Arguments

**Price — Option prices**

numeric

Option prices, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput.

**StrikeOut — Strikes corresponding to Price**

numeric

Strikes corresponding to Price, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput.



## Definitions

### Merton Jump Diffusion Model

The Merton jump diffusion model (Merton (1976)) is a different extension of the Black-Scholes model, where sudden asset price movements (both up and down) are modeled by adding the jump diffusion parameters with the Poisson process.

The stochastic differential equation is:

$$dS_t = (r - q - \lambda_p \mu_j) S_t dt + \sigma S_t dW_t + JS_t dP_t$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$W_t$  is the Weiner process.

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_j) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp\left\{-\frac{\left[\ln(1+J) - \left(\ln(1 + \mu_j) - \frac{\delta^2}{2}\right)\right]^2}{2\delta^2}\right\}$$

$\mu_j$  is the mean of  $J$  for ( $\mu_j > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

$\sigma$  is the volatility of the asset price for ( $\sigma > 0$ ).

The characteristic function  $f_{Merton76_j}(\phi)$  for  $j = 1$  (asset prices measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Merton76_j} = f_{BS_j} \exp \left( \lambda_p \tau (1 + \mu_j)^{m_j} + \frac{1}{2} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 \left( m_j i\phi + \frac{(i\phi)^2}{2} \right)} - 1 \right] - \lambda_p \tau \mu_j i\phi \right)$$

where for  $j = 1, 2$ :

$$f_{BS_1}(\phi) = \frac{f_{BS_2}(\phi - i)}{f_{BS_2}(-i)}$$

$$f_{BS_2}(\phi) = \exp \left( i\phi \left[ \ln S_t + \left( r - q - \frac{\sigma^2}{2} \right) \tau \right] - \frac{\phi^2 \sigma^2}{2} \tau \right)$$

$$m_1 = \frac{1}{2}, m_2 = -\frac{1}{2}$$

where

$\phi$  is the characteristic function variable.

$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

## Carr-Madan Formulation

The Carr and Madan (1999) formulation is a popular modified implementation of Heston (1993) framework.

Rather than computing the probabilities  $P_1$  and  $P_2$  as intermediate steps, Carr and Madan developed an alternative expression so that taking its inverse Fourier transform gives the option price itself directly.

$$Call(k) = \frac{e^{-\alpha k}}{\pi} \int_0^\infty \text{Re} \left[ e^{-iuk} \psi(u) \right] du$$

$$\psi(u) = \frac{e^{-r\tau} f_2(\phi = (u - (\alpha + 1)i))}{\alpha^2 + \alpha - u^2 + iu(2\alpha + 1)}$$

$$Put(K) = Call(K) + Ke^{-r\tau} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$

$i$  is a unit imaginary number ( $i^2 = -1$ )

$\phi$  is the characteristic function variable.

$\alpha$  is the damping factor.

$u$  is the characteristic function variable for integration, where  $\phi = (u - (\alpha+1)i)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

To apply FFT or FRFT to this formulation, the characteristic function variable for integration,  $u$ , is discretized into  $\text{NumFFT}(N)$  points with the step size  $\text{CharacteristicFcnStep}(\Delta u)$ , and the log-strike  $k$  is discretized into  $N$  points with the step size  $\text{LogStrikeStep}(\Delta k)$ .

The discretized characteristic function variable for integration,  $u_j$  (for  $j = 1, 2, 3, \dots, N$ ), has a minimum value of 0 and a maximum value of  $(N-1)(\Delta u)$ , and it approximates the continuous integration range from 0 to infinity.

The discretized log-strike grid,  $k_n$  (for  $n = 1, 2, 3, N$ ) is approximately centered around  $\ln(S_t)$ , with a minimum value of

$$\ln(S_t) - \frac{N}{2}\Delta k$$

and a maximum value of

$$\ln(S_t) + \left(\frac{N}{2} - 1\right)\Delta k$$

Where the minimum allowable strike is

$$S_t \exp\left(-\frac{N}{2}\Delta k\right)$$

and the maximum allowable strike is

$$S_t \exp\left[\left(\frac{N}{2} - 1\right)\Delta k\right]$$

As a result of the discretization, the expression for the call option becomes

$$Call(k_n) = \Delta u \frac{e^{-\alpha k_n}}{\pi} \sum_{j=1}^N \operatorname{Re} \left[ e^{-i\Delta k \Delta u (j-1)(n-1)} e^{iu_j \left[ \frac{N\Delta k}{2} - \ln(S_t) \right]} \psi(u_j) \right] w_j$$

where

$\Delta u$  is the step size of discretized characteristic function variable for integration.

$\Delta k$  is the step size of discretized log-strike.

$N$  is the number of FFT or FRFT points.

$w_j$  is the weights for quadrature used for approximating the integral.

FFT is used to evaluate the above expression if  $\Delta k$  and  $\Delta u$  are subject to the following constraint:

$$\Delta k \Delta u = \left( \frac{2\pi}{N} \right)$$

otherwise, the functions use the FRFT method described in Chourdakis (2005).

## References

- [1] Bates, D. S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [2] Carr, P., and D.B. Madan. "Option Valuation Using the Fast Fourier Transform." *Journal of Computational Finance*. Vol 2. No. 4. 1999.
- [3] Cont, R. and P. Tankov. *Financial Modeling with Jump Processes*. Chapman & Hall/CRC Press, 2004.

[4] Chourdakis, K. "Option Pricing Using Fractional FFT." *Journal of Computational Finance*. 2005.

[5] Merton, R. "Option Pricing When Underlying Stock Returns are Discontinuous." *Journal of Financial Economics*. Vol 3. 1976.

## **See Also**

[optByBatesFFT](#) | [optByBatesNI](#) | [optByHestonFFT](#) | [optByHestonNI](#) |  
[optByMertonNI](#) | [optSensByBatesFFT](#) | [optSensByBatesNI](#) | [optSensByHestonFFT](#)  
| [optSensByHestonNI](#) | [optSensByMertonFFT](#) | [optSensByMertonNI](#)

**Introduced in R2018a**

## optSensByMertonFFT

Option price and sensitivities by Merton76 model using FFT and FRFT

### Syntax

```
[PriceSens,StrikeOut] = optSensByMertonFFT(Rate,AssetPrice,Settle,  
Maturity,OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq)  
[PriceSens,StrikeOut] = optSensByMertonFFT( ____,Name,Value)
```

### Description

[PriceSens,StrikeOut] = optSensByMertonFFT(Rate,AssetPrice,Settle, Maturity,OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq) computes vanilla European option price and sensitivities by Merton76 model, using Carr-Madan FFT and Chourdakis FRFT methods.

[PriceSens,StrikeOut] = optSensByMertonFFT( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Workflow for Plotting an Option Sensitivity Surface Using the Merton76 Model

Use `optSensByMertonFFT` to calibrate the FFT strike grid for sensitivities, compute option sensitivities, and plot option sensitivity surfaces.

#### Define Option Variables and Merton76 Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
Sigma = 0.16;
```

```
MeanJ = 0.02;
JumpVol = 0.08;
JumpFreq = 2;
```

### Compute the Option Prices for the Entire FFT (or FRFT) Strike Grid, Without Specifying "Strike"

Compute option sensitivities and also output the corresponding strikes. If the Strike input is empty ( [] ), option sensitivities will be computed on the entire FFT (or FRFT) strike grid. The FFT (or FRFT) strike grid is determined as  $\exp(\log\text{-strike grid})$ , where each column of the log-strike grid has NumFFT points with LogStrikeStep spacing that are roughly centered around each element of  $\log(\text{AssetPrice})$ . The default value for NumFFT is  $2^{12}$ . In addition to the sensitivities in the first output, the optional last output contains the corresponding strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = []; % Strike is not specified
```

```
[Delta, Kout] = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta")
```

```
% Show the lowest and highest strike values on the FFT strike grid
format
[Kout(1) Kout(end)]
```

```
ans = 1x2
10138 ×
```

```
0.0000    1.8798
```

```
% Show a subset of the strikes and corresponding option sensitivities
```

```
Range = (2046:2052);
[Kout(Range) Delta(Range)]
```

```
ans = 7x2
```

```
50.4929    0.9895
58.8640    0.9801
68.6231    0.8816
80.0000    0.5283
93.2631    0.1551
108.7251    0.0241
```

126.7505      0.0025

### Change the Number of FFT (or FRFT) Points and Compare with optSensByMertonNI

Try a different number of FFT (or FRFT) points, and compare the results with numerical integration. Unlike `optSensByMertonFFT`, which uses FFT (or FRFT) techniques for fast computation across the whole range of strikes, the `optSensByMertonNI` function uses direct numerical integration and it is typically slower, especially for multiple strikes. However, the values computed by `optSensByMertonNI` can serve as a benchmark for adjusting the settings for `optSensByMertonFFT`.

```
% Try a smaller number of FFT points
% (e.g. for faster performance or smaller memory footprint)
NumFFT = 2^10; % Smaller than the default value of 2^12
Strike = []; % Strike is not specified (will use the entire FFT strike grid)
[Delta, Kout] = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT);

% Compare with numerical integration method
Range = (510:516);
Strike = Kout(Range);
DeltaFFT = Delta(Range);
DeltaNI = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT);
Error = abs(DeltaFFT-DeltaNI);
table(Strike, DeltaFFT, DeltaNI, Error)
```

ans=7x4 table

Strike	DeltaFFT	DeltaNI	Error
12.696	0.89726	0.99002	0.092766
23.449	0.93421	0.99002	0.05581
43.312	0.94691	0.99001	0.043093
80	0.50983	0.52827	0.018446
147.76	0.004147	0.00019101	0.003956
272.93	0.001071	1.547e-09	0.001071
504.11	0.00030521	5.7578e-10	0.00030521



### Make Further Adjustments to FFT (or FRFT)

If the values in the output DeltaFFT are significantly different from those in DeltaNI, try making adjustments to optSensByMertonFFT settings, such as CharacteristicFcnStep, LogStrikeStep, NumFFT, DampingFactor, and so on. Note that if  $(\text{LogStrikeStep} * \text{CharacteristicFcnStep})$  is  $2 * \pi / \text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

```
Strike = []; % Strike is not specified (will use the entire FFT or FRFT strike grid)
[Delta, Kout] = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001);
```

```
% Compare with numerical integration method
```

```
Strike = Kout(Range);
DeltaFFT = Delta(Range);
DeltaNI = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta");
Error = abs(DeltaFFT-DeltaNI);
table(Strike, DeltaFFT, DeltaNI, Error)
```

```
ans=7x4 table
```

Strike	DeltaFFT	DeltaNI	Error
79.76	0.53701	0.53701	5.6407e-12
79.84	0.5341	0.5341	5.3257e-12
79.92	0.53119	0.53119	5.0099e-12
80	0.52827	0.52827	4.6956e-12
80.08	0.52536	0.52536	4.3811e-12
80.16	0.52245	0.52245	4.0653e-12
80.24	0.51953	0.51953	3.7503e-12

```
% Save the final FFT (or FRFT) strike grid for future reference. For
% example, it provides information about the range of Strike inputs for
% which the FFT (or FRFT) operation is valid.
```

```
FFTStrikeGrid = Kout;
MinStrike = FFTStrikeGrid(1) % Strike cannot be less than MinStrike
```

```
MinStrike = 47.9437
```

```
MaxStrike = FFTStrikeGrid(end) % Strike cannot be greater than MaxStrike
```

```
MaxStrike = 133.3566
```

### Compute the Option Sensitivity for a Single Strike

Once the desired FFT (or FRFT) settings are determined, use the `Strike` input to specify the strikes rather than providing an empty array. If the specified strikes do not match a value on the FFT (or FRFT) strike grid, the outputs are interpolated on the specified strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;
```

```
Delta = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)
```

```
Delta = 0.5283
```

### Compute the Option Sensitivities for a Vector of Strikes

Use the `Strike` input to specify the strikes.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';
```

```
Delta = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001)
```

```
Delta = 5×1
```

```
0.6727
0.6013
0.5283
0.4565
0.3883
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the `Strike` input to specify the strikes. Also, the `Maturity` input can be a vector, but it must match the length of the `Strike` vector if the `ExpandOutput` name-value pair argument is not set to `"true"`.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
Strike = [76 78 80 82 84]'; % Five strikes

Delta = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001) % Five values in vector output

```

```
Delta = 5x1
```

```

0.6419
0.5907
0.5565
0.5311
0.5110

```

### Expand the Outputs for a Surface

Set the `ExpandOutput` name-value pair argument to "true" to expand the outputs into `N`Strikes-by-`N`Maturities matrices. In this case, they are square matrices.

```

[Delta, Kout] = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike,
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 5) matrix output

```

```
Delta = 5x5
```

```

0.6419    0.6305    0.6245    0.6204    0.6173
0.5922    0.5907    0.5905    0.5905    0.5905
0.5422    0.5507    0.5565    0.5607    0.5637
0.4927    0.5112    0.5229    0.5311    0.5372
0.4447    0.4725    0.4898    0.5020    0.5110

```

```
Kout = 5x5
```

```

76    76    76    76    76
78    78    78    78    78
80    80    80    80    80
82    82    82    82    82
84    84    84    84    84

```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is "true", `NStrikes` do not have to match `NMaturities`. That is, the output `NStrikes-by-NMaturities` matrix can be rectangular.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes
```

```
Delta = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 6) matrix output
```

Delta = 5×6

0.6727	0.6419	0.6305	0.6245	0.6204	0.6173
0.6013	0.5922	0.5907	0.5905	0.5905	0.5905
0.5283	0.5422	0.5507	0.5565	0.5607	0.5637
0.4565	0.4927	0.5112	0.5229	0.5311	0.5372
0.3883	0.4447	0.4725	0.4898	0.5020	0.5110

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes
```

```
Delta = optSensByMertonFFT(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, 'OutSpec', "delta",
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, ...
    'LogStrikeStep', 0.001, 'ExpandOutput', true) % (5 x 4) matrix output
```

Delta = 5×4

0.3796	0.5157	0.6419	0.7472
0.3315	0.4637	0.5922	0.7043
0.2874	0.4137	0.5422	0.6592

```
0.2474    0.3664    0.4927    0.6128
0.2117    0.3224    0.4447    0.5657
```

## Plot Option Sensitivity Surfaces

Use the `Strike` input to specify the strikes. Increase the value for `NumFFT` to support a wider range of strikes. Also, the `Maturity` input can be a vector. Set `ExpandOutput` to `"true"` to output the surfaces as `NStrikes-by-NMaturities` matrices.

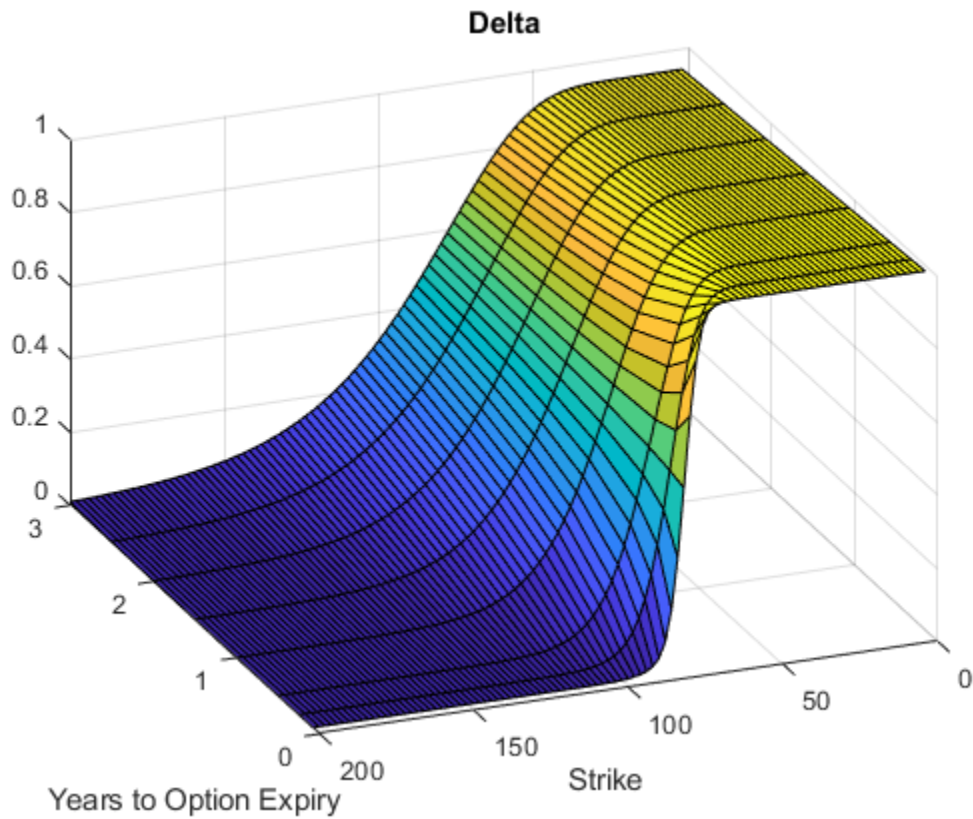
```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

% Increase 'NumFFT' to support a wider range of strikes
NumFFT = 2^13;

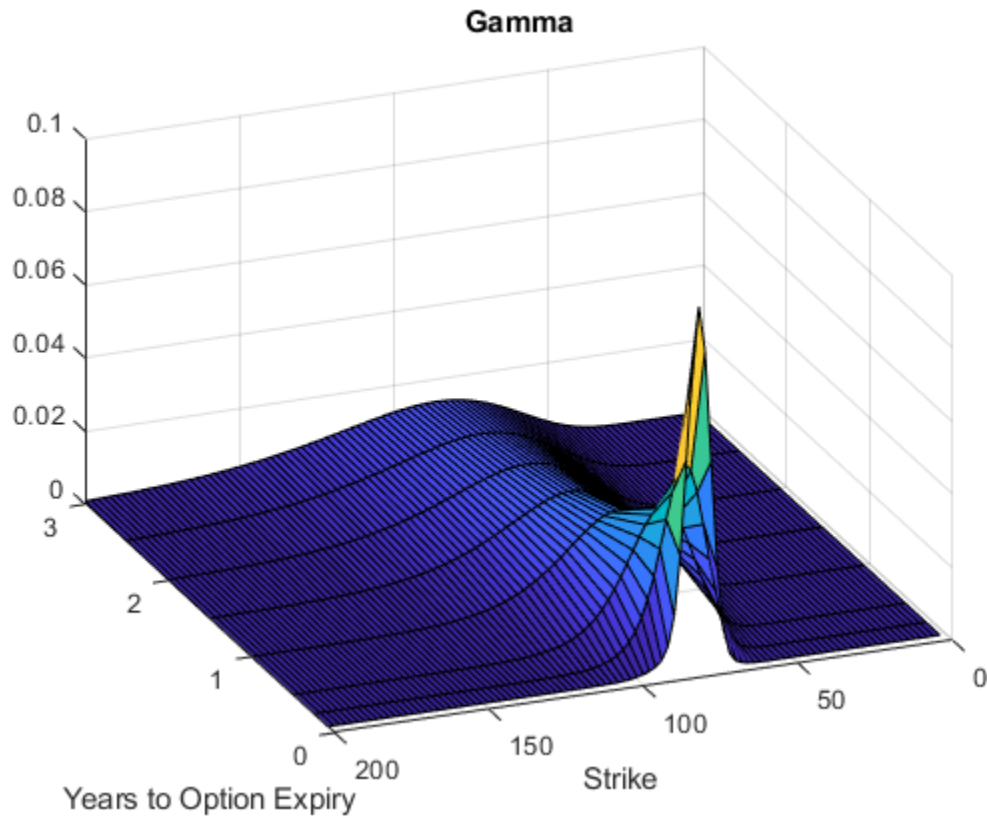
[Delta, Gamma, Rho, Theta, Vega] = optSensByMertonFFT(...
    Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'NumFFT', NumFFT, 'CharacteristicFcnStep', 0.065, 'LogStrikeStep', 0.001, ...
    'OutSpec', ["delta", "gamma", "rho", "theta", "vega"], ...
    'ExpandOutput', true);

[X,Y] = meshgrid(Times,Strike);

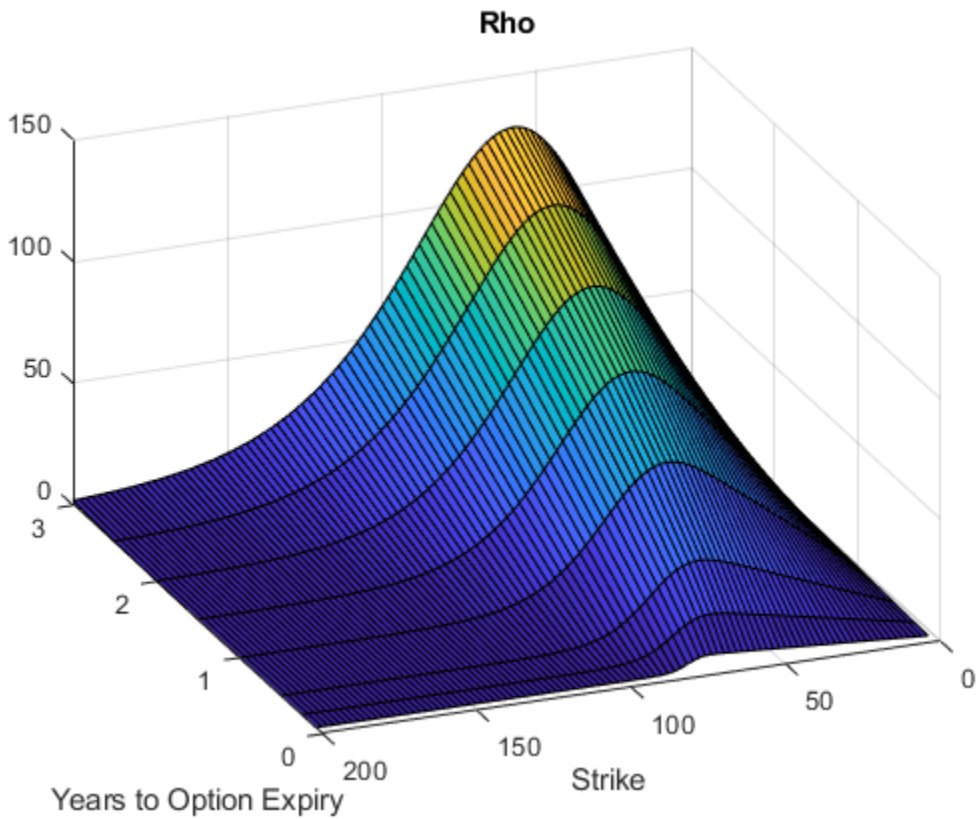
figure;
surf(X,Y,Delta);
title('Delta');
xlabel('Years to Option Expiry');
ylabel('Strike');
view(-112,34);
xlim([0 Times(end)]);
```



```
figure;
surf(X,Y,Gamma)
title('Gamma')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```

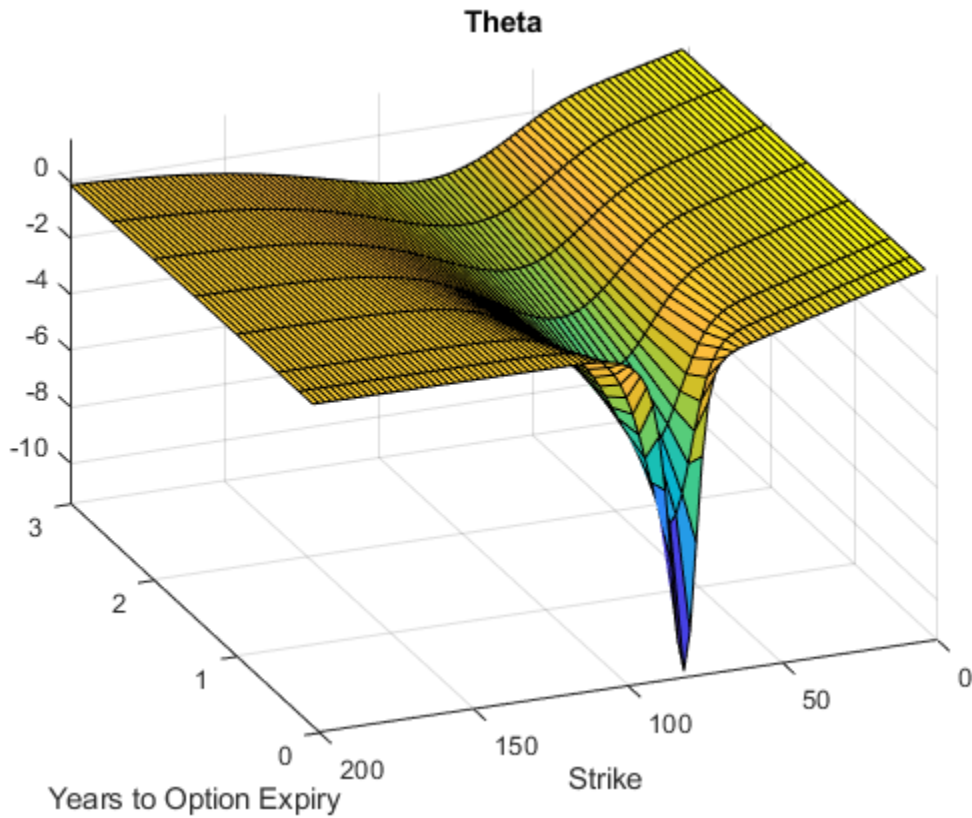


```
figure;  
surf(X,Y,Rho)  
title('Rho')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```

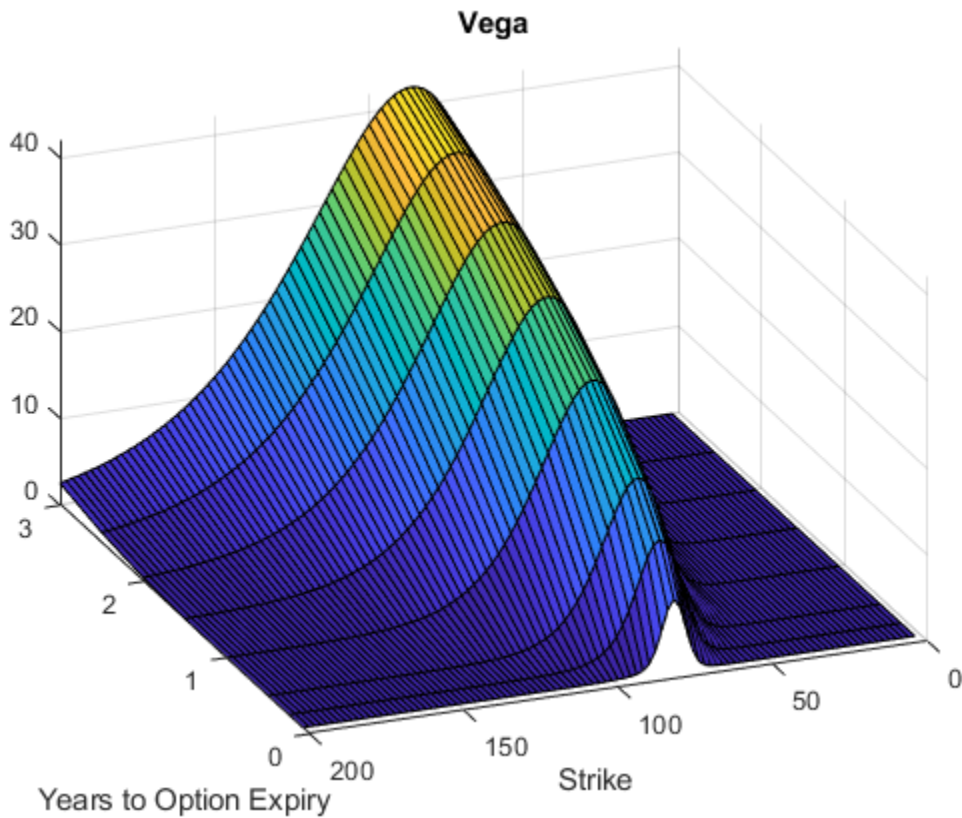


```
figure;
surf(X,Y,Theta)
title('Theta')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```





```
figure;  
surf(X,Y,Vega)  
title('Vega')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



## Input Arguments

**Rate — Continuously compounded risk-free interest rate**

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

**AssetPrice — Current underlying asset price**

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

If this input is an empty array ([]) , option prices are computed on the entire FFT (or FRFT) strike grid, which is determined as  $\exp(\log\text{-strike grid})$ . Each column of the log-strike grid has 'NumFFT' points with 'LogStrikeStep' spacing that are roughly centered around each element of  $\log(\text{AssetPrice})$ .

For more information on the proper dimensions for Strike, see the name-value pair argument ExpandOutput.

Data Types: double

### **Sigma — Volatility of underlying asset**

numeric

Volatility of the underling asset, specified as a scalar numeric value.

Data Types: double

### **MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation  $\text{JumpVol}$ .

Data Types: double

### **JumpVol — Standard deviation of $\log(1+J)$**

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

### **JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[PriceSens, StrikeOut] = optSensByMertonFFT(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, Sigma, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

### Basis — Day-count basis of instrument

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### DividendYield — Continuously compounded underlying asset yield

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

**OutSpec — Define outputs**

["price"] (default) | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegalt" | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegalt'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

---

**Note** "vega" is the sensitivity with respect to the initial volatility  $\sqrt{V_0}$ . In contrast, "vegalt" is the sensitivity with respect to the long-term volatility  $\sqrt{\text{Theta}V}$ .

---

Example: OutSpec =  
["price", "delta", "gamma", "vega", "rho", "theta", "vegalt"]

Data Types: string | cell

**NumFFT — Number of grid points in the characteristic function variable**

4096 (default) | numeric

Number of grid points in the characteristic function variable and in each column of the log-strike grid, specified as the comma-separated pair consisting of 'NumFFT' and a scalar numeric value.

Data Types: double

**CharacteristicFcnStep — Characteristic function variable grid spacing**

0.01 (default) | numeric

Characteristic function variable grid spacing, specified as the comma-separated pair consisting of 'CharacteristicFcnStep' and a scalar numeric value.

Data Types: double

**LogStrikeStep — Log-strike grid spacing**

$2\pi/\text{NumFFT}/\text{CharacteristicFcnStep}$  (default) | numeric

Log-strike grid spacing, specified as the comma-separated pair consisting of 'LogStrikeStep' and a scalar numeric value.

---

**Note** If  $(\text{LogStrikeStep} * \text{CharacteristicFcnStep})$  is  $2 * \pi / \text{NumFFT}$ , FFT is used. Otherwise, FRFT is used.

---

Data Types: double

### **DampingFactor — Damping factor for Carr-Madan formulation**

1.5 (default) | numeric

Damping factor for Carr-Madan formulation, specified as the comma-separated pair consisting of 'DampingFactor' and a scalar numeric value.

Data Types: double

### **Quadrature — Type of quadrature**

"simpson" (default) | character vector with values: 'simpson' or 'trapezoidal' | string array with values: "simpson" or "trapezoidal"

Type of quadrature, specified as the comma-separated pair consisting of 'Quadrature' and a single character vector or string array with a value of 'simpson' or 'trapezoidal'.

Data Types: char | string

### **ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- **true** — If true, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. If Strike is empty, NRows is equal to NumFFT. NColumns is determined by the sizes of AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.
- **false** — If false, the outputs are NINST-by-1 vectors. Also, the inputs Strike, AssetPrice, Settle, Maturity, and OptSpec must all be either scalar or NINST-by-1 vectors.

Data Types: `logical`

## Output Arguments

### **PriceSens** — Option prices or sensitivities

`numeric`

Option prices or sensitivities, returned as a `NINST-by-1`, or `NRows-by-NColumns`, depending on `ExpandOutput`. The name-value pair argument `OutSpec` determines the types and order of the outputs.

### **StrikeOut** — Strikes corresponding to Price

`numeric`

Strikes corresponding to `Price`, returned as a `NINST-by-1`, or `NRows-by-NColumns`, depending on `ExpandOutput`.

## Definitions

### **Merton Jump Diffusion Model**

The Merton jump diffusion model (Merton (1976)) is a different extension of the Black-Scholes model, where sudden asset price movements (both up and down) are modeled by adding the jump diffusion parameters with the Poisson process.

The stochastic differential equation is:

$$dS_t = (r - q - \lambda_p \mu_j) S_t dt + \sigma S_t dW_t + JS_t dP_t$$
$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$W_t$  is the Weiner process.



$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp\left\{-\frac{\left[\ln(1+J) - \left(\ln(1 + \mu_J) - \frac{\delta^2}{2}\right)\right]^2}{2\delta^2}\right\}$$

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

$\sigma$  is the volatility of the asset price for ( $\sigma > 0$ ).

The characteristic function  $f_{Merton76_j}(\phi)$  for  $j = 1$  (asset prices measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Merton76_j} = f_{BS_j} \exp\left(\lambda_p \tau (1 + \mu_j)^{m_j} + \frac{1}{2} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 \left(m_j i\phi + \frac{(i\phi)^2}{2}\right)} - 1 \right] - \lambda_p \tau \mu_j i\phi \right)$$

where for  $j = 1, 2$ :

$$f_{BS_1}(\phi) = \frac{f_{BS_2}(\phi - i)}{f_{BS_2}(-i)}$$

$$f_{BS_2}(\phi) = \exp\left(i\phi \left[\ln S_t + \left(r - q - \frac{\sigma^2}{2}\right)\tau\right] - \frac{\phi^2 \sigma^2}{2} \tau\right)$$

$$m_1 = \frac{1}{2}, m_2 = -\frac{1}{2}$$

where

$\phi$  is the characteristic function variable.

$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

## Carr-Madan Formulation

The Carr and Madan (1999) formulation is a popular modified implementation of Heston (1993) framework.

Rather than computing the probabilities  $P_1$  and  $P_2$  as intermediate steps, Carr and Madan developed an alternative expression so that taking its inverse Fourier transform gives the option price itself directly.

$$Call(k) = \frac{e^{-\alpha k}}{\pi} \int_{-\infty}^{\infty} \text{Re}[e^{-iuk} \psi(u)] du$$

$$\psi(u) = \frac{e^{-r\tau} f_2(\phi = (u - (\alpha + 1)i))}{\alpha^2 + \alpha - u^2 + iu(2\alpha + 1)}$$

$$Put(K) = Call(K) + Ke^{-r\tau} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$\alpha$  is the damping factor.

$u$  is the characteristic function variable for integration, where  $\phi = (u - (\alpha+1)i)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

To apply FFT or FRFT to this formulation, the characteristic function variable for integration,  $u$ , is discretized into  $\text{NumFFT}(N)$  point with the step size  $\text{CharacteristicFcnStep}(\Delta u)$ , and the log-strike  $k$  is discretized into  $N$  points with the step size  $\text{LogStrikeStep}(\Delta k)$ .

The discretized characteristic function variable for integration,  $u_j$  (for  $j = 1, 2, 3, \dots, N$ ), has a minimum value of 0 and a maximum value of  $(N-1)(\Delta u)$ , and it approximates the continuous integration range from 0 to infinity.

The discretized log-strike grid,  $k_n$  (for  $n = 1, 2, 3, N$ ) is approximately centered around  $\ln(S_t)$ , with a minimum value of

$$\ln(S_t) - \frac{N}{2}\Delta k$$

and a maximum value of

$$\ln(S_t) + \left(\frac{N}{2} - 1\right)\Delta k$$

Where the minimum allowable strike is

$$S_t \exp\left(-\frac{N}{2}\Delta k\right)$$

and the maximum allowable strike is

$$S_t \exp\left[\left(\frac{N}{2} - 1\right)\Delta k\right]$$

As a result of the discretization, the expression for the call option becomes

$$\text{Call}(k_n) = \Delta u \frac{e^{-\alpha k_n}}{\pi} \sum_{j=1}^N \text{Re} \left[ e^{-i\Delta k \Delta u (j-1)(n-1)} e^{iu_j \left[ \frac{N\Delta k}{2} - \ln(S_t) \right]} \psi(u_j) \right] w_j$$

where

$\Delta u$  is the step size of discretized characteristic function variable for integration.

$\Delta k$  is the step size of discretized log-strike.

$N$  is the number of FFT or FRFT points.

$w_j$  is the weights for quadrature used for approximating the integral.

FFT is used to evaluate the above expression if  $\Delta k$  and  $\Delta u$  are subject to the following constraint:

$$\Delta k \Delta u = \left( \frac{2\pi}{N} \right)$$

otherwise, the functions use the FRFT method described in Chourdakis (2005).

## References

- [1] Bates, D. S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [2] Carr, P., and D.B. Madan. "Option Valuation Using the Fast Fourier Transform." *Journal of Computational Finance*. Vol 2. No. 4. 1999.
- [3] Cont, R. and P. Tankov. *Financial Modeling with Jump Processes*. Chapman & Hall/CRC Press, 2004.
- [4] Chourdakis, K. "Option Pricing Using Fractional FFT." *Journal of Computational Finance*. 2005.
- [5] Merton, R. "Option Pricing When Underlying Stock Returns are Discontinuous." *Journal of Financial Economics*. Vol 3. 1976.

## See Also

[optByBatesFFT](#) | [optByBatesNI](#) | [optByHestonFFT](#) | [optByHestonNI](#) | [optByMertonFFT](#) | [optByMertonNI](#) | [optSensByBatesFFT](#) | [optSensByBatesNI](#) | [optSensByHestonFFT](#) | [optSensByHestonNI](#) | [optSensByMertonNI](#)

**Introduced in R2018a**

# optByMertonNI

Option price by Merton76 model using numerical integration

## Syntax

```
Price = optByMertonNI(Rate,AssetPrice,Settle,Maturity,OptSpec,  
Strike,Sigma,MeanJ,JumpVol,JumpFreq)  
Price = optByMertonNI( ____,Name,Value)
```

## Description

Price = optByMertonNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq) computes vanilla European option price by the Merton76 model, using numerical integration.

Price = optByMertonNI( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Workflow for Plotting an Option Price Surface Using the Merton76 Model

optByMertonNI uses numerical integration to compute option prices and then plot an option price surface.

### Define Option Variables and Merton76 Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';
```

```
Sigma = 0.16;  
MeanJ = 0.02;
```

```
JumpVol = 0.08;  
JumpFreq = 2;
```

### Compute the Option Price for a Single Strike

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = 80;
```

```
Call = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...  
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield)
```

```
Call = 4.5600
```

### Compute the Option Prices for a Vector of Strikes

The Strike input can be a vector.

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, 6);  
Strike = (76:2:84)';
```

```
Call = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...  
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield)
```

```
Call = 5×1
```

```
6.7410  
5.5762  
4.5600  
3.6891  
2.9551
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the Strike input to specify the strikes. Also, the Maturity input can be a vector, but it must match the length of the Strike vector if the ExpandOutput name-value pair argument is not set to "true".

```
Settle = datenum('29-Jun-2017');  
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities  
Strike = [76 78 80 82 84]'; % Five strikes
```

```
Call = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield)
```

```
Call = 5×1
```

```
8.5589
8.9439
9.2316
9.4653
9.6565
```

```
% Five values in vector output
```

### Expand the Output for a Surface

Set the ExpandOutput name-value pair argument to "true" to expand the output into a NStrikes-by-NMaturities matrix. In this case, it is a square matrix.

```
Call = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'ExpandOutput', true) % (5 x 5) matrix output
```

```
Call = 5×5
```

```
8.5589    9.9675    11.1343    12.1492    13.0464
7.4844    8.9439    10.1481    11.1939    12.1181
6.5125    8.0023     9.2316    10.2999    11.2449
5.6401    7.1402     8.3827     9.4653    10.4249
4.8630    6.3545     7.5990     8.6881     9.6565
```

### Compute the Option Prices for a Vector of Strikes and a Vector of Dates of Different Lengths

When ExpandOutput is "true", NStrikes do not have to match NMaturities. That is, the output NStrikes-by-NMaturities matrix can be rectangular.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes
```

```
Call = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'ExpandOutput', true) % (5 x 6) matrix output
```

```
Call = 5×6
```

```

6.7410    8.5589    9.9675    11.1343    12.1492    13.0464
5.5762    7.4844    8.9439    10.1481    11.1939    12.1181
4.5600    6.5125    8.0023    9.2316    10.2999    11.2449
3.6891    5.6401    7.1402    8.3827    9.4653    10.4249
2.9551    4.8630    6.3545    7.5990    8.6881    9.6565

```

### Compute the Option Prices for a Vector of Strikes and a Vector of Asset Prices

When `ExpandOutput` is "true", the output can also be a `NStrikes-by-NAssetPrices` rectangular matrix by accepting a vector of asset prices.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes

```

```

Call = optByMertonNI(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'ExpandOutput', true) % (5 x 4) matrix output

```

```
Call = 5×4
```

```

3.4186    5.6579    8.5589    12.0417
2.8538    4.8401    7.4844    10.7343
2.3718    4.1205    6.5125    9.5230
1.9635    3.4922    5.6401    8.4090
1.6198    2.9476    4.8630    7.3921

```

### Plot an Option Price Surface

The `Strike` and `Maturity` inputs can be vectors. Set `ExpandOutput` to "true" to output the surface as a `NStrikes-by-NMaturities` matrix.

```

Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
Strike = (2:2:200)';

```

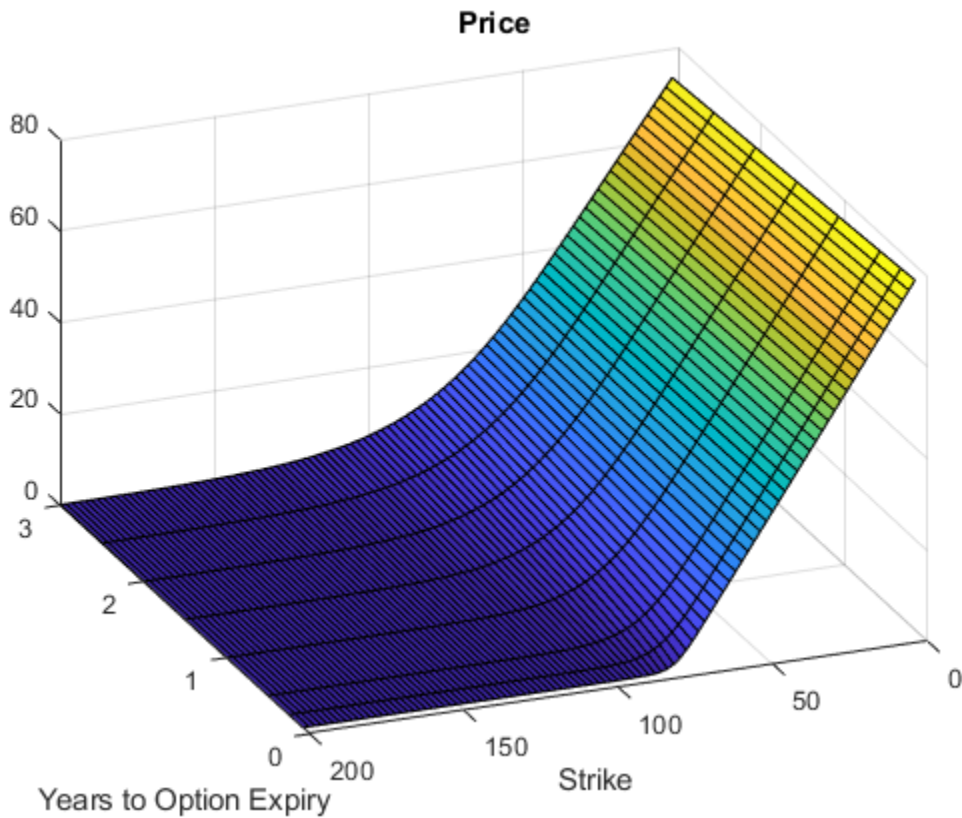
```

Call = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'ExpandOutput', true);

```



```
[X,Y] = meshgrid(Times,Strike);  
  
figure;  
surf(X,Y,Call);  
title('Price');  
xlabel('Years to Option Expiry');  
ylabel('Strike');  
view(-112,34);  
xlim([0 Times(end)]);  
zlim([0 80]);
```



## Input Arguments

### **Rate — Continuously compounded risk-free interest rate**

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice — Current underlying asset price**

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

For more information on the proper dimensions for `Strike`, see the name-value pair argument `ExpandOutput`.

Data Types: double

**Sigma — Volatility of underlying asset**

numeric

Volatility of the underling asset, specified as a scalar numeric value.

Data Types: double

**MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation `JumpVol`.

Data Types: double

**JumpVol — Standard deviation of  $\log(1+J)$** 

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

**JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = optByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, Sigma, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

### **Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double

**AbsTol — Absolute error tolerance for numerical integration**`1e-10 (default) | numeric`

Absolute error tolerance for numerical integration, specified as the comma-separated pair consisting of 'AbsTol' and a scalar numeric value.

Data Types: double

**RelTol — Relative error tolerance for numerical integration**`1e-6 (default) | numeric`

Relative error tolerance for numerical integration, specified as the comma-separated pair consisting of 'RelTol' and a scalar numeric value.

Data Types: double

**IntegrationRange — Numerical integration range used to approximate continuous integral over [0 Inf]**`[1e-9 Inf] (default) | vector`

Numerical integration range used to approximate the continuous integral over [0 Inf], specified as the comma-separated pair consisting of 'IntegrationRange' and a 1-by-2 vector representing [LowerLimit UpperLimit].

Data Types: double

**Framework — Framework for computing option prices and sensitivities using numerical integration of models**`"heston1993" (default) | string with values "heston1993" or "lewis2001" | character vector with values 'heston1993' or 'lewis2001'`

Framework for computing option prices and sensitivities using numerical integration of models, specified as the comma-separated pair consisting of 'Framework' and a scalar string or character vector with the following values:

- "heston1993" or 'heston1993' — Method used in Heston (1993)
- "lewis2001" or 'lewis2001' — Method used in Lewis (2001)

Data Types: char | string

**ExpandOutput — Flag to expand the outputs**`false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false`

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- `true` — If `true`, the outputs are `NRows`-by-`NColumns` matrices. `NRows` is the number of strikes for each column and it is determined by the `Strike` input. For example, `Strike` can be a `NRows`-by-1 vector, or a `NRows`-by-`NColumns` matrix. `NColumns` is determined by the sizes of `AssetPrice`, `Settle`, `Maturity`, and `OptSpec`, which must all be either scalar or `NColumns`-by-1 vectors.
- `false` — If `false`, the outputs are `NINST`-by-1 vectors. Also, the inputs `Strike`, `AssetPrice`, `Settle`, `Maturity`, and `OptSpec` must all be either scalar or `NINST`-by-1 vectors.

Data Types: `logical`

## Output Arguments

### Price — Option prices

`numeric`

Option prices, returned as a `NINST`-by-1, or `NRows`-by-`NColumns`, depending on `ExpandOutput`.

## Definitions

### Merton Jump Diffusion Model

The Merton jump diffusion model (Merton (1976)) is a different extension of the Black-Scholes model, where sudden asset price movements (both up and down) are modeled by adding the jump diffusion parameters with the Poisson process.

The stochastic differential equation is:

$$dS_t = (r - q - \lambda_p \mu_j) S_t dt + \sigma S_t dW_t + JS_t dP_t$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$W_t$  is the Weiner process.

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_J) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1 + J)\delta\sqrt{2\pi}} \exp \left\{ - \frac{\left[ \ln(1 + J) - \left( \ln(1 + \mu_J) - \frac{\delta^2}{2} \right) \right]^2}{2\delta^2} \right\}$$

$\mu_J$  is the mean of  $J$  for ( $\mu_J > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

$\sigma$  is the volatility of the asset price for ( $\sigma > 0$ ).

The characteristic function  $f_{Merton76_j}(\phi)$  for  $j = 1$  (asset prices measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Merton76_j} = f_{BS_j} \exp \left( \lambda_p \tau (1 + \mu_j)^{m_j} + \frac{1}{2} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 \left( m_j i\phi + \frac{(i\phi)^2}{2} \right)} - 1 \right] - \lambda_p \tau \mu_j i\phi \right)$$

where for  $j = 1, 2$ :

$$f_{BS_1}(\phi) = \frac{f_{BS_2}(\phi - i)}{f_{BS_2}(-i)}$$

$$f_{BS_2}(\phi) = \exp \left( i\phi \left[ \ln S_t + \left( r - q - \frac{\sigma^2}{2} \right) \tau \right] - \frac{\phi^2 \sigma^2}{2} \tau \right)$$

$$m_1 = \frac{1}{2}, m_2 = -\frac{1}{2}$$

where

$\phi$  is the characteristic function variable



$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

## Numerical Integration Method Under Heston (1993) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Heston (1993) framework is based on the following expressions:

$$\begin{aligned} Call(K) &= S_t e^{-q\tau} P_1 - K e^{-r\tau} P_2 \\ Put(K) &= Call(K) + K e^{-r\tau} - S_t e^{-q\tau} \\ P_j &= \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \operatorname{Re} \left[ \frac{e^{-i\phi \ln(K)} f_j(\phi)}{i\phi} \right] d\phi \end{aligned}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T - t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ )

$\phi$  is the characteristic function variable.

$f_j(\phi)$  is the characteristic function for  $P_j$  ( $j = 1, 2$ ).

$P_1$  is the probability of  $S_t > K$  under the asset price measure for the model.

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

Where  $j = 1, 2$  so that  $f_1(\phi)$  and  $f_2(\phi)$  are the characteristic functions for probabilities  $P_1$  and  $P_2$ , respectively.

This framework is chosen with the default value “Heston1993” for the Framework name-value pair argument.

## Numerical Integration Method Under Lewis (2001) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Lewis (2001) framework is based on the following expressions:

$$Call(k) = S_t e^{-q\tau} - \frac{\sqrt{K} e^{-\tau t}}{\pi} \int_0^{\infty} \operatorname{Re} \left[ K^{-iu} f_2 \left( \phi = \left( u - \frac{i}{2} \right) \frac{1}{u^2 + \frac{1}{4}} \right) \right] du$$

$$Put(K) = Call(K) = Ke^{-\tau t} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$

$i$  is a unit imaginary number ( $i^2 = -1$ )

$\phi$  is the characteristic function variable.

$u$  is the characteristic function variable for integration, where  $\phi = \left(u - \frac{i}{2}\right)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

This framework is chosen with the value "Lewis2001" for the Framework name-value pair argument.

## References

- [1] Bates, D. S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [2] Cont, R. and P. Tankov. *Financial Modeling with Jump Processes*. Chapman & Hall/CRC Press, 2004.
- [3] Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *The Review of Financial Studies*. Vol 6. No. 2. 1993.
- [4] Lewis, A. L. "A Simple Option Formula for General Jump-Diffusion and Other Exponential Levy Processes." Envision Financial Systems and OptionCity.net, 2001.
- [5] Merton, R. "Option Pricing When Underlying Stock Returns are Discontinuous." *Journal of Financial Economics*. Vol 3. 1976.

## See Also

optByBatesFFT | optByBatesNI | optByHestonFFT | optByHestonNI |  
 optByMertonFFT | optSensByBatesFFT | optSensByBatesNI |  
 optSensByHestonFFT | optSensByHestonNI | optSensByMertonFFT |  
 optSensByMertonNI

**Introduced in R2018a**

## optSensByMertonNI

Option price and sensitivities by Merton76 model using numerical integration

### Syntax

```
PriceSens = optSensByMertonNI(Rate,AssetPrice,Settle,Maturity,  
OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq)  
PriceSens = optSensByMertonNI( ____,Name,Value)
```

### Description

`PriceSens = optSensByMertonNI(Rate,AssetPrice,Settle,Maturity,OptSpec,Strike,Sigma,MeanJ,JumpVol,JumpFreq)` computes vanilla European option price and sensitivities by the Merton76 model, using numerical integration.

`PriceSens = optSensByMertonNI( ____,Name,Value)` adds optional name-value pair arguments.

### Examples

#### Workflow for Plotting an Option Sensitivity Surface Using the Merton76 Model

`optSensByMertonNI` uses numerical integration to compute option sensitivities and then plot option sensitivity surfaces.

#### Define Option Variables and Merton76 Model Parameters

```
AssetPrice = 80;  
Rate = 0.03;  
DividendYield = 0.02;  
OptSpec = 'call';  
  
Sigma = 0.16;  
MeanJ = 0.02;
```

```
JumpVol = 0.08;
JumpFreq = 2;
```

### Compute the Option Sensitivity for a Single Strike

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = 80;
```

```
Delta = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', "delta")
```

```
Delta = 0.5283
```

### Compute the Option Sensitivities for a Vector of Strikes

The Strike input can be a vector.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 6);
Strike = (76:2:84)';
```

```
Delta = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', "delta")
```

```
Delta = 5×1
```

```
0.6727
0.6013
0.5283
0.4565
0.3883
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of the Same Lengths

Use the Strike input to specify the strikes. Also, the Maturity input can be a vector, but it must match the length of the Strike vector if the ExpandOutput name-value pair argument is not set to "true".

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, [12 18 24 30 36]); % Five maturities
```

```
Strike = [76 78 80 82 84]'; % Five strikes

Delta = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', "delta")

Delta = 5x1

    0.6419
    0.5907
    0.5565
    0.5311
    0.5110

% Five values in vector output
```

### Expand the Output for a Surface

Set the `ExpandOutput` name-value pair argument to `"true"` to expand the output into a `NStrikes-by-NMaturities` matrix. In this case, it is a square matrix.

```
Delta = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'ExpandOutput', true) % (5 x 5) matrix output

Delta = 5x5

    0.6419    0.6305    0.6245    0.6204    0.6173
    0.5922    0.5907    0.5905    0.5905    0.5905
    0.5422    0.5507    0.5565    0.5607    0.5637
    0.4927    0.5112    0.5229    0.5311    0.5372
    0.4447    0.4725    0.4898    0.5020    0.5110
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Dates of Different Lengths

When `ExpandOutput` is `"true"`, `NStrikes` do not have to match `NMaturities`. That is, the output `NStrikes-by-NMaturities` matrix can be rectangular.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*(0.5:0.5:3)); % Six maturities
Strike = (76:2:84)'; % Five strikes
```

```
Delta = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'ExpandOutput', true) % (5 x 6) matrix output
```

```
Delta = 5x6
```

```
    0.6727    0.6419    0.6305    0.6245    0.6204    0.6173
    0.6013    0.5922    0.5907    0.5905    0.5905    0.5905
    0.5283    0.5422    0.5507    0.5565    0.5607    0.5637
    0.4565    0.4927    0.5112    0.5229    0.5311    0.5372
    0.3883    0.4447    0.4725    0.4898    0.5020    0.5110
```

### Compute the Option Sensitivities for a Vector of Strikes and a Vector of Asset Prices

When ExpandOutput is "true", the output can also be a NStrikes-by-NAssetPrices rectangular matrix by accepting a vector of asset prices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12); % Single maturity
ManyAssetPrices = [70 75 80 85]; % Four asset prices
Strike = (76:2:84)'; % Five strikes
```

```
Delta = optSensByMertonNI(Rate, ManyAssetPrices, Settle, Maturity, OptSpec, Strike, ..
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', "delta", 'ExpandOutput', true) % (5 x 4) matrix output
```

```
Delta = 5x4
```

```
    0.3796    0.5157    0.6419    0.7472
    0.3315    0.4637    0.5922    0.7043
    0.2874    0.4137    0.5422    0.6592
    0.2474    0.3664    0.4927    0.6128
    0.2117    0.3224    0.4447    0.5657
```

### Plot Option Sensitivity Surfaces

The Strike and Maturity inputs can be vectors. Set ExpandOutput to "true" to output the surfaces as NStrikes-by-NMaturities matrices.

```
Settle = datenum('29-Jun-2017');
Maturity = datemnth(Settle, 12*[1/12 0.25 (0.5:0.5:3)]');
Times = yearfrac(Settle, Maturity);
```

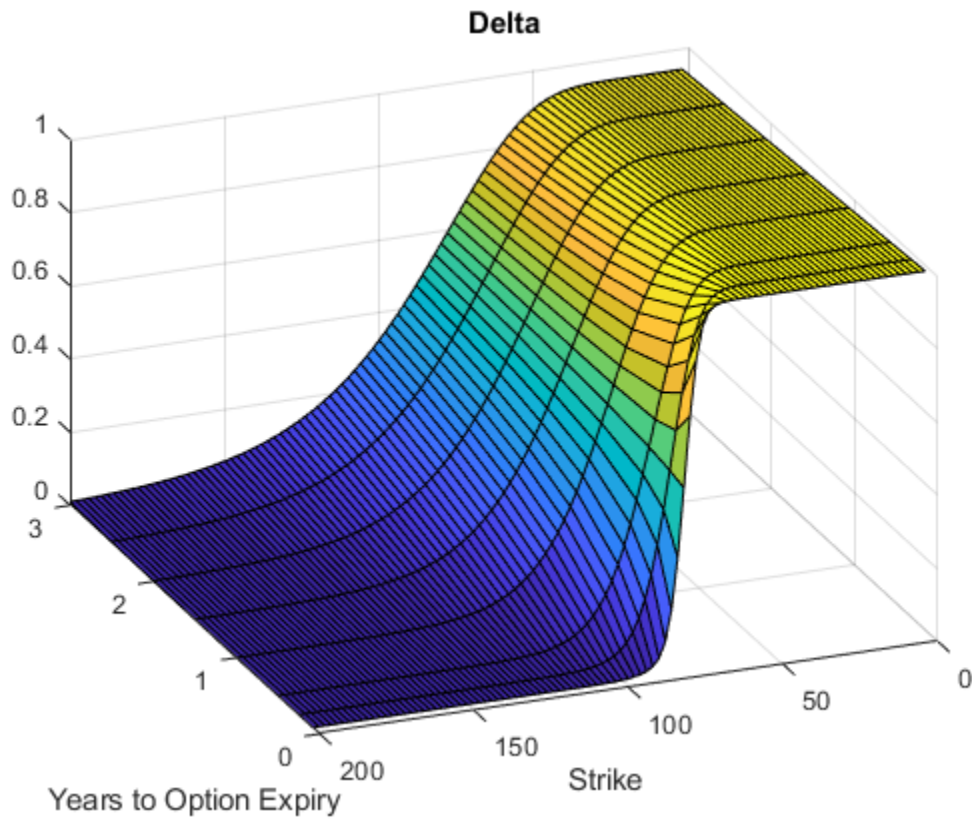
```
Strike = (2:2:200)';

[Delta, Gamma, Rho, Theta, Vega] = optSensByMertonNI(...
    Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, ...
    Sigma, MeanJ, JumpVol, JumpFreq, 'DividendYield', DividendYield, ...
    'OutSpec', ["delta", "gamma", "rho", "theta", "vega"], ...
    'ExpandOutput', true);

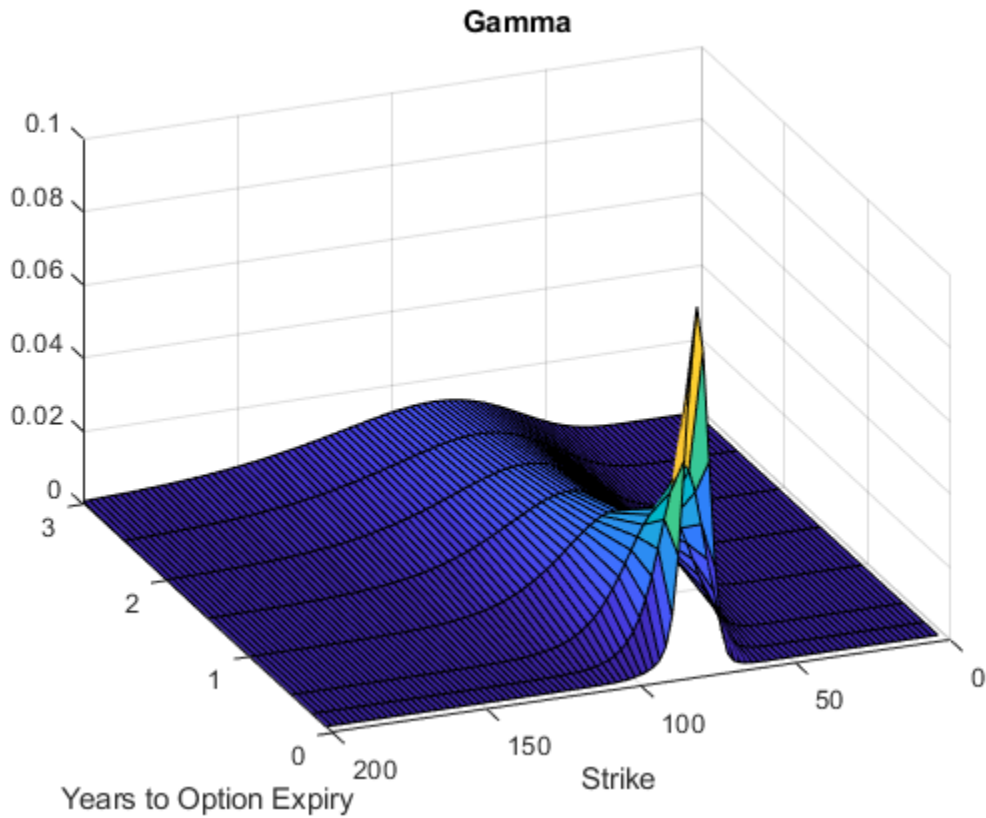
[X,Y] = meshgrid(Times,Strike);

figure;
surf(X,Y,Delta);
title('Delta');
xlabel('Years to Option Expiry');
ylabel('Strike');
view(-112,34);
xlim([0 Times(end)]);
```

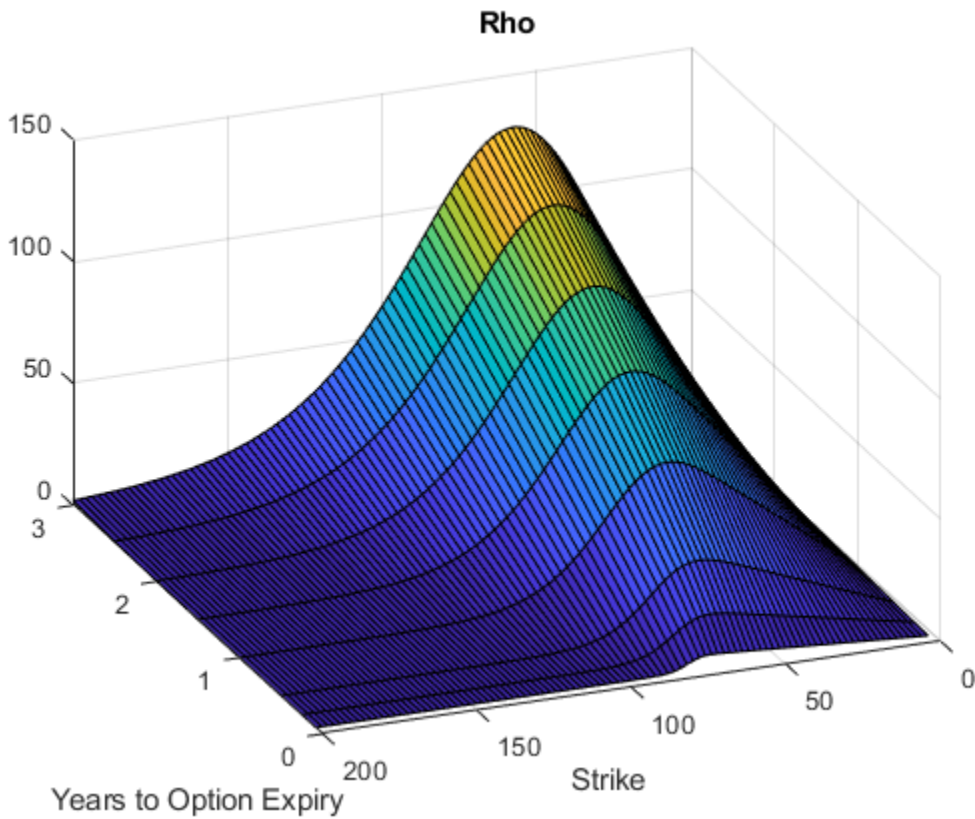




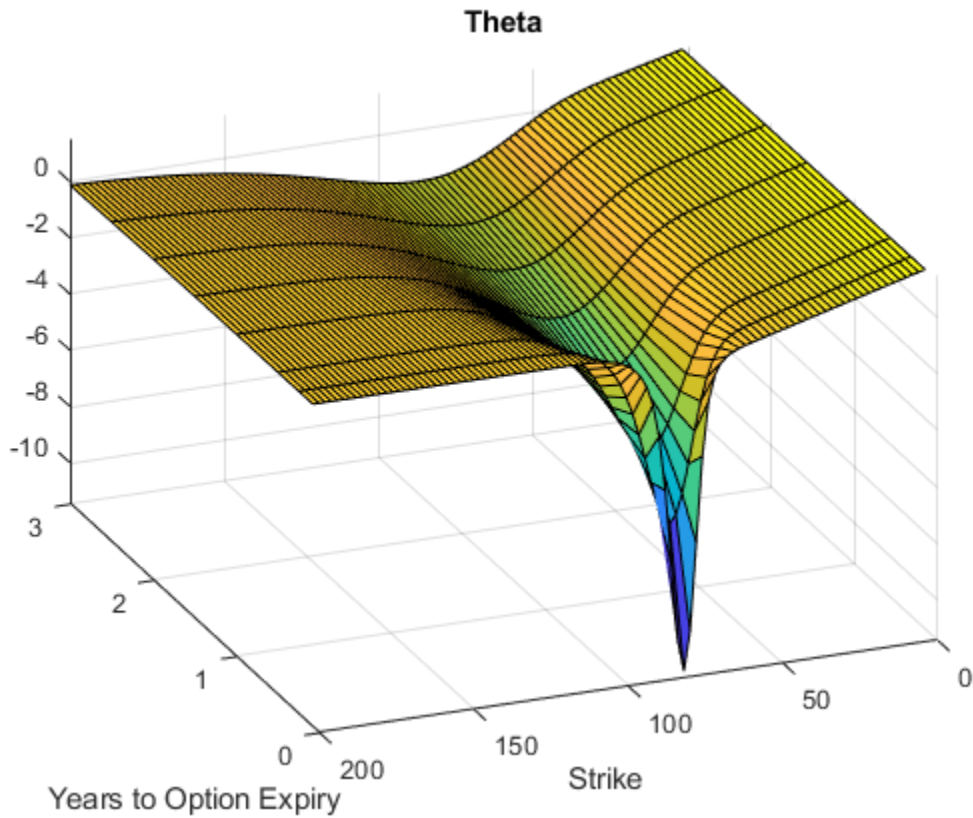
```
figure;  
surf(X,Y,Gamma)  
title('Gamma')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



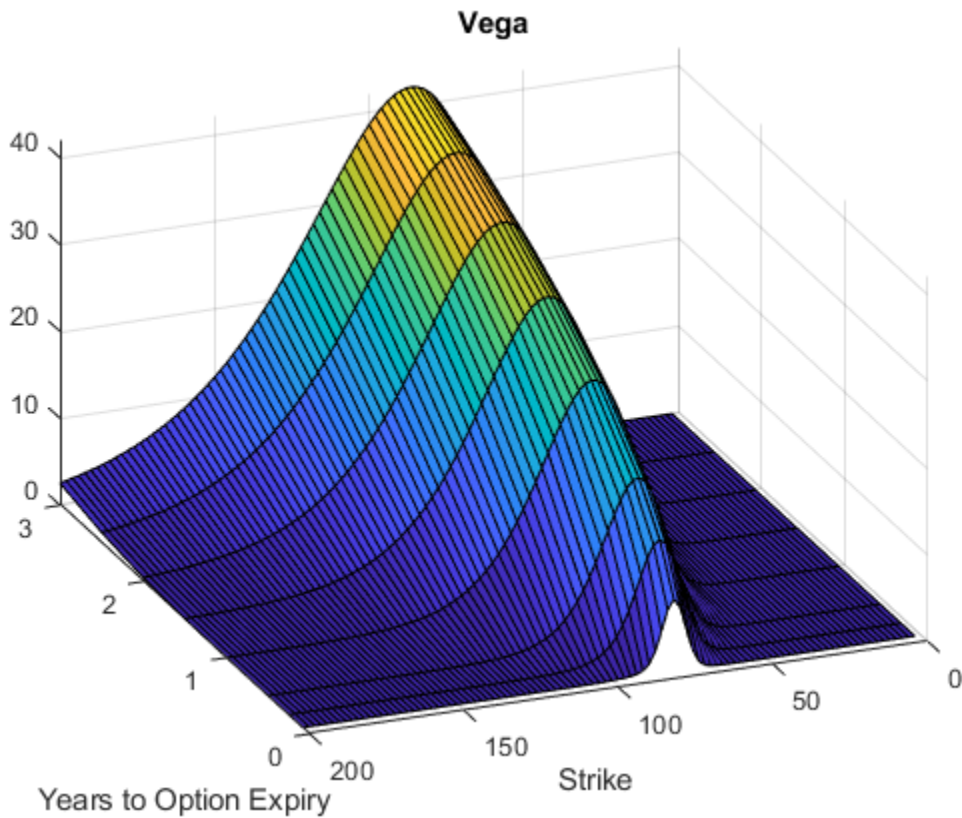
```
figure;
surf(X,Y,Rho)
title('Rho')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



```
figure;  
surf(X,Y,Theta)  
title('Theta')  
xlabel('Years to Option Expiry')  
ylabel('Strike')  
view(-112,34);  
xlim([0 Times(end)]);
```



```
figure;
surf(X,Y,Vega)
title('Vega')
xlabel('Years to Option Expiry')
ylabel('Strike')
view(-112,34);
xlim([0 Times(end)]);
```



## Input Arguments

### **Rate** — Continuously compounded risk-free interest rate

decimal

Continuously compounded risk-free interest rate, specified as a scalar decimal value.

Data Types: double

### **AssetPrice** — Current underlying asset price

numeric

Current underlying asset price, specified as numeric value using a scalar or a NINST-by-1 or NColumns-by-1 vector.

For more information on the proper dimensions for `AssetPrice`, see the name-value pair argument `ExpandOutput`.

Data Types: `double`

### **Settle — Option settlement date**

serial date number | date character vector | datetime | string array

Option settlement date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays. The `Settle` date must be before the `Maturity` date.

For more information on the proper dimensions for `Settle`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **Maturity — Option maturity date**

serial date number | date character vector | datetime | string array

Option maturity date, specified as a NINST-by-1 or NColumns-by-1 vector using serial date numbers, date character vectors, datetime arrays, or string arrays.

For more information on the proper dimensions for `Maturity`, see the name-value pair argument `ExpandOutput`.

Data Types: `double` | `char` | `datetime` | `string`

### **OptSpec — Definition of option**

cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option, specified as a NINST-by-1 or NColumns-by-1 vector using a cell array of character vectors or string arrays with values 'call' or 'put'.

For more information on the proper dimensions for `OptSpec`, see the name-value pair argument `ExpandOutput`.

Data Types: `cell` | `string`

### **Strike — Option strike price value**

numeric

Option strike price value, specified as a NINST-by-1, NRows-by-1, NRows-by-NColumns vector of strike prices.

For more information on the proper dimensions for `Strike`, see the name-value pair argument `ExpandOutput`.

Data Types: double

### **Sigma — Volatility of underlying asset**

numeric

Volatility of the underling asset, specified as a scalar numeric value.

Data Types: double

### **MeanJ — Mean of the random percentage jump size**

decimal

Mean of the random percentage jump size ( $J$ ), specified as a scalar decimal value where  $\log(1+J)$  is normally distributed with mean  $(\log(1+\text{MeanJ})-0.5*\text{JumpVol}^2)$  and the standard deviation `JumpVol`.

Data Types: double

### **JumpVol — Standard deviation of $\log(1+J)$**

decimal

Standard deviation of  $\log(1+J)$  where  $J$  is the random percentage jump size, specified as a scalar decimal value.

Data Types: double

### **JumpFreq — Annual frequency of Poisson jump process**

numeric

Annual frequency of Poisson jump process, specified as a scalar numeric value.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = optSensByMertonNI(Rate, AssetPrice, Settle, Maturity, OptSpec, Strike, Sigma, MeanJ, JumpVol, JumpFreq, 'Basis', 7)`

**Basis — Day-count basis of instrument**

0 (default) | numeric values: 0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar using a supported value:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**DividendYield — Continuously compounded underlying asset yield**

0 (default) | numeric

Continuously compounded underlying asset yield, specified as the comma-separated pair consisting of 'DividendYield' and a scalar numeric value.

Data Types: double



**OutSpec — Define outputs**

["price"] (default) | string array with values "price", "delta", "gamma", "vega", "rho", "theta", and "vegalt" | cell array of character vectors with values 'price', 'delta', 'gamma', 'vega', 'rho', 'theta', and 'vegalt'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT string array or cell array of character vectors with supported values.

---

**Note** "vega" is the sensitivity with respect the initial volatility  $\sqrt{V_0}$ . In contrast, "vegalt" is the sensitivity with respect to the long-term volatility  $\sqrt{\text{Theta}V}$ .

---

Example: OutSpec =  
["price", "delta", "gamma", "vega", "rho", "theta", "vegalt"]

Data Types: string | cell

**AbsTol — Absolute error tolerance for numerical integration**

1e-10 (default) | numeric

Absolute error tolerance for numerical integration, specified as the comma-separated pair consisting of 'AbsTol' and a scalar numeric value.

Data Types: double

**RelTol — Relative error tolerance for numerical integration**

1e-6 (default) | numeric

Relative error tolerance for numerical integration, specified as the comma-separated pair consisting of 'RelTol' and a scalar numeric value.

Data Types: double

**IntegrationRange — Numerical integration range used to approximate continuous integral over [0 Inf]**

[1e-9 Inf] (default) | vector

Numerical integration range used to approximate the continuous integral over [0 Inf], specified as the comma-separated pair consisting of 'IntegrationRange' and a 1-by-2 vector representing [LowerLimit UpperLimit].

Data Types: double

**Framework — Framework for computing option prices and sensitivities using numerical integration of models**

"heston1993" (default) | string with values "heston1993" or "lewis2001" | character vector with values 'heston1993' or 'lewis2001'

Framework for computing option prices and sensitivities using numerical integration of models, specified as the comma-separated pair consisting of 'Framework' and a scalar string or character vector with the following values:

- "heston1993" or 'heston1993' — Method used in Heston (1993)
- "lewis2001" or 'lewis2001' — Method used in Lewis (2001)

Data Types: char | string

**ExpandOutput — Flag to expand the outputs**

false (outputs are NINST-by-1 vectors) (default) | logical with value of true or false

Flag to expand the outputs, specified as the comma-separated pair consisting of 'ExpandOutput' and a logical:

- true — If true, the outputs are NRows-by- NColumns matrices. NRows is the number of strikes for each column and it is determined by the Strike input. For example, Strike can be a NRows-by-1 vector, or a NRows-by-NColumns matrix. NColumns is determined by the sizes of AssetPrice, Settle, Maturity, and OptSpec, which must all be either scalar or NColumns-by-1 vectors.
- false — If false, the outputs are NINST-by-1 vectors. Also, the inputs Strike, AssetPrice, Settle, Maturity, and OptSpec must all be either scalar or NINST-by-1 vectors.

Data Types: logical

## Output Arguments

**PriceSens — Option prices or sensitivities**

numeric

Option prices or sensitivities, returned as a NINST-by-1, or NRows-by-NColumns, depending on ExpandOutput. The name-value pair argument OutSpec determines the types and order of the outputs.

## Definitions

### Merton Jump Diffusion Model

The Merton jump diffusion model (Merton (1976)) is a different extension of the Black-Scholes model, where sudden asset price movements (both up and down) are modeled by adding the jump diffusion parameters with the Poisson process.

The stochastic differential equation is:

$$dS_t = (r - q - \lambda_p \mu_j) S_t dt + \sigma S_t dW_t + JS_t dP_t$$

$$\text{prob}(dP_t = 1) = \lambda_p dt$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$W_t$  is the Weiner process.

$J$  is the random percentage jump size conditional on the jump occurring, where  $\ln(1+J)$  is normally distributed with mean  $\ln(1 + \mu_j) - \frac{\delta^2}{2}$  and the standard deviation  $\delta$ , and  $(1+J)$  has a lognormal distribution:

$$\frac{1}{(1+J)\delta\sqrt{2\pi}} \exp\left\{-\frac{\left[\ln(1+J) - \left(\ln(1 + \mu_j) - \frac{\delta^2}{2}\right)\right]^2}{2\delta^2}\right\}$$

$\mu_j$  is the mean of  $J$  for ( $\mu_j > -1$ ).

$\delta$  is the standard deviation of  $\ln(1+J)$  for ( $\delta \geq 0$ ).

$\lambda_p$  is the annual frequency (intensity) of Poisson process  $P_t$  for ( $\lambda_p \geq 0$ ).

$\sigma$  is the volatility of the asset price for ( $\sigma > 0$ ).

The characteristic function  $f_{Merton76_j}(\phi)$  for  $j = 1$  (asset prices measure) and  $j = 2$  (risk-neutral measure) is:

$$f_{Merton76_j} = f_{BS_j} \exp \left( \lambda_p \tau (1 + \mu_j)^{m_j} + \frac{1}{2} \left[ (1 + \mu_j)^{i\phi} e^{\delta^2 \left( m_j i\phi + \frac{(i\phi)^2}{2} \right)} - 1 \right] - \lambda_p \tau \mu_j i\phi \right)$$

where for  $j = 1, 2$ :

$$f_{BS_1}(\phi) = \frac{f_{BS_2}(\phi - i)}{f_{BS_2}(-i)}$$

$$f_{BS_2}(\phi) = \exp \left( i\phi \left[ \ln S_t + \left( r - q - \frac{\sigma^2}{2} \right) \tau \right] - \frac{\phi^2 \sigma^2}{2} \tau \right)$$

$$m_1 = \frac{1}{2}, m_2 = -\frac{1}{2}$$

where

$\phi$  is the characteristic function variable.

$\tau$  is the time to maturity ( $\tau = T - t$ ).

$i$  is the unit imaginary number ( $i^2 = -1$ ).

## Numerical Integration Method Under Heston (1993) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Heston (1993) framework is based on the following expressions:

$$Call(K) = S_t e^{-q\tau} P_1 - K e^{-r\tau} P_2$$

$$Put(K) = Call(K) + K e^{-r\tau} - S_t e^{-q\tau}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \operatorname{Re} \left[ \frac{e^{-i\phi \ln(K)} f_j(\phi)}{i\phi} \right] d\phi$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$f_j(\phi)$  is the characteristic function for  $P_j$  ( $j = 1, 2$ ).

$P_1$  is the probability of  $S_t > K$  under the asset price measure for the model.

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

Where  $j = 1, 2$  so that  $f_1(\phi)$  and  $f_2(\phi)$  are the characteristic functions for probabilities  $P_1$  and  $P_2$ , respectively.

This framework is chosen with the default value "Heston1993" for the Framework name-value pair argument.

## Numerical Integration Method Under Lewis (2001) Framework

Numerical integration is used to evaluate the continuous integral for the inverse Fourier transform.

The numerical integration method under the Lewis (2001) framework is based on the following expressions:

$$Call(k) = S_t e^{-q\tau} - \frac{\sqrt{K} e^{-\tau t}}{\pi} \int_0^{\infty} \operatorname{Re} \left[ K^{-iu} f_2 \left( \phi = \left( u - \frac{i}{2} \right) \right) \frac{1}{u^2 + \frac{1}{4}} \right] du$$

$$Put(K) = Call(K) = K e^{-\tau t} - S_t e^{-q\tau}$$

where

$r$  is the continuous risk-free rate.

$q$  is the continuous dividend yield.

$S_t$  is the asset price at time  $t$ .

$K$  is the strike.

$\tau$  is time to maturity ( $\tau = T-t$ ).

$Call(K)$  is the call price at strike  $K$ .

$Put(K)$  is the put price at strike  $K$ .

$i$  is a unit imaginary number ( $i^2 = -1$ ).

$\phi$  is the characteristic function variable.

$u$  is the characteristic function variable for integration, where  $\phi = \left(u - \frac{i}{2}\right)$ .

$f_2(\phi)$  is the characteristic function for  $P_2$ .

$P_2$  is the probability of  $S_t > K$  under the risk-neutral measure for the model.

This framework is chosen with the value “Lewis2001” for the Framework name-value pair argument.

## References

- [1] Bates, D. S. “Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options.” *The Review of Financial Studies*. Vol 9. No. 1. 1996.
- [2] Cont, R. and P. Tankov. *Financial Modeling with Jump Processes*. Chapman & Hall/CRC Press, 2004.
- [3] Heston, S. L. “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options.” *The Review of Financial Studies*. Vol 6. No. 2. 1993.

- [4] Lewis, A. L. "A Simple Option Formula for General Jump-Diffusion and Other Exponential Levy Processes." *Envision Financial Systems and OptionCity.net*, 2001.
- [5] Merton, R. "Option Pricing When Underlying Stock Returns are Discontinuous." *Journal of Financial Economics*. Vol 3. 1976.

## **See Also**

[optByBatesFFT](#) | [optByBatesNI](#) | [optByHestonFFT](#) | [optByHestonNI](#) | [optByMertonFFT](#) | [optByMertonNI](#) | [optSensByBatesFFT](#) | [optSensByBatesNI](#) | [optSensByHestonFFT](#) | [optSensByHestonNI](#) | [optSensByMertonFFT](#)

**Introduced in R2018a**

## optembndbybdt

Price bonds with embedded options by Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = optembndbybdt(BDTree,CouponRate,Settle,  
Maturity,OptSpec,Strike,ExerciseDates)  
[Price,PriceTree] = optembndbybdt( ____,Name,Value)
```

### Description

[Price,PriceTree] = optembndbybdt(BDTree,CouponRate,Settle, Maturity,OptSpec,Strike,ExerciseDates) calculates price for bonds with embedded options from a Black-Derman-Toy interest-rate tree.

optembndbybdt computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions. For more information, see “Definitions” on page 11-1658.

[Price,PriceTree] = optembndbybdt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a Callable Bond Using a BDT Interest-Rate Tree Model

Create a BDTree with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];  
Compounding = 1;  
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];  
EndDates   = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];  
ValuationDate = 'jan-1-2007';
```

Create a RateSpec.



```
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, 'EndDates', ...
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [3x1 double]
    Rates: [3x1 double]
    EndTimes: [3x1 double]
    StartTimes: [3x1 double]
    EndDates: [3x1 double]
    StartDates: 733043
    ValuationDate: 733043
    Basis: 0
    EndMonthRule: 1
```

Create a VolSpec.

```
Volatility = 0.10 * ones(3,1);
VolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)
```

```
VolSpec = struct with fields:
    FinObj: 'BDTVolSpec'
    ValuationDate: 733043
    VolDates: [3x1 double]
    VolCurve: [3x1 double]
    VolInterpMethod: 'linear'
```

Create a TimeSpec.

```
TimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
```

Build the BDTTree.

```
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)
```

```
BDTTree = struct with fields:
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2]
    dObs: [733043 733408 733774]
    TFwd: {[3x1 double] [2x1 double] [2]}
```

```

CFlowT: {[3x1 double] [2x1 double] [3]}
FwdTree: {[1.0350] [1.0406 1.0495] [1.0447 1.0546 1.0667]}

```

To compute the price of an American callable bond that pays a 5.25% annual coupon, matures in Jan-1-2010, and is callable on Jan-1-2008 and 01-Jan-2010.

```

BondSettlement = 'jan-1-2007';
BondMaturity    = 'jan-1-2010';
CouponRate     = 0.0525;
Period         = 1;
OptSpec        = 'call';
Strike         = [100];
ExerciseDates   = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt     = 1;

PriceCallBond = optembndbybdt(BDTree, CouponRate, BondSettlement, BondMaturity,...
    OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOp', 1)

PriceCallBond = 101.4750

```

### Price Single Stepped Callable Bonds Using a BDT Interest-Rate Tree Model

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};
OptSpec='call';
Strike=100;

```

```

ExerciseDates='01-Jan-2012'; %Callable in two years

% Build the tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable bond
% with maturity of four years.
PBDT= optembndbybdt(BDTT, CouponRate, Settle, Maturity ,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PBDT = 2×1

    100.0945
    100.0297

```

### Price a Sinking Fund Bond Using a BDT Interest-Rate Tree Model

A corporation issues a three year bond with a sinking fund obligation requiring the company to sink 1/3 of face value after the first year and 1/3 after the second year. The company has the option to buy the bonds in the market or call them at \$98. The following data describes the details needed for pricing the sinking fund bond:

```

Rates = [0.1;0.1;0.1;0.1];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

% Create RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Build the BDT tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);

```

```

BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);

% Instrument
% The bond has a coupon rate of 9%, a period of one year and matures in
% 1-Jan-2014. Face decreases 1/3 after the first year and 1/3 after the
% second year.
CouponRate = 0.09;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
Period = 1;
Face = { ...
        {'Jan-1-2012' 100; ...
         'Jan-1-2013' 66.6666; ...
         'Jan-1-2014' 33.3333};
};

% Option provision
OptSpec = 'call';
Strike = [98 98];
ExerciseDates = {'Jan-1-2012', 'Jan-1-2013'};

% Price of non-sinking fund bond.
PNSF = bondbybdt(BDTT, CouponRate, Settle, Maturity, Period)

PNSF = 97.5131

Price of the bond with the option sinking provision.

PriceSF = optembndbybdt(BDTT, CouponRate, Settle, Maturity, ...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'Face', Face)

PriceSF = 96.8364

```

## Input Arguments

### **BDTTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdttree`.

Data Types: `struct`

**CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

**Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond is set to the `ValuationDate` of the BDT tree. The bond argument `Settle` is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

### **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = optembndbybdt(BDTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOpt', 1)`

### **AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers date or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated



at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: `char` | `double`

### **StartDate — Forward starting date of payments**

`serial date number` | `date character vector`

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: `char` | `double`

### **Face — Face value**

`100` (default) | `nonnegative value` | `cell array of nonnegative values`

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: `double`

### **Options — Derivatives pricing options**

`structure`

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices of embedded option at time 0**

`matrix`

Expected price of the embedded option at time 0, returned as a NINST-by-1 matrix.

### **PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

`structure`

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

### Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

### Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision.

This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market

or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer buys back the requirement amount of bonds from the market since bonds are cheap, but if interest rates are low (bond prices are high), then most likely the issuer is buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## See Also

[bdtpprice](#) | [bdttree](#) | [cfamounts](#) | [instoptembnd](#)

## Topics

[“Pricing a Portfolio Using the Black-Derman-Toy Model”](#) on page 1-13

[“Understanding Interest-Rate Tree Models”](#) on page 2-79

[“Pricing Options Structure”](#) on page B-2

[“Supported Interest-Rate Instruments”](#) on page 2-2

## Introduced in R2008a

## optembndbybk

Price bonds with embedded options by Black-Karasinski interest-rate tree

### Syntax

```
[Price,PriceTree] = optembndbybk(BKTree,CouponRate,Settle,Maturity,  
OptSpec,Strike,ExerciseDates)  
[Price,PriceTree] = optembndbybk( ____,Name,Value)
```

### Description

[Price,PriceTree] = optembndbybk(BKTree,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates) calculates price for bonds with embedded options from a Black-Karasinski interest-rate tree.

optembndbybk computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions. For more information, see “Definitions” on page 11-1669.

[Price,PriceTree] = optembndbybk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a Callable Bond Using a BK Interest-Rate Tree Model

Create a BKTtree with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];  
Compounding = 1;  
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];  
EndDates   = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];  
ValuationDate = 'jan-1-2007';
```

Create a RateSpec.

```
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, 'EndDates', ...
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [3x1 double]
    Rates: [3x1 double]
    EndTimes: [3x1 double]
    StartTimes: [3x1 double]
    EndDates: [3x1 double]
    StartDates: 733043
    ValuationDate: 733043
    Basis: 0
    EndMonthRule: 1
```

Create a VolSpec.

```
VolDates = ['jan-1-2008'; 'jan-1-2009'; 'jan-1-2010'];
VolCurve = 0.01;
AlphaDates = 'jan-1-2010';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve)
```

```
BKVolSpec = struct with fields:
    FinObj: 'BKVolSpec'
    ValuationDate: 733043
    VolDates: [3x1 double]
    VolCurve: [3x1 double]
    AlphaCurve: 0.1000
    AlphaDates: 734139
    VolInterpMethod: 'linear'
```

Create a TimeSpec.

```
BKTimeSpec = bktimespec(ValuationDate, EndDates, Compounding);
```

Build the BKTree.

```
BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec)
```

```
BKTree = struct with fields:
    FinObj: 'BKFwdTree'
    VolSpec: [1x1 struct]
```

```
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
  tObs: [0 1 2]
  dObs: [733043 733408 733774]
  CFlowT: {[3x1 double] [2x1 double] [3]}
  Probs: {[3x1 double] [3x3 double]}
Connect: {[2] [2 3 4]}
FwdTree: {[1.0350] [1.0458 1.0450 1.0442] [1x5 double]}
```

To compute the price of an American puttable bond that pays an annual coupon of 5.25% , matures on January 1, 2010, and is callable on January 1, 2008 and January 1, 2010.

```
BondSettlement = 'jan-1-2007';
BondMaturity   = 'jan-1-2010';
CouponRate    = 0.0525;
Period        = 1;
OptSpec       = 'put';
Strike        = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt   = 1;

PricePutBondBK = optembndbybk(BKTree, CouponRate, BondSettlement, BondMaturity,...
    OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOpt', 1)

PricePutBondBK = 102.3820
```

### Price Single Stepped Callable Bonds Using a BK Interest-Rate Tree Model

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);
```

```

% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012'; %Callable in two years

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;

BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable bond
% with maturity of four years.
PBK= optembndbybk(BKT, CouponRate, Settle, Maturity ,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PBK = 2×1

    100.0945
    100.0945

```

## Input Arguments

### **BKTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

**Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond is set to the `ValuationDate` of the BK tree. The bond argument `Settle` is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.



- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

Data Types: double | char

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: Price =  
optembndbybk(BKTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOpt', 1)

### **AmericanOpt — Option type**

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated

at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

Data Types: `char` | `double`

### **StartDate — Forward starting date of payments**

`serial date number` | `date character vector`

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify `StartDate`, the effective start date is the `Settle` date.

Data Types: `char` | `double`

### **Face — Face value**

`100` (default) | `nonnegative value` | `cell array of nonnegative values`

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: `double`

### **Options — Derivatives pricing options**

`structure`

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices of embedded option at time 0**

`matrix`

Expected price of the embedded option at time 0, returned as a NINST-by-1 matrix.

### **PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

`structure`

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

### Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

### Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision.

This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market

or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer buys back the requirement amount of bonds from the market since bonds are cheap, but if interest rates are low (bond prices are high), then most likely the issuer is buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## **See Also**

`bkprice` | `bktree` | `cfamounts` | `instoptembnd`

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## **Introduced in R2008a**

# optembndbycir

Price bonds with embedded options by Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = optembndbycir(CIRTree,CouponRate,Settle,
Maturity,OptSpec,Strike,ExerciseDates)
[Price,PriceTree] = optembndbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = optembndbycir(CIRTree,CouponRate,Settle, Maturity,OptSpec,Strike,ExerciseDates) calculates price for bonds with embedded options from a Cox-Ingersoll-Ross (CIR) interest-rate tree.

optembndbycir computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions using a CIR++ model with the Nawalka-Beliaeva (NB) approach. For more information, see “Definitions” on page 11-1678.

[Price,PriceTree] = optembndbycir( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a Callable Bond Using a CIR Interest-Rate Tree

Create a RateSpec using the intenvset function.

```
Rates = [0.025; 0.032; 0.037; 0.042];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
```

```
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'EndDates', EndDates);
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Maturity = '01-Jan-2018';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);
CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)
```

```
CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tobs: [0 0.2500 0.5000 0.7500]
        dobs: [736696 736787 736878 736969]
    FwdTree: {[1.0062] [1.0090 1.0062 1.0039] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

Define the bond with embedded option instrument.

```
BondSettlement = 'Jan-1-2017';
BondMaturity = 'Jan-1-2020';
CouponRate = 0.035;
Period = 1;
OptSpec = 'put';
Strike = 100;
ExerciseDates = {'Jan-1-2018' '01-Jan-2019'};
```

Price the bond.

```
[Price, PriceTree] = optembndbycir(CIRT, CouponRate, BondSettlement, BondMaturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', 1, 'Period', 1)
```

```
Price = 103.3099
```

```
PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
```



```
tObs: [0 0.2500 0.5000 0.7500 1]
Ptree: {1x5 cell}
Connect: {[3x1 double] [3x3 double] [3x5 double]}
```

## Input Arguments

### **CIRtree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an `NINST`-by-1 decimal annual rate or `NINST`-by-1 cell array, where each element is a `NumDates`-by-2 cell array. The first column of the `NumDates`-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle — Settlement date**

serial date number | date character vector | string array | datetime

Settlement date for the bond option, specified as a `NINST`-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

---

**Note** The `Settle` date for every bond is set to the `ValuationDate` of the CIR tree. The bond argument `Settle` is ignored.

---

Data Types: `double` | `char` | `string` | `datetime`

### **Maturity — Maturity date**

serial date number | date character vector | string array | datetime

Maturity date, specified as an `NINST`-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string array with value "call" or "put"

Definition of option, specified as a NINST-by-1 cell array of character vectors or string arrays with a value of 'call' or 'put'.

Data Types: char | cell | string

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

**ExerciseDates — Option exercise dates**

serial date number | date character vector | string array | datetime

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers, date character vectors, string arrays, or datetime arrays depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a NINST-by-1 vector, the option can be exercised between ValuationDate of the stock tree and the single listed ExerciseDates.

Data Types: double | char | string | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = optembndbycir(BDTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOpt', 1)`

### AmericanOpt — Option type

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and a NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### Period — Coupons per year

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of `'Period'` and a NINST-by-1 vector.

Data Types: double

### Basis — Day-count basis

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of `'Basis'` and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)

- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector | string array | datetime

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: double | char | string | datetime

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector | string array | datetime

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | string | datetime

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector | string array | datetime

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: char | double | string | datetime

### **StartDate — Forward starting date of payments**

serial date number | date character vector | string array | datetime

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers, date character vectors, string arrays, or datetime arrays.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double | string | datetime

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: double

## Output Arguments

### **Price — Expected prices of embedded option at time 0**

matrix

Expected price of the embedded option at time 0, returned as a NINST-by-1 matrix.

### **PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.tObs` contains the observation times.
- `PriceTree.PTree` contains the clean prices.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.

## Definitions

### **Vanilla Bond with Embedded Option**

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

## Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (calls and puts).

## Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision.

This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer buys back the requirement amount of bonds from the market since bonds are cheap, but if interest rates are low (bond prices are high), then most likely the issuer is buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirsa, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.

- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

### See Also

bondbycir | capbycir | cfbycir | fixedbycir | floatbycir | floorbycir | instoptembnd | oasbycir | optbndbycir | optemfloatbycir | optfloatbycir | rangefloatbycir | swapbycir | swaptionbycir

### Topics

"Pricing a Portfolio Using the Black-Derman-Toy Model" on page 1-13

"Understanding Interest-Rate Tree Models" on page 2-79

"Pricing Options Structure" on page B-2

"Supported Interest-Rate Instruments" on page 2-2

**Introduced in R2018a**



# optembndbyhjm

Price bonds with embedded options by Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price,PriceTree] = optembndbyhjm(HJMTtree,CouponRate,Settle,
Maturity,OptSpec,Strike,ExerciseDates)
[Price,PriceTree] = optembndbyhjm( ____,Name,Value)
```

## Description

[Price,PriceTree] = optembndbyhjm(HJMTtree,CouponRate,Settle, Maturity,OptSpec,Strike,ExerciseDates) calculates price for bonds with embedded options from a Heath-Jarrow-Morton interest-rate tree.

optembndbyhjm computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions. For more information, see “Definitions” on page 11-1692.

[Price,PriceTree] = optembndbyhjm( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a Callable Bond Using an HJM Interest-Rate Tree Model

Create a HJMTtree with the following data:

```
Rates = [0.05;0.06;0.07];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates    = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
```

Create a RateSpec.

```
RateSpec = intenvset('Rates', Rates, 'StartDates', ValuationDate, 'EndDates', ...  
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
        Disc: [3x1 double]  
        Rates: [3x1 double]  
        EndTimes: [3x1 double]  
        StartTimes: [3x1 double]  
        EndDates: [3x1 double]  
    StartDates: 733043  
    ValuationDate: 733043  
        Basis: 0  
    EndMonthRule: 1
```

Create a VolSpec.

```
VolSpec = hjmvolspec('Constant', 0.01)
```

```
VolSpec = struct with fields:  
    FinObj: 'HJMVolSpec'  
    FactorModels: {'Constant'}  
    FactorArgs: {{1x1 cell}}  
    SigmaShift: 0  
    NumFactors: 1  
    NumBranch: 2  
        PBranch: [0.5000 0.5000]  
    Fact2Branch: [-1 1]
```

Create a TimeSpec.

```
TimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding)
```

```
TimeSpec = struct with fields:  
    FinObj: 'HJMTimeSpec'  
    ValuationDate: 733043  
        Maturity: [3x1 double]  
    Compounding: 1  
        Basis: 0  
    EndMonthRule: 1
```

Build the HJMTree.

```
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
```

```
HJMTree = struct with fields:
```

```
  FinObj: 'HJMFwdTree'
  VolSpec: [1x1 struct]
  TimeSpec: [1x1 struct]
  RateSpec: [1x1 struct]
  tObs: [0 1 2]
  dObs: [733043 733408 733774]
  TFwd: {[3x1 double] [2x1 double] [2]}
  CFlowT: {[3x1 double] [2x1 double] [3]}
  FwdTree: {[3x1 double] [2x1x2 double] [1x2x2 double]}
```

To compute the price of an American callable bond that pays a 6% annual coupon and matures and is callable on January 1, 2010.

```
BondSettlement = 'jan-1-2007';
BondMaturity    = 'jan-1-2010';
CouponRate     = 0.06;
Period         = 1;
OptSpec        = 'call';
Strike         = [98];
ExerciseDates   = '01-Jan-2010';
AmericanOpt    = 1;
```

```
[PriceCallBond,PT] = optembndbyhjm(HJMTree, CouponRate, BondSettlement, BondMaturity, .
OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOp', 1)
```

```
PriceCallBond = 95.9492
```

```
PT = struct with fields:
```

```
  FinObj: 'HJMPriceTree'
  tObs: [0 1 2 3]
  PBush: {[95.9492] [1x1x2 double] [1x2x2 double] [98 98 98 98]}
```

### Price Single Stepped Callable Bonds Using an HJM Interest-Rate Tree Model

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};;
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012'; %Callable in two years

% Build the tree with the following data
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RS, HJMTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable
% bond with maturity of four years.
PHJM= optembndbyhjm(HJMT, CouponRate, Settle, Maturity ,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PHJM = 2×1

    100.0484
     99.8009

```

### Price a Sinking Fund Bond Using an HJM Interest-Rate Tree Model

A corporation issues a three year bond with a sinking fund obligation requiring the company to sink 1/3 of face value after the first year and 1/3 after the second year. The

company has the option to buy the bonds in the market or call them at \$99. The following data describes the details needed for pricing the sinking fund bond:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x1 double]
    Rates: [4x1 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734504
    ValuationDate: 734504
    Basis: 0
    EndMonthRule: 1
```

Build the HJM tree.

```
Sigma = 0.1;
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);
HJMVolSpec = hjmvolspec('Constant', Sigma);
HJMT = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec)
```

```
HJMT = struct with fields:
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734504 734869 735235 735600]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFLOWT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    FwdTree: {[4x1 double] [3x1x2 double] [2x2x2 double] [1x4x2 double]}
```

Define the sinking fund instrument. The bond has a coupon rate of 4.5%, a period of one year and matures in 1-Jan-2013. Face decreases 1/3 after the first year.

```
CouponRate = 0.045;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2013';
Period = 1;
Face = { {'Jan-1-2012' 100; ...
         'Jan-1-2013' 66.6666}};
```

Define the option provision.

```
OptSpec = 'call';
Strike = 99;
ExerciseDates = 'Jan-1-2012';
```

Price of non-sinking fund bond.

```
PNSF = bondbyhjm(HJMT, CouponRate, Settle, Maturity, Period)
PNSF = 100.5663
```

Price of the bond with the option sinking provision.

```
PriceSF = optembndbyhjm(HJMT, CouponRate, Settle, Maturity, ...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'Face', Face)
PriceSF = 98.8736
```

## Input Arguments

### **HJMTtree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `hjmtree`.

Data Types: `struct`

### **CouponRate — Bond coupon rate**

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the

NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: double | cell

### **Settle — Settlement date**

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The **Settle** date for every bond is set to the **ValuationDate** of the HJM tree. The bond argument **Settle** is ignored.

---

Data Types: double | char

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

### **OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char

### **Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.

- American option — `NINST-by-1` vector of strike price values for each option.

Data Types: `double`

### **ExerciseDates — Option exercise dates**

`serial date number` | `date character vector`

Option exercise dates, specified as a `NINST-by-1`, `NINST-by-2`, or `NINST-by-NSTRIKES` using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a `NINST-by-1` vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a `NINST-by-NSTRIKES` vector of dates.
- For an American option, use a `NINST-by-2` vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a `NINST-by-1` vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = optembndbyhjm(HJMTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOp', 1)`

### **AmericanOpt — Option type**

`0` European/Bermuda (default) | integer with values `0` or `1`

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and `NINST-by-1` positive integer flags with values:

- `0` — European/Bermuda
- `1` — American

Data Types: `double`



**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

**FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: char | double

**StartDate — Forward starting date of payments**

serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

**Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: double

**Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: struct

## Output Arguments

**Price — Expected prices of embedded option at time 0**

matrix

Expected price of the embedded option at time 0, returned as a NINST-by-1 matrix.

**PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PBush` contains the clean prices.

- `PriceTree.tObs` contains the observation times.

## Definitions

### Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

### Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

### Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision.

This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer buys back the requirement amount of bonds from the market since bonds are cheap, but if interest rates are low (bond prices are high), then most likely the

issuer is buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## **See Also**

[cfamounts](#) | [hjprice](#) | [hjmtree](#) | [instoptembnd](#)

## **Topics**

["Pricing Using Interest-Rate Tree Models"](#) on page 2-99

["Understanding Interest-Rate Tree Models"](#) on page 2-79

["Pricing Options Structure"](#) on page B-2

["Supported Interest-Rate Instruments"](#) on page 2-2

**Introduced in R2008a**

## optembndbyhw

Price bonds with embedded options by Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = optembndbyhw(HWTtree,CouponRate,Settle,Maturity,  
OptSpec,Strike,ExerciseDates)  
[Price,PriceTree] = optembndbyhw( ____,Name,Value)
```

### Description

[Price,PriceTree] = optembndbyhw(HWTtree,CouponRate,Settle,Maturity,OptSpec,Strike,ExerciseDates) calculates price for bonds with embedded options from a Hull-White interest-rate tree.

optembndbyhw computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions. For more information, see “Definitions” on page 11-1704.

[Price,PriceTree] = optembndbyhw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a Callable Bond Using an HW Interest-Rate Tree Model

Create a HWTtree with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];  
Compounding = 1;  
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];  
EndDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];  
ValuationDate = 'jan-1-2007';
```

Create a RateSpec.

```
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, 'EndDates', ...
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [3x1 double]
    Rates: [3x1 double]
    EndTimes: [3x1 double]
    StartTimes: [3x1 double]
    EndDates: [3x1 double]
    StartDates: 733043
    ValuationDate: 733043
    Basis: 0
    EndMonthRule: 1
```

Create a VolSpec.

```
VolDates = ['jan-1-2008'; 'jan-1-2009'; 'jan-1-2010'];
VolCurve = 0.01;
AlphaDates = 'jan-1-2010';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve)
```

```
HWVolSpec = struct with fields:
    FinObj: 'HWVolSpec'
    ValuationDate: 733043
    VolDates: [3x1 double]
    VolCurve: [3x1 double]
    AlphaCurve: 0.1000
    AlphaDates: 734139
    VolInterpMethod: 'linear'
```

Create a TimeSpec.

```
HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding)
```

```
HWTimeSpec = struct with fields:
    FinObj: 'HWTimeSpec'
    ValuationDate: 733043
    Maturity: [3x1 double]
    Compounding: 1
    Basis: 0
```

```
EndMonthRule: 1
```

Build the HWTtree.

```
HWTtree = hwtree(HWVolSpec, RateSpec, HWTTimeSpec)
```

```
HWTtree = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2]
    dObs: [733043 733408 733774]
    CFlowT: {[3x1 double] [2x1 double] [3]}
    Probs: {[3x1 double] [3x3 double]}
    Connect: {[2] [2 3 4]}
    FwdTree: {[1.0350] [1.0633 1.0451 1.0271] [1x5 double]}
```

Compute the price of an American puttable bond that pays an annual coupon of 5.25%, matures on January 1, 2010, and is puttable from January 1, 2008 to January 1, 2010.

```
BondSettlement = 'jan-1-2007';
BondMaturity = 'jan-1-2010';
CouponRate = 0.0525;
Period = 1;
OptSpec = 'put';
Strike = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt = 1;

PricePutBondHW = optembndbyhw(HWTtree, CouponRate, BondSettlement, BondMaturity,...
    OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOpt', 1)

PricePutBondHW = 102.9127
```

### **Price Single Stepped Callable Bonds Using an HW Interest-Rate Tree Model**

Price the following single stepped callable bonds using the following data: The data for the interest rate term structure is as follows:



```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013'; '01-Jan-2014'};
CouponRate = {'01-Jan-2012' .0425; '01-Jan-2014' .0750};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012'; %Callable in two years

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;

HWVolSpec = hwwolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable
% bond with maturity of four years.

PHW= optembndbyhw(HWT, CouponRate, Settle, Maturity,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PHW = 2x1

    100.0326
     99.7987

```

### Price a Sinking Fund Bond Using an HW Interest-Rate Tree Model

A corporation issues a two year bond with a sinking fund obligation requiring the company to sink 1/3 of face value after the first year. The company has the option to buy the bonds in the market or call them at \$99. The following data describes the details needed for pricing the sinking fund bond:

```
Rates = [0.1;0.1;0.1;0.1];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

% Create RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Build the HW tree
% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);

% Instrument
% The bond has a coupon rate of 9%, a period of one year and matures in
% 1-Jan-2013. Face decreases 1/3 after the first year.
CouponRate = 0.09;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2013';
Period = 1;
Face = { ...
        {'Jan-1-2012' 100; ...
         'Jan-1-2013' 66.6666}; ...
};

% Option provision
OptSpec = 'call';
```

```

Strike = 99;
ExerciseDates = 'Jan-1-2012';

% Price of non-sinking fund bond.
PNSF = bondbyhw(HWT, CouponRate, Settle, Maturity, Period)

PNSF = 98.2645

Price of the bond with the option sinking provision.

PriceSF = optembndbyhw(HWT, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'Face', Face)

PriceSF = 98.1553

```

## Input Arguments

### **HWTtree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using hwt tree.

Data Types: `struct`

### **CouponRate** — Bond coupon rate

positive decimal value

Bond coupon rate, specified as an NINST-by-1 decimal annual rate or NINST-by-1 cell array, where each element is a NumDates-by-2 cell array. The first column of the NumDates-by-2 cell array is dates and the second column is associated rates. The date indicates the last day that the coupon rate is valid.

Data Types: `double` | `cell`

### **Settle** — Settlement date

serial date number | date character vector

Settlement date for the bond option, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

---

**Note** The `Settle` date for every bond is set to the `ValuationDate` of the HW tree. The bond argument `Settle` is ignored.

---

Data Types: double | char

**Maturity — Maturity date**

serial date number | date character vector

Maturity date, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char

**OptSpec — Definition of option**

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors.

Data Types: char

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified as a NINST-by-1 or NINST-by-NSTRIKES depending on the type of option:

- European option — NINST-by-1 vector of strike price values.
- Bermuda option — NINST by number of strikes (NSTRIKES) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- American option — NINST-by-1 vector of strike price values for each option.

Data Types: double

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the type of option:

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates.

- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: double | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = optembndbyhw(HWTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Period', 1, 'AmericanOp', 1)`

### AmericanOpt — Option type

0 European/Bermuda (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and NINST-by-1 positive integer flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### Period — Coupons per year

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of `'Period'` and a NINST-by-1 vector.

Data Types: double

### Basis — Day-count basis

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of `'Basis'` and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with values 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer using a NINST-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **IssueDate — Bond issue date**

serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | char

### **FirstCouponDate — Irregular first coupon date**

serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: char | double

### **StartDate — Forward starting date of payments**

serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector using serial date numbers or date character vectors.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: char | double

### **Face — Face value**

100 (default) | nonnegative value | cell array of nonnegative values

Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector.

Data Types: `double`

### **Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure that is created with `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices of embedded option at time $\theta$**

matrix

Expected price of the embedded option at time  $\theta$ , returned as a NINST-by-1 matrix.

### **PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## **Definitions**

### **Vanilla Bond with Embedded Option**

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time.

The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value)



and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

## **Stepped Coupon Bond with Callable and Puttable Features**

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

## **Sinking Fund Bond with Embedded Option**

A sinking fund bond is a coupon bond with a sinking fund provision.

This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer buys back the requirement amount of bonds from the market since bonds are cheap, but if interest rates are low (bond prices are high), then most likely the issuer is buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## **See Also**

[cfamounts](#) | [hwprice](#) | [hwtree](#) | [instoptembnd](#)

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2008a**

## optemfloatbybdt

Price embedded option on floating-rate note for Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = optemfloatbybdt(BDTree,Spread,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
[Price,PriceTree] = optemfloatbybdt( ____,Name,Value)
```

### Description

[Price,PriceTree] = optemfloatbybdt(BDTree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) prices embedded options on floating-rate notes from a Black-Derman-Toy interest rate tree. optemfloatbybdt computes prices of vanilla floating-rate notes with embedded options.

[Price,PriceTree] = optemfloatbybdt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price Callable Embedded Option for Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x1 double]
    Rates: [4x1 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734869
    ValuationDate: 734869
    Basis: 0
    EndMonthRule: 1
```

Build the BDT tree and assume a volatility of 10%.

```
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Define the floater instruments with the embedded call option.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = {'Jan-1-2015'; 'Jan-1-2016'};
Period = 1;
OptSpec = {'call'};
Strike = 101;
ExerciseDates = 'Jan-1-2015';
```

Compute the price of the floaters with the embedded call.

```
Price= optemfloatbybdt(BDTT, Spread, Settle, Maturity, OptSpec, Strike,...
ExerciseDates)
```

```
Price = 2x1
```

```
100.2800
100.3655
```

## Input Arguments

### **BDTTree — Interest-rate tree structure**

binomial tree structure

Interest-rate tree specified as a structure by using `bdttree`.

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: `single` | `double`

### **Settle — Settlement dates of floating-rate note**

ValuationDate of BDT Tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The `Settle` date for every floating-rate note with an embedded option is set to the `ValuationDate` of the BDT tree. The floating-rate note argument `Settle` is ignored.

---

Data Types: `double` | `cell` | `char`

### **Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: `double` | `cell` | `char`

### **OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `cell` | `char`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: `single` | `double`

### **ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- If an American option, then `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = optemfloatbybdt(BDTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', 1, 'FloatReset', 6, 'Basis', 8)`

**AmericanOpt — Option type**

scalar | vector of positive integers[0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

**FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 3, 4, 6, 12] in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360

- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.



Data Types: double | cell

### **Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 are returned as a scalar or an NINST-by-1 vector.

### **PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PTree` contains option prices.
- `PriceTree.tObs` contains the observation times.

## **See Also**

`instoptemfloat` | `optembndbybdt` | `optemfloatbybk` | `optemfloatbyhjm` | `optemfloatbyhw`

## **Topics**

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# optemfloatbybk

Price embedded option on floating-rate note for Black-Karasinski interest-rate tree

## Syntax

```
[Price,PriceTree] = optemfloatbybk(BKTree,Spread,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
[Price,PriceTree] = optemfloatbybk( ____,Name,Value)
```

## Description

[Price,PriceTree] = optemfloatbybk(BKTree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) prices embedded options on floating-rate notes from a Black-Karasinski interest rate tree. optemfloatbybk computes prices of vanilla floating-rate notes with embedded options.

[Price,PriceTree] = optemfloatbybk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price European Callable Embedded Option for Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: 734869
    ValuationDate: 734869
        Basis: 0
    EndMonthRule: 1
```

Build the BK tree.

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
    AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec)
```

```
BKT = struct with fields:
    FinObj: 'BKFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
        CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
        Probs: {[3x1 double] [3x3 double] [3x5 double]}
        Connect: {[2] [2 3 4] [2 3 4 5 6]}
        FwdTree: {[1.0300] [1.0387 1.0380 1.0373] [1x5 double] [1x7 double]}
```

The floater instrument has a spread of 15, a period of one year, and matures and is callable on Jan-1-2015.

```
Spread = 15;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2015';
```

```

Period = 1;
OptSpec = {'call'};
Strike =101;
ExerciseDates = 'Jan-1-2015';

```

Compute the price of the floater with the embedded call.

```

Price= optemfloatbybk(BKT, Spread, Settle, Maturity,...
OptSpec, Strike, ExerciseDates)

```

```

Price = 100.4201

```

## Input Arguments

### **BKTree — Interest-rate tree structure**

binomial tree structure

Interest-rate tree specified as a structure by using `bktree`.

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: `single` | `double`

### **Settle — Settlement dates of floating-rate note**

ValuationDate of BDT Tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The `Settle` date for every floating-rate note with an embedded option is set to the `ValuationDate` of the BK tree. The floating-rate note argument `Settle` is ignored.

---

Data Types: `double` | `cell` | `char`

**Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: double | cell | char

**OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: cell | char

**Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: single | double

**ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.

Data Types: double | char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = optemfloatbybk(BKTree, Spread, Settle, Maturity, OptSpec, Strike, Exercise Dates, 'AmericanOpt', 1, 'FloatReset', 6, 'Basis', 8)`

### AmericanOpt — Option type

scalar | vector of positive integers [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### FloatReset — Frequency of payments per year

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 3, 4, 6, 12] in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.



- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell

### **Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 are returned as a scalar or an NINST-by-1 vector.

### **PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PTree` contains option prices.
- `PriceTree.tObs` contains the observation times.

## **See Also**

instoptemfloat | optembndbybk | optemfloatbybdt | optemfloatbyhjm |  
optemfloatbyhw

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99  
“Understanding Interest-Rate Tree Models” on page 2-79  
“Pricing Options Structure” on page B-2  
“Supported Interest-Rate Instruments” on page 2-2

## **Introduced in R2013a**

# optemfloatbycir

Price embedded option on floating-rate note for Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = optemfloatbycir(CIRTree,Spread,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
[Price,PriceTree] = optemfloatbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = optemfloatbycir(CIRTree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) prices embedded options on floating-rate notes from a Cox-Ingersoll-Ross (CIR) interest rate tree. `optemfloatbycir` computes prices of vanilla floating-rate notes with embedded options using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = optemfloatbycir( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price European Callable Embedded Option for a Floating-Rate Note Using a CIR Interest-Rate Tree

Create a `RateSpec` using the `intenvset` function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2020';
CIRTimeSpec = cirtimespec(Settle, Maturity, 3);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)
```

```
CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2]
    dObs: [736696 737061 737426]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double]}
    Connect: {[3x1 double] [3x3 double]}
    Probs: {[3x1 double] [3x3 double]}
```

Define the floater instruments with the embedded call option.

```
Spread = 10;
Settle = 'Jan-1-2017';
Maturity = {'Jan-1-2019'; 'Jan-1-2020'};
Period = 1;
OptSpec = {'call'};
Strike = 101;
ExerciseDates = 'Jan-1-2019';
```

Compute the price of the floaters with the embedded call.

```
[Price, PriceTree] = optemfloatbycir(CIRT, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates);

Price = 2x1

    100.1887
    100.2757
```

```
PriceTree = struct with fields:
  FinObj: 'CIRPriceTree'
  tObs: [0 1 2 3]
  PTree: {[2x1 double] [2x3 double] [2x5 double] [2x5 double]}
```

## Input Arguments

### **CIRTree — Interest-rate tree structure**

structure

Interest-rate tree specified as a structure by using `cirtree`.

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST-by-1).

Data Types: `single` | `double`

### **Settle — Settlement dates of floating-rate note**

ValuationDate of CIR tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors | string array | datetime

Settlement dates of floating-rate note specified as serial date numbers, date character vectors, string arrays, or datetime arrays using a NINST-by-1 vector of dates.

---

**Note** The `Settle` date for every floating-rate note with an embedded option is set to the `ValuationDate` of the CIR tree. The floating-rate note argument `Settle` is ignored.

---

Data Types: `double` | `cell` | `char` | `string` | `datetime`

### **Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors | string array | datetime

Floating-rate note maturity date specified as serial date numbers, date character vectors, string arrays, or datetime arrays using a NINST-by-1 vector of dates.

Data Types: double | cell | char | string | datetime

### **OptSpec — Definition of option**

character vector with value of 'call' or 'put' | cell array of character vectors with value of 'call' or 'put' | string array with value of "call" or "put"

Definition of option, specified as a NINST-by-1 cell array of character vectors or string arrays with a value of 'call' or 'put'.

Data Types: cell | char | string

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: single | double

### **ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors | string array | datetime

Exercise date for option (European, Bermuda, or American) specified as serial date numbers, date character vectors, string arrays, or datetime arrays using an NINST-by-NSTRIKES or an NINST-by-2 vector of for the option exercise dates.

- For a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For an American option, the ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDates.

Data Types: double | char | cell | string | datetime

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = optemfloatbycir(CIRTree, Spread, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', 1, 'FloatReset', 6, 'Basis', 8)`

### **AmericanOpt — Option type**

scalar | vector of positive integers [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a NINST-by-1 vector of flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 3, 4, 6, 12] in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

### **Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double



**Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts.

When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates, and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell

**Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and the output from derivset.

Data Types: struct

**Output Arguments****Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 are returned as a scalar or an NINST-by-1 vector.

**PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- PriceTree.tObs contains the observation times.
- PriceTree.PTree contains option prices.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

`bondbycir` | `capbycir` | `cfbycir` | `fixedbycir` | `floatbycir` | `floorbycir` | `instoptemfloat` | `oasbycir` | `optbndbycir` | `optembndbycir` | `optfloatbycir` | `rangefloatbycir` | `swapbycir` | `swaptionbycir`

## Topics

- "Pricing Using Interest-Rate Tree Models" on page 2-99
- "Understanding Interest-Rate Tree Models" on page 2-79
- "Pricing Options Structure" on page B-2
- "Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

# optemfloatbyhjm

Price embedded option on floating-rate note for Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price,PriceTree] = optemfloatbyhjm(HJMTree,Spread,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
[Price,PriceTree] = optemfloatbyhjm( ____,Name,Value)
```

## Description

[Price,PriceTree] = optemfloatbyhjm(HJMTree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) prices embedded options on floating-rate notes from a Heath-Jarrow-Morton interest rate tree. optemfloatbyhjm computes prices of vanilla floating-rate notes with embedded options.

[Price,PriceTree] = optemfloatbyhjm( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price European Callable Embedded Option for a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.035;0.040;0.045];
ValuationDate = 'Jan-1-2012';
StartDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: [4x1 double]
    ValuationDate: 734869
        Basis: 0
    EndMonthRule: 1
```

Build the HJM tree.

```
VolSpec = hjmvolspec('Constant', 0.01);
TimeSpec = hjmtimespec(RateSpec.ValuationDate, EndDates, Compounding);
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
```

```
HJMTree = struct with fields:
    FinObj: 'HJMFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734869 735235 735600 735965]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    FwdTree: {[4x1 double] [3x1x2 double] [2x2x2 double] [1x4x2 double]}
```

The floater instrument has a spread of 15, a period of one year, and matures and is callable on Jan-1-2016.

```
Spread = 15;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2016';
Period = 1;
OptSpec = {'call'};
Strike = 95;
ExerciseDates = 'Jan-1-2016';
```

Compute the price of the floater with the embedded call.

```
Price = optemfloatbyhjm(HJMTree, Spread, Settle, Maturity, ...
OptSpec, Strike, ExerciseDates)
```

```
Price = 96.2355
```

## Input Arguments

### **HJMTree — Interest-rate tree structure**

binomial tree structure

Interest-rate tree specified as a structure by using `hjm tree`.

Data Types: `struct`

### **Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: `single` | `double`

### **Settle — Settlement dates of floating-rate note**

ValuationDate of BDT Tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The `Settle` date for every floating-rate note with an embedded option is set to the `ValuationDate` of the HJM tree. The floating-rate note argument `Settle` is ignored.

---

Data Types: `double` | `cell` | `char`

### **Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: `double` | `cell` | `char`

**OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `cell` | `char`

**Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: `single` | `double`

**ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- If an American option, then `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-`NaN` date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `cell` | `char`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: [Price,PriceTree] =  
optemfloatbyhjm(HJMTTree,Spread,Settle,Maturity,OptSpec,Strike,ExerciseDates,'AmericanOpt',1,'FloatReset',6,'Basis',8)

### **AmericanOpt — Option type**

scalar | vector of positive integers [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### **FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 3, 4, 6, 12] in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

### **Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values



Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell

### **Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 are returned as a scalar or an NINST-by-1 vector.

### **PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PBush` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## **See Also**

`instoptemfloat` | `optembndbyhjm` | `optemfloatbybdt` | `optemfloatbybk` | `optemfloatbyhw`

## **Topics**

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# optemfloatbyhw

Price embedded option on floating-rate note for Hull-White interest-rate tree

## Syntax

```
[Price,PriceTree] = optemfloatbyhw(HWTtree,Spread,Settle,Maturity,
OptSpec,Strike,ExerciseDates)
[Price,PriceTree] = optemfloatbyhw( ____,Name,Value)
```

## Description

[Price,PriceTree] = optemfloatbyhw(HWTtree,Spread,Settle,Maturity, OptSpec,Strike,ExerciseDates) prices embedded options on floating-rate notes from a Hull-White interest rate tree. optemfloatbyhw computes prices of vanilla floating-rate notes with embedded options.

[Price,PriceTree] = optemfloatbyhw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price European Callable Embedded Option for Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: 734869
    ValuationDate: 734869
        Basis: 0
    EndMonthRule: 1
```

Build the HW tree using the following:

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
```

```
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
```

```
HWT = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734869 735235 735600 735965]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0300] [1.0562 1.0381 1.0202] [1x5 double] [1x7 double]}
```

Define the floater instruments with the embedded call option.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = {'Jan-1-2015'; 'Jan-1-2016'};
Period = 1;
```

```
OptSpec = {'call'};
Strike = 101;
ExerciseDates = 'Jan-1-2015';
```

Compute the price of the floaters with the embedded call.

```
Price= optemfloatbyhw(HWT, Spread, Settle, Maturity, OptSpec, Strike,...
ExerciseDates)
```

```
Price = 2×1
```

```
100.2800
100.3655
```

## Input Arguments

### **HWTtree — Interest-rate tree structure**

binomial tree structure

Interest-rate tree specified as a structure by using hwtree.

Data Types: struct

### **Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: single | double

### **Settle — Settlement dates of floating-rate note**

ValuationDate of BDT Tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The Settle date for every floating-rate note with an embedded option is set to the ValuationDate of the HW tree. The floating-rate note argument Settle is ignored.

---

Data Types: double | cell | char

**Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: double | cell | char

**OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: cell | char

**Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: single | double

**ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the ExerciseDates is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one ExerciseDate on the option expiry date.
- If an American option, then ExerciseDates is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if ExerciseDates is 1-by-1, the option exercises between the Settle date and the single listed ExerciseDate.

Data Types: double | char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `[Price,PriceTree] = optemfloatbyhw(HWTree,Spread,Settle,Maturity,OptSpec,Strike,ExerciseDates,'AmericanOpt',1,'FloatReset',6,'Basis',8)`

### AmericanOpt — Option type

scalar | vector of positive integers [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

### FloatReset — Frequency of payments per year

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 3, 4, 6, 12] in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.



- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

### **Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell

### **Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 are returned as a scalar or an NINST-by-1 vector.

### **PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PTree` contains embedded option prices.
- `PriceTree.tObs` contains the observation times.

## **See Also**

instoptemfloat | optembndbyhw | optemfloatbybdt | optemfloatbybk |  
optemfloatbyhjm

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99  
“Calibrating Hull-White Model Using Market Data” on page 2-111  
“Understanding Interest-Rate Tree Models” on page 2-79  
“Pricing Options Structure” on page B-2  
“Supported Interest-Rate Instruments” on page 2-2

## **Introduced in R2013a**

## optfloatbybdt

Price options on floating-rate notes for Black-Derman-Toy interest-rate tree

### Syntax

```
[Price,PriceTree] = optfloatbybdt(BDTree,OptSpec,Strike,
ExerciseDates,AmericanOpt,Spread,Settle,Maturity)
[Price,PriceTree] = optfloatbybdt( ____,Name,Value)
```

### Description

[Price,PriceTree] = optfloatbybdt(BDTree,OptSpec,Strike, ExerciseDates,AmericanOpt,Spread,Settle,Maturity) prices options on floating-rate notes from a Black-Derman-Toy interest rate tree. optfloatbybdt computes prices of options on vanilla floating-rate notes.

[Price,PriceTree] = optfloatbybdt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Price of American Call and Put Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: 734869
    ValuationDate: 734869
        Basis: 0
    EndMonthRule: 1
```

Build the BDT tree and assume a volatility of 10%.

```
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)));
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
```

```
BDTT = struct with fields:
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    dObs: [734869 735235 735600 735965]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    FwdTree: {1x4 cell}
```

The floater instrument has a spread of 10, a period of one year, and matures on Jan-1-2016.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2016';
Period = 1;
```

Define the option for the floating-rate note.

```
OptSpec = {'call'; 'put'};
Strike = [100;101];
```

```
ExerciseDates = 'Jan-1-2015';
AmericanOpt = 1;
```

Compute the price of the call and put options.

```
Price= optfloatbybdt(BDTree, OptSpec, Strike, ExerciseDates,AmericanOpt, Spread,...
Settle, Maturity)
```

```
Price = 2×1
```

```
    0.3655
    0.8087
```

## Input Arguments

### **BDTree — Interest-rate tree structure**

binomial tree structure

Interest-rate tree specified as a structure by using `bdtree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `cell` | `char`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: `single` | `double`

### **ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- If an American option, then `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

### **AmericanOpt — Option type**

scalar | vector of positive integers [0, 1]

Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: `single` | `double`

### **Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: `single` | `double`

### **Settle — Settlement dates of floating-rate note**

`ValuationDate` of BDT Tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The `Settle` date for every floating-rate note is set to the `ValuationDate` of the BDT tree. The floating-rate note argument `Settle` is ignored.

---

Data Types: double | cell | char

### **Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: double | cell | char

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example:

```
[Price, PriceTree]=optfloatbybdt(BDTree, OptSpec, Strike, ExerciseDates, AmericanOpt, Spread, Settle, Maturity, 'FloatReset', 4, 'Basis', 7)
```

### **FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 3, 4, 6, 12] in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell



**Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and structure obtained from using `derivset`.

Data Types: `struct`**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: `double`

## Output Arguments

**Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

**PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PTree` contains option prices.
- `PriceTree.t0bs` contains the observation times.

## See Also

bdttree | bondbybdt | capbybdt | cfbybdt | floatbybdt | floorbybdt | instoptfloat | swapbybdt

## Topics

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## Introduced in R2013a

# optfloatbybk

Price options on floating-rate notes for Black-Karasinski interest-rate tree

## Syntax

```
[Price,PriceTree] = optfloatbybdt(BKTree,OptSpec,Strike,
ExerciseDates,AmericanOpt,Spread,Settle,Maturity)
[Price,PriceTree] = optfloatbybdt( ____,Name,Value)
```

## Description

[Price,PriceTree] = optfloatbybdt(BKTree,OptSpec,Strike, ExerciseDates,AmericanOpt,Spread,Settle,Maturity) prices options on floating-rate notes from a Black-Karasinski interest rate tree. optfloatbybk computes prices of options on vanilla floating-rate notes.

[Price,PriceTree] = optfloatbybdt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Compute the Price of American and European Call Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',StartDates,...
'EndDates',EndDates,'Rates',Rates,'Compounding',Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
    StartDates: 734869
    ValuationDate: 734869
        Basis: 0
    EndMonthRule: 1
```

Build the BK tree.

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolSpec(RateSpec.ValuationDate,VolDates,VolCurve,...
AlphaDates,AlphaCurve);
BKTimeSpec = bktimeSpec(RateSpec.ValuationDate,VolDates,Compounding);
BKT = bktree(BKVolSpec,RateSpec,BKTimeSpec)
```

```
BKT = struct with fields:
    FinObj: 'BKFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0300] [1.0387 1.0380 1.0373] [1x5 double] [1x7 double]}
```

The floater instrument has a spread of 10, a period of one year, and matures on Jan-1-2016.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2016';
Period = 1;
```

Define the option for the floating-rate note.

```
OptSpec = {'call'};
Strike = 95;
ExerciseDates = 'Jan-1-2016';
AmericanOpt = [0;1];
```

Compute the price of the call options.

```
Price = optfloatbybk(BKT,OptSpec,Strike,ExerciseDates,AmericanOpt,...
Spread,Settle,Maturity)
```

```
Price = 2×1
```

```
4.2740
5.3655
```

## Input Arguments

### **BKTree** — Interest-rate tree structure

binomial tree structure

Interest-rate tree specified as a structure by using `bktree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `cell` | `char`

### **Strike** — Option strike price values

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: `single` | `double`

**ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- If an American option, then `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-`NaN` date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

**AmericanOpt — Option type**

scalar | vector of positive integers[0, 1]

Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: `single` | `double`

**Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: `single` | `double`

**Settle — Settlement dates of floating-rate note**

ValuationDate of BK tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The **Settle** date for every floating-rate note is set to the **ValuationDate** of the BK tree. The floating-rate note argument **Settle** is ignored.

---

Data Types: double | cell | char

**Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: double | cell | char

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1**, **Value1**, . . . , **NameN**, **ValueN**.

Example: [Price,PriceTree] =  
optfloatbybk(BKTree,OptSpec,Strike,ExerciseDates,AmericanOpt,Spread,  
Settle,Maturity,'FloatReset',4,'Basis',7)

**FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1,2,3,4,6,12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1,2,3,4,6,12] in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double



**Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell

**Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using derivset.

Data Types: struct

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

**Output Arguments****Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

**PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

**See Also**

`bktree` | `bondbybk` | `capbybk` | `cfbybk` | `floatbybk` | `floorbybk` | `instoptfloat` | `swapbybk`

**Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

# optfloatbycir

Price options on floating-rate notes for Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = optfloatbycir(CIRTree,OptSpec,Strike,
ExerciseDates,AmericanOpt,Spread,Settle,Maturity)
[Price,PriceTree] = optfloatbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = optfloatbycir(CIRTree,OptSpec,Strike, ExerciseDates,AmericanOpt,Spread,Settle,Maturity) prices options on floating-rate notes from a Cox-Ingersoll-Ross (CIR) interest-rate tree. optfloatbycir computes prices of options on vanilla floating-rate notes using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = optfloatbycir( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Compute the Price of American and European Call Options on a Floating-Rate Note Using a CIR Interest-Rate Tree

Create a RateSpec using the intenvset function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2021';
CIRTimeSpec = cirtimespec(Settle, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tobs: [0 1 2 3]
    dobs: [736696 737061 737426 737791]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double] [1x7 double]}
    Connect: {[3x1 double] [3x3 double] [3x5 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

The floater instrument has a spread of 10, a period of one year, and matures on Jan-1-2018.

```
Spread = 10;
Settle = 'Jan-1-2017';
Maturity = 'Jan-1-2019';
Period = 1;
```

Define the option for the floating-rate note.

```
OptSpec = {'call'};
Strike = 95;
ExerciseDates = 'Jan-1-2018';
AmericanOpt = [0;1];
```

Compute the price of the call options.

```
[Price,PriceTree] = optfloatbycir(CIRT, OptSpec,Strike,ExerciseDates,AmericanOpt,...
Spread, Settle, Maturity)
```

```
Price = 2x1
```

```
4.9230
5.1887
```

```
PriceTree = struct with fields:
```

```
  FinObj: 'CIRPriceTree'
  PTree: {1x5 cell}
  AITree: {1x5 cell}
  tObs: [0 1 2 3 4]
  Connect: {[3x1 double] [3x3 double] [3x5 double]}
  Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

## Input Arguments

### **CIRTree — Interest-rate tree structure**

structure

Interest-rate tree specified as a structure by using `cirtree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values of 'call' or 'put' | string array with values 'call' or 'put'

Definition of option, specified as a NINST-by-1 cell array of character vectors or string arrays with values of 'call' or 'put'.

Data Types: `cell` | `char` | `string`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values, specified as nonnegative integers using an NINST-by-NSTRIKES vector of strike price values.

Data Types: `single` | `double`

**ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors | string array | datetime

Exercise date for option (European, Bermuda, or American) specified as serial date numbers, date character vectors, string arrays, or datetime arrays using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- For a European or Bermuda option, the `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, the `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: double | char | cell | string | datetime

**AmericanOpt — Option type**

scalar | vector of positive integers[0, 1]

Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: single | double

**Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: single | double

**Settle — Settlement dates of floating-rate note**

ValuationDate of CIR tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors | string array | datetime

Settlement dates of floating-rate note specified as serial date numbers, date character vectors, string arrays, or datetime arrays using a NINST-by-1 vector of dates.

---

**Note** The `Settle` date for every floating-rate note is set to the `ValuationDate` of the CIR tree. The floating-rate note argument `Settle` is ignored.

---

Data Types: `double` | `cell` | `char` | `string`

### **Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors | string array | `datetime`

Floating-rate note maturity date specified as serial date numbers, date character vectors, string arrays, or `datetime` arrays using a NINST-by-1 vector of dates.

Data Types: `double` | `cell` | `char` | `string` | `datetime`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `[Price,PriceTree] = optfloatbybk(CIRTree,OptSpec,Strike,ExerciseDates,AmericanOpt,Spread,Settle,Maturity,'FloatReset',4,'Basis',7)`

### **FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set `[1,2,3,4,6,12]` | vector of positive integers from the set `[1,2,3,4,6,12]`

Frequency of payments per year, specified as the comma-separated pair consisting of `'FloatReset'` and positive integers for the values `[1,2,3,4,6,12]` in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

**Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts.



When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates, and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell

### **Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and the output from derivset.

Data Types: struct

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

## **Output Arguments**

### **Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

### **PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

`bondbycir` | `capbycir` | `cfbycir` | `fixedbycir` | `floatbycir` | `floorbycir` | `instoptfloat` | `oasbycir` | `optbndbycir` | `optembndbycir` | `optemfloatbycir` | `rangefloatbycir` | `swapbycir` | `swaptionbycir`

## Topics

"Pricing Using Interest-Rate Tree Models" on page 2-99

"Understanding Interest-Rate Tree Models" on page 2-79

"Pricing Options Structure" on page B-2  
"Supported Interest-Rate Instruments" on page 2-2

**Introduced in R2018a**

## optfloatbyhjm

Price options on floating-rate notes for Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = optfloatbyhjm(HJMTtree,OptSpec,Strike,  
ExerciseDates,AmericanOpt,Spread,Settle,Maturity)  
[Price,PriceTree] = optfloatbyhjm( ____,Name,Value)
```

### Description

[Price,PriceTree] = optfloatbyhjm(HJMTtree,OptSpec,Strike,ExerciseDates,AmericanOpt,Spread,Settle,Maturity) prices options on floating-rate notes from a Heath-Jarrow-Morton interest rate tree. optfloatbyhjm computes prices of options on vanilla floating-rate notes.

[Price,PriceTree] = optfloatbyhjm( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Price of American and European Call Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.035;0.040;0.045];  
ValuationDate = 'Jan-1-2012';  
StartDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};  
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};  
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
        StartDates: [4x1 double]
    ValuationDate: 734869
        Basis: 0
    EndMonthRule: 1
```

Build the HJM tree.

```
VolSpec = hjmvolspec('Constant', 0.01);
TimeSpec = hjmtimespec(RateSpec.ValuationDate, EndDates, Compounding);
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
```

```
HJMTree = struct with fields:
    FinObj: 'HJMTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
        TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
        CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
        FwdTree: {[4x1 double] [3x1x2 double] [2x2x2 double] [1x4x2 double]}
```

The floater instrument has a spread of 10, a period of one year, and matures on Jan-1-2015.

```
Spread = 10;
Settle = 'Jan-1-2012';
Maturity = 'Jan-1-2015';
Period = 1;
```

Define the option for the floating-rate note.

```
OptSpec = {'call'};  
Strike = 95;  
ExerciseDates = 'Jan-1-2015';  
AmericanOpt = [0;1];
```

Compute the price of the call options.

```
Price= optfloatbyhjm(HJMTree, OptSpec, Strike, ExerciseDates,AmericanOpt,...  
Spread, Settle, Maturity)
```

```
Price = 2×1
```

```
4.5098  
5.2811
```

## Input Arguments

### **HJMTree — Interest-rate tree structure**

binomial tree structure

Interest-rate tree specified as a structure by using `hjm tree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: `cell` | `char`

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: `single` | `double`

**ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- If an American option, then `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: double | char | cell

**AmericanOpt — Option type**

scalar | vector of positive integers[0, 1]

Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: single | double

**Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: single | double

**Settle — Settlement dates of floating-rate note**

ValuationDate of HJM tree (default) | serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The `Settle` date for every floating-rate note is set to the `ValuationDate` of the HJM tree. The floating-rate note argument `Settle` is ignored.

---

Data Types: `double` | `cell` | `char`

### **Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: `double` | `cell` | `char`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = optfloatbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, AmericanOpt, Spread, Settle, Maturity, 'FloatReset', 4, 'Basis', 7)`

### **FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set `[1, 2, 3, 4, 6, 12]` | vector of positive integers from the set `[1, 2, 3, 4, 6, 12]`

Frequency of payments per year, specified as the comma-separated pair consisting of `'FloatReset'` and positive integers for the values `[1, 2, 3, 4, 6, 12]` in a NINST-by-1 vector.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---



Data Types: double

### **Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: `double` | `cell`

### **Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: `struct`

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: `double`

## **Output Arguments**

### **Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

### **PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PBush` contains the clean prices.

- `PriceTree.AIBush` contains the accrued interest.
- `PriceTree.t0bs` contains the observation times.

## See Also

`bondbyhjm` | `capbyhjm` | `cfbyhjm` | `floatbyhjm` | `floorbyhw` | `hjmtree` |  
`instoptfloat` | `swapbyhjm`

## Topics

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

## Introduced in R2013a

## optfloatbyhw

Price options on floating-rate notes for Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = optfloatbyhw(HWTree,OptSpec,Strike,
ExerciseDates,AmericanOpt,Spread,Settle,Maturity)
[Price,PriceTree] = optfloatbyhw( ____,Name,Value)
```

### Description

[Price,PriceTree] = optfloatbyhw(HWTree,OptSpec,Strike,ExerciseDates,AmericanOpt,Spread,Settle,Maturity) prices options on floating-rate notes from a Hull-White interest rate tree. optfloatbyhw computes prices of options on vanilla floating-rate notes.

[Price,PriceTree] = optfloatbyhw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Price of American and European Call Options on a Floating-Rate Note

Define the interest-rate term structure.

```
Rates = [0.03;0.034;0.038;0.04];
ValuationDate = 'Jan-1-2012';
StartDates = ValuationDate;
EndDates = {'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'; 'Jan-1-2016'};
Compounding = 1;
```

Create the RateSpec.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
        Disc: [4x1 double]
        Rates: [4x1 double]
        EndTimes: [4x1 double]
        StartTimes: [4x1 double]
        EndDates: [4x1 double]
    StartDates: 734869
    ValuationDate: 734869
        Basis: 0
    EndMonthRule: 1
```

Build the HW tree using the following:

```
VolDates = ['1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'];
VolCurve = 0.01;
AlphaDates = '01-01-2016';
AlphaCurve = 0.1;
```

```
HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
    AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec)
```

```
HWT = struct with fields:
    FinObj: 'HWFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [734869 735235 735600 735965]
    CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}
    Connect: {[2] [2 3 4] [2 3 4 5 6]}
    FwdTree: {[1.0300] [1.0562 1.0381 1.0202] [1x5 double] [1x7 double]}
```

The floater instrument has a spread of 10, a period of one year, and matures on Jan-1-2016.

```
Spread = 10;  
Settle = 'Jan-1-2012';  
Maturity = 'Jan-1-2016';  
Period = 1;
```

Define the option for the floating-rate note.

```
OptSpec = {'call'};  
Strike = 95;  
ExerciseDates = 'Jan-1-2016';  
AmericanOpt = [0;1];
```

Compute the price of the call options.

```
Price= optfloatbyhw(HWT, OptSpec, Strike, ExerciseDates,AmericanOpt,...  
Spread, Settle, Maturity)
```

```
Price = 2×1
```

```
4.2740  
5.3655
```

## Input Arguments

### **HWTtree — Interest-rate tree structure**

binomial tree structure

Interest-rate tree specified as a structure by using hwtree.

Data Types: struct

### **OptSpec — Definition of option**

character vector | cell array of character vectors

Definition of option as 'call' or 'put' specified as a NINST-by-1 cell array of character vectors for 'call' or 'put'.

Data Types: cell | char

### **Strike — Option strike price values**

nonnegative integer | vector of nonnegative integers

Option strike price values specified nonnegative integers using as NINST-by-NSTRIKES vector of strike price values.

Data Types: `single` | `double`

### **ExerciseDates — Exercise date for option (European, Bermuda, or American)**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Exercise date for option (European, Bermuda, or American) specified as serial date numbers or date character vectors using a NINST-by-NSTRIKES or NINST-by-2 vector of for the option exercise dates.

- If a European or Bermuda option, the `ExerciseDates` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- If an American option, then `ExerciseDates` is a 1-by-2 vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-NaN date, or if `ExerciseDates` is 1-by-1, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

### **AmericanOpt — Option type**

scalar | vector of positive integers [0, 1]

Option type specified as NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: `single` | `double`

### **Spread — Number of basis points over the reference rate**

nonnegative integer | vector of nonnegative integers

Number of basis points over the reference rate specified as a vector of nonnegative integers for the number of instruments (NINST)-by-1).

Data Types: `single` | `double`

**Settle — Settlement dates of floating-rate note**

ValuationDate of HW tree (default) | serial date number | vector of serial date numbers  
| date character vector | cell array of date character vectors

Settlement dates of floating-rate note specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

---

**Note** The **Settle** date for every floating-rate note is set to the **ValuationDate** of the HW tree. The floating-rate note argument **Settle** is ignored.

---

Data Types: double | cell | char

**Maturity — Floating-rate note maturity date**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Floating-rate note maturity date specified as serial date numbers or date character vectors using a NINST-by-1 vector of dates.

Data Types: double | cell | char

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1**, **Value1**, . . . , **NameN**, **ValueN**.

Example:

```
[Price,PriceTree]=optfloatbyhw(HWTree,OptSpec,Strike,ExerciseDates,AmericanOpt,Spread,Settle,Maturity,'FloatReset',4,'Basis',7)
```

**FloatReset — Frequency of payments per year**

1 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Frequency of payments per year, specified as the comma-separated pair consisting of 'FloatReset' and positive integers for the values [1, 2, 3, 4, 6, 12] in a NINST-by-1 vector.



---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for an FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

Data Types: double

### **Basis — Day-count basis of the instrument**

0 (actual/actual) (default) | positive integers of the set [1 . . . 13] | vector of positive integers of the set [1 . . . 13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. The Basis value represents the basis used when annualizing the input forward-rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Principal values**

100 (default) | vector of nonnegative values | cell array of nonnegative values

Principal values, specified as the comma-separated pair consisting of 'Principal' and nonnegative values using a NINST-by-1 vector or NINST-by-1 cell array of notional principal amounts. When using a NINST-by-1 cell array, each element is a NumDates-by-2 cell array where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

Data Types: double | cell

**Options — Structure containing derivatives pricing options**

structure

Structure containing derivatives pricing options, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

## Output Arguments

**Price — Expected prices of the floating-rate note option at time 0**

scalar | vector

Expected prices of the floating-rate note option at time 0 is returned as a scalar or an NINST-by-1 vector.

## **PriceTree — Structure of trees containing vectors of option prices at each node**

tree structure

Structure of trees containing vectors of instrument prices and accrued interest and a vector of observation times for each node returned as:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicates where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## **See Also**

`bondbyhw` | `capbyhw` | `cfbyhw` | `floatbyhw` | `floorbyhw` | `hwtree` | `instoptfloat` | `swapbyhw`

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

## optsensbysabr

Calculate option sensitivities using SABR model

### Syntax

```
Sens = optsensbysabr(ZeroCurve,Alpha,Beta,Rho,Nu,Settle,  
ExerciseDate,ForwardValue,Strike,OptSpec)  
Sens = optsensbysabr( ____,Name,Value)
```

### Description

`Sens = optsensbysabr(ZeroCurve,Alpha,Beta,Rho,Nu,Settle,ExerciseDate,ForwardValue,Strike,OptSpec)` returns the sensitivities of an option value by using the SABR stochastic volatility model.

`Sens = optsensbysabr( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Calculate the Sensitivity Values for an Interest-Rate Swaption

Define the interest rate and the model parameters.

```
SwapRate = 0.0357;  
Strike = 0.03;  
Alpha = 0.036;  
Beta = 0.5;  
Rho = -0.25;  
Nu = 0.35;  
Rates = 0.05;
```

Define the `Settle`, `ExerciseDate`, and `OptSpec` for an interest-rate swaption.

```
Settle = datenum('15-Sep-2013');
ExerciseDate = datenum('15-Sep-2015');
OptSpec = 'call';
```

Define the RateSpec for the interest-rate curve.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', ExerciseDate, 'Rates', Rates, 'Compounding', -1, 'Basis', 1)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9048
    Rates: 0.0500
    EndTimes: 2
    StartTimes: 0
    EndDates: 736222
    StartDates: 735492
    ValuationDate: 735492
    Basis: 1
    EndMonthRule: 1
```

Calculate the Delta and Vega sensitivity values for the interest-rate swaption.

```
[SABRDelta, SABRVega] = optsensbysabr(RateSpec, Alpha, Beta, Rho, Nu, Settle, ...
ExerciseDate, SwapRate, Strike, OptSpec, 'OutSpec', {'Delta', 'Vega'})
```

```
SABRDelta = 0.7025
```

```
SABRVega = 0.0772
```

### Calculate the Sensitivity Values for a Swaption Using the Shifted SABR Model

Define the interest rate and the model parameters.

```
SwapRate = 0.0002;
Strike = -0.001; % -0.1% strike.
Alpha = 0.01;
Beta = 0.5;
Rho = -0.1;
Nu = 0.15;
```

```
Shift = 0.005; % 0.5 percent shift
Rates = 0.0002;
```

Define the `Settle`, `ExerciseDate`, and `OptSpec` for the swaption.

```
Settle = datenum('1-Mar-2016');
ExerciseDate = datenum('1-Mar-2017');
OptSpec = 'call';
```

Define the `RateSpec` for the interest-rate curve.

```
RateSpec = intenvset('ValuationDate',Settle,'StartDates',Settle, ...
'EndDates',ExerciseDate,'Rates',Rates,'Compounding',-1,'Basis', 1)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9998
    Rates: 2.0000e-04
    EndTimes: 1
    StartTimes: 0
    EndDates: 736755
    StartDates: 736390
    ValuationDate: 736390
    Basis: 1
    EndMonthRule: 1
```

Calculate the `Delta` and `Vega` sensitivity values for the swaption.

```
[ShiftedSABRDelta,ShiftedSABRVega] = optsensbysabr(RateSpec, ...
Alpha,Beta,Rho,Nu,Settle,ExerciseDate,SwapRate,Strike,OptSpec, ...
'OutSpec',{'Delta','Vega'},'Shift',Shift)
```

```
ShiftedSABRDelta = 0.9628
```

```
ShiftedSABRVega = 0.0060
```

## Calculate the Sensitivity Values for an Interest-Rate Swaption with Normal (Bachelier) Implied Volatility

This example shows how to use `optsensbysabr` to calculate sensitivities for an interest-rate swaption using the Normal model for the case where the Beta parameter is  $> 0$  and where Beta = 0.

For the case where the Beta parameter is  $> 0$ , select the Normal (Bachelier) implied volatility model in `optsensbysabr`, specify the 'Model' name-value pair to 'normal'.

```
% Define the interest rate and the model parameters.
```

```
SwapRate = 0.025;
Strike = 0.02;
Alpha = 0.044;
Beta = 0.5;
Rho = -0.21;
Nu = 0.31;
Rates = 0.028;
```

```
% Define the Settle, ExerciseDate, and OptSpec for the swaption.
```

```
Settle = datenum('7-Mar-2018');
ExerciseDate = datenum('7-Mar-2020');
OptSpec = 'call';
```

```
% Define the RateSpec for the interest-rate curve.
```

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', ExerciseDate, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

```
% Calculate the Delta and Vega sensitivity values for the swaption. Set the
% 'Model' name-value pair to 'normal' in order to select the Normal
% (Bachelier) implied volatility model.
```

```
[SABRDelta, SABRVega] = optsensbysabr(RateSpec, Alpha, Beta, Rho, Nu, ...
    Settle, ExerciseDate, SwapRate, Strike, OptSpec, ...
    'OutSpec', {'Delta', 'Vega'}, 'Model', 'normal')
```

```
SABRDelta = 0.7171
```

```
SABRVega = 0.0686
```

### Calculate Sensitivities for a Swaption with Normal Implied Volatility Using the Normal SABR Model

When the Beta parameter is set to zero, the SABR model becomes the Normal SABR model. Negative interest rates are allowed when the Normal SABR model is used in combination with Normal (Bachelier) implied volatility. To select the Normal (Bachelier) implied volatility model in `optsensbysabr`, specify the 'Model' name-value pair to 'normal'.

```
% Define the interest rate and the model parameters.
```

```
SwapRate = -0.00254;  
Strike = -0.002;  
Alpha = 0.0047;  
Beta = 0; % Set the Beta parameter to zero to use the Normal SABR model  
Rho = -0.20;  
Nu = 0.28;  
Rates = 0.0001;
```

```
% Define the Settle, ExerciseDate, and OptSpec for the swaption.
```

```
Settle = datenum('11-Apr-2018');  
ExerciseDate = datenum('11-Apr-2019');  
OptSpec = 'call';
```

```
% Define the RateSpec for the interest-rate curve.
```

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...  
    'EndDates', ExerciseDate, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);
```

```
% Calculate the Delta and Vega sensitivity values for the swaption. Set the  
% 'Model' name-value pair to 'normal' in order to select the Normal  
% (Bachelier) implied volatility model.
```

```
[SABRDelta, SABRVega] = optsensbysabr(RateSpec, Alpha, Beta, Rho, Nu, ...  
    Settle, ExerciseDate, SwapRate, Strike, OptSpec, ...  
    'OutSpec', {'Delta', 'Vega'}, 'Model', 'normal')
```

```
SABRDelta = 0.4644
```



SABRVega = 0.3987

## Input Arguments

**ZeroCurve — Annualized interest-rate term structure for zero-coupon bonds**  
structure

Annualized interest-rate term structure for zero-coupon bonds, specified by using the RateSpec obtained from `intenvset` or an IRDataCurve with multiple rates using the IRDataCurve constructor.

Data Types: struct

**Alpha — Current SABR volatility**  
scalar numeric

Current SABR volatility, specified as a scalar numeric.

Data Types: double

**Beta — SABR constant elasticity of variance (CEV) exponent**  
scalar numeric

SABR CEV exponent, specified as a scalar numeric.

Data Types: double

**Rho — Correlation between forward value and volatility**  
scalar numeric

Correlation between forward value and volatility, specified as a scalar numeric.

Data Types: double

**Nu — Volatility of volatility**  
scalar numeric

Volatility of volatility, specified as a scalar numeric.

Data Types: double

**Settle — Settlement date**  
scalar for serial nonnegative date number | scalar for date character vector

Settlement date, specified as a scalar using a serial nonnegative date number or date character vector.

Data Types: double | char

**ExerciseDate — Option exercise date**

scalar for serial nonnegative date number | scalar for date character vector

Option exercise date, specified as a scalar using a serial nonnegative date number or date character vector.

Data Types: double | char

**ForwardValue — Current forward value of underlying asset**

scalar numeric | vector

Current forward value of the underlying asset, specified as a scalar numeric or vector of size NINST-by-1.

Data Types: double

**Strike — Option strike price values**

scalar numeric | vector

Option strike price values, specified as a scalar numeric or a vector of size NINST-by-1.

Data Types: double

**OptSpec — Definition of option**

character vector with value of 'call' or 'put'

Definition of the option, specified as 'call' or 'put' using a character vector.

Data Types: char

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `ModifiedSABRDelta = optsensbysabr(RateSpec, Alpha, Beta, Rho, Nu, Settle, ExerciseDate, ForwardValue, Strike, OptSpec, 'OutSpec', 'ModifiedDelta')`

**OutSpec — Sensitivity outputs**

'Delta' (default) | character vector with values 'Delta', 'Vega', 'ModifiedDelta', 'ModifiedVega', 'dSigmaF', 'dSigmaAlpha' | cell array of character vectors with values 'Delta', 'Vega', 'ModifiedDelta', 'ModifiedVega', 'dSigmaF', 'dSigmaAlpha'

Sensitivity outputs, specified as the comma-separated pair consisting of 'OutSpec' and an NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Delta', 'Vega', 'ModifiedDelta', 'ModifiedVega', 'dSigmaF', and 'dSigmaAlpha' where:

- 'Delta' is SABR Delta by Hagan et al. (2002).
- 'Vega' is SABR Vega by Hagan et al. (2002).
- 'ModifiedDelta' is SABR Delta modified by Bartlett (2006).
- 'ModifiedVega' is SABR Vega modified by Bartlett (2006).
- 'dSigmaF' is the sensitivity of implied volatility with respect to the underlying current forward value,  $F$ . The implied volatility type depends on Shift and Model.
- 'dSigmaAlpha' is the sensitivity of implied volatility with respect to the Alpha parameter. The implied volatility type depends on Shift and Model.

Example: OutSpec =

```
{'Delta','Vega','ModifiedDelta','ModifiedVega','dSigmaF','dSigmaAlpha'}
```

Data Types: char | cell

**Shift — Shift in decimals for shifted SABR model**

0 (no shift) (default) | scalar positive decimal

Shift in decimals for the shifted SABR model (to be used with the Shifted Black model), specified as the comma-separated pair consisting of 'Shift' and a scalar positive decimal value. Set this parameter to a positive shift in decimals to add a positive shift to ForwardValue and Strike, which effectively sets a negative lower bound for ForwardValue and Strike. For example, a Shift value of 0.01 is equal to a 1% positive shift.

---

**Note** If the Model is set to 'normal', the Shift parameter must be 0.

---

Data Types: double

**Model — Model used by the implied volatility sigma**

'lognormal' (default) | character vector with value 'lognormal' or 'normal' | string with value "lognormal" or "normal"

Model used by the implied volatility sigma, specified as the comma-separated pair consisting of 'Model' and a character vector with a value of 'lognormal' or 'normal', or a string with a value of "lognormal" or "normal".

---

**Note** The setting for `Model` affects the interpretation of the implied volatility “sigma”. Depending on the setting for `Model`, the “sigma” has the following interpretations:

- If `Model` is 'lognormal' (default), “sigma” can be either Implied Black (no shift) or Implied Shifted Black volatility.
  - If `Model` is 'normal', “sigma” is the Implied Normal (Bachelier) volatility and `Shift` must be zero.
- 

Data Types: char | string

## Output Arguments

**Sens — Sensitivity values**

array

Sensitivity values, returned as an NINST-by-1 array as defined by the `OutSpec`.

## Algorithms

In the SABR model, an option with value  $V$  is defined by the modified Black formula  $B$ , where  $\sigma_B$  is the SABR implied Black volatility.

$$V = B(F, K, T, \sigma_B(\alpha, \beta, \rho, \nu, F, K, T))$$

The `Delta` and `Vega` sensitivities under the SABR model are expressed in terms of partial derivatives in the original paper by Hagan (2002).

$$\text{SABR } \Delta = \frac{\partial V}{\partial F} = \frac{\partial B}{\partial F} + \frac{\partial B}{\partial \sigma_B} \frac{\partial \sigma_B}{\partial F}$$

$$\text{SABR Vega} = \frac{\partial V}{\partial \alpha} = \frac{\partial B}{\partial \sigma_B} \frac{\partial \sigma_B}{\partial \alpha}$$

Later, Bartlett (2006) made better use of the model dynamics by incorporating the correlated changes between  $F$  and  $\alpha$

$$\text{Modified SABR Delta} = \frac{\partial B}{\partial F} + \frac{\partial B}{\partial \sigma_B} \left( \frac{\partial \sigma_B}{\partial F} + \frac{\partial \sigma_B}{\partial \alpha} \frac{\rho v}{F^\beta} \right)$$

$$\text{Modified SABR Vega} = \frac{\partial B}{\partial \sigma_B} \left( \frac{\partial \sigma_B}{\partial \alpha} + \frac{\partial \sigma_B}{\partial F} \frac{\rho F^\beta}{v} \right)$$

where  $\frac{\partial B}{\partial F}$  is the classic Black Delta and  $\frac{\partial B}{\partial \sigma_B}$  is the classic Black Vega. The Black implied volatility  $\sigma_B$  is computed internally by calling `blackvolbysabr`, while its partial derivatives  $\frac{\partial \sigma_B}{\partial F}$  and  $\frac{\partial \sigma_B}{\partial \alpha}$  are computed using closed-form expressions by `optsensbysabr`.

Similar expressions apply to the implied Normal volatility  $\sigma_N$ . For more information, see `normalvolbysabr`.

## References

- [1] Hagan, P. S., D. Kumar, A. S. Lesniewski, and D. E. Woodward. "Managing Smile Risk." *Wilmott Magazine*, 2002.
- [2] Bartlett, B. "Hedging under SABR Model." *Wilmott Magazine*, 2006.

## See Also

`IRDataCurve` | `blackvolbysabr` | `intenvset` | `normalvolbysabr` | `toRateSpec`

## Topics

- "Calibrate the SABR Model" on page 2-36
- "Price a Swaption Using the SABR Model" on page 2-42
- "Price Swaptions with Negative Strikes Using the Shifted SABR Model" on page 2-27
- "Work with Negative Interest Rates" on page 2-22

“Supported Interest-Rate Instruments” on page 2-2

**External Websites**

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

**Introduced in R2014b**

# optstockbybaw

Calculate American options prices using Barone-Adesi and Whaley option pricing model

## Syntax

Price = optstockbybaw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

## Description

Price = optstockbybaw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) calculates American options prices using the Barone-Adesi and Whaley option pricing model.

## Examples

### Compute American Option Prices Using the Barone-Adesi and Whaley Option Pricing Model

Consider an American call option with an exercise price of \$120. The option expires on Jan 1, 2018. The stock has a volatility of 14% per annum, and the annualized continuously compounded risk-free rate is 4% per annum as of Jan 1, 2016. Using this data, calculate the price of the American call, assuming the price of the stock is \$125 and pays a dividend of 2%.

```
StartDate = 'Jan-1-2016';
EndDate = 'jan-1-2018';
Basis = 1;
Compounding = -1;
Rates = 0.04;
```

Define the RateSpec.

```
RateSpec = intenvset('ValuationDate', StartDate, 'StartDate', StartDate, 'EndDate', EndDate, 'Rates', Rates, 'Basis', Basis, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9231
    Rates: 0.0400
    EndTimes: 2
    StartTimes: 0
    EndDates: 737061
    StartDates: 736330
    ValuationDate: 736330
    Basis: 1
    EndMonthRule: 1
```

Define the StockSpec.

```
Dividend = 0.02;
AssetPrice = 125;
Volatility = 0.14;
```

```
StockSpec = stockspec(Volatility,AssetPrice,'Continuous',Dividend)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1400
    AssetPrice: 125
    DividendType: {'continuous'}
    DividendAmounts: 0.0200
    ExDividendDates: []
```

Define the American option.

```
OptSpec = 'call';
Strike = 120;
Settle = 'Jan-1-2016';
Maturity = 'jan-1-2018';
```

Compute the price for the American option.

```
Price = optstockbybaw(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike)
```

```
Price = 14.5180
```



## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement date**

serial date number | date character vector | datetime object

Settlement date for the American option, specified as a NINST-by-1 matrix using a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

### **Maturity — Maturity date**

serial date number | date character vector | datetime object

Maturity date for the American option, specified as a NINST-by-1 matrix using a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors with values 'call' or 'put' | string array with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors or string arrays with values 'call' or 'put'.

Data Types: `char` | `cell` | `string`

**Strike — American option strike price value**

`nonnegative scalar` | `nonnegative vector`

American Option strike price value, specified as a nonnegative scalar or NINST-by-1 matrix of strike price values. Each row is the schedule for one option.

Data Types: `single` | `double`

## Output Arguments

**Price — Expected prices for American options**

`vector`

Expected prices for American options, returned as a NINST-by-1 vector.

## References

- [1] Barone-Aclesi, G. and Robert E. Whaley. "Efficient Analytic Approximation of American Option Values." *The Journal of Finance*. Volume 42, Issue 2 (June 1987), 301-320.
- [2] Haug, E. *The Complete Guide to Option Pricing Formulas. Second Edition*. McGraw-Hill Education, January 2007.

## See Also

`impvbybaw` | `optstocksensbybaw`

## Topics

"Supported Equity Derivatives" on page 3-24

**Introduced in R2017a**

# optstocksensbybaw

Calculate American options prices and sensitivities using Barone-Adesi and Whaley option pricing model

## Syntax

```
PriceSens = optstocksensbybaw(RateSpec,StockSpec,Settle,Maturity,  
OptSpec,Strike)  
PriceSens = optstocksensbybaw( ____,Name,Value)
```

## Description

PriceSens = optstocksensbybaw(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike) calculates American options prices using the Barone-Adesi and Whaley option pricing model.

PriceSens = optstocksensbybaw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Compute an American Option Price and Sensitivities Using the Barone-Adesi and Whaley Option Pricing Model

Consider an American call option with an exercise price of \$120. The option expires on Jan 1, 2018. The stock has a volatility of 14% per annum, and the annualized continuously compounded risk-free rate is 4% per annum as of Jan 1, 2016. Using this data, calculate the price of the American call, assuming the price of the stock is \$125 and pays a dividend of 2%.

```
StartDate = 'Jan-1-2016';  
EndDate = 'jan-1-2018';  
Basis = 1;
```

```
Compounding = -1;  
Rates = 0.04;
```

Define the RateSpec.

```
RateSpec = intenvset('ValuationDate',StartDate,'StartDate',StartDate,'EndDate',EndDate,  
'Rates',Rates,'Basis',Basis,'Compounding',Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9231  
    Rates: 0.0400  
    EndTimes: 2  
    StartTimes: 0  
    EndDates: 737061  
    StartDates: 736330  
    ValuationDate: 736330  
    Basis: 1  
    EndMonthRule: 1
```

Define the StockSpec.

```
Dividend = 0.02;  
AssetPrice = 125;  
Volatility = 0.14;
```

```
StockSpec = stockspec(Volatility,AssetPrice,'Continuous',Dividend)
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
    Sigma: 0.1400  
    AssetPrice: 125  
    DividendType: {'continuous'}  
    DividendAmounts: 0.0200  
    ExDividendDates: []
```

Define the American option.

```
OptSpec = 'call';  
Strike = 120;  
Settle = 'Jan-1-2016';  
Maturity = 'jan-1-2018';
```

Compute the price and sensitivities for the American option.

```
OutSpec = {'price'; 'delta'; 'theta'};
```

```
[Price,Delta,Theta] = optstocksensbybaw(RateSpec,StockSpec,Settle,Maturity,OptSpec,Str
```

```
Price = 14.5180
```

```
Delta = 0.6672
```

```
Theta = -3.1861
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle** — Settlement date

serial date number | date character vector | datetime object

Settlement date for the American option, specified as a NINST-by-1 matrix using a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

**Maturity — Maturity date**

serial date number | date character vector | datetime object

Maturity date for the American option, specified as a NINST-by-1 matrix using a serial date number, a date character vector, or a datetime object.

Data Types: double | char | datetime

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors or string arrays with values 'call' or 'put'.

Data Types: char | string

**Strike — American option strike price value**

nonnegative scalar | nonnegative vector

American option strike price value, specified as a nonnegative scalar or NINST-by-1 matrix of strike price values. Each row is the schedule for one option.

Data Types: single | double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price,Delta,Theta] = optstocksensbybaw(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike,'OutSpec',OutSpec)`

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.

Example: OutSpec =  
{'delta','gamma','vega','lambda','rho','theta','price'}

Data Types: char | cell

## Output Arguments

### PriceSens — Expected prices or sensitivities for American options

matrix

Expected prices or sensitivities for American options, returned as a NINST-by-1 matrix.

---

**Note** All sensitivities are evaluated by computing a discrete approximation of the partial derivative. This means that the option is revalued with a fractional change for each relevant parameter. The change in the option value divided by the increment is the approximated sensitivity value.

---

## References

- [1] Barone-Aclesi, G. and Robert E. Whaley. "Efficient Analytic Approximation of American Option Values." *The Journal of Finance*. Volume 42, Issue 2 (June 1987), 301-320.
- [2] Haug, E. *The Complete Guide to Option Pricing Formulas. Second Edition*. McGraw-Hill Education, January 2007.

## See Also

impvbybaw | optstockbybaw

**Topics**

“Supported Equity Derivatives” on page 3-24

**Introduced in R2017a**



# optstockbybjs

Price American options using Bjerksund-Stensland 2002 option pricing model

## Syntax

Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

## Description

Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) computes American option prices with continuous dividend yield using the Bjerksund-Stensland 2002 option pricing model.

---

**Note** optstockbybjs computes prices of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model.

---

## Examples

### Compute the American Option Prices With Continuous Dividend Yield Using the Bjerksund-Stensland 2002 Option Pricing Model

This example shows how to compute the American option prices with continuous dividend yield using the Bjerksund-Stensland 2002 option pricing model. Consider two American stock options (a call and a put) with an exercise price of \$100. The options expire on April 1, 2008. Assume the underlying stock pays a continuous dividend yield of 4% as of January 1, 2008. The stock has a volatility of 20% per annum and the annualized continuously compounded risk-free rate is 8% per annum. Using this data, calculate the price of the American call and put, assuming the following current prices of the stock: \$90 (for the call) and \$120 (for the put).

```
Settle = 'Jan-1-2008';  
Maturity = 'April-1-2008';
```

```
Strike = 100;
AssetPrice = [90;120];
DivYield = 0.04;
Rate = 0.08;
Sigma = 0.20;

% define the RateSpec and StockSpec
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);

% define the option type
OptSpec = {'call'; 'put'};

Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Price = 2×1

    0.8420
    0.1108
```

The first element of the Price vector represents the price of the call (\$0.84); the second element represents the price of the put option (\$0.11).

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspect`.

stockspec handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: `double` | `char`

### **Maturity — Maturity date for option**

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: `double` | `char`

### **OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: `char` | `cell`

### **Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: `double`

## **Output Arguments**

### **Price — Expected option prices**

vector

Expected option prices, returned as a NINST-by-1 vector.

Data Types: double

## References

- [1] Bjerksund, P. and G. Stensland. “Closed-Form Approximation of American Options.” *Scandinavian Journal of Management*. Vol. 9, 1993, Suppl., pp. S88-S99.
- [2] Bjerksund, P. and G. Stensland. “*Closed Form Valuation of American Options.*” Discussion paper 2002 (<https://www.scribd.com/doc/215619796/Closed-form-Valuation-of-American-Options-by-Bjerksund-and-Stensland#scribd>)

## See Also

IRDataCurve | blackvolbysabr | intenvset | toRateSpec

## Topics

- “Calibrate the SABR Model” on page 2-36
- “Price a Swaption Using the SABR Model” on page 2-42
- “Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2008b**

# optstockbyblk

Price options on futures and forwards using Black option pricing model

## Syntax

```
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec,  
Strike)  
Price = optstockbyblk( ____, Name, Value)
```

## Description

Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) computes option prices on futures using the Black option pricing model.

---

**Note** optstockbyblk calculates option prices on futures and forwards. If ForwardMaturity is not passed, the function calculates prices of future options. If ForwardMaturity is passed, the function computes prices of forward options. This function handles several types of underlying assets, for example, stocks and commodities. For more information on the underlying asset specification, see `stockspec`.

---

Price = optstockbyblk( \_\_\_\_, Name, Value) adds an optional name-value pair argument for ForwardMaturity to compute option prices on forwards using the Black option pricing model.

## Examples

### Compute Option Prices on Futures Using the Black Option Pricing Model

This example shows how to compute option prices on futures using the Black option pricing model. Consider two European call options on a futures contract with exercise prices of \$20 and \$25 that expire on September 1, 2008. Assume that on May 1, 2008 the

contract is trading at \$20, and has a volatility of 35% per annum. The risk-free rate is 4% per annum. Using this data, calculate the price of the call futures options using the Black model.

```
Strike = [20; 25];
AssetPrice = 20;
Sigma = .35;
Rates = 0.04;
Settle = 'May-01-08';
Maturity = 'Sep-01-08';

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(Sigma, AssetPrice);

% define the call options
OptSpec = {'call'};

Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity,...
    OptSpec, Strike)

Price = 2×1

    1.5903
    0.3037
```

### Compute Option Prices on a Forward

This example shows how to compute option prices on forwards using the Black pricing model. Consider two European options, a call and put on the Brent Blend forward contract that expires on January 1, 2015. The options expire on October 1, 2014 with an exercise price of \$200 and \$90 respectively. Assume that on January 1, 2014 the forward price is at \$107, the annualized continuously compounded risk-free rate is 3% per annum and volatility is 28% per annum. Using this data, compute the price of the options.

Define the RateSpec.

```
ValuationDate = 'Jan-1-2014';
EndDates = 'Jan-1-2015';
```

```

Rates = 0.03;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, 'Rates', Rates, ...
'Compounding', Compounding, 'Basis', Basis')

```

```

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9704
    Rates: 0.0300
    EndTimes: 1
    StartTimes: 0
    EndDates: 735965
    StartDates: 735600
    ValuationDate: 735600
    Basis: 1
    EndMonthRule: 1

```

Define the StockSpec.

```

AssetPrice = 107;
Sigma = 0.28;
StockSpec = stockspec(Sigma, AssetPrice);

```

Define the options.

```

Settle = 'Jan-1-2014';
Maturity = 'Oct-1-2014'; %Options maturity
Strike = [200;90];
OptSpec = {'call'; 'put'};

```

Price the forward call and put options.

```

ForwardMaturity = 'Jan-1-2015'; % Forward contract maturity
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, ...
'ForwardMaturity', ForwardMaturity)

```

```

Price = 2×1
    0.0535
    3.2111

```

### Compute the Option Price on a Future

Consider a call European option on the Crude Oil Brent futures. The option expires on December 1, 2014 with an exercise price of \$120. Assume that on April 1, 2014 futures price is at \$105, the annualized continuously compounded risk-free rate is 3.5% per annum and volatility is 22% per annum. Using this data, compute the price of the option.

Define the RateSpec.

```
ValuationDate = 'January-1-2014';  
EndDates = 'January-1-2015';  
Rates = 0.035;  
Compounding = -1;  
Basis = 1;  
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate,...  
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9656  
    Rates: 0.0350  
    EndTimes: 1  
    StartTimes: 0  
    EndDates: 735965  
    StartDates: 735600  
    ValuationDate: 735600  
    Basis: 1  
    EndMonthRule: 1
```

Define the StockSpec.

```
AssetPrice = 105;  
Sigma = 0.22;  
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
    Sigma: 0.2200  
    AssetPrice: 105  
    DividendType: []
```



```
DividendAmounts: 0
ExDividendDates: []
```

Define the option.

```
Settle = 'April-1-2014';
Maturity = 'Dec-1-2014';
Strike = 120;
OptSpec = {'call'};
```

Price the futures call option.

```
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)
```

```
Price = 2.5847
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### StockSpec — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### Settle — Settlement or trade date

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: double | cell

**Maturity — Maturity date for option**

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: double | cell

**OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: cell

**Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'ForwardMaturity', ForwardMaturity)`

**ForwardMaturity — Maturity date or delivery date of forward contract**

Maturity of option (default) | serial date number | date character vector

Maturity date or delivery date of forward contract, specified as the comma-separated pair consisting of 'ForwardMaturity' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | cell

## Output Arguments

### Price — Expected option prices

vector

Expected option prices, returned as a NINST-by-1 vector.

## See Also

impvbyblk | intenvset | optstocksensbyblk | stockspec

## Topics

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing Using the Black Model” on page 3-156

“Forwards Option” on page 3-53

“Futures Option” on page 3-54

“Black Model” on page 3-151

“Supported Equity Derivatives” on page 3-24

## Introduced in R2008b

## optstockbybls

Price options using Black-Scholes option pricing model

### Syntax

```
Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec,  
Strike)
```

### Description

Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) returns option prices using the Black-Scholes option pricing model.

---

**Note** When using StockSpec with optstockbybls, you can modify StockSpec to handle other types of underliers when pricing instruments that use the Black-Scholes model.

When pricing Futures (Black model), enter the following in StockSpec:

```
DivType = 'Continuous';  
DivAmount = RateSpec.Rates;
```

For example, see “Compute Option Prices Using the Black-Scholes Option Pricing Model” on page 11-1821.

When pricing Foreign Currencies (Garman-Kohlhagen model), enter the following in StockSpec:

```
DivType = 'Continuous';  
DivAmount = ForeignRate;
```

where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country. For example, see “Compute Option Prices on Foreign Currencies Using the Garman-Kohlhagen Option Pricing Model” on page 11-1821.

---

## Examples

### Compute Option Prices Using the Black-Scholes Option Pricing Model

This example shows how to compute option prices using the Black-Scholes option pricing model. Consider two European options, a call and a put, with an exercise price of \$29 on January 1, 2008. The options expire on May 1, 2008. Assume that the underlying stock for the call option provides a cash dividend of \$0.50 on February 15, 2008. The underlying stock for the put option provides a continuous dividend yield of 4.5% per annum. The stocks are trading at \$30 and have a volatility of 25% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, compute the price of the options using the Black-Scholes model.

```
Strike = 29;
AssetPrice = 30;
Sigma = .25;
Rates = 0.05;
Settle = 'Jan-01-2008';
Maturity = 'May-01-2008';

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rates, 'Compounding', -1);

DividendType = {'cash'; 'continuous'};
DividendAmounts = [0.50; 0.045];
ExDividendDates = {'Feb-15-2008'; NaN};

StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts,...
ExDividendDates);

OptSpec = {'call'; 'put'};

Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Price = 2×1

    2.2030
    1.2025
```

### Compute Option Prices on Foreign Currencies Using the Garman-Kohlhagen Option Pricing Model

This example shows how to compute option prices on foreign currencies using the Garman-Kohlhagen option pricing model. Consider a European put option on a currency with an exercise price of \$0.50 on October 1, 2015. The option expires on June 1, 2016. Assume that the current exchange rate is \$0.52 and has a volatility of 12% per annum. The annualized continuously compounded domestic risk-free rate is 4% per annum and the foreign risk-free rate is 8% per annum. Using this data, compute the price of the option using the Garman-Kohlhagen model.

```
Settle = 'October-01-2015';  
Maturity = 'June-01-2016';  
AssetPrice = 0.52;  
Strike = 0.50;  
Sigma = .12;  
Rates = 0.04;  
ForeignRate = 0.08;
```

Define the RateSpec.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates', ...  
Maturity, 'Rates', Rates, 'Compounding', -1)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: -1  
    Disc: 0.9737  
    Rates: 0.0400  
    EndTimes: 0.6667  
    StartTimes: 0  
    EndDates: 736482  
    StartDates: 736238  
    ValuationDate: 736238  
    Basis: 0  
    EndMonthRule: 1
```

Define the StockSpec.

```
DividendType = 'Continuous';  
DividendAmounts = ForeignRate;
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts)
```

```

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1200
    AssetPrice: 0.5200
    DividendType: {'continuous'}
    DividendAmounts: 0.0800
    ExDividendDates: []

```

Price the European put option.

```

OptSpec = {'put'};
Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Price = 0.0162

```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### StockSpec — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### Settle — Settlement or trade date

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: double | char

**Maturity — Maturity date for option**

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: double | char

**OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

**Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: double

## Output Arguments

**Price — Expected option prices**

vector

Expected option prices, returned as a NINST-by-1 vector.

Data Types: double

## See Also

impvbybls | intenvset | optstocksensbybls | stockspec



## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing European Call Options Using Different Equity Models”

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

## **Introduced in R2008b**

## optstockbycrr

Price stock option from Cox-Ross-Rubinstein tree

### Syntax

```
[Price,PriceTree] = optstockbycrr(CRRTree,OptSpec,Strike,Settle,ExerciseDates)
[Price,PriceTree] = optstockbycrr( ____,AmericanOpt)
```

### Description

[Price,PriceTree] = optstockbycrr(CRRTree,OptSpec,Strike,Settle,ExerciseDates) returns the price of a European, Bermuda, or American stock option from a Cox-Ross-Rubinstein tree.

[Price,PriceTree] = optstockbycrr( \_\_\_\_,AmericanOpt) adds an optional argument for AmericanOpt.

### Examples

#### Price an American Stock Option Using a CRR Binomial Tree

This example shows how to price an American stock option using a CRR binomial tree by loading the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the American option.

```
load deriv.mat;

OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2005';
AmericanOpt = 1;
```

```
Price = optstockbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates, AmericanOpt)
```

```
Price = 8.2863
```

## Price a Bermudan Stock Option Using a CRR Binomial Tree

Load the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the Bermudan option.

```
load deriv.mat;
```

```
% Option
```

```
OptSpec = 'Call';
```

```
Strike = 105;
```

```
Settle = '01-Jan-2003';
```

```
ExerciseDatesBerm={'01-Jan-2004', '01-Jul-2004', '01-Jan-2005', '01-Jul-2005'};
```

Price the Bermudan option.

```
Price = optstockbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDatesBerm)
```

```
Warning: Some ExerciseDates are not aligned with tree nodes. Result will be approximate
```

```
Price = 9.6381
```

## Input Arguments

### **CRRTree** — Stock tree structure

structure

Stock tree structure, specified by using `crrtree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with value `'call'` or `'put'` | cell array of character vectors with values `'call'` or `'put'`

Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

nonnegative integer

Option strike price value, specified with a NINST-by-1 or NINST-by-NSTRIKES depending on the option type:

- For a European option, use a NINST-by-1 vector of strike prices.
- For a Bermuda option, use a NINST-by-NSTRIKES matrix of strike prices. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American option, use a NINST-by-1 of strike prices.

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt` = 0, NaN, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt` = 1, the option is an American option.

---

Data Types: double

**Settle — Settlement date or trade date**

serial date number | date character vector

Settlement date or trade date, specified as a NINST-by-1 vector of date character vectors or serial date numbers.

---

**Note** The `Settle` date for every option is set to the `ValuationDate` of the stock tree. The option argument `Settle` is ignored.

---

Data Types: char | double

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the option type:

- For a European option, use a NINST-by-1 vector of dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates. Each row is the schedule for one option.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt` = 0, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt` = 1, the option is an American option.

---

Data Types: double | char

### **AmericanOpt — Option type**

0 European or Bermuda (default) | integer with values of 0 or 1

(Optional) Option type, specified as NINST-by-1 vector of integer flags with values:

- 0 — European or Bermuda
- 1 — American

Data Types: single | double

## **Output Arguments**

### **Price — Expected price of option at time 0**

vector

Expected price of the vanilla option at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Structure containing trees of vectors of instrument prices for each node**

structure

Structure containing trees of vectors of instrument prices and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.t0bs` contains the observation times.
- `PriceTree.d0bs` contains the observation dates.

### **See Also**

`crrtree` | `instoptstock`

### **Topics**

“Computing Prices Using CRR” on page 3-131

“Examining Output from the Pricing Functions” on page 3-139

“Computing Equity Instrument Sensitivities” on page 3-144

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Computing Instrument Prices” on page 3-130

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

# optstockbyeqp

Price stock option from Equal Probabilities binomial tree

## Syntax

```
[Price,PriceTree] = optstockbyeqp(EQPTree,OptSpec,Strike,Settle,  
ExerciseDates)  
[Price,PriceTree] = optstockbyeqp( ____,AmericanOpt)
```

## Description

[Price,PriceTree] = optstockbyeqp(EQPTree,OptSpec,Strike,Settle,ExerciseDates) returns the price of a European, Bermuda, or American stock option from an Equal Probabilities binomial tree.

[Price,PriceTree] = optstockbyeqp( \_\_\_\_,AmericanOpt) adds an optional argument for AmericanOpt.

## Examples

### Price an American Stock Option Using an EQP Equity Tree

This example shows how to price an American stock option using an EQP equity tree by loading the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the American option.

```
load deriv.mat  
  
OptSpec = 'Call';  
Strike = 105;  
Settle = '01-Jan-2003';  
ExerciseDates = '01-Jan-2006';  
AmericanOpt = 1;
```

```
Price = optstockbyeqp(EQPTree, OptSpec, Strike, Settle, ...  
ExerciseDates, AmericanOpt)
```

```
Price = 12.2632
```

### Price a Bermudan Stock Option Using a EQP Equity Tree

Load the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the Bermudan option.

```
load deriv.mat;
```

```
% Option
```

```
OptSpec = 'Call';
```

```
Strike = 105;
```

```
Settle = '01-Jan-2003';
```

```
ExerciseDatesBerm={'15-Jan-2004', '15-Jul-2004', '15-Jan-2005', '15-Jul-2005'};
```

Price the Bermudan option.

```
Price= optstockbyeqp(EQPTree, OptSpec, Strike, Settle, ExerciseDatesBerm)
```

```
Warning: Some ExerciseDates are not aligned with tree nodes. Result will be approximate
```

```
Price = 12.0255
```

## Input Arguments

### EQPTree — Stock tree structure

structure

Stock tree structure, specified by using `eqptree`.

Data Types: `struct`

### OptSpec — Definition of option

character vector with value `'call'` or `'put'` | cell array of character vectors with values `'call'` or `'put'`



Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of character vectors.

Data Types: char | cell

### **Strike — Option strike price values**

nonnegative integer

Option strike price value, specified with a NINST-by-1 or NINST-by-NSTRIKES depending on the option type:

- For a European option, use a NINST-by-1 vector of strike prices.
- For a Bermuda option, use a NINST-by-NSTRIKES matrix of strike prices. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American option, use a NINST-by-1 of strike prices.

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt` = 0, NaN, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt` = 1, the option is an American option.

---

Data Types: double

### **Settle — Settlement date or trade date**

serial date number | date character vector

Settlement date or trade date, specified as a NINST-by-1 vector of date character vectors or serial date numbers.

---

**Note** The `Settle` date for every option is set to the `ValuationDate` of the stock tree. The option argument `Settle` is ignored.

---

Data Types: char | double

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the option type:

- For a European option, use a NINST-by-1 vector of dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates. Each row is the schedule for one option.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt` = 0, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt` = 1, the option is an American option.

---

Data Types: `double` | `char`

### **AmericanOpt — Option type**

0 European or Bermuda (default) | integer with values of 0 or 1

(Optional) Option type, specified as NINST-by-1 vector of integer flags with values:

- 0 — European or Bermuda
- 1 — American

Data Types: `single` | `double`

## **Output Arguments**

### **Price — Expected price of option at time 0**

vector

Expected price of the vanilla option at time 0, returned as a NINST-by-1 vector.

## **PriceTree — Structure containing trees of vectors of instrument prices for each node**

structure

Structure containing trees of vectors of instrument prices and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.t0bs` contains the observation times.
- `PriceTree.d0bs` contains the observation dates.

## **See Also**

`eqptree` | `instoptstock`

## **Topics**

“Computing Prices Using EQP” on page 3-133

“Examining Output from the Pricing Functions” on page 3-139

“Computing Equity Instrument Sensitivities” on page 3-144

“Graphical Representation of Equity Derivative Trees” on page 3-142

“Computing Instrument Prices” on page 3-130

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

## optstockbyfd

Calculate vanilla option prices using finite difference method

### Syntax

```
[Price,PriceGrid,AssetPrices,Times] = optstockbyfd(RateSpec,  
StockSpec,OptSpec,Strike,Settle,ExerciseDates)  
[Price,PriceGrid,AssetPrices,Times] = optstockbyfd( ____,Name,Value)
```

### Description

[Price,PriceGrid,AssetPrices,Times] = optstockbyfd(RateSpec, StockSpec,OptSpec,Strike,Settle,ExerciseDates) calculates vanilla option prices using the finite difference method.

[Price,PriceGrid,AssetPrices,Times] = optstockbyfd( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a Vanilla Call Option Using Finite Difference Method

Create a RateSpec.

```
AssetPrice = 50;  
Strike = 45;  
Rate = 0.035;  
Volatility = 0.30;  
Settle = '01-Jan-2015';  
Maturity = '01-Jan-2016';  
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',Settle,'StartDates',Settle,'EndDates',...  
Maturity,'Rates',Rate,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9656
    Rates: 0.0350
    EndTimes: 1
    StartTimes: 0
    EndDates: 736330
    StartDates: 735965
    ValuationDate: 735965
    Basis: 1
    EndMonthRule: 1
```

Create a StockSpec.

```
StockSpec = stockspec(Volatility,AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
    AssetPrice: 50
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Calculate the price of a European vanilla call option using the finite difference method.

```
ExerciseDates = 'may-1-2015';
OptSpec = 'Call';
Price = optstockbyfd(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)
```

```
Price = 6.7352
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a character vector or string array with values 'call' or 'put'.

Data Types: `char` | `string`

**Strike — Option strike price value**

nonnegative scalar | nonnegative vector

Option strike price value, specified as a nonnegative scalar or vector.

- For a European option, use a scalar of strike price.
- For a Bermuda option, use a 1-by-NSTRIKES vector of strike prices.
- For an American option, use a scalar of strike price.

Data Types: `single` | `double`

**Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: `double` | `char` | `datetime`

**ExerciseDates — Option exercise dates**

serial date number | date character vector | datetime object

Option exercise dates, specified as a serial date number, a date character vector, or a datetime object:

- For a European option, use a 1-by-1 vector of dates, specified as a nonnegative scalar integer, a date character vector, or a datetime object. For a Bermuda option, use a 1-by-NSTRIKES vector of dates, specified as a nonnegative scalar integer, date character vector, or datetime object.
- For an American option, use a 1-by-2 cell array of date character vectors. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a 1-by-1 vector of serial date numbers or a cell array of date character vectors, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: double | char | cell | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price = optstockbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'AssetGridSize', 1000)`

### **AssetGridSize** — Size of asset grid used for finite difference grid

400 (default) | positive scalar

Size of the asset grid used for a finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a positive scalar.

Data Types: double

### **AssetPriceMax** — Maximum price for price grid boundary

if unspecified, `StockSpec` values are calculated using asset distributions at maturity (default) | positive scalar

Maximum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' as a positive scalar.

Data Types: single | double

**TimeGridSize — Size of time grid used for finite difference grid**

100 (default) | positive scalar

Size of the time grid used for a finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive scalar.

Data Types: double

**AmericanOpt — Option type**

0 (European/Bermuda) (default) | scalar with values [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

## Output Arguments

**Price — Expected prices for vanilla options**

scalar

Expected prices for vanilla options, returned as a 1-by-1 matrix.

**PriceGrid — Grid containing prices calculated by finite difference method**

grid

Grid containing prices calculated by the finite difference method, returned as a grid that is two-dimensional with size PriceGridSize\*length(Times). The number of columns does not have to be equal to the TimeGridSize, because ex-dividend dates in the StockSpec are added to the time grid. The price for  $t = 0$  is contained in PriceGrid(:, end).

**AssetPrices — Prices of asset defined by StockSpec**

vector

Prices of the asset defined by the StockSpec corresponding to the first dimension of PriceGrid, returned as a vector.



## **Times — Times corresponding to second dimension of PriceGrid** vector

Times corresponding to second dimension of the PriceGrid, returned as a vector.

## **References**

- [1] Haug, E. G., J. Haug, and A. Lewis. "*Back to basics: a new approach to the discrete dividend problem.*" Vol. 9, Wilmott magazine, 2003, pp. 37-47.
- [2] Wu, L. and Y. K. Kwok. "A front-fixing finite difference method for the valuation of American options." *Journal of Financial Engineering*. Vol. 6.4, 1997, pp. 83-97.

## **See Also**

optstockbyblk | optstockbylr | optstockbyls | optstocksensbyfd

## **Topics**

"Supported Equity Derivatives" on page 3-24

**Introduced in R2016b**

## optstocksensbyfd

Calculate vanilla option prices or sensitivities using finite difference method

### Syntax

```
[PriceSens,PriceGrid,AssetPrices,Times] = optstocksensbyfd(RateSpec,  
StockSpec,OptSpec,Strike,Settle,ExerciseDates)  
[PriceSens,PriceGrid,AssetPrices,Times] = optstocksensbyfd( ____,  
Name,Value)
```

### Description

[PriceSens,PriceGrid,AssetPrices,Times] = optstocksensbyfd(RateSpec, StockSpec,OptSpec,Strike,Settle,ExerciseDates) calculates vanilla option prices or sensitivities using the finite difference method.

[PriceSens,PriceGrid,AssetPrices,Times] = optstocksensbyfd( \_\_\_\_, Name,Value) adds optional name-value pair arguments.

### Examples

#### Calculate the Price and Sensitivities for a Vanilla Call Option Using Finite Difference Method

Create a RateSpec.

```
AssetPrice = 50;  
Strike = 45;  
Rate = 0.035;  
Volatility = 0.30;  
Settle = '01-Jan-2015';  
Maturity = '01-Jan-2016';  
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',Settle,'StartDates',Settle,'EndDates',...
Maturity,'Rates',Rate,'Compounding',-1,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9656
    Rates: 0.0350
    EndTimes: 1
    StartTimes: 0
    EndDates: 736330
    StartDates: 735965
    ValuationDate: 735965
    Basis: 1
    EndMonthRule: 1
```

Create a StockSpec.

```
StockSpec = stockspec(Volatility,AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3000
    AssetPrice: 50
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Calculate the price and sensitivities for of a European vanilla call option using the finite difference method.

```
ExerciseDates = 'may-1-2015';
OptSpec = 'Call';
OutSpec = {'price'; 'delta'; 'theta'};
[PriceSens, Delta, Theta] = optstocksensbyfd(RateSpec,StockSpec,OptSpec,Strike,Settle,
ExerciseDates,'OutSpec',OutSpec)
```

```
PriceSens = 6.7352
```

```
Delta = 0.7765
```

```
Theta = -4.9999
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | string array with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a character vector or string array with values 'call' or 'put'.

Data Types: `char` | `string`

### **Strike — Option strike price value**

nonnegative scalar | nonnegative vector

Option strike price value, specified as a nonnegative scalar or vector.

- For a European option, use a scalar of strike price.
- For a Bermuda option, use a 1-by-NSTRIKES vector of strike prices.
- For an American option, use a scalar of strike price.

Data Types: `single` | `double`

**Settle — Settlement or trade date**

serial date number | date character vector | datetime object

Settlement or trade date for the barrier option, specified as a serial date number, a date character vector, or a datetime object.

Data Types: double | char | datetime

**ExerciseDates — Option exercise dates**

date character vector | nonnegative scalar integer | datetime object

Option exercise dates, specified as a nonnegative scalar integer, date character vector, or datetime object:

- For a European option, use a 1-by-1 vector of dates, specified as a nonnegative scalar integer, a date character vector, or a datetime object. For a Bermuda option, use a 1-by-NSTRIKES vector of dates, specified as a nonnegative scalar integer, date character vector, or datetime object.
- For an American option, use a 1-by-2 cell array of date character vectors. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a 1-by-1 vector of serial date numbers or a cell array of date character vectors, the option can be exercised between `Settle` and the single listed date in `ExerciseDates`.

Data Types: double | char | cell | datetime

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `PriceSens = optstocksensbyfd(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'OutSpec', {'All'}, 'AssetGridSize', 1000)`

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output is Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity.

Example: OutSpec =  
{'delta','gamma','vega','lambda','rho','theta','price'}

Data Types: char | cell

### **AssetGridSize — Size of asset grid used for finite difference grid**

400 (default) | positive scalar

Size of asset grid used for finite difference grid, specified as the comma-separated pair consisting of 'AssetGridSize' and a positive scalar.

Data Types: double

### **AssetPriceMax — Maximum price for price grid boundary**

if unspecified, StockSpec values are calculated using asset distributions at maturity (default) | positive scalar

Maximum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a positive scalar.

Data Types: single | double

### **TimeGridSize — Size of time grid used for finite difference grid**

100 (default) | positive scalar

Size of the time grid used for a finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a positive scalar.

Data Types: double

### **AmericanOpt — Option type**

0 (European/Bermuda) (default) | scalar with values [0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- 0 — European/Bermuda
- 1 — American

Data Types: double

## Output Arguments

### **PriceSens — Expected prices or sensitivities for vanilla options**

scalar

Expected price or sensitivities (defined by OutSpec) of the vanilla option, returned as a 1-by-1 array.

### **PriceGrid — Grid containing prices calculated by finite difference method**

grid

Grid containing prices calculated by the finite difference method, returned as a two-dimensional grid with size PriceGridSize\*length(Times). The number of columns does not have to be equal to the TimeGridSize, because ex-dividend dates in the StockSpec are added to the time grid. The price for  $t = 0$  is contained in PriceGrid(:, end).

### **AssetPrices — Prices of asset defined by StockSpec**

vector

Prices of the asset defined by the StockSpec corresponding to the first dimension of PriceGrid, returned as a vector.

### **Times — Times corresponding to second dimension of PriceGrid**

vector

Times corresponding to second dimension of the PriceGrid, returned as a vector.

## References

- [1] Haug, E. G., J. Haug, and A. Lewis. "Back to basics: a new approach to the discrete dividend problem." Vol. 9, Wilmott magazine, 2003, pp. 37-47.
- [2] Wu, L. and Y. K. Kwok. "A front-fixing finite difference method for the valuation of American options." *Journal of Financial Engineering*. Vol. 6.4, 1997, pp. 83-97.

## **See Also**

optstockbyblk | optstockbyfd | optstockbylr | optstockbyls

## **Topics**

“Supported Equity Derivatives” on page 3-24

**Introduced in R2016b**



# optstockbyitt

Price options on stocks using implied trinomial tree (ITT)

## Syntax

```
[Price,PriceTree] = optstockbyitt(ITTTree,OptSpec,Strike,Settle,  
ExerciseDates)  
[Price,PriceTree] = optstockbyitt( ____,AmericanOpt)
```

## Description

[Price,PriceTree] = optstockbyitt(ITTTree,OptSpec,Strike,Settle,ExerciseDates) returns the price of a European, Bermuda, or American stock option from an implied trinomial tree (ITT).

[Price,PriceTree] = optstockbyitt( \_\_\_\_,AmericanOpt) adds an optional argument for AmericanOpt.

## Examples

### Price an American Stock Option Using an ITT Equity Tree

This example shows how to price an American stock option using an ITT equity tree by loading the file `deriv.mat`, which provides the `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the American option.

```
load deriv.mat  
  
OptSpec = 'Put';  
Strike = 30;  
Settle = '01-Jan-2006';  
ExerciseDates = ' 01-Jan-2010 ';  
AmericanOpt = 1;
```

```
Price = optstockbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)
Price = 0.1271
```

### Price a Bermudan Stock Option Using an ITT Equity Tree

Load the file `deriv.mat`, which provides an `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the Bermudan option.

```
load deriv.mat;

% Option
OptSpec = 'Put';
Strike = 30;
Settle = '01-Jan-2006';
ExerciseDatesBerm={'1-Jan-2007', '1-Jul-2007', '1-Jan-2008', '1-Jul-2008'};
```

Price the Bermudan option.

```
Price = optstockbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDatesBerm)

Warning: Some ExerciseDates are not aligned with tree nodes. Result will be approximated.
> In procoptions at 171
   In optstockbystocktree at 22
   In optstockbyitt at 68

Price =

    0.0664
```

## Input Arguments

### ITTTree — Stock tree structure

structure

Stock tree structure, specified by using `itttree`.

Data Types: `struct`

### OptSpec — Definition of option

character vector with value `'call'` or `'put'` | cell array of character vectors with values `'call'` or `'put'`

Definition of option, specified as 'call' or 'put' using a NINST-by-1 cell array of character vectors.

Data Types: char | cell

### **Strike — Option strike price values**

nonnegative integer

Option strike price value, specified with a NINST-by-1 or NINST-by-NSTRIKES depending on the option type:

- For a European option, use a NINST-by-1 vector of strike prices.
- For a Bermuda option, use a NINST-by-NSTRIKES matrix of strike prices. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American option, use a NINST-by-1 of strike prices.

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt` = 0, NaN, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt` = 1, the option is an American option.

---

Data Types: double

### **Settle — Settlement date or trade date**

serial date number | date character vector

Settlement date or trade date, specified as a NINST-by-1 vector of date character vectors or serial date numbers.

---

**Note** The `Settle` date for every option is set to the `ValuationDate` of the stock tree. The option argument `Settle` is ignored.

---

Data Types: char | double

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a NINST-by-1, NINST-by-2, or NINST-by-NSTRIKES using serial date numbers or date character vectors, depending on the option type:

- For a European option, use a NINST-by-1 vector of dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of dates. Each row is the schedule for one option.
- For an American option, use a NINST-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a NINST-by-1 vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt` = 0, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt` = 1, the option is an American option.

---

Data Types: `double` | `char`

### **AmericanOpt — Option type**

0 European or Bermuda (default) | integer with values of 0 or 1

(Optional) Option type, specified as NINST-by-1 vector of integer flags with values:

- 0 — European or Bermuda
- 1 — American

Data Types: `single` | `double`

## **Output Arguments**

### **Price — Expected price of option at time 0**

vector

Expected price of the vanilla option at time 0, returned as a NINST-by-1 vector.

## **PriceTree — Structure containing trees of vectors of instrument prices for each node**

structure

Structure containing trees of vectors of instrument prices and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.t0bs` contains the observation times.
- `PriceTree.d0bs` contains the observation dates.

## **References**

[1] Chriss, Neil A., E. Derman, and I. Kani. "Implied trinomial trees of the volatility smile." *Journal of Derivatives*. 1996.

## **See Also**

`instoptstock` | `itttree`

## **Topics**

"Computing Prices Using ITT" on page 3-135

"Examining Output from the Pricing Functions" on page 3-139

"Computing Equity Instrument Sensitivities" on page 3-144

"Graphical Representation of Equity Derivative Trees" on page 3-142

"Computing Instrument Prices" on page 3-130

"Supported Equity Derivatives" on page 3-24

**Introduced in R2007a**

## optstockbylr

Price options on stocks using Leisen-Reimer binomial tree model

### Syntax

```
[Price,PriceTree] = optstockbylr(LRTree,OptSpec,Strike,Settle,
ExerciseDates)
[Price,PriceTree] = optstockbylr( ____,Name,Value)
```

### Description

[Price,PriceTree] = optstockbylr(LRTree,OptSpec,Strike,Settle,ExerciseDates) computes option prices on stocks using the Leisen-Reimer binomial tree model.

[Price,PriceTree] = optstockbylr( \_\_\_\_,Name,Value) adds an optional name-value pair argument for AmericanOpt.

### Examples

#### Price Options on Stocks Using the Leisen-Reimer Binomial Tree Model

This example shows how to price options on stocks using the Leisen-Reimer binomial tree model. Consider European call and put options with an exercise price of \$95 that expire on July 1, 2010. The underlying stock is trading at \$100 on January 1, 2010, provides a continuous dividend yield of 3% per annum and has a volatility of 20% per annum. The annualized continuously compounded risk-free rate is 8% per annum. Using this data, compute the price of the options using the Leisen-Reimer model with a tree of 15 and 55 time steps.

```
AssetPrice = 100;
Strike = 95;
```

```
ValuationDate = 'Jan-1-2010';
```

```

Maturity = 'July-1-2010';

% define StockSpec
Sigma = 0.2;
DividendType = 'continuous';
DividendAmounts = 0.03;

StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);

% define RateSpec
Rates = 0.08;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

% build the Leisen-Reimer (LR) tree with 15 and 55 time steps
LRTimeSpec15 = lrtimespec(ValuationDate, Maturity, 15);
LRTimeSpec55 = lrtimespec(ValuationDate, Maturity, 55);

% use the PP2 method
LRMethod = 'PP2';

LRTree15 = lrtree(StockSpec, RateSpec, LRTimeSpec15, Strike, 'method', LRMethod);
LRTree55 = lrtree(StockSpec, RateSpec, LRTimeSpec55, Strike, 'method', LRMethod);

% price the call and the put options using the LR model:
OptSpec = {'call'; 'put'};

PriceLR15 = optstockbylr(LRTree15, OptSpec, Strike, Settle, Maturity);
PriceLR55 = optstockbylr(LRTree55, OptSpec, Strike, Settle, Maturity);

% calculate price using the Black-Scholes model (BLS) to compare values with
% the LR model:
PriceBLS = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike);

% compare values of BLS and LR
[PriceBLS PriceLR15 PriceLR55]

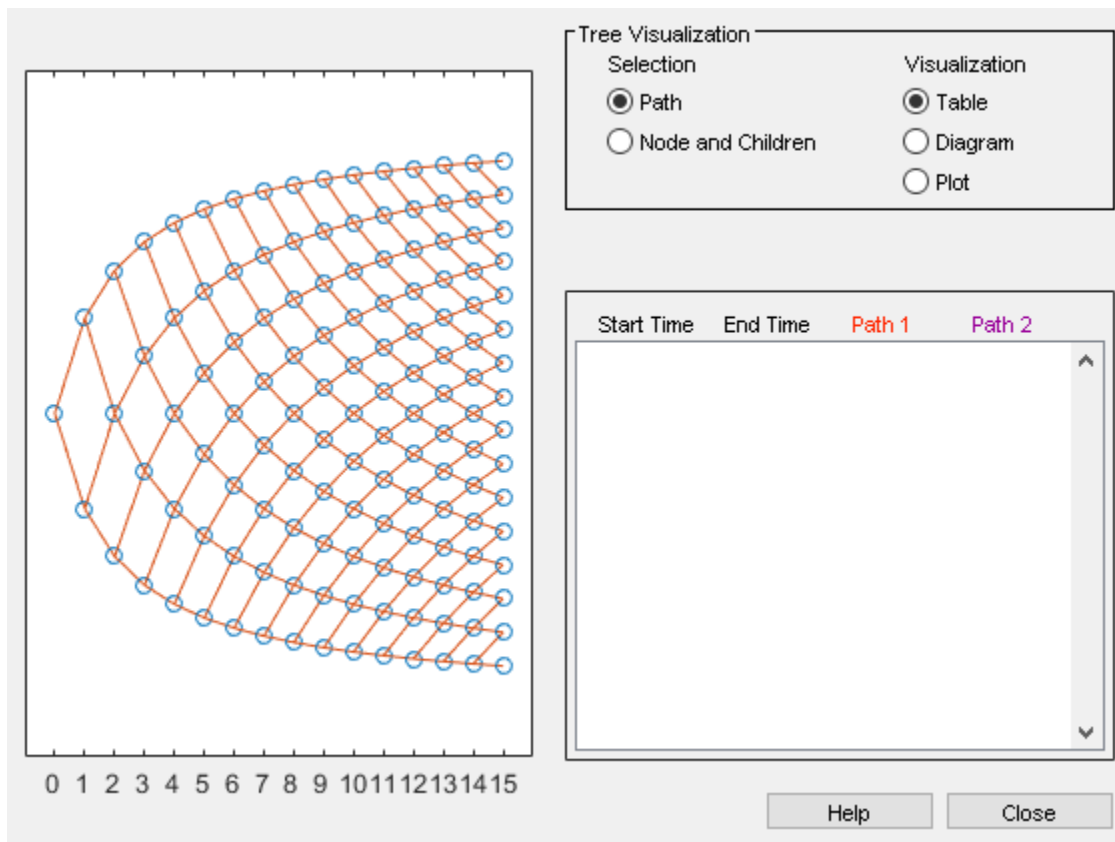
ans = 2x3

    9.7258    9.7252    9.7257

```

2.4896    2.4890    2.4895

```
% use treeviewer to display LRtree of 15 time steps
treeviewer(LRtree15)
```



## Input Arguments

**LRtree** — Stock tree structure  
structure



Stock tree structure, specified by `lrtree`.

Data Types: `struct`

### **OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: `char` | `cell`

### **Strike — Option strike price values**

vector of nonnegative integers

Option strike price value, specified with nonnegative integer:

- For a European option, use a NINST-by-1 vector of strike prices.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of strike prices. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American option, use a NINST-by-1 vector of strike prices.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date, specified as an NINST-by-1 matrix using serial date numbers or date character vectors.

Data Types: `double` | `char`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a vector of date character vectors or serial date numbers where each row is the schedule for one option and the last element of each row must be the same as the maturity of the tree.

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one `ExerciseDate` on the option expiry date.

- For a Bermuda option, use a NINST-by-NSTRIKEDATES vector of dates.
- For an American option, use a NINST-by-1 vector of exercise dates. For the American type, the option can be exercised on any tree data between the ValuationDate and tree maturity.

Data Types: double | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

```
Example: [Price, PriceTree] =  
optstockbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates, 'AmericanOpt'  
, '1')
```

### AmericanOpt — Option type

0 European or Bermuda (default) | values: [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a NINST-by-1 vector of flags with values:

- 0 — European or Bermuda
- 1 — American

Data Types: double

## Output Arguments

### Price — Expected prices at time 0

vector

expected prices at time 0, returned as a NINST-by-1 vector.

### PriceTree — Tree structure

structure

Tree structure, returned as a vector of instrument prices at each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.t0bs` contains the observation times.
- `PriceTree.d0bs` contains the observation dates.

## References

- [1] Leisen D.P., M. Reimer. "Binomial Models for Option Valuation - Examining and Improving Convergence." *Applied Mathematical Finance*. Number 3, 1996, pp. 319-346.

## See Also

`instoptstock` | `lrtree`

## Topics

- "Pricing Equity Derivatives Using Trees" on page 3-130  
"Pricing European Call Options Using Different Equity Models"  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2010b**

## optstockbyls

Price European, Bermudan, or American vanilla options using Monte Carlo simulations

### Syntax

```
Price = optstockbyls(RateSpec, StockSpec, OptSpec, Strike, Settle,  
ExerciseDates)
```

```
Price = optstockbyls( ____, Name, Value)
```

```
[Price, Path, Times, Z] = optstockbyls(RateSpec, StockSpec, OptSpec,  
Strike, Settle, ExerciseDates)
```

```
[Price, Path, Times, Z] = optstockbyls( ____, Name, Value)
```

### Description

`Price = optstockbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates)` returns vanilla option prices using the Longstaff-Schwartz model. `optstockbyls` computes prices of European, Bermudan, and American vanilla options.

For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

`Price = optstockbyls( ____, Name, Value)` adds optional name-value pair arguments.

`[Price, Path, Times, Z] = optstockbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates)` returns vanilla option prices using the Longstaff-Schwartz model.

`[Price, Path, Times, Z] = optstockbyls( ____, Name, Value)` adds optional name-value pair arguments.

### Examples

## Compute the Price of a Vanilla Option

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.05;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
    Disc: 0.9060
    Rates: 0.0500
    EndTimes: 4
    StartTimes: 0
    EndDates: 735965
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.1;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1000
    AssetPrice: 100
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Define the vanilla option.

```
OptSpec = 'put';
Settle = 'Jan-1-2013';
ExerciseDates = 'Jan-1-2015';
Strike = 105;
```

Compute the vanilla option price using the Longstaff-Schwartz model.

```
Antithetic = true;  
Price = optstockbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ...  
ExerciseDates, 'Antithetic', Antithetic)
```

```
Price = 3.2292
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with value 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector.

Data Types: `char`

### **Strike — Option strike price values**

nonnegative scalar integer

Option strike price value, specified with nonnegative scalar integer:

- For a European option, use a scalar of strike price.
- For a Bermuda option, use a 1-by-NSTRIKES vector of strike prices.
- For an American option, use a scalar of strike price.

Data Types: `single` | `double`

### **Settle — Settlement date or trade date**

date character vector | nonnegative scalar integer

Settlement date or trade date for the vanilla option, specified as a date character vector or nonnegative scalar integer.

Data Types: `double` | `char`

### **ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a date character vector or serial date number:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a 1-by-NSTRIKES vector of dates.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is a 1-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between `Settle` and the single listed `ExerciseDates`.

Data Types: `double` | `char`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
optstockbyls(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates,
'AmericanOpt', '1', 'NumTrials', '2000')
```

### **AmericanOpt — Option type**

0 European or Bermuda (default) | scalar with values [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and positive integer scalar flags with values:

- 0 — European or Bermuda
- 1 — American

---

**Note** For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: single | double

### **NumTrials — Simulation trials**

1000 (default) | scalar

Simulation trials, specified as a scalar number of independent sample paths.

Data Types: double

### **NumPeriods — Simulation periods per trial**

100 (default) | scalar

Simulation periods per trial, specified as a scalar number. NumPeriods is considered only when pricing European vanilla options. For American and Bermuda vanilla options, NumPeriod is equal to the number of Exercise days during the life of the option.

Data Types: double

### **Z — Dependent random variates**

scalar | nonnegative integer

Dependent random variates used to generate the Brownian motion vector (that is, Wiener processes) that drive the simulation, specified as a be NumPeriods-by-1-by-NumTrials 3-D time series array.

Data Types: single | double



**Antithetic — Indicator for antithetic sampling**

false (default) | logical flag with value of true or false

Indicator for antithetic sampling, specified with a value of true or false.

Data Types: logical

## Output Arguments

**Price — Expected price of vanilla option**

scalar

Expected price of the vanilla option, returned as a 1-by-1 scalar.

**Path — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a (NumPeriods + 1)-by-1-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

**Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a (NumPeriods + 1)-by-1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths.

**Z — Dependent random variates**

vector

Dependent random variates, if Z is specified as an optional input argument, the same value is returned. Otherwise, Z contains the random variates generated internally.

## See Also

optstocksensbyls

## Topics

“Pricing Asian Options”

“Vanilla Option” on page 3-48

“Supported Equity Derivatives” on page 3-24

**Introduced in R2013b**

# optstocksensbyls

Calculate price and sensitivities for European, Bermudan, or American vanilla options using Monte Carlo simulations

## Syntax

```
PriceSens = optstocksensbyls(RateSpec,StockSpec,OptSpec,Strike,
Settle,ExerciseDates)
```

```
PriceSens = optstocksensbyls( ____,Name,Value)
```

```
[PriceSens,Path,Times,Z] = optstocksensbyls(RateSpec,StockSpec,
OptSpec,Strike,Settle,ExerciseDates)
```

```
[PriceSens,Path,Times,Z] = optstocksensbyls( ____,Name,Value)
```

## Description

`PriceSens = optstocksensbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)` returns vanilla option prices or sensitivities using the Longstaff-Schwartz model. `optstocksensbyls` computes prices or sensitivities of European, Bermudan, and American vanilla options.

For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

`PriceSens = optstocksensbyls( ____,Name,Value)` adds optional name-value pair arguments.

`[PriceSens,Path,Times,Z] = optstocksensbyls(RateSpec,StockSpec,OptSpec,Strike,Settle,ExerciseDates)` returns vanilla option prices or sensitivities using the Longstaff-Schwartz model.

`[PriceSens,Path,Times,Z] = optstocksensbyls( ____,Name,Value)` adds optional name-value pair arguments.

## Examples

### Compute the Price and Sensitivities of a Vanilla Option

Define the RateSpec.

```
StartDates = 'Jan-1-2013';
EndDates = 'Jan-1-2015';
Rates = 0.05;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
    Disc: 0.9060
    Rates: 0.0500
    EndTimes: 4
    StartTimes: 0
    EndDates: 735965
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Define the StockSpec for the asset.

```
AssetPrice = 100;
Sigma = 0.1;
DivType = 'continuous';
DivAmounts = 0.04;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmounts)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1000
    AssetPrice: 100
    DividendType: {'continuous'}
    DividendAmounts: 0.0400
    ExDividendDates: []
```

Define the vanilla option.

```

OptSpec = 'call';
Settle = 'jan-1-2013';
ExerciseDates = 'jan-1-2015';
Strike = 105;

```

Compute the Delta sensitivity for the vanilla option using the Longstaff-Schwartz model.

```

Antithetic = true;
OutSpec = {'Delta'};
PriceSens = optstocksensbyls(RateSpec, StockSpec, OptSpec, Strike, ...
Settle, ExerciseDates, 'Antithetic', Antithetic, 'OutSpec', OutSpec)

PriceSens = 0.3945

```

To display the output for Price, Delta, Path, and Times, use the following:

```

OutSpec = {'Price', 'Delta'};
[Price, Delta, Path, Times] = optstocksensbyls(RateSpec, StockSpec, OptSpec, Strike, .
Settle, ExerciseDates, 'Antithetic', Antithetic, 'OutSpec', OutSpec);

```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### StockSpec — Stock specification for underlying asset

structure

Stock specification for underlying asset, specified using StockSpec obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities.

Data Types: `struct`

**OptSpec — Definition of option**

character vector values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector.

Data Types: char

**Strike — Option strike price value**

nonnegative scalar integer

Option strike price value, specified with a nonnegative scalar integer:

- For a European option, use a scalar of strike price.
- For a Bermuda option, use a 1-by-NSTRIKES vector of strike price.
- For an American option, use a scalar of strike price.

Data Types: single | double

**Settle — Settlement date or trade date**

serial date number | date character vector

Settlement date or trade date for the vanilla option, specified as a date character vector or a serial date number.

Data Types: double | char

**ExerciseDates — Option exercise date**

serial date number | date character vector

Option exercise date, specified as a date character vector or serial date number:

- For a European option, use a 1-by-1 vector of dates. For a European option, there is only one ExerciseDates on the option expiry date.
- For a Bermuda option, use a 1-by-NSTRIKES vector of dates.
- For an American option, use a 1-by-2 vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-NaN date is listed, or if ExerciseDates is a 1-by-1 vector of serial date numbers or cell array of character vectors, the option can be exercised between Settle and the single listed ExerciseDates.

Data Types: double | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: Price = optstocksensbyls(RateSpec, StockSpec,
    OptSpec, Strike, Settle, ExerciseDates, 'AmericanOpt', '1', 'NumTrials', '2
    000', 'OutSpec', {'Price', 'Delta', 'Gamma'})
```

### AmericanOpt — Option type

0 European or Bermuda (default) | scalar with values [0, 1]

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and a positive integer scalar flag with values:

- 0 — European or Bermuda
- 1 — American

---

**Note** For American and Bermudan options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: single | double

### NumTrials — Simulation trials

1000 (default) | scalar

Simulation trials, specified as the comma-separated pair consisting of `'NumTrials'` and a scalar number of independent sample paths.

Data Types: double

### NumPeriods — Simulation periods per trial

100 (default) | scalar

Simulation periods per trial, specified as the comma-separated pair consisting of `'NumPeriods'` and a scalar number. `NumPeriods` is considered only when pricing

European vanilla options. For American and Bermuda vanilla options, NumPeriod is equal to the number of Exercise days during the life of the option.

Data Types: double

### **Z — Dependent random variates**

scalar | nonnegative integer

Dependent random variates used to generate the Brownian motion vector (that is, Wiener processes) that drive the simulation, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-1-by-NumTrials 3-D time series array.

Data Types: single | double

### **Antithetic — Indicator for antithetic sampling**

false (default) | logical flag with value of true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

### **OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}

Data Types: char | cell



## Output Arguments

### **PriceSens — Expected price or sensitivities of vanilla option**

scalar

Expected price or sensitivities (defined by `OutSpec`) of the vanilla option, returned as a 1-by-1 array.

### **Path — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a  $(\text{NumPeriods} + 1)$ -by-1-by-`NumTrials` 3-D time series array. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

### **Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a  $(\text{NumPeriods} + 1)$ -by-1 column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

### **Z — Dependent random variates**

vector

Dependent random variates, if `Z` is specified as an optional input argument, the same value is returned. Otherwise, `Z` contains the random variates generated internally.

## See Also

optstockbyls

## Topics

“Pricing Asian Options”

“Vanilla Option” on page 3-48

“Supported Equity Derivatives” on page 3-24

**Introduced in R2013b**

## optstockbyrgw

Determine American call option prices using Roll-Geske-Whaley option pricing model

### Syntax

```
Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike)
```

### Description

Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike) computes the American call option prices using the Roll-Geske-Whaley option pricing model.

---

**Note** optstockbyrgw computes prices of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model.

---

### Examples

#### Determine American Call Option Prices Using Roll-Geske-Whaley Option Pricing Model

This example shows how to determine American call option prices using Roll-Geske-Whaley option pricing model. Consider an American call option with an exercise price of \$22 that expires on February 1, 2009. The underlying stock is trading at \$20 on June 1, 2008 and has a volatility of 20% per annum. The annualized continuously compounded risk-free rate is 6.77% per annum. The stock pays a single dividend of \$2 on September 1, 2008. Using this data, compute price of the American call option using the Roll-Geske-Whaley option pricing model.

```
Settle = 'Jun-01-2008';  
Maturity = 'Feb-01-2009';  
AssetPrice = 20;  
Strike = 22;
```

```

Sigma = 0.2;
Rate = 0.0677;
DivAmount = 2;
DivDate = 'Sep-01-2008';

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 0);

StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);

Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity,Strike)

Price = 0.3359

```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types: struct

### StockSpec — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see stockspec.

stockspec handles several types of underlying assets. For example, for physical commodities the price is StockSpec.Asset, the volatility is StockSpec.Sigma, and the convenience yield is StockSpec.DividendAmounts.

Data Types: struct

### Settle — Settlement or trade date

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: double | char

### **Maturity — Maturity date for option**

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: double | char

### **Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: double

## **Output Arguments**

### **Price — Expected call option prices**

vector

Expected call option prices, returned as a NINST-by-1 vector.

Data Types: double

## **See Also**

impvbyrgw | intenvset | optstocksensbyrgw | stockspec

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing Using the Roll-Geske-Whaley Model” on page 3-157

“Roll-Geske-Whaley Model” on page 3-152

“Supported Equity Derivatives” on page 3-24

## **Introduced in R2008b**

## optstocksensbybjs

Determine American option prices or sensitivities using Bjerksund-Stensland 2002 option pricing model

### Syntax

```
PriceSens = optstocksensbybjs(RateSpec,StockSpec,Settle,Maturity,  
OptSpec,Strike)
```

```
PriceSens = optstocksensbybjs( ____,Name,Value)
```

### Description

`PriceSens = optstocksensbybjs(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike)` computes American option prices or sensitivities using the Bjerksund-Stensland 2002 option pricing model.

---

**Note** `optstocksensbybjs` computes prices of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model. All sensitivities are evaluated by computing a discrete approximation of the partial derivative. This means that the option is revalued with a fractional change for each relevant parameter, and the change in the option value divided by the increment, is the approximated sensitivity value.

---

`PriceSens = optstocksensbybjs( ____,Name,Value)` adds an optional name-value pair argument for `OutSpec`.

### Examples

#### Compute American Option Prices and Sensitivities Using the Bjerksund-Stensland 2002 Option Pricing Model

This example shows how to compute American option prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model. Consider four American put options with

an exercise price of \$100. The options expire on October 1, 2008. Assume the underlying stock pays a continuous dividend yield of 4% and has a volatility of 40% per annum. The annualized continuously compounded risk-free rate is 8% per annum. Using this data, calculate the delta, gamma, and price of the American put options, assuming the following current prices of the stock on July 1, 2008: \$90, \$100, \$110 and \$120.

```
Settle = 'July-1-2008';
Maturity = 'October-1-2008';
Strike = 100;
AssetPrice = [90;100;110;120];
Rate = 0.08;
Sigma = 0.40;
DivYield = 0.04;

% define the RateSpec and StockSpec
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);

% define the option type
OptSpec = {'put'};

OutSpec = {'Delta', 'Gamma', 'Price'};

[Delta, Gamma, Price] = optstocksensbybjs(RateSpec, StockSpec, Settle, Maturity, ...
OptSpec, Strike, 'OutSpec', OutSpec)

Delta = 4×1

    -0.6572
    -0.4434
    -0.2660
    -0.1442

Gamma = 4×1

    0.0217
    0.0202
    0.0150
    0.0095
```

Price = 4×1

12.9467  
7.4571  
3.9539  
1.9495

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle** — Settlement or trade date

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a `NINST-by-1` vector.

Data Types: `double` | `char`

### **Maturity** — Maturity date for option

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: double | char

**OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

**Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Delta, Gamma, Price] = optstocksensbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'OutSpec', OutSpec)`

**OutSpec — Define outputs**

{'Price'} (default) | cell array of character vectors with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` to include each sensitivity:



```
Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}
```

Data Types: char | cell

## Output Arguments

### **PriceSens** — Expected future prices or sensitivities values

vector

Expected future prices or sensitivities values, returned as a NINST-by-1 vector.

Data Types: double

## References

- [1] Bjerksund, P. and G. Stensland. "Closed-Form Approximation of American Options." *Scandinavian Journal of Management*. Vol. 9, 1993, Suppl., pp. S88-S99.
- [2] Bjerksund, P. and G. Stensland. "Closed Form Valuation of American Options." Discussion paper 2002 (<https://www.scribd.com/doc/215619796/Closed-form-Valuation-of-American-Options-by-Bjerksund-and-Stensland#scribd>)

## See Also

impvbybjs | intenvset | optstockbybjs | stockspec

## Topics

"Equity Derivatives Using Closed-Form Solutions" on page 3-150

"Pricing Using the Bjerksund-Stensland Model" on page 3-158

"Bjerksund-Stensland 2002 Model" on page 3-153

"Supported Equity Derivatives" on page 3-24

**Introduced in R2008b**

## optstocksensbyblk

Determine option prices or sensitivities using Black-Scholes option pricing model

### Syntax

```
PriceSens = optstocksensbyblk(RateSpec,StockSpec,Settle,Maturity,  
OptSpec,Strike)  
PriceSens = optstocksensbyblk( ____,Name,Value)
```

### Description

PriceSens = optstocksensbyblk(RateSpec,StockSpec,Settle,Maturity,OptSpec,Strike) computes option prices on futures using the Black option pricing model.

---

**Note** optstocksensbyblk calculates option prices or sensitivities on futures and forwards. If ForwardMaturity is not passed, the function calculates prices or sensitivities of future options. If ForwardMaturity is passed, the function computes prices or sensitivities of forward options. This function handles several types of underlying assets, for example, stocks and commodities. For more information on the underlying asset specification, see `stockspec`.

---

PriceSens = optstocksensbyblk( \_\_\_\_,Name,Value) adds optional name-value pair arguments for ForwardMaturity and OutSpec to compute option prices or sensitivities on forwards using the Black option pricing model.

### Examples

#### Compute Option Prices and Sensitivities on Futures Using the Black Pricing Model

This example shows how to compute option prices and sensitivities on futures using the Black pricing model. Consider a European put option on a futures contract with an

exercise price of \$60 that expires on June 30, 2008. On April 1, 2008 the underlying stock is trading at \$58 and has a volatility of 9.5% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, compute delta, gamma, and the price of the put option.

```

AssetPrice = 58;
Strike = 60;
Sigma = .095;
Rates = 0.05;
Settle = 'April-01-08';
Maturity = 'June-30-08';

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);

StockSpec = stockspec(Sigma, AssetPrice);

% define the options
OptSpec = {'put'};

OutSpec = {'Delta', 'Gamma', 'Price'};
[Delta, Gamma, Price] = optstocksensbyblk(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)

Delta = -0.7469

Gamma = 0.1130

Price = 2.3569

```

### Compute Forward Option Prices and Delta Sensitivities

This example shows how to compute option prices and sensitivities on forwards using the Black pricing model. Consider two European call options on the Brent Blend forward contract that expires on January 1, 2015. The options expire on October 1, 2014 and Dec 1, 2014 with an exercise price % of \$120 and \$150 respectively. Assume that on January 1, 2014 the forward price is at \$107, the annualized continuously compounded risk-free rate is 3% per annum and volatility is 28% per annum. Using this data, compute the price and delta of the options.

Define the RateSpec.

```
ValuationDate = 'Jan-1-2014';
EndDates = 'Jan-1-2015';
Rates = 0.03;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ...
ValuationDate, 'EndDates', EndDates, 'Rates', Rates, ...
'Compounding', Compounding, 'Basis', Basis');
```

Define the StockSpec.

```
AssetPrice = 107;
Sigma = 0.28;
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the options.

```
Settle = 'Jan-1-2014';
Maturity = {'Oct-1-2014'; 'Dec-1-2014'}; %Options maturity
Strike = [120;150];
OptSpec = {'call'; 'call'};
```

Price the forward call options and return the Delta sensitivities.

```
ForwardMaturity = 'Jan-1-2015'; % Forward contract maturity
OutSpec = {'Delta'; 'Price'};
[Delta, Price] = optstocksensbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, ...
Strike, 'ForwardMaturity', ForwardMaturity, 'OutSpec', OutSpec)
```

```
Delta = 2×1
```

```
0.3518
0.1262
```

```
Price = 2×1
```

```
5.4808
1.6224
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a `NINST-by-1` vector.

Data Types: `double` | `char`

### **Maturity — Maturity date for option**

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a `NINST-by-1` vector.

Data Types: `double` | `char`

### **OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a `NINST-by-1` cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

**Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Delta, Gamma, Price] =  
optstocksensbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike,  
'OutSpec', OutSpec)

**ForwardMaturity — Maturity date or delivery date of forward contract**

Maturity of option (default) | serial date number | date character vector

Maturity date or delivery date of forward contract, specified as the comma-separated pair consisting of 'ForwardMaturity' and a NINST-by-1 vector using serial date numbers or date character vectors.

Data Types: double | cell

**OutSpec — Define outputs**

{'Price'} (default) | cell array of character vectors with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}

Data Types: char | cell

## Output Arguments

**PriceSens** — Expected future prices or sensitivities values  
vector

Expected future prices or sensitivities values, returned as a NINST-by-1 vector.

Data Types: double

## See Also

impvbyblk | intenvset | optstockbyblk | stockspec

## Topics

“Pricing Asian Options”

“Forwards Option” on page 3-53

“Futures Option” on page 3-54

“Supported Equity Derivatives” on page 3-24

“Supported Energy Derivatives” on page 3-44

**Introduced in R2008b**

## optstocksensbybls

Determine option prices or sensitivities using Black-Scholes option pricing model

### Syntax

```
PriceSens = optstocksensbybls(RateSpec, StockSpec, Settle, Maturity,  
OptSpec, Strike)  
PriceSens = optstocksensbybls( ____, Name, Value)
```

### Description

`PriceSens = optstocksensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)` computes option prices or sensitivities using the Black-Scholes option pricing model.

---

**Note** When using `StockSpec` with `optstocksensbybls`, you can modify `StockSpec` to handle other types of underliers when pricing instruments that use the Black-Scholes model.

When pricing Futures (Black model), enter the following in `StockSpec`:

```
DivType = 'Continuous';  
DivAmount = RateSpec.Rates;
```

When pricing Foreign Currencies (Garman-Kohlhagen model), enter the following in `StockSpec`:

```
DivType = 'Continuous';  
DivAmount = ForeignRate;
```

where `ForeignRate` is the continuously compounded, annualized risk free interest rate in the foreign country.

---

`PriceSens = optstocksensbybls( ____, Name, Value)` adds an optional name-value pair argument for `OutSpec`.



## Examples

### Compute Option Prices and Sensitivities Using the Black-Scholes Option Pricing Model

This example shows how to compute option prices and sensitivities using the Black-Scholes option pricing model. Consider a European call and put options with an exercise price of \$30 that expires on June 1, 2008. The underlying stock is trading at \$30 on January 1, 2008 and has a volatility of 30% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, compute the delta, gamma, and price of the options using the Black-Scholes model.

```

AssetPrice = 30;
Strike = 30;
Sigma = .30;
Rates = 0.05;
Settle = 'January-01-2008';
Maturity = 'June -01-2008';

% define the RateSpec and StockSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rates, 'Compounding',-1, 'Basis', 1);

StockSpec = stockspec(Sigma, AssetPrice);

% define the options
OptSpec = {'call', 'put'};

OutSpec = {'Delta','Gamma','Price'};
[Delta, Gamma, Price] = optstocksensbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)

Delta = 2×1

    0.5810
   -0.4190

Gamma = 2×1

    0.0673
    0.0673

```

Price = 2×1

2.6126  
1.9941

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec** — Stock specification for underlying asset

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle** — Settlement or trade date

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a `NINST-by-1` vector.

Data Types: `double` | `char`

### **Maturity** — Maturity date for option

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a `NINST-by-1` vector.

Data Types: double | char

### **OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: char | cell

### **Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Delta, Gamma, Price] = optstocksensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'OutSpec', OutSpec)`

### **OutSpec — Define outputs**

`{'Price'}` (default) | cell array of character vectors with values: 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` to include each sensitivity:

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: char | cell

## Output Arguments

### **PriceSens** — Expected future prices or sensitivities values

vector

Expected future prices or sensitivities values, returned as a NINST-by-1 vector.

Data Types: double

## See Also

`impvbybls` | `intenvset` | `optstockbybls` | `stockspec`

## Topics

“Pricing Asian Options”

“Forwards Option” on page 3-53

“Futures Option” on page 3-54

“Supported Equity Derivatives” on page 3-24

“Supported Energy Derivatives” on page 3-44

## Introduced in R2008b

# optstocksensbylr

Determine option prices or sensitivities using Leisen-Reimer binomial tree model

## Syntax

```
PriceSens = optstockbylr(LRTree,OptSpec,Strike,Settle,ExerciseDates)
PriceSens = optstockbylr( ____,Name,Value)
```

## Description

`PriceSens = optstockbylr(LRTree,OptSpec,Strike,Settle,ExerciseDates)` calculates option prices or sensitivities using a Leisen-Reimer binomial tree model.

`PriceSens = optstockbylr( ____,Name,Value)` adds optional name-value pair arguments for `AmericanOpt` and `OutSpec`.

## Examples

### Compute Option Prices and Sensitivities Using a Leisen-Reimer Binomial Tree Model

This example shows how to compute option prices and sensitivities using a Leisen-Reimer binomial tree model. Consider European call and put options with an exercise price of \$100 that expire on December 1, 2010. The underlying stock is trading at \$100 on June 1, 2010 and has a volatility of 30% per annum. The annualized continuously compounded risk-free rate is 7% per annum. Using this data, compute the price, delta and gamma of the options using the Leisen-Reimer model with a tree of 25 time steps and the PP2 method.

```
AssetPrice = 100;
Strike = 100;
```

```
ValuationDate = 'June-1-2010';
Maturity = 'December-1-2010';
```

```
% define StockSpec
Sigma = 0.3;

StockSpec = stockspec(Sigma, AssetPrice);

% define RateSpec
Rates = 0.07;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

% build the Leisen-Reimer (LR) tree with 25 time steps
LRTimeSpec = lrtimespec(ValuationDate, Maturity, 25);

% use the PP2 method
LRMethod = 'PP2';

TreeLR = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike, 'method', LRMethod);

% compute prices and sensitivities using the LR model:
OptSpec = {'call'; 'put'};
OutSpec = {'Price', 'Delta', 'Gamma'};

[Price, Delta, Gamma] = optstocksensbylr(TreeLR, OptSpec, Strike, Settle, ...
    Maturity, 'OutSpec', OutSpec)

Price = 2×1

    10.1332
     6.6937

Delta = 2×1

     0.6056
    -0.3944

Gamma = 2×1

     0.0185
```

0.0185

## Input Arguments

### **LRTree — Stock tree structure**

structure

Stock tree structure, specified by `lrtree`.

Data Types: `struct`

### **OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: `char` | `cell`

### **Strike — Option strike price values**

vector of nonnegative integers

Option strike price value, specified with nonnegative integer:

- For a European option, use a NINST-by-1 vector of strike prices.
- For a Bermuda option, use a NINST-by-NSTRIKES vector of strike prices. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.
- For an American option, use a NINST-by-1 vector of strike prices.

Data Types: `double`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date, specified as an NINST-by-1 matrix using serial date numbers or date character vectors.

Data Types: `double` | `char`

**ExerciseDates — Option exercise dates**

serial date number | date character vector

Option exercise dates, specified as a vector of date character vectors or serial date numbers where each row is the schedule for one option and the last element of each row must be the same as the maturity of the tree.

- For a European option, use a NINST-by-1 vector of dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- For a Bermuda option, use a NINST-by-NSTRIKEDATES vector of dates.
- For an American option, use a NINST-by-1 vector of exercise dates. For the American type, the option can be exercised on any tree data between the `ValuationDate` and tree maturity.

Data Types: double | char

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

```
Example: [Price,Delta,Gamma] =  
optstocksensbylr(LRTree,OptSpec,Strike,Settle,ExerciseDates,'OutSpec'  
,OutSpec)
```

**AmericanOpt — Option type**

0 European or Bermuda (default) | values: [0,1]

Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and a NINST-by-1 vector of flags with values:

- 0 — European or Bermuda
- 1 — American

Data Types: double

**OutSpec — Define outputs**

```
{'Price'} (default) | cell array of character vectors with values: 'Price', 'Delta',  
'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'
```



Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

```
Example: OutSpec =
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}
```

Data Types: char | cell

## Output Arguments

### PriceSens — Expected future prices or sensitivities values

vector

Expected future prices or sensitivities values, returned as a NINST-by-1 vector.

Data Types: double

## References

- [1] Leisen D.P., M. Reimer. “Binomial Models for Option Valuation – Examining and Improving Convergence.” *Applied Mathematical Finance*. Number 3, 1996, pp. 319-346.

## See Also

lrtree | optstockbylr

## Topics

- “Pricing Equity Derivatives Using Trees” on page 3-130
- “Pricing European Call Options Using Different Equity Models”
- “Supported Equity Derivatives” on page 3-24

**Introduced in R2010b**

## optstocksensbyrgw

Determine American call option prices or sensitivities using Roll-Geske-Whaley option pricing model

### Syntax

```
PriceSens = optstocksensbyrgw(RateSpec, StockSpec, Settle, Maturity,  
OptSpec, Strike)  
PriceSens = optstocksensbyrgw( ____, Name, Value)
```

### Description

PriceSens = optstocksensbyrgw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike) computes American call option prices or sensitivities using the Roll-Geske-Whaley option pricing model.

---

**Note** optstocksensbyrgw computes prices of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model. All sensitivities are evaluated by computing a discrete approximation of the partial derivative. This means that the option is revalued with a fractional change for each relevant parameter, and the change in the option value divided by the increment, is the approximated sensitivity value.

---

PriceSens = optstocksensbyrgw( \_\_\_\_, Name, Value) adds an optional name-value pair argument for OutSpec.

### Examples

#### Compute American Call Option Prices and Sensitivities Using the Roll-Geske-Whaley Option Pricing Model

This example shows how to compute American call option prices and sensitivities using the Roll-Geske-Whaley option pricing model. Consider an American stock option with an

exercise price of \$82 on January 1, 2008 that expires on May 1, 2008. Assume the underlying stock pays dividends of \$4 on April 1, 2008. The stock is trading at \$80 and has a volatility of 30% per annum. The risk-free rate is 6% per annum. Using this data, calculate the price and the value of delta and gamma of the American call using the Roll-Geske-Whaley option pricing model.

```

AssetPrice = 80;
Settle = 'Jan-01-2008';
Maturity = 'May-01-2008';
Strike = 82;
Rate = 0.06;
Sigma = 0.3;
DivAmount = 4;
DivDate = 'Apr-01-2008';

% define the RateSpec and StockSpec
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 1);

% define the OutSpec
OutSpec = {'Price', 'Delta', 'Gamma'};

[Price, Delta, Gamma] = optstocksensbyrgw(RateSpec, StockSpec, Settle, ...
    Maturity, Strike, 'OutSpec', OutSpec)

Price = 4.3860
Delta = 0.5022
Gamma = 0.0336

```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from intenvset. For information on the interest-rate specification, see intenvset.

Data Types: `struct`

**StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: `double` | `char`

**Maturity — Maturity date for option**

serial date number | date character vector

Maturity date for option, specified as serial date number or date character vector using a NINST-by-1 vector.

Data Types: `double` | `char`

**OptSpec — Definition of option**

cell array of character vectors with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors with values 'call' or 'put'.

Data Types: `char` | `cell`

**Strike — Option strike price value**

nonnegative vector

Option strike price value, specified as a nonnegative NINST-by-1 vector.

Data Types: `double`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Delta, Gamma, Price] = optstocksensbyrgw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'OutSpec', OutSpec)`

### OutSpec — Define outputs

`{'Price'}` (default) | cell array of character vectors with values: `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` to include each sensitivity:

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: `char` | `cell`

## Output Arguments

### PriceSens — Expected future prices or sensitivities values

vector

Expected future prices or sensitivities values, returned as a NINST-by-1 vector.

Data Types: `double`

## See Also

`impvbyrgw` | `intenvset` | `optstockbyrgw` | `stockspec`

**Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing Using the Roll-Geske-Whaley Model” on page 3-157

“Supported Equity Derivatives” on page 3-24

**Introduced in R2008b**

# optstockbystt

Price vanilla options on stocks using standard trinomial tree

## Syntax

```
[Price,PriceTree] = optstockbystt(STTTree,OptSpec,Strike,Settle,
ExerciseDates)
[Price,PriceTree] = optstockbystt( ____,Name,Value)
```

## Description

[Price,PriceTree] = optstockbystt(STTTree,OptSpec,Strike,Settle, ExerciseDates) returns vanilla option (American, European, or Bermudan) prices on stocks using a standard trinomial (STT) tree.

[Price,PriceTree] = optstockbystt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price Call and Put Stock Options Using the Standard Trinomial Tree Model

Create a RateSpec.

```
StartDates = 'Jan-1-2009';
EndDates = 'Jan-1-2013';
Rates = 0.035;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
```

```
Compounding: -1
  Disc: 0.8694
  Rates: 0.0350
  EndTimes: 4
  StartTimes: 0
  EndDates: 735235
  StartDates: 733774
ValuationDate: 733774
  Basis: 1
  EndMonthRule: 1
```

Create a `StockSpec`.

```
AssetPrice = 85;
Sigma = 0.15;
StockSpec = stockspec(Sigma, AssetPrice)
```

```
StockSpec = struct with fields:
  FinObj: 'StockSpec'
  Sigma: 0.1500
  AssetPrice: 85
  DividendType: []
  DividendAmounts: 0
  ExDividendDates: []
```

Create an `STTtree`.

```
NumPeriods = 4;
TimeSpec = stttimespec(StartDates, EndDates, 4);
STTtree = stttree(StockSpec, RateSpec, TimeSpec)
```

```
STTtree = struct with fields:
  FinObj: 'STStockTree'
  StockSpec: [1x1 struct]
  TimeSpec: [1x1 struct]
  RateSpec: [1x1 struct]
  tObs: [0 1 2 3 4]
  dObs: [733774 734139 734504 734869 735235]
  STree: {1x5 cell}
  Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
```

Define the call and put options and compute the price.



```

Settle = '1/1/09';
ExerciseDates = [datenum('1/1/11');datenum('1/1/12')];
OptSpec = {'call';'put'};
Strike = [100;80];

Price = optstockbystt(STTtree, OptSpec, Strike, Settle, ExerciseDates)

Price = 2×1

    4.5025
    3.0603

```

## Input Arguments

**STTtree** — Stock tree structure for standard trinomial tree structure

Stock tree structure for a standard trinomial tree, specified by using `stttree`.

Data Types: `struct`

**OptSpec** — Definition of option

character vector with value 'call' or 'put' | cell array of character vectors with values 'call' or 'put'

Definition of option, specified as 'call' or 'put' using a character vector.

Data Types: `char` | `cell`

**Strike** — Option strike price values

nonnegative integer

Option strike price value, specified with a NINST-by-1 or NINST-by-NSTRIKES depending on the option type:

- For a European option, use a NINST-by-1 vector of strike prices.
- For a Bermuda option, use aNINST-by-NSTRIKES matrix of strike prices. Each row is the schedule for one option. If an option has fewer than NSTRIKES exercise opportunities, the end of the row is padded with NaNs.

- For an American option, use a `NINST-by-1` of strike prices.

Data Types: `double`

**Settle — Settlement date or trade date**

`serial date number` | `date character vector`

Settlement date or trade date for the vanilla option, specified as a `NINST-by-1` vector of date character vectors or serial date numbers.

---

**Note** The `Settle` date for every vanilla option is set to the `ValuationDate` of the stock tree. The vanilla option argument `Settle` is ignored.

---

Data Types: `char` | `double`

**ExerciseDates — Option exercise dates**

`serial date number` | `date character vector`

Option exercise dates, specified as a `NINST-by-1`, `NINST-by-2`, or `NINST-by-NSTRIKES` using serial date numbers or date character vectors, depending on the option type:

- For a European option, use a `NINST-by-1` vector of dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDates` on the option expiry date.
- For a Bermuda option, use a `NINST-by-NSTRIKES` vector of dates. Each row is the schedule for one option.
- For an American option, use a `NINST-by-2` vector of exercise date boundaries. The option can be exercised on any date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is a `NINST-by-1` vector, the option can be exercised between `ValuationDate` of the stock tree and the single listed `ExerciseDates`.

Data Types: `double` | `char`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = optstockbystt(RateSpec, StockSpec, OptSpec, Strike, Settle, ExerciseDates, 'AmericanOpt', '1')`

### **AmericanOpt — Option type**

0 European or Bermuda (default) | integer with values of 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a NINST-by-1 vector of integer flags with values:

- 0 — European or Bermuda
- 1 — American

Data Types: `single` | `double`

## **Output Arguments**

### **Price — Expected price of vanilla option at time 0**

vector

Expected price of the vanilla option at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Structure containing trees of vectors of instrument prices and accrued interest for each node**

structure

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.dObs` contains the observation dates.

## **See Also**

`instoptstock` | `sttprice` | `sttsens` | `stttimespec` | `stttree`

## **Topics**

“Vanilla Option” on page 3-48

“Supported Equity Derivatives” on page 3-24

**Introduced in R2015b**

# optpricebysim

Price option given simulated underlying values

## Syntax

```
Price = optpricebysim(RateSpec, SimulatedPrices, Times, OptSpec, Strike,
ExerciseTimes)
Price = optpricebysim( ____, Name, Value)
```

## Description

Price = optpricebysim(RateSpec, SimulatedPrices, Times, OptSpec, Strike, ExerciseTimes) calculates the price of European, American, and Bermudan call/put options based on risk-neutral simulation of the underlying asset. For American and Bermudan options, the Longstaff-Schwartz least squares method calculates the early exercise premium.

Price = optpricebysim( \_\_\_\_, Name, Value) adds optional name-value pair arguments.

## Examples

### Compute the Price of an American Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
S0 = 100; % Initial price of underlying asset
Sigma = .2; % Volatility of underlying asset
Strike = 110; % Strike
OptSpec = 'call'; % Call option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
```

```
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the `gbm` object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the `simBySolution` method from Financial Toolbox™.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', S0);
[Paths, Times, Z] = simBySolution(OptionGBM, NPERIODS, ...
    'NTRIALS',NTRIALS, 'DeltaTime',dt,'Antithetic',true);
```

Create the interest-rate term structure to define `RateSpec`.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rate', r, 'Compounding', Compounding, ...
    'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Price an American option.

```
SimulatedPrices = squeeze(Paths);
OptPrice = optpricebysim(RateSpec, SimulatedPrices, Times, OptSpec, ...
    Strike, T, 'AmericanOpt', 1)
```

```
OptPrice = 6.2028
```

## Compute the Price of an American Asian Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
S0 = 100; % Initial price of underlying asset
Sigma = .2; % Volatility of underlying asset
Strike = 110; % Strike
OptSpec = 'call'; % Call option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the `gbm` object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the `simBySolution` method from Financial Toolbox™.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', S0);
[Paths, Times, Z] = simBySolution(OptionGBM, NPERIODS, ...
'NTRIALS', NTRIALS, 'DeltaTime', dt, 'Antithetic', true);
```

Create the interest-rate term structure to define `RateSpec`.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rate', r, 'Compounding', Compounding, ...
'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Price an American Asian option (arithmetic mean) by finding the average price over periods.

```
AvgPrices = zeros(NPERIODS+1, NTRIALS);
    for i = 1:NPERIODS+1
        AvgPrices(i,:) = mean(squeeze(Paths(1:i,:,:)));
    end
    AsianPrice = optpricebysim(RateSpec, AvgPrices, Times, OptSpec, ...
        Strike, T, 'AmericanOpt', 1)

AsianPrice = 1.8540
```

### Compute the Price of an American Lookback Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
S0 = 100; % Initial price of underlying asset
Sigma = .2; % Volatility of underlying asset
Strike = 110; % Strike
OptSpec = 'call'; % Call option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the gbm object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the `simBySolution` method from Financial Toolbox™.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', S0);
[Paths, Times, Z] = simBySolution(OptionGBM, NPERIODS, ...
    'NTRIALS', NTRIALS, 'DeltaTime', dt, 'Antithetic', true);
```

Create the interest-rate term structure to define `RateSpec`.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rate', r, 'Compounding', Compounding, ...
    'Basis', Basis)
```



```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Price an American lookback option by finding the maximum price over periods.

```
MaxPrices = zeros(NPERIODS+1, NTRIALS);
LastPrice = squeeze(Paths(1,:,:))';
for i = 1:NPERIODS+1;
    MaxPrices(i,:) = max([LastPrice; Paths(i,:)]);
    LastPrice = MaxPrices(i,:);
end
LookbackPrice = optpricebysim(RateSpec, MaxPrices, Times, OptSpec, ...
    Strike, T, 'AmericanOpt', 1)

LookbackPrice = 10.4084
```

### Compute the Price of a Bermudan Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
S0 = 80; % Initial price of underlying asset
Sigma = .3; % Volatility of underlying asset
Strike = 75; % Strike
OptSpec = 'put'; % Put option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
ExerciseDates = {'1-Jun-2013', '1-Jan-2014'}; % Exercise dates of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
```

```
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
ExerciseTimes = yearfrac(Settle, ExerciseDates, Basis)'; % Exercise times
```

Set up the `gbm` object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the `simBySolution` method from Financial Toolbox™.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
OptionGBM = gbm(r, Sigma, 'StartState', S0);
[Paths, Times, Z] = simBySolution(OptionGBM, NPERIODS, ...
    'NTRIALS',NTRIALS, 'DeltaTime',dt,'Antithetic',true);
```

Create the interest-rate term structure to define `RateSpec`.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rate', r, 'Compounding', Compounding, ...
    'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
    ValuationDate: 735235
    Basis: 0
    EndMonthRule: 1
```

Price the Bermudan option.

```
SimulatedPrices = squeeze(Paths);
BermudanPrice = optpricebysim(RateSpec, SimulatedPrices, Times, ...
    OptSpec, Strike, ExerciseTimes)
```

```
BermudanPrice = 5.3950
```

## Compute the Price of an American Spread Option Using Monte Carlo Simulation Based on Geometric Brownian Motion

Define the option.

```
S1 = 110; % Price of first underlying asset
S2 = 100; % Price of second underlying asset
Sigma1 = .1; % Volatility of first underlying asset
Sigma2 = .15; % Volatility of second underlying asset
Strike = 15; % Strike
Rho = .3; % Correlation between underlyings
OptSpec = 'put'; % Put option
Settle = '1-Jan-2013'; % Settlement date of option
Maturity = '1-Jan-2014'; % Maturity date of option
r = .05; % Risk-free rate (annual, continuous compounding)
Compounding = -1; % Continuous compounding
Basis = 0; % Act/Act day count convention
T = yearfrac(Settle, Maturity, Basis); % Time to expiration in years
```

Set up the `gbm` object and run the Monte Carlo simulation based on Geometric Brownian Motion (GBM) using the `simBySolution` method from Financial Toolbox™.

```
NTRIALS = 1000;
NPERIODS = daysact(Settle, Maturity);
dt = T/NPERIODS;
SpreadGBM = gbm(r*eye(2), diag([Sigma1;Sigma2]), 'Correlation', ...
[1 Rho;Rho 1], 'StartState', [S1;S2]);
[Paths, Times, Z] = simBySolution(SpreadGBM, NPERIODS, 'NTRIALS', NTRIALS, ...
'DeltaTime', dt, 'Antithetic', true);
```

Create the interest-rate term structure to define `RateSpec`.

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rate', r, 'Compounding', Compounding, ...
'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9512
    Rates: 0.0500
    EndTimes: 1
    StartTimes: 0
    EndDates: 735600
    StartDates: 735235
```

```
ValuationDate: 735235
              Basis: 0
              EndMonthRule: 1
```

Price the American spread option.

```
Spread = squeeze(Paths(:,1,:) - Paths(:,2,:));
SpreadPrice = optpricebysim(RateSpec, Spread, Times, OptSpec, Strike, ...
T, 'AmericanOpt', 1)
```

```
SpreadPrice = 9.0007
```

## Input Arguments

### **RateSpec** — Interest-rate term structure of risk-free rates

structure

Interest-rate term structure of risk-free rates (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. The valuation date must be at the settlement date of the option, and the day-count basis and end-of-month rule must be the same as those used to calculate the `Times` input. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **SimulatedPrices** — Simulated prices

matrix

Simulated prices, specified using a  $(\text{NumPeriods} + 1)$ -by-`NumTrials` matrix of risk-neutral simulated prices. The first element of `SimulatedPrices` is the initial value at time 0.

Data Types: `single` | `double`

### **Times** — Annual time factors associated with simulated prices

vector

Annual time factors associated with simulated prices, specified using a  $(\text{NumPeriods} + 1)$ -by-1 column vector. Each element of `Times` is associated with the corresponding row of `SimulatedPrices`. The first element of `Times` must be 0 (current time).

Data Types: `single` | `double`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put'

Definition of option as 'call' or 'put', specified as a character vector.

Data Types: `char`

### **Strike — Option strike price values**

scalar | function handle

Option strike price values, specified as a scalar value `Strike` price. `Strike` for Bermudan options can be specified as a 1-by-NSTRIKES vector or a function handle that returns the value of the strike given the time of the strike.

Data Types: `single` | `double` | `function_handle`

### **ExerciseTimes — Exercise time for option**

vector of exercise times

Exercise time for the option, specified as a vector of exercise times as follows:

- For a European or Bermudan option, `ExerciseTimes` is a 1-by-1 (European) or 1-by-NSTRIKES (Bermudan) vector of exercise times. For a European option, there is only one `ExerciseTimes` on the option expiry date.
- For an American option, `ExerciseTimes` is a 1-by-2 vector of exercise time boundaries. The option exercises on any date between, or including, the pair of times on that row. If `ExerciseTimes` is 1-by-1, the option exercises between time 0 and the single listed `ExerciseTimes`.

Data Types: `double`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
optpricebysim(RateSpec, Prices, Times, OptSpec, Settle, Strike, ExerciseTimes, 'AmericanOpt', 1)
```

### **AmericanOpt — Option type**

0 European or Bermudan (default) | scalar flag with value [0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and an integer scalar flag with values:

- 0 — European or Bermudan
- 1 — American

For American options, the Longstaff-Schwartz least squares method calculates the early exercise premium.

Data Types: single | double

## **Output Arguments**

### **Price — Price of option**

scalar

Price of the option, returned as a scalar value.

## **See Also**

gbm | intenvset | simBySolution

## **Topics**

“Pricing Asian Options”

“Creating Geometric Brownian Motion (GBM) Models” (Financial Toolbox)

Supported Equity Derivatives on page 3-24

**Introduced in R2014a**

## rangefloatbybdt

Price range floating note using Black-Derman-Toy tree

### Syntax

```
[Price,PriceTree] = rangefloatbybdt(BDTree,Spread,Settle,Maturity,
RateSched)
[Price,PriceTree] = rangefloatbybdt( ____,Name,Value)
```

### Description

[Price,PriceTree] = rangefloatbybdt(BDTree,Spread,Settle,Maturity,RateSched) prices range floating note using a Black-Derman-Toy tree.

Payments on range floating notes are determined by the effective interest-rate between reset dates. If the reset period for a range spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

[Price,PriceTree] = rangefloatbybdt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Price of a Range Note Using a Black-Derman-Toy Tree

This example shows how to compute the price of a range note using a Black-Derman-Toy tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```

```
Compounding = 1;

% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% range note instrument matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];

% data to build the tree is as follows:
% assume the volatility is 10%.
Sigma = 0.1;
BDTTS = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates)'));
BDTT = bdttree(BDTV, RS, BDTTS);

% price the instrument
Price = rangefloatbybdt(BDTT, Spread, Settle, Maturity, RateSched)

Price = 97.5267
```

## Input Arguments

### **BDTT** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bdttree`.

Data Types: `struct`

### **Spread** — Number of basis points over reference rate

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle** — Settlement date for floating range note

serial date number | date character vector | cell array of date character vectors



Settlement date for the floating range note, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The `Settle` date for every range floating instrument is set to the `ValuationDate` of the BDT tree. The floating range note argument `Settle` is ignored.

Data Types: `double` | `char` | `cell`

### **Maturity — Maturity date for floating range note**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floating-rate note, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

### **RateSched — Range of rates within which cash flows are nonzero**

structure

Range of rates within which cash flows are nonzero, specified as a NINST-by-1 vector of structures. Each element of the structure array contains two fields:

- `RateSched.Dates` — NDates-by-1 cell array of dates corresponding to the range schedule.
- `RateSched.Rates` — NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates(n)` is nonzero for rates in the range `RateSched.Rates(n,1) < Rate < RateSched.Rates(n,2)`.

Data Types: `struct`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = rangefloatbybdt(BDTree, Spread, Settle, Maturity, RateSched, 'Reset', 4, 'Basis', 5, 'Principal', 10000)`

### **Reset — Frequency payment per year**

1 (default) | numeric

Frequency of payments per year, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

**Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: `struct`**EndMonthRule — End-of-month rule flag for generating caplet dates**

1 (in effect) (default) | nonnegative integer with value 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer with a value of 0 or 1 using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: `logical`

## Output Arguments

**Price — Expected prices of range floating notes at time 0**

vector

Expected prices of the range floating notes at time 0, returned as a NINST-by-1 vector.

**PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node.

Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range.

If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets direct exposure to the reference rate. In return for the drawback that no interest is paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

### References

[1] Jarrow, Robert. “Modelling Fixed Income Securities and Interest Rate Options.” *Stanford Economics and Finance*. 2nd Edition. 2002.

### See Also

bdttree | bondbybdt | cfbybdt | fixedbybdt | floatbybdt | floorbybdt | instrangefloat | rangefloatbybk | rangefloatbyhjm | rangefloatbyhw | swapbybdt

### Topics

“Computing Instrument Prices” on page 2-99

“Pricing a Portfolio Using the Black-Derman-Toy Model” on page 1-13

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

### Introduced in R2012a

# rangefloatbybk

Price range floating note using Black-Karasinski tree

## Syntax

```
[Price,PriceTree] = rangefloatbybk(BKTree,Spread,Settle,Maturity,
RateSched)
[Price,PriceTree] = rangefloatbybk( ____,Name,Value)
```

## Description

[Price,PriceTree] = rangefloatbybk(BKTree,Spread,Settle,Maturity,RateSched) prices range floating note using a Black-Karasinski tree.

Payments on range floating notes are determined by the effective interest-rate between reset dates. If the reset period for a range spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

[Price,PriceTree] = rangefloatbybk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Compute the Price of a Range Note Using a Black-Karasinski Tree

This example shows how to compute the price of a range note using a Black-Karasinski tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```

```
Compounding = 1;

% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% range note instrument matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];

% data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

BKVS = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTS = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVS, RS, BKTS);

% price the instrument
Price = rangefloatbybk(BKT, Spread, Settle, Maturity, RateSched)

Price = 102.7574
```

## Input Arguments

### **BKTree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: double

### **Settle — Settlement date for floating range note**

serial date number | date character vector | cell array of date character vectors

Settlement date for the floating range note, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The `Settle` date for every range floating instrument is set to the `ValuationDate` of the BK tree. The floating range note argument `Settle` is ignored.

Data Types: double | char | cell

### **Maturity — Maturity date for floating range note**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floating-rate note, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **RateSched — Range of rates within which cash flows are nonzero**

structure

Range of rates within which cash flows are nonzero, specified as a NINST-by-1 vector of structures. Each element of the structure array contains two fields:

- `RateSched.Dates` — NDates-by-1 cell array of dates corresponding to the range schedule.
- `RateSched.Rates` — NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates(n)` is nonzero for rates in the range `RateSched.Rates(n,1) < Rate < RateSched.Rate(n,2)`.

Data Types: struct

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price,PriceTree] = rangefloatbybk(BKTree,Spread,Settle,Maturity,RateSched,'Reset',4,'Basis',5,'Principal',10000)`

### **Reset — Frequency payment per year**

1 (default) | numeric

Frequency of payments per year, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double



**Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

**Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

**EndMonthRule — End-of-month rule flag for generating caplet dates**

1 (in effect) (default) | nonnegative integer with value 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer with a value of 0 or 1 using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**Output Arguments****Price — Expected prices of range floating notes at time 0**

vector

Expected prices of the range floating notes at time 0, returned as a NINST-by-1 vector.

**PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range.

If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets direct exposure to the reference rate. In return for the drawback that no interest is paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

### References

[1] Jarrow, Robert. “Modelling Fixed Income Securities and Interest Rate Options.” *Stanford Economics and Finance*. 2nd Edition. 2002.

### See Also

`bktree` | `bondbybk` | `capbybk` | `cfbybk` | `fixedbybk` | `floorbybk` | `instrangefloat` | `rangefloatbybdt` | `rangefloatbyhjm` | `rangefloatbyhw` | `swapbybk`

### Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2012a**

## rangefloatbycir

Price range floating note using Cox-Ingersoll-Ross tree

### Syntax

```
[Price,PriceTree] = rangefloatbycir(CIRTree,Spread,Settle,Maturity,RateSched)
[Price,PriceTree] = rangefloatbycir( ____,Name,Value)
```

### Description

[Price,PriceTree] = rangefloatbycir(CIRTree,Spread,Settle,Maturity,RateSched) prices range floating note with a Cox-Ingersoll-Ross (CIR) interest-rate tree using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = rangefloatbycir( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price a Range Floating-Rate Note Using a CIR Interest-Rate Tree

Create a RateSpec using the `intenvset` function.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
```

```

Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2020';
CIRTimeSpec = cirtimespec(Settle, Maturity, 3);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

```

```

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2]
    dObs: [736696 737061 737426]
    FwdTree: {[1.0350] [1.0790 1.0500 1.0298] [1x5 double]}
    Connect: {[3x1 double] [3x3 double]}
    Probs: {[3x1 double] [3x3 double]}

```

Define the range note instrument that matures in Jan-1-2014 and has the following RateSchedule:

```

Spread = 100;
Settle = 'Jan-1-2017';
Maturity = 'Jan-1-2020';
RateSched(1).Dates = {'Jan-1-2018'; 'Jan-1-2019' ; 'Jan-1-2020'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];

```

Compute the price of the range floating note.

```

[Price,PriceTree] = rangefloatbycir(CIRT,Spread,Settle,Maturity,RateSched)

```

```

Price = 91.6849

```

```

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    PTree: {1x4 cell}
    AITree: {[0] [0 0 0] [0 0 0 0 0] [0 0 0 0 0]}
    tObs: [0 1 2 3]
    Connect: {[3x1 double] [3x3 double]}
    Probs: {[3x1 double] [3x3 double]}

```

## Input Arguments

### **CIRTree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date for floating range note**

serial date number | date character vector | cell array of date character vectors | string array | datetime

Settlement date for the floating range note, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays. The `Settle` date for every range floating instrument is set to the `ValuationDate` of the CIR tree. The floating range note argument `Settle` is ignored.

Data Types: `double` | `char` | `cell` | `string` | `datetime`

### **Maturity — Maturity date for floating range note**

serial date number | date character vector | cell array of date character vectors | string array | datetime

Maturity date for the floating-rate note, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

Data Types: `double` | `char` | `cell` | `string` | `datetime`

### **RateSched — Range of rates within which cash flows are nonzero**

structure

Range of rates within which cash flows are nonzero, specified as a NINST-by-1 vector of structures. Each element of the structure array contains two fields:

- `RateSched.Dates` — `NDates`-by-1 cell array of dates corresponding to the range schedule.

- `RateSched.Rates` — `NDates`-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates(n)` is nonzero for rates in the range `RateSched.Rates(n,1) < Rate < RateSched.Rate(n,2)`.

Data Types: `struct`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = rangefloatbycir(CIRTree, Spread, Settle, Maturity, RateSched, 'Reset', 4, 'Basis', 5, 'Principal', 10000)`

### Reset — Frequency payment per year

1 (default) | numeric

Frequency of payments per year, specified as the comma-separated pair consisting of 'Reset' and a `NINST`-by-1 vector.

---

**Note** Payments on range floating notes are determined by the effective interest-rate between reset dates. If the reset period for a range spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

---

Data Types: `double`

### Basis — Day-count basis of instrument

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a `NINST`-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating caplet dates**

1 (in effect) (default) | nonnegative integer with value 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer with a value of 0 or 1 using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.



Data Types: logical

## Output Arguments

### Price — Expected prices of range floating notes at time 0

vector

Expected prices of the range floating notes at time 0, returned as a NINST-by-1 vector.

### PriceTree — Tree structure of instrument prices

structure

Tree structure of instrument prices, returned as a structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range.

If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference

interest rate and are floored to be positive. The note holder gets direct exposure to the reference rate. In return for the drawback that no interest is paid for the time the range is left, they offer higher coupon rates than comparable standard products, vanilla floating notes.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirsa, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

`bondbycir` | `capbycir` | `cfbycir` | `fixedbycir` | `floatbycir` | `floorbycir` | `instrangefloat` | `oasbycir` | `optbndbycir` | `optembndbycir` | `optemfloatbycir` | `optfloatbycir` | `swapbycir` | `swaptionbycir`

## Topics

- "Pricing Using Interest-Rate Tree Models" on page 2-99
- "Pricing Options Structure" on page B-2
- "Understanding Interest-Rate Tree Models" on page 2-79
- "Supported Interest-Rate Instruments" on page 2-2

## Introduced in R2018a

# rangefloatbyhjm

Price range floating note using Heath-Jarrow-Morton tree

## Syntax

```
[Price,PriceTree] = rangefloatbyhjm(HJMTree,Spread,Settle,Maturity,
RateSched)
[Price,PriceTree] = rangefloatbyhjm( ___ Name,Value)
```

## Description

[Price,PriceTree] = rangefloatbyhjm(HJMTree,Spread,Settle,Maturity,RateSched) prices range floating note using a Heath-Jarrow-Morton tree.

Payments on range floating notes are determined by the effective interest-rate between reset dates. If the reset period for a range spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

[Price,PriceTree] = rangefloatbyhjm( \_\_\_ Name,Value) adds optional name-value pair arguments.

## Examples

### Compute the Price of a Range Note Using a Heath-Jarrow-Morton Tree

This example shows how to compute the price of a range note using a Heath-Jarrow-Morton tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```

```
Compounding = 1;

% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% range note instrument matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];

% data to build the tree is as follows:
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1; 2; 3; 4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
HJMTree = hjmtimespec(ValuationDate, MaTree);
HJMVS = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVS, RS, HJMTree);

% price the instrument
Price = rangefloatbyhjm(HJMT, Spread, Settle, Maturity, RateSched)

Price = 90.2348
```

## Input Arguments

### **HJMTree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `hjmTree`.

Data Types: `struct`

### **Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

**Settle — Settlement date for floating range note**

serial date number | date character vector | cell array of date character vectors

Settlement date for the floating range note, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The **Settle** date for every range floating instrument is set to the **ValuationDate** of the HJM tree. The floating range note argument **Settle** is ignored.

Data Types: double | char | cell

**Maturity — Maturity date for floating range note**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floating-rate note, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**RateSched — Range of rates within which cash flows are nonzero**

structure

Range of rates within which cash flows are nonzero, specified as a NINST-by-1 vector of structures. Each element of the structure array contains two fields:

- **RateSched.Dates** — NDates-by-1 cell array of dates corresponding to the range schedule.
- **RateSched.Rates** — NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date **RateSched.Dates(n)** is nonzero for rates in the range  $\text{RateSched.Rates}(n,1) < \text{Rate} < \text{RateSched.Rate}(n,2)$ .

Data Types: struct

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1**, **Value1**, ..., **NameN**, **ValueN**.

Example: `[Price,PriceTree] = rangefloatbyhjm(HJMTree,Spread,Settle,Maturity,RateSched,'Reset',4,'Basis',5,'Principal',10000)`

**Reset — Frequency payment per year**

1 (default) | numeric

Frequency of payments per year, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

### **EndMonthRule — End-of-month rule flag for generating caplet dates**

1 (in effect) (default) | nonnegative integer with value 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer with a value of 0 or 1 using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

## **Output Arguments**

### **Price — Expected prices of range floating notes at time 0**

vector

Expected prices of the range floating notes at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.

- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range.

If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets direct exposure to the reference rate. In return for the drawback that no interest is paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

### References

- [1] Jarrow, Robert. “Modelling Fixed Income Securities and Interest Rate Options.” *Stanford Economics and Finance*. 2nd Edition. 2002.

### See Also

`bondbyhjm` | `cfbyhjm` | `fixedbyhjm` | `floatbyhjm` | `floorbyhjm` | `hjmtree` | `instrangefloat` | `rangefloatbybdt` | `rangefloatbybk` | `rangefloatbyhw` | `swapbyhjm`

### Topics

- “Computing Instrument Prices” on page 2-99  
“Pricing Options Structure” on page B-2  
“Understanding Interest-Rate Tree Models” on page 2-79  
“Supported Interest-Rate Instruments” on page 2-2

### Introduced in R2012a



# rangefloatbyhw

Price range floating note using Hull-White tree

## Syntax

```
[Price,PriceTree] = rangefloatbyhw(HWTtree,Spread,Settle,Maturity,
RateSched)
[Price,PriceTree] = rangefloatbyhw( ____,Name,Value)
```

## Description

[Price,PriceTree] = rangefloatbyhw(HWTtree,Spread,Settle,Maturity,RateSched) prices range floating note using a Hull-White tree.

Payments on range floating notes are determined by the effective interest-rate between reset dates. If the reset period for a range spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates cannot be uniquely determined because there is more than one possible path for connecting the two payment dates.

[Price,PriceTree] = rangefloatbyhw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Compute the Price of a Range Note Using a Hull-White Tree

This example shows how to compute the price of a range note using a Hull-White tree with the following interest-rate term structure data.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
```

```
Compounding = 1;

% define RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% range note instrument matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525 0.0675; 0.06 0.08];

% data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

HWVS = hwwolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTS = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVS, RS, HWTS);

% price the instrument
Price = rangefloatbyhw(HWT, Spread, Settle, Maturity, RateSched)

Price = 96.6107
```

## Input Arguments

### **HWTtree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using hwtree.

Data Types: struct

### **Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: double

### **Settle — Settlement date for floating range note**

serial date number | date character vector | cell array of date character vectors

Settlement date for the floating range note, specified as a NINST-by-1 vector of serial date numbers or date character vectors. The **Settle** date for every range floating instrument is set to the **ValuationDate** of the HW tree. The floating range note argument **Settle** is ignored.

Data Types: double | char | cell

### **Maturity — Maturity date for floating range note**

serial date number | date character vector | cell array of date character vectors

Maturity date for the floating-rate note, specified as a NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **RateSched — Range of rates within which cash flows are nonzero**

structure

Range of rates within which cash flows are nonzero, specified as a NINST-by-1 vector of structures. Each element of the structure array contains two fields:

- **RateSched.Dates** — NDates-by-1 cell array of dates corresponding to the range schedule.
- **RateSched.Rates** — NDates-by-2 array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date **RateSched.Dates(n)** is nonzero for rates in the range **RateSched.Rates(n,1) < Rate < RateSched.Rates(n,2)**.

Data Types: struct

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1, Value1, ..., NameN, ValueN**.

Example: `[Price,PriceTree] = rangefloatbyhw(HWTree,Spread,Settle,Maturity,RateSched,'Reset',4,'Basis',5,'Principal',10000)`

### **Reset — Frequency payment per year**

1 (default) | numeric

Frequency of payments per year, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

**Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

**EndMonthRule — End-of-month rule flag for generating caplet dates**

1 (in effect) (default) | nonnegative integer with value 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer with a value of 0 or 1 using a NINST-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**Output Arguments****Price — Expected prices of range floating notes at time 0**

vector

Expected prices of the range floating notes at time 0, returned as a NINST-by-1 vector.

**PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range.

If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets direct exposure to the reference rate. In return for the drawback that no interest is paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

### References

[1] Jarrow, Robert. “Modelling Fixed Income Securities and Interest Rate Options.” *Stanford Economics and Finance*. 2nd Edition. 2002.

### See Also

`bondbyhw` | `capbyhw` | `cfbyhw` | `fixedbyhw` | `floorbyhw` | `hwtree` | `instrangefloat` | `rangefloatbybdt` | `rangefloatbybk` | `rangefloatbyhjm` | `swapbyhw`

### Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2012a**

## rate2disc

Discount factors from interest rates

### Syntax

**Usage 1:** Interval points are input as times in periodic units.

```
Disc = rate2disc(Compounding,Rates,EndTimes)
```

```
Disc = rate2disc(Compounding,Rates,EndTimes,StartTimes)
```

**Usage 2:** ValuationDate is passed and interval points are input as dates.

```
[Disc,EndTimes,StartTimes] = rate2disc(Compounding,Rates,EndDates,StartDates,ValuationDate)
```

```
[Disc,EndTimes,StartTimes] = rate2disc(Compounding,Rates,EndDates,StartDates,ValuationDate,DayCountBasis)
```



## Arguments

Compounding	<p>Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors (<b>Disc</b>):</p> <ul style="list-style-type: none"> <li>• <b>Compounding</b> = 0 for simple interest <ul style="list-style-type: none"> <li>• <math>\text{Disc} = 1 / (1 + Z * T)</math>, where T is time in years and simple interest assumes annual times <math>F = 1</math>.</li> </ul> </li> <li>• <b>Compounding</b> = 1, 2, 3, 4, 6, 12 <ul style="list-style-type: none"> <li>• <math>\text{Disc} = (1 + Z/F)^{-T}</math>, where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units, for example, <math>T = F</math> is one year.</li> </ul> </li> <li>• <b>Compounding</b> = 365 <ul style="list-style-type: none"> <li>• <math>\text{Disc} = (1 + Z/F)^{-T}</math>, where F is the number of days in the basis year and T is a number of days elapsed computed by basis.</li> </ul> </li> <li>• <b>Compounding</b> = -1 <ul style="list-style-type: none"> <li>• <math>\text{Disc} = \exp(-T*Z)</math>, where T is time in years.</li> </ul> </li> </ul>
Rates	<p>Number of points (<b>NPOINTS</b>) by number of curves (<b>NCURVES</b>) matrix of rates in decimal form. For example, 5% is 0.05 in <b>Rates</b>. <b>Rates</b> are the yields over investment intervals from <b>StartTimes</b>, when the cash flow is valued, to <b>EndTimes</b>, when the cash flow is received.</p>
EndTimes	<p><b>NPOINTS</b>-by-1 vector or scalar of times in periodic units ending the interval to discount over.</p> <hr/> <p><b>Note</b> When <b>ValuationDate</b> is not passed, the third and fourth arguments (<b>EndTimes</b> and <b>StartTimes</b>) are interpreted as times.</p>
StartTimes	<p>(Optional) <b>NPOINTS</b>-by-1 vector or scalar of times in periodic units starting the interval to discount over. Default = 0.</p>

<p>EndDates</p>	<p>NPOINTS-by-1 vector or scalar of serial maturity dates ending the interval to discount over.</p> <hr/> <p><b>Note</b> : When ValuationDate is passed, the third and fourth arguments (EndDates and StartDates) are interpreted as dates. The date ValuationDate is used as the zero point for computing the times.</p>
<p>StartDates</p>	<p>(Optional) NPOINTS-by-1 vector or scalar of serial dates starting the interval to discount over. StartDates must be earlier than EndDates.</p> <p>Default = ValuationDate.</p>
<p>ValuationDate</p>	<p>Scalar value in serial date number form representing the observation date of the investment horizons entered in StartDates and EndDates. Required in <b>Usage 2</b>. Omitted or passed as an empty matrix to invoke <b>Usage 1</b>.</p>

Basis	<p>(Optional) Day-count basis of the instrument when using <b>Usage 2</b>. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
EndMonthRule	<p>(Optional) End-of-month rule when using <b>Usage 2</b>. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>

## Description

**Usage 1:**  $\text{Disc} = \text{rate2disc}(\text{Compounding}, \text{Rates}, \text{EndTimes})$  or  $\text{Disc} = \text{rate2disc}(\text{Compounding}, \text{Rates}, \text{EndTimes}, \text{StartTimes})$  where interval points are input as times in periodic units.

**Usage 2:**  $[\text{Disc}, \text{EndTimes}, \text{StartTimes}] = \text{rate2disc}(\text{Compounding}, \text{Rates}, \text{EndDates}, \text{StartDates}, \text{ValuationDate})$  or

[Disc, EndTimes, StartTimes] =  
rate2disc(Compounding, Rates, EndDates, StartDates, ValuationDate, Basis,  
EndMonthRule) where ValuationDate is passed and interval points are input as dates.

rate2disc computes the discounts over a series of NPOINTS time intervals given the annualized yield over those intervals. NCURVES different rate curves can be translated at once if they have the same time structure. The time intervals can represent a zero curve or a forward curve.

The output Disc is an NPOINTS-by-NCURVES column vector of discount factors in decimal form representing the value at time StartTime of a unit cash flow received at time EndTime.

You can specify the investment intervals either with input times (**Usage 1**) or with input dates (**Usage 2**). Entering ValuationDate invokes the date interpretation; omitting ValuationDate invokes the default time interpretations.

For **Usage 1**:

- StartTimes is an NPOINTS-by-1 column vector of times starting the interval to discount over, measured in periodic units.
- EndTimes is an NPOINTS-by-1 column vector of times ending the interval to discount over, measured in periodic units.

For **Usage 2**:

- StartDates is an NPOINTS-by-1 column vector of serial dates starting the interval to discount over, measured in days.
- EndDates is an NPOINTS-by-1 column vector of serial dates ending the interval to discount over, measured in days.

If Compounding = 365 (daily), StartDates and EndDates are measured in days as in **Usage 2**. Otherwise, in **Usage 1**, the arguments contain values, T, computed from SIA semiannual time factors, Tsemi, by the formula  $T = Tsemi/2 * F$ , where F is the compounding frequency.

## Examples

**Example 1** demonstrates **Usage 1**. Compute discounts from a zero curve at 6 months, 12 months, and 24 months. The times to the cash flows are 1, 2, and 4. You are computing the present value (at time 0) of the cash flows.

```
Compounding = 2;
Rates = [0.05; 0.06; 0.065];
EndTimes = [1; 2; 4];
Disc = rate2disc(Compounding, Rates, EndTimes)
```

```
Disc =
    0.9756
    0.9426
    0.8799
```

**Example 2** demonstrates **Usage 2**. Compute discounts from a zero curve at 6 months, 12 months, and 24 months. Use dates to specify the ending time horizon.

```
Compounding = 2;
Rates = [0.05; 0.06; 0.065];
EndDates = ['10/15/97'; '04/15/98'; '04/15/99'];
ValuationDate = '4/15/97';
Disc = rate2disc(Compounding, Rates, EndDates, [], ValuationDate)
```

```
Disc =
    0.9756
    0.9426
    0.8799
```

**Example 3** demonstrates **Usage 1**. Compute discounts from the 1-year forward rates beginning now, in six months, and in 12 months. Use monthly compounding. The times to the cash flows are 12, 18, 24, and the forward times are 0, 6, 12.

```
Compounding = 12;
Rates = [0.05; 0.04; 0.06];
EndTimes = [12; 18; 24];
StartTimes = [0; 6; 12];
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
```

```
Disc =
    0.9513
    0.9609
    0.9419
```

## **See Also**

disc2rate | ratetimes

## **Topics**

“Modeling the Interest-Rate Term Structure” on page 2-67

“Interest-Rate Term Conversions” on page 2-62

“Interest Rates Versus Discount Factors” on page 2-55

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

# ratetimes

Change time intervals defining interest-rate environment

## Syntax

Usage 1: ValuationDate not passed; third through sixth arguments are interpreted as times.

```
[Rates,EndTimes,StartTimes] = ratetimes(Compounding,RefRates,RefEndTimes,RefStartTimes)
```

Usage 2: ValuationDate passed and interval points input as dates.

```
[Rates,EndTimes,StartTimes] = ratetimes(Compounding,RefRates,RefEndDates,RefStartDates)
```

## Arguments

Compounding	<p>Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors (Disc):</p> <ul style="list-style-type: none"> <li>• Compounding = 0 for simple interest <ul style="list-style-type: none"> <li>• <math>Disc = 1 / (1 + Z * T)</math>, where T is time in years and simple interest assumes annual times <math>F = 1</math>.</li> </ul> </li> <li>• Compounding = 1, 2, 3, 4, 6, 12 <ul style="list-style-type: none"> <li>• <math>Disc = (1 + Z/F)^{-T}</math>, where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units, for example, <math>T = F</math> is one year.</li> </ul> </li> <li>• Compounding = 365 <ul style="list-style-type: none"> <li>• <math>Disc = (1 + Z/F)^{-T}</math>, where F is the number of days in the basis year and T is a number of days elapsed computed by basis.</li> </ul> </li> <li>• Compounding = -1 <ul style="list-style-type: none"> <li>• <math>Disc = \exp(-T*Z)</math>, where T is time in years.</li> </ul> </li> </ul>
RefRates	NREFPTS-by-NCURVES matrix of reference rates in decimal form. RefRates are the yields over investment intervals from RefStartTimes, when the cash flow is valued, to RefEndTimes, when the cash flow is received.
RefEndTimes	NREFPTS-by-1 vector or scalar of times in periodic units ending the intervals corresponding to RefRates.
RefStartTimes	(Optional) NREFPTS-by-1 vector or scalar of times in periodic units starting the intervals corresponding to RefRates. Default = 0.
EndTimes	NPOINTS-by-1 vector or scalar of times in periodic units ending the interval to discount over.
StartTimes	(Optional) NPOINTS-by-1 vector or scalar of times in periodic units starting the interval to discount over. Default = 0.



RefEndDates	NREFPTS-by-1 vector or scalar of serial dates ending the intervals corresponding to RefRates.
RefStartDates	(Optional) NREFPTS-by-1 vector or scalar of serial dates starting the intervals corresponding to RefRates. Default = ValuationDate.
EndDates	NPOINTS-by-1 vector or scalar of serial maturity dates ending the interval to discount over.
StartDates	(Optional) NPOINTS-by-1 vector or scalar of serial dates starting the interval to discount over. StartDates must be earlier than EndDates.  Default = ValuationDate.
ValuationDate	Scalar value in serial date number form representing the observation date of the investment horizons entered in StartDates and EndDates. Required in Usage 2. Omitted or passed as an empty matrix to invoke Usage 1.

## Description

[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes) and [Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate) change time intervals defining an interest-rate environment.

ratetimes takes an interest-rate environment defined by yields over one collection of time intervals and computes the yields over another set of time intervals. The zero rate is assumed to be piecewise linear in time.

Rates is an NPOINTS-by-NCURVES matrix of rates implied by the reference interest-rate structure and sampled at new intervals.

StartTimes is an NPOINTS-by-1 column vector of times starting the new intervals where rates are desired, measured in periodic units.

EndTimes is an NPOINTS-by-1 column vector of times ending the new intervals, measured in periodic units.

If `Compounding = 365` (daily), `StartTimes` and `EndTimes` are measured in days. The arguments otherwise contain values, `T`, computed from SIA semiannual time factors, `Tsemi`, by the formula  $T = Tsemi/2 * F$ , where `F` is the compounding frequency.

You can specify the investment intervals either with input times (Usage 1) or with input dates (Usage 2). Entering the argument `ValuationDate` invokes the date interpretation; omitting `ValuationDate` invokes the default time interpretations.

## Examples

**Example 1.** The reference environment is a collection of zero rates at 6, 12, and 24 months. Create a collection of 1-year forward rates beginning at 0, 6, and 12 months.

```
RefRates = [0.05; 0.06; 0.065];
RefEndTimes = [1; 2; 4];
StartTimes = [0; 1; 2];
EndTimes = [2; 3; 4];
Rates = ratetimes(2, RefRates, RefEndTimes, 0, EndTimes,...
StartTimes)
```

```
Rates =
    0.0600
    0.0688
    0.0700
```

**Example 2.** Interpolate a zero yield curve to different dates. Zero curves start at the default date of `ValuationDate`.

```
RefRates = [0.04; 0.05; 0.052];
RefDates = [729756; 729907; 730121];
Dates = [730241; 730486];
ValuationDate = 729391;
Rates = ratetimes(2, RefRates, RefDates, [], Dates, [],...
ValuationDate)
```

```
Rates =
    0.0520
    0.0520
```

## See Also

[disc2rate](#) | [rate2disc](#)

## **Topics**

“Modeling the Interest-Rate Term Structure” on page 2-67

“Interest-Rate Term Conversions” on page 2-62

“Interest Rates Versus Discount Factors” on page 2-55

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

## spreadbykirk

Price European spread options using Kirk pricing model

### Syntax

```
Price = spreadbykirk(RateSpec, StockSpec1, StockSpec2, Settle, Maturity,  
OptSpec, Strike, Corr)
```

### Description

Price = spreadbykirk(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr) returns the price for a European spread option using the Kirk pricing model.

### Examples

#### Compute the Price of a Spread Option Using the Kirk Model

Define the spread option dates.

```
Settle = '01-Jan-2012';  
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon  
Price1 = Price1gallon * 42;   % $/barrel  
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;               % $/barrel  
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', rates, ...
'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
```

```
    FinObj: 'RateSpec'
    Compounding: -1
           Disc: 0.9876
           Rates: 0.0500
           EndTimes: 0.2500
           StartTimes: 0
           EndDates: 734960
           StartDates: 734869
           ValuationDate: 734869
           Basis: 1
           EndMonthRule: 1
```

Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

```
StockSpec1 = struct with fields:
```

```
    FinObj: 'StockSpec'
           Sigma: 0.2900
           AssetPrice: 119.7000
           DividendType: []
           DividendAmounts: 0
           ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
```

```
    FinObj: 'StockSpec'
```

```
Sigma: 0.3600
AssetPrice: 93.2000
DividendType: []
DividendAmounts: 0
ExDividendDates: []
```

Compute the European spread option price based on the Kirk model.

```
Price = spreadbykirk(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr)
```

```
Price = 11.1904
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec1 — Stock specification for underlying asset 1**

structure

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **StockSpec2 — Stock specification for underlying asset 2**

structure

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement dates for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates for the spread option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: `double` | `char` | `cell`

### **Maturity — Maturity date for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of date character vectors

Maturity date for spread option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: `double` | `char` | `cell`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: `cell` | `char`

### **Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

If `Strike` is equal to 0, the function computes the price of an exchange option.

Data Types: `single` | `double`

### **Corr — Correlation between underlying asset prices**

integer | vector of integers

Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types: single | double

## **Output Arguments**

### **Price — Expected prices of spread option**

vector

Expected prices of the spread option, returned as a NINST-by-1 vector.

## **References**

[1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

## **See Also**

spreadbybjs | spreadbyfd | spreadbyls | spreadsensbykirk

## **Topics**

"Pricing European and American Spread Options"

"Pricing Asian Options"

"Spread Option" on page 3-50

"Supported Energy Derivatives" on page 3-44

## **External Websites**

Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## **Introduced in R2013b**



# spreadbybjs

Price European spread options using Bjerksund-Stensland pricing model

## Syntax

Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)

## Description

Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr) returns the price for a European spread option using the Bjerksund-Stensland pricing model.

## Examples

### Compute the Price of a Spread Option Using the Bjerksund-Stensland Model

Define the spread option dates.

```
Settle = '01-Jan-2012';
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon
Price1 = Price1gallon * 42;    % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;               % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';  
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;  
Compounding = -1;  
Basis = 1;  
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...  
    'EndDates', Maturity, 'Rates', rates, ...  
    'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
```

```
    FinObj: 'RateSpec'  
    Compounding: -1  
        Disc: 0.9876  
        Rates: 0.0500  
        EndTimes: 0.2500  
        StartTimes: 0  
        EndDates: 734960  
        StartDates: 734869  
    ValuationDate: 734869  
        Basis: 1  
    EndMonthRule: 1
```

Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

```
StockSpec1 = struct with fields:
```

```
    FinObj: 'StockSpec'  
        Sigma: 0.2900  
    AssetPrice: 119.7000  
    DividendType: []  
    DividendAmounts: 0  
    ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
```

```
    FinObj: 'StockSpec'
```

```

        Sigma: 0.3600
    AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []

```

Compute the European spread option price based on the Bjerk Sund-Stensland model.

```

Price = spreadbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr)

```

```

Price = 11.2000

```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec1 — Stock specification for underlying asset 1**

structure

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **StockSpec2 — Stock specification for underlying asset 2**

structure

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement dates for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates for the spread option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: `double` | `char` | `cell`

### **Maturity — Maturity date for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Maturity date for spread option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: `double` | `char` | `cell`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: `char` | `cell`

### **Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

If `Strike` is equal to zero, the function computes the price of an exchange option.

Data Types: `single` | `double`

**Corr — Correlation between underlying asset prices**

integer | vector of integers

Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types: single | double

**Output Arguments****Price — Expected prices of spread option**

vector

Expected prices of the spread option, returned as a NINST-by-1 vector.

**References**

- [1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.
- [2] Bjerksund, Petter, Stensland, Gunnar. "Closed form spread option valuation." Department of Finance, NHH, 2006.

**See Also**

spreadbybjs | spreadbyfd | spreadbyls | spreadsensbykirk

**Topics**

- "Pricing European and American Spread Options"
- "Pricing Asian Options"
- "Spread Option" on page 3-50
- "Supported Energy Derivatives" on page 3-44

**External Websites**

Energy Trading &amp; Risk Management with MATLAB (47 min 31 sec)

**Introduced in R2013b**

## spreadbyfd

Price European or American spread options using finite difference method

### Syntax

```
Price = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity,  
OptSpec, Strike, Corr)
```

```
Price = spreadbyfd( ____, Name, Value)
```

```
[Price, PriceGrid, AssetPrice1, AssetPrice2, Times] = spreadbyfd(  
RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)  
[Price, PriceGrid, AssetPrice1, AssetPrice2, Times] = spreadbyfd( ____,  
Name, Value)
```

### Description

`Price = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` returns the price of European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method. The spread is between the asset defined in `StockSpec1` minus the asset defined in `StockSpec2`.

`Price = spreadbyfd( ____, Name, Value)` adds optional name-value pair arguments.

`[Price, PriceGrid, AssetPrice1, AssetPrice2, Times] = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` returns the `Price`, `PriceGrid`, `AssetPrice1`, `AssetPrice2`, and `Times` for a European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method. The spread is between the asset defined in `StockSpec1` minus the asset defined in `StockSpec2`.

`[Price, PriceGrid, AssetPrice1, AssetPrice2, Times] = spreadbyfd( ____, Name, Value)` returns the `Price`, `PriceGrid`, `AssetPrice1`, `AssetPrice2`, and `Times` and adds optional name-value pair arguments.

## Examples

### Compute the Price of a Spread Option Using the Alternate Direction Implicit (ADI) Finite Difference Method

Define the spread option dates.

```
Settle = '01-Jan-2012';
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon
Price1 = Price1gallon * 42;    % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;               % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', rates, ...
    'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9876
    Rates: 0.0500
```

```
EndTimes: 0.2500
StartTimes: 0
EndDates: 734960
StartDates: 734869
ValuationDate: 734869
Basis: 1
EndMonthRule: 1
```

Define the `StockSpec` for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

```
StockSpec1 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2900
    AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3600
    AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the spread option price based on the Alternate Direction Implicit (ADI) finite difference method.

```
[Price, PriceGrid, AssetPrice1, AssetPrice2, Times] = ...
spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr);
```

Display the price.

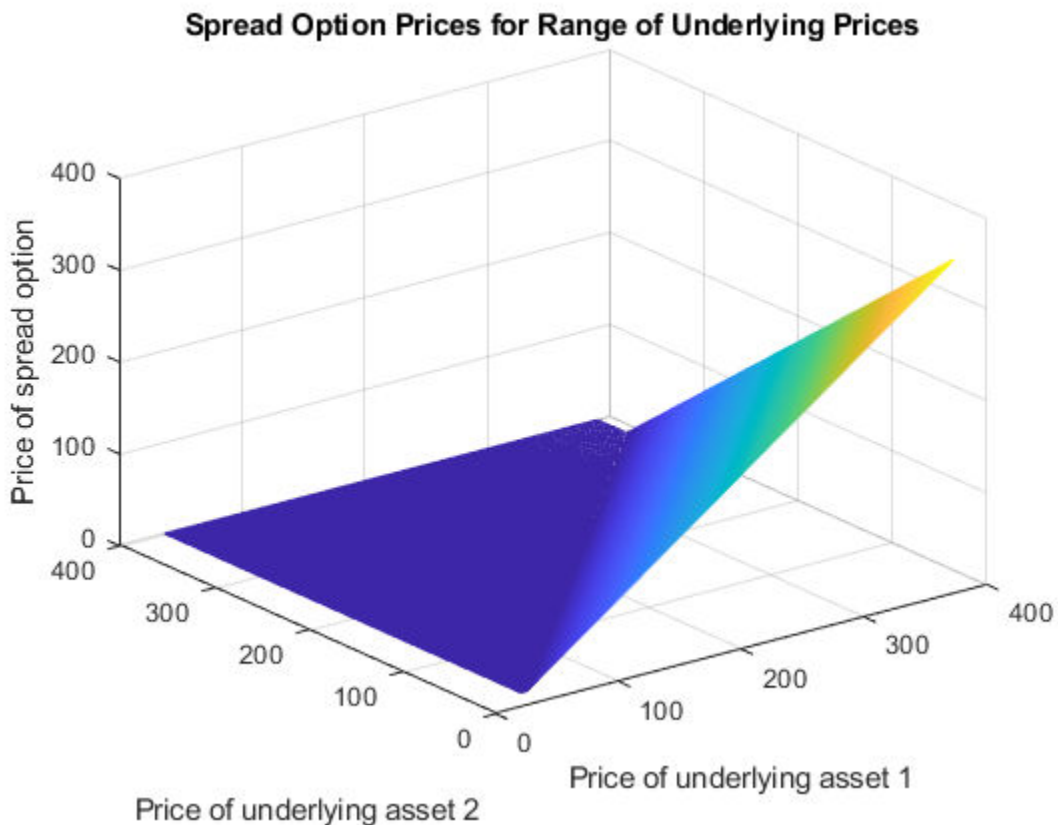
```
Price
```

```
Price = 11.1998
```



Plot the finite difference grid.

```
mesh(AssetPrice1, AssetPrice2, PriceGrid(:, :, 1));  
title('Spread Option Prices for Range of Underlying Prices');  
xlabel('Price of underlying asset 1');  
ylabel('Price of underlying asset 2');  
zlabel('Price of spread option');
```



## Input Arguments

**RateSpec** – Interest-rate term structure  
structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec1 — Stock specification for underlying asset 1** `structure`

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **StockSpec2 — Stock specification for underlying asset 2** `structure`

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement dates for spread option**

`serial date number` | `vector of serial date numbers` | `date character vector` | `cell array of character vectors`

Settlement dates for the spread option, specified as `date character vectors` or as `serial date numbers` using a `NINST-by-1` vector or `cell array of character vector dates`.

Data Types: `double` | `char` | `cell`

### **Maturity — Maturity date for spread option**

`serial date number` | `vector of serial date numbers` | `date character vector` | `cell array of character vectors`

Maturity date for spread option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: double | char | cell

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

### **Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types: single | double

### **Corr — Correlation between underlying asset prices**

integer | vector of integers

Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types: single | double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

Example: [Price, PriceGrid, AssetPrice1, AssetPrice2, Times] = spreadbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, 'AssetPriceMin', 'AssetPriceMax', 'PriceGridSize', 'TimeGridSize', 'AmericanOpt', 0)

### **AssetPriceMin — Minimum price for price grid boundary**

if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array

Minimum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMin' and a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: single | double

### **AssetPriceMax — Maximum price for price grid boundary**

if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array

Maximum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: single | double

### **PriceGridSize — Size for finite difference grid**

[300,300] (default) | array

Size for finite difference grid, specified as the comma-separated pair consisting of 'PriceGridSize' and a 1-by-2 array. The first entry corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: single | double

### **TimeGridSize — Size of the time grid for finite difference grid**

100 (default) | scalar | nonnegative integer

Size of the time grid for finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a nonnegative integer.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments `AssetPriceMax`, `PriceGridSize`, and `TimeGridSize` to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: single | double

### **AmericanOpt — Option type**

0 European (default) | scalar | vector of positive integers[0,1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- 0 — European
- 1 — American

Data Types: single | double

## **Output Arguments**

### **Price — Expected prices of spread option**

vector

Expected prices of the spread option, returned as a NINST-by-1 vector.

### **PriceGrid — Grid containing prices calculated by finite difference method**

array

Grid containing prices calculated by finite difference method, returned as a 3-D grid with a size of `PriceGridSize(1) * PriceGridSize(2) * TimeGridSize`. The price for  $t = 0$  is contained in `PriceGrid(:, :, 1)`.

**AssetPrice1 — Prices for first asset defined by StockSpec1**

vector

Prices for first asset defined by StockSpec1, corresponding to the first dimension of PriceGrid, returned as a vector.

**AssetPrice2 — Prices for second asset defined by StockSpec2**

vector

Prices for second asset defined by StockSpec2, corresponding to the second dimension of PriceGrid, returned as a vector.

**Times — Times corresponding to third dimension of PriceGrid**

vector

Times corresponding to third dimension of PriceGrid, returned as a vector.

**References**

- [1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.
- [2] Villeneuve, S., Zannett, A. "Parabolic ADI Methods for Pricing American Options on Two Stocks." *Mathematics of Operations Research*. Vol. 27, No. 1, pp. 121-149, INFORMS, 2002.
- [3] Ikonen, S., Toivanen, J. *Efficient Numerical Methods for Pricing American Options Under Stochastic Volatility*. Wiley InterScience, 2007.

**See Also**

spreadbybjs | spreadbykirk | spreadbyls | spreadsensbyfd

**Topics**

- "Pricing European and American Spread Options"
- "Pricing Asian Options"
- "Spread Option" on page 3-50
- "Supported Energy Derivatives" on page 3-44

## **External Websites**

Energy Trading & Risk Management with MATLAB (47 min 31 sec)

**Introduced in R2013b**

## spreadbyls

Price European or American spread options using Monte Carlo simulations

### Syntax

```
Price = spreadbyls(RateSpec, StockSpec1, StockSpec2, Settle, Maturity,  
OptSpec, Strike, Corr)
```

```
Price = spreadbyls( ____, Name, Value)
```

```
[Price, Paths, Times, Z] = spreadbyls(RateSpec, StockSpec1, StockSpec2,  
Settle, Maturity, OptSpec, Strike, Corr)
```

```
[Price, Paths, Times, Z] = spreadbyls( ____, Name, Value)
```

### Description

`Price = spreadbyls(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` returns the price of a European or American call or put spread option using Monte Carlo simulations.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

`Price = spreadbyls( ____, Name, Value)` returns the price of a European or American call or put spread option using Monte Carlo simulations using optional name-value pair arguments.

`[Price, Paths, Times, Z] = spreadbyls(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` returns the Price, Paths, Times, and Z of a European or American call or put spread option using Monte Carlo simulations.

`[Price, Paths, Times, Z] = spreadbyls( ____, Name, Value)` returns the Price, Paths, Times, and Z of a European or American call or put spread option using Monte Carlo simulations using optional name-value pair arguments.



## Examples

### Compute the Price of a Spread Option Using Monte Carlo Simulation

Define the spread option dates.

```
Settle = '01-Jan-2012';
Maturity = '01-April-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon
Price1 = Price1gallon * 42;    % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;              % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', rates, ...
    'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9876
    Rates: 0.0500
    EndTimes: 0.2500
```

```
StartTimes: 0
EndDates: 734960
StartDates: 734869
ValuationDate: 734869
Basis: 1
EndMonthRule: 1
```

Define the `StockSpec` for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

```
StockSpec1 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2900
    AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3600
    AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the spread option price using Monte Carlo simulation based on the Longstaff-Schwartz model.

```
Price = spreadbyls(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr)
```

```
Price = 11.0799
```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec1 — Stock specification for underlying asset 1**

structure

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **StockSpec2 — Stock specification for underlying asset 2**

structure

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement date for spread option**

date character vector | nonnegative scalar integer

Settlement date for the spread option specified, as a date character vector or nonnegative scalar integer.

Data Types: `char` | `double`

**Maturity — Maturity date for spread option**

date character vector | nonnegative scalar integer

Maturity date for spread option, specified as a date character vector or a nonnegative scalar integer.

Data Types: char | double

**OptSpec — Definition of option**

character vector with values 'call' or 'put'

Definition of option as 'call' or 'put', specified as a character vector.

Data Types: char

**Strike — Option strike price value**

nonnegative scalar integer

Option strike price value, specified, as a nonnegative scalar integer.

Data Types: single | double

**Corr — Correlation between underlying asset prices**

scalar integer

Correlation between underlying asset prices, specified as a scalar integer.

Data Types: single | double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
spreadbyls(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, 'AmericanOpt', 1)
```

**AmericanOpt — Option type**

0 European (default) | scalar with value [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and an integer scalar flag with value:

- 0 — European
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/%7Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: single | double

### **NumTrials — Scalar number of independent sample paths**

1000 (default) | nonnegative scalar integer

Scalar number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of 'NumTrials' and a nonnegative integer.

Data Types: single | double

### **NumPeriods — Scalar number of simulation periods per trial**

100 (default) | nonnegative scalar integer

Scalar number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a nonnegative integer. NumPeriods is considered only when pricing European basket options. For American spread options, NumPeriod is equal to the number of exercise days during the life of the option.

Data Types: single | double

### **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-2-by-NumTrials 3-D array. The Z value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: single | double

**Antithetic — Indicator for antithetic sampling**

false (default) | scalar logical flag with value of true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

## Output Arguments

**Price — Expected price of spread option**

scalar

Expected price of the spread option, returned as a 1-by-1 scalar.

**Paths — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a NumPeriods + 1-by-2-by-NumTrials 3-D time series array. Each row of Paths is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

**Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a NumPeriods + 1-by-1 column vector of observation times associated with the simulated paths. Each element of Times is associated with the corresponding row of Paths.

**Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, returned as a NumPeriods-by-2-by-NumTrials 3-D array when Z is specified as an input argument. If the Z input argument is not specified, then the Z output argument contains the random variates generated internally.

## References

- [1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

## **See Also**

spreadbybjs | spreadbyfd | spreadbykirk | spreadsensbyls

## **Topics**

“Pricing European and American Spread Options”

“Pricing Asian Options”

“Spread Option” on page 3-50

“Supported Energy Derivatives” on page 3-44

## **External Websites**

Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## **Introduced in R2013b**

## spreadsensbykirk

Calculate European spread option prices or sensitivities using Kirk pricing model

### Syntax

```
PriceSens = spreadbykirk(RateSpec,StockSpec1,StockSpec2,Settle,  
Maturity,OptSpec,Strike,Corr)  
PriceSens = spreadsensbykirk( ____,Name,Value)
```

### Description

`PriceSens = spreadbykirk(RateSpec,StockSpec1,StockSpec2,Settle, Maturity,OptSpec,Strike,Corr)` returns the European spread option prices or sensitivities using the Kirk pricing model.

`PriceSens = spreadsensbykirk( ____,Name,Value)` adds optional name-value pair arguments.

### Examples

#### Compute the Price and Sensitivities of a Spread Option Using the Kirk Model

Define the spread option dates.

```
Settle = '01-Jun-2012';  
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon  
Price1 = Price1gallon * 42;    % $/barrel  
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil



```
Price2 = 93.20;           % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', rates, ...
    'Compounding', Compounding, 'Basis', Basis)
```

RateSpec = struct with fields:

```
    FinObj: 'RateSpec'
    Compounding: -1
        Disc: 0.9876
        Rates: 0.0500
    EndTimes: 0.2500
    StartTimes: 0
        EndDates: 735113
        StartDates: 735021
    ValuationDate: 735021
        Basis: 1
    EndMonthRule: 1
```

Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

StockSpec1 = struct with fields:

```
    FinObj: 'StockSpec'
        Sigma: 0.2900
    AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
```

```
ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3600
    AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the spread option price and sensitivities based on the Kirk model.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
[Price, Delta, Gamma] = spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)
```

```
Price = 11.1904
```

```
Delta = 1×2
```

```
    0.6722    -0.6067
```

```
Gamma = 1×2
```

```
    0.0191    0.0217
```

## Input Arguments

### **RateSpec** — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: struct

**StockSpec1 — Stock specification for underlying asset 1**

structure

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`**StockSpec2 — Stock specification for underlying asset 2**

structure

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`**Settle — Settlement dates for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates for the spread option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: `char` | `cell` | `double`**Maturity — Maturity date for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Maturity date for spread option, specified as date character vectors or as serial date numbers using a NINST-by-1 vector or cell array of character vector dates.

Data Types: `char` | `cell` | `double`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using as NINST-by-1 vector of strike price values.

If **Strike** is equal to zero, this function computes the price and sensitivities of an exchange option.

Data Types: single | double

**Corr — Correlation between underlying asset prices**

integer | vector of integers

Correlation between underlying asset prices, specified as an integer using as NINST-by-1 vector.

Data Types: single | double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1**, **Value1**, ..., **NameN**, **ValueN**.

Example: `PriceSens = spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, OutSpec, {'All'})`

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =  
{'delta','gamma','vega','lambda','rho','theta','price'}

Data Types: char | cell

## Output Arguments

### PriceSens — Expected price or sensitivities values of spread option

vector

Expected price or sensitivities values (defined by OutSpec) of the spread option, returned as a NINST-by-1 or NINST-by-2 vector.

## References

- [1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

## See Also

spreadbybjs | spreadbyfd | spreadbykirk | spreadbyls

## Topics

"Pricing European and American Spread Options"

"Pricing Asian Options"

"Spread Option" on page 3-50

"Supported Energy Derivatives" on page 3-44

## External Websites

Energy Trading & Risk Management with MATLAB (47 min 31 sec)

**Introduced in R2013b**

## spreadsensbybjs

Calculate European spread option prices or sensitivities using Bjerksund-Stensland pricing model

### Syntax

```
PriceSens = spreadbybjs(RateSpec,StockSpec1,StockSpec2,Settle,
Maturity,OptSpec,Strike,Corr)
PriceSens = spreadsensbybjs( ____,Name,Value)
```

### Description

PriceSens = spreadbybjs(RateSpec,StockSpec1,StockSpec2,Settle, Maturity,OptSpec,Strike,Corr) returns the European spread option prices or sensitivities using the Bjerksund-Stensland pricing model.

PriceSens = spreadsensbybjs( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Compute the Price and Sensitivities of a Spread Option Using the Bjerksund-Stensland Model

Define the spread option dates.

```
Settle = '01-Jun-2012';
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon
Price1 = Price1gallon * 42;    % $/barrel
Voll = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;           % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', rates, ...
    'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9876
    Rates: 0.0500
    EndTimes: 0.2500
    StartTimes: 0
    EndDates: 735113
    StartDates: 735021
    ValuationDate: 735021
    Basis: 1
    EndMonthRule: 1
```

Define the StockSpec for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

```
StockSpec1 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2900
    AssetPrice: 119.7000
    DividendType: []
```



```
DividendAmounts: 0
ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3600
    AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the spread option price and sensitivities based on the Kirk model.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
[Price, Delta, Gamma] = spreadsensbybjs(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)
```

```
Price = 11.2000
```

```
Delta = 1×2
```

```
    0.6737    -0.6082
```

```
Gamma = 1×2
```

```
    0.0190    0.0216
```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: struct

### **StockSpec1 — Stock specification for underlying asset 1**

structure

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **StockSpec2 — Stock specification for underlying asset 2**

structure

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement dates for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates for the spread option, specified as date character vectors or as serial date numbers using a `NINST-by-1` vector or cell array of character vector dates.

Data Types: `char` | `cell` | `double`

### **Maturity — Maturity date for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Maturity date for spread option, specified as date character vectors or as serial date numbers using a `NINST-by-1` vector or cell array of character vector dates.

Data Types: `char` | `cell` | `double`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

If **Strike** is equal to zero the function computes the price and sensitivities of an exchange option.

Data Types: single | double

**Corr — Correlation between underlying asset prices**

integer | vector of integers

Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types: single | double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1**, **Value1**, ..., **NameN**, **ValueN**.

Example: `PriceSens = spreadsensbykirk(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, OutSpec, {'All'})`

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =  
{'delta','gamma','vega','lambda','rho','theta','price'}

Data Types: char | cell

## Output Arguments

### **PriceSens** — Expected prices or sensitivities values of spread option

vector

Expected prices or sensitivities values (defined by OutSpec) of the spread option, returned as a NINST-by-1 or NINST-by-2 vector.

## References

- [1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options," *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.
- [2] Bjerksund, Petter, Stensland, Gunnar. "Closed form spread option valuation." Department of Finance, NHH, 2006.

## See Also

spreadbybjs | spreadbyfd | spreadbykirk | spreadbyls

## Topics

- "Pricing European and American Spread Options"
- "Pricing Asian Options"
- "Spread Option" on page 3-50
- "Supported Energy Derivatives" on page 3-44

## **External Websites**

Energy Trading & Risk Management with MATLAB (47 min 31 sec)

**Introduced in R2013b**

## spreadsensbyls

Calculate price and sensitivities for European or American spread options using Monte Carlo simulations

### Syntax

```
PriceSens = spreadsensbyls(RateSpec,StockSpec1,StockSpec2,Settle,  
Maturity,OptSpec,Strike,Corr)
```

```
PriceSens = spreadsensbyls( ____,Name,Value)
```

```
[PriceSens,Paths,Times,Z] = spreadsensbyls(RateSpec,StockSpec1,  
StockSpec2,Settle,Maturity,OptSpec,Strike,Corr)
```

```
[PriceSens,Paths,Times,Z] = spreadsensbyls( ____,Name,Value)
```

### Description

`PriceSens = spreadsensbyls(RateSpec,StockSpec1,StockSpec2,Settle, Maturity,OptSpec,Strike,Corr)` returns the price of a European or American call or put spread option using Monte Carlo simulations.

For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium.

`PriceSens = spreadsensbyls( ____,Name,Value)` adds optional name-value pair arguments.

`[PriceSens,Paths,Times,Z] = spreadsensbyls(RateSpec,StockSpec1, StockSpec2,Settle,Maturity,OptSpec,Strike,Corr)` returns the `PriceSens`, `Paths`, `Times`, and `Z` of a European or American call or put spread option using Monte Carlo simulations.

`[PriceSens,Paths,Times,Z] = spreadsensbyls( ____,Name,Value)` returns the `PriceSens`, `Paths`, `Times`, and `Z` and adds optional name-value pair arguments.

## Examples

### Compute the Price and Sensitivities of a Spread Option Using Monte Carlo Simulation

Define the spread option dates.

```
Settle = '01-Jun-2012';
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon
Price1 = Price1gallon * 42;    % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;              % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', rates, ...
    'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9876
    Rates: 0.0500
```

```
EndTimes: 0.2500
StartTimes: 0
EndDates: 735113
StartDates: 735021
ValuationDate: 735021
Basis: 1
EndMonthRule: 1
```

Define the `StockSpec` for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

```
StockSpec1 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2900
    AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3600
    AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the spread option price and sensitivities using Monte Carlo simulation based on the Longstaff-Schwartz model.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
[Price, Delta, Gamma] = spreadsensbyls(RateSpec, StockSpec1, StockSpec2, ...
Settle, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)
```

```
Price = 11.0799
```

```
Delta = 1×2
```

```
0.6626    -0.5972
```



Gamma =  $1 \times 2$

0.0209      0.0240

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec1 — Stock specification for underlying asset 1**

structure

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **StockSpec2 — Stock specification for underlying asset 2**

structure

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement date for spread option**

date character vector | nonnegative scalar integer

Settlement date for the spread option, specified as a date character vector or as a nonnegative scalar integer.

Data Types: char | double

### **Maturity — Maturity date for spread option**

date character vector | nonnegative scalar integer

Maturity date for spread option, specified as a date character vector or as a nonnegative scalar integer.

Data Types: char | double

### **OptSpec — Definition of option**

character vector with values 'call' or 'put'

Definition of option as 'call' or 'put', specified as a character vector.

Data Types: char

### **Strike — Option strike price value**

nonnegative scalar integer

Option strike price value, specified as a scalar integer.

Data Types: single | double

### **Corr — Correlation between underlying asset prices**

scalar integer

Correlation between underlying asset prices, specified as a scalar integer.

Data Types: single | double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `PriceSens = spreadbyls(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, 'AmericanOpt', 1)`

### **AmericanOpt — Option type**

0 European (default) | scalar with values [0, 1]

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and a scalar integer flag with values:

- 0 — European
- 1 — American

---

**Note** For American options, the Longstaff-Schwartz least squares method is used to calculate the early exercise premium. For more information on the least squares method, see <https://people.math.ethz.ch/~Ehjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf>.

---

Data Types: single | double

### **NumTrials — Number of independent sample paths**

1000 (default) | nonnegative scalar integer

Number of independent sample paths (simulation trials), specified as the comma-separated pair consisting of 'NumTrials' and a nonnegative scalar integer.

Data Types: single | double

### **NumPeriods — Number of simulation periods per trial**

100 (default) | nonnegative scalar integer

Number of simulation periods per trial, specified as the comma-separated pair consisting of 'NumPeriods' and a nonnegative scalar integer. NumPeriods is considered only when pricing European basket options. For American spread options, NumPeriods is equal to the number of exercise days during the life of the option.

Data Types: single | double

### **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, specified as the comma-separated pair consisting of 'Z' and a NumPeriods-by-2-by-NumTrials 3-D array. The Z value generates the Brownian motion vector (that is, Wiener processes) that drives the simulation.

Data Types: single | double

**Antithetic — Indicator for antithetic sampling**

false (default) | logical flag with value of true or false

Indicator for antithetic sampling, specified as the comma-separated pair consisting of 'Antithetic' and a value of true or false.

Data Types: logical

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

Example: OutSpec =  
{'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}

Data Types: char | cell

## Output Arguments

**PriceSens — Expected price or sensitivities of spread option**

scalar

Expected price or sensitivities of the spread option, returned as a 1-by-1 array as defined by OutSpec.

**Paths — Simulated paths of correlated state variables**

vector

Simulated paths of correlated state variables, returned as a `NumPeriods + 1-by-2-by-NumTrials` 3-D time series array. Each row of `Paths` is the transpose of the state vector  $X(t)$  at time  $t$  for a given trial.

### **Times — Observation times associated with simulated paths**

vector

Observation times associated with simulated paths, returned as a `NumPeriods + 1-by-1` column vector of observation times associated with the simulated paths. Each element of `Times` is associated with the corresponding row of `Paths`.

### **Z — Time series array of dependent random variates**

vector

Time series array of dependent random variates, returned as a `NumPeriods-by-2-by-NumTrials` 3-D array when `Z` is specified as an input argument. If the `Z` input argument is not specified, then the `Z` output argument contains the random variates generated internally.

## **References**

- [1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

## **See Also**

`spreadbybjs` | `spreadbyfd` | `spreadbykirk` | `spreadbyls`

## **Topics**

"Pricing European and American Spread Options"

"Pricing Asian Options"

"Spread Option" on page 3-50

"Supported Energy Derivatives" on page 3-44

## **External Websites**

Energy Trading & Risk Management with MATLAB (47 min 31 sec)

**Introduced in R2013b**

## spreadsensbyfd

Calculate price and sensitivities of European or American spread options using finite difference method

### Syntax

```
PriceSens = spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle,  
Maturity, OptSpec, Strike, Corr)  
PriceSens = spreadsensbyfd( ____, Name, Value)
```

```
[PriceSens, PriceGrid, AssetPrice1, AssetPrice2, Times] =  
spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity,  
OptSpec, Strike, Corr)  
[PriceSens, PriceGrid, AssetPrice1, AssetPrice2, Times] =  
spreadsensbyfd( ____, Name, Value)
```

### Description

`PriceSens = spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` returns the price and sensitivities of European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method. The spread is between the asset defined in `StockSpec1` minus the asset defined in `StockSpec2`.

`PriceSens = spreadsensbyfd( ____, Name, Value)` adds optional name-value pair arguments.

`[PriceSens, PriceGrid, AssetPrice1, AssetPrice2, Times] = spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` returns the `PriceSens`, `PriceGrid`, `AssetPrice1`, `AssetPrice2`, and `Times` for European or American call or put spread options using the Alternate Direction Implicit (ADI) finite difference method. The spread is between the asset defined in `StockSpec1` minus the asset defined in `StockSpec2`.

[PriceSens,PriceGrid,AssetPrice1,AssetPrice2,Times] = spreadsensbyfd( \_\_\_\_,Name,Value) returns the PriceSens, PriceGrid, AssetPrice1, AssetPrice2, and Times and adds optional name-value pair arguments.

## Examples

### Compute the Price of a Spread Option Using the Alternate Direction Implicit (ADI) Finite Difference Method

Define the spread option dates.

```
Settle = '01-Jun-2012';
Maturity = '01-Sep-2012';
```

Define asset 1. Price and volatility of RBOB gasoline

```
Price1gallon = 2.85;           % $/gallon
Price1 = Price1gallon * 42;    % $/barrel
Vol1 = 0.29;
```

Define asset 2. Price and volatility of WTI crude oil

```
Price2 = 93.20;               % $/barrel
Vol2 = 0.36;
```

Define the correlation between the underlying asset prices of asset 1 and asset 2.

```
Corr = 0.42;
```

Define the spread option.

```
OptSpec = 'call';
Strike = 20;
```

Define the RateSpec.

```
rates = 0.05;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, ...
    'EndDates', Maturity, 'Rates', rates, ...
    'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9876
    Rates: 0.0500
    EndTimes: 0.2500
    StartTimes: 0
    EndDates: 735113
    StartDates: 735021
    ValuationDate: 735021
    Basis: 1
    EndMonthRule: 1
```

Define the `StockSpec` for the two assets.

```
StockSpec1 = stockspec(Vol1, Price1)
```

```
StockSpec1 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2900
    AssetPrice: 119.7000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

```
StockSpec2 = stockspec(Vol2, Price2)
```

```
StockSpec2 = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.3600
    AssetPrice: 93.2000
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Compute the spread option price and sensitivities based on the Alternate Direction Implicit (ADI) finite difference method.

```
OutSpec = {'Price', 'Delta', 'Gamma'};
[Price, Delta, Gamma, PriceGrid, AssetPrice1, AssetPrice2, Times] = ...
spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, ...
Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec);
```



Display the price and sensitivities.

Price

Price = 11.1998

Delta

Delta = 1×2

0.6736 -0.6082

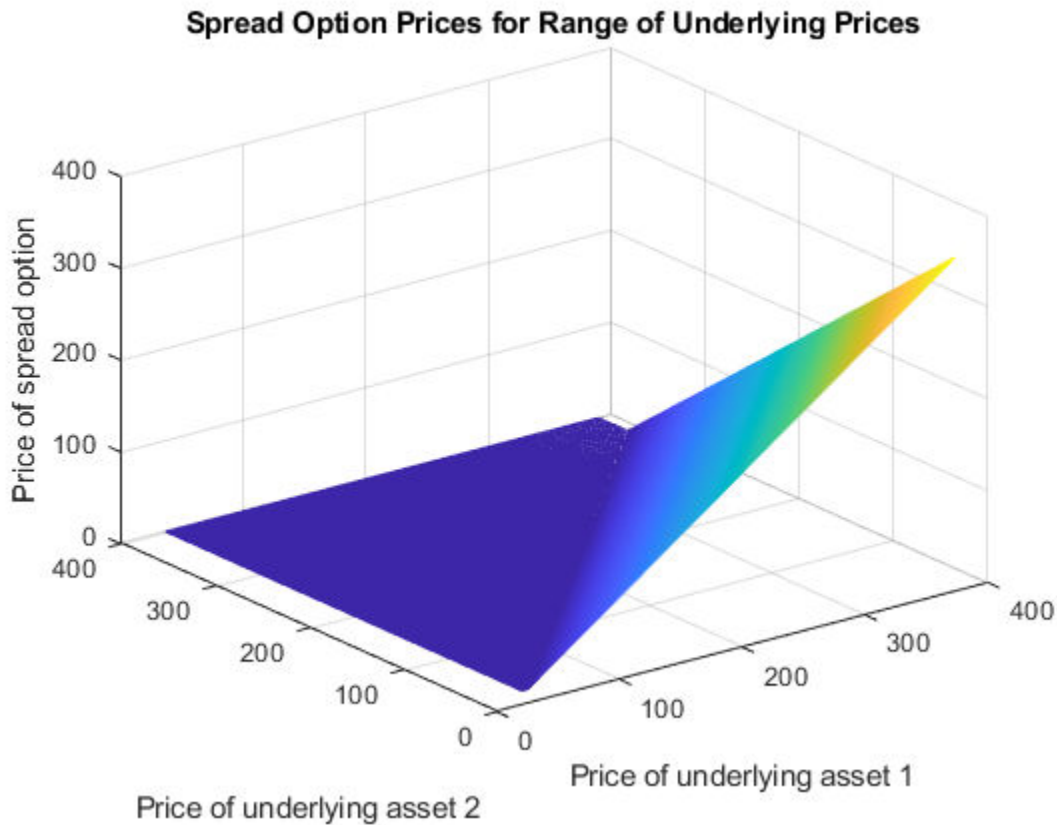
Gamma

Gamma = 1×2

0.0190 0.0214

Plot the finite difference grid.

```
mesh(AssetPrice1, AssetPrice2, PriceGrid(:, :, 1));  
title('Spread Option Prices for Range of Underlying Prices');  
xlabel('Price of underlying asset 1');  
ylabel('Price of underlying asset 2');  
zlabel('Price of spread option');
```



## Input Arguments

**RateSpec** — Interest-rate term structure  
structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec1 — Stock specification for underlying asset 1**

structure

Stock specification for underlying asset 1. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`**StockSpec2 — Stock specification for underlying asset 2**

structure

Stock specification for underlying asset 2. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, for physical commodities the price is represented by `StockSpec.Asset`, the volatility is represented by `StockSpec.Sigma`, and the convenience yield is represented by `StockSpec.DividendAmounts`.

Data Types: `struct`**Settle — Settlement dates for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Settlement dates for the spread option, specified as date character vectors or as serial date numbers using a `NINST-by-1` vector or cell array of character vector dates.

Data Types: `char` | `cell` | `double`**Maturity — Maturity date for spread option**

serial date number | vector of serial date numbers | date character vector | cell array of character vectors

Maturity date for spread option, specified as date character vectors or as serial date numbers using a `NINST-by-1` vector or cell array of character vector dates.

Data Types: `char` | `cell` | `double`

**OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vectors

Definition of option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

Data Types: char | cell

**Strike — Option strike price values**

integer | vector of integers

Option strike price values, specified as an integer using a NINST-by-1 vector of strike price values.

Data Types: single | double

**Corr — Correlation between underlying asset prices**

integer | vector of integers

Correlation between underlying asset prices, specified as an integer using a NINST-by-1 vector.

Data Types: single | double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: [PriceSens, PriceGrid, AssetPrice1, AssetPrice2, Times] =  
spreadsensbyfd(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec,  
Strike, Corr,  
'AssetPriceMin', 'AssetPriceMax', 'PriceGridSize', 'TimeGridSize', 'AmericanOpt', 0, 'OutSpec', {'All'})
```

**OutSpec — Define outputs**

{'Price'} (default) | character vector with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All' | cell array of character vectors with values 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'

Define outputs, specified as the comma-separated pair consisting of 'OutSpec' and a NOUT-by-1 or 1-by-NOUT cell array of character vectors with possible values of 'Price', 'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', and 'All'.

OutSpec = {'All'} specifies that the output should be Delta, Gamma, Vega, Lambda, Rho, Theta, and Price, in that order. This is the same as specifying OutSpec to include each sensitivity:

```
Example: OutSpec =
{'delta','gamma','vega','lambda','rho','theta','price'}
```

Data Types: char | cell

### **AssetPriceMin — Minimum price for price grid boundary**

if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array

Minimum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMin' and a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: single | double

### **AssetPriceMax — Maximum price for price grid boundary**

if unspecified, StockSpec values are calculated based on asset distributions at maturity (default) | array

Maximum price for price grid boundary, specified as the comma-separated pair consisting of 'AssetPriceMax' and a 1-by-2 array. The first entry in the array corresponds to the first asset defined by StockSpec1 and the second entry corresponds to the second asset defined by StockSpec2.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments AssetPriceMin, AssetPriceMax, PriceGridSize, and TimeGridSize to control the

composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: `single` | `double`

### **PriceGridSize — Size for finite difference grid**

`[300,300]` (default) | `array`

Size for finite difference grid, specified as the comma-separated pair consisting of 'PriceGridSize' and a 1-by-2 array. The first entry corresponds to the first asset defined by `StockSpec1` and the second entry corresponds to the second asset defined by `StockSpec2`.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments `AssetPriceMax`, `PriceGridSize`, and `TimeGridSize` to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: `single` | `double`

### **TimeGridSize — Size of time grid for finite difference grid**

`100` (default) | `scalar` | `nonnegative integer`

Size of time grid for finite difference grid, specified as the comma-separated pair consisting of 'TimeGridSize' and a nonnegative integer.

For the finite difference method, the composition of the grid affects the quality of the output and the execution time. It is highly recommended to use the optional arguments `AssetPriceMax`, `PriceGridSize`, and `TimeGridSize` to control the composition of the grid to ensure the quality of the output and a reasonable execution time.

Data Types: `single` | `double`

### **AmericanOpt — Option type**

`0` `European` (default) | `scalar` | `vector of positive integers` `[0,1]`

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer scalar flags with values:

- `0` — European

- 1 — American

Data Types: `single` | `double`

## Output Arguments

### **PriceSens** — Expected prices or sensitivities of spread option

scalar

Expected price or sensitivities of the spread option, returned as a 1-by-1 array as defined by `OutSpec`.

### **PriceGrid** — Grid containing prices calculated by finite difference method

array

Grid containing prices calculated by finite difference method, returned as a 3-D grid with a size of `PriceGridSize(1) * PriceGridSize(2) * TimeGridSize`. The price for  $t = 0$  is contained in `PriceGrid(:, :, 1)`.

### **AssetPrice1** — Prices for first asset defined by `StockSpec1`

vector

Prices for first asset defined by `StockSpec1`, corresponding to the first dimension of `PriceGrid`, returned as a vector.

### **AssetPrice2** — Prices for second asset defined by `StockSpec2`

vector

Prices for second asset defined by `StockSpec2`, corresponding to the second dimension of `PriceGrid`, returned as a vector.

### **Times** — Times corresponding to third dimension of `PriceGrid`

vector

Times corresponding to third dimension of `PriceGrid`, returned as a vector.

## References

- [1] Carmona, R., Durrleman, V. "Pricing and Hedging Spread Options." *SIAM Review*. Vol. 45, No. 4, pp. 627-685, Society for Industrial and Applied Mathematics, 2003.

- [2] Villeneuve, S., Zanette, A. "Parabolic ADI Methods for Pricing American Options on Two Stocks." *Mathematics of Operations Research*. Vol. 27, No. 1, pp. 121-149, INFORMS, 2002.
- [3] Ikonen, S., Toivanen, J. *Efficient Numerical Methods for Pricing American Options Under Stochastic Volatility*. Wiley InterScience, 2007.

## See Also

spreadbybjs | spreadbyfd | spreadbykirk | spreadbyls

## Topics

"Pricing European and American Spread Options"  
"Pricing Asian Options"  
"Spread Option" on page 3-50  
"Supported Energy Derivatives" on page 3-44

## External Websites

Energy Trading & Risk Management with MATLAB (47 min 31 sec)

## Introduced in R2013b



# stockoptspec

Specify European stock option structure

## Syntax

```
StockOptSpec = stockoptspec(OptPrice,Strike,Settle,Maturity,OptSpec,  
InterpMethod)  
StockOptSpec = stockoptspec( ____,InterpMethod)
```

## Description

StockOptSpec = stockoptspec(OptPrice,Strike,Settle,Maturity,OptSpec,InterpMethod) creates a structure encapsulating the properties of a stock option structure.

StockOptSpec = stockoptspec( \_\_\_\_,InterpMethod) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Specify a European Stock Option Structure

This example shows how to specify a European stock option structure using the following data quoted from liquid options in the market with varying strikes and maturity.

```
Settle = '01/01/06';  
Maturity = ['07/01/06';  
           '07/01/06';  
           '07/01/06';  
           '01/01/07';  
           '01/01/07';  
           '01/01/07'];
```

```
'01/01/07';
'07/01/07';
'07/01/07';
'07/01/07';
'07/01/07';
'01/01/08';
'01/01/08';
'01/01/08';
'01/01/08'];

Strike = [113;
101;
100;
88;
128;
112;
100;
78;
144;
112;
100;
69;
162;
112;
100;
61];

OptPrice =[
4.807905472659144;
1.306321897011867;
0.048039195057173;
0;
2.310953054191461;
1.421950392866235;
0.020414826276740;
0;
5.091986935627730;
1.346534812295291;
0.005101325584140;
0;
8.047628153217246;
1.219653432150932;
0.001041436654748];
0;
```

```
OptSpec = { 'call';  
            'call';  
            'put';  
            'put';  
            'call';  
            'call';  
            'put';  
            'put';  
            'call';  
            'call';  
            'put';  
            'put';  
            'call';  
            'call';  
            'put';  
            'put'};
```

```
StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)
```

```
StockOptSpec = struct with fields:  
    FinObj: 'StockOptSpec'  
    OptPrice: [16x1 double]  
    Strike: [16x1 double]  
    Settle: 732678  
    Maturity: [16x1 double]  
    OptSpec: {16x1 cell}  
    InterpMethod: 'price'
```

## Input Arguments

### **OptPrice** — European option prices

vector

European option prices, specified as an NINST-by-1 vector.

Data Types: double

### **Strike** — Strike prices

vector

Strike prices, specified as an NINST-by-1 vector.

Data Types: double

### **Settle — Settlement date**

scalar numeric date

Settlement date, specified as a scalar numeric date.

Data Types: double

### **Maturity — Maturity dates**

vector

Maturity dates, specified as an NINST-by-1 vector.

Data Types: double

### **OptSpec — Option type**

cell array of character vectors with a value of 'call' or 'put'

Option type, specified as an NINST-by-1 cell array of character vectors with a value of 'call' or 'put'.

Data Types: cell

### **InterpMethod — Interpolation method for option prices**

'price' (default) | scalar character vector with value of 'price' or 'vol'

(Optional) Interpolation method for option prices, specified as a scalar character vector with one of the following values:

- 'price' indicates that prices are used for interpolation purposes.
- 'vol' indicates that implied volatilities are used for interpolation purposes. The interpolated values are then used to calculate the implicit interpolated prices.

.

Data Types: char

## **Output Arguments**

### **StockOptSpec — Structure encapsulating the properties of a stock options structure**

structure

Structure encapsulating the properties of a stock options structure, returned as a structure.

## **See Also**

`ittprice` | `itttree` | `stockspec`

## **Topics**

“Building Implied Trinomial Trees” on page 3-8

“Examining Equity Trees” on page 3-18

“Understanding Equity Trees” on page 3-2

“Supported Equity Derivatives” on page 3-24

**Introduced in R2007a**

## stockspec

Create stock structure

### Syntax

```
StockSpec = stockspec(Sigma,AssetPrice)
StockSpec = stockspec( ____,DividendType,DividendAmounts,
ExDividendDates)
```

### Description

StockSpec = stockspec(Sigma,AssetPrice) creates a MATLAB structure containing the properties of a stock.

---

**Note** StockSpec handles other types of underliers when pricing instruments other than equities.

---

StockSpec = stockspec( \_\_\_\_,DividendType,DividendAmounts, ExDividendDates) adds optional arguments for DividendType, DividendAmounts, and ExDividendDates.

### Examples

#### Create a StockSpec for Stocks With Cash Dividends

Consider a stock that provides four cash dividends of \$0.50 on January 3, 2008, April 1, 2008, July 5, 2008 and October 1, 2008. The stock is trading at \$50, and has a volatility of 20% per annum. Using this data, create the structure StockSpec:

```
AssetPrice = 50;
Sigma = 0.20;
```

```

DividendType = {'cash'};
DividendAmounts = [0.50, 0.50, 0.50, 0.50];
ExDividendDates = {'03-Jan-2008', '01-Apr-2008', '05-July-2008', '01-Oct-2008'};

StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts, ExDividendDates)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2000
    AssetPrice: 50
    DividendType: {'cash'}
    DividendAmounts: [0.5000 0.5000 0.5000 0.5000]
    ExDividendDates: [733410 733499 733594 733682]

```

Examine the StockSpec structure.

```

datedisp(StockSpec.ExDividendDates)

03-Jan-2008    01-Apr-2008    05-Jul-2008    01-Oct-2008

StockSpec.DividendType

ans = 1x1 cell array
    {'cash'}

```

The StockSpec structure encapsulates the information of the stock and its four cash dividends.

### Create a StockSpec for Stocks With Cash and Continuous Dividends

Consider two stocks that are trading at \$40 and \$35. The first one provides two cash dividends of \$0.25 on March 1, 2008 and June 1, 2008. The second stock provides a continuous dividend yield of 3%. The stocks have a volatility of 30% per annum. Using this data, create the structure StockSpec:

```

AssetPrice = [40; 35];
Sigma = .30;

DividendType = {'cash'; 'continuous'};
DividendAmount = [0.25, 0.25 ; 0.03 NaN];

```

```
DividendDate1 = 'March-01-2008';  
DividendDate2 = 'Jun-01-2008';
```

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmount,...  
{ DividendDate1, DividendDate2 ; NaN NaN})
```

```
StockSpec = struct with fields:  
    FinObj: 'StockSpec'  
    Sigma: [2x1 double]  
    AssetPrice: [2x1 double]  
    DividendType: {2x1 cell}  
    DividendAmounts: [2x2 double]  
    ExDividendDates: [2x2 double]
```

Examine the StockSpec structure.

```
datedisp(StockSpec.ExDividendDates)
```

```
01-Mar-2008    01-Jun-2008  
    NaN            NaN
```

```
StockSpec.DividendType
```

```
ans = 2x1 cell array  
    {'cash'      }  
    {'continuous'}
```

The StockSpec structure encapsulates the information of the two stocks and their dividends.

## Input Arguments

### **Sigma — Annual price volatility of underlying security**

decimal

Annual price volatility of underlying security, specified as a NINST-by-1 decimal.

Data Types: double



**AssetPrice — Underlying asset price values at time 0**

vector

Underlying asset price values at time 0, specified as a NINST-by-1 vector.

Data Types: double

**DividendType — Stock dividend type**

cell array of date character vectors

(Optional) Stock dividend type, specified as a NINST-by-1 cell array of character vectors.

Dividend type must be either `cash` for actual dollar dividends, `constant` for constant dividend yield, or `continuous` for continuous dividend yield. This function does not handle stock option dividends.

---

**Note** Dividends are assumed to be paid in cash. Noncash dividends (stock) are not allowed. When combining two or more types of dividends, shorter rows should be padded with the value `NaN`.

---

Data Types: char | cell

**DividendAmounts — Dividend amounts**

matrix | vector

(Optional) Dividend amounts, specified as a NINST-by-NDIV matrix of cash dividends or NINST-by-1 vector representing a constant or continuous annualized dividend yield.

Data Types: double

**ExDividendDates — Ex-dividend dates**

matrix | vector

(Optional) Ex-dividend dates, specified as a NINST-by-NDIV matrix of ex-dividend dates for a cash `DividendType` or NINST-by-1 vector of ex-dividend dates for constant `DividendType`. For continuous `DividendType`, this argument should be ignored.

Data Types: double | cell

## Output Arguments

### **StockSpec — Properties of stock structure**

structure

Properties of stock structure, returned as a structure encapsulating the properties of a stock.

## See Also

crrprice | crrtree | intenvset | optstockbybjs | optstockbyblk |  
optstockbybls | optstockbyls | optstockbyrgw | spreadbybjs | spreadbyfd |  
spreadbykirk | spreadbyls

## Topics

“Portfolio Creation” on page 1-8

“Supported Equity Derivatives” on page 3-24

**Introduced before R2006a**

# sttprice

Price instruments using standard trinomial tree

## Syntax

```
[Price,PriceTree] = sttprice(STTTree,InstSet)
[Price,PriceTree] = sttprice( ____,Name,Value)
```

## Description

[Price,PriceTree] = sttprice(STTTree,InstSet) prices instruments using a standard trinomial (STT) tree.

[Price,PriceTree] = sttprice( \_\_\_\_,Name,Value) prices instruments using a standard trinomial (STT) tree with an optional name-value pair argument for Options.

## Examples

### Price a stttree Instrument Set

Load the data into the MATLAB® workspace.

```
load deriv.mat
```

STTTree and STTInstSet are the input arguments required to call the function sttprice. Use the command instdisp to examine the set of instruments contained in the variable STTInstSet.

```
instdisp(STTInstSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
1	OptStock	call	100	01-Jan-2009	01-Jan-2011	1	Call1	10
2	OptStock	put	80	01-Jan-2009	01-Jan-2012	0	Put1	5

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Bar
-------	------	---------	--------	--------	---------------	-------------	-------------	-----

```

3   Barrier call    105   01-Jan-2009   01-Jan-2012   1           ui           115
Index Type      UOptSpec UStrike USettle      UExerciseDates UAmericanOpt COptSpec CS
4   Compound call    95     01-Jan-2009   01-Jan-2012   1           put          5
Index Type      OptSpec Strike Settle      ExerciseDates AmericanOpt Name      Quant
5   Lookback call    90     01-Jan-2009   01-Jan-2012   0           Lookback1 7
6   Lookback call    95     01-Jan-2009   01-Jan-2013   0           Lookback2 9
Index Type  OptSpec Strike Settle      ExerciseDates AmericanOpt AvgType  AvgPri
7   Asian call    100    01-Jan-2009   01-Jan-2012   0           arithmetic NaN
8   Asian call    100    01-Jan-2009   01-Jan-2013   0           arithmetic NaN

```

The instrument set contains eight instruments:

- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Use `sttprice` to calculate the price of each instrument in the instrument set.

```
Price = sttprice(STTtree, STTInstSet)
```

```
Price = 8×1
```

```

4.5025
3.0603
3.7977
1.7090
11.7296
12.9120
1.6905
2.6203

```

## Input Arguments

### **STTTree — Stock tree structure for standard trinomial tree**

structure

Stock tree structure for a standard trinomial tree, specified by using `stttree`.

Data Types: `struct`

### **InstSet — Variable containing a collection instruments**

structure

Variable containing a collection of NINST instruments, specified as a structure. Instruments are broken down by type and each type can have different data fields.

Data Types: `struct`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = sttprice(STTTree, InstSet, 'Options', deriv)`

### **Options — Derivatives pricing options**

structure

Derivatives pricing options, specified as the comma-separated pair consisting of `'Options'` and a structure that is created with `derivset`.

Data Types: `struct`

## Output Arguments

### **Price — Expected prices for each instrument at time 0**

matrix

Expected prices for each instrument at time 0, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the standard trinomial (STT) stock tree. If an instrument cannot be priced, a NaN is returned in that entry.

### **PriceTree — Structure with vector of instrument prices at each node**

tree structure

Structure with a vector of instrument prices at each node, returned as a tree structure.

PriceTree is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

PriceTree.PTree contains the prices.

PriceTree.tObs contains the observation times.

PriceTree.dObs contains the observation dates.

### **See Also**

sttsens | stttimespec | stttree

### **Topics**

“Convertible Bond” on page 2-3

**Introduced in R2015b**

## sttsens

Instrument sensitivities and prices using standard trinomial tree

### Syntax

```
[Delta,Gamma,Vega,Price] = sttsens(STTTree,InstSet)
[Delta,Gamma,Vega,Price] = sttsens( ____,Name,Value)
```

### Description

[Delta,Gamma,Vega,Price] = sttsens(STTTree,InstSet) to generate instrument sensitivities and prices using a standard trinomial (STT) tree.

[Delta,Gamma,Vega,Price] = sttsens( \_\_\_\_,Name,Value) to generate instrument sensitivities and prices using a standard trinomial (STT) tree with an optional name-value pair argument for Options.

### Examples

#### Determine the Price and Sensitivities for a stttree Instrument Set

Load the data into the MATLAB® workspace.

```
load deriv.mat
```

STTTree and STTInstSet are the input arguments required to call the function sttprice. Use the command instdisp to examine the set of instruments contained in the variable STTInstSet.

```
instdisp(STTInstSet)
```

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quantity
1	OptStock	call	100	01-Jan-2009	01-Jan-2011	1	Call1	10
2	OptStock	put	80	01-Jan-2009	01-Jan-2012	0	Put1	5

Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	BarrierSpec	Bar
3	Barrier	call	105	01-Jan-2009	01-Jan-2012	1	ui	115
Index	Type	UOptSpec	UStrike	USettle	UExerciseDates	UAmericanOpt	COptSpec	CS
4	Compound	call	95	01-Jan-2009	01-Jan-2012	1	put	5
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	Name	Quant
5	Lookback	call	90	01-Jan-2009	01-Jan-2012	0	Lookback1	7
6	Lookback	call	95	01-Jan-2009	01-Jan-2013	0	Lookback2	9
Index	Type	OptSpec	Strike	Settle	ExerciseDates	AmericanOpt	AvgType	AvgPri
7	Asian	call	100	01-Jan-2009	01-Jan-2012	0	arithmetic	NaN
8	Asian	call	100	01-Jan-2009	01-Jan-2013	0	arithmetic	NaN

The instrument set contains eight instruments:

- Two vanilla options (Call1, Put1)
- One barrier option (Barrier1)
- One compound option (Compound1)
- Two lookback options (Lookback1, Lookback2)
- Two Asian options (Asian1, Asian2)

Use `sttsens` to calculate the price and sensitivities for each instrument in the instrument set.

```
[Delta,Gamma,Vega,Price] = sttsens(STTtree, STTInstSet)
```

```
Delta = 8×1
```

```

0.5267
-0.0943
0.4726
-0.0624
0.2313
0.3266
0.5706
0.6646
```

```
Gamma = 8×1
105 ×
```



```
0.0000
0.0000
0.0000
0.0000
-1.8650
-1.9119
1.8650
1.9119
```

Vega = 8×1

```
52.8980
42.4369
25.9792
-9.5266
70.3758
92.9226
25.8122
37.8757
```

Price = 8×1

```
4.5025
3.0603
3.7977
1.7090
11.7296
12.9120
1.6905
2.6203
```

### **Determine Price and Sensitivities for Convertible Bond Instruments Using a sttree**

Create a RateSpec.

```
StartDates = 'Jan-1-2015';
EndDates = 'Jan-1-2020';
```

```
Rates = 0.025;
Basis = 1;

RateSpec = intenvset('ValuationDate',StartDates,'StartDates',StartDates,...
'EndDates',EndDates,'Rates',Rates,'Compounding',-1,'Basis',Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.8825
    Rates: 0.0250
    EndTimes: 5
    StartTimes: 0
    EndDates: 737791
    StartDates: 735965
    ValuationDate: 735965
    Basis: 1
    EndMonthRule: 1
```

Create a StockSpec.

```
AssetPrice = 80;
Sigma = 0.12;
StockSpec = stockspec(Sigma,AssetPrice)

StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.1200
    AssetPrice: 80
    DividendType: []
    DividendAmounts: 0
    ExDividendDates: []
```

Create a STTTree.

```
TimeSpec = stttimespec(StartDates, EndDates, 20);
STTTree = stttree(StockSpec, RateSpec, TimeSpec)

STTTree = struct with fields:
    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
```

```

    tObs: [1x21 double]
    dObs: [1x21 double]
    STree: {1x21 cell}
    Probs: {1x20 cell}

```

Define the convertible bond. The convertible bond can be called starting on Jan 1, 2016 with a strike price of 95.

```

CouponRate = 0.03;
Settle = 'Jan-1-2015';
Maturity = 'April-1-2018';
Period = 1;
CallStrike = 95;
CallExDates = [datenum('Jan-1-2016') datenum('April-1-2018')];
ConvRatio = 1;
Spread = 0.025;

```

Price the convertible bond using the standard trinomial tree model.

```

[Price,PriceTree,EqTree,DbtTree] = cbondbystt(STTTree,CouponRate,Settle,Maturity,ConvR
'Period',Period,'Spread',Spread,'CallExDates',CallExDates,'CallStrike',CallStrike,'Ame

```

```

Price = 90.2511

```

```

PriceTree = struct with fields:

```

```

    FinObj: 'TrinPriceTree'
    PTree: {1x21 cell}
    tObs: [1x21 double]
    dObs: [1x21 double]

```

```

EqTree = struct with fields:

```

```

    FinObj: 'TrinPriceTree'
    PTree: {1x21 cell}
    tObs: [1x21 double]
    dObs: [1x21 double]

```

```

DbtTree = struct with fields:

```

```

    FinObj: 'TrinPriceTree'
    PTree: {1x21 cell}
    tObs: [1x21 double]
    dObs: [1x21 double]

```

Compute the delta and gamma of the convertible bond.

```
InstSet= instcbond(CouponRate,Settle,Maturity,ConvRatio,'Spread',Spread,...  
'CallExDates',CallExDates,'CallStrike',CallStrike,'AmericanCall',1);  
[Delta,Gamma] = sttsens(STTtree,InstSet)
```

```
Delta = 0.3945
```

```
Gamma = 0.0324
```

## Input Arguments

### **STTtree** — Stock tree structure for standard trinomial tree

structure

Stock tree structure for a standard trinomial tree, specified by using `sttree`.

Data Types: `struct`

### **InstSet** — Variable containing a collection instruments

structure

Variable containing a collection of NINST instruments, specified as a structure. Instruments are broken down by type and each type can have different data fields.

Data Types: `struct`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

```
Example: [Delta,Gamma,Vega,Price] =  
sttsens(STTtree,InstSet,'Options',deriv)
```

### **Options** — Derivatives pricing options

structure

Derivatives pricing options, specified as the comma-separated pair consisting of `'Options'` and a structure that is created with `derivset`.

Data Types: struct

## Output Arguments

### **Delta — Rate of change of instruments prices with respect to changes in the stock price**

vector of deltas

Rate of change of instrument prices with respect to changes in the stock price, returned as a NINST-by-1 vector of deltas. For more information on the stock tree, see `stttree`.

### **Gamma — Rate of change of instrument deltas with respect to changes in the stock price**

vector of gammas

Rate of change of instrument deltas with respect to changes in the stock price, returned as a NINST-by-1 vector of gammas.

### **Vega — Rate of change of instrument prices with respect to changes in the volatility of the stock price**

vector of vegas

Rate of change of instrument prices with respect to changes in the volatility of the stock price, returned as a NINST-by-1 vector of vegas. For more information on the stock tree, see `stttree`.

### **Price — Expected prices for each instrument at time 0**

matrix

Expected prices for each instrument at time 0, returned as a NINST-by-1 vector. The prices are computed by backward dynamic programming on the standard trinomial (STT) stock tree. If an instrument cannot be priced, a NaN is returned in that entry.

## See Also

`derivset` | `sttsens` | `stttimespec` | `stttree`

## Topics

“Convertible Bond” on page 2-3

**Introduced in R2015b**

# stttimespec

Specify time structure for standard trinomial tree

## Syntax

```
TimeSpec = stttimespec(ValuationDate,Maturity,NumPeriods)
```

## Description

TimeSpec = stttimespec(ValuationDate,Maturity,NumPeriods) creates a time spec for a standard trinomial (STT) tree.

## Examples

### Create a stttimespec to Build a STTTree

Create a RateSpec.

```
StartDates = 'Jan-1-2014';
EndDates = 'Jan-1-2018';
Rates = 0.025;
Basis = 1;
Compounding = -1;
```

```
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: 0.9048
    Rates: 0.0250
    EndTimes: 4
    StartTimes: 0
    EndDates: 737061
```

```
    StartDates: 735600
ValuationDate: 735600
    Basis: 1
    EndMonthRule: 1
```

Create a StockSpec.

```
AssetPrice = 110;
Sigma = 0.22;
Div = 0.02;
StockSpec = stockspec(Sigma, AssetPrice, 'continuous', Div)
```

```
StockSpec = struct with fields:
    FinObj: 'StockSpec'
    Sigma: 0.2200
    AssetPrice: 110
    DividendType: {'continuous'}
    DividendAmounts: 0.0200
    ExDividendDates: []
```

Create a STTTimespec and STTTree.

```
NumPeriods = length(cfdates(StartDates,EndDates,12));
TimeSpec = stttimespec(StartDates, EndDates, NumPeriods)
```

```
TimeSpec = struct with fields:
    FinObj: 'STTTTimeSpec'
    ValuationDate: 735600
    Maturity: 737061
    NumPeriods: 48
    Basis: 0
    EndMonthRule: 1
    tobs: [1x49 double]
    dobs: [1x49 double]
```

```
STTT = stttree(StockSpec, RateSpec, TimeSpec)
```

```
STTT = struct with fields:
    FinObj: 'STStockTree'
    StockSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tobs: [1x49 double]
```



```
dObs: [1x49 double]  
STree: {1x49 cell}  
Probs: {1x48 cell}
```

## Input Arguments

### **ValuationDate** — Date marking the pricing date and first observation tree

serial date number | date character vector

Date marking the pricing date and first observation in the tree, specified as a scalar using a serial date number or date character vector.

Data Types: double | char

### **Maturity** — Date marking the depth of tree

serial date number | date character vector

Date marking the depth of the tree, specified as a scalar using a serial date number or date character vector.

Data Types: double | char

### **NumPeriods** — Determines how many time steps are in tree

nonnegative integer

Determines how many time steps are in tree, specified as a scalar using a nonnegative integer value.

Data Types: double

## Output Arguments

### **TimeSpec** — Time layout for standard trinomial (STT) tree

structure

Time layout for standard trinomial (STT) tree, returned as a structure.

## **See Also**

stttree

**Introduced in R2015b**

# stttree

Build standard trinomial tree

## Syntax

```
STTTree = stttree(StockSpec,RateSpec,TimeSpec)
```

## Description

STTTree = stttree(StockSpec,RateSpec,TimeSpec) builds a standard trinomial (STT) tree.

## Examples

### Build a STTTree

Create a RateSpec.

```
StartDates = 'Jan-1-2014';
EndDates = 'Jan-1-2018';
Rates = 0.025;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', StartDates, 'StartDates', StartDates,...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis)
```

```
RateSpec = struct with fields:
```

```
    FinObj: 'RateSpec'
  Compounding: -1
         Disc: 0.9048
         Rates: 0.0250
    EndTimes: 4
  StartTimes: 0
    EndDates: 737061
  StartDates: 735600
```

```
ValuationDate: 735600
      Basis: 1
      EndMonthRule: 1
```

Create a `StockSpec`.

```
AssetPrice = 55;
Sigma = 0.22;
Div = 0.02;
StockSpec = stockspec(Sigma, AssetPrice, 'continuous', Div)
```

```
StockSpec = struct with fields:
      FinObj: 'StockSpec'
      Sigma: 0.2200
      AssetPrice: 55
      DividendType: {'continuous'}
      DividendAmounts: 0.0200
      ExDividendDates: []
```

Create a Standard Trinomial Tree (`STTTTree`).

```
NumSteps = 8;
TimeSpec = stttimespec(StartDates, EndDates, NumSteps);
STTT = stttree(StockSpec, RateSpec, TimeSpec)
```

```
STTT = struct with fields:
      FinObj: 'STStockTree'
      StockSpec: [1x1 struct]
      TimeSpec: [1x1 struct]
      RateSpec: [1x1 struct]
      tObs: [0 0.5000 1 1.5000 2 2.5000 3 3.5000 4]
      dObs: [1x9 double]
      STree: {1x9 cell}
      Probs: {1x8 cell}
```

## Input Arguments

**StockSpec** — Stock specification for underlying asset structure

Stock specification for underlying asset, specified using `StockSpec` obtained from `stockspec`. For information on the stock specification, see `stockspec`.

`stockspec` can handle other types of underlying assets. For example, stocks, stock indices, and commodities. If dividends are not specified in `StockSpec`, dividends are assumed to be 0.

Data Types: `struct`

### **RateSpec — Interest-rate term specification of initial risk-free rate curve** structure

Interest-rate term specification of initial risk-free rate curve, specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **TimeSpec — Tree time layout specification** structure

Tree time layout specification, specified using `stttimespec` to define the observation dates of the standard trinomial (STT) tree.

Data Types: `struct`

## **Output Arguments**

### **STTtree — Tree specifying stock and time information for a standard trinomial (STT) tree**

tree structure

Tree specifying stock and time information for a standard trinomial (STT) tree, returned as a tree structure.

## **See Also**

`stttimespec`

**Introduced in R2015b**

## supersharebybls

Determine price of supershare digital options using Black-Scholes model

### Syntax

```
Price = supersharebybls(RateSpec,StockSpec,Settle,Maturity,  
StrikeLow,StrikeHigh)
```

### Description

Price = supersharebybls(RateSpec,StockSpec,Settle,Maturity,StrikeLow,StrikeHigh) computes supershare digital options using the Black-Scholes option pricing model.

### Examples

#### Compute the Price of Supershare Digital Options Using Black-Scholes Model

This example shows how to compute the price of supershare digital options using Black-Scholes model. Consider a supershare based on a portfolio of nondividend paying stocks with a lower strike of 350 and an upper strike of 450. The value of the portfolio on November 1, 2008 is 400. The risk-free rate is 4.5% and the volatility is 18%. Using this data, calculate the price of the supershare option on February 1, 2009.

```
Settle = 'Nov-1-2008';  
Maturity = 'Feb-1-2009';  
Rates = 0.045;  
Basis = 1;  
Compounding = -1;  
  
% create the RateSpec  
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...  
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
```

```

% define the StockSpec
AssetPrice = 400;
Sigma = .18;
StockSpec = stockspec(Sigma, AssetPrice);

% define the high and low strike points
StrikeLow = 350;
StrikeHigh = 450;

% calculate the price
Pssh = supersharebybls(RateSpec, StockSpec, Settle, Maturity,...
StrikeLow, StrikeHigh)

Pssh = 0.9411

```

## Input Arguments

### **RateSpec — Interest-rate term structure**

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

### **StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

### **Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **StrikeLow — Low strike price values**

vector

Low strike price values, specified as an NINST-by-1 vector.

Data Types: double

### **StrikeHigh — High strike price values**

vector

High strike price values, specified as an NINST-by-1 vector.

Data Types: double

## **Output Arguments**

### **Price — Expected prices for supershare option**

vector

Expected prices for supershare option, returned as a NINST-by-1 vector.

## **See Also**

assetbybls | cashbybls | gapbybls | supersharesensbybls

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing European Call Options Using Different Equity Models”



"Pricing Using the Black-Scholes Model" on page 3-154  
"Supported Equity Derivatives" on page 3-24

**Introduced in R2009a**

## supersharesensbyls

Determine price or sensitivities of supershare digital options using Black-Scholes model

### Syntax

```
PriceSens = supersharesensbyls(RateSpec,StockSpec,Settle,Maturity,  
StrikeLow,StrikeHigh)  
PriceSens = supersharesensbyls( ____,Name,Value)
```

### Description

`PriceSens = supersharesensbyls(RateSpec,StockSpec,Settle,Maturity,StrikeLow,StrikeHigh)` computes price or sensitivities of supershare digital options using the Black-Scholes option pricing model.

`PriceSens = supersharesensbyls( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute Price and Sensitivities of Supershare Digital Options Using Black-Scholes Model

This example shows how to compute price and sensitivities of supershare digital options using a Black-Scholes model. Consider a supershare based on a portfolio of nondividend paying stocks with a lower strike of 350 and an upper strike of 450. The value of the portfolio on November 1, 2008 is 400. The risk-free rate is 4.5% and the volatility is 18%. Using this data, calculate the price and sensitivity of the supershare option on February 1, 2009.

```
Settle = 'Nov-1-2008';  
Maturity = 'Feb-1-2009';  
Rates = 0.045;
```

```

Basis = 1;
Compounding = -1;

% define the RateSpec
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

% define the StockSpec
AssetPrice = 400;
Sigma = .18;
StockSpec = stockspec(Sigma, AssetPrice);

% define the high and low strike points
StrikeLow = 350;
StrikeHigh = 450;

% calculate the price
Pssh = supersharebybls(RateSpec, StockSpec, Settle, Maturity,...
StrikeLow, StrikeHigh)

Pssh = 0.9411

% compute the delta and theta of the supershare option
OutSpec = { 'delta'; 'theta' };
[Delta, Theta]= supersharesensbybls(RateSpec, StockSpec, Settle,...
Maturity, StrikeLow, StrikeHigh, 'OutSpec', OutSpec)

Delta = -0.0010

Theta = -1.0102

```

## Input Arguments

### RateSpec — Interest-rate term structure

structure

Interest-rate term structure (annualized and continuously compounded), specified by the RateSpec obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

Data Types: `struct`

**StockSpec — Stock specification for underlying asset**

structure

Stock specification for the underlying asset. For information on the stock specification, see `stockspec`.

`stockspec` handles several types of underlying assets. For example, for physical commodities the price is `StockSpec.Asset`, the volatility is `StockSpec.Sigma`, and the convenience yield is `StockSpec.DividendAmounts`.

Data Types: `struct`

**Settle — Settlement or trade date**

serial date number | date character vector

Settlement or trade date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**Maturity — Maturity date**

serial date number | date character vector

Maturity date for the basket option, specified as an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**StrikeLow — Low strike price values**

vector

Low strike price values, specified as an NINST-by-1 vector.

Data Types: `double`

**StrikeHigh — High strike price values**

vector

High strike price values, specified as an NINST-by-1 vector.

Data Types: `double`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Gamma, Theta, Price] = supersharesensbyls(RateSpec, StockSpec, Settle, Maturity, StrikeLow, StrikeHigh, 'OutSpec', {'gamma'; 'theta'; 'price'})`

### OutSpec — Define outputs

`{'Price'}` (default) | character vector with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'` | cell array of character vectors with values `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`

Define outputs, specified as the comma-separated pair consisting of `'OutSpec'` and a NOUT-by-1 or a 1-by-NOUT cell array of character vectors with possible values of `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`.

`OutSpec = {'All'}` specifies that the output is `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` to include each sensitivity.

Example: `OutSpec = {'delta', 'gamma', 'vega', 'lambda', 'rho', 'theta', 'price'}`

Data Types: `char` | `cell`

## Output Arguments

### PriceSens — Expected prices or sensitivities for supershare option

vector

Expected prices or sensitivities for supershare option, returned as a NINST-by-1 vector.

## See Also

`supersharebybls`

## **Topics**

“Equity Derivatives Using Closed-Form Solutions” on page 3-150

“Pricing European Call Options Using Different Equity Models”

“Pricing Using the Black-Scholes Model” on page 3-154

“Supported Equity Derivatives” on page 3-24

## **Introduced in R2009a**

# swapbybdt

Price swap instrument from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price,PriceTree,CFTree,SwapRate] = swapbybdt(BDTree,LegRate,
Settle,Maturity)
[Price,PriceTree,CFTree,SwapRate] = swapbybdt( ____,Name,Value)
```

## Description

[Price,PriceTree,CFTree,SwapRate] = swapbybdt(BDTree,LegRate,Settle,Maturity) prices a swap instrument from a Black-Derman-Toy interest-rate tree. swapbybdt computes prices of vanilla swaps, amortizing swaps and forward swaps.

[Price,PriceTree,CFTree,SwapRate] = swapbybdt( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \$100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.15 (15%)
- Spread for floating leg: 10 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.15 10]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Price the swap using the `BDTTree` included in the MAT-file `deriv.mat`. `BDTTree` contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use `swapbybdt` to compute the price of the swap.

```
Price = swapbybdt(BDTTree, LegRate, Settle, Maturity, ...
LegReset, Basis, Principal, LegType)
```

```
Price = 7.4222
```

Using the previous data, calculate the swap rate, the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];
```

```
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree, ...
LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price = -1.4211e-14
```

```
PriceTree = struct with fields:
```

```
  FinObj: 'BDTPriceTree'
   tObs: [0 1 2 3 4]
   PTree: {1x5 cell}
```

```
CFTree = struct with fields:
```

```
  FinObj: 'BDTCFTree'
   tObs: [0 1 2 3 4]
  CFTree: {[NaN] [NaN NaN] [NaN NaN NaN] [NaN NaN NaN NaN] [1x4 double]}
```

```
SwapRate = 0.1205
```



## Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: 0.8135
    Rates: 0.0350
    EndTimes: 6
    StartTimes: 0
    EndDates: 736696
    StartDates: 734504
    ValuationDate: 734504
    Basis: 0
    EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle = '1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal = {'1-Jan-2013' 100; '1-Jan-2014' 80; '1-Jan-2015' 60; '1-Jan-2016' 40; '1-Jan-2017' 0};
```

Build the BDT tree and assume volatility is 10%.

```
MatDates = {'1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
```

```
Volatility = 0.10;  
BDTVolSpec = bdtvolSpec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates)))  
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbybdt(BDTT, LegRate, Settle, Maturity, 'Principal', Principal)
```

```
Price = 1.4574
```

### Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0325;  
ValuationDate = '1-Jan-2012';  
StartDates = ValuationDate;  
EndDates = '1-Jan-2018';  
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: 0.8254  
    Rates: 0.0325  
    EndTimes: 6  
    StartTimes: 0  
    EndDates: 737061  
    StartDates: 734869  
    ValuationDate: 734869  
    Basis: 0  
    EndMonthRule: 1
```

Build the tree with a volatility of 10%.

```

MatDates = {'1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'; '1-Jan-2018'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates)))';
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);

```

Compute the price of a forward swap that starts in two years (Jan 1, 2014) and matures in three years with a forward swap rate of 3.85%.

```

Settle = '1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2014';
LegRate = [0.0385 10];

Price = swapbybdt(BDTT, LegRate, Settle, Maturity, 'StartDate', StartDate)

Price = 1.3203

```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```

LegRate = [NaN 10];
[Price, ~,~, SwapRate] = swapbybdt(BDTT, LegRate, Settle, Maturity, 'StartDate', StartDate)

Price = -4.9738e-12

SwapRate = 0.0335

```

## Input Arguments

### **BDTTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `bdttree`

Data Types: `struct`

### **LegRate** — Leg rate

matrix

Leg rate, specified as a NINST-by-2 matrix, with each row defined as one of the following:

- [CouponRate Spread] (fixed-float)
- [Spread CouponRate] (float-fixed)
- [CouponRate CouponRate] (fixed-fixed)
- [Spread Spread] (float-float)

CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

### Settle — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The Settle date for every swap is set to the ValuationDate of the BDT tree. The swap argument Settle is ignored.

Data Types: char | double

### Maturity — Maturity date

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each swap.

Data Types: char | double

## Name-Value Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)

**LegReset — Reset frequency per year for each swap**

[1 1] (default) | vector

Reset frequency per year for each swap, specified as the comma-separated pair consisting of 'LegReset' and a NINST-by-2 vector.

Data Types: double

**Basis — Day-count basis representing the basis for each leg**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis for each leg, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 array (or NINST-by-2 if Basis is different for each leg).

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts or principal value schedules, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

`Principal` accepts a NINST-by-1 vector or NINST-by-1 cell array (or NINST-by-2 if `Principal` is different for each leg) of the notional principal amounts or principal value schedules. For schedules, each element of the cell array is a NumDates-by-2 array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: `cell` | `double`

### **LegType — Leg type**

[1 0] for each instrument (default) | matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float)

Leg type, specified as the comma-separated pair consisting of 'LegType' and a NINST-by-2 matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float). Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in `LegRate`. `LegType` allows [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float) swaps

Data Types: `double`

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: `struct`

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when `Maturity` is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 (or NINST-by-2 if `EndMonthRule` is different for each leg).

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 (or NINST-by-2 if AdjustCashFlowsBasis is different for each leg) of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: `double`

**StartDate — Date swap actually starts**

Settle date (default) | serial date number | character vector

Date swap actually starts, specified as the comma-separated pair consisting of 'StartDate' and a `NINST-by-1` vector of dates using a serial date number or a character vector.

Use this argument to price forward swaps, that is, swaps that start in a future date

Data Types: `char` | `double`

## Output Arguments

**Price — Expected swap prices at time 0**

vector

Expected swap prices at time 0, returned as a `NINST-by-1` vector.

**PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node.

Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

**CFTree — Swap cash flows**

structure

Swap cash flows, returned as a tree structure with a vector of the swap cash flows at each node. This structure contains only NaNs because with binomial recombining trees, cash flows cannot be computed accurately at each node of a tree.

**SwapRate — Rates applicable to fixed leg**

matrix



Rates applicable to the fixed leg, returned as a NINST-by-1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. The SwapRate output is padded with NaN for those instruments in which CouponRate is not set to NaN.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## See Also

bdttree | capbybdt | cfbybdt | floorbybdt

## Topics

"Computing Instrument Prices" on page 2-99

"Pricing a Portfolio Using the Black-Derman-Toy Model" on page 1-13

"Understanding Interest-Rate Tree Models" on page 2-79

"Pricing Options Structure" on page B-2

"Supported Interest-Rate Instruments" on page 2-2

**Introduced before R2006a**

## swapbybk

Price swap instrument from Black-Karasinski interest-rate tree

### Syntax

```
[Price,PriceTree,CFTree,SwapRate] = swapbybk(BKTree,LegRate,Settle,
Maturity)
[Price,PriceTree,CFTree,SwapRate] = swapbybk( ____,Name,Value)
```

### Description

[Price,PriceTree,CFTree,SwapRate] = swapbybk(BKTree,LegRate,Settle, Maturity) prices a swap instrument from a Black-Karasinski interest-rate tree. swapbybk computes prices of vanilla swaps, amortizing swaps and forward swaps.

[Price,PriceTree,CFTree,SwapRate] = swapbybk( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \$100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2004
- Swap maturity date: Jan. 01, 2006

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:

```

Settle = '01-Jan-2004';
Maturity = '01-Jan-2006';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year

```

Price the swap using the BKTtree included in the MAT-file `deriv.mat`. BKTtree contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use `swapbybk` to price of the swap.

```
Price = swapbybk(BKTtree, LegRate,...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price = 5.0425
```

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];
```

```
[Price, PriceTree, SwapRate] = swapbybk(BKTtree, LegRate,...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price = -2.8422e-14
```

```
PriceTree = struct with fields:
```

```

    FinObj: 'BKPriceTree'
    PTree: {1x5 cell}
    tObs: [0 1 2 3 4]
    Connect: {[2] [2 3 4] [2 2 3 4 4]}
    Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

```
SwapRate = 0.0336
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: 0.8135
    Rates: 0.0350
    EndTimes: 6
    StartTimes: 0
    EndDates: 736696
    StartDates: 734504
    ValuationDate: 734504
    Basis: 0
    EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle = '1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal = {'1-Jan-2013' 100; '1-Jan-2014' 80; '1-Jan-2015' 60; '1-Jan-2016' 40; '1-Jan-2017' 0};
```

Build the BK tree and assume volatility is 10%.

```
MatDates = {'1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'};
BKTimeSpec = bktimespec(ValuationDate, MatDates);
```

```

Volatility = 0.10;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;
BKVolspec = bkvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates))',..
AlphaDates, AlphaCurve);
BKT = bktree(BKVolspec, RateSpec, BKTimeSpec);

```

Compute the price of the amortizing swap.

```
Price = swapbybk(BKT, LegRate, Settle, Maturity, 'Principal' , Principal)
```

```
Price = 1.4574
```

## Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```

Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;

```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: 0.8023
    Rates: 0.0374
    EndTimes: 6
    StartTimes: 0
    EndDates: 737061
    StartDates: 734869
    ValuationDate: 734869
    Basis: 0
    EndMonthRule: 1

```

Build a BK tree.

```
VolDates = {'1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'; '1-Jan-2018'};
VolCurve = 0.1;
AlphaDates = '01-01-2018';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of 4.25%.

```
Settle = '1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];

Price = swapbybk(BKT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price = 1.4434
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
[Price, ~, SwapRate] = swapbybk(BKT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price = 2.8422e-14
```

```
SwapRate = 0.0384
```

## Input Arguments

### **BKTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `bktree`

Data Types: `struct`

**LegRate — Leg rate**

matrix

Leg rate, specified as a NINST-by-2 matrix, with each row defined as one of the following:

- [CouponRate Spread] (fixed-float)
- [Spread CouponRate] (float-fixed)
- [CouponRate CouponRate] (fixed-fixed)
- [Spread Spread] (float-float)

CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

**Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The **Settle** date for every swap is set to the **ValuationDate** of the BK tree. The swap argument **Settle** is ignored.

Data Types: char | double

**Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each swap.

Data Types: char | double

**Name-Value Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1, Value1, . . . , NameN, ValueN**.

Example: `[Price, PriceTree, CFTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, Leg Type)`

**LegReset — Reset frequency per year for each swap**

`[1 1]` (default) | vector

Reset frequency per year for each swap, specified as the comma-separated pair consisting of 'LegReset' and a NINST-by-2 vector.

Data Types: double

**Basis — Day-count basis representing the basis for each leg**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis for each leg, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 array (or NINST-by-2 if Basis is different for each leg).

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double



**Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts or principal value schedules, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array (or NINST-by-2 if Principal is different for each leg) of the notional principal amounts or principal value schedules. For schedules, each element of the cell array is a NumDates-by-2 array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

**LegType — Leg type**

[1 0] for each instrument (default) | matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float)

Leg type, specified as the comma-separated pair consisting of 'LegType' and a NINST-by-2 matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float). Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate. LegType allows [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float) swaps

Data Types: double

**Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using derivset.

Data Types: struct

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 (or NINST-by-2 if EndMonthRule is different for each leg).

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 (or NINST-by-2 if AdjustCashFlowsBasis is different for each leg) of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

### **StartDate — Date swap actually starts**

Settle date (default) | serial date number | character vector

Date swap actually starts, specified as the comma-separated pair consisting of 'StartDate' and a `NINST-by-1` vector of dates using a serial date number or a character vector.

Use this argument to price forward swaps, that is, swaps that start in a future date

Data Types: char | double

## **Output Arguments**

### **Price — Expected swap prices at time 0**

vector

Expected swap prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value

indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.

- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

### **CFTree — Swap cash flows**

structure

Swap cash flows, returned as a tree structure with a vector of the swap cash flows at each node. This structure contains only NaNs because with binomial recombining trees, cash flows cannot be computed accurately at each node of a tree.

### **SwapRate — Rates applicable to fixed leg**

matrix

Rates applicable to the fixed leg, returned as a NINST-by-1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is NaN. The `SwapRate` output is padded with NaN for those instruments in which `CouponRate` is not set to NaN.

## Definitions

### **Amortizing Swap**

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### **Forward Swap**

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## See Also

`bktree` | `bondbybk` | `capbybk` | `fixedbybk` | `floorbybk`

## **Topics**

"Pricing Using Interest-Rate Tree Models" on page 2-99

"Understanding Interest-Rate Tree Models" on page 2-79

"Pricing Options Structure" on page B-2

"Supported Interest-Rate Instruments" on page 2-2

**Introduced before R2006a**

## swapbycir

Price swap instrument from Cox-Ingersoll-Ross interest-rate tree

### Syntax

```
[Price,PriceTree,SwapRate] = swapbycir(CIRTree,LegRate,Settle,  
Maturity)  
[Price,PriceTree,SwapRate] = swapbycir( ____,Name,Value)
```

### Description

[Price,PriceTree,SwapRate] = swapbycir(CIRTree,LegRate,Settle, Maturity) prices a swap instrument from a Cox-Ingersoll-Ross (CIR) interest-rate tree. swapbycir computes prices of vanilla swaps, amortizing swaps, and forward swaps using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree,SwapRate] = swapbycir( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price an Interest-Rate Swap Using a CIR Interest-Rate Tree

Define an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year and the notional principal amount is \$100.

```
Basis = 0;  
Principal = 100;  
LegRate = [0.06 20]; % [CouponRate Spread]  
LegType = [1 0]; % [Fixed Float]  
LegReset = [1 1]; % Payments once per year
```

Create a RateSpec using the intenvset function.

```

Rates = [0.035; 0.042147; 0.047345; 0.052707];
Dates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
ValuationDate = 'Jan-1-2017';
EndDates = Dates(2:end)';
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'EndDates', EndDates);

```

Create a CIR tree.

```

NumPeriods = 5;
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Settle = '01-Jan-2017';
Maturity = '01-Jan-2022';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

```

```
CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)
```

```

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 1 2 3 4]
    dObs: [736696 737061 737426 737791 738156]
    FwdTree: {1x5 cell}
    Connect: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}
    Probs: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}

```

Price the interest-rate swap.

```
[Price, PriceTree] = swapbycir(CIRT, LegRate, Settle, Maturity, 'LegReset', LegReset, 'Basis');
```

```
Price = 2.5522
```

```

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
    tObs: [0 1 2 3 4 5]
    PTree: {1x6 cell}
    Connect: {[3x1 double] [3x3 double] [3x5 double] [3x7 double]}

```

## Input Arguments

### **CIRTree — Interest-rate structure**

structure

Interest-rate tree structure, created by `cirtree`

Data Types: `struct`

### **LegRate — Leg rate**

matrix

Leg rate, specified as a NINST-by-2 matrix, with each row defined as one of the following:

- [CouponRate Spread] (fixed-float)
- [Spread CouponRate] (float-fixed)
- [CouponRate CouponRate] (fixed-fixed)
- [Spread Spread] (float-float)

CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

Data Types: `double`

### **Settle — Settlement date**

serial date number | character vector | string array | datetime

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays.

The `Settle` date for every swap is set to the `ValuationDate` of the CIR tree. The swap argument `Settle` is ignored.

Data Types: `char` | `double` | `string` | `datetime`

### **Maturity — Maturity date**

serial date number | character vector | string array | datetime

Maturity date, specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or datetime arrays representing the maturity date for each swap.

Data Types: `char` | `double` | `string` | `datetime`



## Name-Value Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree, SwapRate] = swapbycir(CIRTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)`

### **LegReset — Reset frequency per year for each swap**

`[1 1]` (default) | vector

Reset frequency per year for each swap, specified as the comma-separated pair consisting of 'LegReset' and a NINST-by-2 vector.

Data Types: double

### **Basis — Day-count basis representing the basis for each leg**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis for each leg, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 array (or NINST-by-2 if Basis is different for each leg).

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts or principal value schedules, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array (or NINST-by-2 if Principal is different for each leg) of the notional principal amounts or principal value schedules. For schedules, each element of the cell array is a NumDates-by-2 array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **LegType — Leg type**

[1 0] for each instrument (default) | matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float)

Leg type, specified as the comma-separated pair consisting of 'LegType' and a NINST-by-2 matrix with values:

- [1 1] (fixed-fixed) swap
- [1 0] (fixed-float) swap
- [0 1] (float-fixed) swap
- [0 0] (float-float) swap

Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate.

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when **Maturity** is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 (or NINST-by-2 if EndMonthRule is different for each leg).

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 (or NINST-by-2 if AdjustCashFlowsBasis is different for each leg) of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.

- `previous` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- `modifiedprevious` — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: `char` | `cell`

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: `double`

### **StartDate — Date swap actually starts**

Settle date (default) | serial date number | character vector | string array | `datetime`

Date swap actually starts, specified as the comma-separated pair consisting of 'StartDate' and a `NINST-by-1` vector of dates using a serial date number, character vector, string array, or string array.

Use this argument to price forward swaps, that is, swaps that start in a future date

Data Types: `char` | `double` | `string` | `datetime`

## **Output Arguments**

### **Price — Expected swap prices at time 0**

vector

Expected swap prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.tObs` contains the observation times.
- `PriceTree.PTree` contains the clean prices.

### **SwapRate — Rates applicable to fixed leg**

matrix

Rates applicable to the fixed leg, returned as a NINST-by-1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is NaN. The `SwapRate` output is padded with NaN for those instruments in which `CouponRate` is not set to NaN.

## **Definitions**

### **Amortizing Swap**

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### **Forward Swap**

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## **References**

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.

- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

### See Also

bondbycir | capbycir | cfbycir | fixedbycir | floatbycir | floorbycir | instswap | oasbycir | optbndbycir | optembndbycir | optemfloatbycir | optfloatbycir | rangefloatbycir | swaptionbycir

### Topics

"Pricing Using Interest-Rate Tree Models" on page 2-99  
"Understanding Interest-Rate Tree Models" on page 2-79  
"Pricing Options Structure" on page B-2  
"Supported Interest-Rate Instruments" on page 2-2

**Introduced in R2018a**

# swapbyhjm

Price swap instrument from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price,PriceTree,CFTree,SwapRate] = swapbyhjm(HJMTree,LegRate,Settle,Maturity)
[Price,PriceTree,CFTree,SwapRate] = swapbyhjm( ____,Name,Value)
```

## Description

[Price,PriceTree,CFTree,SwapRate] = swapbyhjm(HJMTree,LegRate,Settle,Maturity) prices a swap instrument from a Heath-Jarrow-Morton interest-rate tree. swapbyhjm computes prices of vanilla swaps, amortizing swaps and forward swaps.

[Price,PriceTree,CFTree,SwapRate] = swapbyhjm( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

## Examples

### Price an Interest-Rate Swap

This example shows how to price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \$100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Price the swap using the HJMTree included in the MAT-file `deriv.mat`. The HJMTree structure contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use `swapbyhjm` to compute the price of the swap.

```
[Price, PriceTree, CFTree] = swapbyhjm(HJMTree, LegRate, ...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price =
```

```
3.6923
```

```
PriceTree =
```

```
FinObj: 'HJMPriceTree'
tObs: [0 1 2 3 4]
PBush: {1x5 cell}
```

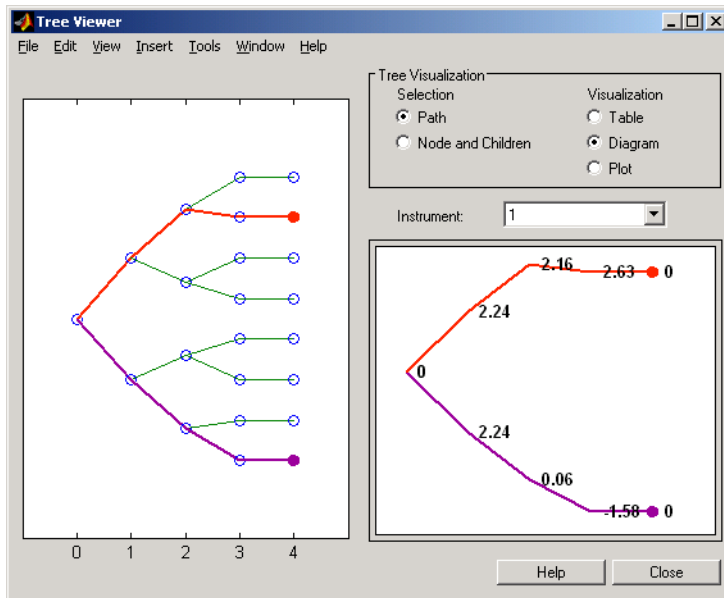
```
CFTree =
```

```
FinObj: 'HJMCFTree'
tObs: [0 1 2 3 4]
CFBush: {[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}
```

Use `treeview` to examine `CFTree` graphically and see the cash flows from the swap along both the up and the down branches. A positive cash flow indicates an inflow (income - payments > 0), while a negative cash flow indicates an outflow (income - payments < 0).

```
treeview(CFTree)
```





In this example, you have sold a swap (receive fixed rate and pay floating rate). At time  $t = 3$ , if interest rates go down, your cash flow is positive (\$2.63), meaning that you receive this amount. But if interest rates go up, your cash flow is negative (-\$1.58), meaning that you owe this amount.

treeviewer price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, so, decreasing prices appear on the lower branch. Conversely, for interest-rate displays, *decreasing* interest rates appear on the upper branch (prices are rising) and *increasing* interest rates on the lower branch (prices are falling).

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];
```

```
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, ...
LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price =
```

```
0
```

```
PriceTree =  
  
FinObj: 'HJMPriceTree'  
  tObs: [0 1 2 3 4]  
  PBush:{[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}  
  
CFTree =  
  
FinObj: 'HJMCFTree'  
  tObs: [0 1 2 3 4]  
  CFBush:{[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}  
  
SwapRate =  
  
    0.0466
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;  
ValuationDate = '1-Jan-2011';  
StartDates = ValuationDate;  
EndDates = '1-Jan-2017';  
Compounding = 1;  
  
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
  'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)  
  
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: 0.8135  
    Rates: 0.0350  
    EndTimes: 6  
    StartTimes: 0  
    EndDates: 736696  
    StartDates: 734504  
    ValuationDate: 734504
```

```
Basis: 0
EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle = '1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal = {'1-Jan-2013' 100; '1-Jan-2014' 80; '1-Jan-2015' 60; '1-Jan-2016' 40; '1-Jan-2017' 0};
```

Build the HJM tree using the following data:

```
MatDates = {'1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'Principal', Principal)
```

```
Price = 1.4574
```

## Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: 0.8023
    Rates: 0.0374
    EndTimes: 6
    StartTimes: 0
    EndDates: 737061
    StartDates: 734869
    ValuationDate: 734869
    Basis: 0
    EndMonthRule: 1
```

Build an HJM tree.

```
MatDates = {'1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'; '1-Jan-2018'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec);
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of 4.25%.

```
Settle = '1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];

Price = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'StartDate', StartDate)

Price = 1.4434
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
[Price, ~,~, SwapRate] = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'StartDate', StartDate)

Price = 0
```

SwapRate = 0.0384

## Input Arguments

### **HJMTree** — Interest-rate structure

structure

Interest-rate tree structure, created by `hjmtree`

Data Types: `struct`

### **LegRate** — Leg rate

matrix

Leg rate, specified as a NINST-by-2 matrix, with each row defined as one of the following:

- [CouponRate Spread] (fixed-float)
- [Spread CouponRate] (float-fixed)
- [CouponRate CouponRate] (fixed-fixed)
- [Spread Spread] (float-float)

CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

Data Types: `double`

### **Settle** — Settlement date

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The `Settle` date for every swap is set to the `ValuationDate` of the HJM tree. The swap argument `Settle` is ignored.

Data Types: `char` | `double`

### **Maturity** — Maturity date

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each swap.

Data Types: char | double

## Name-Value Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)

### LegReset — Reset frequency per year for each swap

[1 1] (default) | vector

Reset frequency per year for each swap, specified as the comma-separated pair consisting of 'LegReset' and a NINST-by-2 vector.

Data Types: double

### Basis — Day-count basis representing the basis for each leg

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis for each leg, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 array (or NINST-by-2 if Basis is different for each leg).

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)

- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts or principal value schedules, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array (or NINST-by-2 if Principal is different for each leg) of the notional principal amounts or principal value schedules. For schedules, each element of the cell array is a NumDates-by-2 array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **LegType — Leg type**

[1 0] for each instrument (default) | matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float)

Leg type, specified as the comma-separated pair consisting of 'LegType' and a NINST-by-2 matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float). Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate. LegType allows [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float) swaps

Data Types: double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: `struct`

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when `Maturity` is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 (or NINST-by-2 if EndMonthRule is different for each leg).

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: `logical`

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 (or NINST-by-2 if AdjustCashFlowsBasis is different for each leg) of logicals with values of 0 (false) or 1 (true).

Data Types: `logical`

**BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:



- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

### **StartDate — Date swap actually starts**

Settle date (default) | serial date number | character vector

Date swap actually starts, specified as the comma-separated pair consisting of 'StartDate' and a `NINST-by-1` vector of dates using a serial date number or a character vector.

Use this argument to price forward swaps, that is, swaps that start in a future date

Data Types: char | double

## **Output Arguments**

### **Price — Expected swap prices at time 0**

vector

Expected swap prices at time 0, returned as a `NINST-by-1` vector.

**PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within PriceTree:

- PriceTree.tObs contains the observation times.
- PriceTree.PBush contains the clean prices.

**CFTree — Swap cash flows**

structure

Swap cash flows, returned as a tree structure with a vector of the swap cash flows at each node. This structure contains only NaNs because with binomial recombining trees, cash flows cannot be computed accurately at each node of a tree.

**SwapRate — Rates applicable to fixed leg**

matrix

Rates applicable to the fixed leg, returned as a NINST-by-1 vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. The SwapRate output is padded with NaN for those instruments in which CouponRate is not set to NaN.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## **See Also**

capbyhjm | cfbyhjm | floorbyhjm | hjmtree | treeviewer

## **Topics**

“Computing Instrument Prices” on page 2-99

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## swapbyhw

Price swap instrument from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree,CFTree,SwapRate] = swapbyhw(HWTtree,LegRate,Settle,
Maturity)
[Price,PriceTree,CFTree,SwapRate] = swapbyhw( ____,Name,Value)
```

### Description

[Price,PriceTree,CFTree,SwapRate] = swapbyhw(HWTtree,LegRate,Settle, Maturity) prices a swap instrument from a Hull-White interest-rate tree. swapbyhw computes prices of vanilla swaps, amortizing swaps and forward swaps.

[Price,PriceTree,CFTree,SwapRate] = swapbyhw( \_\_\_\_,Name,Value) adds additional name-value pair arguments.

### Examples

#### Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \$100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2005
- Swap maturity date: Jan. 01, 2008

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:

```

Settle = '01-Jan-2005';
Maturity = '01-Jan-2008';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year

```

Price the swap using the HWTtree included in the MAT-file `deriv.mat`. The HWTtree structure contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use `swapbyhw` to compute the price of the swap.

```
[Price, PriceTree, SwapRate] = swapbyhw(HWTtree, LegRate, ...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

Warning: Swaps are valued at Tree ValuationDate rather than Settle

```
Price = 5.9109
```

```
PriceTree = struct with fields:
```

```

  FinObj: 'HWPriceTree'
  PTree: {1x5 cell}
  tObs: [0 1 2 3 4]
  Connect: {[2] [2 3 4] [2 2 3 4 4]}
  Probs: {[3x1 double] [3x3 double] [3x5 double]}

```

```
SwapRate = NaN
```

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];
```

```
[Price, PriceTree, SwapRate] = swapbyhw(HWTtree, LegRate, ...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

Warning: Swaps are valued at Tree ValuationDate rather than Settle

```
Price = 1.4211e-14
```

```
PriceTree = struct with fields:
```

```
  FinObj: 'HWPriceTree'
```

```
PTree: {1x5 cell}
tObs: [0 1 2 3 4]
Connect: {[2] [2 3 4] [2 2 3 4 4]}
Probs: {[3x1 double] [3x3 double] [3x5 double]}
```

```
SwapRate = 0.0438
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: 0.8135
    Rates: 0.0350
    EndTimes: 6
    StartTimes: 0
    EndDates: 736696
    StartDates: 734504
    ValuationDate: 734504
    Basis: 0
    EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle = '1-Jan-2011';
Maturity = '1-Jan-2017';
```

```
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal = {'1-Jan-2013' 100; '1-Jan-2014' 80; '1-Jan-2015' 60; '1-Jan-2016' 40; '1-Jan-2017' 20};
```

Build the HW tree using the following data:

```
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'];
VolCurve = 0.1;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;
```

```
HWVolSpec = hwwolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbyhw(HWT, LegRate, Settle, Maturity, 'Principal', Principal)

Price = 1.4574
```

## Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
```

```
Compounding: 1
  Disc: 0.8023
  Rates: 0.0374
  EndTimes: 6
  StartTimes: 0
  EndDates: 737061
  StartDates: 734869
ValuationDate: 734869
  Basis: 0
EndMonthRule: 1
```

Build an HW tree.

```
VolDates = {'1-Jan-2013'; '1-Jan-2014'; '1-Jan-2015'; '1-Jan-2016'; '1-Jan-2017'; '1-Jan-2018'};
VolCurve = 0.1;
AlphaDates = '01-01-2018';
AlphaCurve = 0.1;
```

```
HWVolSpec = hwwolspec(RateSpec.ValuationDate, VolDates, VolCurve, ...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of 4.25%.

```
Settle = '1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];
```

```
Price = swapbyhw(HWT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price = 1.4434
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
[Price, ~, SwapRate] = swapbyhw(HWT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price = 0
```

```
SwapRate = 0.0384
```



## Input Arguments

### **HWTree — Interest-rate structure**

structure

Interest-rate tree structure, created by hwtree

Data Types: struct

### **LegRate — Leg rate**

matrix

Leg rate, specified as a NINST-by-2 matrix, with each row defined as one of the following:

- [CouponRate Spread] (fixed-float)
- [Spread CouponRate] (float-fixed)
- [CouponRate CouponRate] (fixed-fixed)
- [Spread Spread] (float-float)

CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

### **Settle — Settlement date**

serial date number | character vector

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors.

The Settle date for every swap is set to the ValuationDate of the HW tree. The swap argument Settle is ignored.

Data Types: char | double

### **Maturity — Maturity date**

serial date number | character vector

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each swap.

Data Types: char | double

## **Name-Value Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree, CFTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)`

### **LegReset — Reset frequency per year for each swap**

`[1 1]` (default) | vector

Reset frequency per year for each swap, specified as the comma-separated pair consisting of 'LegReset' and a NINST-by-2 vector.

Data Types: double

### **Basis — Day-count basis representing the basis for each leg**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis for each leg, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 array (or NINST-by-2 if `Basis` is different for each leg).

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)

- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts or principal value schedules, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array (or NINST-by-2 if Principal is different for each leg) of the notional principal amounts or principal value schedules. For schedules, each element of the cell array is a NumDates-by-2 array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

### **LegType — Leg type**

[1 0] for each instrument (default) | matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float)

Leg type, specified as the comma-separated pair consisting of 'LegType' and a NINST-by-2 matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float). Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate. LegType allows [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float) swaps

Data Types: double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using derivset.

Data Types: struct

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 (or NINST-by-2 if EndMonthRule is different for each leg).

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 (or NINST-by-2 if AdjustCashFlowsBasis is different for each leg) of logicals with values of 0 (false) or 1 (true).

Data Types: logical

**BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- `actual` — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- `follow` — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.

- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use `holidays.m` (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a `NHolidays-by-1` vector.

Data Types: double

### **StartDate — Date swap actually starts**

Settle date (default) | serial date number | character vector

Date swap actually starts, specified as the comma-separated pair consisting of 'StartDate' and a `NINST-by-1` vector of dates using a serial date number or a character vector.

Use this argument to price forward swaps, that is, swaps that start in a future date

Data Types: char | double

## **Output Arguments**

### **Price — Expected swap prices at time 0**

vector

Expected swap prices at time 0, returned as a `NINST-by-1` vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

### **CFTree — Swap cash flows**

structure

Swap cash flows, returned as a tree structure with a vector of the swap cash flows at each node. This structure contains only NaNs because with binomial recombining trees, cash flows cannot be computed accurately at each node of a tree.

### **SwapRate — Rates applicable to fixed leg**

matrix

Rates applicable to the fixed leg, returned as a `NINST-by-1` vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is NaN. The `SwapRate` output is padded with NaN for those instruments in which `CouponRate` is not set to NaN.

## **Definitions**

### **Amortizing Swap**

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

## **Forward Swap**

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## **See Also**

bondbyhw | capbyhw | cfbyhw | fixedbyhw | floorbyhw | hwtree

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Calibrating Hull-White Model Using Market Data” on page 2-111

“Understanding Interest-Rate Tree Models” on page 2-79

“Pricing Options Structure” on page B-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## swapbyzero

Price swap instrument from set of zero curves and price cross-currency swaps

### Syntax

```
[Price,SwapRate,AI,RecCF,RecCFDates,PayCF,PayCFDates] = swapbyzero(
RateSpec,LegRate,Settle,Maturity)
[Price,SwapRate,AI,RecCF,RecCFDates,PayCF,PayCFDates] = swapbyzero(
RateSpec,LegRate,Settle,Maturity,Name,Value)
```

### Description

[Price,SwapRate,AI,RecCF,RecCFDates,PayCF,PayCFDates] = swapbyzero(RateSpec,LegRate,Settle,Maturity) prices a swap instrument. You can use swapbyzero to compute prices of vanilla swaps, amortizing swaps, and forward swaps. All inputs are either scalars or NINST-by-1 vectors unless otherwise specified. Any date can be a serial date number or date character vector. An optional argument can be passed as an empty matrix [ ].

[Price,SwapRate,AI,RecCF,RecCFDates,PayCF,PayCFDates] = swapbyzero(RateSpec,LegRate,Settle,Maturity,Name,Value) prices a swap instrument with additional options specified by one or more Name,Value pair arguments. You can use swapbyzero to compute prices of vanilla swaps, amortizing swaps, forward swaps, and cross-currency swaps. For more information on the name-value pairs for vanilla swaps, amortizing swaps, and forward swaps, see [Vanilla Swaps, Amortizing Swaps, Forward Swaps](#) on page 11-0 .

Specifically, you can use name-value pairs for `FXRate`, `ExchangeInitialPrincipal`, and `ExchangeMaturityPrincipal` to compute the price for cross-currency swaps. For more information on the name-value pairs for cross-currency swaps, see [Cross-Currency Swaps](#) on page 11-0 .

### Examples



## Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is \$100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the LegRate, LegType, and LegReset matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Load the file `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure needed to price the bond.

```
load deriv.mat;
```

Use `swapbyzero` to compute the price of the swap.

```
Price = swapbyzero(ZeroRateSpec, LegRate, Settle, Maturity, ...
LegReset, Basis, Principal, LegType)
```

```
Price = 3.6923
```

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];
```

```
[Price, SwapRate] = swapbyzero(ZeroRateSpec, LegRate, Settle, ...
Maturity, LegReset, Basis, Principal, LegType)
```

```
Price = 0
```

```
SwapRate = 0.0466
```

In `swapbyzero`, if `Settle` is not on a reset date (and `'StartDate'` is not specified), the effective date is assumed to be the previous reset date before `Settle` in order to compute the accrued interest and dirty price. In this example, the effective date is ( `'15-Sep-2009'` ), which is the previous reset date before the ( `'08-Jun-2010'` ) `Settle` date.

Use `swapbyzero` with name-value pair arguments for `LegRate`, `LegType`, `LatestFloatingRate`, `AdjustCashFlowsBasis`, and `BusinessDayConvention` to calculate output for `Price`, `SwapRate`, `AI`, `RecCF`, `RecCFDates`, `PayCF`, and `PayCFDates`:

```
Settle = datenum('08-Jun-2010');
RateSpec = intenvset('Rates', [.005 .0075 .01 .014 .02 .025 .03]',...
'StartDates',Settle, 'EndDates',{'08-Dec-2010','08-Jun-2011',...
'08-Jun-2012','08-Jun-2013','08-Jun-2015','08-Jun-2017','08-Jun-2020'});
Maturity = datenum('15-Sep-2020');
LegRate = [.025 50];
LegType = [1 0]; % fixed/floating
LatestFloatingRate = .005;
```

```
[Price, SwapRate, AI, RecCF, RecCFDates, PayCF,PayCFDates] = ...
swapbyzero(RateSpec, LegRate, Settle, Maturity,'LegType',LegType,...
'LatestFloatingRate',LatestFloatingRate,'AdjustCashFlowsBasis',true,...
'BusinessDayConvention','modifiedfollow')
```

```
Price = -6.7259
```

```
SwapRate = NaN
```

```
AI = 1.4575
```

```
RecCF = 1×12
```

```
    -1.8219    2.5000    2.5000    2.5137    2.4932    2.4932    2.5000    2.5000    2.5
```

```
RecCFDates = 1×12
```

```
    734297    734396    734761    735129    735493    735857    736222
```

```
PayCF = 1×12
```

```
-0.3644    0.5000    1.4048    1.9961    2.8379    3.2760    3.8218    4.1733    4.5
```

```
PayCFDates = 1x12
```

```
734297    734396    734761    735129    735493    735857    736222
```

### Price Swaps By Specifying Multiple Term Structures Using RateSpec

Price three swaps using two interest-rate curves. First, define the data for the interest-rate term structure:

```
StartDates = '01-May-2012';
EndDates = {'01-May-2013'; '01-May-2014'; '01-May-2015'; '01-May-2016'};
Rates = [[0.0356;0.041185;0.04489;0.047741],[0.0366;0.04218;0.04589;0.04974]];
```

Create the RateSpec using intenvset.

```
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Compounding', 1)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [4x2 double]
    Rates: [4x2 double]
    EndTimes: [4x1 double]
    StartTimes: [4x1 double]
    EndDates: [4x1 double]
    StartDates: 734990
    ValuationDate: 734990
    Basis: 0
    EndMonthRule: 1
```

Look at the Rates for the two interest-rate curves.

```
RateSpec.Rates
```

```
ans = 4x2
```

```
0.0356    0.0366
```

```
0.0412    0.0422
0.0449    0.0459
0.0477    0.0497
```

Define the swap instruments.

```
Settle = '01-May-2012';
Maturity = '01-May-2015';
LegRate = [0.06 10];
Principal = [100;50;100]; % Three notional amounts
```

Price three swaps using two curves.

```
Price = swapybyzero(RateSpec, LegRate, Settle, Maturity, 'Principal', Principal)
```

```
Price = 3x2
```

```
3.9688    3.6869
1.9844    1.8434
3.9688    3.6869
```

### Price Swap By Specifying Multiple Term Structures Using a 1-by-2 RateSpec

Price a swap using two interest-rate curves. First, define data for the two interest-rate term structures:

```
StartDates = '01-May-2012';
EndDates = {'01-May-2013'; '01-May-2014'; '01-May-2015'; '01-May-2016'};
Rates1 = [0.0356;0.041185;0.04489;0.047741];
Rates2 = [0.0366;0.04218;0.04589;0.04974];
```

Create the RateSpec using `intenvset`.

```
RateSpecReceiving = intenvset('Rates', Rates1, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Compounding', 1);
RateSpecPaying = intenvset('Rates', Rates2, 'StartDates', StartDates, ...
    'EndDates', EndDates, 'Compounding', 1);
RateSpec = [RateSpecReceiving RateSpecPaying]
```

```
RateSpec = 1x2 struct array with fields:
    FinObj
```

```

Compounding
Disc
Rates
EndTimes
StartTimes
EndDates
StartDates
ValuationDate
Basis
EndMonthRule

```

Define the swap instruments.

```

Settle = '01-May-2012';
Maturity = '01-May-2015';
LegRate = [0.06 10];
Principal = [100;50;100];

```

Price three swaps using the two curves.

```

Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'Principal', Principal)

Price = 3x1

    3.9693
    1.9846
    3.9693

```

### Compute a Forward Par Swap Rate

To compute a forward par swap rate, set the `StartDate` parameter to a future date and set the fixed coupon rate in the `LegRate` input to `NaN`.

Define the zero curve data and build a zero curve using `IRDataCurve`.

```

ZeroRates = [2.09 2.47 2.71 3.12 3.43 3.85 4.57]'/100;
Settle = datenum('1-Jan-2012');
EndDates = datemnth(Settle,12*[1 2 3 5 7 10 20]');
Compounding = 1;

```

```

ZeroCurve = IRDataCurve('Zero',Settle,EndDates,ZeroRates,'Compounding',Compounding)

```

```
ZeroCurve =  
    Type: Zero  
    Settle: 734869 (01-Jan-2012)  
    Compounding: 1  
    Basis: 0 (actual/actual)  
    InterpMethod: linear  
    Dates: [7x1 double]  
    Data: [7x1 double]
```

Create a `RateSpec` structure using the `toRateSpec` method.

```
RateSpec = ZeroCurve.toRateSpec(EndDates)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
    Disc: [7x1 double]  
    Rates: [7x1 double]  
    EndTimes: [7x1 double]  
    StartTimes: [7x1 double]  
    EndDates: [7x1 double]  
    StartDates: 734869  
    ValuationDate: 734869  
    Basis: 0  
    EndMonthRule: 1
```

Compute the forward swap rate (the coupon rate for the fixed leg), such that the forward swap price at time = 0 is zero. The forward swap starts in a month (1-Feb-2012) and matures in 10 years (1-Feb-2022).

```
StartDate = datenum('1-Feb-2012');  
Maturity = datenum('1-Feb-2022');  
LegRate = [NaN 0];
```

```
[Price, SwapRate] = swapbyzero(RateSpec, LegRate, Settle, Maturity, ...  
    'StartDate', StartDate)
```

```
Price = 0
```

```
SwapRate = 0.0378
```

## Compute a Forward Swap Rate Using the Optional Input BusinessDayConvention

The `swapbyzero` function generates the cash flow dates based on the `Settle` and `Maturity` dates, while using the `Maturity` date as the "anchor" date from which to count backwards in regular intervals. By default, `swapbyzero` does not distinguish non-business days from business days. To make `swapbyzero` move non-business days to the following business days, you can set the optional name-value input argument `BusinessDayConvention` with a value of `follow`.

Define the zero curve data and build a zero curve using `IRDataCurve`.

```
ZeroRates = [2.09 2.47 2.71 3.12 3.43 3.85 4.57]'/100;
Settle = datenum('5-Jan-2012');
EndDates = datemnth(Settle,12*[1 2 3 5 7 10 20]');
Compounding = 1;
ZeroCurve = IRDataCurve('Zero',Settle,EndDates,ZeroRates,'Compounding',Compounding);
RateSpec = ZeroCurve.toRateSpec(EndDates);
StartDate = datenum('5-Feb-2012');
Maturity = datenum('5-Feb-2022');
LegRate = [NaN 0];
```

To demonstrate the optional input `BusinessDayConvention`, `swapbyzero` is first used without and then with the optional name-value input argument `BusinessDayConvention`. Notice that when using `BusinessDayConvention`, all days are business days.

```
[Price1,SwapRate1,~,~,RecCFDates1,~,PayCFDates1] = swapbyzero(RateSpec,LegRate,Settle,
    'StartDate',StartDate);
datestr(RecCFDates1)
```

```
ans = 11x11 char array
    '05-Jan-2012'
    '05-Feb-2013'
    '05-Feb-2014'
    '05-Feb-2015'
    '05-Feb-2016'
    '05-Feb-2017'
    '05-Feb-2018'
    '05-Feb-2019'
    '05-Feb-2020'
    '05-Feb-2021'
    '05-Feb-2022'
```

```
isbusday(RecCFDates1)
```

```
ans = 11x1 logical array
```

```
1
1
1
1
1
1
0
1
1
1
1
:
```

```
[Price2,SwapRate2,~,~,RecCFDates2,~,PayCFDates2] = swapbyzero(RateSpec,LegRate,Settle,
    'StartDate',StartDate,'BusinessDayConvention','follow');
datestr(RecCFDates2)
```

```
ans = 12x11 char array
```

```
'05-Jan-2012'
'06-Feb-2012'
'05-Feb-2013'
'05-Feb-2014'
'05-Feb-2015'
'05-Feb-2016'
'06-Feb-2017'
'05-Feb-2018'
'05-Feb-2019'
'05-Feb-2020'
'05-Feb-2021'
'07-Feb-2022'
```

```
isbusday(RecCFDates2)
```

```
ans = 12x1 logical array
```

```
1
1
1
1
1
1
1
1
```



```

1
1
1
1
:

```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```

Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;

```

```

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

```

Create the swap instrument using the following data:

```

Settle = '1-Jan-2011';
Maturity = '1-Jan-2017';
LegRate = [0.04 10];

```

Define the swap amortizing schedule.

```

Principal = {'1-Jan-2013' 100; '1-Jan-2014' 80; '1-Jan-2015' 60; '1-Jan-2016' 40; '1-Jan-2017' 0};

```

Compute the price of the amortizing swap.

```

Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'Principal', Principal)

```

```

Price = 1.4574

```

### Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0325;  
ValuationDate = '1-Jan-2012';  
StartDates = ValuationDate;  
EndDates = '1-Jan-2018';  
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, ...  
    'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 1  
        Disc: 0.8254  
        Rates: 0.0325  
    EndTimes: 6  
    StartTimes: 0  
    EndDates: 737061  
    StartDates: 734869  
    ValuationDate: 734869  
        Basis: 0  
    EndMonthRule: 1
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in three years with a forward swap rate of 4.27%.

```
Settle = '1-Jan-2012';  
StartDate = '1-Jan-2013';  
Maturity = '1-Jan-2016';  
LegRate = [0.0427 10];
```

```
Price = swapybyzero(RateSpec, LegRate, Settle, Maturity, 'StartDate' , StartDate)
```

```
Price = 2.5083
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```

LegRate = [NaN 10];
[Price, SwapRate] = swapbyzero(RateSpec, LegRate, Settle, Maturity, ...
'StartDate' , StartDate)

Price = 0

SwapRate = 0.0335

```

### Specify the Rate at the Instrument's Starting Date When It Cannot Be Obtained from the RateSpec

If `Settle` is not on a reset date of a floating-rate note, `swapbyzero` attempts to obtain the latest floating rate before `Settle` from `RateSpec` or the `LatestFloatingRate` parameter. When the reset date for this rate is out of the range of `RateSpec` (and `LatestFloatingRate` is not specified), `swapbyzero` fails to obtain the rate for that date and generates an error. This example shows how to use the `LatestFloatingRate` input parameter to avoid the error.

Create the error condition when a swap instrument's `StartDate` cannot be determined from the `RateSpec`.

```

Settle = '01-Jan-2000';
Maturity = '01-Dec-2003';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year

load deriv.mat;

Price = swapbyzero(ZeroRateSpec,LegRate,Settle,Maturity,...
'LegReset',LegReset,'Basis',Basis,'Principal',Principal, ...
'LegType',LegType)

Error using floatbyzero (line 256)
The rate at the instrument starting date cannot be obtained from RateSpec.
Its reset date (01-Dec-1999) is out of the range of dates contained in RateSpec.
This rate is required to calculate cash flows at the instrument starting date.
Consider specifying this rate with the 'LatestFloatingRate' input parameter.

Error in swapbyzero (line 289)
[FloatFullPrice, FloatPrice,FloatCF,FloatCFDates] = floatbyzero(FloatRateSpec, Spreads, Settle,...

```

Here, the reset date for the rate at `Settle` was 01-Dec-1999, which was earlier than the valuation date of `ZeroRateSpec` (01-Jan-2000). This error can be avoided by

specifying the rate at the swap instrument's starting date using the LatestFloatingRate input parameter.

Define LatestFloatingRate and calculate the floating-rate price.

```
Price = swapbyzero(ZeroRateSpec,LegRate,Settle,Maturity,...  
'LegReset',LegReset,'Basis',Basis,'Principal',Principal,...  
'LegType',LegType,'LatestFloatingRate',0.03)
```

```
Price =
```

```
4.7594
```

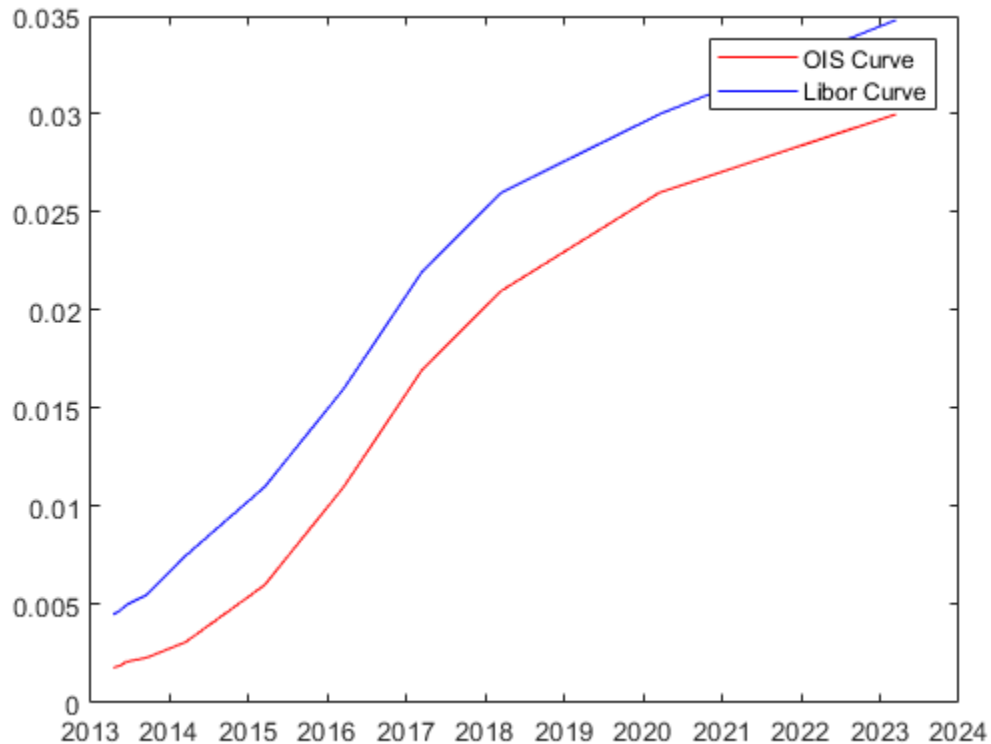
### **Price a Swap Using a Different Curve to Generate the Cash Flows of the Floating Leg**

Define the OIS and Libor rates.

```
Settle = datenum('15-Mar-2013');  
CurveDates = daysadd(Settle,360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10],1);  
OISRates = [.0018 .0019 .0021 .0023 .0031 .006 .011 .017 .021 .026 .03]';  
LiborRates = [.0045 .0047 .005 .0055 .0075 .011 .016 .022 .026 .030 .0348]';
```

Plot the dual curves.

```
figure,plot(CurveDates,OISRates,'r');hold on;plot(CurveDates,LiborRates,'b')  
datetick  
legend({'OIS Curve', 'Libor Curve'})
```



Create an associated RateSpec for the OIS and Libor curves.

```
OISCurve = intenvset('Rates',OISRates,'StartDate',Settle,'EndDates',CurveDates);
LiborCurve = intenvset('Rates',LiborRates,'StartDate',Settle,'EndDates',CurveDates);
```

Define the swap.

```
Maturity = datenum('15-Mar-2018'); % Five year swap
FloatSpread = 0;
FixedRate = .025;
LegRate = [FixedRate FloatSpread];
```

Compute the price of the swap instrument. The `LiborCurve` term structure will be used to generate the cash flows of the floating leg. The `OISCurve` term structure will be used for discounting the cash flows.

```
Price = swapybyzero(OISCurve, LegRate, Settle,...
Maturity, 'ProjectionCurve', LiborCurve)
```

```
Price = -0.3697
```

Compare results when the term structure `OISCurve` is used both for discounting and also generating the cash flows of the floating leg.

```
PriceSwap = swapybyzero(OISCurve, LegRate, Settle, Maturity)
```

```
PriceSwap = 2.0517
```

### Price a Fixed-Fixed Currency Swap

Price an existing cross currency swap that receives a fixed rate of JPY and pays a fixed rate of USD at an annual frequency.

```
Settle = datenum('15-Aug-2015');
Maturity = datenum('15-Aug-2018');
Reset = 1;
LegType = [1 1]; % Fixed-Fixed
```

```
r_USD = .09;
r_JPY = .04;
```

```
FixedRate_USD = .08;
FixedRate_JPY = .05;
```

```
Principal_USD = 10000000;
Principal_JPY = 1200000000;
```

```
S = 1/110;
```

```
RateSpec_USD = intenvset('StartDate',Settle,'EndDate', Maturity,'Rates',r_USD,'Compound
RateSpec_JPY = intenvset('StartDate',Settle,'EndDate', Maturity,'Rates', r_JPY,'Compound
```

```
Price = swapybyzero([RateSpec_JPY RateSpec_USD], [FixedRate_JPY FixedRate_USD],...
Settle, Maturity, 'Principal',[Principal_JPY Principal_USD], 'FXRate',[S 1], 'LegType',L
```

```
Price = 1.5430e+06
```

## Price a Float-Float Currency Swap

Price a new swap where you pay a EUR float and receive a USD float.

```
Settle = datenum('22-Dec-2015');
Maturity = datenum('15-Aug-2018');
LegRate = [0 -50/10000];
LegType = [0 0]; % Float Float
LegReset = [4 4];
FXRate = 1.1;
Notional = [10000000 8000000];

USD_Dates = datemnth(Settle,[1 3 6 12*[1 2 3 5 7 10 20 30]]');
USD_Zero = [0.03 0.06 0.08 0.13 0.36 0.76 1.63 2.29 2.88 3.64 3.89]'/100;
Curve_USD = intenvset('StartDate',Settle,'EndDates',USD_Dates,'Rates',USD_Zero);

EUR_Dates = datemnth(Settle,[3 6 12*[1 2 3 5 7 10 20 30]]');
EUR_Zero = [0.017 0.033 0.088 .27 .512 1.056 1.573 2.183 2.898 2.797]'/100;
Curve_EUR = intenvset('StartDate',Settle,'EndDates',EUR_Dates,'Rates',EUR_Zero);

Price = swapbyzero([Curve_USD Curve_EUR], ...
    LegRate, Settle, Maturity, 'LegType', LegType, 'LegReset', LegReset, 'Principal', Notional,
    'FXRate', [1 FXRate], 'ExchangeInitialPrincipal', false)

Price = 1.2002e+06
```

## Input Arguments

### RateSpec — Interest-rate structure

structure

Interest-rate structure, specified using `intenvset` to create a `RateSpec`.

`RateSpec` can also be a 1-by-2 input variable of `RateSpecs`, with the second `RateSpec` structure containing one or more discount curves for the paying leg. If only one `RateSpec` structure is specified, then this `RateSpec` is used to discount both legs.

Data Types: `struct`

**LegRate — Leg rate**

matrix

Leg rate, specified as a NINST-by-2 matrix, with each row defined as one of the following:

- [CouponRate Spread] (fixed-float)
- [Spread CouponRate] (float-fixed)
- [CouponRate CouponRate] (fixed-fixed)
- [Spread Spread] (float-float)

CouponRate is the decimal annual rate. Spread is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

**Settle — Settlement date**

serial date number | character vector | cell array of character vectors

Settlement date, specified either as a scalar or NINST-by-1 vector of serial date numbers or date character vectors of the same value which represent the settlement date for each swap. Settle must be earlier than Maturity.

Data Types: char | cell | double

**Maturity — Maturity date**

serial date number | character vector | cell array of character vectors

Maturity date, specified as a NINST-by-1 vector of serial date numbers or date character vectors representing the maturity date for each swap.

Data Types: char | cell | double

**Name-Value Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Price, SwapRate, AI, RecCF, RecCFDates, PayCF, PayCFDates] = swapbyzero(RateSpec, LegRate, Settle,



```
Maturity, 'LegType', LegType, 'LatestFloatingRate', LatestFloatingRate, '
AdjustCashFlowsBasis', true,
'BusinessDayConvention', 'modifiedfollow')
```

### Vanilla Swaps, Amortizing Swaps, Forward Swaps

#### LegReset — Reset frequency per year for each swap

[1 1] (default) | vector

Reset frequency per year for each swap, specified as the comma-separated pair consisting of 'LegReset' and a NINST-by-2 vector.

Data Types: double

#### Basis — Day-count basis of instrument

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis for each leg, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 array (or NINST-by-2 if Basis is different for each leg).

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**Principal — Notional principal amounts or principal value schedules**

100 (default) | vector or cell array

Notional principal amounts or principal value schedules, specified as the comma-separated pair consisting of 'Principal' and a vector or cell array.

Principal accepts a NINST-by-1 vector or NINST-by-1 cell array (or NINST-by-2 if Principal is different for each leg) of the notional principal amounts or principal value schedules. For schedules, each element of the cell array is a NumDates-by-2 array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

Data Types: cell | double

**LegType — Leg type**

[1 0] for each instrument (default) | matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float)

Leg type, specified as the comma-separated pair consisting of 'LegType' and a NINST-by-2 matrix with values [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float). Each row represents an instrument. Each column indicates if the corresponding leg is fixed (1) or floating (0). This matrix defines the interpretation of the values entered in LegRate. LegType allows [1 1] (fixed-fixed), [1 0] (fixed-float), [0 1] (float-fixed), or [0 0] (float-float) swaps

Data Types: double

**EndMonthRule — End-of-month rule flag for generating dates when Maturity is end-of-month date for month having 30 or fewer days**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag for generating dates when Maturity is an end-of-month date for a month having 30 or fewer days, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a NINST-by-1 (or NINST-by-2 if EndMonthRule is different for each leg).

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **AdjustCashFlowsBasis — Flag to adjust cash flows based on actual period day count**

false (default) | value of 0 (false) or 1 (true)

Flag to adjust cash flows based on actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a NINST-by-1 (or NINST-by-2 if AdjustCashFlowsBasis is different for each leg) of logicals with values of 0 (false) or 1 (true).

Data Types: logical

### **BusinessDayConvention — Business day conventions**

actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a N-by-1 (or NINST-by-2 if BusinessDayConvention is different for each leg) cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

### **Holidays — Holidays used in computing business days**

if not specified, the default is to use holidays.m (default) | MATLAB date numbers

Holidays used in computing business days, specified as the comma-separated pair consisting of 'Holidays' and MATLAB date numbers using a NHolidays-by-1 vector.

Data Types: double

**StartDate — Dates when swaps actually start**

If not specified, date is `Settle` (default) | serial date number | character vector | cell array of character vectors

Dates when the swaps actually start, specified as the comma-separated pair consisting of 'StartDate' and a NINST-by-1 vector of serial date numbers, character vectors, or cell array of character vectors.

Data Types: char | cell | double

**LatestFloatingRate — Rate for the next floating payment**

If not specified, then `RateSpec` must contain this information (default) | scalar numeric

Rate for the next floating payment, set at the last reset date, specified as the comma-separated pair consisting of 'LatestFloatingRate' and a scalar numeric value.

`LatestFloatingRate` accepts a Rate for the next floating payment, set at the last reset date. `LatestFloatingRate` is a NINST-by-1 (or NINST-by-2 if `LatestFloatingRate` is different for each leg).

Data Types: double

**ProjectionCurve — Rate curve used in generating cash flows for the floating leg of the swap**

if `ProjectionCurve` is not specified, then `RateSpec` is used both for discounting and generating cash flows for the floating leg (default) | `RateSpec` or vector

Rate curve used in generating cash flows for the floating leg of the swap, specified as the comma-separated pair consisting of 'ProjectionCurve' and a `RateSpec`.

If specifying a fixed-float or a float-fixed swap, the `ProjectionCurve` rate curve is used in generating cash flows for the floating leg of the swap. This structure must be created using `intenvset`.

If specifying a fixed-fixed or a float-float swap, then `ProjectionCurve` is NINST-by-2 vector because each floating leg could have a different projection curve.

Data Types: struct

## Cross-Currency Swaps

### **FXRate — Foreign exchange (FX) rate applied to cash flows**

if not specified, both legs of swapbyzero are in same currency (default) | array

Foreign exchange (FX) rate applied to cash flows, specified as the comma-separated pair consisting of 'FXRate' and a NINST-by-2 array of doubles. Since the foreign exchange rate could be applied to either the payer or receiver leg, there are 2 columns in the input array and you must specify which leg has the foreign currency.

Data Types: double

### **ExchangeInitialPrincipal — Flag to indicate if initial Principal is exchanged**

0 (false) (default) | array

Flag to indicate if initial Principal is exchanged, specified as the comma-separated pair consisting of 'ExchangeInitialPrincipal' and a NINST-by-1 array of logicals.

Data Types: logical

### **ExchangeMaturityPrincipal — Flag to indicate if Principal exchanged at Maturity**

1 (true) (default) | array

Flag to indicate if Principal is exchanged at Maturity, specified as the comma-separated pair consisting of 'ExchangeMaturityPrincipal' and a NINST-by-1 array of logicals. While in practice most single currency swaps do not exchange principal at maturity, the default is true to maintain backward compatibility.

Data Types: logical

## Output Arguments

### **Price — Swap prices**

matrix

Swap prices, returned as the number of instruments (NINST) by number of curves (NUMCURVES) matrix. Each column arises from one of the zero curves. Price output is the dirty price. To compute the clean price, subtract the accrued interest (AI) from the dirty price.

**SwapRate — Rates applicable to fixed leg**

matrix

Rates applicable to the fixed leg, returned as a NINST-by-NUMCURVES matrix of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in LegRate is NaN. The SwapRate output is padded with NaN for those instruments in which CouponRate is not set to NaN.

**AI — Accrued interest**

matrix

Accrued interest, returned as a NINST-by-NUMCURVES matrix.

**RecCF — Cash flows for receiving leg**

matrix

Cash flows for the receiving leg, returned as a NINST-by-NUMCURVES matrix.

---

**Note** If there is more than one curve specified in the RateSpec input, then the first NCURVES row corresponds to the first swap, the second NCURVES row correspond to the second swap, and so on.

---

**RecCFDates — Payment dates for receiving leg**

matrix

Payment dates for the receiving leg, returned as an NINST-by-NUMCURVES matrix.

**PayCF — Cash flows for paying leg**

matrix

Cash flows for the paying leg, returned as an NINST-by-NUMCURVES matrix.

**PayCFDates — Payment dates for paying leg**

matrix

Payment dates for the paying leg, returned as an NINST-by-NUMCURVES matrix.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

### Cross-currency Swap

Swaps where the payment legs of the swap are denominated in different currencies.

One difference between cross-currency swaps and standard swaps is that an exchange of principal may occur at the beginning and/or end of the swap. The exchange of initial principal will only come into play in pricing a cross-currency swap at inception (in other words, pricing an existing cross-currency swap will occur after this cash flow has happened). Furthermore, these exchanges of principal typically do not affect the value of the swap (since the principal values of the two legs are chosen based on the currency exchange rate) but affect the cash flows for each leg.

## References

[1] Hull, J. *Options, Futures and Other Derivatives* Fourth Edition. Prentice Hall, 2000.

## See Also

[bondbyzero](#) | [cfbyzero](#) | [fixedbyzero](#) | [floatbyzero](#) | [intenvset](#)

## Topics

[“Pricing Using Interest-Rate Term Structure”](#) on page 2-72

[“Understanding the Interest-Rate Term Structure”](#) on page 2-55

[“Supported Interest-Rate Instruments”](#) on page 2-2

**Introduced before R2006a**



# swaptionbybdt

Price swaption from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price,PriceTree] = swaptionbybdt(BDTree,OptSpec,Strike,
ExerciseDates,Spread,Settle,Maturity)
[Price,PriceTree] = swaptionbybdt( ____,Name,Value)
```

## Description

[Price,PriceTree] = swaptionbybdt(BDTree,OptSpec,Strike,ExerciseDates,Spread,Settle,Maturity) prices swaption using a Black-Derman-Toy tree.

[Price,PriceTree] = swaptionbybdt( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a 5-Year Call Swaption Using a BDT Interest-Rate Tree

This example shows how to price a 5-year call swaption using a BDT interest-rate tree. Assume that interest rate and volatility are fixed at 6% and 20% annually between the valuation date of the tree until its maturity. Build a tree with the following data.

```
Rates = 0.06 * ones (10,1);
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011';...
'jan-1-2012';'jan-1-2013';'jan-1-2014';'jan-1-2015';'jan-1-2016'];

EndDates =['jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011';'jan-1-2012';...
'jan-1-2013';'jan-1-2014';'jan-1-2015';'jan-1-2016';'jan-1-2017'];
ValuationDate = 'jan-1-2007';
Compounding = 1;
```

```
% define the RateSpec
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', EndDates, .
'Compounding', Compounding);

% use VolSpec to compute interest-rate volatility
Volatility = 0.20 * ones (10,1); VolSpec = bdtvolspec(ValuationDate,...
EndDates, Volatility);

% use TimeSpec to specify the structure of the time layout for a BDT tree
TimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);

% build the BDT tree
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec);

% use the following swaption arguments
ExerciseDates = 'jan-1-2012';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2015';
Spread = 0;
SwapReset = 1;
Principal = 100;
OptSpec = 'call';
Strike=.062;
Basis=1;

% price the swaption
[Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, ...
'Basis', Basis, 'Principal', Principal)

Price = 2.0592

PriceTree = struct with fields:
  FinObj: 'BDTPriceTree'
  tObs: [0 1 2 3 4 5 6 7 8 9 10]
  PTree: {1x11 cell}
```

## Price a 5-Year Call Swaption with Receiving and Paying Legs Using a BDT Interest-Rate Tree

This example shows how to price a 5-year call swaption with receiving and paying legs using a BDT interest-rate tree. Assume that interest rate and volatility are fixed at 6% and 20% annually between the valuation date of the tree until its maturity. Build a tree with the following data.

```
Rates = 0.06 * ones (10,1);
StartDates = ['jan-1-2007'; 'jan-1-2008'; 'jan-1-2009'; 'jan-1-2010'; 'jan-1-2011'; ...
'jan-1-2012'; 'jan-1-2013'; 'jan-1-2014'; 'jan-1-2015'; 'jan-1-2016'];

EndDates = ['jan-1-2008'; 'jan-1-2009'; 'jan-1-2010'; 'jan-1-2011'; 'jan-1-2012'; ...
'jan-1-2013'; 'jan-1-2014'; 'jan-1-2015'; 'jan-1-2016'; 'jan-1-2017'];
ValuationDate = 'jan-1-2007';
Compounding = 1;
```

### Define the RateSpec.

```
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', EndDates,
'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [10x1 double]
    Rates: [10x1 double]
    EndTimes: [10x1 double]
    StartTimes: [10x1 double]
    EndDates: [10x1 double]
    StartDates: [10x1 double]
    ValuationDate: 733043
    Basis: 0
    EndMonthRule: 1
```

### Use VolSpec to compute interest-rate volatility.

```
Volatility = 0.20 * ones (10,1);
VolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
```

### Use TimeSpec to specify the structure of the time layout for a BDT tree.

```
TimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
```

**Build the BDT tree.**

```
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)
```

```
BDTTree = struct with fields:
    FinObj: 'BDTFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3 4 5 6 7 8 9]
        dObs: [1x10 double]
        TFwd: {1x10 cell}
    CFlowT: {1x10 cell}
    FwdTree: {1x10 cell}
```

**Define the swaption arguments.**

```
ExerciseDates = 'jan-1-2012';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2015';
Spread = 0;
SwapReset = [1 1]; % 1st column represents receiving leg, 2nd column represents paying
Principal = 100;
OptSpec = 'call';
Strike=.062;
Basis= [2 4]; % 1st column represents receiving leg, 2nd column represents paying leg
```

**Price the swaption.**

```
[Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, ...
'Basis', Basis, 'Principal', Principal)
```

```
Price = 2.0592
```

```
PriceTree = struct with fields:
    FinObj: 'BDTPriceTree'
    tObs: [0 1 2 3 4 5 6 7 8 9 10]
```

Ptree: {1x11 cell}

## Input Arguments

### **BDTtree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `bdttree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vector with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors. For more information, see “Definitions” on page 11-2153.

Data Types: `char` | `cell`

### **Strike — Strike swap rate values**

decimal

Strike swap rate values, specified as a NINST-by-1 vector.

Data Types: `double`

### **ExerciseDates — Exercise dates for swaption**

serial date number | date character vector | cell array of date character vectors

Exercise dates for the swaption, specified as a NINST-by-1 vector or NINST-by-2 using serial date numbers or date character vectors, depending on the option type.

- For a European option, `ExerciseDates` are a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. When using a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` are a NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between the `ValuationDate` of the tree and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

**Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

**Settle — Settlement date**

serial date number | date character vector | cell array of date character vectors

Settlement date (representing the settle date for each swap), specified as a NINST-by-1 vector of serial date numbers or a date character vectors. The `Settle` date for every swaption is set to the `ValuationDate` of the BDT tree. The swap argument `Settle` is ignored. The underlying swap starts at the maturity of the swaption.

Data Types: `double` | `char`

**Maturity — Maturity date for swap**

serial date number | date character vector | cell array of date character vectors

Maturity date for each swap, specified as a NINST-by-1 vector of dates using serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = swaptionbybdtd(BDTree, OptSpec, ExerciseDates, Spread, Settle, Maturity, 'SwapReset', 4, 'Basis', 5, 'Principal', 10000)`

**AmericanOpt — Option type**

0 (European) (default) | integer with values 0 or 1

(Optional) Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and NINST-by-1 positive integer flags with values:

- 0 — European
- 1 — American

Data Types: double

### **SwapReset — Reset frequency per year for underlying swap**

1 (default) | numeric

Reset frequency per year for the underlying swap, specified as the comma-separated pair consisting of 'SwapReset' and a NINST-by-1 vector or NINST-by-2 matrix representing the reset frequency per year for each leg. If SwapReset is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree for each instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If Basis is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)

- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices of swaptions at time 0**

vector

Expected prices of the swaptions at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node.

Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.



## Definitions

### Call Swaption

A call swaption or payer swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

### Put Swaption

A put swaption or receiver swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

## See Also

[bdttree](#) | [instswaption](#) | [swapbybdt](#)

## Topics

["Computing Instrument Prices"](#) on page 2-99

["Pricing Options Structure"](#) on page B-2

["Understanding Interest-Rate Tree Models"](#) on page 2-79

["Supported Interest-Rate Instruments"](#) on page 2-2

**Introduced before R2006a**

## swaptionbybk

Price swaption from Black-Karasinski interest-rate tree

### Syntax

```
[Price,PriceTree] = swaptionbybk(BKTree,OptSpec,Strike,  
ExerciseDates,Spread,Settle,Maturity)  
[Price,PriceTree] = swaptionbybk( ____,Name,Value)
```

### Description

[Price,PriceTree] = swaptionbybk(BKTree,OptSpec,Strike,ExerciseDates,Spread,Settle,Maturity) prices swaption using a Black-Karasinski tree.

[Price,PriceTree] = swaptionbybk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a 4-Year Call and Put Swaption Using a BK Interest-Rate Tree

This example shows how to price a 4-year call and put swaption using a BK interest-rate tree, assuming the interest rate is fixed at 7% annually.

```
Rates = 0.07 * ones (10,1);  
Compounding = 2;  
StartDates = ['jan-1-2007'; 'jul-1-2007'; 'jan-1-2008'; 'jul-1-2008'; 'jan-1-2009'; ...  
             'jul-1-2009'; 'jan-1-2010'; 'jul-1-2010'; 'jan-1-2011'; 'jul-1-2011'];  
EndDates = ['jul-1-2007'; 'jan-1-2008'; 'jul-1-2008'; 'jan-1-2009'; 'jul-1-2009'; ...  
           'jan-1-2010'; 'jul-1-2010'; 'jan-1-2011'; 'jul-1-2011'; 'jan-1-2012'];  
ValuationDate = 'jan-1-2007';  
  
% define the RateSpec
```

```

RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', EndDates, ..
'Compounding', Compounding);

% use BKVolSpec to compute the interest-rate volatility
Volatility = 0.10*ones(10,1);
AlphaCurve = 0.05*ones(10,1);
AlphaDates = EndDates;
BKVolSpec = bkvolspec(ValuationDate, EndDates, Volatility, AlphaDates, AlphaCurve);

% use BKTimeSpec to specify the structure of the time layout for the BK interest-rate t
BKTimeSpec = bktimespec(ValuationDate, EndDates, Compounding);

% build the BK tree
BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec);

% use the following arguments for a 1-year swap and 4-year swaption
ExerciseDates = 'jan-1-2011';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2012';
Spread = 0;
SwapReset = 2 ;
Principal = 100;
OptSpec = {'call' ;'put'};
Strike= [ 0.07 ; 0.0725];
Basis=1;

% price the swaption
PriceSwaption = swaptionbybk(BKTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, 'Basis', Basis, ...
'Principal', Principal)

PriceSwaption = 2×1

    0.3634
    0.4798

```

### Price a 4-Year Call and Put Swaption with Receiving and Paying Legs Using a BK Interest-Rate Tree

This example shows how to price a 4-year call and put swaption with receiving and paying legs using a BK interest-rate tree, assuming the interest rate is fixed at 7% annually. Build a tree with the following data.

```
Rates = 0.07 * ones (10,1);
Compounding = 2;
StartDates = ['jan-1-2007'; 'jul-1-2007'; 'jan-1-2008'; 'jul-1-2008'; 'jan-1-2009'; ...
'jul-1-2009'; 'jan-1-2010'; 'jul-1-2010'; 'jan-1-2011'; 'jul-1-2011'];
EndDates = ['jul-1-2007'; 'jan-1-2008'; 'jul-1-2008'; 'jan-1-2009'; 'jul-1-2009'; ...
'jan-1-2010'; 'jul-1-2010'; 'jan-1-2011'; 'jul-1-2011'; 'jan-1-2012'];
ValuationDate = 'jan-1-2007';
```

Define the RateSpec.

```
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', EndDates, ...
'Compounding', Compounding)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
    Disc: [10x1 double]
    Rates: [10x1 double]
    EndTimes: [10x1 double]
    StartTimes: [10x1 double]
    EndDates: [10x1 double]
    StartDates: [10x1 double]
    ValuationDate: 733043
    Basis: 0
    EndMonthRule: 1
```

Use BKVolSpec to compute interest-rate volatility.

```
Volatility = 0.10*ones(10,1);
AlphaCurve = 0.05*ones(10,1);
AlphaDates = EndDates;
BKVolSpec = bkvolspec(ValuationDate, EndDates, Volatility, AlphaDates, AlphaCurve);
```

Use BKTimeSpec to specify the structure of the time layout for a BK tree.

```
BKTimeSpec = bktimespec(ValuationDate, EndDates, Compounding);
```

Build the BK tree.

```
BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Define the arguments for a 1-year swap and 4-year swaption.

```
ExerciseDates = 'jan-1-2011';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2012';
Spread = 0;
SwapReset = [2 2]; % 1st column represents swaption receiving leg, 2nd column represents
Principal = 100;
OptSpec = {'call' ; 'put'};
Strike= [ 0.07 ; 0.0725];
Basis= [1 3]; % 1st column represents swaption receiving leg, 2nd column represents sw
```

Price the swaption.

```
PriceSwaption = swaptionbybk(BKTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, 'Basis', Basis, ...
'Principal', Principal)
```

```
PriceSwaption = 2×1
```

```
0.3634
0.4798
```

## Input Arguments

### **BKTree** — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `bktree`.

Data Types: `struct`

### **OptSpec** — Definition of option

character vector with values 'call' or 'put' | cell array of character vector with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors. For more information, see “Definitions” on page 11-2161.

Data Types: char | cell

**Strike — Strike swap rate values**

decimal

Strike swap rate values, specified as a NINST-by-1 vector.

Data Types: double

**ExerciseDates — Exercise dates for swaption**

serial date number | date character vector | cell array of date character vectors

Exercise dates for the swaption, specified as a NINST-by-1 vector or NINST-by-2 using serial date numbers or date character vectors, depending on the option type.

- For a European option, `ExerciseDates` are a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. When using a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` are a NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-NaN date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between the `ValuationDate` of the tree and the single listed `ExerciseDate`.

Data Types: double | char | cell

**Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector | cell array of date character vectors

Settlement date (representing the settle date for each swap), specified as a NINST-by-1 vector of serial date numbers or date character vectors. The `Settle` date for every swaption is set to the `ValuationDate` of the BK tree. The swap argument `Settle` is ignored. The underlying swap starts at the maturity of the swaption.

Data Types: double | char

**Maturity — Maturity date for swap**

serial date number | date character vector | cell array of date character vectors

Maturity date for each swap, specified as a NINST-by-1 vector of dates using serial date numbers or date character vectors.

Data Types: double | char | cell

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = swaptionbybk(BKTree, OptSpec, ExerciseDates, Spread, Settle, Maturity, 'SwapReset', 4, 'Basis', 5, 'Principal', 10000)`

**AmericanOpt — Option type**

0 (European) (default) | integer with values 0 or 1

(Optional) Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer flags with values:

- 0 — European
- 1 — American

Data Types: double

**SwapReset — Reset frequency per year for underlying swap**

1 (default) | numeric

Reset frequency per year for the underlying swap, specified as the comma-separated pair consisting of 'SwapReset' and a NINST-by-1 vector or NINST-by-2 matrix representing the reset frequency per year for each leg. If `SwapReset` is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

**Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree for each instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If Basis is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.



Data Types: struct

## Output Arguments

### **Price — Expected prices of swaptions at time 0**

vector

Expected prices of the swaptions at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node.

Within PriceTree:

- PriceTree.PTree contains the clean prices.
- PriceTree.tObs contains the observation times.

## Definitions

### **Call Swaption**

A call swaption or payer swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

### **Put Swaption**

A put swaption or receiver swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

## See Also

bktree | instswaption | swapbybk

## **Topics**

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

## **Introduced before R2006a**

# swaptionbycir

Price swaption from Cox-Ingersoll-Ross interest-rate tree

## Syntax

```
[Price,PriceTree] = swaptionbycir(CIRTree,OptSpec,Strike,
ExerciseDates,Spread,Settle,Maturity)
[Price,PriceTree] = swaptionbycir( ____,Name,Value)
```

## Description

[Price,PriceTree] = swaptionbycir(CIRTree,OptSpec,Strike,ExerciseDates,Spread,Settle,Maturity) prices swaption with a Cox-Ingersoll-Ross (CIR) tree using a CIR++ model with the Nawalka-Beliaeva (NB) approach.

[Price,PriceTree] = swaptionbycir( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a Swaption Using a CIR Interest-Rate Tree

Define a 3-year put swaption.

```
Rates =0.075 * ones (10,1);
Compounding = 2;
StartDates = ['Jan-1-2017';'Jul-1-2017';'Jan-1-2018';'Jul-1-2018';'Jan-1-2019';...
'Jul-1-2019';'Jan-1-2020';'Jul-1-2020';'Jan-1-2021';'Jul-1-2021'];
EndDates =['Jul-1-2017';'Jan-1-2018';'Jul-1-2018';'Jan-1-2019';'Jul-1-2019';...
'Jan-1-2020';'Jul-1-2020';'Jan-1-2021';'Jul-1-2021';'Jan-1-2022'];
ValuationDate = 'Jan-1-2017';
```

Create a RateSpec using the intenvset function.

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, 'End
```

Create a CIR tree.

```
NumPeriods = length(EndDates);
Alpha = 0.03;
Theta = 0.02;
Sigma = 0.1;
Maturity = '01-jan-2023';
CIRTimeSpec = cirtimespec(ValuationDate, Maturity, NumPeriods);
CIRVolSpec = cirvolspec(Sigma, Alpha, Theta);

CIRT = cirtree(CIRVolSpec, RateSpec, CIRTimeSpec)

CIRT = struct with fields:
    FinObj: 'CIRFwdTree'
    VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
    tObs: [0 0.6000 1.2000 1.8000 2.4000 3 3.6000 4.2000 4.8000 5.4000]
    dObs: [1x10 double]
    FwdTree: {1x10 cell}
    Connect: {1x9 cell}
    Probs: {1x9 cell}
```

Use the following arguments for a 1-year swap and a 3-year swaption.

```
ExerciseDates = 'Jan-1-2020';
SwapSettlement = ExerciseDates;
SwapMaturity = 'Jan-1-2022';
Spread = 0;
SwapReset = 2 ;
Principal = 100;
OptSpec = 'put';
Strike = 0.04;
Basis=1;
```

Price the swaption.

```
[Price,PriceTree] = swaptionbycir(CIRT,OptSpec,Strike,ExerciseDates,Spread,SwapSettlement,
'Basis',Basis,'Principal',Principal)

Price = 3.1537

PriceTree = struct with fields:
    FinObj: 'CIRPriceTree'
```

```

PTree: {1x11 cell}
  tObs: [0 0.6000 1.2000 1.8000 2.4000 3 3.6000 4.2000 4.8000 5.4000 6]
Connect: {1x9 cell}
  Probs: {1x9 cell}

```

## Input Arguments

### **CIRTree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `cirtree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vector with values 'call' or 'put' | string array with values "call" or "put"

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors or string arrays. For more information, see “Definitions” on page 11-2161.

Data Types: `char` | `cell` | `string`

### **Strike — Strike swap rate values**

decimal

Strike swap rate values, specified as a NINST-by-1 vector.

Data Types: `double`

### **ExerciseDates — Exercise dates for swaption**

serial date number | date character vector | cell array of date character vectors | string array | `datetime`

Exercise dates for the swaption, specified as a NINST-by-1 vector or NINST-by-2 using serial date numbers, date character vectors, string arrays, or `datetime` arrays depending on the option type.

- For a European option, `ExerciseDates` are a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. When using a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` are a NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between the `ValuationDate` of the tree and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell` | `string` | `datetime`

### **Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | date character vector | cell array of date character vectors | string array | `datetime`

Settlement date (representing the settle date for each swap), specified as a NINST-by-1 vector of serial date numbers, date character vectors, string arrays, or `datetime` arrays. The `Settle` date for every swaption is set to the `ValuationDate` of the CIR tree. The swap argument `Settle` is ignored. The underlying swap starts at the maturity of the swaption.

Data Types: `double` | `char` | `string` | `datetime`

### **Maturity — Maturity date for swap**

serial date number | date character vector | cell array of date character vectors | string array | `datetime`

Maturity date for each swap, specified as a NINST-by-1 vector of dates using serial date numbers, date character vectors, string arrays, or `datetime` arrays.

Data Types: `double` | `char` | `cell` | `string` | `datetime`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `[Price, PriceTree] = swaptionbycir(CIRTree, OptSpec, ExerciseDates, Spread, Settle, Maturity, 'SwapReset', 4, 'Basis', 5, 'Principal', 10000)`

### **AmericanOpt — Option type**

0 (European) (default) | integer with values 0 or 1

Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer flags with values:

- 0 — European
- 1 — American

Data Types: double

### **SwapReset — Reset frequency per year for underlying swap**

1 (default) | numeric

Reset frequency per year for the underlying swap, specified as the comma-separated pair consisting of 'SwapReset' and a NINST-by-1 vector or NINST-by-2 matrix representing the reset frequency per year for each leg. If `SwapReset` is NINST-by-2, the first column represents the receiving leg, and the second column represents the paying leg.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward-rate tree for each instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If `Basis` is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365

- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

## **Output Arguments**

### **Price — Expected prices of swaptions at time 0**

vector

Expected prices of the swaptions at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within PriceTree:

- PriceTree.PTree contains the clean prices.



- `PriceTree.tObs` contains the observation times.
- `PriceTree.Connect` contains the connectivity vectors. Each element in the cell array describes how nodes in that level connect to the next. For a given tree level, there are `NumNodes` elements in the vector, and they contain the index of the node at the next level that the middle branch connects to. Subtracting 1 from that value indicates where the up-branch connects to, and adding 1 indicated where the down branch connects to.
- `PriceTree.Probs` contains the probability arrays. Each element of the cell array contains the up, middle, and down transition probabilities for each node of the level.

## Definitions

### Call Swaption

A call swaption or payer swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

### Put Swaption

A put swaption or receiver swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

## References

- [1] Cox, J., Ingersoll, J., and S. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*. Vol. 53, 1985.
- [2] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.
- [3] Hirt, A. *Computational Methods in Finance*. CRC Press, 2012.
- [4] Nawalka, S., Soto, G., and N. Beliaeva. *Dynamic Term Structure Modeling*. Wiley, 2007.
- [5] Nelson, D. and K. Ramaswamy. "Simple Binomial Processes as Diffusion Approximations in Financial Models." *The Review of Financial Studies*. Vol 3. 1990, pp. 393-430.

## See Also

bondbycir | capbycir | cfbycir | fixedbycir | floatbycir | floorbycir |  
instswaption | oasbycir | optbndbycir | optembndbycir | optemfloatbycir |  
optfloatbycir | rangefloatbycir | swapbycir

## Topics

“Pricing Using Interest-Rate Tree Models” on page 2-99

“Pricing Options Structure” on page B-2

“Understanding Interest-Rate Tree Models” on page 2-79

“Supported Interest-Rate Instruments” on page 2-2

## Introduced in R2018a

# swaptionbyblk

Price European swaption instrument using Black model

## Syntax

```
Price = swaptionbyblk(RateSpec,OptSpec,Strike,Settle,ExerciseDates,  
Maturity,Volatility)  
Price = swaptionbyblk( ____,Name,Value)
```

## Description

Price = swaptionbyblk(RateSpec,OptSpec,Strike,Settle,ExerciseDates, Maturity,Volatility) prices swaptions using the Black option pricing model.

Price = swaptionbyblk( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

## Examples

### Price a European Swaption Using the Black Model Where the Yield Curve is Flat at 6%

Price a European swaption that gives the holder the right to enter in five years into a three-year paying swap where a fixed-rate of 6.2% is paid and floating is received. Assume that the yield curve is flat at 6% per annum with continuous compounding, the volatility of the swap rate is 20%, the principal is \$100, and payments are exchanged semiannually.

Create the RateSpec.

```
Rate = 0.06;  
Compounding = -1;  
ValuationDate = 'Jan-1-2010';  
EndDates = 'Jan-1-2020';
```

```
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, ...  
'EndDates', EndDates, 'Rates', Rate, 'Compounding', Compounding, 'Basis', Basis);
```

Price the swaption using the Black model.

```
Settle = 'Jan-1-2011';  
ExerciseDates = 'Jan-1-2016';  
Maturity = 'Jan-1-2019';  
Reset = 2;  
Principal = 100;  
Strike = 0.062;  
Volatility = 0.2;  
OptSpec = 'call';
```

```
Price= swaptionbyblk(RateSpec, OptSpec, Strike, Settle, ExerciseDates, Maturity, ...  
Volatility, 'Reset', Reset, 'Principal', Principal, 'Basis', Basis)
```

```
Price = 2.0710
```

### **Price a European Swaption with Receiving and Paying Legs Using the Black Model Where the Yield Curve is 6%**

This example shows Price a European swaption with receiving and paying legs that gives the holder the right to enter in five years into a three-year paying swap where a fixed-rate of 6.2% is paid and floating is received. Assume that the yield curve is flat at 6% per annum with continuous compounding, the volatility of the swap rate is 20%, the principal is \$100, and payments are exchanged semiannually.

```
Rate = 0.06;  
Compounding = -1;  
ValuationDate = 'Jan-1-2010';  
EndDates = 'Jan-1-2020';  
Basis = 1;
```

#### **Define the RateSpec.**

```
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, ...  
'EndDates', EndDates, 'Rates', Rate, 'Compounding', Compounding, 'Basis', Basis);
```

**Define the swaption arguments.**

```

Settle = 'Jan-1-2011';
ExerciseDates = 'Jan-1-2016';
Maturity = 'Jan-1-2019';
Reset = [2 4]; % 1st column represents receiving leg, 2nd column represents paying leg
Principal = 100;
Strike = 0.062;
Volatility = 0.2;
OptSpec = 'call';
Basis = [1 3]; % 1st column represents receiving leg, 2nd column represents paying leg

```

**Price the swaption.**

```

Price= swaptionbyblk(RateSpec,OptSpec,Strike,Settle,ExerciseDates,Maturity,Volatility,
'Reset',Reset,'Principal',Principal,'Basis',Basis)

```

```

Price = 1.6494

```

**Price a European Swaption Using the Black Model Where the Yield Curve Is Incrementally Increasing**

Price a European swaption that gives the holder the right to enter into a 5-year receiving swap in a year, where a fixed rate of 3% is received and floating is paid. Assume that the 1-year, 2-year, 3-year, 4-year and 5-year zero rates are 3%, 3.4%, 3.7%, 3.9% and 4% with continuous compounding. The swap rate volatility is 21%, the principal is \$1000, and payments are exchanged semiannually.

Create the RateSpec.

```

ValuationDate = 'Jan-1-2010';
EndDates = {'Jan-1-2011';'Jan-1-2012';'Jan-1-2013';'Jan-1-2014';'Jan-1-2015'};
Rates = [0.03; 0.034 ; 0.037; 0.039; 0.04;];
Compounding = -1;
Basis = 1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding,'Basis', Basis)

RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: -1
    Disc: [5x1 double]

```

```
        Rates: [5x1 double]
        EndTimes: [5x1 double]
        StartTimes: [5x1 double]
        EndDates: [5x1 double]
        StartDates: 734139
ValuationDate: 734139
        Basis: 1
        EndMonthRule: 1
```

Price the swaption using the Black model.

```
Settle = 'Jan-1-2011';
ExerciseDates = 'Jan-1-2012';
Maturity = 'Jan-1-2017';
Strike = 0.03;
Volatility = 0.21;
Principal = 1000;
Reset = 2;
OptSpec = 'put';
```

```
Price = swaptionbyblk(RateSpec, OptSpec, Strike, Settle, ExerciseDates, ...
Maturity, Volatility, 'Basis', Basis, 'Reset', Reset, 'Principal', Principal)
```

```
Price = 0.5771
```

### Price a Swaption Using a Different Curve to Generate the Future Forward Rates

Define the OIS and Libor curves.

```
Settle = datenum('15-Mar-2013');
CurveDates = daysadd(Settle, 360*[1/12 2/12 3/12 6/12 1 2 3 4 5 7 10], 1);
OISRates = [.0018 .0019 .0021 .0023 .0031 .006 .011 .017 .021 .026 .03]';
LiborRates = [.0045 .0047 .005 .0055 .0075 .0109 .0162 .0216 .0262 .0309 .0348]';
```

Create an associated RateSpec for the OIS and Libor curves.

```
OISCurve = intenvset('Rates', OISRates, 'StartDate', Settle, 'EndDates', CurveDates, 'Compounding', 'Annual');
LiborCurve = intenvset('Rates', LiborRates, 'StartDate', Settle, 'EndDates', CurveDates, 'Compounding', 'Annual');
```

Define the swaption instruments.

```
ExerciseDate = '15-Mar-2018';
Maturity = {'15-Mar-2020'; '15-Mar-2023'};
```

```
OptSpec = 'call';
Strike = 0.04;
BlackVol = 0.2;
```

Price the swaption instruments using the term structure `OISCurve` both for discounting the cash flows and generating the future forward rates.

```
Price = swaptionbyblk(OISCurve, OptSpec, Strike, Settle, ExerciseDate, Maturity, BlackVol);
```

```
Price = 2×1
```

```
1.0956
2.6944
```

Price the swaption instruments using the term structure `LiborCurve` to generate the future forward rates. The term structure `OISCurve` is used for discounting the cash flows.

```
PriceLC = swaptionbyblk(OISCurve, OptSpec, Strike, Settle, ExerciseDate, Maturity, BlackVol, LiborCurve);
```

```
PriceLC = 2×1
```

```
1.5346
3.8142
```

## Price a Swaption Using the Shifted Black Model

Create the `RateSpec`.

```
ValuationDate = 'Jan-1-2016';
EndDates = {'Jan-1-2017'; 'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'; 'Jan-1-2021'};
Rates = [-0.02; 0.024 ; 0.047; 0.090; 0.12;]/100;
Compounding = 1;
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',ValuationDate, ...
'EndDates',EndDates,'Rates',Rates,'Compounding',Compounding,'Basis',Basis)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
```

```
Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 736330
ValuationDate: 736330
    Basis: 1
EndMonthRule: 1
```

Price the swaption with a negative strike using the Shifted Black model.

```
Settle = 'Jan-1-2016';
ExerciseDates = 'Jan-1-2017';
Maturity = 'Jan-1-2020';
Strike = -0.003; % Set -0.3 percent strike.
ShiftedBlackVolatility = 0.31;
Principal = 1000;
Reset = 1;
OptSpec = 'call';
Shift = 0.008; % Set 0.8 percent shift.
```

```
Price = swaptionbyblk(RateSpec,OptSpec,Strike,Settle,ExerciseDates, ...
Maturity,ShiftedBlackVolatility,'Basis',Basis,'Reset',Reset,...
'Principal',Principal,'Shift',Shift)
```

```
Price = 12.8301
```

### Price Swaptions Using the Shifted Black Model with a Vector of Shifts

Create the RateSpec.

```
ValuationDate = 'Jan-1-2016';
EndDates = {'Jan-1-2017';'Jan-1-2018';'Jan-1-2019';'Jan-1-2020';'Jan-1-2021'};
Rates = [-0.02; 0.024 ; 0.047; 0.090; 0.12;]/100;
Compounding = 1;
Basis = 1;
```

```
RateSpec = intenvset('ValuationDate',ValuationDate,'StartDates',ValuationDate, ...
'EndDates',EndDates,'Rates',Rates,'Compounding',Compounding,'Basis',Basis)
```



```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 1
    Disc: [5x1 double]
    Rates: [5x1 double]
    EndTimes: [5x1 double]
    StartTimes: [5x1 double]
    EndDates: [5x1 double]
    StartDates: 736330
    ValuationDate: 736330
    Basis: 1
    EndMonthRule: 1
```

Price the swaptions with using the Shifted Black model.

```
Settle = 'Jan-1-2016';
ExerciseDates = 'Jan-1-2017';
Maturities = {'Jan-1-2018'; 'Jan-1-2019'; 'Jan-1-2020'};
Strikes = [-0.0034; -0.0032; -0.003];
ShiftedBlackVolatilities = [0.33; 0.32; 0.31]; % A vector of volatilities.
Principal = 1000;
Reset = 1;
OptSpec = 'call';
Shifts = [0.0085; 0.0082; 0.008]; % A vector of shifts.
```

```
Prices = swaptionbyblk(RateSpec, OptSpec, Strikes, Settle, ExerciseDates, ...
    Maturities, ShiftedBlackVolatilities, 'Basis', Basis, 'Reset', Reset, ...
    'Principal', Principal, 'Shift', Shifts)
```

```
Prices = 3x1
```

```
    4.1117
    8.0577
   12.8301
```

## Input Arguments

**RateSpec** — Interest-rate term structure  
structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

If the paying leg is different than the receiving leg, the `RateSpec` can be a NINST-by-2 input variable of `RateSpecs`, with the second input being the discount curve for the paying leg. If only one curve is specified, then it is used to discount both legs.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vector with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

A 'call' swaption, or Payer swaption, allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A 'put' swaption, or Receiver swaption, allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

Data Types: `char` | `cell`

### **Strike — Strike swap rate values**

decimal

Strike swap rate values, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

### **Settle — Settlement date**

serial date number | date character vector | cell array of date character vectors

Settlement date (representing the settle date for each swaption), specified as a NINST-by-1 vector of serial date numbers or date character vectors. `Settle` must not be later than `ExerciseDates`.

The `Settle` date input for `swaptionbyblk` is the valuation date on which the swaption (an option to enter into a swap) is priced. The swaption buyer pays this price on this date to hold the swaption.

Data Types: double | char

### **ExerciseDates — Dates on which swaption expires and underlying swap starts**

serial date number | date character vector | cell array of date character vectors

Dates, specified as serial date numbers or date character vectors, on which the swaption expires and the underlying swap starts. The swaption holder can choose to enter into the swap on this date if the situation is favorable.

For a European option, `ExerciseDates` are a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. When using a European option, there is only one `ExerciseDate` on the option expiry date.

Data Types: double | char | cell

### **Maturity — Maturity date for each forward swap**

serial date number | date character vector | cell array of date character vectors

Maturity date for each forward swap, specified as a NINST-by-1 vector of dates using serial date numbers or date character vectors.

Data Types: double | char | cell

### **Volatility — Annual volatilities values**

numeric

Annual volatilities values, specified as a NINST-by-1 vector of numeric values.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `Price =`

```
swaptionbyblk(OISCurve,OptSpec,Strike,Settle,ExerciseDate,Maturity,B  
lackVol,'Reset',1,'Shift',.5)
```

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If Basis is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg. Default is 0 (actual/actual).

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

### **Reset — Reset frequency per year for underlying forward swap**

1 (default) | numeric

Reset frequency per year for the underlying forward swap, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector or NINST-by-2 matrix

representing the reset frequency per year for each leg. If `Reset` is `NINST-by-2`, the first column represents the receiving leg, while the second column represents the paying leg.

Data Types: `double`

### **ProjectionCurve — Rate curve used in generating future forward rates**

if `ProjectionCurve` is not specified, then `RateSpec` is used both for discounting cash flows and projecting future forward rates (default) | `structure`

The rate curve to be used in generating the future forward rates, specified as the comma-separated pair consisting of `'ProjectionCurve'` and a structure created using `intenvset`. Use this optional input if the forward curve is different from the discount curve.

Data Types: `struct`

### **Shift — Shift in decimals for shifted Black model**

0 (no shift) (default) | positive decimal

Shift in decimals for the shifted Black model, specified as the comma-separated pair consisting of `'Shift'` and a scalar or `NINST-by-1` vector of rate shifts in positive decimals. Set this parameter to a positive rate shift in decimals to add a positive shift to the forward swap rate and strike, which effectively sets a negative lower bound for the forward swap rate and strike. For example, a `Shift` of `0.01` is equal to a 1% shift.

Data Types: `double`

## **Output Arguments**

### **Price — Prices for swaptions at time 0**

`vector`

Prices for the swaptions at time 0, returned as a `NINST-by-1` vector of prices.

## **Definitions**

### **Forward Swap**

A forward swap is a swap that starts at a future date.

## Shifted Black

The Shifted Black model is essentially the same as the Black's model, except that it models the movements of  $(F + Shift)$  as the underlying asset, instead of  $F$  (which is the forward swap rate in the case of swaptions).

This model allows negative rates, with a fixed negative lower bound defined by the amount of shift; that is, the zero lower bound of Black's model has been shifted.

## Algorithms

### Black Model

$$dF = \sigma_{Black} F dw$$

$$call = e^{-\nu T} [FN(d_1) - KN(d_2)]$$

$$put = e^{-\nu T} [KN(-d_2) - FN(-d_1)]$$

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \left(\frac{\sigma_B^2}{2}\right)T}{\sigma_B \sqrt{T}}, \quad d_2 = d_1 - \sigma_B \sqrt{T}$$

$$\sigma_B = \sigma_{Black}$$

Where  $F$  is the forward value and  $K$  is the strike.

### Shifted Black Model

$$dF = \sigma_{Shifted\_Black} (F + Shift) dw$$

$$call = e^{-\nu T} [(F + Shift)N(d_{s1}) - (K + Shift)N(d_{s2})]$$

$$put = e^{-\nu T} [(K + Shift)N(-d_{s2}) - (F + Shift)N(-d_{s1})]$$

$$d_{s1} = \frac{\ln\left(\frac{F + Shift}{K + Shift}\right) + \left(\frac{\sigma_{sB}^2}{2}\right)T}{\sigma_{sB} \sqrt{T}}, \quad d_{s2} = d_{s1} - \sigma_{sB} \sqrt{T}$$

$$\sigma_{sB} = \sigma_{Shifted\_Black}$$

Where  $F+Shift$  is the forward value and  $K+Shift$  is the strike for the shifted version.

## See Also

blackvolbysabr | bondbyzero | capbyblk | cfbyzero | fixedbyzero | floatbyzero | floorbyblk | intenvset | swaptionbynormal

## Topics

“Calibrate the SABR Model” on page 2-36

“Price a Swaption Using the SABR Model” on page 2-42

“Price Swaptions with Negative Strikes Using the Shifted SABR Model” on page 2-27

“Work with Negative Interest Rates” on page 2-22

“Supported Interest-Rate Instruments” on page 2-2

## External Websites

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

**Introduced before R2006a**

## swaptionbyhjm

Price swaption from Heath-Jarrow-Morton interest-rate tree

### Syntax

```
[Price,PriceTree] = swaptionbyhjm(HJMTree,OptSpec,Strike,  
ExerciseDates,Spread,Settle,Maturity)  
[Price,PriceTree] = swaptionbyhjm( ____,Name,Value)
```

### Description

[Price,PriceTree] = swaptionbyhjm(HJMTree,OptSpec,Strike,ExerciseDates,Spread,Settle,Maturity) prices swaption using a Heath-Jarrow-Morton tree.

[Price,PriceTree] = swaptionbyhjm( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a 1-Year Call Swaption Using an HJM Interest-Rate Tree

This example shows how to price a 1-year call swaption using an HJM interest-rate tree. Assume that interest rate is fixed at 5% annually between the valuation date of the tree until its maturity. Build a tree with the following data.

```
Rates = [ 0.05;0.05;0.05;0.05];  
StartDates = 'jan-1-2007';  
EndDates =['jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011'];  
ValuationDate = StartDates;  
Compounding = 1;  
  
% define the RateSpec  
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', ...
```



```

EndDates, 'Compounding', Compounding);

% use VolSpec to compute the interest-rate volatility
VolSpec=hjmvolspec('Constant',0.01);

% use TimeSpec to specify the structure of the time layout for the HJM interest-rate t
TimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);

% build the HJM tree
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec);

% use the following swaption arguments
ExerciseDates = '01-Jan-2008';
SwapSettlement = ExerciseDates;
SwapMaturity   = 'jan-1-2010';
Spread = [0];
SwapReset = 1;
Basis = 1;
Principal = 100;
OptSpec = 'call';
Strike=0.05;

% price the swaption

[Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity,'SwapReset', SwapReset, ...
'Basis', Basis, 'Principal', Principal)

Price = 0.9296

PriceTree = struct with fields:
    FinObj: 'HJMPriceTree'
    tObs: [5x1 double]
    PBush: {1x5 cell}

```

### Price a 1-Year Call Swaption with Receiving and Paying Legs Using an HJM Interest-Rate Tree

This example shows how to price a 1-year call swaption with receiving and paying legs using an HJM interest-rate tree. Assume that interest rate is fixed at 5% annually between the valuation date of the tree until its maturity. Build a tree with the following data.

```

Rates = [ 0.05;0.05;0.05;0.05];
StartDates = 'jan-1-2007';
EndDates =['jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011'];
ValuationDate = StartDates;
Compounding = 1;

```

Define the RateSpec.

```

RateSpec = intenvset('Rates',Rates,'StartDates',StartDates,'EndDates',...
EndDates,'Compounding',Compounding);

```

Use VolSpec to compute the interest-rate volatility.

```

VolSpec=hjmvolspec('Constant',0.01);

```

Use TimeSpec to specify the structure of the time layout for the HJM interest-rate tree.

```

TimeSpec = hjmtimespec(ValuationDate,EndDates,Compounding);

```

Build the HJM tree.

```

HJMTree = hjmtree(VolSpec,RateSpec,TimeSpec);

```

Use the following swaption arguments

```

ExerciseDates = '01-Jan-2008';
SwapSettlement = ExerciseDates;
SwapMaturity   = 'jan-1-2010';
Spread = [0];
SwapReset = [1 1]; % 1st column represents receiving leg, 2nd column represents paying
Basis = [1 3]; % 1st column represents receiving leg, 2nd column represents paying
Principal = 100;
OptSpec = 'call';
Strike=0.05;

```

Price the swaption.

```

[Price, PriceTree] = swaptionbyhjm(HJMTree,OptSpec,Strike,ExerciseDates, ...
Spread,SwapSettlement,SwapMaturity,'SwapReset',SwapReset, ...
'Basis',Basis,'Principal',Principal)

```

```

Price = 0.9296

```

```

PriceTree = struct with fields:
    FinObj: 'HJMPriceTree'
    tObs: [5x1 double]

```

PBush: {1x5 cell}

## Input Arguments

### HJMTtree — Interest-rate tree structure

structure

Interest-rate tree structure, specified by using `hjm tree`.

Data Types: `struct`

### OptSpec — Definition of option

character vector with values 'call' or 'put' | cell array of character vector with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors. For more information, see “Definitions” on page 11-2191.

Data Types: `char` | `cell`

### Strike — Strike swap rate values

decimal

Strike swap rate values, specified as a NINST-by-1 vector.

Data Types: `double`

### ExerciseDates — Exercise dates for swaption

serial date number | date character vector | cell array of date character vectors

Exercise dates for the swaption, specified as a NINST-by-1 vector or NINST-by-2 using serial date numbers or date character vectors, depending on the option type.

- For a European option, `ExerciseDates` are a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. When using a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` are a NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed,

or if `ExerciseDates` is `NINST-by-1`, the option can be exercised between the `ValuationDate` of the tree and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

### **Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a `NINST-by-1` vector.

Data Types: `double`

### **Settle — Settlement date**

serial date number | date character vector | cell array of date character vectors

Settlement date (representing the settle date for each swap), specified as a `NINST-by-1` vector of serial date numbers or date character vectors. The `Settle` date for every swaption is set to the `ValuationDate` of the HJM tree. The swap argument `Settle` is ignored. The underlying swap starts at the maturity of the swaption.

Data Types: `double` | `char`

### **Maturity — Maturity date for swap**

serial date number | date character vector | cell array of date character vectors

Maturity date for each swap, specified as a `NINST-by-1` vector of dates using serial date numbers or date character vectors.

Data Types: `double` | `char` | `cell`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, ExerciseDates, Spread, Settle, Maturity, 'SwapReset', 4, 'Basis', 5, 'Principal', 10000)`

### **AmericanOpt — Option type**

0 (European) (default) | integer with values 0 or 1

(Optional) Option type, specified as the comma-separated pair consisting of 'AmericanOpt' and NINST-by-1 positive integer flags with values:

- 0 — European
- 1 — American

Data Types: double

### **SwapReset — Reset frequency per year for underlying swap**

1 (default) | numeric

Reset frequency per year for the underlying swap, specified as the comma-separated pair consisting of 'SwapReset' and a NINST-by-1 vector or NINST-by-2 matrix representing the reset frequency per year for each leg. If SwapReset is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree for each instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If Basis is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)

- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: `double`

### **Principal — Notional principal amount**

100 (default) | `numeric`

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: `double`

### **Options — Derivatives pricing options structure**

`structure`

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: `struct`

## **Output Arguments**

### **Price — Expected prices of swaptions at time 0**

`vector`

Expected prices of the swaptions at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

`structure`

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Call Swaption

A call swaption or payer swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

### Put Swaption

A put swaption or receiver swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

## See Also

[hjmtree](#) | [instswaption](#) | [swapbyhjm](#)

## Topics

["Computing Instrument Prices"](#) on page 2-99

["Pricing Options Structure"](#) on page B-2

["Understanding Interest-Rate Tree Models"](#) on page 2-79

["Supported Interest-Rate Instruments"](#) on page 2-2

**Introduced before R2006a**

## swaptionbyhw

Price swaption from Hull-White interest-rate tree

### Syntax

```
[Price,PriceTree] = swaptionbyhw(HWTree,OptSpec,Strike,  
ExerciseDates,Spread,Settle,Maturity)  
[Price,PriceTree] = swaptionbyhw( ____,Name,Value)
```

### Description

[Price,PriceTree] = swaptionbyhw(HWTree,OptSpec,Strike,ExerciseDates,Spread,Settle,Maturity) prices swaption using a Hull-White tree.

[Price,PriceTree] = swaptionbyhw( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a 3-Year Put Swaption Using an HW Interest-Rate Tree

This example shows how to price a 3-year put swaption using an HW interest-rate tree with the following data.

```
Rates =0.075 * ones (10,1);  
Compounding = 2;  
StartDates = ['jan-1-2007';'jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009';...  
'jul-1-2009';'jan-1-2010'; 'jul-1-2010';'jan-1-2011';'jul-1-2011'];  
EndDates =['jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009';'jul-1-2009';...  
'jan-1-2010';'jul-1-2010';'jan-1-2011';'jul-1-2011';'jan-1-2012'];  
ValuationDate = 'jan-1-2007';  
  
% define the RatesSpec  
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates',...
```



```

EndDates, 'Compounding', Compounding);

% use HWVolSpec to compute the interest-rate volatility
Volatility = 0.05*ones(10,1);
AlphaCurve = 0.01*ones(10,1);
AlphaDates = EndDates;
HWVolSpec = hwwolspec(ValuationDate, EndDates, Volatility, AlphaDates, AlphaCurve);

% use HWTTimeSpec to specify the structure of the time layout for an HW interest-rate tree
HWTTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding);

% build the HW tree
HWTTree = hwtree(HWVolSpec, RateSpec, HWTTimeSpec);

% use the following arguments for a 1-year swap and 3-year swaption
ExerciseDates = 'jan-1-2010';
SwapSettlement = ExerciseDates;
SwapMaturity = 'jan-1-2012';
Spread = 0;
SwapReset = 2 ;
Principal = 100;
OptSpec = 'put';
Strike= 0.04;
Basis=1;

% price the swaption
PriceSwaption = swaptionbyhw(HWTTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, ...
'Basis', Basis, 'Principal', Principal)

PriceSwaption = 2.9201

```

### Price a 3-Year Put Swaption with Receiving and Paying Legs Using an HW Interest-Rate Tree

This example shows how to price a 3-year put swaption with receiving and paying legs using an HW interest-rate tree with the following data.

```

Rates =0.075 * ones (10,1);
Compounding = 2;
StartDates = ['jan-1-2007'; 'jul-1-2007'; 'jan-1-2008'; 'jul-1-2008'; 'jan-1-2009'; ...
'jul-1-2009'; 'jan-1-2010'; 'jul-1-2010'; 'jan-1-2011'; 'jul-1-2011'];

```

```
EndDates = ['jul-1-2007'; 'jan-1-2008'; 'jul-1-2008'; 'jan-1-2009'; 'jul-1-2009'; ...  
'jan-1-2010'; 'jul-1-2010'; 'jan-1-2011'; 'jul-1-2011'; 'jan-1-2012'];  
ValuationDate = 'jan-1-2007';
```

Define the RatesSpec.

```
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', ...  
EndDates, 'Compounding', Compounding);
```

Use HWVolSpec to compute the interest-rate volatility.

```
Volatility = 0.05*ones(10,1);  
AlphaCurve = 0.01*ones(10,1);  
AlphaDates = EndDates;  
HWVolSpec = hwwolspec(ValuationDate, EndDates, Volatility, AlphaDates, AlphaCurve);
```

Use HWTimeSpec to specify the structure of the time layout for an HW interest-rate tree.

```
HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding);
```

Build the HW tree.

```
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Use the following arguments for a 1-year swap and 3-year swaption

```
ExerciseDates = 'jan-1-2010';  
SwapSettlement = ExerciseDates;  
SwapMaturity = 'jan-1-2012';  
Spread = 0;  
SwapReset = [2 2]; % 1st column represents receiving leg, 2nd column represents paying leg  
Principal = 100;  
OptSpec = 'put';  
Strike = 0.04;  
Basis = [1 3]; % 1st column represents receiving leg, 2nd column represents paying leg
```

Price the swaption.

```
PriceSwaption = swaptionbyhw(HWTree, OptSpec, Strike, ExerciseDates, ...  
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, ...  
'Basis', Basis, 'Principal', Principal)
```

```
PriceSwaption = 2.9201
```

## Input Arguments

### **HWTtree — Interest-rate tree structure**

structure

Interest-rate tree structure, specified by using `hwtree`.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vector with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors. For more information, see “Definitions” on page 11-2199.

Data Types: `char` | `cell`

### **Strike — Strike swap rate values**

decimal

Strike swap rate values, specified as a NINST-by-1 vector.

Data Types: `double`

### **ExerciseDates — Exercise dates for swaption**

serial date number | date character vector | cell array of date character vectors

Exercise dates for the swaption, specified as a NINST-by-1 vector or NINST-by-2 using serial date numbers or date character vectors, depending on the option type.

- For a European option, `ExerciseDates` are a NINST-by-1 vector of exercise dates. Each row is the schedule for one option. When using a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` are a NINST-by-2 vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is NINST-by-1, the option can be exercised between the `ValuationDate` of the tree and the single listed `ExerciseDate`.

Data Types: `double` | `char` | `cell`

**Spread — Number of basis points over reference rate**

numeric

Number of basis points over the reference rate, specified as a NINST-by-1 vector.

Data Types: double

**Settle — Settlement date**

serial date number | date character vector | cell array of date character vectors

Settlement date (representing the settle date for each swap), specified as a NINST-by-1 vector of serial date numbers or date character vectors. The `Settle` date for every swaption is set to the `ValuationDate` of the HW tree. The swap argument `Settle` is ignored. The underlying swap starts at the maturity of the swaption.

Data Types: double | char

**Maturity — Maturity date for swap**

serial date number | date character vector | cell array of date character vectors

Maturity date for each swap, specified as a NINST-by-1 vector of dates using serial date numbers or date character vectors.

Data Types: double | char | cell

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

```
Example: [Price, PriceTree] = swaptionbyhw(HWTree, OptSpec,  
ExerciseDates, Spread, Settle, Maturity, 'SwapReset', 4, 'Basis', 5, 'Princi  
pal', 10000)
```

**AmericanOpt — Option type**

0 (European) (default) | integer with values 0 or 1

(Optional) Option type, specified as the comma-separated pair consisting of `'AmericanOpt'` and NINST-by-1 positive integer flags with values:

- 0 — European

- 1 — American

Data Types: double

### **SwapReset — Reset frequency per year for underlying swap**

1 (default) | numeric

Reset frequency per year for the underlying swap, specified as the comma-separated pair consisting of 'SwapReset' and a NINST-by-1 vector or NINST-by-2 matrix representing the reset frequency per year for each leg. If SwapReset is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis representing the basis used when annualizing the input forward rate tree for each instrument, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If Basis is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

### **Options — Derivatives pricing options structure**

structure

Derivatives pricing options structure, specified as the comma-separated pair consisting of 'Options' and a structure obtained from using `derivset`.

Data Types: struct

## **Output Arguments**

### **Price — Expected prices of swaptions at time 0**

vector

Expected prices of the swaptions at time 0, returned as a NINST-by-1 vector.

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, returned as a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node.

Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Call Swaption

A call swaption or payer swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

### Put Swaption

A put swaption or receiver swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

## See Also

[hwtree](#) | [instswaption](#) | [swapbyhw](#)

## Topics

["Pricing Using Interest-Rate Tree Models"](#) on page 2-99

["Calibrating Hull-White Model Using Market Data"](#) on page 2-111

["Pricing Options Structure"](#) on page B-2

["Understanding Interest-Rate Tree Models"](#) on page 2-79

["Supported Interest-Rate Instruments"](#) on page 2-2

**Introduced before R2006a**

## swaptionbylg2f

Price European swaption using Linear Gaussian two-factor model

### Syntax

```
Price = swaptionbylg2f(ZeroCurve,a,b,sigma,eta,rho,Strike,  
ExerciseDate,Maturity)  
Price = swaptionbylg2f(____,Name,Value)
```

### Description

Price = swaptionbylg2f(ZeroCurve,a,b,sigma,eta,rho,Strike,ExerciseDate,Maturity) returns the European swaption price for a two-factor additive Gaussian interest-rate model.

Price = swaptionbylg2f(\_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a European Swaption Using a Linear Gaussian Two-Factor Model

Define the ZeroCurve, a, b, sigma, eta, and rho parameters to compute the price of the swaption.

```
Settle = datenum('15-Dec-2007');  
  
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';  
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';  
CurveDates = daysadd(Settle,360*ZeroTimes,1);  
  
irdc = IRDataCurve('Zero',Settle,CurveDates,ZeroRates);  
  
a = .07;
```



```

b = .5;
sigma = .01;
eta = .006;
rho = -.7;

Reset = 1;
ExerciseDate = daysadd(Settle,360*5,1);
Maturity = daysadd(ExerciseDate,360*[3;4],1);
Strike = .05;

Price = swaptionbylg2f(irdc,a,b,sigma,eta,rho,Strike,ExerciseDate,Maturity,'Reset',Rese

Price = 2×1

    1.1870
    1.5633

```

## Input Arguments

### ZeroCurve — Zero-curve for Linear Gaussian two-factor model

structure

Zero-curve for the Linear Gaussian two-factor model, specified using IRDataCurve or RateSpec.

Data Types: struct

### a — Mean reversion for first factor for Linear Gaussian two-factor model

scalar

Mean reversion for first factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: single | double

### b — Mean reversion for second factor for Linear Gaussian two-factor model

scalar

Mean reversion for second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: single | double

**sigma — Volatility for first factor for Linear Gaussian two-factor model**

scalar

Volatility for first factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: `single` | `double`**eta — Volatility for second factor for Linear Gaussian two-factor model**

scalar

Volatility for second factor for the Linear Gaussian two-factor model, specified as a scalar.

Data Types: `single` | `double`**rho — Scalar correlation of the factors**

scalar

Scalar correlation of the factors, specified as a scalar.

Data Types: `single` | `double`**Strike — Swaption strike price**

nonnegative integer | vector of nonnegative integers

Swaption strike price, specified as a nonnegative integer using a `NumSwaptions-by-1` vector.Data Types: `single` | `double`**ExerciseDate — Swaption exercise dates**

vector of serial date numbers | character vector of dates

Swaption exercise dates, specified as a `NumSwaptions-by-1` vector of serial date numbers or date character vectors.Data Types: `single` | `double` | `char` | `cell`**Maturity — Underlying swap maturity date**

vector of serial date numbers | character vector of dates

Underlying swap maturity date, specified using a `NumSwaptions-by-1` vector of serial date numbers or date character vectors.Data Types: `single` | `double` | `char` | `cell`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = swaptionbylg2f(ir,dc,a,b,sigma,eta,rho,Strike,ExerciseDate,Maturity,'Reset',1,'Notional',100,'OptSpec','call')`

### Reset — Frequency of swaption payments per year

2 (default) | positive integer from the set [1,2,3,4,6,12] | vector of positive integers from the set [1,2,3,4,6,12]

Frequency of swaption payments per year, specified as the comma-separated pair consisting of 'Reset' and positive integers for the values 1,2,4,6,12 in a `NumSwaptions-by-1` vector.

Data Types: `single` | `double`

### Notional — Notional value of swaption

100 (default) | nonnegative integer | vector of nonnegative integers

Notional value of swaption, specified as the comma-separated pair consisting of 'Notional' and a nonnegative integer using a `NumSwaptions-by-1` vector of notional amounts.

Data Types: `single` | `double`

### OptSpec — Option specification for the swaption

'call' (default) | character vector with value of 'call' or 'put' | cell array of character vectors with values of 'call' or 'put'

Option specification for the swaption, specified as the comma-separated pair consisting of 'OptSpec' and a character vector or a `NumSwaptions-by-1` cell array of character vectors with a value of 'call' or 'put'.

A 'call' swaption or Payer swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A 'put' swaption or Receiver swaption allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

Data Types: char | cell

## Output Arguments

### Price — Swaption price

scalar | vector

Swaption price, returned as a scalar or an NumSwaptions-by-1 vector.

## Algorithms

The following defines the swaption price for a two-factor additive Gaussian interest-rate model, given the ZeroCurve, a, b, sigma, eta, and rho parameters:

$$r(t) = x(t) + y(t) + \phi(t)$$

$$dx(t) = -ax(t)dt + \sigma dW_1(t), \quad x(0) = 0$$

$$dy(t) = -by(t)dt + \eta dW_2(t), \quad y(0) = 0$$

where  $dW_1(t)dW_2(t) = \rho dt$  is a two-dimensional Brownian motion with correlation  $\rho$  and  $\phi$  is a function chosen to match the initial zero curve.

## References

- [1] Brigo, D. and F. Mercurio. *Interest Rate Models - Theory and Practice*. Springer Finance, 2006.

## See Also

LinearGaussian2F | capbylg2f | floorbylg2f

## Topics

“Price Swaptions with Interest-Rate Models Using Simulation” on page 2-123

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2013a**

## swaptionbynormal

Price swaptions using Normal or Bachelier option pricing model

### Syntax

```
Price = swaptionbynormal(RateSpec,OptSpec,Strike,Settle,  
ExerciseDates,Maturity,Volatility)  
Price = swaptionbynormal( ____,Name,Value)
```

### Description

Price = swaptionbynormal(RateSpec,OptSpec,Strike,Settle,ExerciseDates,Maturity,Volatility) prices swaptions using the Normal or Bachelier option pricing model.

Price = swaptionbynormal( \_\_\_\_,Name,Value) adds optional name-value pair arguments.

### Examples

#### Price a Swaption Using the Normal Model

Define the zero curve, and create a RateSpec.

```
Settle = datenum('20-Jan-2016');  
ZeroTimes = [.5 1 2 3 4 5 7 10 20 30]';  
ZeroRates = [0.0052 0.0055 0.0061 0.0073 0.0094 0.0119 0.0168 0.0222 0.0293 0.0307]';  
ZeroDates = datemnth(Settle,12*ZeroTimes);  
RateSpec = intenvset('StartDate',Settle,'EndDates',ZeroDates,'Rates',ZeroRates)  
  
RateSpec = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 2  
    Disc: [10x1 double]
```

```

        Rates: [10x1 double]
        EndTimes: [10x1 double]
        StartTimes: [10x1 double]
        EndDates: [10x1 double]
        StartDates: 736349
ValuationDate: 736349
        Basis: 0
        EndMonthRule: 1

```

Define the swaption.

```

ExerciseDate = datenum('20-Jan-2021');
Maturity = datenum('20-Jan-2026');
OptSpec = 'call';
LegReset = [1 1];

```

Compute the par swap rate.

```

[~,ParSwapRate] = swapybyzero(RateSpec,[NaN 0],Settle,Maturity,'LegReset',LegReset)
ParSwapRate = 0.0216

Strike = ParSwapRate;
BlackVol = .3;
NormalVol = BlackVol*ParSwapRate;

```

Price with Black volatility.

```

Price = swaptionbyblk(RateSpec,OptSpec,Strike,Settle,ExerciseDate,Maturity,BlackVol)
Price = 5.9756

```

Price with Normal volatility.

```

Price_Normal = swaptionbynormal(RateSpec,OptSpec,Strike,Settle,ExerciseDate,Maturity,NormalVol)
Price_Normal = 5.5537

```

### Price a Swaption with a Receiving and Paying Leg Using the Normal Model

Create a RateSpec.

```
Rate = 0.06;
Compounding = -1;
ValuationDate = 'Jan-1-2010';
EndDates = 'Jan-1-2020';
Basis = 1;
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, ...
    'EndDates', EndDates, 'Rates', Rate, 'Compounding', Compounding, 'Basis', Basis);
```

Define the swaption.

```
ExerciseDate = datenum('20-Jan-2021');
Maturity = datenum('20-Jan-2026');
Settle = 'Jan-1-2010';
OptSpec = 'call';
Strike = .09;
NormalVol = .03;
Reset = [1 4]; % 1st column represents receiving leg, 2nd column represents paying leg
Basis = [1 7]; % 1st column represents receiving leg, 2nd column represents paying leg
```

Price with Normal volatility.

```
Price_Normal = swaptionbynormal(RateSpec, OptSpec, Strike, Settle, ExerciseDate, Maturity, NormalVol);
Price_Normal = 5.9084
```

### Price a Swaption Using `swaptionbynormal` and Compare to `swaptionbyblk`

Define the RateSpec.

```
Settle = datenum('20-Jan-2016');
ZeroTimes = [.5 1 2 3 4 5 7 10 20 30]';
ZeroRates = [0.0052 0.0055 0.0061 0.0073 0.0094 0.0119 0.0168 0.0222 0.0293 0.0307]';
ZeroDates = datemnth(Settle, 12*ZeroTimes);
RateSpec = intenvset('StartDate', Settle, 'EndDates', ZeroDates, 'Rates', ZeroRates)
```

```
RateSpec = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
    Disc: [10x1 double]
    Rates: [10x1 double]
    EndTimes: [10x1 double]
    StartTimes: [10x1 double]
    EndDates: [10x1 double]
```



```

    StartDates: 736349
    ValuationDate: 736349
        Basis: 0
    EndMonthRule: 1

```

Define the swaption instrument and price with `swaptionbyblk`.

```

ExerciseDate = datenum('20-Jan-2021');
Maturity = datenum('20-Jan-2026');
OptSpec = 'call';

[~,ParSwapRate] = swapybyzero(RateSpec,[NaN 0],Settle,Maturity,'StartDate',ExerciseDate);

ParSwapRate = 0.0326

Strike = ParSwapRate;
BlackVol = .3;
NormalVol = BlackVol*ParSwapRate;

Price = swaptionbyblk(RateSpec,OptSpec,Strike,Settle,ExerciseDate,Maturity,BlackVol)

Price = 3.6908

```

Price the swaption instrument using `swaptionbynormal`.

```

Price_Normal = swaptionbynormal(RateSpec,OptSpec,Strike,Settle,ExerciseDate,Maturity,NormalVol);

Price_Normal = 3.7602

```

Price the swaption instrument using `swaptionbynormal` for a negative strike.

```

Price_Normal = swaptionbynormal(RateSpec,OptSpec,-.005,Settle,ExerciseDate,Maturity,NormalVol);

Price_Normal = 16.3674

```

## Input Arguments

**RateSpec** — Interest-rate term structure  
structure

Interest-rate term structure (annualized and continuously compounded), specified by the `RateSpec` obtained from `intenvset`. For information on the interest-rate specification, see `intenvset`.

If the discount curve for the paying leg is different than the receiving leg, `RateSpec` can be a NINST-by-2 input variable of `RateSpecs`, with the second input being the discount curve for the paying leg. If only one curve is specified, then it is used to discount both legs.

Data Types: `struct`

### **OptSpec — Definition of option**

character vector with values 'call' or 'put' | cell array of character vector with values 'call' or 'put'

Definition of the option as 'call' or 'put', specified as a NINST-by-1 cell array of character vectors.

A 'call' swaption, or Payer swaption, allows the option buyer to enter into an interest-rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A 'put' swaption, or Receiver swaption, allows the option buyer to enter into an interest-rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

Data Types: `char` | `cell`

### **Strike — Strike swap rate values**

decimal

Strike swap rate values, specified as a NINST-by-1 vector of decimal values.

Data Types: `double`

### **Settle — Settlement date**

serial date number | date character vector | cell array of date character vectors | datetime object | string object

Settlement date (representing the settle date for each swaption), specified as a NINST-by-1 vector of serial date numbers, or cell array of date character vectors, datetime objects, or string objects. `Settle` must not be later than `ExerciseDates`.

The `Settle` date input for `swaptionbynormal` is the valuation date on which the swaption (an option to enter into a swap) is priced. The swaption buyer pays this price on this date to hold the swaption.

Data Types: `double` | `char` | `cell` | `datetime` | `string`

### **ExerciseDates — Dates on which swaption expires and underlying swap starts**

serial date number | date character vector | cell array of date character vectors | datetime object

Dates on which the swaption expires and the underlying swap starts, specified as a NINST-by-1 vector of serial date numbers, or cell array of date character vectors, datetime objects, or string objects. There is only one `ExerciseDate` on the option expiry date. This is also the `StartDate` of the underlying forward swap.

Data Types: `double` | `char` | `cell` | `datetime` | `string`

### **Maturity — Maturity date for each forward swap**

serial date number | date character vector | cell array of date character vectors | datetime object

Maturity date for each forward swap, specified as a NINST-by-1 vector of dates using serial date numbers, cell array of date character vectors, datetime objects, or string objects.

Data Types: `double` | `char` | `cell` | `datetime` | `string`

### **Volatility — Volatilities values**

numeric

Volatilities values (for normal volatility), specified as a NINST-by-1 vector of numeric values.

For more information on the Normal model, see “Work with Negative Interest Rates” on page 2-22.

Data Types: `double`

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes.

You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `Price = swaptionbynormal(OISCurve, OptSpec, Strike, Settle, ExerciseDate, Maturity, NormalVol, 'Reset', 4)`

### **Reset — Reset frequency per year for underlying forward swap**

1 (default) | numeric

Reset frequency per year for the underlying forward swap, specified as the comma-separated pair consisting of 'Reset' and a NINST-by-1 vector or NINST-by-2 matrix representing the reset frequency per year for each leg. If `Reset` is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

Data Types: double

### **Basis — Day-count basis of instrument**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis of the instrument representing the basis used when annualizing the input term structure, specified as the comma-separated pair consisting of 'Basis' and a NINST-by-1 vector or NINST-by-2 matrix representing the basis for each leg. If `Basis` is NINST-by-2, the first column represents the receiving leg, while the second column represents the paying leg.

Values are:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)

- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **Principal — Notional principal amount**

100 (default) | numeric

Notional principal amount, specified as the comma-separated pair consisting of 'Principal' and a NINST-by-1 vector.

Data Types: double

### **ProjectionCurve — Rate curve used in projecting future cash flows**

if `ProjectionCurve` is not specified, then `RateSpec` is used both for discounting cash flows and projecting future cash flows (default) | structure

The rate curve to be used in projecting the future cash flows, specified as the comma-separated pair consisting of 'ProjectionCurve' and a rate curve structure. This structure must be created using `intenvset`. Use this optional input if the forward curve is different from the discount curve.

Data Types: struct

## **Output Arguments**

### **Price — Prices for swaptions at time 0**

vector

Prices for the swaptions at time 0, returned as a NINST-by-1 vector of prices.

## Definitions

### Call Swaption

A Call swaption or Payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

### Put Swaption

A Put swaption or Receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

## See Also

`capbynormal` | `floorbynormal` | `intenvset` | `swaptionbyblk`

## Topics

“Work with Negative Interest Rates” on page 2-22

“Supported Interest-Rate Instruments” on page 2-2

## External Websites

How to Price Interest Rate Options with Negative Interest Rates (3 min 05 sec)

## Introduced in R2017a

# time2date

Dates from time and frequency

## Syntax

Dates = time2date(Settle,Times,Compounding,Basis,EndMonthRule)

## Arguments

Settle	Settlement date. A vector of serial date numbers or date character vectors.
Times	Vector of times corresponding to the compounding value. Times must be equal to or greater than 0.
Compounding	<p>(Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = 2. This argument determines the formula for the discount factors:</p> <p>Compounding = 1, 2, 3, 4, 6, 12 = F</p> <p><math>Disc = (1 + Z/F)^{-T}</math>, where F is the compounding frequency, Z is the zero rate, and T is the time in periodic units; for example, T = F is 1 year.</p> <p>Compounding = 365</p> <p><math>Disc = (1 + Z/F)^{-T}</math>, where F is the number of days in the basis year and T is a number of days elapsed computed by basis.</p> <p>Compounding = -1</p> <p><math>Disc = \exp(-T*Z)</math>, where T is time in years.</p>

<p><b>Basis</b></p>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>EndMonthRule</b></p>	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>

## Description

`Dates = time2date(Settle, Times, Compounding, Basis, EndMonthRule)` computes dates corresponding to compounded rate quotes between `Settle` and time factors.

---

**Note** To obtain accurate results from this function, the `Basis` and `Dates` arguments must be consistent. If the `Dates` argument contains months that have 31 days, `Basis`



must be one of the values that allow months to contain more than 30 days; for example, `Basis = 0, 3, or 7`.

---

The `time2date` function is the inverse of `date2time`.

## Examples

Show that `date2time` and `time2date` are the inverse of each other. First compute the time factors using `date2time`.

```
Settle = '1-Sep-2002';
Dates = datenum(['31-Aug-2005'; '28-Feb-2006'; '15-Jun-2006';
                '31-Dec-2006']);
Compounding = 2;
Basis = 0;
EndMonthRule = 1;
Times = date2time(Settle, Dates, Compounding, Basis,...
                  EndMonthRule)
```

Times =

```
5.9945
6.9945
7.5738
8.6576
```

Now use the calculated `Times` in `time2date` and compare the calculated dates with the original set.

```
Dates_calc = time2date(Settle, Times, Compounding, Basis,...
                       EndMonthRule)
```

Dates\_calc =

```
732555
732736
732843
733042
```

datestr(Dates\_calc)

ans =

```
31-Aug-2005
28-Feb-2006
15-Jun-2006
31-Dec-2006
```

## **See Also**

cfamounts | cftimes | date2time | disc2rate | rate2disc

## **Topics**

“Modeling the Interest-Rate Term Structure” on page 2-67

“Interest-Rate Term Conversions” on page 2-62

“Interest Rates Versus Discount Factors” on page 2-55

“Understanding the Interest-Rate Term Structure” on page 2-55

**Introduced before R2006a**

# treepath

Entries from node of recombining binomial tree

## Syntax

Values = treepath(Tree,BranchList)

## Arguments

Tree	Recombining binomial tree.
BranchList	Number of paths (NUMPATHS) by path length (PATHLENGTH) matrix containing the sequence of branchings.

## Description

Values = treepath(Tree,BranchList) extracts entries of a node of a recombining binomial tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number one, the second-to-top is two, and so on. Set the branch sequence to zero to obtain the entries at the root node.

Values is a number of values (NUMVALS)-by-NUMPATHS matrix containing the retrieved entries of a recombining tree.

## Examples

Create a BDT tree by loading the example file.

```
load deriv.mat;
```

Then

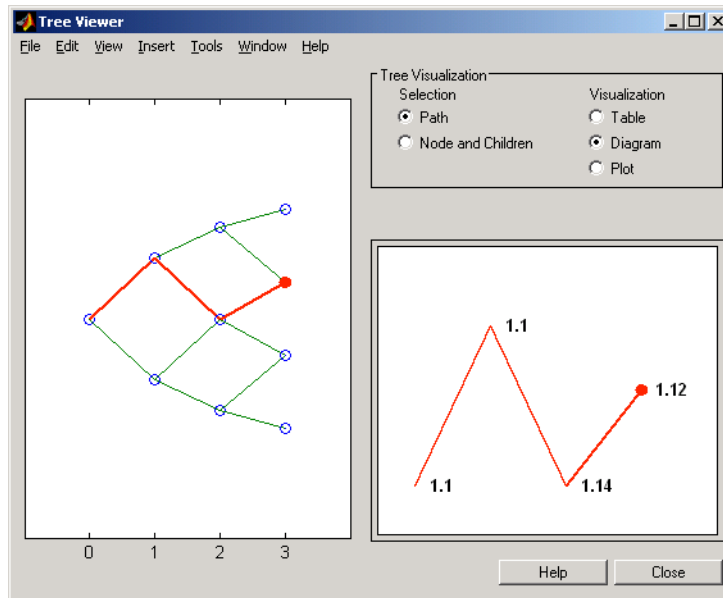
```
FwdRates = treepath(BDTree.FwdTree, [1 2 1])
```

returns the rates at the tree nodes located by taking the up branch, then the down branch, and finally the up branch again.

```
FwdRates =
    1.1000
    1.0979
    1.1377
    1.1183
```

You can visualize this with the `treeviewer` function.

```
treeviewer(BDTree)
```



## See Also

`mktree` | `treeshape`

## Topics

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50

**Introduced before R2006a**

## treeshape

Shape of recombining binomial tree

### Syntax

```
[NumLevels, NumPos, IsPriceTree] = treeshape(Tree)
```

### Arguments

Tree	Recombining binomial tree.
------	----------------------------

### Description

`[NumLevels, NumPos, IsPriceTree] = treeshape(Tree)` returns information on a recombining binomial tree's shape.

`NumLevels` is the number of time levels of the tree.

`NumPos` is a 1-by-`NUMLEVELS` vector containing the length of the state vectors in each level.

`IsPriceTree` is a Boolean determining if a final horizontal branch is present in the tree.

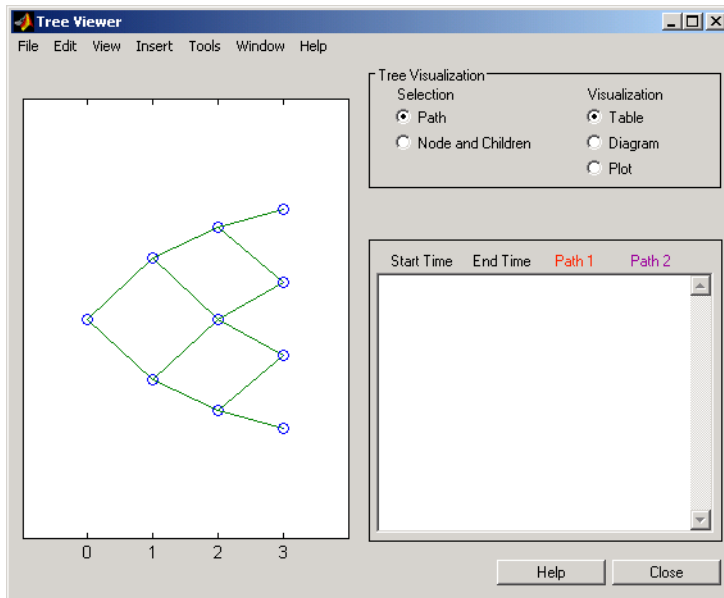
### Examples

Create a BDT tree by loading the example file.

```
load deriv.mat;
```

With `treeviewer` you can see the general shape of the BDT interest-rate tree.

```
treeviewer(BDTree)
```



With this tree

```
[NumLevels, NumPos, IsPriceTree] = treeshape(BDTree.FwdTree)
```

returns

```
NumLevels =
    4
```

```
NumPos =
    1    1    1    1
```

```
IsPriceTree =
    0
```

## See Also

[mktree](#) | [treepath](#)

## Topics

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50

**Introduced before R2006a**



# treeviewer

Tree information

## Syntax

```
treeviewer(Tree)
treeviewer(PriceTree,InstSet)
treeviewer(CFTree,InstSet)
```

## Description

`treeviewer(Tree)` displays an interest rate tree, stock price tree, or money-market tree.

`treeviewer(PriceTree,InstSet)` displays a tree of instrument prices.

If you provide the name of an instrument set (`InstSet`) and you have named the instruments using the field `Name`, the `treeviewer` display identifies the instrument being displayed with its name. (See Example 3 on page 11-2231 for a description.) If you do not provide the optional `InstSet` input argument, the instruments are identified by their sequence number in the instrument set. (See Example 6 on page 11-2235 for a description.)

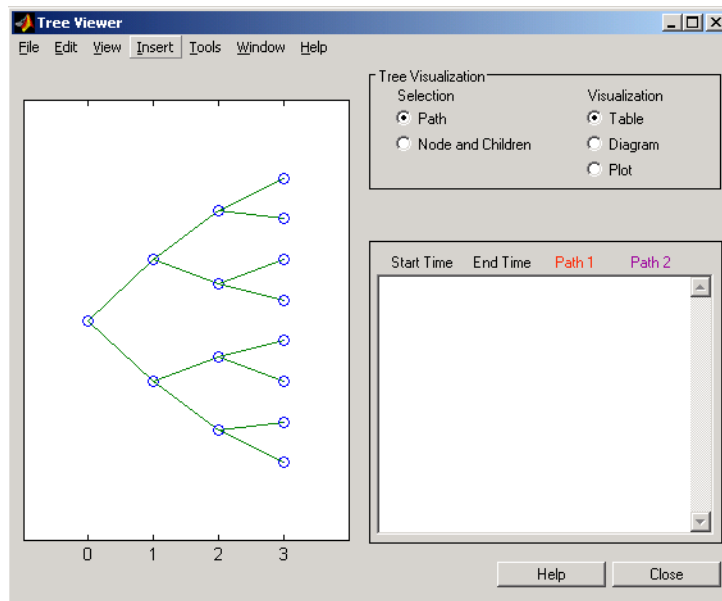
`treeviewer(CFTree,InstSet)` displays a cash flow tree that has been created with `swapbybdt` or `swapbyhjm`. If you provide the name of an instrument set (`InstSet`) containing cash flow names, the `treeviewer` display identifies the instrument being displayed with its name. (See Example 3 on page 11-2231 for a description.) If the optional `InstSet` argument is not present, the instruments are identified by their sequence number in the instrument set. See Example 6 on page 11-2235 for a description.)

## Examples

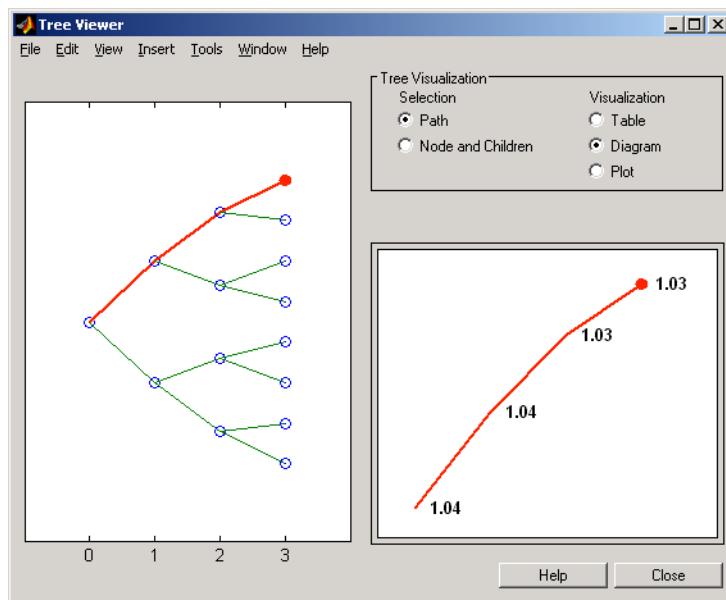
### Display an HJM Interest-Rate Tree

```
load deriv.mat
treeviewer(HJMTree)
```

The `treeviewer` function displays the structure of an HJM tree in the left pane. The tree visualization in the right pane is blank.

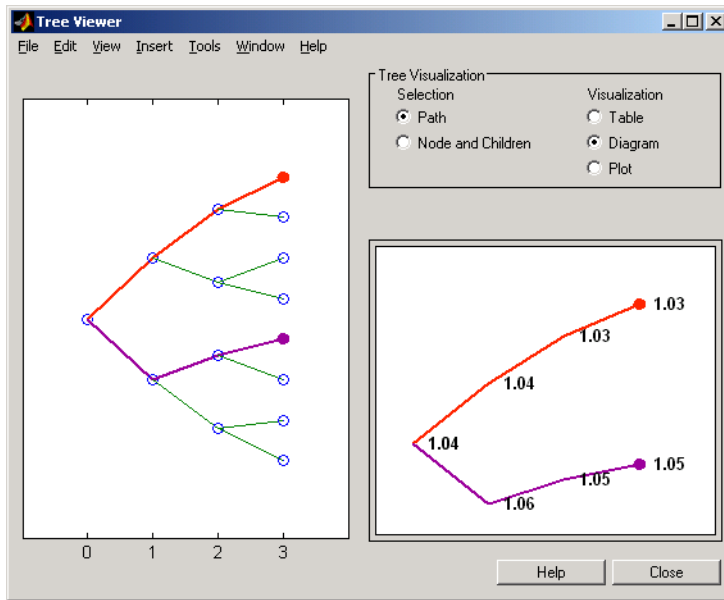


To visualize the actual interest-rate tree, go to the **Tree Visualization** pane and click **Path** (the default) and **Diagram**. Now, select the first path by clicking the last node ( $t = 3$ ) of the upper branch.



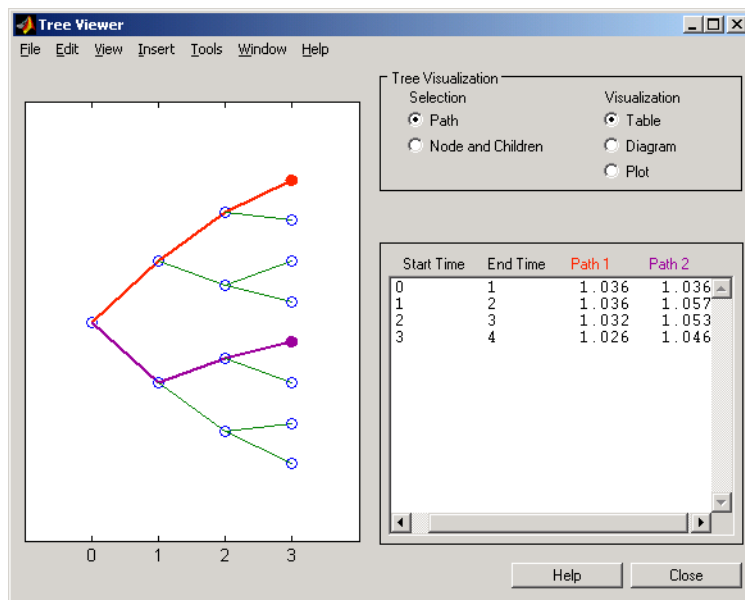
The entire upper path is highlighted in red.

To complete the process, select a second path by clicking the last node ( $t = 3$ ) of another branch. The second path is highlighted in purple. The final display looks like this.

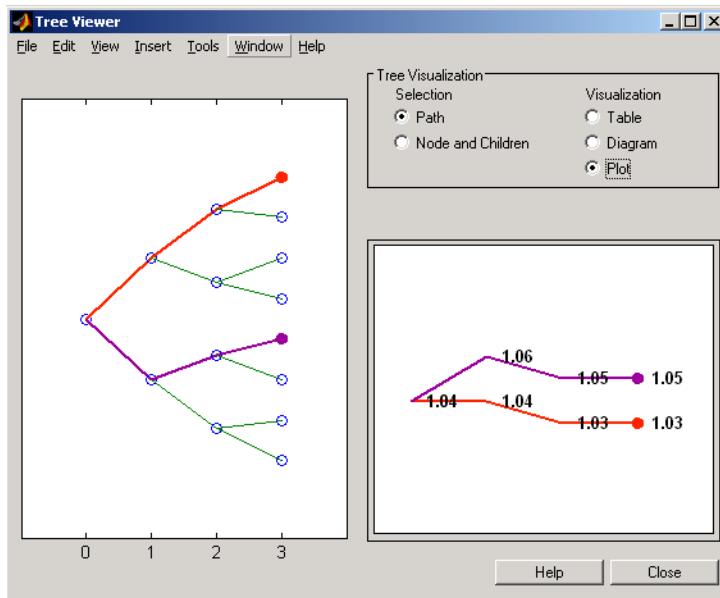


## Alternative Forms of Display

The **Tree Visualization** pane allows you to select alternative ways to display tree data. For example, if you select **Path** and **Table** as your visualization choices, the final display above instead appears in tabular form.

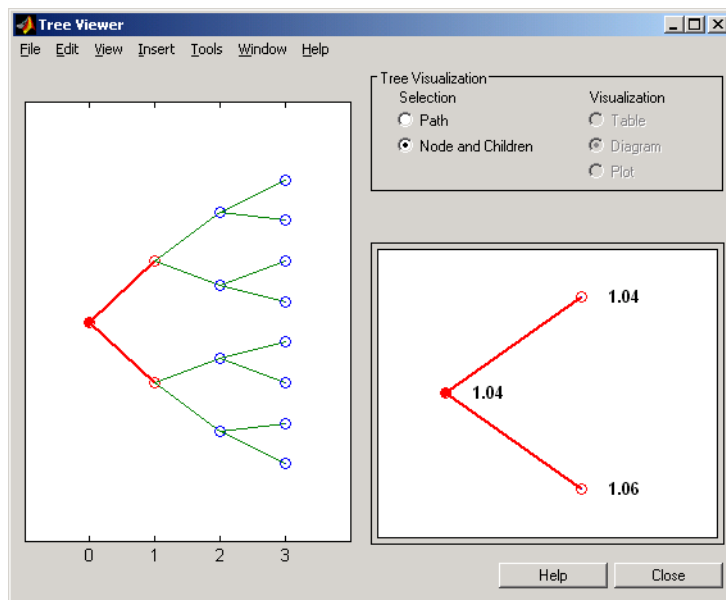


To see a plot of interest rates along the chosen branches, click **Path** and **Plot** in the **Tree Visualization** pane.



With **Plot** selected, rising interest rates are shown on the upper branch and declining interest rates on the lower.

Finally, if you clicked **Node and Children** under **Tree Visualization**, you restrict the data displayed to just the selected parent node and its children.

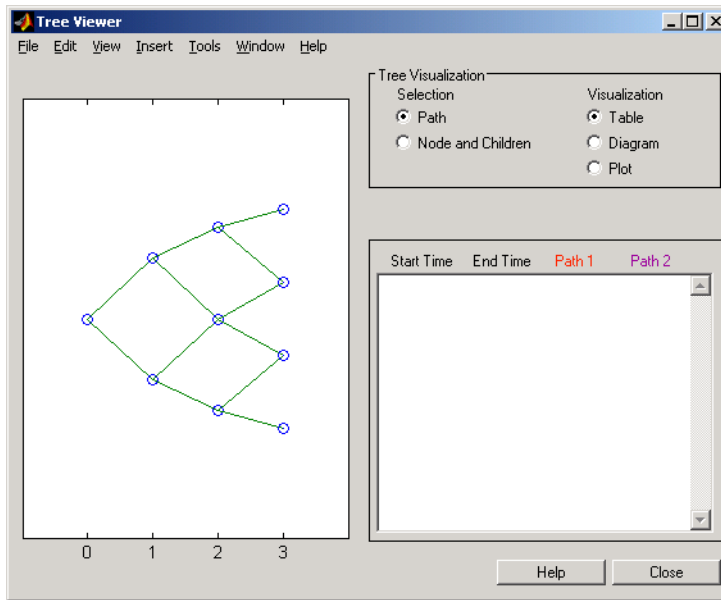


With **Node and Children** selected, the choices under **Visualization** are unavailable.

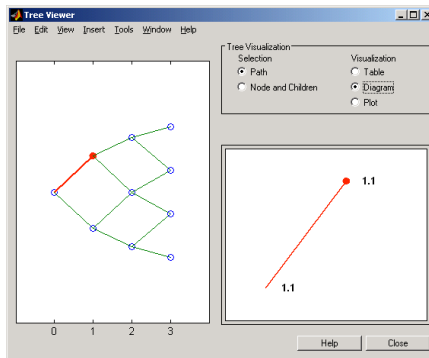
## Display a BDT Interest-Rate Tree

```
load deriv.mat
treeviewer(BDTTree)
```

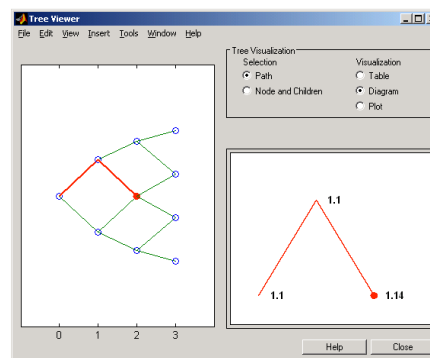
The `treeviewer` function displays the structure of a BDT tree in the left pane. The tree visualization in the right pane is blank.



To visualize the actual interest-rate tree, go to the **Tree Visualization** pane and click **Path** (the default) and **Diagram**. Now, select the first path by clicking the first node of the up branch ( $t = 1$ ). Continue by clicking the down branch at the next node ( $t = 2$ ). The two figures below show the treeviewer path diagrams for these selections.



$t = 1$

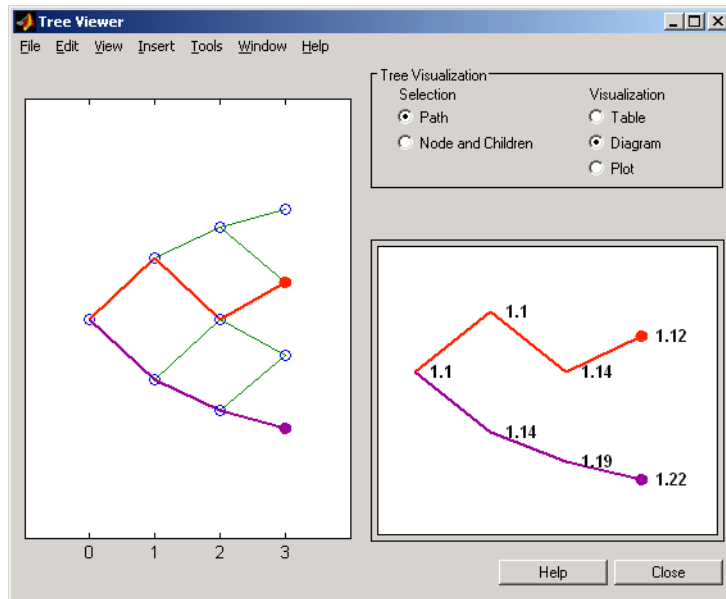


$t = 2$

Continue clicking all nodes in succession until you reach the end of the branch. The entire path you have selected is highlighted in red.

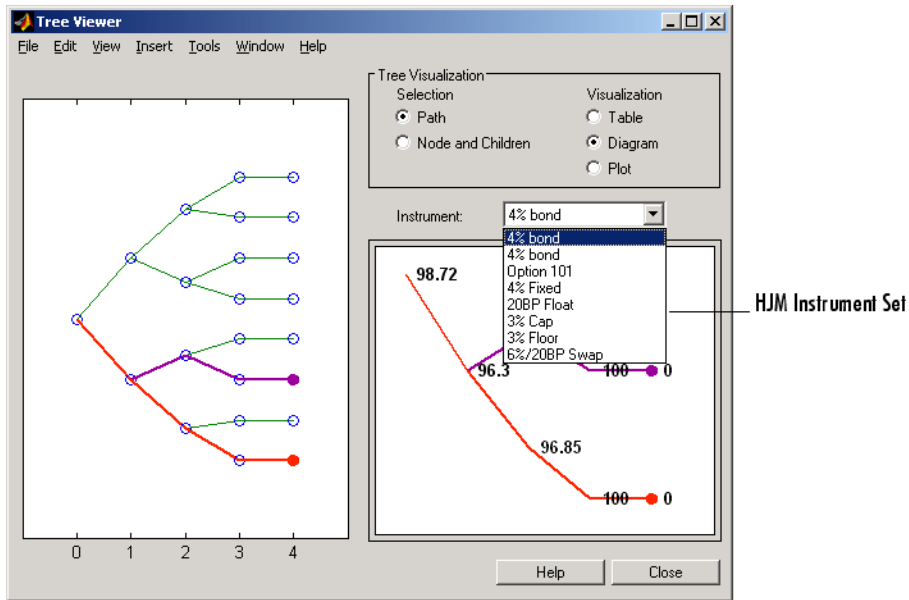


Select a second path by clicking the first node of the lower branch ( $t = 1$ ). Continue clicking lower nodes as you did on the first branch. The second branch is highlighted in purple. The final display looks like this.



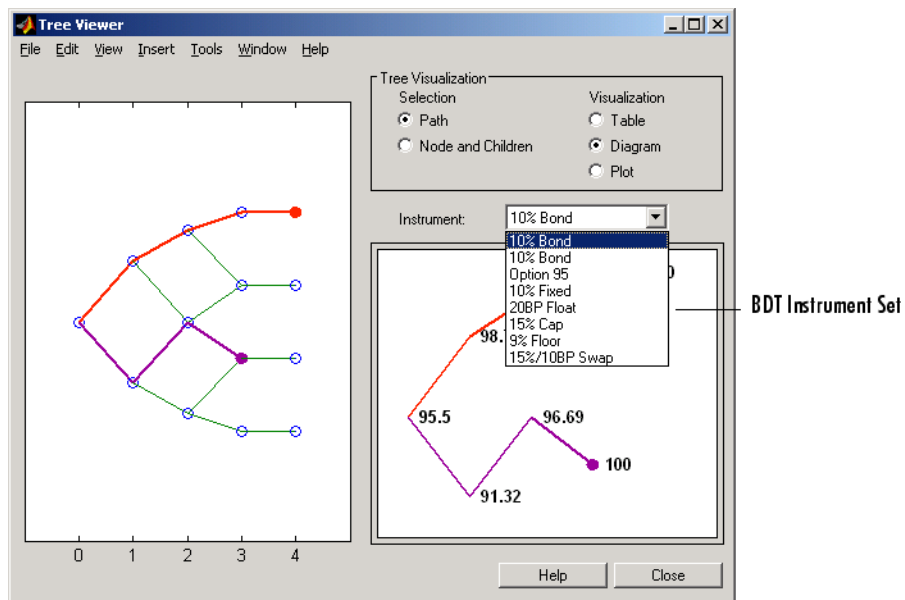
## Display an HJM Price Tree for Named Instruments

```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
treeviewer(PriceTree, HJMInstSet)
```



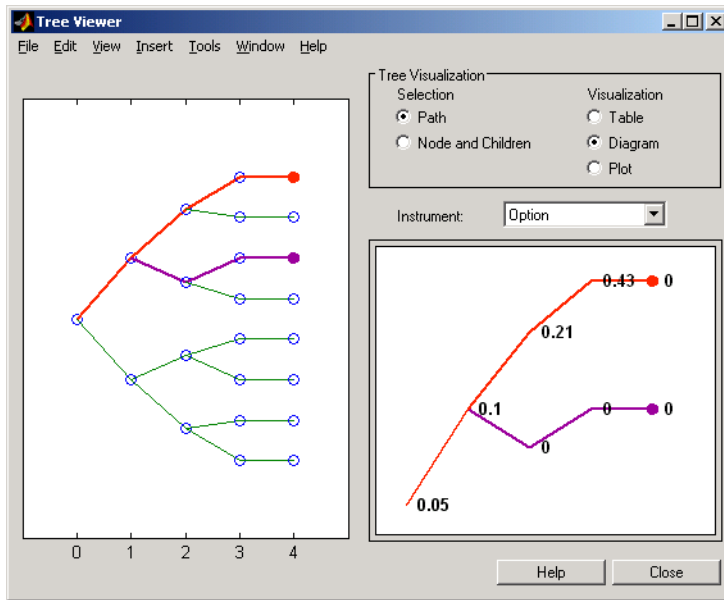
## Display a BDT Price Tree for Named Instruments

```
load deriv.mat
[Price, PriceTree] = bdtprice(BDTTree, BDTInstSet);
treeviewer(PriceTree, BDTInstSet)
```



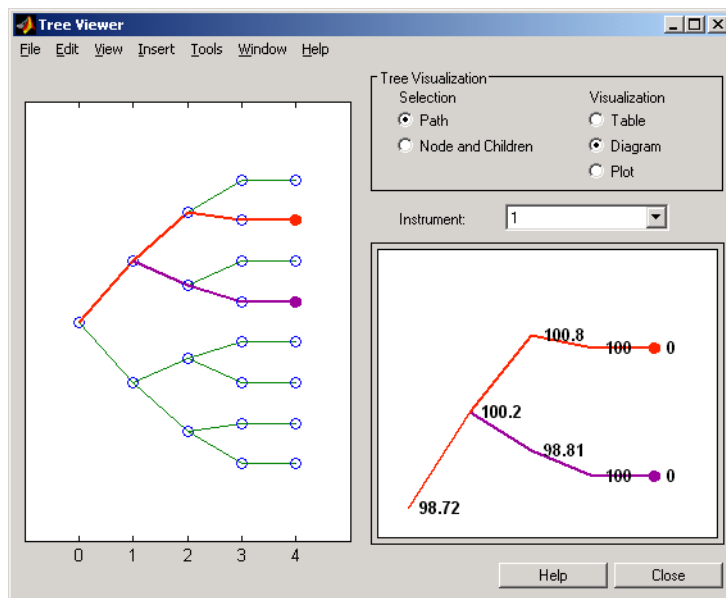
## Display an HJM Price Tree with Renamed Instruments

```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
Names = {'Bond1', 'Bond2', 'Option', 'Fixed', 'Float', 'Cap', ...
        'Floor', 'Swap'};
treeviewer(PriceTree, Names)
```



## Display an HJM Price Tree Using Default Instrument Names (Numbers)

```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
treeviewer(PriceTree)
```



## Input Arguments

### Tree — Interest-rate tree, stock price tree, or money-market tree

[interest-rate tree structure](#) | [stock price tree structure](#) | [money-market tree structure](#)

Interest-rate tree, stock price tree, or money-market tree, specified using the associated tree function.

*Interest-rate trees:*

- Black-Derman-Toy (BDTtree) obtained from `bdttree`
- Black-Karasinski (BKTtree) obtained from `bktree`
- Heath-Jarrow-Morton (HJMTtree) obtained from `hjmtree`
- Hull-White (HWTtree) obtained from `hwtree`
- Cox-Ingersoll-Ross (CIRTtree) obtained from `cirtree`

*Money market trees:*

- Black-Derman-Toy (BDTMMktTree) obtained from `mmktbybdt` for a money-market tree from a BDT interest-rate tree.
- Heath-Jarrow-Morton (HJMMMktTree) obtained from `mmktbyhjm` for a money-market tree from an HJM interest-rate tree.

---

**Note** Money market trees cannot be created from BK or HW interest-rate trees.

---

*Stock price trees:*

- Cox-Ross-Rubinstein (CRRTree) obtained from `crrtree`
- Implied Trinomial tree (ITTree) obtained from `itttree`
- Standard Trinomial tree (STTree) obtained from `sttree`
- Leisen-Reimer stock tree (LRTree) obtained from `lrtree`
- Equal probabilities (EQPTree) obtained from `eqptree`

*Cash flow trees:*

- Black-Derman-Toy (BDTCFTree) obtained as output from the swap function `swapbybdt`
- Heath-Jarrow-Morton (HJMCFTree) obtained as output from the swap function `swapbyhjm`

---

**Note** For the function `swapbybdt`, which uses a recombining binomial tree, this structure contains only NaNs because cash flows cannot be accurately calculated at every tree node for floating-rate notes.

---

Data Types: `struct`

### **PriceTree — Tree structure of instrument prices**

structure

Tree structure of instrument prices, specified as:

- Black-Derman-Toy (BDTPriceTree) obtained from the portfolio function `bdtprice` or the individual functions, such as `bondbybdt`, `capbybdt`, and so on.
- Black-Karasinski (BKPriceTree) obtained from the portfolio function `bkprice` or the individual functions, such as `bondbybk`, `capbybk`, and so on.

- Cox-Ingersoll-Ross (CIRPriceTree) obtained from the portfolio function `cirprice` or the individual functions, such as `bondbycir`, `capbycir`, and so on.
- Heath-Jarrow-Morton (HJMPriceTree) obtained from the portfolio function `hjprice` or the individual functions, such as `bondbyhjm`, `capbyhjm`, and so on.
- Hull-White (HWPriceTree) obtained from the portfolio function `hwprice` or the individual functions, such as `bondbyhw`, `capbyhw`, and so on.
- Leisen-Reimer (LRPriceTree) obtained from the individual function `optstockbylr`.
- Cox-Ross-Rubinstein (CRRPriceTree) obtained from the portfolio function `crrprice` or the individual functions, such as `asianbycrr`, `barrierbycrr`, and so on.
- Equal probabilities (EQPPriceTree) obtained from the portfolio function `eqpprice` or the individual functions, such as `asianbyeqp`, `barrierbyeqp`, and so on.
- Implied Trinomial tree (ITTPriceTree) obtained from the portfolio function `ittpprice` or the individual functions, such as `asianbyitt`, `barrierbyitt`, and so on.
- Standard trinomial tree (STTPriceTree) obtained from the portfolio function `sttpprice` or the individual functions, such as `asianbystt`, `barrierbystt`, and so on.

Data Types: `struct`

### **CFTree — Tree of swap cash flows**

structure

CFTree is a tree of swap cash flows, specified when you create cash flow trees by executing the Black-Derman-Toy (obtained as output from the swap function `swapbybdt`) and Heath-Jarrow-Morton (`swapbyhjm`) swap functions. (Black-Derman-Toy cash flow trees contain only NaNs.)

Data Types: `struct`

### **InstSet — Variable containing a collection of instruments whose prices or cash flows are contained in a tree**

structure

(Optional) Variable containing a collection of instruments whose prices or cash flows are contained in a tree, specified using `instadd`. To display the names of the instruments, the field `Name` should exist in `InstSet`. If `InstSet` is not passed, `treeviewer` uses default instruments names (numbers) when displaying prices or cash flows.

Data Types: `struct`

## Definitions

### Treeviewer Conventions

`treeviewer` price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, so, decreasing prices appear on the lower branch.

Conversely, for interest rate displays, *decreasing* interest rates appear on the upper branch (prices are rising) and *increasing* interest rates on the lower branch (prices are falling).

### Using Treeviewer

`treeviewer` provides an interactive display of prices or interest rates.

The `treeviewer` display is activated by clicking the nodes along the price or interest rate path shown in the left pane when the function is called.

- For HJM trees, you select the endpoints of the path, and `treeviewer` displays all data from beginning to end.
- With recombining trees, such as BDT, BK, HW, and CIR you must click *each* node in succession from the beginning ( $t = 1$ ) to the last node ( $t = n$ ). Do not include the *root node*, the node at  $t = 0$ . If you do not click the nodes in the proper order, you are reminded with the message

Parent of selected node must be selected.

---

**Note** The **Help** button is not available for `treeviewer` in MATLAB Online.

---

## See Also

`bdttree` | `bktree` | `cirtree` | `eqptree` | `hjmtree` | `hwtree` | `instadd` | `itttree` | `lrtree` | `mmktbybdt` | `mmktbyhjm` | `stttree` | `swapbybdt` | `swapbyhjm`

## Topics

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50



**Introduced before R2006a**

## trintreepath

Entries from node of recombining trinomial tree

### Syntax

```
Values = trintreepath(TrinTree,BranchList)
```

### Arguments

TrinTree	Recombining price or interest-rate trinomial tree.
BranchList	Number of paths (NUMPATHS) by path length (PATHLENGTH) matrix containing the sequence of branchings.

### Description

`Values = trintreepath(TrinTree,BranchList)` extracts entries of a node of a recombining trinomial tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number 1, the middle branch is 2, and the bottom branch is 3. Set the branch sequence to 0 to obtain the entries at the root node.

`Values` is a number of values (NUMVALS)-by-NUMPATHS matrix containing the retrieved entries of a recombining tree.

### Examples

Create a Hull-White tree by loading the example file.

```
load deriv.mat;
```

Then, for example

```
FwdRates = trintreepath(HWTree, [1 2 3])
```

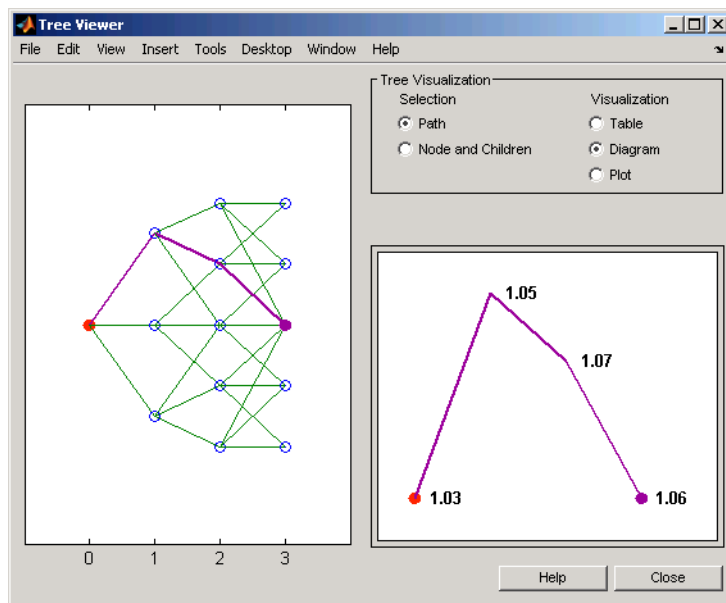
returns the rates at the tree nodes located by starting at 0, taking the up branch at the first node, the middle branch at the second node, and finally the bottom branch at the third node.

FwdRates =

```
1.0279
1.0528
1.0652
1.0591
```

You can visualize this with the `treeviewer` function.

`treeviewer(HWTtree)`



## See Also

`mktrintree`

## Topics

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50

**Introduced before R2006a**

# trintreeshape

Shape of recombining trinomial tree

## Syntax

```
[NumLevels,NumPos,NumStates] = trintreeshape(TrinTree)
```

## Arguments

TrinTree	Recombining price or interest-rate trinomial tree.
----------	--

## Description

`[NumLevels,NumPos,NumStates] = trintreeshape(TrinTree)` returns information on a recombining trinomial tree's shape.

`NumLevels` is the number of time levels of the tree.

`NumPos` is a 1-by-`NUMLEVELS` vector containing the length of the state vectors in each level.

`NumStates` is a 1-by-`NUMLEVELS` vector containing the number of state vectors in each level.

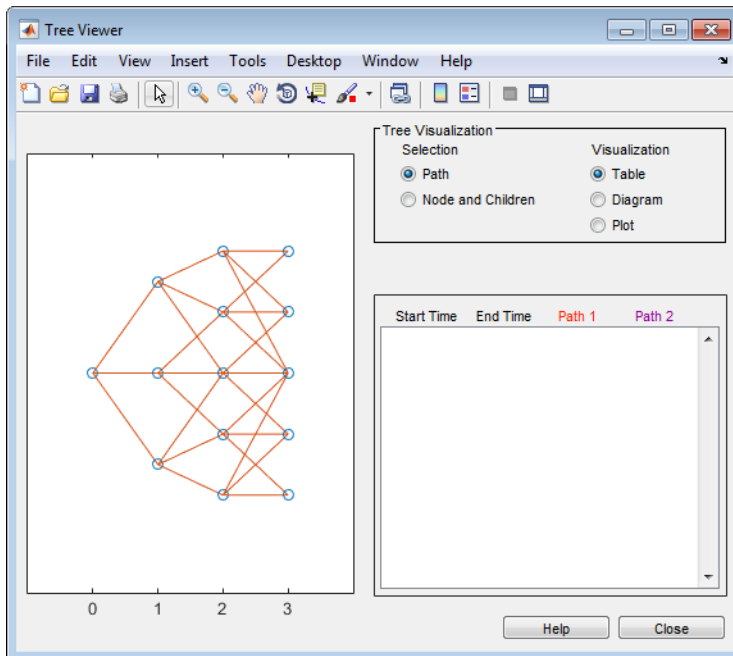
## Examples

Create a Hull-White tree by loading the example file.

```
load deriv.mat;
```

With `treeviewer` you can see the general shape of the HW interest-rate tree.

```
treeviewer(HWTree)
```



With this tree

```
[NumLevels, NumPos, NumStates] = trintreeshape(HWTree)
```

returns

```
NumLevels =
    4
```

```
NumPos =
    1    1    1    1
```

```
NumStates =
    1    3    5    5
```

## See Also

mktrintree | trintreepath

## **Topics**

“Graphical Representation of Trees” on page 2-157

“Overview of Interest-Rate Tree Models” on page 2-50

**Introduced before R2006a**

## agencyoas

Determine option-adjusted spread of callable bond using Agency OAS model

### Syntax

```
OAS = agencyoas(ZeroData,Price,CouponRate,Settle,Maturity,Vol,
CallDate)
OAS = agencyoas( ____,Name,Value)
```

### Description

`OAS = agencyoas(ZeroData,Price,CouponRate,Settle,Maturity,Vol, CallDate)` computes OAS of a callable bond given price using the Agency OAS model.

`OAS = agencyoas( ____,Name,Value)` specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Agency OAS Value

This example shows how to compute the agency OAS value.

```
Settle = datenum('20-Jan-2010');
ZeroRates = [.07 .164 .253 1.002 1.732 2.226 2.605 3.316 ...
3.474 4.188 4.902]'/100;
ZeroDates = daysadd(Settle,360* [.25 .5 1 2 3 4 5 7 10 20 30],1);
ZeroData = [ZeroDates ZeroRates];

Maturity = datenum('30-Dec-2013');
CouponRate = .022;
Price = 99.155;
Vol = .5117;
CallDate = datenum('30-Dec-2010');
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, Vol, CallDate)
```



OAS = 8.5837

## Input Arguments

### **ZeroData — Zero curve**

matrix

Zero curve, specified as an numRates-by-2 matrix where the first column is zero dates and the second column is the accompanying zero rates.

Data Types: double

### **Price — Prices**

vector

Prices specified as an numBonds-by-1 vector.

Data Types: double

### **CouponRate — Coupon rates**

vector in decimals

Coupon rates, specified as an numBonds-by-1 vector in decimals.

Data Types: double

### **Settle — Settlement date**

serial date number

Settlement date, specified as a scalar serial date number.

---

**Note** The `Settle` date must be an identical settlement date for all the bonds and the zero curve.

---

Data Types: double

### **Maturity — Maturity date**

serial date number

Maturity date, specified as a numBonds-by-1 vector.

Data Types: double

**Vol — Volatilities**

vector in decimals

Volatilities specified as a scalar or an `numBonds-by-1` vector in decimals. `Vol` is the volatility of interest rates corresponding to the time of the `CallDate`.

Data Types: double

**CallDate — Call dates**

vector

Call dates, specified as an `numBonds-by-1` vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `OAS =`

```
agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, Vol, CallDate, 'Basis', 7, 'Face', 1000)
```

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of `'Basis'` and a `N-by-1` vector using the following values:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **CurveBasis – Curve basis**

0 (actual/actual) (default) | integer from 0 to 13

Curve basis, specified as the comma-separated pair consisting of 'CurveBasis' and a N-by-1 vector using the following values:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**CurveCompounding — Compounding frequency of the zero curve**

2 (semiannual) (default) | possible values include: -1, 0, 1, 2, 3, 4, 6, 12.

Compounding frequency of the zero curve, specified as the comma-separated pair consisting of 'CurveCompounding' and a N-by-1 vector using the supported values: -1, 0, 1, 2, 3, 4, 6, and 12.

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a N-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

**Face — face value of bond**

100 (default) | vector

Face value of bond, specified as the comma-separated pair consisting of 'Face' and an N-by-1 vector of numeric values.

Data Types: double

**FirstCouponDate — Irregular first coupon date**

if you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs (default) | serial date number

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial date numbers.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Data Types: double

**InterpMethod — Interpolation method**

'linear' (default) | 'cubic', 'pchip'

Interpolation method, specified as the comma-separated pair consisting of 'InterpMethod' and a N-by-1 vector using a supported value. For more information on interpolation methods, see `interp1`.

Data Types: char

**IssueDate — Bond issue date**

if you do not specify an IssueDate, the cash flow payment dates are determined from other inputs (default) | serial date number

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a N-by-1 vector using serial date numbers.

Data Types: double

**LastCouponDate — Irregular last coupon date**

serial date number

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a N-by-1 vector using serial date numbers

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Data Types: double

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and an N-by-1 vector. Values for Period are 0, 1, 2, 3, 4, 6, and 12.

Data Types: double

**StartDate — Forward starting date of payments**

Settle date (default) | serial date number

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a N-by-1 vector using serial date numbers.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: double

## Output Arguments

### OAS — Option-adjusted spreads

matrix

Option-adjusted spreads, returned as an numBonds-by-1 matrix.

## Definitions

### Agency OAS Model

The BMA European Callable Securities Formula provides a standard methodology for computing price and option-adjusted spread for European Callable Securities (ECS).

## References

[1] SIFMA, The BMA European Callable Securities Formula, <https://www.sifma.org>.

## See Also

agencyprice

## Topics

“Computing the Agency OAS for Bonds” on page 6-3

“Agency Option-Adjusted Spreads” on page 6-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# agencyprice

Price callable bond using Agency OAS model

## Syntax

```
Price = agencyprice(ZeroData,OAS,CouponRate,Settle,Maturity,Vol,
CallDate)
Price = agencyprice(____,Name,Value)
```

## Description

Price = agencyprice(ZeroData,OAS,CouponRate,Settle,Maturity,Vol, CallDate) computes the price for a callable bond, given OAS, using the Agency OAS model.

Price = agencyprice(\_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute the Agency Price

This example shows how to compute the agency Price.

```
Settle = datenum('20-Jan-2010');
ZeroRates = [.07 .164 .253 1.002 1.732 2.226 2.605 3.316 ...
3.474 4.188 4.902]'/100;
ZeroDates = daysadd(Settle,360* [.25 .5 1 2 3 4 5 7 10 20 30],1);
ZeroData = [ZeroDates ZeroRates];

Maturity = datenum('30-Dec-2013');
CouponRate = .022;
OAS = 6.53/10000;
Vol = .5117;
```

```
CallDate = datenum('30-Dec-2010');  
Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate)  
  
Price = 99.4212
```

## Input Arguments

### **ZeroData — Zero curve**

matrix

Zero curve, specified as an numRates-by-2 matrix where the first column is zero dates and the second column is the accompanying zero rates.

Data Types: double

### **OAS — Option-adjusted spreads**

vector in decimals

Option-adjusted spreads, specified as an numBonds-by-1 vector expressed as a decimal (that is, 50 basis points is entered as .005).

Data Types: double

### **CouponRate — Coupon rates**

vector in decimals

Coupon rates, specified as an numBonds-by-1 vector in decimals.

Data Types: double

### **Settle — Settlement date**

serial date number

Settlement date, specified as a scalar serial date number.

---

**Note** The Settle date must be an identical settlement date for all the bonds and the zero curve.

---

Data Types: double



**Maturity — Maturity date**

serial date number

Maturity date, specified as a numBonds-by-1 vector.

Data Types: double

**Vol — Volatilities**

vector in decimals

Volatilities specified as a scalar or an numBonds-by-1 vector in decimals. Vol is the volatility of interest rates corresponding to the time of the CallDate.

Data Types: double

**CallDate — Call dates**

vector

Call dates, specified as an numBonds-by-1 vector.

Data Types: double

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Price =

```
agencyprice(ZeroData,OAS,CouponRate,Settle,Maturity,Vol,CallDate,'Basis',7,'Face',1000)
```

**Basis — Day-count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a N-by-1 vector using the following values:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360

- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **CurveBasis — Curve basis**

0 (actual/actual) (default) | integer from 0 to 13

Curve basis, specified as the comma-separated pair consisting of 'CurveBasis' and a N-by-1 vector using the following values:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **CurveCompounding — Compounding frequency of the zero curve**

2 (semiannual) (default) | possible values include: -1, 0, 1, 2, 3, 4, 6, 12.

Compounding frequency of the zero curve, specified as the comma-separated pair consisting of 'CurveCompounding' and a N-by-1 vector using the supported values: -1, 0, 1, 2, 3, 4, 6, and 12.

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer [0, 1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer [0, 1] using a N-by-1 vector.

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **Face — face value of bond**

100 (default) | vector

Face value of bond, specified as the comma-separated pair consisting of 'Face' and an N-by-1 vector of numeric values.

Data Types: double

### **FirstCouponDate — Irregular first coupon date**

if you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs (default) | serial date number

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a NINST-by-1 vector using a serial date numbers.

When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

Data Types: double

**InterpMethod — Interpolation method**

'linear' (default) | 'cubic','pchip'

Interpolation method, specified as the comma-separated pair consisting of 'InterpMethod' and a N-by-1 vector using a supported value. For more information on interpolation methods, see `interp1`.

Data Types: char

**IssueDate — Bond issue date**

if you do not specify an `IssueDate`, the cash flow payment dates are determined from other inputs (default) | serial date number

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a N-by-1 vector using serial date numbers.

Data Types: double

**LastCouponDate — Irregular last coupon date**

serial date number

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a N-by-1 vector using serial date numbers

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Data Types: double

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and an N-by-1 vector. Values for `Period` are 0, 1, 2, 3, 4, 6, and 12.

Data Types: double

**StartDate — Forward starting date of payments**

Settle date (default) | serial date number

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a N-by-1 vector using serial date numbers.

If you do not specify StartDate, the effective start date is the Settle date.

Data Types: double

## Output Arguments

**Price — Prices**

matrix

Prices returned as an numBonds-by-1 matrix.

## Definitions

### Agency OAS Model

The BMA European Callable Securities Formula provides a standard methodology for computing price and option-adjusted spread for European Callable Securities (ECS).

## References

[1] SIFMA, The BMA European Callable Securities Formula, <https://www.sifma.org>.

## See Also

agencyoas

## Topics

“Computing the Agency OAS for Bonds” on page 6-3

“Agency Option-Adjusted Spreads” on page 6-2

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bkcall

Price European call option on bonds using Black model

### Syntax

CallPrice = bkcall(Strike,ZeroData,Sigma,BondData,Settle,Expiry,Period,Basis,EndMonthR

### Arguments

Strike	Scalar or number of options (NOPT)-by-1 vector of strike prices.
ZeroData	Two-column (optionally three-column) matrix containing zero (spot) rate information used to discount future cash flows. <ul style="list-style-type: none"> <li>• Column 1: Serial maturity date associated with the zero rate in the second column.</li> <li>• Column 2: Annualized zero rates, in decimal form, appropriate for discounting cash flows occurring on the date specified in the first column. All dates must occur after <b>Settle</b> (dates must correspond to future investment horizons) and must be in ascending order.</li> <li>• Column 3 (optional): Annual compounding frequency. Values are 1 (annual), 2 (semiannual, default), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).</li> </ul>
Sigma	Scalar or NOPT-by-1 vector of annualized price volatilities required by Black's model.

<p><b>BondData</b></p>	<p>Row vector with three (optionally four) columns or NOPT-by-3 (optionally NOPT-by-4) matrix specifying characteristics of underlying bonds in the form:</p> <p>[CleanPrice CouponRate Maturity Face]</p> <p>CleanPrice is the price excluding accrued interest.</p> <p>CouponRate is the decimal coupon rate.</p> <p>Maturity is the bond maturity date in serial date number format.</p> <p>Face is the face value of the bond. If unspecified, the face value is assumed to be 100.</p>
<p><b>Settle</b></p>	<p>Settlement date of the options, specified as a serial date number or date character vector. <b>Settle</b> also represents the starting reference date for the input zero curve.</p>
<p><b>Expiry</b></p>	<p>Scalar or NOPT-by-1 vector of option maturity dates, specified as a serial date number or date character vector.</p>
<p><b>Period</b></p>	<p>(Optional) Number of coupons per year for the underlying bond. Default = 2 (semiannual). Supported values are 0, 1, 2, 3, 4, 6, and 12.</p>



<b>Basis</b>	<p>(Optional) Day-count basis of the bond. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<b>EndMonthRule</b>	<p>(Optional) End-of-month rule. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
<b>InterpMethod</b>	<p>(Optional) Scalar integer zero curve interpolation method. For cash flows that do not fall on a date found in the <b>ZeroData</b> spot curve, indicates the method used to interpolate the appropriate zero discount rate. Available methods are (0) nearest, (1) linear, and (2) cubic. Default = 1. See <b>interp1</b> for more information.</p>

StrikeConvention	<p>(Optional) Scalar or NOPT-by-1 vector of option contract strike price conventions.</p> <p>StrikeConvention = 0 (default) defines the strike price as the cash (dirty) price paid for the underlying bond.</p> <p>StrikeConvention = 1 defines the strike price as the quoted (clean) price paid for the underlying bond. When evaluating Black's model, the accrued interest of the bond at option expiration is added to the input strike price.</p>
------------------	--

## Description

CallPrice =

bkcall(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention) using Black's model, derives an NOPT-by-1 vector of prices of European call options on bonds.

If cash flows occur beyond the dates spanned by ZeroData, the input zero curve, the appropriate zero rate for discounting such cash flows is obtained by extrapolating the nearest rate on the curve (that is, if a cash flow occurs before the first or after the last date on the input zero curve, a flat curve is assumed).

In addition, you can use the Financial Instruments Toolbox method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for bkcall. For more information, see “Converting an IRDataCurve or IRFunctionCurve Object” on page 9-41.

## Examples

### Price a European Call Option On Bonds Using the Black Model

This example shows how to price a European call option on bonds using the Black model. Consider a European call option on a bond maturing in 9.75 years. The underlying bond has a clean price of \$935, a face value of \$1000, and pays 10% semiannual coupons. Since the bond matures in 9.75 years, a \$50 coupon will be paid in 3 months and again in 9 months. Also, assume that the annualized volatility of the forward bond price is 9%.

Furthermore, suppose the option expires in 10 months and has a strike price of \$1000, and that the annualized continuously compounded risk-free discount rates for maturities of 3, 9, and 10 months are 9%, 9.5%, and 10%, respectively.

```
% specify the option information
Settle      = '15-Mar-2004';
Expiry      = '15-Jan-2005'; % 10 months from settlement
Strike      = 1000;
Sigma       = 0.09;
Convention   = [0 1]';

% specify the interest-rate environment
ZeroData    = [datenum('15-Jun-2004') 0.09 -1; % 3 months
               datenum('15-Dec-2004') 0.095 -1; % 9 months
               datenum(Expiry)        0.10 -1]; % 10 months

% specify the bond information
CleanPrice  = 935;
CouponRate  = 0.1;
Maturity    = '15-Dec-2013'; % 9.75 years from settlement
Face        = 1000;
BondData    = [CleanPrice CouponRate datenum(Maturity) Face];
Period      = 2;
Basis       = 1;

% call Black's model
CallPrices = bkcall(Strike, ZeroData, Sigma, BondData, Settle,...
                   Expiry, Period, Basis, [], [], Convention)

CallPrices = 2×1

    9.4873
    7.9686
```

When the strike price is the dirty price ( $\text{Convention} = 0$ ), the call option value is \$9.49. When the strike price is the clean price ( $\text{Convention} = 1$ ), the call option value is \$7.97.

## References

- [1] Hull, John C. *Options, Futures, and Other Derivatives*. 5th Edition, Prentice Hall, 2003, pp. 287–288, 508–515.

## **See Also**

bkput

## **Topics**

“Analysis of Bond Futures” on page 7-13

“Managing Interest-Rate Risk with Bond Futures”

“Fitting the Diebold Li Model”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bkcaplet

Price interest-rate caplet using Black model

---

**Note** `bkcaplet` has been removed. Use `capbyblk` instead.

---

## Syntax

`CapPrices = bkcaplet(CapData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma)`

## Arguments

CapData	<p>Number of caps (NCAP)-by-2 matrix containing cap rates and bases: [CapRates Basis].</p> <p>Values for bases are:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
FwdRates	Scalar or NCAP-by-1 vector containing forward rates in decimal. FwdRates accrue on the same basis as CapRates.
ZeroPrice	Scalar or NCAP-by-1 vector containing zero coupon prices with maturities corresponding to those of each cap in CapData, per \$100 nominal value.
Settle	Scalar or NCAP-by-1 vector of identical elements containing settlement date of caplets.
StartDate	Scalar or NCAP-by-1 vector containing start dates of the caplets.
EndDate	Scalar or NCAP-by-1 vector containing maturity dates of caplets.

Sigma	Scalar or NCAP-by-1 vector containing volatility of forward rates in decimal, corresponding to each caplet.
-------	---

## Description

CapPrices =  
 bkcaplet(CapData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma)  
 computes the prices of interest-rate caplets for every \$100 face value of principal.

## Examples

### Compute the Price of Interest-Rate Caplets for Every \$100 Face Value of Principal

This example shows how to compute the price of interest-rate caplets for every \$100 face value of principal. Given a notional amount of \$1,000,000, compute the value of a caplet on October 15, 2002 that starts on October 15, 2003 and ends on January 15, 2004.

```
CapData = [0.08, 1];
FwdRates = 0.07;
ZeroPrice = 100*exp(-0.065*1.25);
Settle = datenum('15-Oct-2002');
BeginDates = datenum('15-Oct-2003');
EndDates = datenum('15-Jan-2004');
Sigma = 0.20;

% because the caplet is $100 notional, divide $1,000,000 by $100
Notional = 1000000/100;

CapPrice = Notional*bkcaplet(CapData, FwdRates, ZeroPrice, ...
  Settle, BeginDates, EndDates, Sigma)
```

Error using bkcapiet (line 117)  
BKCAPLET has been removed. Use CAPBYBLK instead.

## See Also

### Topics

“Analysis of Bond Futures” on page 7-13  
“Managing Interest-Rate Risk with Bond Futures”  
“Fitting the Diebold Li Model”  
“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**



## bkfloorlet

Price interest-rate floorlet using Black model

---

**Note** bkfloorlet has been removed. Use floorbyblk instead.

---

## Syntax

FloorPrices = bkfloorlet(FloorData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma)

## Arguments

FloorData	<p>Number of floors (NFLR)-by-2 matrix containing floor rates and bases: [FloorRate Basis].</p> <p>Values for bases are:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
FwdRates	<p>Scalar or NFLR-by-1 vector containing forward rates in decimal. FwdRates accrue on the same basis as FloorRates.</p>
ZeroPrice	<p>Scalar or NFLR-by-1 vector containing zero coupon prices with maturities corresponding to those of each floor in FloorData, per \$100 nominal value.</p>
Settle	<p>Scalar or NFLR-by-1 vector of identical elements containing settlement date of floorlets.</p>
StartDate	<p>Scalar or NFLR-by-1 vector containing start dates of the floorlets.</p>
EndDate	<p>Scalar or NFLR-by-1 vector containing maturity dates of floorlets.</p>

Sigma	Scalar or NFLR-by-1 vector containing volatility of forward rates in decimal, corresponding to each floorlet.
-------	---

## Description

FloorPrices =  
 bkfloorlet(FloorData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma) computes the prices of interest-rate floorlets for every \$100 of notional value.

## Examples

### Price an Interest-Rate Floorlet For Every \$100 of Notional Value Using the Black Model

This example shows how to price an interest-rate floorlet for every \$100 of notional value using the Black model. Given a notional amount of \$1,000,000, compute the value of a floorlet on October 15, 2002 that starts on October 15, 2003 and ends on January 15, 2004.

```
FloorData = [0.08, 1];
FwdRates = 0.07;
ZeroPrice = 100*exp(-0.065*1.25);
Settle = datenum('15-Oct-2002');
BeginDates = datenum('15-Oct-2003');
EndDates = datenum('15-Jan-2004');
Sigma = 0.20;

% because floorlet is $100 notional, divide $1,000,000 by $100
Notional = 1000000/100;

FloorPrice = Notional*bkfloorlet(FloorData, FwdRates, ...
ZeroPrice, Settle, BeginDates, EndDates, Sigma)
```

Error using bkfloorlet (line 115)  
BKFL00RLET has been removed. Use FL00RBYBLK instead.

## See Also

### Topics

“Analysis of Bond Futures” on page 7-13

“Managing Interest-Rate Risk with Bond Futures”

“Fitting the Diebold Li Model”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

# bkput

Price European put option on bonds using Black model

## Syntax

PutPrice = bkput(Strike,ZeroData,Sigma,BondData,Settle,Expiry,Period,Basis,EndMonthRule)

## Arguments

Strike	Scalar or number of options (NOPT)-by-1 vector of strike prices.
ZeroData	Two-column (optionally three-column) matrix containing zero (spot) rate information used to discount future cash flows. <ul style="list-style-type: none"> <li>Column 1: Serial maturity date associated with the zero rate in the second column.</li> <li>Column 2: Annualized zero rates, in decimal form, appropriate for discounting cash flows occurring on the date specified in the first column. All dates must occur after <b>Settle</b> (dates must correspond to future investment horizons) and must be in ascending order.</li> <li>Column 3 (optional): Annual compounding frequency. Values are 1 (annual), 2 (semiannual, default), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).</li> </ul>
Sigma	Scalar or NOPT-by-1 vector of annualized price volatilities required by Black's model.

<p><b>BondData</b></p>	<p>Row vector with three (optionally four) columns or NOPT-by-3 (optionally NOPT-by-4) matrix specifying characteristics of underlying bonds in the form [CleanPrice CouponRate Maturity Face] where:</p> <ul style="list-style-type: none"> <li>• CleanPrice is the price excluding accrued interest.</li> <li>• CouponRate is the decimal coupon rate.</li> <li>• Maturity is the bond maturity date in serial date number format.</li> <li>• Face is the face value of the bond. If unspecified, the face value is assumed to be 100.</li> </ul>
<p><b>Settle</b></p>	<p>Settlement date of the options, specified using a serial date number or date character vector. <b>Settle</b> also represents the starting reference date for the input zero curve.</p>
<p><b>Expiry</b></p>	<p>Scalar or NOPT-by-1 vector of option maturity dates, specified using a serial date number or date character vector.</p>
<p><b>Period</b></p>	<p>(Optional) Number of coupons per year for the underlying bond. Default = 2 (semiannual). Supported values are 0, 1, 2, 3, 4, 6, and 12.</p>

Basis	<p>(Optional) Day-count basis of the bond. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
EndMonthRule	<p>(Optional) End-of-month rule. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
InterpMethod	<p>(Optional) Scalar integer zero curve interpolation method. For cash flows that do not fall on a date found in the <b>ZeroData</b> spot curve, indicates the method used to interpolate the appropriate zero discount rate. Available methods are (0) nearest, (1) linear, and (2) cubic. Default = 1. See <b>interp1</b> for more information.</p>

StrikeConvention	<p>(Optional) Scalar or NOPT-by-1 vector of option contract strike price conventions.</p> <p>StrikeConvention = 0 (default) defines the strike price as the cash (dirty) price paid for the underlying bond.</p> <p>StrikeConvention = 1 defines the strike price as the quoted (clean) price paid for the underlying bond. The accrued interest of the bond at option expiration is added to the input strike price when evaluating Black's model.</p>
------------------	---

## Description

PutPrice = bkput(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention) using Black's model, derives an NOPT-by-1 vector of prices of European put options on bonds.

If cash flows occur beyond the dates spanned by ZeroData, the input zero curve, the appropriate zero rate for discounting such cash flows is obtained by extrapolating the nearest rate on the curve (that is, if a cash flow occurs before the first or after the last date on the input zero curve, a flat curve is assumed).

In addition, you can use the Financial Instruments Toolbox method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for bkput. For more information, see “Converting an IRDataCurve or IRFunctionCurve Object” on page 9-41.

## Examples

### Price European Put Options On Bonds Using the Black Model

This example shows how to price European put options on bonds using the Black model. Consider a European put option on a bond maturing in 10 years. The underlying bond has a clean price of \$122.82, a face value of \$100, and pays 8% semiannual coupons. Also, assume that the annualized volatility of the forward bond yield is 20%. Furthermore, suppose the option expires in 2.25 years and has a strike price of \$115, and that the



annualized continuously compounded risk free zero (spot) curve is flat at 5%. For a hypothetical settlement date of March 15, 2004, the following code illustrates the use of Black's model to duplicate the put prices in Example 22.2 of the Hull reference. In particular, it illustrates how to convert a broker's yield volatility to a price volatility suitable for Black's model.

```
% Specify the option information.
Settle      = '15-Mar-2004';
Expiry      = '15-Jun-2006'; % 2.25 years from settlement
Strike      = 115;
YieldSigma  = 0.2;
Convention  = [0; 1];

% Specify the interest-rate environment. Since the
% zero curve is flat, interpolation into the curve always returns
% 0.05. Thus, the following curve is not unique to the solution.
ZeroData    = [datenum('15-Jun-2004') 0.05 -1;
               datenum('15-Dec-2004') 0.05 -1;
               datenum(Expiry)        0.05 -1];

% Specify the bond information.
CleanPrice  = 122.82;
CouponRate  = 0.08;
Maturity    = '15-Mar-2014'; % 10 years from settlement
Face        = 100;
BondData    = [CleanPrice CouponRate datenum(Maturity) Face];
Period      = 2; % semiannual coupons
Basis       = 1; % 30/360 day-count basis

% Convert a broker's yield volatility quote to a price volatility
% required by Black's model. To duplicate Example 22.2 in Hull,
% first compute the periodic (semiannual) yield to maturity from
% the clean bond price.
Yield = bndyield(CleanPrice, CouponRate, Settle, Maturity,...
Period, Basis);

% Compute the duration of the bond at option expiration. Most
% fixed-income sensitivity analyses use the modified duration
% statistic to examine the impact of small changes in periodic
% yields on bond prices. However, Hull's example operates in
% continuous time (annualized instantaneous volatilities and
% continuously compounded zero yields for discounting coupons).
% To duplicate Hull's results, use the second output of BNDDURY,
% the Macaulay duration.
```

```
[Modified, Macaulay] = bnddury(Yield, CouponRate, Expiry,...  
Maturity, Period, Basis);  
  
% Convert the yield-to-maturity from a periodic to a  
% continuous yield.  
Yield = Period .* log(1 + Yield./Period);  
  
% Convert the yield volatility to a price volatility via  
% Hull's Equation 22.6 (page 514).  
PriceSigma = Macaulay .* Yield .* YieldSigma;  
  
% Finally, call Black's model.  
PutPrices = bkput(Strike, ZeroData, PriceSigma, BondData,...  
Settle, Expiry, Period, Basis, [], [], Convention)  
  
PutPrices = 2×1  
  
    1.7838  
    2.4071
```

When the strike price is the dirty price (`Convention = 0`), the call option value is \$1.78.  
When the strike price is the clean price (`Convention = 1`), the call option value is \$2.41.

## References

[1] Hull, John C. *Options, Futures, and Other Derivatives*. 5th Edition, Prentice Hall, 2003, pp. 287-288, 508-515.

## See Also

`bkcall`

## Topics

“Analysis of Bond Futures” on page 7-13  
“Managing Interest-Rate Risk with Bond Futures”  
“Fitting the Diebold Li Model”  
“Supported Interest-Rate Instruments” on page 2-2

**Introduced before R2006a**

## bndfutimprepo

Implied repo rates for bond future given price

### Syntax

```
ImpRepo = bndfutimprepo(Price,FutSettle,FutPrice,Delivery,ConvFactor,  
CouponRate,Maturity)
```

```
ImpRepo = bndfutimprepo( ____,Name,Value)
```

### Description

`ImpRepo = bndfutimprepo(Price,FutSettle,FutPrice,Delivery,ConvFactor,CouponRate,Maturity)` computes the implied repo rate for a bond future given the price of a bond, the bond properties, the price of the bond future, and the bond conversion factor. The default behavior is that the coupon reinvestment rate matches the repo rate. However, you can specify a separate reinvestment rate using optional inputs.

`ImpRepo = bndfutimprepo( ____,Name,Value)` specifies options using one or more optional name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Repo Rate For a Bond Future

This example shows how to compute the repo rate for a bond future using the following data.

```
bndfutimprepo(129,98,'9/21/2000','12/29/2000',1.3136,.0875,'8/15/2020')
```

```
ans = 0.0584
```

## Input Arguments

### **Price — Bond prices**

vector

Bond prices, specified as an `numBonds-by-1` vector in decimals.

Data Types: `double`

### **FutPrice — Future prices**

vector

Future prices, specified as an `numBonds-by-1` vector.

Data Types: `double` | `cell`

### **FutSettle — Future settlement dates**

vector

Future settlement dates, specified as an `numBonds-by-1` vector of serial date numbers or a cell array of character vectors.

Data Types: `double` | `cell`

### **Delivery — Future delivery dates**

vector

Future delivery dates, specified as an `numBonds-by-1` vector.

Data Types: `double` | `cell`

### **ConvFactor — Bond conversion factors**

vector

Bond conversion factors, specified as an `numBonds-by-1` vector. For more information, see `convfactor`.

Data Types: `double`

### **CouponRate — Coupon rates**

vector

Coupon rates, specified as an `numBonds-by-1` vector of numeric decimals.

Data Types: `double`

**Maturity — Maturity dates**

vector

Maturity dates, specified as an `numBonds`-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: `double` | `cell`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `ImpRepo = bndfutimrepo(Price, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'Basis', 5, 'Face', 1000, 'Period', 4)`

**Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis, specified as the comma-separated pair consisting of `'Basis'` and a scalar integer from 0 to 13.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating floorlet dates**

1 (in effect) (default) | scalar of nonnegative integer [0, 1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar with a nonnegative integer [0, 1].

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **Face — Face value of the bond**

100 (default) | scalar numeric

Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar numeric. Face has no impact on key rate duration.

Data Types: double

### **FirstCouponDate — Irregular first coupon date**

if you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs (default) | serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar date using a serial date number or a date character vector.

When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Data Types: double | char

### **IssueDate — Bond issue date**

if you do not specify an IssueDate, the cash flow payment dates are determined from other inputs (default) | serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar date using a serial date number or a date character vector.

Data Types: double | char

**LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar date using a serial date number or a date character vector.

In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Data Types: double | char

**Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of 'Period' and a scalar integer. Values for Period are 0, 1, 2, 3, 4, 6, and 12.

Data Types: double

**ReinvestBasis — Day count basis for reinvestment rate**

identical to RepoBasis (default) | integer from 0 to 13

Day count basis for the reinvestment rate, specified as the comma-separated pair consisting of 'ReinvestBasis' and a scalar integer from 0 to 13.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)



- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **ReinvestRate — Underlying bond annual coupon**

scalar decimal numeric

Underlying bond annual coupon, specified as the comma-separated pair consisting of 'ReinvestRate' and a scalar decimal numeric.

Data Types: double

### **RepoBasis — Day count basis for repo rate**

2 (actual/360) (default) | integer from 0 to 13

Day count basis for repo rate, specified as the comma-separated pair consisting of 'RepoBasis' and a scalar integer from 0 to 13.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)

- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **StartDate — Forward starting date of payments**

Settle date (default) | serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a scalar date using a serial date number or a date character vector.

Data Types: double

## **Output Arguments**

### **ImpRepo — Implied repo rate**

vector

Implied repo rate, or the repo rate that would produce the price input, returned as numBonds-by-1 vector.

## **References**

[1] Burghardt, G., T. Belton, M. Lane, and J. Papa. *The Treasury Bond Basis*. McGraw-Hill, 2005.

[2] Krgin, Dragomir. *Handbook of Global Fixed Income Calculations*. John Wiley & Sons, 2002.

## **See Also**

bndfutprice | convfactor

## **Topics**

"Analysis of Bond Futures" on page 7-13

"Managing Interest-Rate Risk with Bond Futures"

"Fitting the Diebold Li Model"

"Supported Interest-Rate Instruments" on page 2-2

## **Introduced in R2009b**

## bndfutprice

Price bond future given repo rates

### Syntax

```
[FutPrice,AccrInt] = bndfutprice(RepoRatePrice,FutSettle,Delivery,  
ConvFactor,CouponRate,Maturity)  
[FutPrice,AccrInt] = bndfutprice( ____,Name,Value)
```

### Description

[FutPrice,AccrInt] = bndfutprice(RepoRatePrice,FutSettle,Delivery,ConvFactor,CouponRate,Maturity) computes the price of a bond futures contract for one or more bonds given a repo rate, and bond properties, including the bond conversion factor. The default behavior is that the coupon reinvestment rate matches the repo rate. However, you can specify a separate reinvestment rate using optional arguments.

[FutPrice,AccrInt] = bndfutprice( \_\_\_\_,Name,Value) specifies options using one or more optional name-value pair arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Price For a Bond Future

This example shows how to compute the price for a bond future using the following data.

```
bndfutprice(.064, 129, '9/21/2000', '12/29/2000', 1.3136, .0875, '8/15/2020')  
ans = 98.1516
```

## Input Arguments

### **RepoRate — Repo rates**

vector

Repo rates, specified as an numBonds-by-1 vector in decimals.

Data Types: double

### **Price — Bond prices**

vector

Bond prices, specified as an numBonds-by-1 vector in decimals.

Data Types: double

### **FutSettle — Future settlement date**

scalar

Future settlement date, specified as scalar date using a serial date number or a date character vectors.

Data Types: double | char

### **Delivery — Future delivery dates**

vector

Future delivery dates, specified as an numBonds-by-1 vector.

Data Types: double | cell

### **ConvFactor — Bond conversion factors**

vector

Bond conversion factors, specified as an numBonds-by-1 vector. For more information, see [convfactor](#).

Data Types: double

### **CouponRate — Coupon rates**

vector

Coupon rates, specified as an numBonds-by-1 vector of numeric decimals.

Data Types: double

**Maturity — Maturity dates**

vector

Maturity dates, specified as an `numBonds`-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: `double` | `cell`

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

Example: `[FutPrice, AccrInt] = bndfutprice(RepoRate, Price, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'Basis', 5, 'Face', 1000, 'Period', 4)`

**Basis — Day count basis**

0 (actual/actual) (default) | integer from 0 to 13

Day count basis, specified as the comma-separated pair consisting of `'Basis'` and a scalar integer from 0 to 13.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **EndMonthRule — End-of-month rule flag for generating floorlet dates**

1 (in effect) (default) | scalar of nonnegative integer [0, 1]

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar with a nonnegative integer [0, 1].

- 0 = Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.

Data Types: logical

### **IssueDate — Bond issue date**

if you do not specify an IssueDate, the cash flow payment dates are determined from other inputs (default) | serial date number | date character vector

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar date using a serial date number or a date character vector.

Data Types: double | char

### **Face — Face value of the bond**

100 (default) | scalar numeric

Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar numeric. Face has no impact on key rate duration.

Data Types: double

### **FirstCouponDate — Irregular first coupon date**

if you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs (default) | serial date number | date character vector

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar date using a serial date number or a date character vector.

When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

Data Types: `double` | `char`

### **LastCouponDate — Irregular last coupon date**

serial date number | date character vector

Irregular last coupon date, specified as the comma-separated pair consisting of `'LastCouponDate'` and a scalar date using a serial date number or a date character vector.

In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Data Types: `double` | `char`

### **Period — Coupons per year**

2 per year (default) | vector

Coupons per year, specified as the comma-separated pair consisting of `'Period'` and a scalar integer. Values for `Period` are 0, 1, 2, 3, 4, 6, and 12.

Data Types: `double`

### **ReinvestBasis — Day count basis for reinvestment rate**

identical to `RepoBasis` (default) | integer from 0 to 13

Day count basis for the reinvestment rate, specified as the comma-separated pair consisting of `'ReinvestBasis'` and a scalar integer from 0 to 13.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)



- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **ReinvestRate — Underlying bond annual coupon**

scalar decimal numeric

Underlying bond annual coupon, specified as the comma-separated pair consisting of 'ReinvestRate' and a scalar decimal numeric.

Data Types: double

### **RepoBasis — Day count basis for repo rate**

2 (actual/360) (default) | integer from 0 to 13

Day count basis for repo rate, specified as the comma-separated pair consisting of 'RepoBasis' and a scalar integer from 0 to 13.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)

- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

### **StartDate** — Forward starting date of payments

Settle date (default) | serial date number | date character vector

Forward starting date of payments (the date from which a bond cash flow is considered), specified as the comma-separated pair consisting of 'StartDate' and a scalar date using a serial date number or a date character vector.

Data Types: double

## Output Arguments

### **FutPrice** — Quoted futures price, per \$100 notional

vector

Quoted futures price, per \$100 notional, returned as numBonds-by-1 vector.

### **AccrInt** — Accrued interest due at delivery date, per \$100 notional

vector

Accrued interest due at delivery date, per \$100 notional, returned as numBonds-by-1 vector.

## References

- [1] Burghardt, G., T. Belton, M. Lane, and J. Papa. *The Treasury Bond Basis*. McGraw-Hill, 2005.
- [2] Krgin, Dragomir. *Handbook of Global Fixed Income Calculations*. John Wiley & Sons, 2002.

## **See Also**

bndfutimprepo | convfactor

## **Topics**

“Analysis of Bond Futures” on page 7-13

“Managing Interest-Rate Risk with Bond Futures”

“Fitting the Diebold Li Model”

“Supported Interest-Rate Instruments” on page 2-2

**Introduced in R2009b**

## bootstrap

Bootstrap interest-rate curve from market data

### Class

@IRDataCurve on page A-7

### Syntax

```
Dcurve = IRDataCurve.bootstrap(Type,Settle,InstrumentTypes,Instruments)
```

```
Dcurve = IRDataCurve.bootstrap(Type,Settle,InstrumentTypes,Instruments,'Parameter1',Val)
```

### Arguments

Type	Type of interest-rate curve. <b>Type</b> refers to the type of data in the curve that is bootstrapped from the market instruments. Acceptable values are: <b>discount</b> , <b>forward</b> , or <b>zero</b> .  When using the <b>bootstrap</b> method, the choice of the <b>Type</b> parameter can impact the curve construction because it will affect the type of data that will be interpolated on (that is, forward rates, zero rates or discount factors) during the bootstrapping process. So curves that are bootstrapped using different <b>Type</b> parameters undergo different bootstrapping algorithms with different interpolation methods, and they can sometimes produce different results when using the “get” methods (for example, <b>getForwardRates</b> ).
Settle	Scalar or column vector of settlement dates.

InstrumentTypes	N-by-1 cell array (where N is the number of instruments) indicating what kind of instrument is in the Instruments matrix. Acceptable values are deposit, futures, swap, bond, and fra.
Instruments	<p>N-by-3 data matrix for Instruments where the first column is Settle date, the second column is Maturity, and the third column is the market quote (dates must be MATLAB date numbers). The market quote represents the following for each instrument:</p> <ul style="list-style-type: none"><li>• deposit: rate</li><li>• futures: price (e.g., 9628.54)</li><li>• swap: rate</li><li>• bond: clean price</li><li>• fra: forward rate</li></ul> <hr/> <p><b>Note</b> Instruments input for fra and for futures are different. Specifically, the forward rate underlying a fra starts on the start date (column 1 of Instruments) and ends on the end date (column 2 of Instruments). While the forward rate underlying a futures contract starts on the maturity date of the futures contract and ends on a date <math>n</math> months after the futures maturity, where <math>n</math> is the periodicity of the futures contract.</p>

<p>Compounding</p>	<p>(Optional) Scalar that sets the compounding frequency per year for an IRDataCurve object:</p> <ul style="list-style-type: none"> <li>• -1 = Continuous compounding</li> <li>• 0 = Simple interest (no compounding) for “zero” and “discount” curve types only, not supported for “forward” curves</li> <li>• 1 = Annual compounding</li> <li>• 2 = Semiannual compounding (default)</li> <li>• 3 = Compounding three times per year</li> <li>• 4 = Quarterly compounding</li> <li>• 6 = Bimonthly compounding</li> <li>• 12 = Monthly compounding</li> </ul>
<p>Basis</p>	<p>(Optional) Day-count basis of the interest-rate curve. A scalar of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>

InterpMethod	(Optional) Values are: <ul style="list-style-type: none"> <li>'linear' — Linear interpolation (default).</li> <li>'constant' — Piecewise constant interpolation.</li> <li>'pchip' — Piecewise cubic Hermite interpolation.</li> <li>'spline' — Cubic spline interpolation.</li> </ul>
IRBootstrapOptionsObj	(Optional) An IRBootstrapOptions object.
DiscountCurve	(Optional) RateSpec for a curve used to discount the cash flows.

## Instrument Parameters

For each bond Instrument, you can specify the following additional instrument parameters as parameter/value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value. For instruments of type deposit, futures, or swap the Basis and Compounding values must be identical for each instance of the instrument.

InstrumentCouponRate	(Optional) Decimal number indicating the annual percentage rate used to determine the coupons payable on an instrument.
InstrumentPeriod	(Optional) Coupons per year of the instrument. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.

<p><b>InstrumentBasis</b></p>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>InstrumentEndMonthRule</b></p>	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that an instrument's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that an instrument's coupon payment date is always the last actual day of the month.</p>
<p><b>InstrumentIssueDate</b></p>	<p>(Optional) Date when an instrument was issued.</p>
<p><b>InstrumentFirstCouponDate</b></p>	<p>(Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When <b>FirstCouponDate</b> and <b>LastCouponDate</b> are both specified, <b>FirstCouponDate</b> takes precedence in determining the coupon payment structure. If you do not specify a <b>FirstCouponDate</b>, the cash flow payment dates are determined from other inputs.</p>



InstrumentLastCouponDate	(Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentFace	(Optional) Face or par value. Default = 100.

**Note** When using Instrument parameter/value pairs, you can specify simple interest for an Instrument by specifying the InstrumentPeriod value as 0. If InstrumentBasis and InstrumentPeriod are not specified for an Instrument, the following default values are used:

- deposit instrument uses InstrumentBasis as 2 (act/360) and InstrumentPeriod is 0 (simple interest).
- futures instrument uses InstrumentBasis as 2 (act/360) and InstrumentPeriod is 4 (quarterly).
- swap instrument uses InstrumentBasis as 2 (act/360) and InstrumentPeriod is 2.
- bond instrument uses InstrumentBasis as 0 (act/act) and InstrumentPeriod is 2.
- FRA instrument uses InstrumentBasis as 2 (act/360) and InstrumentPeriod is 4 (quarterly).

## Description

Dcurve = IRDataCurve.bootstrap(Type, Settle, InstrumentTypes, Instruments, 'Parameter1', Value1, 'Parameter2', Value2, ...)  
 bootstraps an interest-rate curve from market data. The dates of the bootstrapped curve correspond to the maturity dates of the input instruments. You must enter the optional arguments for Basis, Compounding, Interpmethod, IRBootstrapOptionsObj, and DiscountCurve as parameter/value pairs.

## Examples

### Use the bootstrap Method to Create an IRDataCurve Object

In this bootstrapping example, InstrumentTypes, Instruments, and a Settle date are defined:

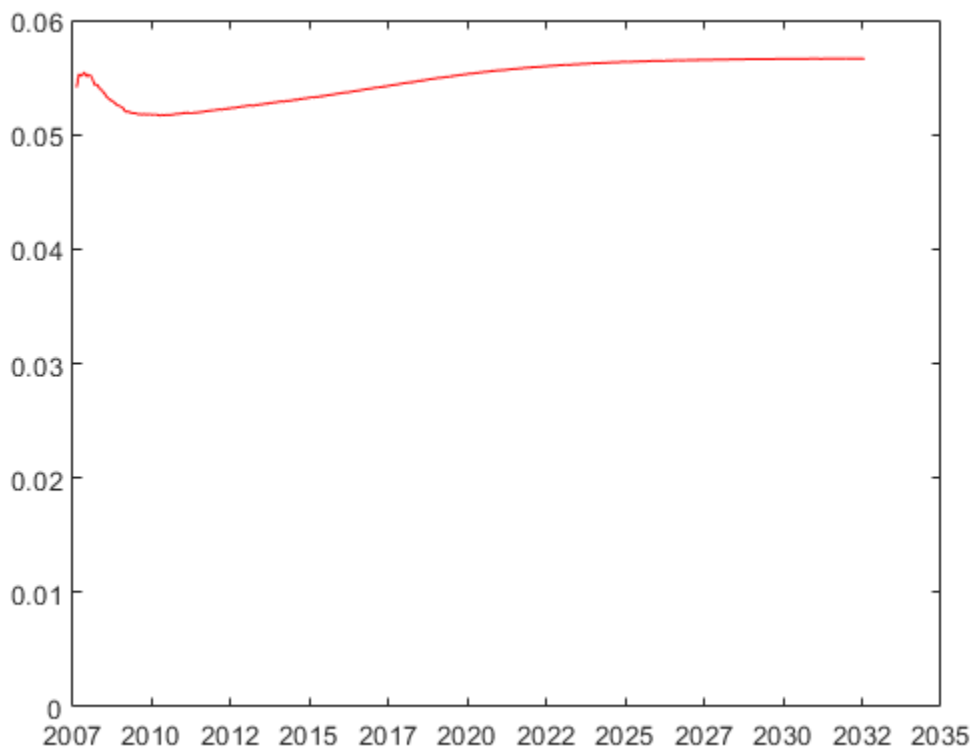
```
InstrumentTypes = {'Deposit';'Deposit';...  
'Futures';'Futures';'Futures';'Futures';'Futures';'Futures';...  
'Swap';'Swap';'Swap';'Swap';};  
  
Instruments = [datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...  
datenum('08/08/2007'),datenum('11/17/2007'),.0535866; ...  
datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...  
datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...  
datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...  
datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...  
datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...  
datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...  
datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...  
datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...  
datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...  
datenum('08/08/2007'),datenum('08/08/2037'),.0566];  
  
CurveSettle = datenum('08/10/2007');
```

Use the bootstrap method to create an IRDataCurve object.

```
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...  
InstrumentTypes, Instruments, 'InterpMethod', 'pchip');
```

To create the plot for the bootstrapped market data:

```
PlottingDates = (datenum('08/11/2007'):30:CurveSettle+365*25)';  
plot(PlottingDates, getParYields(bootModel, PlottingDates), 'r')  
ylim([0 .06])  
datetick
```



### Use the bootstrap Method to Create an IRDataCurve Object That Includes Bonds

In this bootstrapping example, InstrumentTypes, Instruments, and a Settle date are defined:

```
CurveSettle = datenum('8-Mar-2010');
```

```
InstrumentTypes = {'Deposit'; 'Deposit'; 'Deposit'; 'Deposit'; ...  
                  'Futures'; 'Futures'; 'Futures'; 'Futures'; 'Swap'; 'Swap'; 'Bond'; 'Bond'};
```

```
Instruments = [datenum('8-Mar-2010'),datenum('8-Apr-2010'),.003; ...  
    datenum('8-Mar-2010'),datenum('8-Jun-2010'),.005; ...  
    datenum('8-Mar-2010'),datenum('8-Sep-2010'),.007; ...  
    datenum('8-Mar-2010'),datenum('8-Mar-2011'),.009; ...  
    datenum('8-Mar-2010'),datenum('18-Jun-2011'),9840; ...  
    datenum('8-Mar-2010'),datenum('17-Sep-2011'),9820; ...  
    datenum('8-Mar-2010'),datenum('17-Dec-2011'),9810; ...  
    datenum('8-Mar-2010'),datenum('18-Mar-2012'),9800; ...  
    datenum('8-Mar-2010'),datenum('8-Mar-2015'),.025; ...  
    datenum('8-Mar-2010'),datenum('8-Mar-2020'),.035; ...  
    datenum('8-Mar-2010'),datenum('8-Mar-2030'),99; ...  
    datenum('8-Mar-2010'),datenum('8-Mar-2040'),101];
```

When bonds are used, `InstrumentCouponRate` must be specified:

```
InstrumentCouponRate = [zeros(10,1);.045;.05];
```

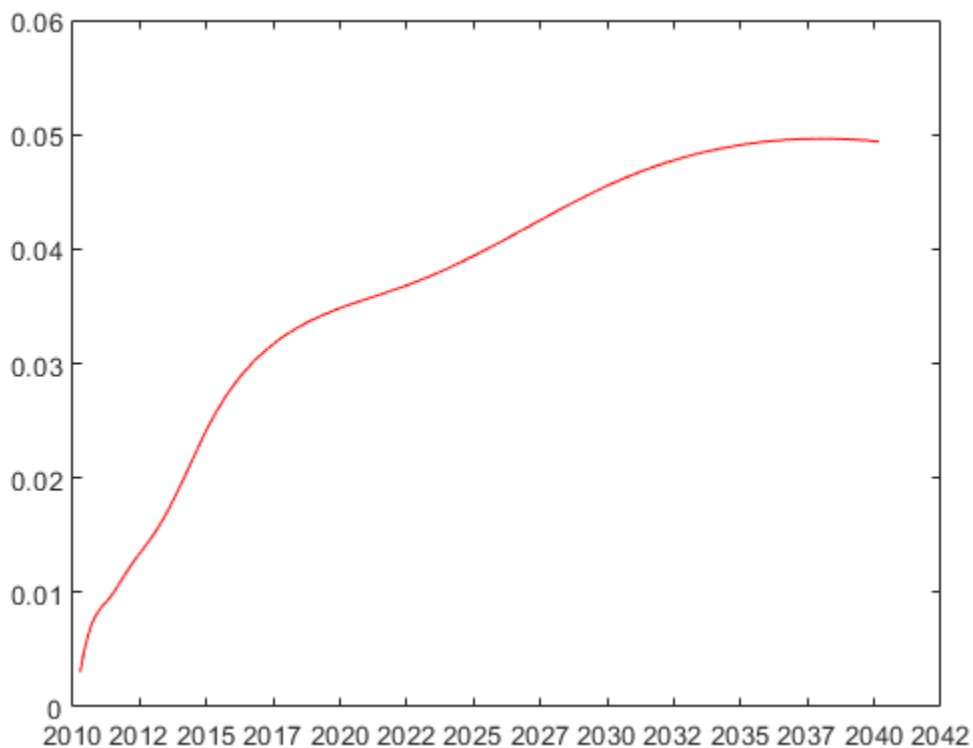
Note, for parameters that are only applicable to bonds (`InstrumentFirstCouponDate`, `InstrumentLastCouponDate`, `InstrumentIssueDate`, `InstrumentFace`) the entries for non-bond instruments (deposits and futures) are ignored.

Use the `bootstrap` method to create an `IRDataCurve` object.

```
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...  
InstrumentTypes, Instruments, 'InterpMethod', 'pchip', ...  
'InstrumentCouponRate', InstrumentCouponRate);
```

Create the plot for the bootstrapped market data.

```
PlottingDates = datemnth(CurveSettle,1:30*12);  
plot(PlottingDates, getParYields(bootModel, PlottingDates), 'r')  
ylim([0 .06])  
datetick
```



### Use IRBootstrapOptionsObj with bootstrap for Negative Zero Interest-Rates

Use the IRBootstrapOptionsObj optional argument with the bootstrap method to allow for negative zero rates when solving for the swap zero points.

```
Settle = datenum('15-Mar-2015');
InstrumentTypes = {'Deposit'; 'Deposit'; 'Swap'; 'Swap'; 'Swap'; 'Swap'};

Instruments = [Settle, datenum('15-Jun-2015'), .001; ...
Settle, datenum('15-Dec-2015'), .0005; ...
Settle, datenum('15-Mar-2016'), -.001; ...
```

```
Settle,datenum('15-Mar-2017'),-0.0005; ...
Settle,datenum('15-Mar-2018'),.0017; ...
Settle,datenum('15-Mar-2020'),.0019];

irbo = IRBootstrapOptions('LowerBound',-1);

bootModel = IRDataCurve.bootstrap('zero', Settle, InstrumentTypes,...
    Instruments,'IRBootstrapOptions',irbo);

bootModel.getZeroRates(datemnth(Settle,1:60))

ans = 60x1

    0.0012
    0.0011
    0.0010
    0.0009
    0.0008
    0.0008
    0.0007
    0.0006
    0.0005
   -0.0000
    :
```

Note that optional argument for `LowerBound` is set to `-1` for negative zero rates when solving the swap zero points.

## See Also

["@IRDataCurve"](#) on page A-7 | ["@IRBootstrapOptions"](#) on page A-2 | [toRateSpec](#)

## Topics

["Creating Interest-Rate Curve Objects"](#) on page 9-4

["IRDataCurve Bootstrapping Based on Market Instruments"](#) on page 9-7

["Bootstrapping a Swap Curve"](#)

["Dual Curve Bootstrapping"](#) on page 9-17

["Interest-Rate Curve Objects and Workflow"](#) on page 9-2

**Introduced in R2008b**

## **cdsoptprice**

Price payer and receiver credit default swap options

### **Syntax**

```
[Payer,Receiver] =  
cdsoptprice(ZeroData,ProbData,Settle,OptionMaturity,CDSMaturity,Strike,SpreadVol)  
[Payer,Receiver] =  
cdsoptprice(ZeroData,ProbData,Settle,OptionMaturity,CDSMaturity,Strike,SpreadVol,Name,Value)
```

### **Description**

[Payer,Receiver] =  
cdsoptprice(ZeroData,ProbData,Settle,OptionMaturity,CDSMaturity,Strike,SpreadVol) computes the price of payer and receiver credit default swap options.

[Payer,Receiver] =  
cdsoptprice(ZeroData,ProbData,Settle,OptionMaturity,CDSMaturity,Strike,SpreadVol,Name,Value) computes the price of payer and receiver credit default swap options with additional options specified by one or more Name, Value pair arguments.

### **Input Arguments**

#### **ZeroData**

M-by-2 vector of dates and zero rates or an IRDataCurve object of zero rates. For more information on an IRDataCurve object, see “Creating an IRDataCurve Object” on page 9-6.

#### **ProbData**

P-by-2 array of dates and default probabilities.



**Settle**

Settlement date is a serial date number or date character vector. **Settle** must be earlier than the maturity date.

**OptionMaturity**

N-by-1 vector of serial date numbers or date character vectors containing the option maturity dates.

**CDSMaturity**

N-by-1 vector of serial date numbers or date character vectors containing the CDS maturity dates.

**Strike**

N-by-1 vector of option strikes expressed in basis points.

**SpreadVol**

N-by-1 vector of annualized credit spread volatilities expressed as a positive decimal number.

**Name-Value Pair Arguments**

Specify optional comma-separated pairs of **Name**, **Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1, Value1, . . . , NameN, ValueN**.

---

**Note** Any optional input of size N-by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N-by-1.

---

**AdjustedForwardSpread**

N-by-1 vector of adjusted forward spreads (in basis points) to be used when pricing CDS index options.

**Default:** unadjusted forward spread normally used for single-name CDS options

## **Basis**

N-by-1 vector of contract day-count basis:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

**Default:** 2 (actual/360)

## **BusDayConvention**

Business day conventions, specified by a character vector or N-by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- **actual** — Non-business days are effectively ignored. Cash flows that fall on non-business days are assumed to be distributed on the actual date.
- **follow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day.

- **modifiedfollow** — Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- **previous** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- **modifiedprevious** — Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

**Default:** actual

### **Knockout**

N-by-1 vector of Boolean flags. If the credit default swaptions is a knockout, the flag is True, otherwise it is False.

**Default:** False

### **PayAccruedPremium**

N-by-1 vector of Boolean flags. If accrued premiums are paid upon default, the flag is True, otherwise it is False.

**Default:** True

### **Period**

N-by-1 vector of the number of premiums per year of the CDS. Allowed values are 1, 2, 3, 4, 6, and 12.

**Default:** 4

### **RecoveryRate**

N-by-1 vector of recovery rates, expressed as a decimal from 0 to 1.

**Default:** 0.4

### **ZeroBasis**

Basis of the zero curve. Choices are identical to **Basis**.

**Default:** 0 (actual/actual)

### ZeroCompounding

Compounding frequency of the zero curve. Allowed values are:

- 1 — Annual compounding
- 2 — Semiannual compounding
- 3 — Compounding three times per year
- 4 — Quarterly compounding
- 6 — Bimonthly compounding
- 12 — Monthly compounding
- -1 — Continuous compounding

---

**Note** When `ZeroData` is an `IRDataCurve` object, the arguments `ZeroCompounding` and `ZeroBasis` are implicit in `ZeroData` and are redundant inside this function. In that case, specify these optional arguments when constructing the `IRDataCurve` object before calling this function.

---

**Default:** 2 (Semiannual compounding)

## Output Arguments

### Payer

N-by-1 vector of prices for payer swap options in `Basis` points.

### Receiver

N-by-1 vector of prices for receiver swap options in `Basis` points.

## Examples

### Obtain Payer and Receiver Values for a Credit Default Swap Option

Use `cdsoptprice` to generate `Payer` and `Receiver` values for a credit default swap option.

```
Settle = datenum('12-Jun-2012');
OptionMaturity = datenum('20-Sep-2012');
CDSMaturity = datenum('20-Sep-2017');
OptionStrike = 200;
SpreadVolatility = .4;

Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [.5 .75 1.5 1.7 1.9 2.2]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];

Market_Time = [1 2 3 5 7 10]';
Market_Rate = [100 120 145 220 245 270]';
Market_Dates = daysadd(Settle,360*Market_Time,1);
MarketData = [Market_Dates Market_Rate];

ProbData = cdsbootstrap(ZeroData, MarketData, Settle);

[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle,...
OptionMaturity, CDSMaturity, OptionStrike, SpreadVolatility)

Payer = 223.5780
Receiver = 22.7460
```

## Definitions

### Credit Default Swap Option

A credit default swap (CDS) option, or credit default swaption, is a contract that provides the option holder with the right, but not the obligation, to enter into a credit default swap in the future.

CDS options can either be payer swaptions or receiver swaptions. In a payer swaption, the option holder has the right to enter into a CDS in which they are paying premiums and in a receiver swaption, the option holder is receiving premiums.

## Algorithms

The payer and receiver credit default swap options are computed using the Black's model as described in O'Kane [1]:

$$V_{Pay(Knockout)} = RPV01(t, t_E, T)(F\Phi(d_1) - K\Phi(d_2))$$

$$V_{Rec(Knockout)} = RPV01(t, t_E, T)(K\Phi(-d_2) - F\Phi(-d_1))$$

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \frac{1}{2}\sigma^2(t_E - t)}{\sigma\sqrt{t_E - t}}$$

$$d_2 = d_1 - \sigma\sqrt{t_E - t}$$

$$V_{Pay(Non - Knockout)} = V_{Pay(Knockout)} + FEP$$

$$V_{Pay(Non - Knockout)} = V_{Rec(Knockout)}$$

where

*RPV01* is the risky present value of a basis point (see `cdsrpv01`).

$\Phi$  is the normal cumulative distribution function.

$\sigma$  is the spread volatility.

$t$  is the valuation date.

$t_E$  is the option expiry date.

$T$  is the CDS maturity date.

$F$  is the forward spread (from option expiry to CDS maturity).

$K$  is the strike spread.

*FEP* is the front-end protection (from option initiation to option expiry).

## References

[1] O'Kane, D. *Modelling Single-name and Multi-name Credit Derivatives*. Wiley, 2008, pp. 156-169.

## See Also

[IRDataCurve](#) | [cdsbootstrap](#) | [cdsprice](#) | [cdsrpv01](#) | [cdsspread](#)

## Topics

["Pricing a Single-Name CDS Option"](#) on page 8-38

["Pricing a CDS Index Option"](#) on page 8-40

["Credit Default Swap Option"](#) on page 8-37

## External Websites

[Pricing and Valuation of Credit Default Swaps \(4 min 22 sec\)](#)

**Introduced in R2011a**

## **cmosched**

Generate principal balance schedule for planned amortization class (PAC) or targeted amortization class (TAC) bond

### **Syntax**

```
[BalanceSchedule,InitialBalance] = cmosched(Principal,Coupon,  
OriginalTerm,TermRemaining,PrepaySpeed)  
[BalanceSchedule,InitialBalance] = cmosched( ____,TranchePrincipal)
```

### **Description**

[BalanceSchedule,InitialBalance] = cmosched(Principal,Coupon,OriginalTerm,TermRemaining,PrepaySpeed) generates a principal balance schedule for planned amortization class (PAC) bonds using two bands of Public Securities Association Prepayment Model (PSA) speeds or targeted amortization class (TAC) bonds using a single PSA speed.

[BalanceSchedule,InitialBalance] = cmosched( \_\_\_\_,TranchePrincipal) adds an optional argument for TranchePrincipal.

### **Examples**

#### **Calculate the Principal Balance Schedule for a CMO PAC Bond**

Define the mortgage pool under consideration and generate a principal balance schedule for planned amortization class (PAC) bonds using two bands of PSA speeds.

```
Principal = 128687000;  
GrossRate = 0.0648;  
OriginalTerm = 360;  
TermRemaining = 325;  
PrepaySpeed = [300 525];  
PacPrincipal = 100250000;
```



```
[BalanceSchedule, InitialBalance] ...
= cmosched(Principal, GrossRate, OriginalTerm, TermRemaining, ...
PrepaySpeed, PacPrincipal)
```

```
BalanceSchedule = 1×325
107 ×
```

```
9.7996 9.5780 9.3602 9.1461 8.9357 8.7289 8.5257 8.3259 8.1
```

```
InitialBalance = 100250000
```

## Input Arguments

### **Principal** — Principal of the underlying mortgage pool

numeric

Principal of the underlying mortgage pool, specified as a scalar numeric value.

Data Types: double

### **Coupon** — Coupon rate of the underlying mortgage pool

decimal

Coupon rate of the underlying mortgage pool, specified as a scalar decimal value.

Data Types: double

### **OriginalTerm** — Original term in months of the underlying mortgage pool

numeric

Original term in months of the underlying mortgage pool, specified as a scalar numeric value.

Data Types: double

### **TermRemaining** — Terms remaining in months of the underlying mortgage pool

numeric

Terms remaining in months of the underlying mortgage pool, specified as a scalar numeric value.

Data Types: double

**PrepaySpeed — PSA speed**

matrix numeric | scalar numeric

PSA speed is specified as follows:

- For planned amortization class (PAC) bonds, PSA speed is specified as a 1-by-2 matrix, where the first element is the lower band and the second element is the upper band.
- For targeted amortization class (TAC) bonds, the PSA speed is specified as a scalar numeric value

Data Types: double

**TranchePrincipal — Principal of the scheduled tranche**

numeric

(Optional) Principal of the scheduled tranche, specified as a scalar numeric value. If `TranchePrincipal` is unspecified or empty [ ], the principal of the scheduled tranche is assumed to be the sum of the payment schedule calculated from the PSA prepayment speeds.

Data Types: double

## Output Arguments

**BalanceSchedule — Number of terms remaining**

matrix

Number of terms remaining, returned as a matrix of size 1-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each column contains the scheduled principal balance for the time period corresponding to the column number.

**InitialBalance — Initial principal balance of the scheduled tranche**

scalar

initial principal balance of the scheduled tranche, returned as a scalar numeric value.

## Definitions

### Planned Amortization Class (PAC) Bond

PAC bonds are a type of CMO bond and are designed to largely eliminate prepayment risk for investors.

They do this by transferring essentially all prepayment risk to other bonds in the CMO that are called support bonds.

### Targeted Amortization Class (TAC) Bond

TAC bonds are analogous to PAC bonds, but are structured differently.

TAC bonds offer one-sided protection, shielding investors from high prepayment rates up to a specified PSA and do not protect against low prepayment rates.

## References

- [1] Hayre, Lakhbir, ed. *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*. John Wiley and Sons, New York, 2001.
- [2] Lyuu, Yuh-Dah. *Financial Engineering and Computation*. Cambridge University Press, 2004.

## See Also

cmoschedcf

## Topics

- “Create PAC and Sequential CMO” on page 5-63
- “What Are CMOs?” on page 5-51
- “Prepayment Risk” on page 5-52
- “CMO Workflow” on page 5-60

## Introduced in R2012a

## **cmoschedcf**

Generate cash flows for scheduled collateralized mortgage obligation (CMO) using PAC or TAC model

### **Syntax**

```
[Balance,Principal,Interest] = cmoschedcf(PrincipalPayments,  
TranchePrincipalsTrancheCoupons,BalanceSchedule)
```

### **Description**

[Balance,Principal,Interest] = cmoschedcf(PrincipalPayments, TranchePrincipalsTrancheCoupons, BalanceSchedule) generates cash flows for a scheduled CMO, such as the planned amortization class (PAC) or targeted amortization class (TAC), given the underlying mortgage pool payments (or payments from another CMO tranche). The output **Balances**, **Principal**, and **Interest** from this function can be used as input into **cmoseqcf** to further divide the PAC, TAC, or support dividing a tranche into sequential tranches.

### **Examples**

#### **Calculate Cash Flows for Each PAC Tranche**

Define the mortgage pool under consideration for CMO structuring using **mbscfamounts** or **mbspassthrough**. Calculate the underlying mortgage cash flow, define the PAC schedule and CMO tranches, and calculate the cash flows for each tranche.

```
MortgagePrincipal = 1000000; % underlying mortgage  
Coupon = 0.12;  
Terms = 6; % months
```

```
[PrincipalBalance, MonthlyPayments, SchedPrincipalPayments, ...  
InterestPayments, Prepayments] = ...
```

```
mbspassthrough(MortgagePrincipal, Coupon, Terms, Terms, 0, []);
PrincipalPayments = SchedPrincipalPayments.' + Prepayments.'
```

```
PrincipalPayments = 1×6
105 ×
```

```
1.6255    1.6417    1.6582    1.6747    1.6915    1.7084
```

Calculate the PAC schedule for CMO using cmosched.

```
PrepaySpeed = [100 300];
[BalanceSchedule, InitialBalance] ...
= cmosched(MortgagePrincipal, Coupon, Terms, Terms, PrepaySpeed, [])
```

```
BalanceSchedule = 1×6
105 ×
```

```
8.3617    6.7180    5.0581    3.3828    1.6955    0
```

```
InitialBalance = 9.9886e+05
```

Define CMO tranches.

```
TranchePrincipals = ...
[InitialBalance; MortgagePrincipal-InitialBalance];
TrancheCoupons = [0.12; 0.12];
```

Calculate cash flows for each tranche.

```
[Balance, Principal, Interest] = ...
cmoschedcf(PrincipalPayments, TranchePrincipals, ...
TrancheCoupons, BalanceSchedule)
```

```
Balance = 2×6
105 ×
```

```
8.3631    6.7213    5.0632    3.3885    1.6970    0
0.0114    0.0114    0.0114    0.0114    0.0114    0.0000
```

```
Principal = 2×6
105 ×
```

```
1.6255    1.6417    1.6582    1.6747    1.6915    1.6970
```

0 0 0 0 0 0.0114

Interest =  $2 \times 6$   
 $10^3 \times$

9.9886	8.3631	6.7213	5.0632	3.3885	1.6970
0.0114	0.0114	0.0114	0.0114	0.0114	0.0114

## Input Arguments

### **PrincipalPayments** — Number of terms remaining for underlying principal payments

numeric matrix

Number of terms remaining for underlying principal payments, specified as a matrix of size 1-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each column contains the underlying principal payment for the time period corresponding to the row number. Calculate underlying principal payments using `mbscfamounts` or `mbspassthrough`. The underlying principal payments can also be outputs from other CMO cash flow functions.

Data Types: double

### **TranchePrincipals** — Initial principal for the scheduled and the support tranche

numeric matrix

Initial principal for the scheduled and the support tranche, specified as a matrix of size 2-by-1.

Data Types: double

### **TrancheCoupons** — Coupons for the schedule tranche and the support tranche

matrix of coupon values

Coupons for the schedule tranche and the support tranche, specified as a matrix of size 2-by-1 of coupon values. The weighted average coupon for the CMO should not exceed the coupon of the underlying mortgage.

Data Types: double

**BalanceSchedule — Number of terms remaining for targeted balance**

numeric matrix

Number of terms remaining for targeted balance, specified as a matrix of size 1-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each element represents the targeted balance schedule for the time period corresponding to that column.

Data Types: double

## Output Arguments

**Balance — Number of terms remaining and principal balances**

matrix

Number of terms remaining and principal balances, returned as a matrix of size 2-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. The first row is the principal balances of the scheduled tranche, and the second row is the principal balances of the support tranche at the time period corresponding to the column.

**Principal — Number of terms remaining and principal payments**

matrix

Number of terms remaining and principal payments, returned as a matrix of size 2-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. The first row is the principal payments of the scheduled tranche, and the second row is the principal payments of the support tranche at the time period corresponding to the column.

**Interest — Number of terms remaining and interest payments**

scalar

Number of terms remaining and interest payments, returned as a matrix of size 2-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. The first row is the interest payments of the schedule tranche, and the second row is the interest payments of the support tranche at the time period corresponding to the column.

## Definitions

### Planned Amortization Class (PAC) Tranches

In a PAC CMO, there is a main tranche, known as the schedule tranche, and a support tranche.

The main purpose of a schedule tranche is to give investors in the PAC tranche a more certain cash flow.

### Targeted Amortization Class (TAC) Tranches

TACs are like PACs, but principal payment is specified for only one prepayment rate.

If prepayment rates are higher or lower, then the principal payment to TAC holders are higher or lower accordingly.

### Schedule and Support Tranche

The main purpose of a PAC tranche is to give investors in the PAC tranche a more certain cash flow.

The PAC tranche receives priority for receiving payments of principal and interest that gives investors in the PAC tranche a steadier income. If prepayments differ from what was expected, then the support tranche gets the variable portion of the payments. While income to the support tranche is more variable, it is also higher yielding. Estimates of the yield, average life, and lockout periods of the PAC tranche is more certain.

## References

- [1] Hayre, Lakhbir, ed. *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*. John Wiley and Sons, New York, 2001.
- [2] Lyuu, Yuh-Dah. *Financial Engineering and Computation*. Cambridge University Press, 2004.

## See Also

cmosched | cmoseqcf | mbscfamounts | mbspassthrough



## **Topics**

“Create PAC and Sequential CMO” on page 5-63

“What Are CMOs?” on page 5-51

“Prepayment Risk” on page 5-52

“CMO Workflow” on page 5-60

## **Introduced in R2012a**

## cmoseqcf

Generate cash flows for sequential collateralized mortgage obligation (CMO)

### Syntax

```
[Balance,Principal,Interest] = cmoseqcf(PrincipalPayments,  
TranchePrincipalsTrancheCoupons)  
[Balance,Principal,Interest] = cmoseqcf( ____,HasZ)
```

### Description

[Balance,Principal,Interest] = cmoseqcf(PrincipalPayments, TranchePrincipalsTrancheCoupons) generates cash flows for a sequential CMO without a Z-bond, given the underlying mortgage pool payments.

[Balance,Principal,Interest] = cmoseqcf( \_\_\_\_,HasZ) generates cash flows for a sequential CMO with a Z-bond, given the underlying mortgage pool payments, by adding an additional optional input for HasZ.

### Examples

#### Calculate Cash Flows for a Sequential Collateralized Mortgage Obligation (CMO)

Define the mortgage pool under consideration for CMO structuring using `mbscfamounts` or `mbspassthrough` and calculate the cash flows with an A and B tranche for a sequential CMO.

```
MortgagePrincipal = 1000000;  
Coupon = 0.12;  
Terms = 6; % months
```

```
% Calculate underlying mortgage cash flows
```

```
[PrincipalBalance, MonthlyPayments, SchedPrincipalPayments, ...  
InterestPayments, Prepayments] = ...
```

```
mbspassthrough(MortgagePrincipal, Coupon, Terms, Terms, 0, []);
PrincipalPayments = SchedPrincipalPayments.' + Prepayments.'
```

```
PrincipalPayments = 1×6
105 ×
```

```
1.6255    1.6417    1.6582    1.6747    1.6915    1.7084
```

Define CMO tranches, A and B.

```
TranchePrincipals = [500000; 500000];
TrancheCoupons = [0.12; 0.12];
```

Calculate cash flows for each tranche.

```
[Balance, Principal, Interest] = ...
cmoseqcf(PrincipalPayments, TranchePrincipals, TrancheCoupons, false)
```

```
Balance = 2×6
105 ×
```

```
3.3745    1.7328    0.0746         0         0         0
5.0000    5.0000    5.0000    3.3999    1.7084    0.0000
```

```
Principal = 2×6
105 ×
```

```
1.6255    1.6417    1.6582    0.0746         0         0
         0         0         0    1.6001    1.6915    1.7084
```

```
Interest = 2×6
103 ×
```

```
5.0000    3.3745    1.7328    0.0746         0         0
5.0000    5.0000    5.0000    5.0000    3.3999    1.7084
```

## Input Arguments

### **PrincipalPayments** — Number of terms remaining for underlying principal payments

numeric matrix

Number of terms remaining for underlying principal payments, specified as a matrix of size 1-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each column contains the underlying principal payment for the time period corresponding to the row number. Calculate underlying principal payments using `mbscfamounts` or `mbspassthrough`. The underlying principal payments can also be outputs from other CMO cash flow functions.

Data Types: `double`

### **TranchePrincipals** — Initial principal for each tranche

numeric matrix

Initial principal for each tranche, specified as a matrix of size `NUMTRANCHES`-by-1, where `NUMTRANCHES` is the number of tranches in the sequential CMO. Each element of the matrix represents the initial principal for each tranche. If the sequential CMO includes a Z-bond (`HasZ` is `true`), the last element of this matrix is the principal of the Z-bond.

Data Types: `double`

### **TrancheCoupons** — Coupon for each tranche

matrix of coupon values

Coupon for each tranche, specified as a matrix of size `NUMTRANCHES`-by-1, where `NUMTRANCHES` is the number of tranches in the sequential CMO. Each element of the matrix represents the coupon for each tranche. If the sequential CMO includes a Z-bond (`HasZ` is `true`), the last element of this matrix is the coupon of the Z-bond. The weighted average coupon for the CMO should not exceed the coupon of the underlying mortgage.

Data Types: `double`

### **HasZ** — Indicates that the sequential CMO contains a Z-bond

`false` (default) | `true` | `false`

(Optional) Indicates that the sequential CMO contains a Z-bond, specified as a Boolean (`true` or `false`). A value of `true` indicates that the sequential CMO contains a Z-bond, and the last element of `TranchePrincipals` and `TrancheCoupons` is treated as that of

the Z-bond. A value of false indicates that there is no Z-bond in the sequential CMO, and the last element of TranchePrincipals and TrancheCoupons is treated as an ordinary tranche.

Data Types: logical

## Output Arguments

### **Balance — Principal balance for time period and tranche**

matrix

Principal balance for time period and tranche, returned as a matrix of size NUMTRANCHES-by-NUMTERMS, where NUMTRANCHES is the number of terms remaining and NUMTRANCHES is the number of tranches. Each element represents the principal balance at the time period corresponding to the column, and for the tranche corresponding to the row.

### **Principal — Principal payments for time period and tranche**

matrix

Principal payments for time period and tranche, returned as a matrix of size NUMTRANCHES-by-NUMTERMS, where NUMTRANCHES is the number of terms remaining and NUMTRANCHES is the number of tranches. Each element represents the principal payments made at the time period corresponding to the column, and to the tranche corresponding to the row.

### **Interest — Interest payments for time period and tranche**

matrix

Interest payments for time period and tranche, returned as a matrix of size NUMTRANCHES-by-NUMTERMS, where NUMTRANCHES is the number of terms remaining and NUMTRANCHES is the number of tranches. Each element represents the interest payments made at the time period corresponding to the column, and to the tranche corresponding to the row.

## Definitions

### **Sequential Pay CMO**

A sequential pay CMO involves tranches that pay off principal sequentially.

For example, consider the following case, where all principal from the underlying mortgage pool is repaid on tranche A first, then tranche B, then tranche C. Interest is paid on each tranche as long as the principal for the tranche has not been retired.

### **CMO Tranche**

Tranche is a term often used to describe a specific class of bonds within an offering wherein each tranche offers varying degrees of risk to the investor.

### **References**

- [1] Hayre, Lakhbir, ed. *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*. John Wiley and Sons, New York, 2001.
- [2] Lyuu, Yuh-Dah. *Financial Engineering and Computation*. Cambridge University Press, 2004.

### **See Also**

[cmosched](#) | [cmoschedcf](#) | [mbscfamounts](#) | [mbspassthrough](#)

### **Topics**

- “Create PAC and Sequential CMO” on page 5-63
- “What Are CMOs?” on page 5-51
- “Prepayment Risk” on page 5-52
- “CMO Workflow” on page 5-60

### **Introduced in R2012a**

# convfactor

Bond conversion factors

## Syntax

```
CF = convfactor(RefDate,Maturity,CouponRate)
CF = convfactor( ____,Name,Value)
```

## Description

CF = convfactor(RefDate,Maturity,CouponRate) computes a conversion factor for a bond futures contract.

CF = convfactor( \_\_\_\_,Name,Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute the Conversion Factors For a Bond Futures Contract

This example shows how to calculate CF, given the following RefDate, Maturity, and CouponRate.

```
RefDate = {'1-Dec-2002';
           '1-Mar-2003';
           '1-Jun-2003';
           '1-Sep-2003';
           '1-Dec-2003';
           '1-Sep-2003';
           '1-Dec-2002';
           '1-Jun-2003'};

Maturity = {'15-Nov-2012';
           '15-Aug-2012'};
```

```
'15-Feb-2012';  
'15-Feb-2011';  
'15-Aug-2011';  
'15-Aug-2010';  
'15-Aug-2009';  
'15-Feb-2010'};
```

```
CouponRate = [0.04; 0.04375; 0.04875; 0.05; 0.05; 0.0575; 0.06; 0.065];
```

```
CF = convfactor(RefDate, Maturity, CouponRate)
```

```
CF = 8×1
```

```
0.8539  
0.8858  
0.9259  
0.9418  
0.9403  
0.9862  
1.0000  
1.0266
```

### **Compute the Conversion Factor For a German Bond**

This example shows how to calculate `cf`, given the following `RefDate`, `Maturity`, and `CouponRate` for a German bond.

```
cf = convfactor('3/10/2009', '1/04/2018', .04, .06, 3)
```

```
cf = 0.8659
```

## **Input Arguments**

### **RefDate — Reference dates**

serial date number

Reference dates for which conversion factor is computed (usually the first day of delivery months), specified as an N-by-1 vector of serial date numbers.



Data Types: double

### **Maturity — Maturity date**

serial date number

Maturity date, specified as a N-by-1 vector of serial date numbers.

Data Types: double

### **CouponRate — Annual coupon rates for underlying bond**

vector in decimals

Annual coupon rates for underlying bond, specified as an numBonds-by-1 vector in decimals.

Data Types: double

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of `Name`, `Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, . . . , NameN, ValueN`.

Example: `CF = convfactor(RefDate, Maturity, CouponRate, 'Convention', 2)`

### **Convention — Conversion factor convention**

1 US Treasury bond (30-year) and Treasury note (10-year) futures contract (default) | integer from 1 to 5

Conversion factor convention, specified as the comma-separated pair consisting of 'Convention' and a N-by-1 vector using the following values:

- 1 = US Treasury bond (30-year) and Treasury note (10-year) futures contract
- 2 = US 2-year and 5-year Treasury note futures contract
- 3 = German Bobl, Bund, Buxl, and Schatz
- 4 = UK gilts
- 5 = Japanese Government Bonds (JGBs)

Data Types: double

**FirstCouponDate — Irregular first coupon date**

serial date number

Irregular first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a N-by-1 vector using a serial date numbers.

Data Types: double

**RefYield — Reference semiannual yield**

0.06 (6%) (default) | vector in decimals

Reference semiannual yield, specified as the comma-separated pair consisting of 'RefYield' and an N-by-1 vector in decimals.

Data Types: double

**StartDate — Forward starting date of payments**

serial date number

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a N-by-1 vector using serial date numbers.

Data Types: double

## Output Arguments

**CF — Conversion factors against the 6% yield par-bond**

vector

Conversion factors against the 6% yield par-bond, returned as an N-by-1 vector.

## Definitions

### Conversion Factors

Conversion factors of US Treasury bonds and other government bonds are based on a bond yielding 6%.

Optionally, you can specify other types of bonds and yields using inputs for RefYield and Convention. For US Treasury bonds, verify the output of convfactor by comparing the

output against the quotations provided by the Chicago Board of Trade (<https://www.cmegroup.com/company/cbot.html>).

For German bonds, verify the output of `convfactor` by comparing the output against the quotations provided by Eurex (<https://www.eurexchange.com>).

For UK Gilts, verify the output of `convfactor` by comparing the output against the quotations provided by Euronext (<https://www.euronext.com>).

For Japanese Government Bonds, verify the output of `convfactor` by comparing the output against the quotations provided by the Tokyo Stock Exchange (<https://www.jpx.co.jp/english/>).

## References

- [1] Burghardt, G., T. Belton, M. Lane, and J. Papa. *The Treasury Bond Basis*. McGraw-Hill, 2005.
- [2] Krgin, Dragomir. *Handbook of Global Fixed Income Calculations*. John Wiley & Sons, 2002.

## See Also

`bndfutimprepo` | `bndfutprice` | `tfutbyprice` | `tfutbyyield` | `tfutimprepo`

## Topics

- “Analysis of Bond Futures” on page 7-13
- “Managing Interest-Rate Risk with Bond Futures”
- “Fitting the Diebold Li Model”
- “Managing Present Value with Bond Futures” on page 7-16

## Introduced in R2009b

## fitFunction

Custom fit interest-rate curve object to bond market data

### Class

@IRFunctionCurve on page A-13

### Syntax

```
CurveObj = IRFunctionCurve.fitFunction(Type,Settle,FunctionHandle,Instruments,IRFitOpt:
```

```
CurveObj = IRFunctionCurve.fitFunction(Type,Settle,FunctionHandle,Instruments,IRFitOpt:
```

### Arguments

Type	Type of interest-rate curve for a bond: zero, forward, or discount.
Settle	Scalar for the Settle date of the curve.
FunctionHandle	Function handle that defines the interest-rate curve. The function handle takes two numeric vectors (time-to-maturity and a vector of function coefficients) and returns one numeric output (interest rate or discount factor). For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.
Instruments	N-by-4 data matrix for Instruments where the first column is Settle date, the second column is Maturity, the third column is the clean price, and the fourth column is a CouponRate for the bond.
IRFitOptionsObj	Object constructed from IRFitOptions.

Compounding	<p>(Optional) Scalar that sets the compounding frequency per year for the IRFunctionCurve object:</p> <ul style="list-style-type: none"> <li>• -1 = Continuous compounding</li> <li>• 1 = Annual compounding</li> <li>• 2 = Semiannual compounding (default)</li> <li>• 3 = Compounding three times per year</li> <li>• 4 = Quarterly compounding</li> <li>• 6 = Bimonthly compounding</li> <li>• 12 = Monthly compounding</li> </ul>
Basis	<p>(Optional) Day-count basis of the bond. A scalar of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>

## Instrument Parameters

For each bond `Instrument`, you can specify the following additional instrument parameters as parameter/value pairs. For example, `InstrumentBasis` distinguishes a bond instrument's `Basis` value from the curve's `Basis` value.

<code>InstrumentPeriod</code>	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.
<code>InstrumentBasis</code>	<p>(Optional) Day-count basis of the bond. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<code>InstrumentEndMonthRule</code>	(Optional) End-of-month rule. A vector. This rule applies only when <code>Maturity</code> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
<code>InstrumentIssueDate</code>	(Optional) Date when an instrument was issued.

InstrumentFirstCouponDate	(Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentLastCouponDate	(Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentFace	(Optional) Face or par value. Default = 100.

---

**Note** When using Instrument name-value pairs, you can specify simple interest for a bond by specifying the InstrumentPeriod value as 0. If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2.

---

## Description

CurveObj = IRFunctionCurve.fitFunction(Type, Settle, FunctionHandle, Instruments, IRFitOptionsObj, Name, Value) fits a bond to a custom fitting function. You must enter the optional arguments for Basis and Compounding as comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## Examples

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009'); datenum('07-Mar-2011'); ...
datenum('07-Mar-2013'); datenum('07-Sep-2016'); ...
```

```

datenum('07-Mar-2025');datenum('07-Mar-2036');
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
CurveSettle = datenum('30-Apr-2008');
OptOptions = optimoptions('lsqnonlin','display','iter');
functionHandle = @(t,theta) polyval(theta,t);

CustomModel = IRFunctionCurve.fitFunction('Zero', CurveSettle, ...
functionHandle,Instruments, ...
IRFitOptions([.05 .05 .05],'FitType','price',...
'OptOptions',OptOptions));

```

Iteration	Func-count	f(x)	Norm of step	First-order optimality	CG-iterations
0	4	38036.7		4.92e+04	
1	8	38036.7	10	4.92e+04	0
2	12	38036.7	2.5	4.92e+04	0
3	16	38036.7	0.625	4.92e+04	0
4	20	38036.7	0.15625	4.92e+04	0
5	24	30741.5	0.0390625	1.72e+05	0
6	28	30741.5	0.078125	1.72e+05	0
7	32	30741.5	0.0195312	1.72e+05	0
8	36	28713.6	0.00488281	2.33e+05	0
9	40	20323.3	0.00976562	9.47e+05	0
10	44	20323.3	0.0195312	9.47e+05	0
11	48	20323.3	0.00488281	9.47e+05	0
12	52	20323.3	0.0012207	9.47e+05	0
13	56	19698.8	0.000305176	1.08e+06	0
14	60	17493	0.000610352	7e+06	0
15	64	17493	0.0012207	7e+06	0
16	68	17493	0.000305176	7e+06	0
17	72	15455.1	7.62939e-05	2.25e+07	0
18	76	15455.1	0.000177499	2.25e+07	0
19	80	13317.1	3.8147e-05	3.18e+07	0
20	84	12865.3	7.62939e-05	7.83e+07	0
21	88	11779.8	7.62939e-05	7.58e+06	0
22	92	11747.6	0.000152588	1.45e+05	0
23	96	11720.9	0.000305176	2.33e+05	0
24	100	11667.2	0.000610352	1.48e+05	0
25	104	11558.6	0.0012207	3.55e+05	0
26	108	11335.5	0.00244141	1.57e+05	0
27	112	10863.8	0.00488281	6.36e+05	0
28	116	9797.14	0.00976562	2.53e+05	0
29	120	6882.83	0.0195312	9.18e+05	0
30	124	6882.83	0.0373993	9.18e+05	0
31	128	3218.45	0.00934981	1.96e+06	0
32	132	612.703	0.0186996	3.01e+06	0
33	136	13.0998	0.0253882	3.05e+06	0
34	140	0.0762922	0.00154002	5.05e+04	0
35	144	0.0731652	3.61102e-06	29.9	0
36	148	0.0731652	6.32335e-08	0.063	0

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the default value of the function tolerance.



## See Also

"@IRFitOptions" on page A-11 | "@IRFunctionCurve" on page A-13

## Topics

"Creating an IRFunctionCurve Object" on page 9-22

"Fitting Interest Rate Curve Functions" on page 9-33

"Using fitFunction to Create Custom Fitting Function" on page 9-29

"Interest-Rate Curve Objects and Workflow" on page 9-2

"Creating Interest-Rate Curve Objects" on page 9-4

## External Websites

Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)

Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk Applications (30 min 00 sec)

## Introduced in R2008b

## fitNelsonSiegel

Fit Nelson-Siegel function to bond market data

### Class

@IRFunctionCurve on page A-13

### Syntax

`CurveObj = IRFunctionCurve.fitNelsonSiegel(Type,Settle,Instruments)`

`CurveObj = IRFunctionCurve.fitNelsonSiegel(Type,Settle,Instruments,Name,Value)`

### Arguments

Type	Type of interest-rate curve for a bond: zero or forward.
Settle	Scalar for the <code>Settle</code> date of the curve.
Instruments	N-by-4 data matrix for <code>Instruments</code> where the first column is <code>Settle</code> date, the second column is <code>Maturity</code> , the third column is the clean price, and the fourth column is a <code>CouponRate</code> for the bond.
Compounding	(Optional) Scalar that sets the compounding frequency per year for the <code>IRFunctionCurve</code> object: <ul style="list-style-type: none"> <li>-1 = Continuous compounding</li> </ul>

<b>Basis</b>	<p>(Optional) Day-count basis of the interest-rate curve. A scalar of integers.</p> <ul style="list-style-type: none"><li>• 0 = actual/actual (default)</li><li>• 1 = 30/360 (SIA)</li><li>• 2 = actual/360</li><li>• 3 = actual/365</li><li>• 4 = 30/360 (BMA)</li><li>• 5 = 30/360 (ISDA)</li><li>• 6 = 30/360 (European)</li><li>• 7 = actual/365 (Japanese)</li><li>• 8 = actual/actual (ICMA)</li><li>• 9 = actual/360 (ICMA)</li><li>• 10 = actual/365 (ICMA)</li><li>• 11 = 30/360E (ICMA)</li><li>• 12 = actual/365 (ISDA)</li><li>• 13 = BUS/252</li></ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	--

IRFitOptions	<p>(Optional) Object constructed from IRFitOptions. When using IRFitOption, the default FitType is DurationWeightedPrice. Duration weighted price refers to the form of the objective function that needs to be minimized to find the optimal Nelson-Siegel parameters. Specifically, this objective function minimizes using the following three algorithms:</p> <ul style="list-style-type: none"> <li>• The difference between observed and model-predicted yields for each bond, <math>ObsY_i - PredY_i</math></li> <li>• The difference between observed and model-predicted prices for each bond, <math>ObsP_i - PredP_i</math></li> <li>• The difference between observed and model-predicted prices, weighted by the inverse of the duration of each bond <math>(ObsP_i - PredP_i) / D_i</math>. Weighting price by inverse duration converts the pricing errors into yield fitting errors, to a first approximation.</li> </ul>
--------------	--

## Instrument Parameters

For each bond Instrument, you can specify the following additional instrument parameters as name-value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value.

InstrumentPeriod	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.
------------------	---

InstrumentBasis	<p>(Optional) Day-count basis of the bond. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> (Financial Toolbox).</p>
InstrumentEndMonthRule	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. <b>0</b> = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. <b>1</b> = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
InstrumentIssueDate	<p>(Optional) Date when an instrument was issued.</p>
InstrumentFirstCouponDate	<p>(Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When <b>FirstCouponDate</b> and <b>LastCouponDate</b> are both specified, <b>FirstCouponDate</b> takes precedence in determining the coupon payment structure. If you do not specify a <b>FirstCouponDate</b>, the cash flow payment dates are determined from other inputs.</p>

InstrumentLastCouponDate	(Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentFace	(Optional) Face or par value. Default = 100.

---

**Note** When using Instrument name-value pairs, you can specify simple for a bond by specifying the InstrumentPeriod value as 0. If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2.

---

## Description

CurveObj = IRFunctionCurve.fitNelsonSiegel(Type, Settle, Instruments, Name, Value) fits a Nelson-Siegel function to market data for a bond. You must enter the optional arguments for Basis, Compounding, and IRFitOptions as comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

After creating a Nelson-Siegel model, you can view the Nelson-Siegel model parameters using:

```
CurveObj.Parameters
```

where the order of parameters is [Beta0, Beta1, Beta2, tau1].

## Examples

### Use the Nelson-Siegel Function to Fit Bond Market Data

This example shows how to use the Nelson-Siegel function to fit bond market data.

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];

CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);

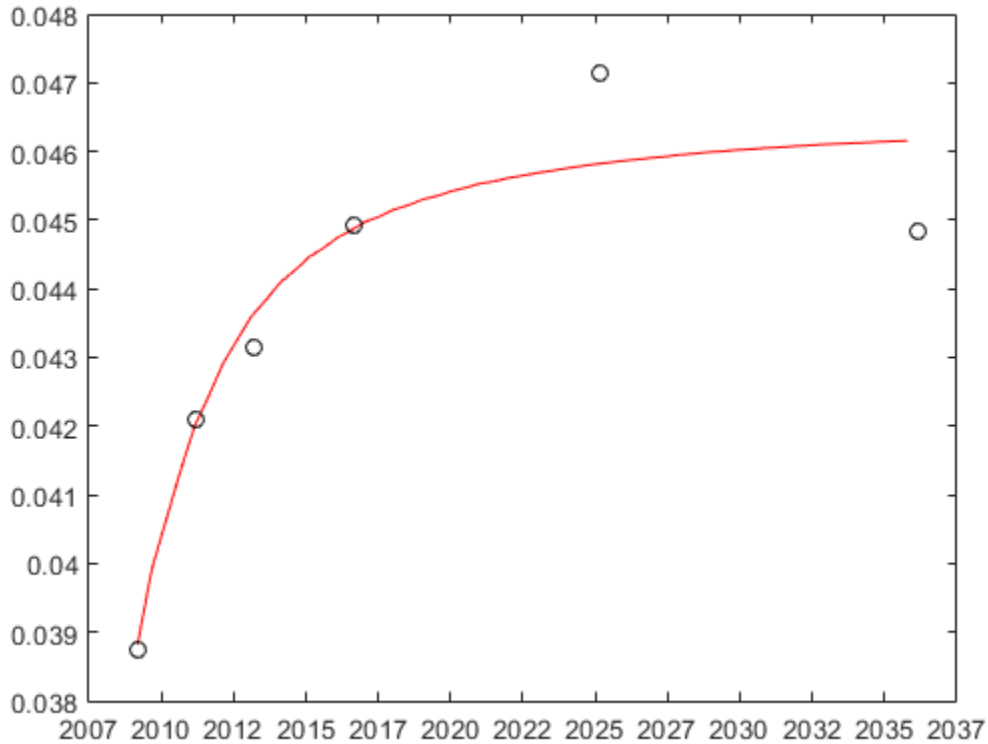
NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',datenum('30-Apr-2008'),Instruments);

NSModel.Parameters

ans = 1x4

    4.6617    -1.0227    -0.3484     1.2386

% create the plot
plot(PlottingPoints, getParYields(NSModel, PlottingPoints),'r')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
```



## Algorithms

The Nelson-Siegel model proposes that the instantaneous forward curve can be modeled with the following:

$$f = \beta_0 + \beta_1 e^{-\frac{m}{\tau}} + \beta_2 \frac{m}{\tau} e^{-\frac{m}{\tau}}$$

This can be integrated to derive an equation for the zero curve (see [6] for more information on the equations and the derivation):



$$s = \beta_0 + (\beta_1 + \beta_2) \frac{\tau}{m} (1 - e^{-\frac{m}{\tau}}) - \beta_2 e^{-\frac{m}{\tau}}$$

See [1] for more information.

## References

- [1] Nelson, C.R., Siegel, A.F. "Parsimonious modelling of yield curves." *Journal of Business*. Vol. 60, 1987, pp 473-89.
- [2] Svensson, L.E.O. "Estimating and interpreting forward interest rates: Sweden 1992-4." International Monetary Fund, IMF Working Paper, 1994/114.
- [3] Fisher, M., Nychka, D., Zervos, D. "Fitting the term structure of interest rates with smoothing splines." Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 1995-1.
- [4] Anderson, N., Sleath, J. "New estimates of the UK real and nominal yield curves." Bank of England Quarterly Bulletin, November, 1999, pp 384-92.
- [5] Waggoner, D. "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices." Federal Reserve Board Working Paper 1997-10.
- [6] "Zero-coupon yield curves: technical documentation." BIS Papers No. 25, October 2005.
- [7] Bolder, D.J., Gusba, S. "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada." Working Papers 2002-29, Bank of Canada.
- [8] Bolder, D.J., Streliski, D. "Yield Curve Modelling at the Bank of Canada." Technical Reports 84, 1999, Bank of Canada.

## See Also

"@IRFitOptions" on page A-11 | "@IRFunctionCurve" on page A-13

## Topics

"Fitting IRFunctionCurve Object Using Nelson-Siegel Method" on page 9-22  
 "Fitting Interest Rate Curve Functions" on page 9-33

“Using fitFunction to Create Custom Fitting Function” on page 9-29

“Interest-Rate Curve Objects and Workflow” on page 9-2

“Creating Interest-Rate Curve Objects” on page 9-4

### **External Websites**

Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)

Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk

Applications (30 min 00 sec)

### **Introduced in R2008b**

# fitSmoothingSpline

Fit smoothing spline to bond market data

## Class

@IRFunctionCurve on page A-13

## Syntax

```
CurveObj = IRFunctionCurve.fitSmoothingSpline(Type,Settle,Instruments,Lambdafun)
```

```
CurveObj = IRFunctionCurve.fitSmoothingSpline(Type,Settle,Instruments,Lambdafun,Name,Val)
```

## Arguments

---

**Note** You must have a license for Curve Fitting Toolbox software to use the `fitSmoothingSpline` method.

---

Type	Type of interest-rate curve for a bond: Forward, Zero, or Discount.
Settle	Scalar for the Settle date of the curve.
Instruments	N-by-4 data matrix for Instruments where the first column is Settle date, the second column is Maturity, the third column is the clean price, and the fourth column is a CouponRate for the bond.

<p>Lambdafun</p>	<p>Penalty function that takes as its input time and returns a penalty value. Use a function handle to support the penalty function. The function handle for the penalty function which takes one numeric input (time-to-maturity) and returns one numeric output (penalty to be applied to the curvature of the spline). For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.</p> <hr/> <p><b>Note</b> The smoothing spline represents the forward curve. The spline is penalized for curvature by specifying a penalty function. This fit may only be done with a <code>FitType</code> of <code>DurationWeightedPrice</code>.</p>
<p>Knots</p>	<p>(Optional) Vector of knot locations (times-to-maturity); by default, knots is set to be a vector comprised of 0 and the time to maturity of all input instruments. The default is for the spline type to be cubic but you can specify any spline type by explicitly specifying the knots. User-defined knots can be specified using the following command, where <code>k</code> is the order: <code>augknt(knots,k)</code>.</p>
<p>Compounding</p>	<p>(Optional) Scalar that sets the compounding frequency per year for the <code>IRFunctionCurve</code> object:</p> <ul style="list-style-type: none"> <li>• -1 = Continuous compounding (default)</li> <li>• 1 = Annual compounding</li> <li>• 2 = Semiannual compounding</li> <li>• 3 = Compounding three times per year</li> <li>• 4 = Quarterly compounding</li> <li>• 6 = Bimonthly compounding</li> <li>• 12 = Monthly compounding</li> </ul>

Basis	<p>(Optional) Day-count basis of the interest-rate curve. A scalar of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
-------	---

## Instrument Parameters

For each bond Instrument, you can specify the following additional instrument parameters as name-value pairs. For example, InstrumentBasis distinguishes a bond instrument's Basis value from the curve's Basis value.

InstrumentPeriod	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.
------------------	---

InstrumentBasis	<p>(Optional) Day-count basis of the bond. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
InstrumentEndMonthRule	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
InstrumentIssueDate	<p>(Optional) Date when an instrument was issued.</p>
InstrumentFirstCouponDate	<p>(Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When <b>FirstCouponDate</b> and <b>LastCouponDate</b> are both specified, <b>FirstCouponDate</b> takes precedence in determining the coupon payment structure. If you do not specify a <b>FirstCouponDate</b>, the cash flow payment dates are determined from other inputs.</p>

InstrumentLastCouponDate	(Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentFace	(Optional) Face or par value. Default = 100.

---

**Note** When using Instrument name-value pairs, you can specify simple interest for a bond by specifying the InstrumentPeriod value as 0. If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2.

---

## Description

Fcurve = IRFunctionCurve.fitSmoothingSpline(Type, Settle, Instruments, Lambdafun, Name, Value) fits a smoothing spline to market data for a bond. You must enter the optional arguments for Basis, Compounding, and Knots as comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

## Examples

### Use a Smoothing Spline Function to Fit Market Data For a Bond

This example shows how to use a smoothing spline function to fit market data for a bond.

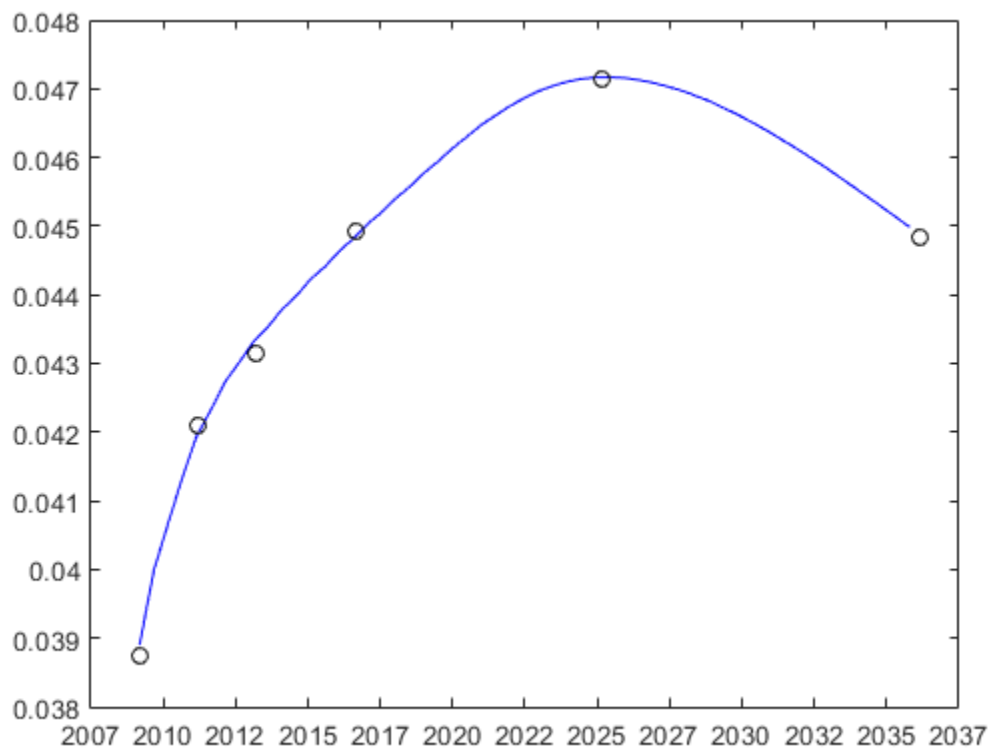
```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];
```

```
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);

% use the AUGKNT function to construct the knots for a cubic spline at every 5 years
CustomKnots = augknt(0:5:30,4);
SmoothingModel = IRFunctionCurve.fitSmoothingSpline('Zero',datenum('30-Apr-2008'),...
Instruments,@(t) 1000,'knots', CustomKnots);

% create the plot
plot(PlottingPoints, getParYields(SmoothingModel, PlottingPoints),'b')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
```





### Fitting an IRFunctionCurve Object Using the Smoothing Spline Method With the Penalty Function

Use the `fitSmoothingSpline` method to fit the interest-rate curve and model the `Lambdafun` penalty function. First, load the data.

```
load ukdata20080430
```

Convert the repo rates to be equivalent zero coupon bonds.

```
RepoCouponRate = repmat(0,size(RepoRates));  
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data.

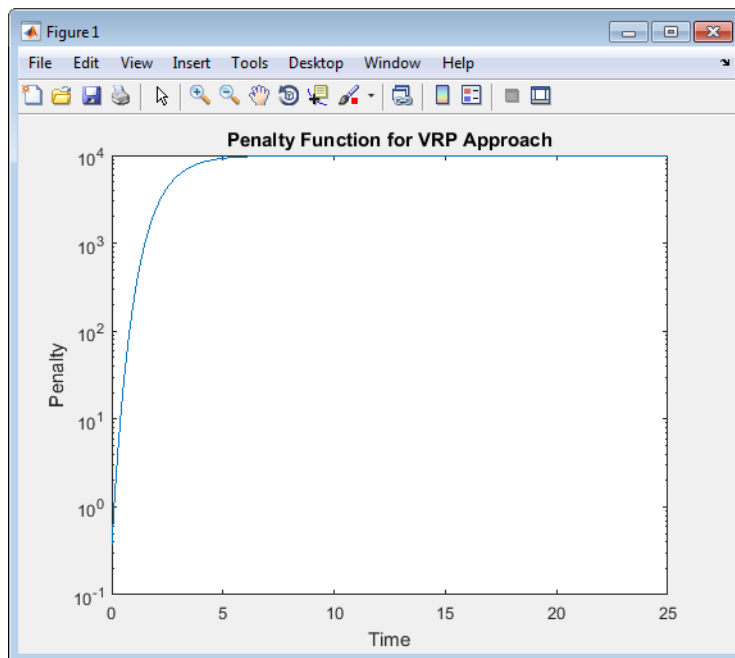
```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

Choose the parameters for the `Lambdafun` input argument.

```
L = 9.2;
S = -1;
mu = 1;
```

Define the `Lambdafun` penalty function.

```
lambdafun = @(t) exp(L - (L-S)*exp(-t/mu));
t = 0:.1:25;
y = lambdafun(t);
figure
semilogy(t,y);
title('Penalty Function for VRP Approach')
ylabel('Penalty')
xlabel('Time')
```



Use the `fitSmoothingSpline` method to fit the interest-rate curve and model the `LambdaFun` penalty function.

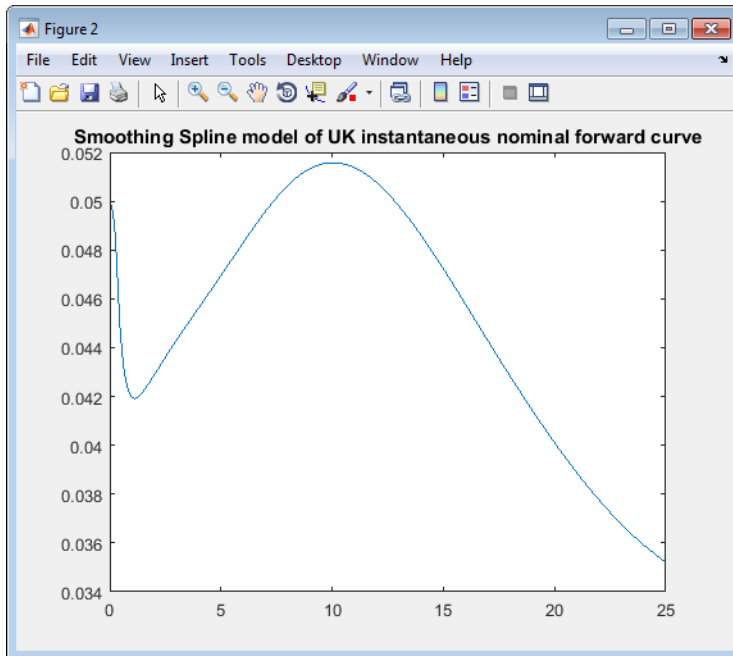
```
VRPModel = IRFunctionCurve.fitSmoothingSpline('Forward',CurveSettle,...
Instruments,lambdafun,'Compounding',-1, 'InstrumentPeriod',InstrumentPeriod)
```

```
VRPModel =
```

```
    Type: Forward
    Settle: 733528 (30-Apr-2008)
    Compounding: -1
    Basis: 0 (actual/actual)
```

Plot the smoothing spline interest-rate curve for the forward rates.

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
VRPForwardRates = getForwardRates(VRPModel, PlottingDates);
figure;plot(TimeToMaturity,VRPForwardRates)
title('Smoothing Spline model of UK instantaneous nominal forward curve')
```



## Algorithms

The term structure can be modeled with a spline — specifically, one way to model the term structure is by representing the forward curve with a cubic spline. To ensure that the spline is sufficiently smooth, a penalty is imposed relating to the curvature (second derivative) of the spline:

$$\sum_{i=1}^N \left[ \frac{P_i - \hat{P}_i(f)}{D_i} \right]^2 + \int_0^M \lambda_t(m) [f''(m)]^2 dm$$

where the first term is the difference between the observed price  $P$  and the predicted price,  $\hat{P}$ , (weighted by the bond's duration,  $D$ ) summed over all bonds in our data set and the second term is the penalty term (where  $\lambda$  is a penalty function and  $f$  is the spline).

See [3], [4], [5] below.

There have been different proposals for the specification of the penalty function  $\lambda$ . One approach, advocated by [4], and currently used by the UK Debt Management Office, is a penalty function of the following form:

$$\log(\lambda(m)) = L - (L - S)e^{\frac{-m}{r}}$$

## References

- [1] Nelson, C.R., Siegel, A.F. "Parsimonious modelling of yield curves." *Journal of Business*. Vol. 60, 1987, pp 473-89.
- [2] Svensson, L.E.O. "Estimating and interpreting forward interest rates: Sweden 1992-4." International Monetary Fund, IMF Working Paper, 1994/114.
- [3] Fisher, M., Nychka, D., Zervos, D. "Fitting the term structure of interest rates with smoothing splines." Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 1995-1.
- [4] Anderson, N., Sleath, J. "New estimates of the UK real and nominal yield curves." Bank of England Quarterly Bulletin, November, 1999, pp 384-92.
- [5] Waggoner, D. "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices." Federal Reserve Board Working Paper 1997-10.
- [6] "Zero-coupon yield curves: technical documentation." BIS Papers No. 25, October 2005.
- [7] Bolder, D.J., Gusba, S. "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada." Working Papers 2002-29, Bank of Canada.
- [8] Bolder, D.J., Streliski, D. "Yield Curve Modelling at the Bank of Canada." Technical Reports 84, 1999, Bank of Canada.

## See Also

"@IRFunctionCurve" on page A-13

## Topics

"Fitting IRFunctionCurve Object Using Smoothing Spline Method" on page 9-26

“Fitting Interest Rate Curve Functions” on page 9-33

“Interest-Rate Curve Objects and Workflow” on page 9-2

“Creating Interest-Rate Curve Objects” on page 9-4

### **External Websites**

Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)

Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk

Applications (30 min 00 sec)

### **Introduced in R2008b**

# fitSvensson

Fit Svensson function to bond market data

## Class

@IRFunctionCurve on page A-13

## Syntax

```
CurveObj = IRFunctionCurve.fitSvensson(Type, Settle, Instruments)
```

```
CurveObj = IRFunctionCurve.fitSvensson(Type, Settle, Instruments, Name, Value)
```

## Arguments

Type	Type of interest-rate curve for a bond: zero or forward.
Settle	Scalar for the <b>Settle</b> date of the curve.
Instruments	N-by-4 data matrix for <b>Instruments</b> where the first column is <b>Settle</b> date, the second column is <b>Maturity</b> , the third column is the clean price, and the fourth column is a <b>CouponRate</b> for the bond.
Compounding	(Optional) Scalar that sets the compounding frequency per year for the <b>IRFunctionCurve</b> object: <ul style="list-style-type: none"> <li>-1 = Continuous compounding</li> </ul>

Basis	<p>(Optional) Day-count basis of the interest-rate curve. A scalar of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
IRFitOptions	(Optional) Object constructed from IRFitOptions.

## Instrument Parameters

For each bond `Instrument`, you can specify the following additional instrument parameters as name-value pairs. For example, `InstrumentBasis` distinguishes a bond instrument's `Basis` value from the curve's `Basis` value.

InstrumentPeriod	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.
------------------	---



InstrumentBasis	<p>(Optional) Day-count basis of the bond. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> (Financial Toolbox).</p>
InstrumentEndMonthRule	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. <b>0</b> = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. <b>1</b> = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
InstrumentIssueDate	<p>(Optional) Date when an instrument was issued.</p>
InstrumentFirstCouponDate	<p>(Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When <b>FirstCouponDate</b> and <b>LastCouponDate</b> are both specified, <b>FirstCouponDate</b> takes precedence in determining the coupon payment structure. If you do not specify a <b>FirstCouponDate</b>, the cash flow payment dates are determined from other inputs.</p>

InstrumentLastCouponDate	(Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
InstrumentFace	(Optional) Face or par value. Default = 100.

---

**Note** When using Instrument name-value pairs, you can specify simple interest for a bond by specifying the InstrumentPeriod value as 0. If InstrumentBasis and InstrumentPeriod are not specified for a bond, the following default values are used: Basis is 0 (act/act) and Period is 2.

---

## Description

CurveObj = IRFunctionCurve.fitSvensson(Type, Settle, Instruments, Name, Value) fits the Svensson function to bond market data. You must enter the optional arguments for Basis, Compounding, and IRFitOptions as comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

After creating a Svensson model, you can view the Svensson model parameters using:

```
CurveObj.Parameters
```

where the order of parameters is [Beta0, Beta1, Beta2, Beta3, tau1, tau2].

## Examples

### Use a Svensson Function to Fit Bond Market Data

This example shows how to use a Svensson function to fit bond market data.

```
Settle = datenum('15-Apr-2014');
Maturity = datemnth(Settle,12*[1 2 3 5 7 10 20 30]');

CleanPrice = [100.1 100.1 100.2 99.0 101.8 99.2 101.7 100.2]';
CouponRate = [0.0200 0.0275 0.035 0.042 0.0475 0.0525 0.055 0.052]';
Instruments = [repmat(Settle,size(Maturity)) Maturity CleanPrice CouponRate];
PlottingPoints = datemnth(Settle,1:360);
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);

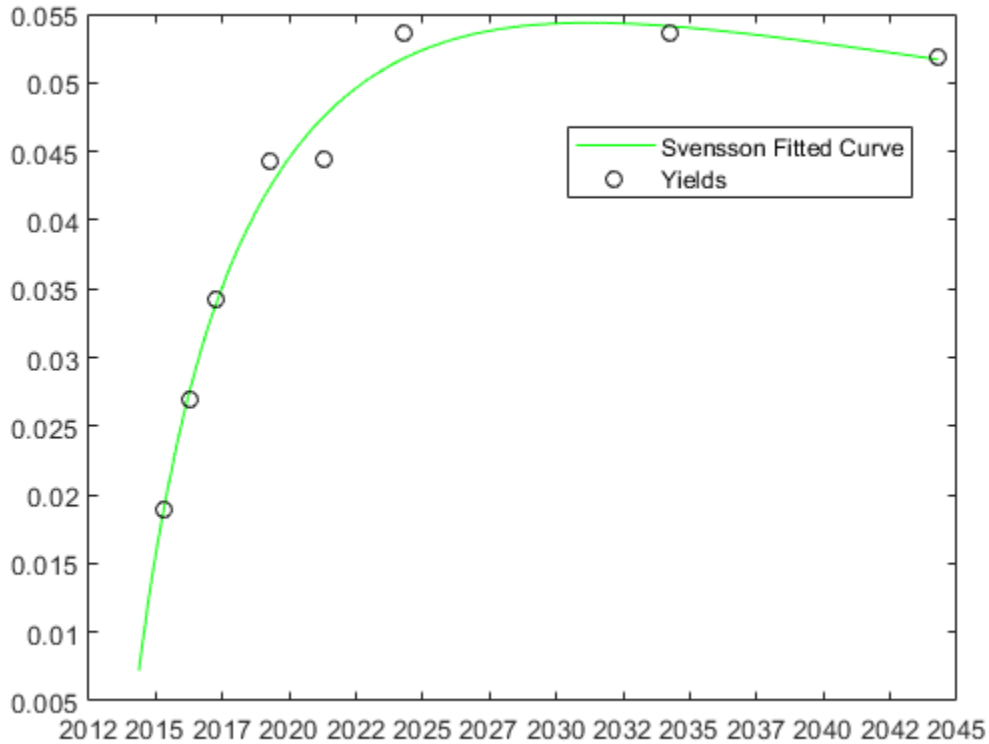
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',Settle,Instruments);

SvenssonModel.Parameters

ans = 1x6

    1.8297    -1.2299     1.6316    12.3891     1.6982     8.9422

% create the plot
plot(PlottingPoints, getParYields(SvenssonModel, PlottingPoints),'g')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
legend({'Svensson Fitted Curve','Yields'},'location','best')
```



## Algorithms

A similar model to the Nelson-Siegel is the Svensson model, which adds two additional parameters to account for greater flexibility in the term structure. This model proposes that the forward rate can be modeled with the following form:

$$f = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 e^{-\frac{m}{\tau_1}} \frac{m}{\tau_1} + \beta_3 e^{-\frac{m}{\tau_2}} \frac{m}{\tau_2}$$

As above, this can be integrated to derive an equation for the zero curve:

$$s = \beta_0 + \beta_1(1 - e^{-\frac{m}{\tau_1}})(-\frac{\tau_1}{m}) + \beta_2((1 - e^{-\frac{m}{\tau_1}})\frac{\tau_1}{m} - e^{-\frac{m}{\tau_1}}) + \beta_3((1 - e^{-\frac{m}{\tau_2}})\frac{\tau_2}{m} - e^{-\frac{m}{\tau_2}})$$

## References

- [1] Nelson, C.R., Siegel, A.F. "Parsimonious modelling of yield curves." *Journal of Business*. Vol. 60, 1987, pp 473-89.
- [2] Svensson, L.E.O. "Estimating and interpreting forward interest rates: Sweden 1992-4." International Monetary Fund, IMF Working Paper, 1994/114.
- [3] Fisher, M., Nychka, D., Zervos, D. "Fitting the term structure of interest rates with smoothing splines." Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 1995-1.
- [4] Anderson, N., Sleath, J. "New estimates of the UK real and nominal yield curves." Bank of England Quarterly Bulletin, November, 1999, pp 384-92.
- [5] Waggoner, D. "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices." Federal Reserve Board Working Paper 1997-10.
- [6] "Zero-coupon yield curves: technical documentation." BIS Papers No. 25, October 2005.
- [7] Bolder, D.J., Gusba, S. "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada." Working Papers 2002-29, Bank of Canada.
- [8] Bolder, D.J., Streliski, D. "Yield Curve Modelling at the Bank of Canada." Technical Reports 84, 1999, Bank of Canada.

## See Also

"@IRFunctionCurve" on page A-13 | "@IRFitOptions" on page A-11

## Topics

- "Fitting IRFunctionCurve Object Using Svensson Method" on page 9-24
- "Fitting Interest Rate Curve Functions" on page 9-33
- "Interest-Rate Curve Objects and Workflow" on page 9-2
- "Creating Interest-Rate Curve Objects" on page 9-4

## **External Websites**

Calibration and Simulation of Interest Rate Models in MATLAB (29 min 03 sec)

Calibration and Simulation Best Practices: Multifactor Interest Rate Models for Risk Applications (30 min 00 sec)

**Introduced in R2008b**

# getDiscountFactors

Get discount factors for input dates for IRDataCurve

## Class

@IRDataCurve on page A-7

## Syntax

```
F = getDiscountFactors(CurveObj,InpDates)
```

## Arguments

CurveObj	Interest-rate curve object that is constructed using IRDataCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.

## Description

F = getDiscountFactors(CurveObj,InpDates) returns discount factors for the input dates.

## Examples

### Get Discount Factors For Input Dates for an IRDataCurve

This example shows how to get discount factors for input dates for an IRDataCurve.

```
CurveSettle = datenum('2-Mar-2016');  
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
```

```
Dates = datemnth(CurveSettle,12*[1 2 3 5 7 10 20 30]);
irdc = IRDataCurve('Zero',CurveSettle,Dates,Data);
getDiscountFactors(irdc, CurveSettle+30:30:CurveSettle+720)
```

```
ans = 24×1
```

```
0.9986
0.9971
0.9956
0.9940
0.9924
0.9907
0.9890
0.9873
0.9855
0.9836
⋮
```

## See Also

“@IRDataCurve” on page A-7

## Topics

“Creating Interest-Rate Curve Objects” on page 9-4

“Creating an IRDataCurve Object” on page 9-6

“Interest-Rate Curve Objects and Workflow” on page 9-2

## Introduced in R2008b



# getDiscountFactors

Get discount factors for input dates for IRFunctionCurve

## Class

@IRFunctionCurve on page A-13

## Syntax

```
F = getDiscountFactors(CurveObj, InpDates)
```

## Arguments

CurveObj	Interest-rate curve object that is constructed using the IRFunctionCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.

## Description

`F = getDiscountFactors(CurveObj, InpDates)` returns discount factors for the input dates.

## Examples

### Get Discount Factors for Input Dates For an IRFunctionCurve

This example shows how to get discount factors for input dates for an IRFunctionCurve.

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));  
getDiscountFactors(irfc, today+30:30:today+720)
```

```
ans = 24×1
```

```
0.9984  
0.9967  
0.9950  
0.9933  
0.9916  
0.9899  
0.9882  
0.9864  
0.9846  
0.9829  
⋮
```

### See Also

“@IRFunctionCurve” on page A-13

### Topics

“Creating an IRFunctionCurve Object” on page 9-22

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**

# getForwardRates

Get forward rates for input dates for IRDataCurve

## Class

@IRDataCurve on page A-7

## Syntax

`F = getForwardRates(CurveObj, InpDates)`

`F = getForwardRates(CurveObj, InpDates, Name, Value)`

## Arguments

CurveObj	Interest-rate curve object that is constructed using IRDataCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.
Compounding	(Optional) Scalar that sets the compounding frequency per year for forward rates. The default Compounding value is CurveObj.Compounding. Acceptable values are: <ul style="list-style-type: none"> <li>• -1 = Continuous compounding</li> <li>• 0 = Simple interest (no compounding)</li> <li>• 1 = Annual compounding</li> <li>• 2 = Semiannual compounding</li> <li>• 3 = Compounding three times per year</li> <li>• 4 = Quarterly compounding</li> <li>• 6 = Bimonthly compounding</li> <li>• 12 = Monthly compounding</li> </ul>

<b>Basis</b>	<p>(Optional) Day-count basis values for the forward rates:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	---

## Description

`F = getForwardRates(CurveObj, InpDates, Name, Value)` returns forward rates for the input dates. `getForwardRates` returns discrete forward rates for the intervals input into this method. For example, running the following code:

```
getForwardRates(irdc, {Date1, Date2, Date3})
```

gives three forwards rates and the three tenors are: `[Settle, Date1]`, `[Date1, Date2]`, and `[Date2, Date3]`.

You must enter the optional arguments for **Basis** and **Compounding** as comma-separated pairs of **Name,Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

The `getForwardRates` method returns forward rates corresponding to the periodicity of the dates input to `getForwardRates`. For example, where the dates are monthly, monthly forward rates are returned. The first element of the output is the forward rate from the `Settle` to one month, the second element is the forward rate from one month to two months, and so on.

## Examples

### Get Forward Rates For Input Dates for an IRDataCurve

This example shows how to get forward rates for input dates for an `IRDataCurve`.

```
CurveSettle = datenum('2-Mar-2016');
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = datemnth(CurveSettle,12*[1 2 3 5 7 10 20 30]);
irdc = IRDataCurve('Zero',CurveSettle,Dates,Data);
getForwardRates(irdc, CurveSettle+30:30:CurveSettle+720)
```

```
ans = 24x1
```

```
0.0174
0.0180
0.0187
0.0193
0.0199
0.0205
0.0212
0.0218
0.0224
0.0230
:
```

### Use `getForwardRates` to Compute the Five Year Forward Rate in Five Years Time

Use `getForwardRates` to compute the forward rate from the `Settle` date to 5 years from March 1, 2017 and then the forward rate for the period from 5 years to 10 years from March 1, 2017.

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(736755,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data);
getForwardRates(irdc,datemnth(irdc.Settle,12*[5 10]))

ans = 2×1

    0.0343
    0.0456
```

The first element (.0312) is the forward rate from the `Settle` to 5 years from March 1, 2017. The second rate (0.0458) is the forward rate for the period from 5 years to 10 years from March 1, 2017, in other words, the 5-year forward rate 5 years from March 1, 2017.

### Compute the Six Month Forward Rates in 1 Month, 2 Months and 3 Months

Use the following data to create an `IRDataCurve` object:

```
Data = [0.1 0.30 0.70 1.05 1.45 1.71 2.12 2.43 2.85 3.57]/100;
Settle = datenum('08-Aug-2016'); % Today's date
Dates = datemnth(Settle,[3 6 9 12*[1 2 3 5 7 10 20]]);
irdc = IRDataCurve('Zero',Settle,Dates,Data)
```

```
irdc =
    Type: Zero
    Settle: 736550 (08-Aug-2016)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [10x1 double]
    Data: [10x1 double]
```

Compute the implied 6 month forward rates in 1 month, 2 months, and 3 months from the `Settle` date.

```
IntervalMonth = 6; % Interval for 6 month forward rates
FwdMonths = [1 2 3]'; % Starting in 1, 2, and 3 months from Settle
N = length(FwdMonths);
FwdRates_6M = zeros(N,1);
```

```
for k = 1:N
    FwdDates = datemnth(irdc.Settle, [FwdMonths(k) FwdMonths(k)+IntervalMonth]);
    f = getForwardRates(irdc,FwdDates);
    FwdRates_6M(k) = f(2);
end
```

```
[FwdMonths FwdRates_6M]
```

```
ans = 3×2
```

```
1.0000    0.0050
2.0000    0.0074
3.0000    0.0101
```

## See Also

"@IRDataCurve" on page A-7

## Topics

"Creating Interest-Rate Curve Objects" on page 9-4

"Creating an IRDataCurve Object" on page 9-6

"Interest-Rate Curve Objects and Workflow" on page 9-2

**Introduced in R2008b**

## getForwardRates

Get forward rates for input dates for IRFunctionCurve

### Class

@IRFunctionCurve on page A-13

### Syntax

F = getForwardRates(CurveObj, InpDates)

F = getForwardRates(CurveObj, InpDates, Name, Value)

### Arguments

CurveObj	Interest-rate curve object that is constructed using IRFunctionCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.
Compounding	(Optional) Scalar that sets the compounding frequency per year for the forward rates are: <ul style="list-style-type: none"><li>• -1 = Continuous compounding</li><li>• 1 = Annual compounding</li><li>• 2 = Semiannual compounding (default)</li><li>• 3 = Compounding three times per year</li><li>• 4 = Quarterly compounding</li><li>• 6 = Bimonthly compounding</li><li>• 12 = Monthly compounding</li></ul>



<b>Basis</b>	<p>(Optional) Day-count basis for the forward rates:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	--

## Description

`F = getForwardRates(CurveObj, InpDates, Name, Value)` returns forward rates for the input dates. You must enter the optional arguments for **Basis** and **Compounding** as comma-separated pairs of **Name, Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1, Value1, ..., NameN, ValueN**.

## Examples

### Get Forward Rates For Input Dates For an IRFunctionCurve

This example shows how to get forward rates for input dates for an `IRFunctionCurve`.

```

irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));
getForwardRates(irfc, today+30:30:today+720)

ans = 24×1

    0.0202
    0.0205
    0.0207
    0.0210
    0.0212
    0.0215
    0.0217
    0.0219
    0.0222
    0.0224
    :

```

### Compute the Implied 2-Year Forward Rates in 1 Year, 2 Years, 5 Years, and 10 Years

This example shows how to compute the implied 2-year forward rates in 1 year, 2 years, 5 years, and 10 years from the Settle date by using the `getForwardRates` method.

Use the following data for an `IRFunctionCurve` object that is created when using the `fitSvensson` method.

```

Settle = datenum('15-Apr-2014');
Maturity = datemnth(Settle,12*[1 2 3 5 7 10 20 30]');

CleanPrice = [100.1 100.1 100.2 99.0 101.8 99.2 101.7 100.2]';
CouponRate = [0.0200 0.0275 0.035 0.042 0.0475 0.0525 0.055 0.052]';
Instruments = [repmat(Settle,size(Maturity)) Maturity CleanPrice CouponRate];

SvenssonModel = IRFunctionCurve.fitSvensson('Zero',Settle,Instruments);

Compute the implied 2-year forward rates in 1 year, 2 years, 5 years, and 10 years from
the Settle date.

IntervalMonth = 12.*2;           % Interval months for 2-year forward rates
FwdMonths = 12.*[1 2 5 10]';    % Starting in 1, 2, 5, and 10 years from Settle
N = length(FwdMonths);

```

```
FwdRates_2Y = zeros(N,1);  
  
for k = 1:N  
    FwdDates = datemnth(SvenssonModel.Settle, [FwdMonths(k) FwdMonths(k)+IntervalMonth]);  
    f = getForwardRates(SvenssonModel,FwdDates);  
    FwdRates_2Y(k) = f(2);  
end
```

```
[FwdMonths FwdRates_2Y]
```

```
ans = 4×2
```

```
    12.0000    0.0418  
    24.0000    0.0504  
    60.0000    0.0620  
   120.0000    0.0629
```

## See Also

“@IRFunctionCurve” on page A-13

## Topics

“Creating an IRFunctionCurve Object” on page 9-22

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**

## getParYields

Get par yields for input dates for IRDataCurve

### Class

@IRDataCurve on page A-7

### Syntax

F = getParYields(CurveObj,InpDates)

F = getParYields(CurveObj,InpDates,Name,Value)

### Arguments

CurveObj	Interest-rate curve object that is constructed using IRDataCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.
Compounding	(Optional) Scalar that sets the compounding frequency per year for the par yield rates are: <ul style="list-style-type: none"><li>• -1 = Continuous compounding</li><li>• 1 = Annual compounding</li><li>• 2 = Semiannual compounding (default)</li><li>• 3 = Compounding three times per year</li><li>• 4 = Quarterly compounding</li><li>• 6 = Bimonthly compounding</li><li>• 12 = Monthly compounding</li></ul>

<b>Basis</b>	<p>(Optional) Day-count basis values for the par yield rates:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	---

## Description

`F = getParYields(CurveObj, InpDates, Name, Value)` returns par yields for the input dates. You must enter the optional arguments for **Basis** and **Compounding** as comma-separated pairs of **Name, Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1, Value1, ..., NameN, ValueN**.

## Examples

### Get Par Yields For Input Dates For an IRDataCurve

This example shows how to get par yields for input dates for an `IRDataCurve`.

```
CurveSettle = datenum('2-Mar-2016');
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = datemnth(CurveSettle,12*[1 2 3 5 7 10 20 30]);
irdc = IRDataCurve('Zero',CurveSettle,Dates,Data);
getParYields(irdc, CurveSettle+30:30:CurveSettle+720)
```

```
ans = 24×1
```

```
0.0175
0.0177
0.0181
0.0183
0.0186
0.0189
0.0194
0.0197
0.0200
0.0203
⋮
```

### Compute Par Yields From a Curve With Simple Interest Compounding

This example shows how set the compounding of an IRDataCurve to Zero (simple interest) and then compute par yields from that curve.

```
CurveSettle = datenum('2-Mar-2016');
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = datemnth(CurveSettle,12*[1 2 3 5 7 10 20 30]);
irdc = IRDataCurve('Zero',CurveSettle,Dates,Data,'Compounding',0);
SimpleInt = irdc.getParYields(Dates(1), 'Basis', 2, 'Compounding', 1)
```

```
SimpleInt = 0.0209
```

## See Also

"@IRDataCurve" on page A-7

## Topics

"Creating Interest-Rate Curve Objects" on page 9-4

"Creating an IRDataCurve Object" on page 9-6

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**

## getParYields

Get par yields for input dates for IRFunctionCurve

### Class

@IRFunctionCurve on page A-13

### Syntax

`F = getParYields(CurveObj, InpDates)`

`F = getParYields(CurveObj, InpDates, Name, Value)`

### Arguments

CurveObj	Interest-rate curve object that is constructed using IRFunctionCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.
Compounding	(Optional) Scalar that sets the compounding frequency per year for par yield rates are: <ul style="list-style-type: none"><li>• -1 = Continuous compounding</li><li>• 1 = Annual compounding</li><li>• 2 = Semiannual compounding (default)</li><li>• 3 = Compounding three times per year</li><li>• 4 = Quarterly compounding</li><li>• 6 = Bimonthly compounding</li><li>• 12 = Monthly compounding</li></ul>



<b>Basis</b>	<p>(Optional) Day-count basis values for par yield rates:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	---

## Description

`F = getParYields(CurveObj, InpDates, Name, Value)` returns par yields for the input dates. You must enter the optional arguments for **Basis** and **Compounding** as comma-separated pairs of **Name, Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

## Examples

### Get Par Yields For Input Dates For an IRFunctionCurve

This example shows how to get par yields for input dates for an `IRFunctionCurve`.

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));  
getParYields(irfc, today+30:30:today+720)
```

```
ans = 24×1
```

```
0.0201  
0.0201  
0.0203  
0.0203  
0.0204  
0.0206  
0.0209  
0.0210  
0.0211  
0.0212  
⋮
```

### See Also

“@IRFunctionCurve” on page A-13

### Topics

“Creating an IRFunctionCurve Object” on page 9-22

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**

# getZeroRates

Get zero rates for input dates for IRDataCurve

## Class

@IRDataCurve on page A-7

## Syntax

`F = getZeroRates(CurveObj, InpDates)`

`F = getZeroRates(CurveObj, InpDates, Name, Value)`

## Arguments

CurveObj	Interest-rate curve object that is constructed using IRDataCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.
Compounding	(Optional) Scalar that sets the compounding frequency per year for zero rates. The default Compounding value is CurveObj.Compounding. Acceptable values are: <ul style="list-style-type: none"> <li>• -1 = Continuous compounding</li> <li>• 0 = Simple interest (no compounding)</li> <li>• 1 = Annual compounding</li> <li>• 2 = Semiannual compounding</li> <li>• 3 = Compounding three times per year</li> <li>• 4 = Quarterly compounding</li> <li>• 6 = Bimonthly compounding</li> <li>• 12 = Monthly compounding</li> </ul>

<b>Basis</b>	<p>(Optional) Day-count basis values for zero rates:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	--

## Description

`F = getZeroRates(CurveObj, InpDates, Name, Value)` returns zero rates for the input dates. You must enter the optional arguments for **Basis** and **Compounding** as comma-separated pairs of **Name, Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

## Examples

### Get Zero Rates For Input Dates For an IRDataCurve

This example shows how to get zero rates for input dates for an `IRDataCurve`.

```
CurveSettle = datenum('2-Mar-2016');  
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;  
Dates = datemnth(CurveSettle,12*[1 2 3 5 7 10 20 30]);  
irdc = IRDataCurve('Zero',CurveSettle,Dates,Data);  
getZeroRates(irdc, CurveSettle+30:30:CurveSettle+720)
```

```
ans = 24x1
```

```
0.0174  
0.0177  
0.0180  
0.0183  
0.0187  
0.0190  
0.0193  
0.0196  
0.0199  
0.0202  
⋮
```

## See Also

"@IRDataCurve" on page A-7

## Topics

"Creating Interest-Rate Curve Objects" on page 9-4

"Creating an IRDataCurve Object" on page 9-6

"Interest-Rate Curve Objects and Workflow" on page 9-2

## Introduced in R2008b

## getZeroRates

Get zero rates for input dates for IRFunctionCurve

### Class

@IRFunctionCurve on page A-13

### Syntax

`F = getZeroRates(CurveObj, InpDates)`

`F = getZeroRates(CurveObj, InpDates, Name, Value)`

### Arguments

CurveObj	Interest-rate curve object that is constructed using IRFunctionCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.
Compounding	(Optional) Scalar that sets the compounding frequency per year for zero rates are: <ul style="list-style-type: none"> <li>• -1 = Continuous compounding</li> <li>• 1 = Annual compounding</li> <li>• 2 = Semiannual compounding (default)</li> <li>• 3 = Compounding three times per year</li> <li>• 4 = Quarterly compounding</li> <li>• 6 = Bimonthly compounding</li> <li>• 12 = Monthly compounding</li> </ul>

<b>Basis</b>	<p>(Optional) Day-count basis value for zero rates:</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
--------------	---

## Description

`F = getZeroRates(CurveObj, InpDates, Name, Value)` returns zero rates for the input dates. You must enter the optional arguments for **Basis** and **Compounding** as comma-separated pairs of **Name, Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1, Value1, ..., NameN, ValueN`.

## Examples

### Get Zero Rates For Input Dates For an IRFunctionCurve

This example shows how to get zero rates for input dates for an `IRFunctionCurve`.

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));  
getZeroRates(irfc, today+30:30:today+720)
```

```
ans = 24×1
```

```
0.0201  
0.0202  
0.0204  
0.0205  
0.0206  
0.0207  
0.0208  
0.0210  
0.0211  
0.0212  
⋮
```

### See Also

“@IRFunctionCurve” on page A-13

### Topics

“Creating an IRFunctionCurve Object” on page 9-22

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**



# IRBootstrapOptions

Construct specific options for bootstrapping interest-rate curve object

## Class

@IRBootstrapOptions on page A-2

## Syntax

```
mybootoptions = IRBootstrapOptions(Name,Value)
```

## Arguments

ConvexityAdjustment	<p>(Optional) Controls the convexity adjustment to interest-rate futures. This can be specified as a function handle that takes one numeric input (time-to-maturity) and returns one numeric output, <code>ConvexityAdjustment</code>. For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.</p> <p>Alternatively, you can define <code>ConvexityAdjustment</code> as an N-by-1 vector of values, where N is the number of interest-rate futures.</p> <p>In either case, the <code>ConvexityAdjustment</code> is subtracted from the futures rate.</p>
---------------------	---

LowerBound	(Optional) Specifies a lower bound for rates associated with bonds or swaps. LowerBound can be a scalar or N-by-1 vector where N is the number of swaps and bonds. By default, LowerBound is 0.
UpperBound	(Optional) Specifies an upper bound for rates associated with bonds or swaps. UpperBound can be a scalar or N-by-1 vector where N is the number of swaps and bonds. By default, UpperBound is 1. Specify an upper bound that is greater than 1 when bootstrapping a discount curve.

## Description

`mybootoptions = IRBootstrapOptions(Name,Value)` constructs an `IRBootstrapOptionsObj` structure. You must enter the optional arguments for `ConvexityAdjustment`, `LowerBound`, and `UpperBound` as comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside quotes. You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

The `IRBootstrapOptionsObj` is used with the `bootstrap` method.

## Examples

### Create `IRBootstrapOptionsObj` to Use With the `bootstrap` Method

Set the `ConvexityAdjustment` to control interest-rate futures.

```
mybootoptions = IRBootstrapOptions('ConvexityAdjustment', repmat(.005,10,1))
```

```
mybootoptions =  
  IRBootstrapOptions with properties:  
    ConvexityAdjustment: [10x1 double]  
      LowerBound: 0
```

UpperBound: 1

Use `mybootoptions` as the optional argument, `IRBootstrapOptionsObj`, to use with the `bootstrap` method.

### Create an `IRBootstrapOptionsObj` to Use With Negative Zero Interest-Rates

Use an `IRBootstrapOptionsObj` optional argument with the `bootstrap` method to allow for negative zero rates when solving the swap zero points.

```
Settle = datenum('15-Mar-2015');
InstrumentTypes = {'Deposit'; 'Deposit'; 'Swap'; 'Swap'; 'Swap'; 'Swap'};

Instruments = [Settle, datenum('15-Jun-2015'), .001; ...
Settle, datenum('15-Dec-2015'), .0005; ...
Settle, datenum('15-Mar-2016'), -.001; ...
Settle, datenum('15-Mar-2017'), -0.0005; ...
Settle, datenum('15-Mar-2018'), .0017; ...
Settle, datenum('15-Mar-2020'), .0019];

irbo = IRBootstrapOptions('LowerBound', -1);

bootModel = IRDataCurve.bootstrap('zero', Settle, InstrumentTypes, ...
    Instruments, 'IRBootstrapOptions', irbo);

bootModel.getZeroRates(datemnth(Settle, 1:60))

ans = 60x1

    0.0012
    0.0011
    0.0010
    0.0009
    0.0008
    0.0008
    0.0007
    0.0006
    0.0005
   -0.0000
    :
```

Note that `IRBootstrapOptions` optional argument for `LowerBound` is set to `-1` for negative zero rates when solving the swap zero points.

### See Also

`@IRDataCurve` on page A-7

### Topics

`“Creating Interest-Rate Curve Objects”` on page 9-4

`“Creating an IRDataCurve Object”` on page 9-6

`“IRDataCurve Bootstrapping Based on Market Instruments”` on page 9-7

`“Bootstrapping a Swap Curve”`

`“Interest-Rate Curve Objects and Workflow”` on page 9-2

### Introduced in R2008b

# IRDataCurve

Construct interest-rate curve object from dates and data

## Class

@IRDataCurve on page A-7

## Syntax

CurveObj = IRDataCurve(Type,Settle,Dates,Data)

CurveObj = IRDataCurve(Type,Settle,Dates,Data,Name,Value)

## Arguments

Type	Type of interest-rate curve. Acceptable values are forward, zero, or discount.
Settle	Scalar for the Settle date of the curve.
Dates	Dates corresponding to rate data.
Data	Interest-rate data for the curve object.

Compounding	<p>(Optional) Scalar that sets the compounding frequency per year for the IRDataCurve object:</p> <ul style="list-style-type: none"><li>• -1 = Continuous compounding</li><li>• 0 = Simple interest (no compounding) for “zero” and “discount” curve types only, not supported for “forward” curves</li><li>• 1 = Annual compounding</li><li>• 2 = Semiannual compounding (default)</li><li>• 3 = Compounding three times per year</li><li>• 4 = Quarterly compounding</li><li>• 6 = Bimonthly compounding</li><li>• 12 = Monthly compounding</li></ul> <hr/> <p><b>Note</b> Simple interest can be specified for an instrument by specifying the Compounding value as 0 and is supported for “zero” and “discount” curve types only (not supported for “forward” curves).</p>
-------------	--

Basis	<p>(Optional) Day-count basis of the interest-rate curve. A scalar of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
InterpMethod	<p>(Optional) Values are:</p> <ul style="list-style-type: none"> <li>• 'linear' — Linear interpolation (default).</li> <li>• 'constant' — Piecewise constant interpolation.</li> <li>• 'pchip' — Piecewise cubic Hermite interpolation.</li> <li>• 'spline' — Cubic spline interpolation.</li> </ul>

## Description

CurveObj = IRDataCurve(Type,Settle,Dates,Data,Name,Value) constructs an interest-rate curve with the specified Dates and Data. You must enter the optional arguments for Basis, Compounding, and InterpMethod as comma-separated pairs of Name,Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1,...,NameN,ValueN.

Alternatively, an `IRDataCurve` object can be bootstrapped from market data using the `bootstrap` method.

After an `IRDataCurve` curve object is constructed, you can use the following methods to determine the forward rates, zero rates, and discount factors. In addition, you can use the `toRateSpec` method to convert the interest-rate curve object to a `RateSpec` structure.

Method	Description
<code>getForwardRates</code>	Returns forward rates for input dates.
<code>getZeroRates</code>	Returns zero rates for input dates.
<code>getDiscountFactors</code>	Returns discount factors for input dates.
<code>getParYields</code>	Returns par yields for input dates.
<code>toRateSpec</code>	Converts to be a <code>RateSpec</code> object; this structure is identical to the <code>RateSpec</code> produced by the Financial Instruments Toolbox function <code>intenvset</code> .
<code>bootstrap</code>	Bootstraps an interest rate curve from market data.

## Examples

```
CurveSettle = datenum('2-Mar-2016');
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = datemnth(CurveSettle,12*[1 2 3 5 7 10 20 30]);
irdc = IRDataCurve('Zero',CurveSettle,Dates,Data)
```

```
irdc =
```

```

    Type: Zero
    Settle: 736391 (02-Mar-2016)
    Compounding: 2
    Basis: 0 (actual/actual)
    InterpMethod: linear
    Dates: [8x1 double]
    Data: [8x1 double]
```

## See Also

"@IRDataCurve" on page A-7



**Topics**

“Creating Interest-Rate Curve Objects” on page 9-4

“Creating an IRDataCurve Object” on page 9-6

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**

## IRFitOptions

Construct specific options for fitting interest-rate curve object

### Class

@IRFitOptions on page A-11

### Syntax

```
myfitoptions = IRFitOptions(InitialGuess)
```

```
myfitoptions = IRFitOptions(InitialGuess,Name,Value)
```

### Arguments

InitialGuess	Initial guess for the parameters of the curve function. Vector of values for the starting point of the optimization.
FitType	(Optional) Price, Yield, or DurationWeightedPrice determines which is minimized in the curve fitting process. The default is DurationWeightedPrice.
UpperBound	(Optional) Lower bound for the parameters of the curve function.
LowerBound	(Optional) Upper bound for the parameters of the curve function.
OptOptions	(Optional) Optimization solver options constructed with optimoptions from the Optimization Toolbox (optimset is also supported).

### Description

`myfitoptions = IRFitOptions(InitialGuess,Name,Value)` constructs the `IRFitOptions` structure with an initial guess or with an initial guess and bounds. You must enter the optional arguments for `FitType`, `UpperBound`, `LowerBound`, and

OptOptions as comma-separated pairs of Name,Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1,...,NameN,ValueN.

---

**Note** IRFitOptions constructor must be used with fitFunction method when building a custom fitting function.

---

## Examples

```
myfitoptions = IRFitOptions([7 2 1 0], 'FitType', 'yield')
```

```
myfitoptions =
```

```
Properties:
  FitType: 'yield'
  InitialGuess: [7 2 1 0]
  UpperBound: []
  LowerBound: []
  OptOptions: []
```

## See Also

“@IRFunctionCurve” on page A-13

## Topics

“Creating an IRFunctionCurve Object” on page 9-22

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**

## IRFunctionCurve

Construct interest-rate curve object from function handle or function and fit to market data

### Class

@IRFunctionCurve on page A-13

### Syntax

CurveObj = IRFunctionCurve(Type,Settle,FunctionHandle)

CurveObj = IRFunctionCurve(Type,Settle,FunctionHandle,Name,Value)

### Arguments

Type	Type of interest-rate curve: zero, forward, or discount.
Settle	Scalar for the Settle date of the curve.
Compounding	(Optional) Scalar that sets the compounding frequency per year for the IRFunctionCurve object: <ul style="list-style-type: none"> <li>• -1 = Continuous compounding</li> <li>• 1 = Annual compounding</li> <li>• 2 = Semiannual compounding (default)</li> <li>• 3 = Compounding three times per year</li> <li>• 4 = Quarterly compounding</li> <li>• 6 = Bimonthly compounding</li> <li>• 12 = Monthly compounding</li> </ul>

Basis	<p>(Optional) Day-count basis of the bond. A scalar of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
FunctionHandle	<p>Function handle that defines the interest-rate curve. The function handle requires one numeric input (time-to-maturity) and returns one numeric output (interest rate or discount factor). For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.</p>
Parameters	<p>Fitted parameters for function.</p>

## Description

`CurveObj = IRFunctionCurve(Type,Settle,FunctionHandle,Name,Value)` constructs an interest-rate curve object directly by specifying a function handle. You must enter the optional arguments for **Basis** and **Compounding** as comma-separated pairs of **Name,Value** arguments. **Name** is the argument name and **Value** is the corresponding value. **Name** must appear inside quotes. You can specify several name and value pair arguments in any order as **Name1,Value1,...,NameN,ValueN**.

After you use the `IRFunctionCurve` constructor to create an `IRFunctionCurve` object, you can fit the bond using the following methods.

Method	Description
<code>getForwardRates</code>	Returns forward rates for input dates.
<code>getZeroRates</code>	Returns zero rates for input dates.
<code>getDiscountFactors</code>	Returns discount factors for input dates.
<code>getParYields</code>	Returns par yields for input dates.
<code>toRateSpec</code>	Converts to be a <code>RateSpec</code> object.  This <code>RateSpec</code> structure is identical to the <code>RateSpec</code> produced by the Financial Instruments Toolbox function <code>intenvset</code> .

Alternatively, you can construct an `IRFunctionCurve` object using the following static methods.

Static Method	Description
<code>fitNelsonSiegel</code>	Fits a Nelson-Siegel function to market data.
<code>fitSvensson</code>	Fits a Svensson function to market data.
<code>fitSmoothingSpline</code>	Fits a smoothing spline function to market data.
<code>fitFunction</code>	Fits a custom function to market data.

## Examples

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t))
```

```
irfc =
```

```

Type: Forward
Settle: 737406 (12-Dec-2018)
Compounding: 2
Basis: 0 (actual/actual)

```

## See Also

“@IRCurve” on page A-4

**Topics**

“Creating Interest-Rate Curve Objects” on page 9-4

“Creating an IRDataCurve Object” on page 9-6

“Creating an IRFunctionCurve Object” on page 9-22

“Interest-Rate Curve Objects and Workflow” on page 9-2

**Introduced in R2008b**

## liborduration

Duration of LIBOR-based interest-rate swap

### Syntax

```
[PayFixDuration,GetFixDuration] = liborduration(SwapFixRate,Tenor,Settle)
```

### Description

[PayFixDuration,GetFixDuration] = liborduration(SwapFixRate,Tenor,Settle) computes the duration of LIBOR-based interest-rate swaps.

### Examples

#### Compute the Duration of LIBOR-Based Interest-Rate Swaps

This example shows how to compute the duration of LIBOR-based interest-rate swaps using the following data.

```
SwapFixRate = 0.0383;  
Tenor = 7;  
Settle = datenum('11-Oct-2002');
```

```
[PayFixDuration GetFixDuration] = liborduration(SwapFixRate,...  
Tenor, Settle)
```

```
PayFixDuration = -4.7567
```

```
GetFixDuration = 4.7567
```



## Input Arguments

### **SwapFixRate — Par swap fixed rate**

vector in decimals

Par swap fixed rate (quarterly compounded), specified as an N-by-1 vector in decimals. The Basis should be actual/360.

Data Types: double

### **Tenor — Swap tenor in years**

vector

Swap tenor in years, specified as a N-by-1 vector. Fractional numbers are rounded upward.

Data Types: double

### **Settle — Settlement date**

vector

Settlement date, specified as an N-by-1 vector using serial date numbers.

Data Types: double

## Output Arguments

### **PayFixDuration — Modified duration, in years, for the pay-fix side of the swap**

vector

Modified duration, in years, for the pay-fix side of the swap, returned as a N-by-1 vector.

### **GetFixDuration — Modified duration, in years, for the receive-fix side of the swap**

vector

Modified duration, in years, for the receive-fix side of the swap, returned as a N-by-1 vector.

## **See Also**

`liborfloat2fixed` | `liborprice`

## **Topics**

“Analysis of Bond Futures” on page 7-13

“Managing Interest-Rate Risk with Bond Futures”

“Fitting the Diebold Li Model”

“Managing Present Value with Bond Futures” on page 7-16

**Introduced before R2006a**

# liborfloat2fixed

Compute par fixed-rate of swap given 3-month LIBOR data

## Syntax

```
[FixedSpec, ForwardDates, ForwardRates] = liborfloat2fixed(  
ThreeMonthRates, Settle, Tenor)  
Price = liborprice( ____, StartDate, Interpolation, ConvexAdj, RateParam,  
InArrears, Sigma, FixedCompound, FixedBasis)
```

## Description

[FixedSpec, ForwardDates, ForwardRates] = liborfloat2fixed(ThreeMonthRates, Settle, Tenor) computes forward rates, dates, and the swap fixed rate.

---

**Note** The liborfloat2fixed function assumes that floating-rate observations occur quarterly on the third Wednesday of a delivery month. The first delivery month is the month of the first third Wednesday after Settle. Floating-side payments occur on the third-month anniversaries of observation dates. Fixed payments start on the same date as the first floating payment, and recur on the same date as the first-coupon date (on anniversary months).

---

Price = liborprice( \_\_\_\_, StartDate, Interpolation, ConvexAdj, RateParam, InArrears, Sigma, FixedCompound, FixedBasis) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute the Par Fixed-Rate of a Swap Given 3-Month LIBOR Data

This example shows how to compute the par fixed-rate of a swap given 3-month LIBOR data. Use the supplied `EDdata.xls` file as input to a `liborfloat2fixed` computation.

```
[EDFutData, textdata] = xlsread('EDdata.xls');  
Settle                = datenum('15-Oct-2002');  
Tenor                 = 2;
```

```
[FixedSpec, ForwardDates, ForwardRates] =...  
liborfloat2fixed(EDFutData(:,1:3), Settle, Tenor)
```

```
FixedSpec = struct with fields:
```

```
    Coupon: 0.0222  
    Settle: '16-Oct-2002'  
    Maturity: '16-Oct-2004'  
    Period: 4  
    Basis: 1
```

```
ForwardDates = 8×1
```

```
731505  
731596  
731687  
731778  
731869  
731967  
732058  
732149
```

```
ForwardRates = 8×1
```

```
0.0177  
0.0166  
0.0170  
0.0188  
0.0214  
0.0248  
0.0279  
0.0305
```

## Input Arguments

### **ThreeMonthRates — Three-month Eurodollar futures data or forward rate agreement data**

matrix

Three-month Eurodollar futures data or forward rate agreement data, specified as an N-by-3 matrix in the form of [month year IMMQuote]. A forward rate agreement stipulates that a certain interest rate applies to a certain principal amount for a given future time period. The floating rate is assumed to compound quarterly and to accrue on an actual/360 basis.

Data Types: double

### **Settle — Settlement date of fixed-rate of swap**

scalar numeric

Settlement date of fixed-rate of swap, specified as a scalar numeric using serial date numbers.

Data Types: double

### **Tenor — Life of the swap contract**

scalar integer

Life of the swap contract, specified as a scalar integer.

Data Types: double

### **StartDate — Reference date for valuation of forward swap**

Settle (default) | scalar numeric

(Optional) Reference date for valuation of forward swap, specified as a scalar numeric using serial date numbers. This in effect allows forward swap valuation.

Data Types: double

### **Interpolation — Interpolation method to determine applicable forward rate for months when no Eurodollar data is available**

'linear' (1) (default) | scalar integer with value of 0, 1, or 2

(Optional) Interpolation method to determine applicable forward rate for months when no Eurodollar data is available, specified as a scalar numeric with values of:

- 0 is 'nearest'
- 1 is 'linear'
- 2 is 'cubic'

Data Types: double

**ConvexAdj** — Indicates whether futures/forward convexity adjustment is required

0 (off) (default) | scalar logical with a value of 0 or 1

(Optional) Indicates whether futures/forward convexity adjustment is required, specified as a scalar logical. Use **ConvexAdj** for forward rate adjustments when those rates are taken from Eurodollar futures data.

Data Types: logical

**RateParam** — Short-rate model's parameters (Hull-White)

[0.05 0.015] (default) | vector

(Optional) Short-rate model's parameters (Hull-White), specified a 1-by-2 vector to denote the parameters [a S], where the short-rate process is:

$$dr = [\theta(t) - ar]dt + Sdz.$$

Data Types: double

**InArrears** — Indicates whether the swap is in arrears

0 (off) (default) | scalar logical with a value of 0 or 1

(Optional) Indicates whether the swap is in arrears, specified as a scalar logical.

Data Types: logical

**Sigma** — Overall annual volatility of caplets

scalar numeric

(Optional) Overall annual volatility of caplets, specified as a scalar numeric.

Data Types: double

**FixedCompound** — Compounding or frequency of payment on the fixed side

4 (quarterly) (default) | scalar numeric with possible values of 1, 2,4, or 12

(Optional) Compounding or frequency of payment on the fixed side, specified as a scalar numeric with one of the following possible values:

- 1 is annual
- 2 is semiannual
- 4 is quarterly
- 12 is monthly

Data Types: double

### **FixedBasis — Basis of the fixed side**

0 (actual/actual ) (default) | scalar numeric

(Optional) Basis of the fixed side, specified as a scalar numeric using one of the supported values:.

- 0 = actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)

For more information, see **basis** on page Glossary-0 .

Data Types: double

## Output Arguments

### **FixedSpec — Structure of the fixed-rate side of the swap**

structure

Structure of the fixed-rate side of the swap, returned as a structure with the following fields:

- **Coupon:** Par-swap rate
- **Settle:** Start date
- **Maturity:** End date
- **Period:** Frequency of payment
- **Basis:** Accrual basis

### **ForwardDates — Dates corresponding to ForwardRates**

numeric

Dates corresponding to `ForwardRates`, returned as serial date numbers. All of the dates are third Wednesdays of the month, spread three months apart. The first element is the third Wednesday immediately after `Settle`.

### **ForwardRates — Forward rates corresponding to the forward dates**

numeric

Forward rates corresponding to the forward dates, quarterly compounded, and on the actual/360 basis, returned as numeric decimal values.

## See Also

`liborduration` | `liborprice`

## Topics

“Analysis of Bond Futures” on page 7-13

“Managing Interest-Rate Risk with Bond Futures”

“Fitting the Diebold Li Model”

“Managing Present Value with Bond Futures” on page 7-16

**Introduced before R2006a**



# liborprice

Price swap given swap rate

## Syntax

```
Price = liborprice(ThreeMonthRates,Settle,Tenor,SwapRate)
Price = liborprice(____,StartDate,Interpolation,ConvexAdj,RateParam,
InArrears,Sigma,FixedCompound,FixedBasis)
```

## Description

`Price = liborprice(ThreeMonthRates,Settle,Tenor,SwapRate)` computes the price per \$100 notional value of a swap given the swap rate. A positive result indicates that fixed side is more valuable than the floating side.

`Price = liborprice(____,StartDate,Interpolation,ConvexAdj,RateParam,InArrears,Sigma,FixedCompound,FixedBasis)` specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute the Price Per \$100 Notional Value of a Swap Given the Swap Rate

This example shows that a swap paying the par swap rate has a value of 0.

```
% load the input data
[EDFutData, textdata] = xlsread('EDdata.xls');
Settle = datenum('15-Oct-2002');
Tenor = 2;

% compute the fixed rate from the Eurodollar data
FixedSpec = liborfloat2fixed(EDFutData(:,1:3), Settle, Tenor)

FixedSpec = struct with fields:
    Coupon: 0.0222
```

```
Settle: '16-Oct-2002'  
Maturity: '16-Oct-2004'  
Period: 4  
Basis: 1
```

```
% compute the price of a par swap
```

```
Price = liborprice(EDFutData(:,1:3), Settle, Tenor, FixedSpec.Coupon)
```

```
Price = 2.7756e-15
```

Price is effectively equal to 0.

## Input Arguments

### **ThreeMonthRates** — Three-month Eurodollar futures data or forward rate agreement data

matrix

Three-month Eurodollar futures data or forward rate agreement data, specified as an N-by-3 matrix in the form of [month year IMMQuote]. A forward rate agreement stipulates that a certain interest rate applies to a certain principal amount for a given future time period. The floating rate is assumed to compound quarterly and to accrue on an actual/360 basis.

Data Types: double

### **Settle** — Settlement date of fixed-rate of swap

scalar numeric

Settlement date of fixed-rate of swap, specified as a scalar numeric using serial date numbers.

Data Types: double

### **Tenor** — Life of the swap contract

scalar integer

Life of the swap contract, specified as a scalar integer.

Data Types: double

**SwapRate — Swap rate**

scalar decimal

Swap rate, specified as a scalar decimal.

Data Types: double

**StartDate — Reference date for valuation of forward swap**

Settle (default) | scalar numeric

(Optional) Reference date for valuation of forward swap, specified as a scalar numeric using serial date numbers. This in effect allows forward swap valuation.

Data Types: double

**Interpolation — Interpolation method to determine applicable forward rate for months when no Eurodollar data is available**

'linear' (1) (default) | scalar integer with value of 0, 1, or 2

(Optional) Interpolation method to determine applicable forward rate for months when no Eurodollar data is available, specified as a scalar numeric with values of:

- 0 is 'nearest'
- 1 is 'linear'
- 2 is 'cubic'

.

Data Types: double

**ConvexAdj — Indicates whether futures/forward convexity adjustment is required**

0 (off) (default) | scalar logical with a value of 0 or 1

(Optional) Indicates whether futures/forward convexity adjustment is required, specified as a scalar logical. Use ConvexAdj for forward rate adjustments when those rates are taken from Eurodollar futures data.

Data Types: logical

**RateParam — Short-rate model's parameters (Hull-White)**

[0.05 0.015] (default) | vector

(Optional) Short-rate model's parameters (Hull-White), specified a 1-by-2 vector to denote the parameters [a S], where the short-rate process is:

$$dr = [\theta(t) - ar]dt + Sdz .$$

Data Types: double

**InArrears — Indicates whether the swap is in arrears**

0 (off) (default) | scalar logical with a value of 0 or 1

(Optional) Indicates whether the swap is in arrears, specified as a scalar logical.

Data Types: logical

**Sigma — Overall annual volatility of caplets**

scalar numeric

(Optional) Overall annual volatility of caplets, specified as a scalar numeric.

Data Types: double

**FixedCompound — Compounding or frequency of payment on the fixed side**

4 (quarterly) (default) | scalar numeric with possible values of 1, 2,4, or 12

(Optional) Compounding or frequency of payment on the fixed side, specified as a scalar numeric with one of the following possible values:

- 1 is annual
- 2 is semiannual
- 4 is quarterly
- 12 is monthly

Data Types: double

**FixedBasis — Basis of the fixed side**

0 (actual/actual ) (default) | scalar numeric

(Optional) Basis of the fixed side, specified as a scalar numeric using one of the supported values:.

- 0 = actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360

- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)

For more information, see **basis** on page Glossary-0 .

Data Types: double

## Output Arguments

**Price** — Present value of the difference between floating and fixed-rate sides of the swap per \$100 notional

numeric

Present value of the difference between floating and fixed-rate sides of the swap per \$100 notional, returned as a numeric value.

## See Also

`liborduration` | `liborfloat2fixed`

## Topics

“Analysis of Bond Futures” on page 7-13

“Managing Interest-Rate Risk with Bond Futures”

“Fitting the Diebold Li Model”

“Managing Present Value with Bond Futures” on page 7-16

**Introduced before R2006a**

## mbscfamounts

Cash flow and time mapping for mortgage pool

### Syntax

```
[CFlowAmounts,CFlowDates,TFactors,Factors,Payment,Principal,  
Interest,Prepayment] = mbscfamounts(Settle,Maturity,IssueDate,  
GrossRate)
```

```
[CFlowAmounts,CFlowDates,TFactors,Factors,Payment,Principal,  
Interest,Prepayment] = mbscfamounts( ____,CouponRate,Delay,  
PrepaySpeed,PrepayMatrix)
```

### Description

[CFlowAmounts,CFlowDates,TFactors,Factors,Payment,Principal,Interest,Prepayment] = mbscfamounts(Settle,Maturity,IssueDate,GrossRate) computes cash flows between Settle and Maturity dates, the corresponding time factors in months from Settle and the mortgage factor (the fraction of loan principal outstanding).

[CFlowAmounts,CFlowDates,TFactors,Factors,Payment,Principal,Interest,Prepayment] = mbscfamounts( \_\_\_\_,CouponRate,Delay,PrepaySpeed,PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### **Calculate Cash Flow Amounts and Dates, Time Factors, and Mortgage Factors for a Single Mortgage**

Given a mortgage with the following characteristics, compute the cash flow amounts and dates, the time factors, and the mortgage factors.

Define the mortgage characteristics.

```

Settle      = datenum('17-April-2002');
Maturity    = datenum('1-Jan-2030');
IssueDate   = datenum('1-Jan-2000');
GrossRate   = 0.08125;
CouponRate  = 0.075;
Delay       = 14;
PrepaySpeed = 100;

```

Use `mbscfamounts` to evaluate the mortgage.

```

[CFlowAmounts, CFlowDates, TFactors, Factors] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, PrepaySpeed)

```

```
CFlowAmounts = 1×334
```

```

-0.0033    0.0118    0.0120    0.0121    0.0120    0.0119    0.0119    0.0118    0.0117

```

```
CFlowDates = 1×334
```

```

731323    731337    731368    731398    731429    731460    731490

```

```
TFactors = 1×334
```

```

0    0.9333    1.9333    2.9333    3.9333    4.9333    5.9333    6.9333    7.9333

```

```
Factors = 1×334
```

```

1.0000    0.9944    0.9887    0.9828    0.9769    0.9711    0.9653    0.9595    0.9537

```

The result is contained in four 334-element row vectors.

### Compute Cash Flow Amounts and Dates, Time Factors, and Mortgage Factors for a Mortgage Portfolio

Given a portfolio of mortgage-backed securities, use `mbscfamounts` to compute the cash flows and other factors from the portfolio.

Define characteristics for a mortgage portfolio.

```

Settle = datenum(['13-Jan-2000'; '17-Apr-2002'; '17-May-2002']);
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = [0.075; 0.07875; 0.0775];
Delay = 14;
PrepaySpeed = 100;

```

Use `mbscfamonts` to evaluate the mortgage.

```

[CFlowAmounts, CFlowDates, TFactors, Factors] = ...
mbscfamonts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, PrepaySpeed)

```

CFlowAmounts = 3×361

-0.0025	0.0071	0.0072	0.0074	0.0076	0.0077	0.0079	0.0080	0.0081	0.0082
-0.0035	0.0121	0.0123	0.0124	0.0123	0.0122	0.0122	0.0121	0.0121	0.0120
-0.0034	0.0122	0.0123	0.0123	0.0122	0.0121	0.0121	0.0120	0.0120	0.0119

CFlowDates = 3×361

730498	730517	730546	730577	730607	730638	730668	730698	730728	730758
731323	731337	731368	731398	731429	731460	731490	731521	731551	731581
731353	731368	731398	731429	731460	731490	731521	731551	731581	731611

TFactors = 3×361

0	1.0667	2.0667	3.0667	4.0667	5.0667	6.0667	7.0667	8.0667	9.0667
0	0.9333	1.9333	2.9333	3.9333	4.9333	5.9333	6.9333	7.9333	8.9333
0	0.9333	1.9333	2.9333	3.9333	4.9333	5.9333	6.9333	7.9333	8.9333

Factors = 3×361

1.0000	0.9992	0.9982	0.9970	0.9957	0.9942	0.9925	0.9907	0.9888	0.9868
1.0000	0.9944	0.9887	0.9828	0.9769	0.9711	0.9653	0.9595	0.9537	0.9478
1.0000	0.9942	0.9883	0.9824	0.9766	0.9707	0.9649	0.9592	0.9534	0.9475

Each output is a 3-by-361 element matrix padded with NaN's wherever elements are missing.



## Calculate Payment, Principal, Interest, and Prepayment for a Single Mortgage

Given a mortgage with the following characteristics, compute payments, principal, interest, and prepayment.

Define the mortgage characteristics.

```
Settle      = datenum('17-April-2002');
Maturity    = datenum('1-Jan-2030');
IssueDate   = datenum('1-Jan-2000');
GrossRate   = 0.08125;
CouponRate  = 0.075;
Delay       = 14;
PrepaySpeed = 100;
```

Use `mbscfamounts` to evaluate the mortgage.

```
[Payment, Principal, Interest, Prepayment] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, PrepaySpeed)
```

```
Payment = 1×334
```

```
-0.0033    0.0118    0.0120    0.0121    0.0120    0.0119    0.0119    0.0118    0.0117
```

```
Principal = 1×334
```

```
731323    731337    731368    731398    731429    731460    731490
```

```
Interest = 1×334
```

```
0    0.9333    1.9333    2.9333    3.9333    4.9333    5.9333    6.9333    7.9333
```

```
Prepayment = 1×334
```

```
1.0000    0.9944    0.9887    0.9828    0.9769    0.9711    0.9653    0.9595    0.9536
```

## Input Arguments

### **Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

### **Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **DeLay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

### **PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

### **PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use PrepayMatrix only when PrepaySpeed is unspecified.

---

Data Types: double

## **Output Arguments**

### **CFlowAmounts — Cash flow amounts**

matrix

Cash flows starting from `Settle` through end of the last month (`Maturity`), returned as a NMBS-by-P matrix.

### **CFlowDates — Cash flow dates**

matrix

Cash flow dates (including at `Settle`), returned as a NMBS-by-P matrix.

### **TFactors — Time factors**

matrix

Time factors (in months from `Settle`), returned as a NMBS-by-P matrix.

### **Factors — Mortgage factors (the fraction of the balance still outstanding at the end of each month)**

matrix

Mortgage factors (the fraction of the balance still outstanding at the end of each month), returned as a NMBS-by-P matrix.

### **Payment — Total monthly payment**

matrix

Total monthly payment, returned as a NMBS-by-P matrix.

### **Principal — Principal portion of the payment**

matrix

Principal portion of the payment, returned as a NMBS-by-P matrix.

### **Interest — Interest portion of the payment**

matrix

Interest portion of the payment, returned as a NMBS-by-P matrix.

### **Prepayment — Unscheduled payment of principal**

matrix

Unscheduled payment of principal, returned as a NMBS-by-P matrix.

## **References**

[1] *PSA Uniform Practices*, SF-49

## **See Also**

`cmosched` | `cmoschedcf` | `cmoseqcf` | `mbsnoprepay` | `mbspassthrough`

## **Topics**

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced in R2012a**

## mbsconvp

Convexity of mortgage pool given price

### Syntax

```
Convexity = mbsconvp(Price,Settle,Maturity,IssueDate,GrossRate)  
Convexity = mbsconvp( ____,CouponRate,Delay,PrepaySpeed,PrepayMatrix)
```

### Description

`Convexity = mbsconvp(Price,Settle,Maturity,IssueDate,GrossRate)` computes mortgage-backed security convexity, given time information, price at settlement, and optionally, a prepayment model.

`Convexity = mbsconvp( ____,CouponRate,Delay,PrepaySpeed,PrepayMatrix)` specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute a Mortgage-Backed Security Convexity

This example shows how to compute a mortgage-backed security convexity, given a mortgage-backed security with the following characteristics.

```
Price      = 101;  
Settle     = '15-Apr-2002';  
Maturity   = '1 Jan 2030';  
IssueDate  = '1-Jan-2000';  
GrossRate  = 0.08125;  
CouponRate = 0.075;  
Delay      = 14;  
Speed      = 100;
```

```
Convexity = mbsconvp(Price, Settle, Maturity, IssueDate,...  
GrossRate, CouponRate, Delay, Speed)
```

```
Convexity = 71.6299
```

## Input Arguments

### **Price — Clean price for every \$100 face value**

vector

Clean price for every \$100 face value, specified as an NMBS-by-1 vector.

Data Types: double

### **Settle — Settlement dates**

vector

Settlement dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

### **Maturity — Maturity dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

### **IssueDate — Issue dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

### **GrossRate — Gross coupon rate (including fees)**

vector

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

### **CouponRate — Net coupon rate**

GrossRate (default) | vector

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

### **Delay — Delay in days**

0 (no delay) (default) | vector

(Optional) Delay in days, specified as an NMBS-by-1 vector.

Data Types: double

### **PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

### **PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size  $\max(\text{TermRemaining})$ -by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use PrepayMatrix only when PrepaySpeed is unspecified.

---

Data Types: double



## Output Arguments

### **Convexity — Periodic convexity of mortgage pool**

scalar numeric

Periodic convexity of mortgage pool, returned as a scalar numeric.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

mbsconvy | mbsdurp | mbsdury | mbsnoprepay | mbspassthrough

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsconvy

Convexity of mortgage pool given yield

### Syntax

```
Convexity = mbsconvy(Yield,Settle,Maturity,IssueDate,GrossRate)
Convexity = mbsconvy( ____,CouponRate,Delay,PrepaySpeed,PrepayMatrix)
```

### Description

`Convexity = mbsconvy(Yield,Settle,Maturity,IssueDate,GrossRate)` computes mortgage-backed security convexity, given time information, semiannual mortgage yield, and optionally, a prepayment model.

`Convexity = mbsconvy( ____,CouponRate,Delay,PrepaySpeed,PrepayMatrix)` specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Convexity of a Mortgage Pool Given Yield

This example shows how to compute the convexity of mortgage pool given yield for a mortgage-backed security with the following characteristics.

```
Yield      = 0.07125;
Settle     = '15-Apr-2002';
Maturity   = '1 Jan 2030';
IssueDate  = '1-Jan-2000';
GrossRate  = 0.08125;
Speed      = 100;
CouponRate = 0.075;
Delay      = 14;
```

```
Convexity = mbsconvy(Yield, Settle, Maturity, IssueDate, ...  
GrossRate, CouponRate, Delay, Speed)
```

```
Convexity = 72.8263
```

## Input Arguments

### **Yield — Mortgage yield, compounded monthly**

vector

Mortgage yield, compounded monthly, specified as an NMBS-by-1 vector in decimals.

Data Types: double

### **Settle — Settlement dates**

vector

Settlement dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | cell

### **Maturity — Maturity dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

### **IssueDate — Issue dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

### **GrossRate — Gross coupon rate (including fees)**

vector

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

### **CouponRate — Net coupon rate**

GrossRate (default) | vector

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

### **Delay — Delay in days**

0 (no delay) (default) | vector

(Optional) Delay in days, specified as an NMBS-by-1 vector.

Data Types: double

### **PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

### **PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size  $\max(\text{TermRemaining})$ -by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use PrepayMatrix only when PrepaySpeed is unspecified.

---

Data Types: double

## Output Arguments

### **Convexity — Periodic convexity of mortgage pool**

scalar numeric

Periodic convexity of mortgage pool, returned as a scalar numeric.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

mbsconvp | mbsdurp | mbsdury | mbsnoprepay | mbspassthrough

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsdurp

Duration of mortgage pool given price

### Syntax

```
[YearDuration,ModDuration] = mbsdurp(Price,Settle,Maturity,
IssueDate,GrossRate)
[YearDuration,ModDuration] = mbsdurp( ____,CouponRate,Delay,
PrepaySpeed,PrepayMatrix)
```

### Description

[YearDuration,ModDuration] = mbsdurp(Price,Settle,Maturity, IssueDate,GrossRate) computes the mortgage-backed security Macaulay (YearDuration) in years and modified (ModDuration) durations in years, given time information, price at settlement, and optionally, a prepayment model.

[YearDuration,ModDuration] = mbsdurp( \_\_\_\_,CouponRate,Delay, PrepaySpeed,PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Find the Duration of a Mortgage Pool

This example shows how to find the duration of mortgage pool given a mortgage-backed security with the following characteristics.

```
Price = 101;
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;;
```

```
Delay = 14;  
Speed = 100;  
  
[YearDuration, ModDuration] = mbsdurp(Price, Settle, Maturity,...  
IssueDate, GrossRate, CouponRate, Delay, Speed)  
  
YearDuration = 6.4380  
ModDuration = 6.2080
```

## Input Arguments

### **Price — Clean price for every \$100 face of issue**

vector

Clean price for every \$100 face of issue, specified as an NMBS-by-1 vector.

Data Types: double

### **Settle — Settlement dates**

vector

Settlement dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | cell

### **Maturity — Maturity dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

### **IssueDate — Issue dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

**GrossRate — Gross coupon rate (including fees)**

vector

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

**CouponRate — Net coupon rate**

GrossRate (default) | vector

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

**Delay — Delay in days**

0 (no delay) (default) | vector

(Optional) Delay in days, specified as an NMBS-by-1 vector.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use PrepayMatrix only when PrepaySpeed is unspecified.

---

Data Types: double



## Output Arguments

### **YearDuration** — Macaulay duration in years

scalar numeric

Macaulay duration in years, returned as a scalar numeric.

### **ModDuration** — Modified duration in years

scalar numeric

Modified duration in years, returned as a scalar numeric.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

mbsconvp | mbsconvy | mbsdury | mbsnoprepay | mbspassthrough

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsdury

Duration of mortgage pool given yield

### Syntax

```
[YearDuration,ModDuration] = mbsdury(Yield,Settle,Maturity,
IssueDate,GrossRate)
[YearDuration,ModDuration] = mbsdury( ____,CouponRate,Delay,
PrepaySpeed,PrepayMatrix)
```

### Description

[YearDuration,ModDuration] = mbsdury(Yield,Settle,Maturity, IssueDate,GrossRate) computes the mortgage-backed security Macaulay (YearDuration) in years and modified (ModDuration) durations in years, given time information, yield to maturity, and optionally, a prepayment model.

[YearDuration,ModDuration] = mbsdury( \_\_\_\_,CouponRate,Delay, PrepaySpeed,PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Find the Duration of a Mortgage Pool Given the Yield

This example shows how to find the duration of mortgage pool given a mortgage-backed security with the following characteristics.

```
Yield = 0.07298413;
Settle = '15-Apr-2002';
Maturity = '1 Jan 2030';
IssueDate = '1-Jan-2000';
GrossRate = 0.08125;
Speed = 100;
```

```
CouponRate = 0.075;  
Delay = 14;  
  
[YearDuration, ModDuration] = mbsdury(Yield, Settle, Maturity,...  
IssueDate, GrossRate, CouponRate, Delay, Speed)  
  
YearDuration = 6.4380  
ModDuration = 6.2080
```

## Input Arguments

### **Yield — Mortgage yield, compounded monthly**

vector

Mortgage yield, compounded monthly, specified as an NMBS-by-1 vector in decimals.

Data Types: double

### **Settle — Settlement dates**

vector

Settlement dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | cell

### **Maturity — Maturity dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

### **IssueDate — Issue dates**

vector

Maturity dates, specified as an NMBS-by-1 vector of serial date numbers or a cell array of character vectors.

Data Types: double | cell

**GrossRate — Gross coupon rate (including fees)**

vector

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

**CouponRate — Net coupon rate**

GrossRate (default) | vector

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of numeric decimals.

Data Types: double

**Delay — Delay in days**

0 (no delay) (default) | vector

(Optional) Delay in days, specified as an NMBS-by-1 vector.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use PrepayMatrix only when PrepaySpeed is unspecified.

---

Data Types: double

## Output Arguments

### **YearDuration — Macaulay duration in years**

scalar numeric

Macaulay duration in years, returned as a scalar numeric.

### **ModDuration — Modified duration in years**

scalar numeric

Modified duration in years, returned as a scalar numeric.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

mbsconvp | mbsconvy | mbsdurp | mbsnoprepay | mbspassthrough

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsnoprepay

End-of-month mortgage cash flows and balances without prepayment

### Syntax

```
[Balance,Interest,Payment,Principal] = mbsnoprepay(OriginalBalance,  
GrossRate,Term)
```

### Description

[Balance,Interest,Payment,Principal] = mbsnoprepay(OriginalBalance, GrossRate,Term) computes end-of-month mortgage balance, interest payments, principal payments, and cash flow payments with zero prepayment rate.

mbsnoprepay returns amortizing cash flows and balances over a specified term with no prepayment. When the lengths of pass-throughs are not the same, MATLAB software pads the shorter ones with NaN.

### Examples

#### Compute an Amortization Schedule from Mortgage Pools

Given mortgage pools with the following characteristics, compute an amortization schedule.

```
OriginalBalance = 400000000;  
CouponRate = 0.08125;  
Term = [357; 355]; % Three- and five-month old mortgage pools.
```

```
[Balance, Interest, Payment, Principal] = mbsnoprepay(OriginalBalance, CouponRate, Term)
```

```
Balance = 357×2  
108 ×
```

3.9973	3.9973
3.9946	3.9946
3.9919	3.9918
3.9892	3.9890
3.9864	3.9862
3.9837	3.9834
3.9809	3.9806
3.9781	3.9778
3.9753	3.9749
3.9724	3.9720
:	

Interest =  $357 \times 2$   
 $10^6 \times$

2.7083	2.7083
2.7065	2.7065
2.7047	2.7046
2.7029	2.7028
2.7010	2.7009
2.6992	2.6990
2.6973	2.6971
2.6954	2.6952
2.6935	2.6933
2.6916	2.6913
:	

Payment =  $357 \times 2$   
 $10^6 \times$

2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
2.9759	2.9799
:	

Principal = 357×2  
10<sup>6</sup> ×

0.2675	0.2715
0.2693	0.2734
0.2712	0.2752
0.2730	0.2771
0.2749	0.2790
0.2767	0.2809
0.2786	0.2828
0.2805	0.2847
0.2824	0.2866
0.2843	0.2885
:	

## Input Arguments

### **OriginalBalance** — Original face value in dollars

vector

Original face value in dollars, specified as a table or an NUMOBS-by-1 matrix.

Data Types: double

### **GrossRate** — Gross coupon rate (including fees)

vector of decimal values

Gross coupon rate (including fees), specified as an NUMOBS-by-1 vector of decimal values.

Data Types: double

### **Term** — Term of the mortgage in months

vector

Term of the mortgage in months, specified as an NUMOBS-by-1 vector.

Data Types: double



## Output Arguments

### **Balance — End-of-month balances over the life of the pass-through**

vector

End-of-month balances over the life of the pass-through, returned as a Term-by-1 vector.

### **Interest — End-of-month interest payments over the life of the pass-through**

vector

End-of-month interest payments over the life of the pass-through, returned as a Term-by-1 vector.

### **Payment — End-of-month payments over the life of the pass-through**

vector

End-of-month payments over the life of the pass-through, returned as a Term-by-1 vector.

### **Principal — All scheduled end-of-month principal payments over the life of the pass-through**

vector

All scheduled end-of-month principal payments over the life of the pass-through, returned as a Term-by-1 vector.

## See Also

[mbsconvp](#) | [mbsconvy](#) | [mbsdurp](#) | [mbspassthrough](#)

## Topics

[“Generating Prepayment Vectors”](#) on page 5-4

[“Mortgage Prepayments”](#) on page 5-6

[“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”](#)

[“What Are Mortgage-Backed Securities?”](#) on page 5-2

**Introduced before R2006a**

## mbsoas2price

Price given option-adjusted spread

### Syntax

```
Price = mbsoas2price(ZeroCurve,OAS,Settle,Maturity,IssueDate,  
GrossRate)  
Price = mbsoas2price( ____,CouponRate,Delay,Interpolation,PrepaySpeed,  
PrepayMatrix)
```

### Description

Price = mbsoas2price(ZeroCurve,OAS,Settle,Maturity,IssueDate,GrossRate) computes the clean price of a pass-through security for each \$100 face value of outstanding principal.

Price = mbsoas2price( \_\_\_\_,CouponRate,Delay,Interpolation,PrepaySpeed,PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Theoretical Price of a Mortgage Pool

Given an option-adjusted spread, a spot curve, and a prepayment assumption, compute theoretical price of a mortgage pool. First, create the bonds matrix:

```
Bonds = [datenum('11/21/2002') 0 100 0 2 1;  
datenum('02/20/2003') 0 100 0 2 1;  
datenum('07/31/2004') 0.03 100 2 3 1;  
datenum('08/15/2007') 0.035 100 2 3 1;  
datenum('08/15/2012') 0.04875 100 2 3 1;  
datenum('02/15/2031') 0.05375 100 2 3 1];
```

Choose a settlement date.

```
Settle = datenum('20-Aug-2002');
```

Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;
           98.58044;
           100.10534;
           98.18054;
           101.38136;
           99.25411];
```

Use the following formula to compute spot compounding for the bonds:

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]
```

```
ZeroCurve = 6×3
105 ×
```

7.3154	0.0000	0.0000
7.3163	0.0000	0.0000
7.3216	0.0000	0.0000
7.3327	0.0000	0.0000
7.3510	0.0000	0.0000
7.4185	0.0000	0.0000

Assign the following parameters:

```
OAS           = [26.0502; 28.6348; 31.2222];
Maturity      = datenum('02-Jan-2030');
IssueDate     = datenum('02-Jan-2000');
GrossRate     = 0.08125;
CouponRate    = 0.075;
Delay         = 14;
Interpolation = 1;
PrepaySpeed   = [0 50 100];
```

Calculate the theoretical price from the option-adjusted spread.

```
Price = mbsoas2price(ZeroCurve, OAS, Settle, Maturity, ...  
IssueDate, GrossRate, CouponRate, Delay, Interpolation, ...  
PrepaySpeed)
```

```
Price = 3×1
```

```
95.0006  
95.0006  
95.0006
```

## Input Arguments

### ZeroCurve — Zero curve

matrix

Zero curve, specified as a three-column matrix, where:

- Column 1 is serial date numbers.
- Column 2 is spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).
- Column 3 is the compounding value of the rates in Column 2. (This is the agency spot rate on the settlement date.) Allowable compounding values are: 1 (annual), 2 (semiannual), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).

For example:

```
[datenum('1-Jan-2003') 0.0154 12;  
 datenum('1-Jan-2004') 0.0250 12;  
 .....  
 datenum('1-Jan-2020') 0.0675 2];
```

Data Types: double | char | cell

### OAS — Option-adjusted spreads

vector

Option-adjusted spreads, in basis points, specified as an NMBS-by-1 vector.

Data Types: double

**Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

**Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

**IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

**GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**DeLay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

**Interpolation — Interpolation method to compute the corresponding spot rates for the bond's cash flow**

1 (linear) (default) | vector

(Optional) Interpolation method to compute the corresponding spot rates for the bond's cash flow, specified as an NMBS-by-1 vector. Available methods are (0) nearest, (1) linear, and (2) cubic spline. For more information on the supported interpolation methods, see `interp1`.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the `PrepaySpeed` to `[]` if you input a customized `PrepayMatrix`.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double

## Output Arguments

**Price** — Clean price of passthrough per \$100 face of principal outstanding vector

Clean price of passthrough per \$100 face of principal outstanding, returned as a NMBS-by-1 vector.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

[mbspassthrough](#) | [mbsprice2oas](#) | [mbsyield2oas](#)

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsoas2yield

Yield given option-adjusted spread

### Syntax

```
[MYield,BEMBSYield] = mbsoas2yield(ZeroCurve,OAS,Settle,Maturity,
IssueDate,GrossRate)
[MYield,BEMBSYield] = mbsoas2yield( ____,CouponRate,Delay,
Interpolation,PrepaySpeed,PrepayMatrix)
```

### Description

[MYield,BEMBSYield] = mbsoas2yield(ZeroCurve,OAS,Settle,Maturity, IssueDate,GrossRate) computes the mortgage and bond-equivalent yields of a pass-through security.

[MYield,BEMBSYield] = mbsoas2yield( \_\_\_\_,CouponRate,Delay, Interpolation,PrepaySpeed,PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Theoretical Yield to Maturity of a Mortgage Pool

Given an option-adjusted spread, a spot curve, and a prepayment assumption, compute the theoretical yield to maturity of a mortgage pool. First, create the bonds matrix:

```
Bonds = [datenum('11/21/2002') 0 100 0 2 1;
datenum('02/20/2003') 0 100 0 2 1;
datenum('07/31/2004') 0.03 100 2 3 1;
datenum('08/15/2007') 0.035 100 2 3 1;
datenum('08/15/2012') 0.04875 100 2 3 1;
datenum('02/15/2031') 0.05375 100 2 3 1];
```



Choose a settlement date.

```
Settle = datenum('20-Aug-2002');
```

Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;
           98.58044;
           100.10534;
           98.18054;
           101.38136;
           99.25411];
```

Use the following formula to compute spot compounding for the bonds:

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]
```

```
ZeroCurve = 6×3
105 ×
```

7.3154	0.0000	0.0000
7.3163	0.0000	0.0000
7.3216	0.0000	0.0000
7.3327	0.0000	0.0000
7.3510	0.0000	0.0000
7.4185	0.0000	0.0000

Assign the following parameters:

```
OAS = [26.0502; 28.6348; 31.2222];
Maturity = datenum('02-Jan-2030');
IssueDate = datenum('02-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0 50 100];
```

Compute the mortgage yield and bond equivalent mortgage yield.

```
[MYield BEMBSYield] = mboas2yield(ZeroCurve, OAS, Settle, ...  
Maturity, IssueDate, GrossRate, CouponRate, Delay, ...  
Interpolation, PrepaySpeed)
```

```
MYield = 3×1
```

```
0.0802  
0.0814  
0.0828
```

```
BEMBSYield = 3×1
```

```
0.0816  
0.0828  
0.0842
```

## Input Arguments

### ZeroCurve — Zero curve

matrix

Zero curve, specified as a three-column matrix, where:

- Column 1 is serial date numbers.
- Column 2 is spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).
- Column 3 is the compounding value of the rates in Column 2. (This is the agency spot rate on the settlement date.) Allowable compounding values are: 1 (annual), 2 (semiannual), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).

For example:

```
[datenum('1-Jan-2003') 0.0154 12;  
datenum('1-Jan-2004') 0.0250 12;  
.....  
datenum('1-Jan-2020') 0.0675 2];
```

Data Types: double | char | cell

**OAS — Option-adjusted spreads**

vector

Option-adjusted spreads, in basis points, specified as an NMBS-by-1 vector.

Data Types: double

**Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

**Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

**IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

**GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**Delay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

**Interpolation — Interpolation method to compute the corresponding spot rates for the bond's cash flow**

1 (linear) (default) | vector

(Optional) Interpolation method to compute the corresponding spot rates for the bond's cash flow, specified as an NMBS-by-1 vector. Available methods are (0) nearest, (1) linear, and (2) cubic spline. For more information on the supported interpolation methods, see `interp1`.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the `PrepaySpeed` to `[]` if you input a customized `PrepayMatrix`.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double

## Output Arguments

**MYield** — Yield to maturity of the mortgage-backed security  
vector

Yield to maturity of the mortgage-backed security, returned as a NMBS-by-1 vector. This yield is compounded monthly (12 times a year).

**BEMBSYield** — Bond equivalent yield of the mortgage-backed security  
vector

Bond equivalent yield of the mortgage-backed security, returned as a NMBS-by-1 vector. This yield is compounded semiannually (two times a year).

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

mbsoas2price | mbspassthrough | mbsprice2oas | mbsyield2oas

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbspassthrough

Mortgage pool cash flows and balances with prepayment

### Syntax

```
[Balance,Payment,Principal,Interest,Prepayment] = mbspassthrough(  
OriginalBalance,GrossRate,OriginalTerm)  
[Balance,Payment,Principal,Interest,Prepayment] = mbspassthrough(____  
,TermRemaining,PrepaySpeed,PrepayMatrix)
```

### Description

[Balance,Payment,Principal,Interest,Prepayment] = mbspassthrough(OriginalBalance,GrossRate,OriginalTerm) calculates mortgage pool cash flows and balances with prepayments.

If a standard (PSA) prepayment is specified, "aging" is applied to standard prepayment vector. Aging is the same amount as the age of the pool (OriginalTerm - TermRemaining).

[Balance,Payment,Principal,Interest,Prepayment] = mbspassthrough(\_\_\_\_,TermRemaining,PrepaySpeed,PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute the Cash Flow of Principal, Interest, and Prepayment of a Pass-Through Security

This example shows how to compute the cash flows and balances of a 3-month old mortgage pool with original term of 360 months, assuming a prepayment speed of 100.

```
OriginalBalance = 100000;  
GrossRate = 0.08125;
```

```
OriginalTerm = 360;  
TermRemaining = 357;  
PrepaySpeed = 100;
```

```
[Balance, Payment, Principal, Interest, Prepayment] =...  
mbspassthrough(OriginalBalance, GrossRate, OriginalTerm,...  
TermRemaining, PrepaySpeed)
```

```
Balance = 357×1  
104 ×
```

```
9.9866  
9.9715  
9.9548  
9.9363  
9.9161  
9.8942  
9.8707  
9.8454  
9.8185  
9.7900  
⋮
```

```
Payment = 357×1
```

```
743.9671  
743.4693  
742.8468  
742.0999  
741.2285  
740.2329  
739.1132  
737.8699  
736.5034  
735.0139  
⋮
```

```
Principal = 357×1
```

```
66.8837  
67.2915  
67.6904  
68.0802
```

68.4607  
68.8317  
69.1929  
69.5442  
69.8854  
70.2163  
⋮

Interest =  $357 \times 1$

677.0833  
676.1777  
675.1564  
674.0196  
672.7678  
671.4012  
669.9203  
668.3257  
666.6179  
664.7976  
⋮

Prepayment =  $357 \times 1$

66.8676  
83.5494  
100.2000  
116.8108  
133.3731  
149.8785  
166.3183  
182.6840  
198.9672  
215.1593  
⋮

## Input Arguments

**OriginalBalance** — Original balance value in dollars  
vector



Original balance value in dollars (balance at the beginning of each TermRemaining), specified as an NMBS-by-1 vector.

Data Types: double

**GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**OriginalTerm — Term of the mortgage in months**

vector

Term of the mortgage in months, specified as an NMBS-by-1 vector.

Data Types: double

**TermRemaining — Number of full months between settlement and maturity**

OriginalTerm (default) | vector

(Optional) Number of full months between settlement and maturity, specified as an NMBS-by-1 vector. For this argument, "full months" means not including fractional first term (if there is one).

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)-by-NMBS`. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double

## Output Arguments

### **Balance — Principal balance at end of month**

vector

Principal balance at end of month, returned as a `TermRemaining-by-1` vectors of end-of-month values.

### **Payment — Total monthly payment**

vector

Total monthly payment, returned as a `TermRemaining-by-1` vectors of end-of-month values.

### **Principal — Principal portion of the payment**

vector

Principal portion of the payment, returned as a `TermRemaining-by-1` vectors of end-of-month values.

### **Interest — Interest portion of the payment**

vector

Interest portion of the payment, returned as a `TermRemaining-by-1` vectors of end-of-month values.

### **Prepayment — Unscheduled principal payment**

vector

Unscheduled principal payment, returned as a `TermRemaining-by-1` vectors of end-of-month values.

## **See Also**

mbswal

## **Topics**

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsprice

Mortgage-backed security price given yield

### Syntax

```
[Price,AccrInt] = mbsprice(Yield,Settle,Maturity,IssueDate,  
GrossRate)  
[Price,AccrInt] = mbsprice( ___ CouponRate,Delay,PrepaySpeed,  
PrepayMatrix)
```

### Description

[Price,AccrInt] = mbsprice(Yield,Settle,Maturity,IssueDate, GrossRate) computes a mortgage-backed security price, given time information and mortgage yield at settlement.

[Price,AccrInt] = mbsprice( \_\_\_ CouponRate,Delay,PrepaySpeed, PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Determine the Mortgage-Backed Security Price Given the Yield

This example shows how to determine the mortgage-backed security price given a mortgage-backed security with the following characteristics.

```
Yield = 0.0725;  
Settle = datenum('15-Apr-2002');  
Maturity = datenum('1 Jan 2030');  
IssueDate = datenum('1-Jan-2000');  
GrossRate = 0.08125;  
CouponRate = 0.075;  
Delay = 14;
```





Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

**GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**Delay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size  $\max(\text{TermRemaining})$ -by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double

## Output Arguments

**Price — Clean price for every \$100 face value of the securities**  
vector

Clean price for every \$100 face value of the securities, returned as a NMBS-by-1 vector.

**AccrInt — Accrued interest of the mortgage-backed securities**  
vector

Accrued interest of the mortgage-backed securities, returned as a NMBS-by-1 vector.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsyield`

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**



# mbsprice2oas

Option-adjusted spread given price

## Syntax

```
OAS = mbsprice2oas(ZeroCurve,Price,Settle,Maturity,IssueDate,
GrossRate)
OAS = mbsprice2oas( ____,CouponRate,Delay,Interpolation,PrepaySpeed,
PrepayMatrix)
```

## Description

OAS = mbsprice2oas(ZeroCurve,Price,Settle,Maturity,IssueDate, GrossRate) computes the option-adjusted spread in basis points.

OAS = mbsprice2oas( \_\_\_\_,CouponRate,Delay,Interpolation,PrepaySpeed, PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Calculate the Option-Adjusted Spread of a 30-Year Fixed-Rate Mortgage

Calculate the option-adjusted spread of a 30-year fixed-rate mortgage with about a 28-year weighted average maturity remaining, given assumptions of 0, 50, and 100 PSA prepayments. First, create the bonds matrix:

```
Bonds = [datenum('11/21/2002') 0 100 0 2 1;
datenum('02/20/2003') 0 100 0 2 1;
datenum('07/31/2004') 0.03 100 2 3 1;
datenum('08/15/2007') 0.035 100 2 3 1;
datenum('08/15/2012') 0.04875 100 2 3 1;
datenum('02/15/2031') 0.05375 100 2 3 1];
```

Choose a settlement date.

```
Settle = datenum('20-Aug-2002');
```

Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;  
          98.58044;  
          100.10534;  
          98.18054;  
          101.38136;  
          99.25411];
```

Use the following formula to compute spot compounding for the bonds:

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);  
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]
```

```
ZeroCurve = 6×3  
105 ×
```

7.3154	0.0000	0.0000
7.3163	0.0000	0.0000
7.3216	0.0000	0.0000
7.3327	0.0000	0.0000
7.3510	0.0000	0.0000
7.4185	0.0000	0.0000

Assign the following parameters:

```
Price          = 95;  
Maturity       = datenum('02-Jan-2030');  
IssueDate      = datenum('02-Jan-2000');  
GrossRate      = 0.08125;  
CouponRate     = 0.075;  
Delay          = 14;  
Interpolation  = 1;  
PrepaySpeed    = [0; 50; 100];  
Interpolation  = 1;
```

Compute the option-adjusted spread.

```
OAS = mbsprice2oas(ZeroCurve, Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Interpolation, ...
PrepaySpeed)
```

```
OAS = 3×1
```

```
26.0508
28.6355
31.2232
```

## Input Arguments

### ZeroCurve — Zero curve

matrix

Zero curve, specified as a three-column matrix, where:

- Column 1 is serial date numbers.
- Column 2 is spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).
- Column 3 is the compounding value of the rates in Column 2. (This is the agency spot rate on the settlement date.) Allowable compounding values are: 1 (annual), 2 (semiannual, 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).

For example:

```
[datenum('1-Jan-2003') 0.0154 12;
 datenum('1-Jan-2004') 0.0250 12;
 .....
 datenum('1-Jan-2020') 0.0675 2];
```

Data Types: double | char | cell

### Price — Clean price for every \$100 face value of bond issue

vector

Clean price for every \$100 face value of bond issue, specified as an NMBS-by-1 vector.

Data Types: double

### **Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

### **Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **DeLay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

**Interpolation — Interpolation method to compute the corresponding spot rates for the bond's cash flow**

1 (linear) (default) | vector

(Optional) Interpolation method to compute the corresponding spot rates for the bond's cash flow, specified as an NMBS-by-1 vector. Available methods are (0) nearest, (1) linear, and (2) cubic spline. For more information on the supported interpolation methods, see `interp1`.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the `PrepaySpeed` to `[]` if you input a customized `PrepayMatrix`.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double

## Output Arguments

### **OAS — Zero volatility OAS**

vector

Zero volatility OAS, in basis point (bp), returned as a NMBS-by-1 vector.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbssoas2price` | `mbssoas2yield` | `mbsyield2oas`

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

# mbsprice2speed

Implied PSA prepayment speeds given price

## Syntax

```
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed(Price, Settle,
Maturity, IssueDate, GrossRate, PrepayMatrix)
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed( ____,
CouponRate, Delay)
```

## Description

[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed(Price, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix) computes PSA prepayment speeds implied by pool prices and projected (user-defined) prepayment vectors. The calculated PSA speed produces the same price, modified duration, or modified convexity, depending upon the output requested.

[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed( \_\_\_\_, CouponRate, Delay) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Compute PSA Prepayment Speeds

This example shows how to compute the equivalent PSA benchmark prepayment speeds for a mortgage pool with the following characteristics and prepayment matrix.

```
Price          = 101;
Settle         = datenum('1-Jan-2000');
Maturity       = datenum('1-Jan-2030');
IssueDate     = datenum('1-Jan-2000');
GrossRate     = 0.08125;
```

```
PrepayMatrix = 0.005*ones(360,1);  
CouponRate   = 0.075;  
Delay        = 14;  
  
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = ...  
mbsprice2speed(Price,Settle, Maturity, IssueDate, ...  
GrossRate, PrepayMatrix, CouponRate, Delay)  
  
ImpSpdOnPrc = 118.5980  
  
ImpSpdOnDur = 118.3946  
  
ImpSpdOnCnv = 109.5115
```

## Input Arguments

### **Price** — Clean price for every \$100 face value

vector

Clean price for every \$100 face value, specified as an NMBS-by-1 vector.

Data Types: double

### **Settle** — Settlement date

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

### **Maturity** — Maturity date

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **IssueDate** — Issue date

cell array of date character vectors | vector of serial date numbers



Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **PrepayMatrix — Customized prepayment vector**

matrix

Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double

### **CouponRate — Net coupon rate**

`GrossRate` (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **Delay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

## Output Arguments

### **ImpSpd0nPrc — Equivalent PSA benchmark prepayment speed for the pass-through to carry the same price**

vector

Equivalent PSA benchmark prepayment speed for the pass-through to carry the same price, returned as a NMBS-by-1 vector.

### **ImpSpd0nDur — Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration**

vector

Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration, returned as a NMBS-by-1 vector.

### **ImpSpd0nCnv — Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity**

vector

Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity, returned as a NMBS-by-1 vector.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsprice` | `mbsyield2speed`

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

# mbswal

Weighted average life of mortgage pool

## Syntax

```
WAL = mbswal(Settle,Maturity,IssueDate,GrossRate)
WAL = mbswal( ___ CouponRate,Delay,PrepaySpeed,PrepayMatrix)
```

## Description

WAL = mbswal(Settle,Maturity,IssueDate,GrossRate) computes the weighted average life, in number of years, of a mortgage pool, as measured from the settlement date.

WAL = mbswal( \_\_\_ CouponRate,Delay,PrepaySpeed,PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Determine the Weighted Average Life of a Mortgage Pool

This example shows how to determine the weighted average life of a mortgage pool, given a pass-through security with the following characteristics.

```
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
```

```
WAL = mbswal(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, Speed)
```

WAL = 10.5477

## Input Arguments

### **Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

### **Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**Delay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size  $\max(\text{TermRemaining})$ -by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use PrepayMatrix only when PrepaySpeed is unspecified.

---

Data Types: double

## Output Arguments

**WAL — Weighted Average Life of MBS, in number of years**

vector

Weighted Average Life (WAL) of MBS, in number of years, returned as a NMBS-by-1 vector.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

mbspassthrough

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

# mbsyield

Mortgage-backed security yields given price

## Syntax

```
[MYield,BEMBSYield] = mbsyield(Price,Settle,Maturity,IssueDate,  
GrossRate)  
[MYield,BEMBSYield] = mbsyield( ___ CouponRate,Delay,PrepaySpeed,  
PrepayMatrix)
```

## Description

[MYield,BEMBSYield] = mbsyield(Price,Settle,Maturity,IssueDate, GrossRate) computes a mortgage-backed security yield to maturity and the bond equivalent yield, given time information, and price at settlement.

[MYield,BEMBSYield] = mbsyield( \_\_\_ CouponRate,Delay,PrepaySpeed, PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Determine a Mortgage-Backed Security Yield Given the Price

This example shows how to determine the mortgage-backed security yield, given a mortgage-backed security with the following characteristics.

```
Price = 102;  
Settle = '15-Apr-2002';  
Maturity = '1 Jan 2030';  
IssueDate = '1-Jan-2000';  
GrossRate = 0.08125;  
CouponRate = 0.075;  
Delay = 14;
```

```
Speed = 100;

[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Speed)

MYield = 0.0715

BEMBSYield = 0.0725
```

### **Determine Multiple Mortgage-Backed Securities Yields Given the Price**

This example shows how to determine multiple mortgage-backed securities yields, given a portfolio of mortgage-backed securities with the following characteristics.

```
Price = 102;
Settle = datenum(['13-Feb-2000'; '17-Apr-2002'; '17-May-2002'; ...
'13-Jan-2000']);
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = [0.075; 0.07875; 0.0775; 0.08125];
Delay = 14;
Speed = 100;
```

```
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Speed)
```

```
MYield = 4×1
```

```
0.0717
0.0751
0.0739
0.0779
```

```
BEMBSYield = 4×1
```

```
0.0728
0.0763
0.0750
0.0791
```



## Input Arguments

### **Price — Clean price for every \$100 face value**

vector

Clean price for every \$100 face value, specified as an NMBS-by-1 vector.

Data Types: double

### **Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

### **Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**Delay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the PrepaySpeed to [ ] if you input a customized PrepayMatrix.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size  $\max(\text{TermRemaining})$ -by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use PrepayMatrix only when PrepaySpeed is unspecified.

---

Data Types: double

## Output Arguments

**MYield — Yield to maturity of the mortgage-backed security**

vector

Yield to maturity of the mortgage-backed security, returned as a NMBS-by-1 vector. This yield is compounded monthly (12 times a year).

### **BEMBSYield — Bond equivalent yield of the mortgage-backed security**

vector

Bond equivalent yield of the mortgage-backed security, returned as a NMBS-by-1 vector. This yield is compounded semiannually (two times a year).

## **References**

[1] *PSA Uniform Practices*, SF-49

## **See Also**

mbsprice

## **Topics**

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsyield2oas

Option-adjusted spread given yield

### Syntax

```
OAS = mbsyield2oas(ZeroCurve,Yield,Settle,Maturity,IssueDate,  
GrossRate)  
OAS = mbsyield2oas( ____,CouponRate,Delay,Interpolation,PrepaySpeed,  
PrepayMatrix)
```

### Description

OAS = mbsyield2oas(ZeroCurve,Yield,Settle,Maturity,IssueDate,  
GrossRate) computes option-adjusted spread in basis points.

OAS = mbsyield2oas( \_\_\_\_,CouponRate,Delay,Interpolation,PrepaySpeed,  
PrepayMatrix) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Calculate the Option-Adjusted Spread of a 30-Year Fixed-Rate Mortgage Pool

Calculate the option-adjusted spread of a 30-year, fixed-rate mortgage pool with about 28-year weighted average maturity left, given assumptions of 0, 50, and 100 PSA prepayments. First, create the bonds matrix:

```
Bonds = [datenum('11/21/2002') 0 100 0 2 1;  
datenum('02/20/2003') 0 100 0 2 1;  
datenum('07/31/2004') 0.03 100 2 3 1;  
datenum('08/15/2007') 0.035 100 2 3 1;  
datenum('08/15/2012') 0.04875 100 2 3 1;  
datenum('02/15/2031') 0.05375 100 2 3 1];
```

Choose a settlement date.

```
Settle = datenum('20-Aug-2002');
```

Assume the following clean prices for the bonds:

```
Prices = [ 98.97467;
           98.58044;
           100.10534;
           98.18054;
           101.38136;
           99.25411];
```

Use the following formula to compute spot compounding for the bonds:

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding]
```

```
ZeroCurve = 6×3
105 ×
```

7.3154	0.0000	0.0000
7.3163	0.0000	0.0000
7.3216	0.0000	0.0000
7.3327	0.0000	0.0000
7.3510	0.0000	0.0000
7.4185	0.0000	0.0000

Assign the following parameters:

```
Price          = 95;
Maturity       = datenum('02-Jan-2030');
IssueDate      = datenum('02-Jan-2000');
GrossRate      = 0.08125;
CouponRate     = 0.075;
Delay          = 14;
Interpolation  = 1;
PrepaySpeed    = [0 50 100];
```

Compute the yield, and from the yield, compute the option-adjusted spread.

```
[mbsyld, beyld] = mbsyield(Price, Settle, ...  
Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed);
```

```
OAS = mbsyield2oas(ZeroCurve, mbsyld, Settle, ...  
Maturity, IssueDate, GrossRate, CouponRate, Delay, ...  
Interpolation, PrepaySpeed)
```

```
OAS = 3×1
```

```
26.0508  
28.6355  
31.2232
```

## Input Arguments

### **ZeroCurve** — Zero curve

matrix

Zero curve, specified as a three-column matrix, where:

- Column 1 is serial date numbers.
- Column 2 is spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).
- Column 3 is the compounding value of the rates in Column 2. (This is the agency spot rate on the settlement date.) Allowable compounding values are: 1 (annual), 2 (semiannual, 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous).

For example:

```
[datenum('1-Jan-2003') 0.0154 12;  
 datenum('1-Jan-2004') 0.0250 12;  
 .....  
 datenum('1-Jan-2020') 0.0675 2];
```

Data Types: double | char | cell

### **Yield** — Mortgage yield, compounded monthly

vector in decimals

Mortgage yield, compounded monthly, specified as an NMBS-by-1 vector in decimals.

Data Types: double

**Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

**Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

**IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

**GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**CouponRate — Net coupon rate**

GrossRate (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

**DeLay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

**Interpolation — Interpolation method to compute the corresponding spot rates for the bond's cash flow**

1 (linear) (default) | vector

(Optional) Interpolation method to compute the corresponding spot rates for the bond's cash flow, specified as an NMBS-by-1 vector. Available methods are (0) nearest, (1) linear, and (2) cubic spline. For more information on the supported interpolation methods, see `interp1`.

Data Types: double

**PrepaySpeed — Speed relative to PSA standard**

0 (no prepayment) (default) | vector

(Optional) Speed relative to PSA standard, specified as an NMBS-by-1 vector. The PSA standard is 100.

---

**Note** Set the `PrepaySpeed` to `[]` if you input a customized `PrepayMatrix`.

---

Data Types: double

**PrepayMatrix — Customized prepayment vector**

matrix

(Optional) Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double



## Output Arguments

### **OAS — Zero volatility OAS**

vector

Zero volatility OAS, in basis point (bp), returned as a NMBS-by-1 vector.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

[mbssoas2price](#) | [mbsoas2yield](#) | [mbsprice2oas](#)

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## mbsyield2speed

Implied PSA prepayment speeds given yield

### Syntax

```
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed(Yield, Settle,
Maturity, IssueDate, GrossRate, PrepayMatrix)
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed( ____,
CouponRate, Delay)
```

### Description

[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed(Yield, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix) computes PSA prepayment speeds implied by pool yields and projected (user-defined) prepayment vectors. The calculated PSA speed produces the same yield, modified duration, or modified convexity, depending upon the output requested.

[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed( \_\_\_\_, CouponRate, Delay) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Compute PSA Prepayment Speeds

Calculate the equivalent PSA benchmark prepayment speeds for a security with these characteristics and prepayment matrix.

```
Yield          = 0.065;
Settle         = datenum('1-Jan-2000');
Maturity       = datenum('1-Jan-2030');
IssueDate     = datenum('1-Jan-2000');
GrossRate     = 0.08125;
```

```
PrepayMatrix = 0.005*ones(360,1);  
CouponRate   = 0.075;  
Delay        = 14;  
  
[ImpSpd0nYld, ImpSpd0nDur, ImpSpd0nCnv] = ...  
mbsyield2speed(Yield, Settle, Maturity, IssueDate, GrossRate, ...  
PrepayMatrix, CouponRate, Delay)  
  
ImpSpd0nYld = 117.7644  
ImpSpd0nDur = 116.7436  
ImpSpd0nCnv = 108.3309
```

## Input Arguments

### **Yield — Mortgage yield, compounded monthly**

vector in decimals

Mortgage yield, compounded monthly, specified as an NMBS-by-1 vector in decimals.

Data Types: double

### **Settle — Settlement date**

cell array of date character vectors | vector of serial date numbers

Settlement date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors. **Settle** must be earlier than **Maturity**.

Data Types: double | char | cell

### **Maturity — Maturity date**

cell array of date character vectors | vector of serial date numbers

Maturity date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **IssueDate — Issue date**

cell array of date character vectors | vector of serial date numbers

Issue date, specified as an NMBS-by-1 vector using serial date numbers or a cell array of date character vectors.

Data Types: double | char | cell

### **GrossRate — Gross coupon rate (including fees)**

vector of decimal values

Gross coupon rate (including fees), specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **PrepayMatrix — Customized prepayment vector**

matrix

Customized prepayment vector, specified as a NaN-padded matrix of size `max(TermRemaining)`-by-NMBS. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement.

---

**Note** Use `PrepayMatrix` only when `PrepaySpeed` is unspecified.

---

Data Types: double

### **CouponRate — Net coupon rate**

`GrossRate` (default) | vector of decimal values

(Optional) Net coupon rate, specified as an NMBS-by-1 vector of decimal values.

Data Types: double

### **Delay — Delay (in days) between payment from homeowner and receipt by bondholder**

0 (no delay between payment and receipt) (default) | vector

(Optional) Delay (in days) between payment from homeowner and receipt by bondholder, specified as an NMBS-by-1 vector.

Data Types: double

## Output Arguments

### **ImpSpd0nPrc — Equivalent PSA benchmark prepayment speed for the pass-through to carry the same price**

vector

Equivalent PSA benchmark prepayment speed for the pass-through to carry the same price, returned as a NMBS-by-1 vector.

### **ImpSpd0nDur — Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration**

vector

Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration, returned as a NMBS-by-1 vector.

### **ImpSpd0nCnv — Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity**

vector

Equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity, returned as a NMBS-by-1 vector.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

mbsprice2speed | mbsyield

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## psaspeed2default

Benchmark default

### Syntax

```
[ADRPSA,MDRPSA] = psaspeed2default(DefaultSpeed)
```

### Description

[ADRPSA,MDRPSA] = psaspeed2default(DefaultSpeed) computes the benchmark default on the performing balance of mortgage-backed securities per PSA benchmark speed.

### Examples

#### **Compute the Benchmark Default Rates on the Performing Balance of Mortgage-Backed Securities Per PSA Benchmark Speed**

This example shows how to compute the benchmark default rates on the performing balance of mortgage-backed securities per PSA benchmark speed, given a mortgage-backed security with annual speed set at the PSA default benchmark.

```
DefaultSpeed = 100;
```

```
[ADRPSA, MDRPSA] = psaspeed2default(DefaultSpeed)
```

```
ADRPSA = 360×1
```

```
0.0002  
0.0004  
0.0006  
0.0008  
0.0010  
0.0012
```

```
0.0014
0.0016
0.0018
0.0020
:
```

MDRPSA =  $360 \times 1$   
 $10^{-3} \times$

```
0.0167
0.0333
0.0500
0.0667
0.0834
0.1001
0.1167
0.1334
0.1501
0.1668
:
```

## Input Arguments

### **DefaultSpeed** — Annual speed relative to the benchmark

vector in decimals

Annual speed relative to the benchmark, specified as an NDEF-by-1 vector. The PSA benchmark is 100.

Data Types: double

## Output Arguments

### **ADRPSA** — PSA default rate

vector

PSA default rate, returned as a 360-by-NDEF vector in decimals.

### **MDRPSA — PSA monthly default rate**

vector

PSA monthly default rate, returned as a 360-by-NDEF vector in decimals.

### **References**

[1] *PSA Uniform Practices*, SF-49

### **See Also**

psaspeed2rate

### **Topics**

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**



## psaspeed2rate

Single monthly mortality rate given PSA speed

### Syntax

```
[CPRPSA, SMMPSA] = psaspeed2rate(PSASpeed)
```

### Description

`[CPRPSA, SMMPSA] = psaspeed2rate(PSASpeed)` calculates vectors of PSA prepayments, each containing 360 prepayment elements, to represent the 360 months in a 30-year mortgage pool.

### Examples

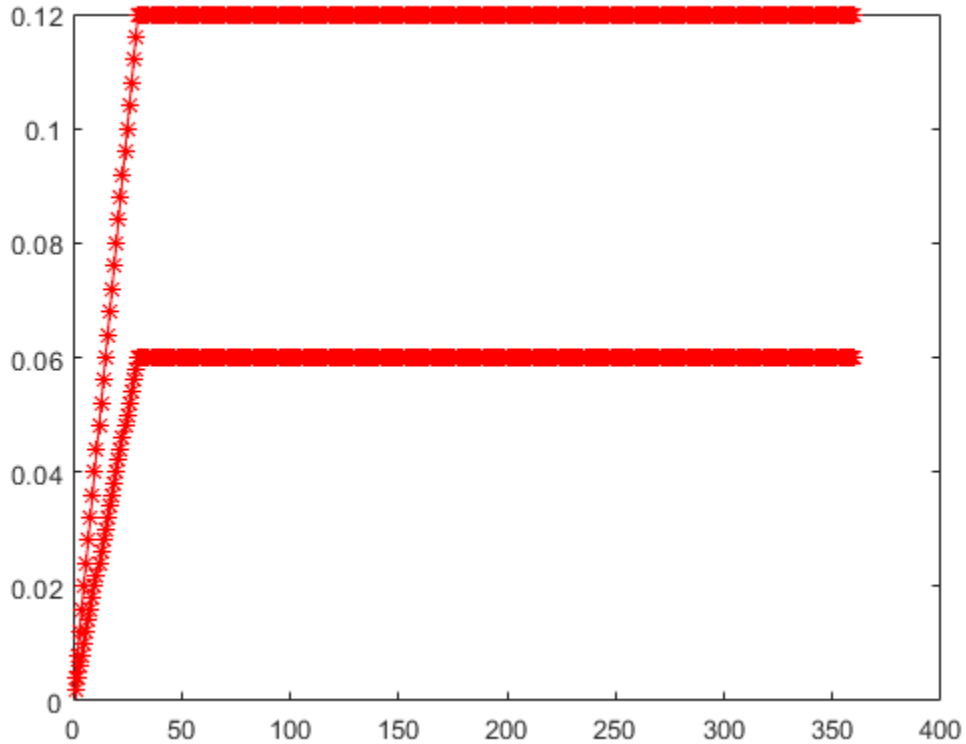
#### Compute the Prepayment and Mortality Rates

This example shows how to compute the prepayment and mortality rates, given a mortgage-backed security with annual speed set at the PSA default benchmark.

```
PSASpeed = [100 200];
```

```
[CPRPSA, SMMPSA] = psaspeed2rate(PSASpeed);
```

```
% view the plot of the output  
psaspeed2rate(PSASpeed)
```



## Input Arguments

### **PSASpeed** — Annual speed relative to the benchmark

vector in decimals

Annual speed relative to the benchmark, specified as any value  $> 0$  using an NSPD-by-1 vector. The PSA benchmark is 100.

Data Types: double

## Output Arguments

### **CPRPSA — PSA conditional prepayment rate**

vector

PSA conditional prepayment rate, returned as a 360-by-NSPD vector in decimals.

### **SMMPSA — PSA single monthly mortality rate**

vector

PSA monthly default rate, returned as a 360-by-NSPD vector in decimals.

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

psaspeed2default

## Topics

“Generating Prepayment Vectors” on page 5-4

“Mortgage Prepayments” on page 5-6

“Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model”

“What Are Mortgage-Backed Securities?” on page 5-2

**Introduced before R2006a**

## stepcpncfamounts

Cash flow amounts and times for bonds and stepped coupons

### Syntax

[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ConvDates, CouponRates, Period)

### Arguments

Settle	Settlement date. A scalar or vector of serial date numbers. <b>Settle</b> must be earlier than <b>Maturity</b> .
Maturity	Maturity date. A scalar or vector of serial date numbers.
ConvDates	Matrix of serial date numbers representing conversion dates after <b>Settle</b> . Size = number of instruments by maximum number of conversions. Fill unspecified entries with <b>NaN</b> .
CouponRates	<p>Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions + 1. First column of this matrix contains rates applicable between <b>Settle</b> and the first conversion date (date in the first column of <b>ConvDates</b>). Fill unspecified entries with <b>NaN</b>.</p> <p><b>ConvDates</b> has the same number of rows as <b>CouponRates</b> to reflect the same number of bonds. However, <b>ConvDates</b> has one less column than <b>CouponRates</b>. This situation is illustrated by</p> <pre> Settle-----ConvDate1-----ConvDate2-----Maturity           Rate1             Rate2             Rate3                     </pre>
Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.

Basis	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
EndMonthRule	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
Face	<p>(Optional) Face value of each bond in the portfolio. Default = 100.</p>

**Note** All arguments must be scalars or number of bonds (NUMBONDS)-by-1 vectors, except for **ConvDates** and **CouponRates**.

## Description

`[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ConvDates, CouponRates, Period, Basis, EndMonthRule, Face)` returns matrices of cash flow amounts, cash flow dates, and time factors for a portfolio of NUMBONDS stepped-coupon bonds.

CFlows is a matrix of cash flow amounts. The first entry in each row vector is a negative number indicating the accrued interest due at settlement. If no accrued interest is due, the first column is 0.

CDates is a matrix of cash flow dates in serial date number form. At least two columns are always present, one for settlement and one for maturity.

CTimes is a matrix of time factors for the SIA semiannual price/yield conversion.

$\text{DiscountFactor} = (1 + \text{Yield}/2)^{-\text{TFactor}}$

Time factors are in units of semiannual coupon periods. In computing time factors, use SIA actual/actual conventions for all time factor calculations.

---

**Note** For bonds with fixed coupons, use `cfamounts`. If you use a fixed-coupon bond with `stepcpncfamounts`, MATLAB software generates an error.

---

## Examples

This example generates stepped cash flows for three different bonds, all paying interest semiannually. Their life span is about 18-19 years each:

- Bond A has two conversions, but the first one occurs on the settlement date and immediately expires.
- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with some conversion dates not on the coupon dates. It has the longest maturity. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.



The following table illustrates the interest rate characteristics of this bond portfolio.

Bond A Dates	Bond A Rates	Bond B Dates	Bond B Rates	Bond C Dates	Bond C Rates
Settle (02-Aug-92)	7.5%	Settle (02-Aug-92)	7.5%	Settle (02-Aug-92)	2.5%
First Conversion (02-Aug-92)	8.875%	First Conversion (15-Jun-97)	8.875%	First Conversion (14-Jun-97)	5.0%
Second Conversion (15-Jun-03)	9.25%	Second Conversion (15-Jun-01)	9.25%	Second Conversion (14-Jun-01)	7.5%
Maturity (15-Jun-10)	NaN	Third Conversion (15-Jun-05)	10.0%	Third Conversion (14-Jun-05)	10.0%
		Maturity (15-Jun-10)	NaN	Maturity (15-Jun-11)	NaN

```

Settle = datenum('02-Aug-1992');

ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'),...
             nan;
             datenum('15-Jun-1997'), datenum('15-Jun-2001'),...
             datenum('15-Jun-2005');
             datenum('14-Jun-1997'), datenum('14-Jun-2001'),...
             datenum('14-Jun-2005')];

Maturity = [datenum('15-Jun-2010');
            datenum('15-Jun-2010');
            datenum('15-Jun-2011')];

CouponRates = [0.075 0.08875 0.0925 nan;
               0.075 0.08875 0.0925 0.1;
               0.025 0.05 0.0750 0.1];

Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;

```

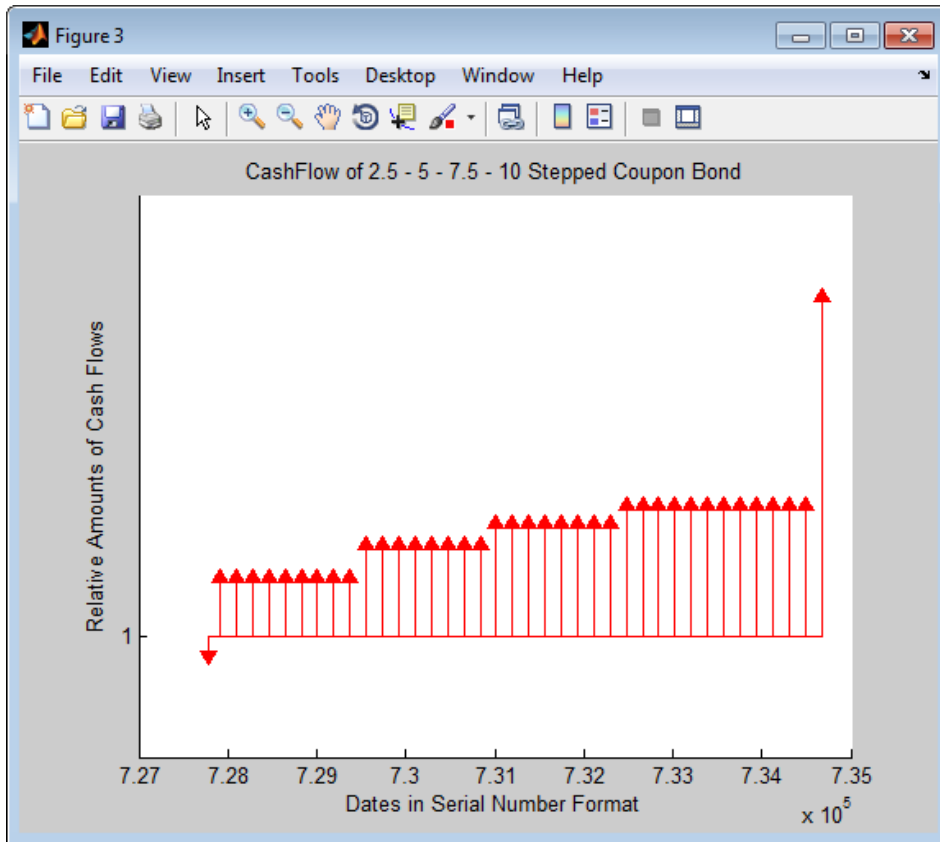
Call `stepcpncfamounts` to compute cash flows and timings.

```
[CFloWS, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ...
ConvDates, CouponRates);
```

Visualize the third bond cash flows (2.5 - 5 - 7.5 - 10) using the `cfplot` function.

```
cfplot(CDates(3,:), CFloWS(3,:));
xlabel('Dates in Serial Number Format')
```

```
ylabel('Relative Amounts of Cash Flows')
title('CashFlow of 2.5 - 5 - 7.5 - 10 Stepped Coupon Bond')
```



## See Also

[cfplot](#) | [stepcpnprice](#) | [stepcpnyield](#)

## Topics

“Cash Flows from Stepped-Coupon Bonds” on page 6-11

“Price and Yield of Stepped-Coupon Bonds” on page 6-12

“Managing Present Value with Bond Futures” on page 7-16



**Introduced before R2006a**

## stepcpnprice

Price bond with stepped coupons

### Syntax

[Price,AccruedInterest] = stepcpnprice(Yield,Settle,Maturity,ConvDates,CouponRates,Per

### Arguments

Yield	Scalar or vector containing yield to maturity of instruments.
Settle	Settlement date. A scalar or vector of serial date numbers. <b>Settle</b> must be earlier than <b>Maturity</b> .
Maturity	Maturity date. A scalar or vector of serial date numbers.
ConvDates	Matrix of serial date numbers representing conversion dates after <b>Settle</b> . Size = number of instruments by maximum number of conversions. Fill unspecified entries with NaN.
CouponRates	<p>Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions + 1. First column of this matrix contains rates applicable between <b>Settle</b> and the first conversion date (date in the first column of <b>ConvDates</b>). Fill unspecified entries with NaN.</p> <p><b>ConvDates</b> has the same number of rows as <b>CouponRate</b> to reflect the same number of bonds. However, <b>ConvDates</b> has one less column than <b>CouponRate</b>. This situation is illustrated by</p> <pre> Settle-----ConvDate1-----ConvDate2-----Maturity           Rate1             Rate2             Rate3                     </pre>
Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.

Basis	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
EndMonthRule	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
Face	<p>(Optional) Face value of each bond in the portfolio. Default = 100.</p>

**Note** All arguments must be scalars or number of bonds (NUMBONDS)-by-1 vectors, except for **ConvDates** and **CouponRates**.

## Description

[Price,AccruedInterest] = stepcpnprice(Yield,Settle,Maturity,ConvDates,CouponRates,Period,Basis,EndMonthRule,Face) computes the price of bonds with stepped coupons given the yield to maturity. The function supports any number of conversion dates.

Price is a NUMBONDS-by-1 vector of clean prices.

AccruedInterest is a NUMBONDS-by-1 vector of accrued interest payable at settlement dates.

---

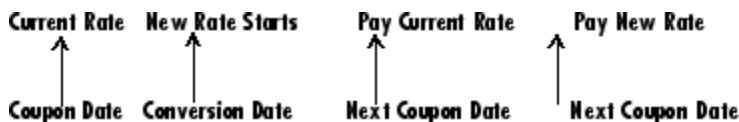
**Note** For bonds with fixed coupons, use bndprice. If you use a fixed-coupon bond with stepcpnprice, you receive the error: incorrect number of inputs.

---

## Examples

Compute the price and accrued interest due on a portfolio of stepped-coupon bonds having a yield of 7.221%, given three conversion scenarios:

- Bond A has two conversions, the first one falling on the settle date and immediately expiring.
- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with one or more conversion dates not on coupon dates. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.



The following table illustrates the interest rate characteristics of this bond portfolio.

Bond A Dates	Bond A Rates	Bond B Dates	Bond B Rates	Bond C Dates	Bond C Rates
Settle (02-Aug-92)	7.5%	Settle (02-Aug-92)	7.5%	Settle (02-Aug-92)	7.5%

Bond A Dates	Bond A Rates	Bond B Dates	Bond B Rates	Bond C Dates	Bond C Rates
First Conversion (02- Aug-92)	8.875%	First Conversion (15- Jun-97)	8.875%	First Conversion (14- Jun-97)	8.875%
Second Conversion (15- Jun-03)	9.25%	Second Conversion (15- Jun-01)	9.25%	Second Conversion (14- Jun-01)	9.25%
Maturity (15- Jun-10)	NaN	Third Conversion (15- Jun-05)	10.0%	Third Conversion (14- Jun-05)	10.0%
		Maturity (15- Jun-10)	NaN	Maturity (15- Jun-10)	NaN

```

Yield = 0.07221;
Settle = datenum('02-Aug-1992');
ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'),...
nan;
datenum('15-Jun-1997'), datenum('15-Jun-2001'),...
datenum('15-Jun-2005');
datenum('14-Jun-1997'), datenum('14-Jun-2001'),...
datenum('14-Jun-2005')];
Maturity = datenum('15-Jun-2010');

CouponRates = [0.075 0.08875 0.0925 nan;
0.075 0.08875 0.0925 0.1;
0.075 0.08875 0.0925 0.1];

Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;

[Price, AccruedInterest] = ...
stepcpnprice(Yield, Settle, Maturity, ConvDates, CouponRates, ...
Period, Basis, EndMonthRule, Face)

```

Price =

```

117.3824
113.4339
113.4339

```

AccruedInterest =

```

1.1587
0.9792
0.9792

```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*. 3rd Edition. Vol. 1, pp. 120-123, on zero-coupon instruments pricing.

## See Also

bndprice | cdprice | stepcpncfamounts | stepcpnprice | stepcpnyield |  
tbillprice

## Topics

“Cash Flows from Stepped-Coupon Bonds” on page 6-11

“Price and Yield of Stepped-Coupon Bonds” on page 6-12

“Managing Present Value with Bond Futures” on page 7-16

**Introduced before R2006a**

# stepcpnyield

Yield to maturity of bond with stepped coupons

## Syntax

Yield = stepcpnyield(Price,Settle,Maturity,ConvDates,CouponRate,Period,Basis,EndMonthR

## Arguments

Price	Vector containing price of the bonds.
Settle	Settlement date. A vector of serial date numbers. <b>Settle</b> must be earlier than <b>Maturity</b> .
Maturity	Maturity date. A vector of serial date numbers.
ConvDates	Matrix of serial date numbers representing conversion dates after <b>Settle</b> . Size = number of instruments by maximum number of conversions. Fill unspecified entries with NaN.
CouponRates	<p>Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions + 1. First column of this matrix contains rates applicable between <b>Settle</b> and the first conversion date (date in the first column of <b>ConvDates</b>). Fill unspecified entries with NaN.</p> <p><b>ConvDates</b> has the same number of rows as <b>CouponRate</b> to reflect the same number of bonds. However, <b>ConvDates</b> has one less column than <b>CouponRate</b>. This situation is illustrated by</p> <pre> Settle-----ConvDate1-----ConvDate2-----Maturity           Rate1             Rate2             Rate3 </pre>
Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.

<p><b>Basis</b></p>	<p>(Optional) Day-count basis of the instrument. A vector of integers.</p> <ul style="list-style-type: none"> <li>• 0 = actual/actual (default)</li> <li>• 1 = 30/360 (SIA)</li> <li>• 2 = actual/360</li> <li>• 3 = actual/365</li> <li>• 4 = 30/360 (BMA)</li> <li>• 5 = 30/360 (ISDA)</li> <li>• 6 = 30/360 (European)</li> <li>• 7 = actual/365 (Japanese)</li> <li>• 8 = actual/actual (ICMA)</li> <li>• 9 = actual/360 (ICMA)</li> <li>• 10 = actual/365 (ICMA)</li> <li>• 11 = 30/360E (ICMA)</li> <li>• 12 = actual/365 (ISDA)</li> <li>• 13 = BUS/252</li> </ul> <p>For more information, see <b>basis</b> on page Glossary-0 .</p>
<p><b>EndMonthRule</b></p>	<p>(Optional) End-of-month rule. A vector. This rule applies only when <b>Maturity</b> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</p>
<p><b>Face</b></p>	<p>(Optional) Face value of each bond in the portfolio. Default = 100.</p>

---

**Note** All arguments must be number of bonds (NUMBONDS)-by-1 vectors, except for **ConvDates** and **CouponRate**.

---



## Description

Yield =  
 stepcpnyield(Price, Settle, Maturity, ConvDates, CouponRate, Period, Basis, EndMonthRule, Face) computes the yield to maturity of bonds with stepped coupons given the price. The function supports any number of conversion dates.

Yield is a NUMBONDS-by-1 vector of yields to maturity in decimal form.

---

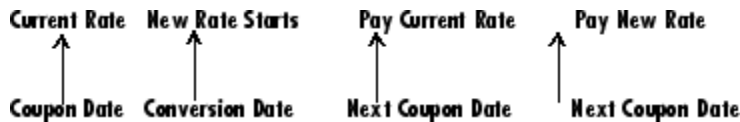
**Note** For bonds with fixed coupons, use bndyield. You receive the error incorrect number of inputs if you use a fixed-coupon bond with stepcpnyield.

---

## Examples

Find the yield to maturity of three stepped-coupon bonds of known price, given three conversion scenarios:

- Bond A has two conversions, the first one falling on the settle date and immediately expiring.
- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with one or more conversion dates not on coupon dates. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.



The following table illustrates the interest rate characteristics of this bond portfolio.

Bond A Dates	Bond A Rates	Bond B Dates	Bond B Rates	Bond C Dates	Bond C Rates
Settle (02-Aug-92)	7.5%	Settle (02-Aug-92)	7.5%	Settle (02-Aug-92)	7.5%
First Conversion (02-Aug-92)	8.875%	First Conversion (15-Jun-97)	8.875%	First Conversion (14-Jun-97)	8.875%

Bond A Dates	Bond A Rates	Bond B Dates	Bond B Rates	Bond C Dates	Bond C Rates
Second Conversion (15-Jun-03)	9.25%	Second Conversion (15-Jun-01)	9.25%	Second Conversion (14-Jun-01)	9.25%
Maturity (15-Jun-10)	NaN	Third Conversion (15-Jun-05)	10.0%	Third Conversion (14-Jun-05)	10.0%
		Maturity (15-Jun-10)	NaN	Maturity (15-Jun-10)	NaN

```

format long
Price = [117.3824; 113.4339; 113.4339];
Settle = datenum('02-Aug-1992');

ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'), nan;
datenum('15-Jun-1997'), datenum('15-Jun-2001'), datenum('15-Jun-2005');
datenum('14-Jun-1997'), datenum('14-Jun-2001'), datenum('14-Jun-2005')];

Maturity = datenum('15-Jun-2010');

CouponRates = [0.075 0.08875 0.0925 nan;
0.075 0.08875 0.0925 0.1;
0.075 0.08875 0.0925 0.1];

Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;

Yield = stepcpnyield(Price, Settle, Maturity, ConvDates, ...
CouponRates, Period, Basis, EndMonthRule, Face)

Yield =

0.07221440204915
0.07221426780036
0.07221426780036

```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*. 3rd Edition. Vol. 1, pp. 120-123, on zero-coupon instruments pricing.

## See Also

bndprice | cdprice | stepcpncfamounts | stepcpnprice | stepcpnprice |  
tbillprice | zeroprice

## **Topics**

"Cash Flows from Stepped-Coupon Bonds" on page 6-11

"Price and Yield of Stepped-Coupon Bonds" on page 6-12

"Managing Present Value with Bond Futures" on page 7-16

**Introduced before R2006a**

## tfutbyprice

Future prices of Treasury bonds given spot price

### Syntax

```
[QtdFutPrice,AccrInt] = tfutbyprice(SpotCurve,Price,SettleFut,  
MatFut,ConvFactor,CouponRate,Maturity)  
[QtdFutPrice,AccrInt] = tfutbyprice(____,Interpolation)
```

### Description

[QtdFutPrice,AccrInt] = tfutbyprice(SpotCurve,Price,SettleFut,MatFut,ConvFactor,CouponRate,Maturity) computes future prices of Treasury notes and bonds given the spot price.

In addition, you can use the Financial Instruments Toolbox method `getZeroRates` for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `tfutbyprice`. For more information, see “Converting an `IRDataCurve` or `IRFunctionCurve` Object” on page 9-41.

[QtdFutPrice,AccrInt] = tfutbyprice(\_\_\_\_,Interpolation) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

### Examples

#### Determine the Future Prices of Treasury Bonds Given the Spot Price

This example shows how to determine the future price of two Treasury bonds based upon a spot rate curve constructed from data for November 14, 2002.

```
% construct spot curve from Nov 14, data  
Bonds = [datenum('02/13/2003'),      0;  
         datenum('05/15/2003'),      0;
```

```

    datenum('10/31/2004'), 0.02125;
    datenum('11/15/2007'), 0.03;
    datenum('11/15/2012'), 0.04;
    datenum('02/15/2031'), 0.05375];

Yields = [1.20; 1.25; 1.86; 2.99; 4.02; 4.93]/100;

Settle = datenum('11/15/2002');

[ZeroRates, CurveDates] = ...
zbyield(Bonds, Yields, Settle);

SpotCurve = [CurveDates, ZeroRates];

% calculate a particular bond's future quoted price
RefDate    = [datenum('1-Dec-2002'); datenum('1-Mar-2003')];
MatFut     = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
Maturity    = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];
CouponRate = [0.06; 0.0575];
ConvFactor = convfactor(RefDate, Maturity, CouponRate);
Price = [114.416; 113.171];
Interpolation = 1;

[QtdFutPrice, AccrInt] = tfutbyprice(SpotCurve, Price, Settle, ...
MatFut, ConvFactor, CouponRate, Maturity, Interpolation)

QtdFutPrice = 2×1

    114.0409
    113.4029

AccrInt = 2×1

    1.9891
    0.4448

```

## Input Arguments

**SpotCurve** — Treasury spot curve  
matrix

Treasury spot curve, specified as a number of futures using one of the following forms:

- NFUT-by-2 matrix in the form of [SpotDates SpotRates] and these spot rates must be quoted as semiannual compounding (2) when the third column is not supplied.
- NFUT-by-3 matrix in the form of [SpotDates SpotRates Compounding], where allowed Compounding values for the third column are -1, 1, 2 (default), 3, 4, and 12, where -1 is continuous compounding.

Data Types: double

**Price — Prices of Treasury bonds or notes per \$100 notional at settlement date**

scalar numeric | vector

Prices of Treasury bonds or notes per \$100 notional at settlement date, specified as a scalar numeric or an NINST-by-1 vector. Use `bndprice` for theoretical value of bond.

Data Types: double

**SettleFut — Settlement date of futures contract**

serial date number | date character vector

Settlement date of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**MatFut — Maturity dates (or anticipated delivery dates) of futures contract**

serial date number | date character vector

Maturity dates (or anticipated delivery dates) of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**ConvFactor — Conversion factor**

numeric

Conversion factor, specified using `convfactor`.

Data Types: double | char | cell

**CouponRate — Underlying bond annual coupon**

scalar numeric in decimal | vector in decimals

Underlying bond annual coupon, specified as a scalar numeric decimal or an NINST-by-1 vector of decimals.

Data Types: double

### **Maturity — Underlying bond maturity date**

serial date number | date character vector

Underlying bond maturity date, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **Interpolation — Interpolation method to compute the corresponding spot rates for the bond's cash flow**

1 (linear) (default) | vector

(Optional) Interpolation method to compute the corresponding spot rates for the bond's cash flow, specified as an NMBS-by-1 vector. Available methods are (0) nearest, (1) linear, and (2) cubic spline. For more information on the supported interpolation methods, see `interp1`.

Data Types: double

## **Output Arguments**

### **QtdFutPrice — Quoted futures price, per \$100 notional**

vector

Quoted futures price, per \$100 notional, returned as a NINST-by-1 vector.

### **AccrInt — Accrued interest due at delivery date, per \$100 notional**

vector

Accrued interest due at delivery date, per \$100 notional, returned as a NINST-by-1 vector.

## **See Also**

`convfactor` | `tfutbyyield`

**Topics**

“Computing Treasury Bill Price and Yield” (Financial Toolbox)

“Treasury Bills Defined” (Financial Toolbox)

**Introduced before R2006a**



# tfutbyyield

Future prices of Treasury bonds given current yield

## Syntax

```
[QtdFutPrice,AccrInt] = tfutbyyield(SpotCurve,Yield,SettleFut,  
MatFut,ConvFactor,CouponRate,Maturity)  
[QtdFutPrice,AccrInt] = tfutbyyield( ____, Interpolation)
```

## Description

[QtdFutPrice,AccrInt] = tfutbyyield(SpotCurve,Yield,SettleFut, MatFut,ConvFactor,CouponRate,Maturity) computes prices of Treasury bond futures given a spot curve and bond yields at settlement.

In addition, you can use the Financial Instruments Toolbox method `getZeroRates` for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `tfutbyyield`. For more information, see “Converting an `IRDataCurve` or `IRFunctionCurve` Object” on page 9-41.

[QtdFutPrice,AccrInt] = tfutbyyield( \_\_\_\_, Interpolation) specifies options using one or more optional arguments in addition to the input arguments in the previous syntax.

## Examples

### Determine Future Prices of Treasury Bonds Given the Current Yield

This example shows how to determine the future price of two Treasury bonds based upon a spot rate curve constructed from data for November 14, 2002.

```
% construct spot curve from Nov 14, data  
Bonds = [datenum('02/13/2003'),      0;  
          datenum('05/15/2003'),      0;
```

```
    datenum('10/31/2004'), 0.02125;
    datenum('11/15/2007'), 0.03;
    datenum('11/15/2012'), 0.04;
    datenum('02/15/2031'), 0.05375];

Yields = [1.20; 1.25; 1.86; 2.99; 4.02; 4.93]/100;

Settle = datenum('11/15/2002');

[ZeroRates, CurveDates] = ...
zbyield(Bonds, Yields, Settle);

SpotCurve = [CurveDates, ZeroRates];

% calculate a particular bond's future quoted price
RefDate    = [datenum('1-Dec-2002'); datenum('1-Mar-2003')];
MatFut     = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
Maturity   = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];
CouponRate = [0.06; 0.0575];
ConvFactor = convfactor(RefDate, Maturity, CouponRate);
Yield      = [0.03576; 0.03773];
Interpolation = 1;

[QtdFutPrice, AccrInt] = tfutbyyield(SpotCurve, Yield, Settle, ...
MatFut, ConvFactor, CouponRate, Maturity, Interpolation)

QtdFutPrice = 2×1

    114.0416
    113.4034

AccrInt = 2×1

    1.9891
    0.4448
```

## Input Arguments

### SpotCurve — Treasury spot curve

matrix

Treasury spot curve, specified as a number of futures using one of the following forms:

- NFUT-by-2 matrix in the form of [SpotDates SpotRates] and these spot rates must be quoted as semiannual compounding (2) when the third column is not supplied.
- NFUT-by-3 matrix in the form of [SpotDates SpotRates Compounding], where allowed Compounding values for the third column are -1, 1, 2 (default), 3, 4, and 12, where -1 is continuous compounding.

Data Types: double

### **Yield — Yield to maturities at settlement date**

scalar numeric | vector

Yield to maturities at settlement date, specified as a scalar numeric or an NINST-by-1 vector. Use `bndyield` for theoretical value of bond yield.

Data Types: double

### **SettleFut — Settlement date of futures contract**

serial date number | date character vector

Settlement date of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **MatFut — Maturity dates (or anticipated delivery dates) of futures contract**

serial date number | date character vector

Maturity dates (or anticipated delivery dates) of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **ConvFactor — Conversion factor**

numeric

Conversion factor, specified using `convfactor`.

Data Types: double | char | cell

### **CouponRate — Underlying bond annual coupon**

scalar numeric in decimal | vector in decimals

Underlying bond annual coupon, specified as a scalar numeric decimal or an NINST-by-1 vector of decimals.

Data Types: double

**Maturity — Underlying bond maturity date**

serial date number | date character vector

Underlying bond maturity date, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**Interpolation — Interpolation method to compute the corresponding spot rates for the bond's cash flow**

1 (linear) (default) | vector

(Optional) Interpolation method to compute the corresponding spot rates for the bond's cash flow, specified as an NMBS-by-1 vector. Available methods are (0) nearest, (1) linear, and (2) cubic spline. For more information on the supported interpolation methods, see `interp1`.

Data Types: double

## Output Arguments

**QtdFutPrice — Quoted futures price, per \$100 notional**

vector

Quoted futures price, per \$100 notional, returned as a NINST-by-1 vector.

**AccrInt — Accrued interest due at delivery date, per \$100 notional**

vector

Accrued interest due at delivery date, per \$100 notional, returned as a NINST-by-1 vector.

## See Also

`convfactor` | `tfutbyprice`

## **Topics**

“Computing Treasury Bill Price and Yield” (Financial Toolbox)

“Treasury Bills Defined” (Financial Toolbox)

**Introduced before R2006a**

## tfutimprepo

Implied repo rates for Treasury bond future given price

### Syntax

```
ImpliedRepo = tfutimprepo(ReinvestData,Price,QtdFutPrice,Settle,  
MatFut,ConvFactor,CouponRate,Maturity)
```

### Description

`ImpliedRepo = tfutimprepo(ReinvestData,Price,QtdFutPrice,Settle,MatFut,ConvFactor,CouponRate,Maturity)` computes the implied repo rate that prevents arbitrage of Treasury bond futures, given the clean price at the settlement and delivery dates.

### Examples

#### Compute the Implied Repo Rates for Treasury Bond Futures Given the Price

This example shows how to compute the implied repo rate given the following set of data.

```
ReinvestData = [0.018 3];  
Price = [114.4160; 113.1710];  
QtdFutPrice = [114.1201; 113.7090];  
Settle = datenum('11/15/2002');  
MatFut = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];  
ConvFactor = [1; 0.9854];  
CouponRate = [0.06; 0.0575];  
Maturity = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];  
  
ImpliedRepo = tfutimprepo(ReinvestData, Price, QtdFutPrice, ...  
Settle, MatFut, ConvFactor, CouponRate, Maturity)  
  
ImpliedRepo = 2×1
```

```
0.0200  
0.0200
```

## Input Arguments

### **ReinvestData — Reinvestment of intervening coupons**

matrix

Reinvestment of intervening coupons, specified as a number of futures NFUT-by-2 matrix of rates and bases in the form of [ReinvestRate ReinvestBasis].

ReinvestRate is the simple reinvestment rate, in decimal. Specify ReinvestBasis as 0 = not reinvested, 2 = actual/360, or 3 = actual/365.

Data Types: double

### **Price — Current bond price per \$100 notional**

scalar numeric | vector

Current bond price per \$100 notional, specified as a scalar numeric or an NINST-by-1 vector.

Data Types: double

### **QtdFutPrice — Quoted bond futures price per \$100 notional**

scalar numeric | vector

Quoted bond futures price per \$100 notional, specified as a scalar numeric or an NINST-by-1 vector.

Data Types: double

### **Settle — Settlement/valuation date of futures contract**

serial date number | date character vector

Settlement/valuation date of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **MatFut — Maturity dates (or anticipated delivery dates) of futures contract**

serial date number | date character vector

Maturity dates (or anticipated delivery dates) of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **ConvFactor — Conversion factor**

numeric

Conversion factor, specified using convfactor.

Data Types: double | char | cell

### **CouponRate — Underlying bond annual coupon**

scalar numeric in decimal | vector in decimals

Underlying bond annual coupon, specified as a scalar numeric decimal or an NINST-by-1 vector of decimals.

Data Types: double

### **Maturity — Underlying bond maturity date**

serial date number | date character vector

Underlying bond maturity date, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

## **Output Arguments**

### **ImpliedRepo — Implied annual repo rate with an actual/360 basis**

vector in decimals

Implied annual repo rate (in decimals) with an actual/360 basis, returned as a NINST-by-1 vector.

## **See Also**

tfutpricebyrepo | tfutyieldbyrepo



## **Topics**

“Computing Treasury Bill Price and Yield” (Financial Toolbox)

“Treasury Bills Defined” (Financial Toolbox)

**Introduced before R2006a**

## tfutpricebyrepo

Calculates Treasury bond futures price given the implied repo rates

### Syntax

```
[QtdFutPrice,AccrInt] = tfutpricebyrepo(RepoData,ReinvestData,Price,  
Settle,MatFut,ConvFactor,CouponRate,Maturity)
```

### Description

[QtdFutPrice,AccrInt] = tfutpricebyrepo(RepoData,ReinvestData,Price,Settle,MatFut,ConvFactor,CouponRate,Maturity) computes the theoretical futures bond price given the settlement price, the repo/funding rates, and the reinvestment rate.

### Examples

#### Compute Treasury Bond Futures Price Given the Implied Repo Rates

This example shows how to compute the quoted futures price and accrued interest due on the target delivery date, given the following data.

```
RepoData      = [0.020  2];  
ReinvestData  = [0.018  3];  
Price         = [114.416; 113.171];  
Settle        = datenum('11/15/2002');  
MatFut        = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];  
ConvFactor    = [1 ; 0.9854];  
CouponRate    = [0.06;0.0575];  
Maturity      = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];  
  
[QtdFutPrice AccrInt] = tfutpricebyrepo(RepoData, ...  
ReinvestData, Price, Settle, MatFut, ConvFactor, CouponRate, ...  
Maturity)
```

QtdFutPrice = 2×1

114.1201  
113.7090

AccrInt = 2×1

1.9891  
0.4448

## Input Arguments

### RepoData — Simple term repo/funding rates

matrix

Simple term repo/funding rates, specified as a number of futures NFUT-by-2 matrix of rates in decimal and their bases in the form of [RepoRate RepoBasis].

Specify RepoBasis as 2 = actual/360 or 3 = actual/365.

Data Types: double

### ReinvestData — Reinvestment of intervening coupons

matrix

Reinvestment of intervening coupons, specified as a number of futures NFUT-by-2 matrix of rates and bases in the form of [ReinvestRate ReinvestBasis].

ReinvestRate is the simple reinvestment rate, in decimal. Specify ReinvestBasis as 0 = not reinvested, 2 = actual/360, or 3 = actual/365.

Data Types: double

### Price — Current bond price per \$100 notional

scalar numeric | vector

Current bond price per \$100 notional, specified as a scalar numeric or an NINST-by-1 vector.

Data Types: double

**Settle — Settlement/valuation date of futures contract**

serial date number | date character vector

Settlement/valuation date of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**MatFut — Maturity dates (or anticipated delivery dates) of futures contract**

serial date number | date character vector

Maturity dates (or anticipated delivery dates) of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

**ConvFactor — Conversion factor**

numeric

Conversion factor, specified using convfactor.

Data Types: double | char | cell

**CouponRate — Underlying bond annual coupon**

scalar numeric in decimal | vector in decimals

Underlying bond annual coupon, specified as a scalar numeric decimal or an NINST-by-1 vector of decimals.

Data Types: double

**Maturity — Underlying bond maturity date**

serial date number | date character vector

Underlying bond maturity date, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

## Output Arguments

**QtdFutPrice — Quoted futures price, per \$100 notional**

vector

Quoted futures price, per \$100 notional, returned as a NINST-by-1 vector.

**AccrInt — Accrued Interest due at delivery date, per \$100 notional**  
vector

Accrued Interest due at delivery date, per \$100 notional, returned as a NINST-by-1 vector.

## See Also

tfutimprepo | tfutyieldbyrepo

## Topics

“Computing Treasury Bill Price and Yield” (Financial Toolbox)

“Treasury Bills Defined” (Financial Toolbox)

**Introduced before R2006a**

## tfutyieldbyrepo

Calculates Treasury bond futures yield given the implied repo rates

### Syntax

```
FwdYield = tfutyieldbyrepo(RepoData,ReinvestData,Yield,Settle,  
MatFut,ConvFactor,CouponRate,Maturity)
```

### Description

`FwdYield = tfutyieldbyrepo(RepoData,ReinvestData,Yield,Settle,MatFut,ConvFactor,CouponRate,Maturity)` computes the theoretical futures bond yield given the settlement yield, the repo/funding rate, and the reinvestment rate.

### Examples

#### Compute the Treasury Bond Futures Yield Given the Implied Repo Rates

This example shows how to compute the quoted futures bond yield, given the following data.

```
RepoData      = [0.020  2];  
ReinvestData = [0.018  3];  
Yield         = [0.0215; 0.0257];  
Settle        = datenum('11/15/2002');  
MatFut        = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];  
ConvFactor    = [1; 0.9854];  
CouponRate    = [0.06; 0.0575];  
Maturity      = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];
```

```
FwdYield = tfutyieldbyrepo(RepoData, ReinvestData, Yield,...  
    Settle, MatFut, ConvFactor, CouponRate, Maturity)
```

```
FwdYield = 2×1
```

```
0.0221  
0.0282
```

## Input Arguments

### **RepoData — Simple term repo/funding rates**

matrix

Simple term repo/funding rates, specified as a number of futures NFUT-by-2 matrix of rates in decimal and their bases in the form of [RepoRate RepoBasis].

Specify RepoBasis as 2 = actual/360 or 3 = actual/365.

Data Types: double

### **ReinvestData — Reinvestment of intervening coupons**

matrix

Reinvestment of intervening coupons, specified as a number of futures NFUT-by-2 matrix of rates and bases in the form of [ReinvestRate ReinvestBasis].

ReinvestRate is the simple reinvestment rate, in decimal. Specify ReinvestBasis as 0 = not reinvested, 2 = actual/360, or 3 = actual/365.

Data Types: double

### **Yield — Yield to maturity of Treasury bonds per \$100 notional at Settle**

scalar numeric | vector

Yield to maturity of Treasury bonds per \$100 notional at Settle, specified as a scalar numeric or an NINST-by-1 vector.

Data Types: double

### **Settle — Settlement/valuation date of futures contract**

serial date number | date character vector

Settlement/valuation date of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **MatFut — Maturity dates (or anticipated delivery dates) of futures contract**

serial date number | date character vector

Maturity dates (or anticipated delivery dates) of futures contract, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

### **ConvFactor — Conversion factor**

numeric

Conversion factor, specified using convfactor.

Data Types: double | char | cell

### **CouponRate — Underlying bond annual coupon**

scalar numeric in decimal | vector in decimals

Underlying bond annual coupon, specified as a scalar numeric decimal or an NINST-by-1 vector of decimals.

Data Types: double

### **Maturity — Underlying bond maturity date**

serial date number | date character vector

Underlying bond maturity date, specified as a scalar or an NINST-by-1 vector of serial date numbers or date character vectors.

Data Types: double | char | cell

## **Output Arguments**

### **FwdYield — Forward yield to maturity compounded semiannually**

vector in decimals

Forward yield to maturity, in decimals, compounded semiannually, returned as a NINST-by-1 vector.

## **See Also**

tfutimprepo | tfutpricebyrepo



## **Topics**

“Computing Treasury Bill Price and Yield” (Financial Toolbox)

“Treasury Bills Defined” (Financial Toolbox)

**Introduced before R2006a**

## toRateSpec

Convert IRDataCurve object to RateSpec

### Class

@IRDataCurve on page A-7

### Syntax

```
F = toratespec(CurveObj, InpDates)
```

### Arguments

CurveObj	Interest-rate curve object that is constructed using IRDataCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.

### Description

`F = toratespec(CurveObj, InpDates)` returns a `RateSpec` object that is identical to the `RateSpec` structure created by the Financial Instruments Toolbox function `intenvset`.

### Examples

#### Convert an IRDataCurve Object to a RateSpec

This example shows how to convert an `IRDataCurve` object to a `RateSpec`. First, an `IRDataCurve` object is created using the function `IRDataCurve` constructor with `Dates`

and Data, then this object is converted to a RateSpec structure using the toRateSpec method.

```
CurveSettle = datenum('2-Mar-2016');
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = datemnth(CurveSettle,12*[1 2 3 5 7 10 20 30]);
irdc = IRDataCurve('Forward',CurveSettle,Dates,Data);
toRateSpec(irdc, CurveSettle+30:30:CurveSettle+365)
```

```
ans = struct with fields:
    FinObj: 'RateSpec'
    Compounding: 2
    Disc: [12x1 double]
    Rates: [12x1 double]
    EndTimes: [12x1 double]
    StartTimes: [12x1 double]
    EndDates: [12x1 double]
    StartDates: 736391
    ValuationDate: 736391
    Basis: 0
    EndMonthRule: 1
```

## See Also

“@IRDataCurve” on page A-7

## Topics

“Creating Interest-Rate Curve Objects” on page 9-4

“Creating an IRDataCurve Object” on page 9-6

“Using the toRateSpec Method” on page 9-41

“Interest-Rate Curve Objects and Workflow” on page 9-2

## Introduced in R2008b

## toRateSpec

Convert IRFunctionCurve object to RateSpec

### Class

@IRFunctionCurve on page A-13

### Syntax

`F = toRateSpec(CurveObj, InpDates)`

### Arguments

CurveObj	Interest-rate curve object that is constructed using IRFunctionCurve.
InpDates	Vector of input dates using MATLAB date format. The input dates must be after the settle date.

### Description

`F = toRateSpec(CurveObj, InpDates)` returns a RateSpec object that is identical to the RateSpec structure created by the Financial Instruments Toolbox function `intenvset`.

### Examples

## Convert an IRFunctionCurve Object to a RateSpec

This example shows how to convert an IRFunctionCurve object to a RateSpec. First, an IRFunctionCurve object is created using the function IRFunctionCurve constructor, then a RateSpec structure is created using the toRateSpec method.

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));  
toRateSpec(irfc, today+30:30:today+365)
```

```
ans = struct with fields:  
    FinObj: 'RateSpec'  
    Compounding: 2  
    Disc: [12x1 double]  
    Rates: [12x1 double]  
    EndTimes: [12x1 double]  
    StartTimes: [12x1 double]  
    EndDates: [12x1 double]  
    StartDates: 737486  
    ValuationDate: 737486  
    Basis: 0  
    EndMonthRule: 1
```

## See Also

“@IRFunctionCurve” on page A-13

## Topics

“Creating an IRFunctionCurve Object” on page 9-22

“Using the toRateSpec Method” on page 9-41

“Interest-Rate Curve Objects and Workflow” on page 9-2

## Introduced in R2008b

## zeroprice

Price zero-coupon instruments given yield

### Syntax

```
Price = zeroprice(Yield,Settle,Maturity)
Price = zeroprice( ____,Period,Basis,EndMonthRule)
```

### Description

Price = zeroprice(Yield,Settle,Maturity) prices zero-coupon instruments given a yield. zeroprice calculates the prices for a portfolio of general short and long-term zero-coupon instruments given the yield of reference bonds. In other words, if the zero-coupon computed with this yield is used to discount the reference bond, the value of that reference bond is equal to its price.

Price = zeroprice( \_\_\_\_,Period,Basis,EndMonthRule) adds optional arguments for Period, Basis, and EndMonthRule.

### Examples

#### Compute the Price of a Short-Term Zero-Coupon Instrument

This example shows how to compute the price of a short-term zero-coupon instrument.

```
Settle = '24-Jun-1993';
Maturity = '1-Nov-1993';
Period = 2;
Basis = 0;
Yield = 0.04;
```

```
Price = zeroprice(Yield, Settle, Maturity, Period, Basis)
```

```
Price = 98.6066
```

## Compute the Prices of a Portfolio of Two Zero-Coupon Instruments

This example shows how to compute the prices of a portfolio of two zero-coupon instruments, one short-term, and the other long-term.

```
Settle = '24-Jun-1993';  
Maturity = ['01-Nov-1993'; '15-Jan-2024'];  
Basis = [0; 1];  
Yield = [0.04; 0.1];  
  
Price = zeroprice(Yield, Settle, Maturity, [], Basis)  
  
Price = 2×1  
  
    98.6066  
    5.0697
```

## Input Arguments

### Yield — Reference bond yield

scalar | vector

Reference bond yield, specified as a scalar or a NZERO-by-1 vector.

Data Types: double

### Settle — Settlement date

serial date number

Settlement date, specified as a NZERO-by-1 vector of serial date numbers.

Data Types: double

### Maturity — Maturity date

serial date number

Maturity date, specified as a NZERO-by-1 vector of serial date numbers.

Data Types: double

**Period — Number of coupons in one year**

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

(Optional) Number of coupons in one year, specified as a positive integer for the values 1, 2, 4, 6, 12 in a NZERO-by-1 vector.

Data Types: double

**Basis — Day-count basis of bond**

0 (actual/actual) (default) | vector of positive integers of the set [1 . . . 13]

(Optional) Day-count basis of the bond, specified as a positive integer using a NZERO-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

Data Types: double

**EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with value of 0 or 1

(Optional) End-of-month rule flag, specified as a nonnegative integer with a value of 0 or 1 using a NZERO-by-1 vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.



- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

## Output Arguments

### Price — Price for each zero-coupon instrument

vector

Price for each zero-coupon instrument (per \$100 notional), returned as a column vector.

## Algorithms

To compute the price when `Period` is 1 or 0 for the quasi-coupon periods to redemption, `zeroprice` uses the formula

$$Price = \frac{RV}{1 + \left(\frac{DSR}{E} \times \frac{Y}{M}\right)}$$

.

*Quasi-coupon periods* are the coupon periods that would exist if the bond were paying interest at a rate other than zero.

When there is more than one quasi-coupon period to the redemption date, `zeroprice` uses the formula

$$Price = \frac{RV}{\left(1 + \frac{Y}{M}\right)^{Nq - 1} + \frac{DSC}{E}}$$

.

The elements of the equations are defined as follows.

<b>Variable</b>	<b>Definition</b>
<i>DSC</i>	Number of days from settlement date to next quasi-coupon date as if the security paid periodic interest.
<i>DSR</i>	Number of days from settlement date to the redemption date (call date, put date, and so on).
<i>E</i>	Number of days in quasi-coupon period.
<i>M</i>	Number of quasi-coupon periods per year (standard for the particular security involved).
<i>Nq</i>	Number of quasi-coupon periods between settlement date and redemption date. If this number contains a fractional part, raise it to the next whole number.
<i>Price</i>	Dollar price per \$100 par value.
<i>RV</i>	Redemption value.
<i>Y</i>	Annual yield (decimal) when held to redemption.

## References

- [1] Mayle, Jan. *Standard Securities Calculation Methods*. 3rd Edition, Vol. 1, Securities Industry Association, Inc., New York, 1993, ISBN 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7.

## See Also

bndprice | cdprice | tbillprice | zeroyield

## Topics

- “Computing Treasury Bill Price and Yield” (Financial Toolbox)
- “Pricing Treasury Notes” on page 6-7
- “Pricing Corporate Bonds” on page 6-8
- “Measuring Zero-Coupon Bond Function Quality” on page 6-6

## Introduced before R2006a

# zeroyield

Yield of zero-coupon instruments given price

## Syntax

```
Yield = zeroyield(Price,Settle,Maturity)
Yield = zeroyield( ____,Period,Basis,EndMonthRule)
```

## Description

`Yield = zeroyield(Price,Settle,Maturity)` computes the yield of zero-coupon instruments given price. `zeroyield` calculates the bond-equivalent yield for a portfolio of general short and long-term zero-coupon instruments given the price of the instruments. In other words, if the zero-coupon computed with this yield is used to discount the reference bond, the value of that reference bond is equal to its price

`Yield = zeroyield( ____,Period,Basis,EndMonthRule)` adds optional arguments for `Period`, `Basis`, and `EndMonthRule`.

## Examples

### Compute the Yield of a Short-Term Zero-Coupon Instrument

This example shows how to compute the yield of a short-term zero-coupon instrument.

```
Settle = '24-Jun-1993';
Maturity = '1-Nov-1993';
Basis = 0;
Price = 95;
```

```
Yield = zeroyield(Price, Settle, Maturity, [], Basis)
```

```
Yield = 0.1490
```

### **Compute the Yield of a Short-Term Zero-Coupon Instrument Using a Day-Count Basis of 30/360 (SIA)**

This example shows how to compute the yield of a short-term zero-coupon instrument using a day-count basis of 30/360 (SIA).

```
Settle = '24-Jun-1993';  
Maturity = '1-Nov-1993';  
Basis = 1;  
Price = 95;
```

```
Yield = zeroyield(Price, Settle, Maturity, [], Basis)
```

```
Yield = 0.1492
```

### **Compute the Yield of a Long-Term Zero-Coupon Instrument**

This example shows how to compute the yield of a long-term zero-coupon instrument.

```
Settle = '24-Jun-1993';  
Maturity = '15-Jan-2024';  
Basis = 0;  
Price = 9;
```

```
Yield = zeroyield(Price, Settle, Maturity, [], Basis)
```

```
Yield = 0.0804
```

## **Input Arguments**

### **Price — Reference bond price**

scalar | vector

Reference bond price, specified as a scalar or a NZERO-by-1 vector.

Data Types: double

**Settle — Settlement date**

serial date number

Settlement date, specified as a NZERO-by-1 vector of serial date numbers.

Data Types: double

**Maturity — Maturity date**

serial date number

Maturity date, specified as a NZERO-by-1 vector of serial date numbers.

Data Types: double

**Period — Number of coupons in one year**

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

(Optional) Number of coupons in one year, specified as a positive integer for the values 1, 2, 4, 6, 12 in a NZERO-by-1 vector.

Data Types: double

**Basis — Day-count basis of bond**

0 (actual/actual) (default) | vector of positive integers of the set [1 . . . 13]

(Optional) Day-count basis of the bond, specified as a positive integer using a NZERO-by-1 vector.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)

- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see **basis** on page Glossary-0 .

---

**Note** When the **Maturity** date is fewer than 182 days away and the **Basis** is **actual/365**, the **zeroyield** uses a simple-interest algorithm. If **Maturity** is more than 182 days away, **zeroyield** uses present value calculations.

When the **Basis** is **actual/360**, the simple interest algorithm gives the money-market yield for short (1-6 months to maturity) Treasury bills.

---

Data Types: double

### **EndMonthRule — End-of-month rule flag**

1 (in effect) (default) | nonnegative integer with value of 0 or 1

(Optional) End-of-month rule flag, specified as a nonnegative integer with a value of 0 or 1 using a **NZERO-by-1** vector. This rule applies only when **Maturity** is an end-of-month date for a month having 30 or fewer days.

- 0 = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

## **Output Arguments**

### **Yield — Bond-equivalent yield for each zero-coupon instrument**

vector

Bond-equivalent yield for each zero-coupon instrument, returned as a column vector.

## Algorithms

To compute the yield when there is zero or one quasi-coupon period to redemption, `zeroyield` uses the formula

$$Yield = \left( \frac{RV - P}{P} \right) \times \left( \frac{M \times E}{DSR} \right)$$

*Quasi-coupon periods* are the coupon periods which would exist if the bond was paying interest at a rate other than zero. The first term calculates the yield on invested dollars. The second term converts this yield to a per annum basis.

When there is more than one quasi-coupon period to the redemption date, `zeroyield` uses the formula

$$Yield = \left( \left( \frac{RV}{P} \right)^{\frac{1}{Nq - 1 + \frac{DSC}{E}}} - 1 \right) \times M$$

The elements of the equations are defined as follows.

Variable	Definition
<i>DSC</i>	Number of days from the settlement date to next quasi-coupon date as if the security paid periodic interest.
<i>DSR</i>	Number of days from the settlement date to redemption date (call date, put date, and so on).
<i>E</i>	Number of days in quasi-coupon period.
<i>M</i>	Number of quasi-coupon periods per year (standard for the particular security involved).
<i>Nq</i>	Number of quasi-coupon periods between the settlement date and redemption date. If this number contains a fractional part, raise it to the next whole number.
<i>P</i>	Dollar price per \$100 par value.
<i>RV</i>	Redemption value.
<i>Yield</i>	Annual yield (decimal) when held to redemption.

## References

[1] Mayle, Jan. *Standard Securities Calculation Methods*. 3rd Edition, Vol. 1, Securities Industry Association, Inc., New York, 1993, ISBN 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7.

## See Also

bndyield | cdyield | tbillyield | zeroprice

## Topics

“Computing Treasury Bill Price and Yield” (Financial Toolbox)

“Pricing Treasury Notes” on page 6-7

“Pricing Corporate Bonds” on page 6-8

“Measuring Zero-Coupon Bond Function Quality” on page 6-6

**Introduced before R2006a**



# **Derivatives Pricing Options**

## Pricing Options Structure

In this section...
“Introduction” on page B-2
“Default Structure” on page B-2
“Customizing the Structure” on page B-4

### Introduction

The MATLAB `Options` structure provides additional input to most pricing functions. The `Options` structure

- Tells pricing functions how to use the interest-rate tree to calculate instrument prices.
- Determines what additional information the Command Window displays along with instrument prices.
- Tells pricing functions which method to use in pricing barrier options.

The pricing options structure is primarily used in the pricing of interest-rate-based financial derivatives. However, the `BarrierMethod` field in the structure allows you to use it in pricing equity barrier options as well.

You provide pricing options in an optional `Options` argument passed to a pricing function. (See, for example, `bondbyhjm`, `bdtprice`, `barrierbycrr`, `barrierbyeqp`, or `barrierbyitt`.)

### Default Structure

If you do not specify the `Options` argument in the call to a pricing function, the function uses a default structure. To observe the default structure, use `derivset` without any arguments.

```
Options = derivset
```

```
Options =
```

```
    Diagnostics: 'off'  
    Warnings:   'on'  
    ConstRate:  'on'  
    BarrierMethod: 'unenhanced'
```

The Options structure has four fields: `Diagnostics` on page B-3, `Warnings` on page B-3, `ConstRate` on page B-3, and `BarrierMethod` on page B-3.

### **Diagnostics Field**

`Diagnostics` indicates whether additional information is displayed if the tree is modified. The default value for this option is 'off'. If `Diagnostics` is set to 'on' and `ConstRate` is set to 'off', the pricing functions display information such as the number of nodes in the last level of the tree generated for pricing purposes.

### **Warnings Field**

`Warnings` indicates whether to display warning messages when the input tree is not adequate for accurately pricing the instruments. The default value for this option is 'on'. If both `ConstRate` and `Warnings` are 'on', a warning is displayed if any of the instruments in the input portfolio have a cash flow date between tree dates. If `ConstRate` is 'off', and `Warnings` is 'on', a warning is displayed if the tree is modified to match the cash flow dates on the instruments in the portfolio.

### **ConstRate Field**

`ConstRate` indicates whether the interest rates should be assumed constant between tree dates. By default this option is 'on', which is not an arbitrage-free assumption. So the pricing functions return an approximate price for instruments featuring cash flows between tree dates. Instruments featuring cash flows only on tree nodes are not affected by this option and return exact (arbitrage-free) prices. When `ConstRate` is 'off', the pricing function finds the cash flow dates for all instruments in the portfolio. If these cash flows do not align exactly with the tree dates, a new tree is generated and used for pricing. This new tree features the same volatility and initial rate specifications of the input tree but contains tree nodes for each date in which at least one instrument in the portfolio has a cash flow. Keep in mind that the number of nodes in a tree grows exponentially with the number of tree dates. So, setting `ConstRate` 'off' dramatically increases the memory and processor demands on the computer.

### **BarrierMethod Field**

When using binomial trees to price barrier options, this may require many tree steps to achieve an accurate result when tree nodes do not align with the barrier level. With the `BarrierMethod` field, the toolbox provides an enhancement method that improves the accuracy of the results without having to use large trees.

The `BarrierMethod` field can be set to 'unenhanced' (default) or 'interp'. If you specify 'unenhanced', no correction calculation is used. Otherwise, if you specify

'interp', the toolbox provides an enhanced valuation by interpolating between nodes on barrier boundaries.

You specify the barrier method in the last input argument, `Options`, of the functions `barrierbycrr`, `barrierbyeqp`, `barrierbyitt`, `crrprice`, `eqpprice`, `ittprice`, `crrsens`, `eqpsens`, or `ittsens`. `Options` is a structure that you create with the function `derivset`. Using `derivset`, you specify whether to use the enhanced or the unenhanced method.

For more information about this algorithm, see Derman, E., I. Kani, D. Ergener and I. Bardhan, "Enhanced Numerical Methods for Options with Barriers," *Financial Analysts Journal*, (Nov. - Dec. 1995), pp. 65-74.

## Customizing the Structure

Customize the `Options` structure by passing property name/property value pairs to the `derivset` function.

As an example, consider an `Options` structure with `ConstRate` 'off' and `Diagnostics` 'on'.

```
Options = derivset('ConstRate', 'off', 'Diagnostics', 'on')
```

```
Options =
```

```
    Diagnostics: 'on'  
    Warnings:   'on'  
    ConstRate:  'off'  
    BarrierMethod: 'unenhanced'
```

To obtain the value of a specific property from the `Options` structure, use `derivget`.

```
CR = derivget(Options, 'ConstRate')
```

```
CR =  
Off
```

---

**Note** Use `derivset` and `derivget` to construct the `Options` structure. These functions are guaranteed to remain unchanged, while the implementation of the structure itself may be modified in the future.

---

Now observe the effects of setting `ConstRate` 'off'. Obtain the tree dates from the HJM tree.

```
TreeDates = [HJMTree.TimeSpec.ValuationDate;...
HJMTree.TimeSpec.Maturity]
```

```
TreeDates =
```

```
    730486
    730852
    731217
    731582
    731947
```

```
datedisp(TreeDates)
```

```
01-Jan-2000
01-Jan-2001
01-Jan-2002
01-Jan-2003
01-Jan-2004
```

All instruments in `HJMInstSet` settle on January 1, 2000, and all have cash flows once a year, except for the second bond, which features a period of 2. This bond has cash flows twice a year, with every other cash flow consequently falling between tree dates. You can extract this bond from the portfolio to compare how its price differs by setting `ConstRate` to 'on' and 'off'.

```
BondPort = instselect(HJMInstSet, 'Index', 2);
```

```
instdisp(BondPort)
```

```
Index Type CouponRate Settle      Maturity      Period Basis...
1      Bond 0.04           01-Jan-2000 01-Jan-2004 2      NaN...
```

First price the bond with `ConstRate` 'on' (default).

```
format long
[BondPrice, BondPriceTree] = hjmprice(HJMTree, BondPort)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
```

```
BondPrice =
```

```
    97.52801411736377
```

```
BondPriceTree =  
FinObj: 'HJMPriceTree'  
PBush: {1x5 cell}  
AIBush: {[0] [1x1x2 double] ... [1x4x2 double] [1x8 double]}  
tObs: [0 1 2 3 4]
```

Now recalculate the price of the bond setting `ConstRate` 'off'.

```
OptionsNoCR = derivset('ConstR', 'off')
```

```
OptionsNoCR =
```

```
Diagnostics: 'off'  
Warnings: 'on'  
ConstRate: 'off'
```

```
[BondPriceNoCR, BondPriceTreeNoCR] = hjmprice(HJMTree, ...  
BondPort, OptionsNoCR)  
Warning: Not all cash flows are aligned with the tree. Rebuilding  
tree.
```

```
BondPriceNoCR =
```

```
97.53342361674437
```

```
BondPriceTreeNoCR =
```

```
FinObj: 'HJMPriceTree'  
PBush: {1x9 cell}  
AIBush: {1x9 cell}  
tObs: [0 0.5000 1 1.5000 2 2.5000 3 3.5000 4]
```

As indicated in the last warning, because the cash flows of the bond did not align with the tree dates, a new tree was generated for pricing the bond. This pricing method returns more accurate results since it guarantees that the process is arbitrage-free. It also takes longer to calculate and requires more memory. The `tObs` field of the price tree structure indicates the increased memory usage. `BondPriceTree.tObs` has only five elements, while `BondPriceTreeNoCR.tObs` has nine. While this may not seem like a large difference, it has a dramatic effect on the number of states in the last node.

```
size(BondPriceTree.PBush{end})
```

```
ans =
```

```
1 8
```

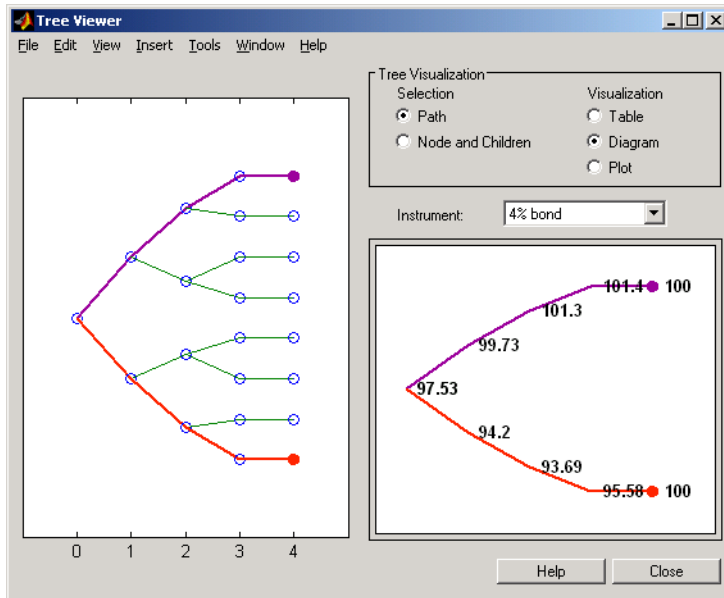
```
size(BondPriceTreeNoCR.PBush{end})
```

```
ans =
```

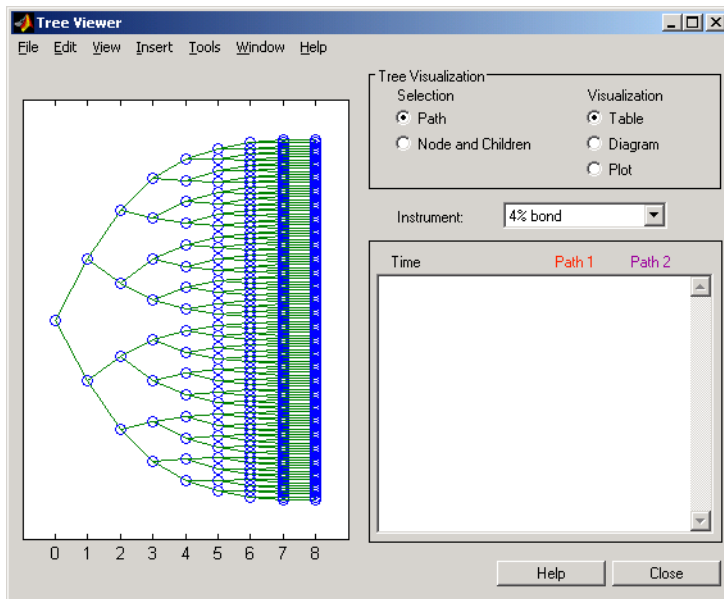
1 128

The differences become more obvious by examining the price trees with `treeviewer`.

```
treeviewer(BondPriceTree, BondPort)
```



```
treeviewer(BondPriceTreeNoCR, BondPort)
```



All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]

All =

-2.76	10.43	0.00	98.72
-3.56	16.64	-0.00	97.53
-166.18	13235.59	700.96	0.05
-2.76	10.43	0.00	98.72
-0.01	0.03	0	100.55
46.95	1090.63	14.91	6.28
-969.85	173969.77	1926.72	0.05
-76.39	287.00	0.00	3.690

## See Also

instasian | instbarrier | instcompound | instlookback | instoptstock

## Related Examples

- “Pricing Equity Derivatives Using Trees” on page 3-130



- “Pricing Options Structure” on page B-2
- “Pricing European Call Options Using Different Equity Models” on page 3-163
- “Pricing Using the Black-Scholes Model” on page 3-154
- “Compute Option Prices on a Forward” on page 11-1814
- “Compute Forward Option Prices and Delta Sensitivities” on page 11-1883
- “Compute the Option Price on a Future” on page 11-1816
- “Pricing Asian Options” on page 3-113

### **More About**

- “Supported Interest-Rate Instruments” on page 2-2
- “Supported Equity Derivatives” on page 3-24
- “Supported Energy Derivatives” on page 3-44



# Bibliography

## Bibliography

- “Black-Derman-Toy (BDT) Modeling” on page C-2
- “Heath-Jarrow-Morton (HJM) Modeling” on page C-2
- “Hull-White (HW) and Black-Karasinski (BK) Modeling” on page C-2
- “Cox-Ross-Rubinstein (CRR) Modeling” on page C-3
- “Implied Trinomial Tree (ITT) Modeling” on page C-3
- “Leisen-Reimer Tree (LR) Modeling” on page C-3
- “Equal Probabilities Tree (EQP) Modeling” on page C-3
- “Closed-Form Solutions Modeling” on page C-4
- “Financial Derivatives” on page C-4
- “Fitting Interest-Rate Curve Functions” on page C-4
- “Interest-Rate Modeling Using Monte Carlo Simulation” on page C-5
- “Bootstrapping a Swap Curve” on page C-5
- “Bond Futures” on page C-5
- “Credit Derivatives” on page C-6
- “Convertible Bonds” on page C-6

### Black-Derman-Toy (BDT) Modeling

A description of the Black-Derman-Toy interest-rate model can be found in:

- [1] Black, Fischer, Emanuel Derman, and William Toy. “A One Factor Model of Interest Rates and its Application to Treasury Bond Options.” *Financial Analysts Journal*. January - February 1990.

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You can find information on the creation of financial derivatives and their role in the marketplace in numerous sources. Among those consulted in the development of Financial Instruments Toolbox software are:

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## Glossary

### **American option**

An option that can be exercised any time until its expiration date. Contrast with **European option** on page Glossary-0 .

### **arbitrary cash flow instrument**

A set of generic cash flow amounts for which a price needs to be established.

### **Asian option**

An option whose payoff depends upon the average price of the underlying asset over a certain period of time.

### **asset-or-nothing option**

A digital option that pays the value of the underlying security if the option expires in the money.

### **barrier option**

An option that is activated or deactivated only if the price of the underlying asset crosses a barrier. See also **knock-in** on page Glossary-0 and **knock-out** on page Glossary-0 . If the option fails to execute, the seller may pay to the purchaser a predetermined **rebate** on page Glossary-0 .

### **barrier option**

An option that is activated or deactivated only if the price of the underlying asset crosses a barrier. See also **knock-in** on page Glossary-0 and **knock-out** on page Glossary-0 . If the option fails to execute, the seller may pay to the purchaser a predetermined **rebate** on page Glossary-0 .

### **basis**

Day count basis determines how interest accrues over time for various instruments and the amount transferred on interest payment dates. The calculation of accrued interest for dates between payments also uses day count basis. Day count basis is a fraction of  $\text{Number of interest accrual days} / \text{Days in the relevant coupon period}$ . Supported day count conventions and basis values are:

<b>B a s i s V a l u e</b>	<b>Day Count Convention</b>
0	actual/actual (default) — Number of days in both a period and a year is the actual number of days.
1	30/360 SIA — Year fraction is calculated based on a 360 day year with 30-day months, after applying the following rules: If the first date and the second date are the last day of February, the second date is changed to the 30th. If the first date falls on the 31st or is the last day of February, it is changed to the 30th. If after the preceding test, the first day is the 30th and the second day is the 31st, then the second day is changed to the 30th.

<b>B a s i s V a l u e</b>	<b>Day Count Convention</b>
2	actual/360 — Number of days in a period is equal to the actual number of days, however the number of days in a year is 360.
3	actual/365 — Number of days in a period is equal to the actual number of days, however the number of days in a year is 365 (even in a leap year).
4	30/360 PSA — Number of days in every month is set to 30 (including February). If the start date of the period is either the 31st of a month or the last day of February, the start date is set to the 30th, while if the start date is the 30th of a month and the end date is the 31st, the end date is set to the 30th. The number of days in a year is 360.

<b>B a s i s V a l u e</b>	<b>Day Count Convention</b>
5	30/360 ISDA — Number of days in every month is set to 30, except for February where it is the actual number of days. If the start date of the period is the 31st of a month, the start date is set to the 30th while if the start date is the 30th of a month and the end date is the 31st, the end date is set to the 30th. The number of days in a year is 360.
6	30E /360 — Number of days in every month is set to 30 except for February where it is equal to the actual number of days. If the start date or the end date of the period is the 31st of a month, that date is set to the 30th. The number of days in a year is 360.

<b>B a s i s V a l u e</b>	<b>Day Count Convention</b>
7	actual/365 Japanese — Number of days in a period is equal to the actual number of days, except for leap days (29th February) which are ignored. The number of days in a year is 365 (even in a leap year).
8	actual/actual ICMA — Number of days in both a period and a year is the actual number of days and the compounding frequency is annual.
9	actual/360 ICMA — Number of days in a period is equal to the actual number of days, however the number of days in a year is 360 and the compounding frequency is annual.

<b>B a s i s V a l u e</b>	<b>Day Count Convention</b>
1 0	actual/365 ICMA — Number of days in a period is equal to the actual number of days, however the number of days in a year is 365 (even in a leap year) and the compounding frequency is annual.
1 1	30/360 ICMA — Number of days in every month is set to 30, except for February where it is equal to the actual number of days. If the start date or the end date of the period is the 31st of a month, that date is set to the 30th. The number of days in a year is 360 and the compounding frequency is annual.

B a s i s V a l u e	Day Count Convention
1 2	actual/365 ISDA — The day count fraction is calculated using the following formula: (Actual number of days in period that fall in a leap year / 366) + (Actual number of days in period that fall in a normal year / 365 ).
1 3	bus/252 — The number of days in a period is equal to the actual number of business days. The number of business days in a year is 252.

**basket option**

An option that provides a payoff dependent on the value of a portfolio of assets.

**beta**

The price volatility of a financial instrument relative to the price volatility of a market or index as a whole. Beta is most commonly used with respect to equities. A high-beta instrument is riskier than a low-beta instrument.

**binomial model**

A method in which the probability over time of each possible price or rate follows a binomial distribution. The

	basic assumption is that prices or rates can move to only two values (one higher and one lower) over any short time period. See also <b>trinomial model</b> on page Glossary-0 .
<b>Black-Derman-Toy (BDT) model</b>	A model for pricing interest rate derivatives where all security prices and rates depend upon the short rate (annualized one-period interest rate).
<b>bond</b>	A long-term debt security with fixed interest payments and fixed maturity date.
<b>bond option</b>	The right to sell a bond back to the issuer (put) or to redeem a bond from its current owner (call) at a specific price and on a specific date.
<b>bushy tree</b>	A tree of prices or interest rates in which the number of branches increases exponentially relative to observation times; branches never recombine. Opposite of a <b>recombining tree</b> on page Glossary-0 .
<b>call</b>	<ol style="list-style-type: none"><li>1. An option to buy a certain quantity of a stock or commodity for a specified price within a specified time. See also <b>put</b> on page Glossary-0 .</li><li>2. A demand to submit bonds to the issuer for redemption before the maturity date.</li></ol>
<b>call swaption</b>	Allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.
<b>callable bond</b>	A bond that allows the issuer to buy back the bond at a predetermined price at specified future dates. The bond contains an embedded call option; that is, the holder has sold a call option to the issuer. See also <b>puttable bond</b> on page Glossary-0 .
<b>cap</b>	Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain level.
<b>caplet</b>	An interim cap component in a multiperiod interest-rate cap agreement.



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<b>cash-or-nothing option</b>	A digital option that pays some fixed amount of cash if the option expires in the money.
<b>compound option</b>	An option on an option, such as a call on a call, a put on a put, a call on a put, or a put on a call.
<b>Delta</b>	Delta measures the rate of change of the price of a derivative security relative to the price of the underlying asset; that is, the first derivative of the curve that relates the price of the derivative to the price of the underlying security.
<b>derivative</b>	A financial instrument that is based on some underlying asset. For example, an option is a derivative instrument based on the right to buy or sell an underlying instrument.
<b>deterministic model</b>	An interest rate model in which the values of the rates in the next time step are determined solely by the values of the rates in the current time step.
<b>digital option</b>	An option whose payout is fixed after the underlying stock exceeds the predetermined threshold or strike price.
<b>discount factor</b>	Coefficient used to compute the present value of future cash flows.
<b>dollar sensitivity</b>	Sensitivity reported as a dollar price change instead of a percentage price change.
<b>down-and-in</b>	A type of <b>barrier option</b> on page Glossary-0 that becomes active if the barrier is reached from above. See also <b>knock-in</b> on page Glossary-0 .
<b>down-and-out</b>	A type of <b>barrier option</b> on page Glossary-0 that becomes deactivated if the barrier is reached from above. See also <b>knock-out</b> on page Glossary-0 .
<b>European option</b>	An option that can be exercised only on its expiration date. Contrast with <b>American option</b> on page Glossary-0 .
<b>ex-dividend date</b>	Date when a declared dividend belongs to the seller rather than the buyer.

<b>exercise price</b>	The price set for buying an asset (call) or selling an asset (put). The strike price.
<b>exotic option</b>	Any nonstandard option. Opposite of <b>vanilla option</b> on page Glossary-0 .
<b>fixed lookback option</b>	Strike price is fixed at purchase. The underlying is priced at its highest or lowest level, depending whether it is a call or put, during the life of the option rather than expiring at market.
<b>fixed-rate note</b>	A long-term debt security with preset interest rate and maturity, by which the interest must be paid. The principal may or may not be paid at maturity.
<b>floating lookback option</b>	Strike price is fixed at maturity. For a call, the price is fixed at the lowest price during the life of the option; for a put it is fixed at the highest price.
<b>floating-rate note</b>	A security similar to a bond, but in which the note's interest rate is reset periodically, relative to a reference index rate, to reflect fluctuations in market interest rates.
<b>floor</b>	Interest-rate option that guarantees that the rate on a floating-rate loan will not fall below a certain level.
<b>floorlet</b>	One of the interim period floors in a multiple period floor agreement.
<b>forward curve</b>	The curve of forward interest rates vs. maturity dates for bonds.
<b>forward rate</b>	The future interest rate of a bond inferred from the term structure, especially from the yield curve of zero-coupon bonds, calculated from the growth factor of an investment in a zero held until maturity.
<b>Gamma</b>	Gamma measures the rate of change of delta for a derivative security relative to the price of the underlying asset; that is, the second derivative of the option price relative to the security price.

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<b>gap option</b>	A digital option in which one strike decides if the option is in or out of money and another strike decides the size the size of the payoff.
<b>Heath-Jarrow-Morton (HJM) model</b>	A model of the interest rate term structure that works with a type of interest rate tree called a <b>bushy tree</b> on page Glossary-0 .
<b>hedge</b>	A securities transaction that reduces or offsets the risk on an existing investment position.
<b>instrument set</b>	A collection of financial assets. A portfolio.
<b>inverse discount</b>	A factor by which the present value of an asset is multiplied to find its future value. The reciprocal of the discount factor.
<b>irregular coupon</b>	A bond interest payment for more or less than six-months' interest. The first coupon on many bonds is irregular because payment is other than six months from the dated date.
<b>knock-in</b>	A <b>barrier option</b> on page Glossary-0 that is activated when the price of the underlying asset achieves a designated target. There are two types: <b>up-and-in</b> on page Glossary-0 and <b>down-and-in</b> on page Glossary-0 .
<b>knock-out</b>	A <b>barrier option</b> on page Glossary-0 that is deactivated when the price of the underlying asset achieves a designated target. There are two types: <b>up-and-out</b> on page Glossary-0 and <b>down-and-out</b> on page Glossary-0 .
<b>Lambda</b>	The percentage change in an option price divided by the percentage change in an underlying price.
<b>least-squares method</b>	A mathematical method of determining the best fit of a curve to a series of observations by choosing the curve that minimizes the sum of the squares of all deviations from the curve.

<b>long rate</b>	The yield on a zero-coupon Treasury bond.
<b>lookback option</b>	An option that reduces uncertainties associated with the timing of market entry. Lookback options can be either <b>fixed lookback option</b> on page Glossary-0 and <b>floating lookback option</b> on page Glossary-0 .
<b>mean reversion</b>	The tendency of a variable to return to its mean value after reaching a point of excessive positive or negative valuation relative to the mean.
<b>option</b>	A right to buy or sell specific securities or commodities at a stated price (exercise or strike price) within a specified time. An option is a type of derivative.
<b>per-dollar sensitivity</b>	The dollar <b>sensitivity</b> on page Glossary-0 divided by the corresponding instrument price.
<b>portfolio</b>	A collection of financial assets. Also called an instrument set.
<b>price tree structure</b>	A MATLAB structure that holds all pricing information.
<b>price vector</b>	A vector of instrument prices.
<b>pricing options structure</b>	A MATLAB structure that defines how the price tree is used to find the price of instruments in the portfolio, and how much additional information is displayed in the command window when the pricing function is called.
<b>put</b>	An option to sell a stipulated amount of stock or securities within a specified time and at a fixed exercise price. See also <b>call</b> on page Glossary-0 .
<b>put swaption</b>	Allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.
<b>puttable bond</b>	A bond that allows the holder to redeem the bond at a predetermined price at specified future dates. The bond contains an embedded put option; that is, the holder has bought a put option. See also <b>callable bond</b> on page Glossary-0 .

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<b>rainbow option</b>	A single option linked to two or more underlying assets. In order for the option to pay off, all the underlying assets must move in the intended direction.
<b>rate specification</b>	A MATLAB structure that holds all information needed to identify completely the evolution of interest rates.
<b>rebate</b>	A predetermined amount of money paid to the purchaser of a <b>barrier option</b> on page Glossary-0 if the option fails to execute.
<b>recombining tree</b>	A tree of prices or interest rates whose branches recombine over time. Opposite of a <b>bushy tree</b> on page Glossary-0 .
<b>Rho</b>	Rho measures sensitivity to the interest rate; it is the derivative of the option value with respect to the risk-free interest rate.
<b>self-financing hedge</b>	A trading strategy whereby the value of a portfolio after rebalancing is equal to its value at any previous time.
<b>sensitivity</b>	The “what if” relationship between variables; the degree to which changes in one variable cause changes in another variable. A specific synonym is volatility. See also <b>dollar sensitivity</b> on page Glossary-0 .
<b>short rate</b>	The annualized one-period interest rate.
<b>sinking fund bond</b>	A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes.
<b>spot curve, spot yield curve</b>	See <b>zero curve, zero-coupon yield curve</b> on page Glossary-0 .
<b>spot rate</b>	The current interest rate appropriate for discounting a cash flow of some given maturity.
<b>spread</b>	For options, a combination of call or put options on the same stock with differing exercise prices or maturity dates.

<b>stepped coupon bond</b>	A step-up and step-down bond is a debt security with a predetermined coupon structure over time.
<b>stochastic model</b>	Involving or containing a random variable or variables; involving chance or probability.
<b>strike</b>	Exercise a put or call option.
<b>strike price</b>	See <b>exercise price</b> on page Glossary-0 .
<b>supershare option</b>	A digital option that pays out a proportion of the assets underlying a portfolio if the asset lies between a lower and an upper bound at the expiry of the option.
<b>swap</b>	A contract between two parties to exchange cash flows in the future according to some formula.
<b>swaption</b>	An option on an interest rate swap. It grants the option buyer the right to enter into an interest rate swap at a future date.
<b>Theta</b>	Theta measures the sensitivity of the value of the derivative to the passage of time.
<b>time specification</b>	A MATLAB structure that represents the mapping between times and dates for interest rate quoting.
<b>trinomial model</b>	A method in which the basic assumption is that prices or rates can move to one of three possible values over any short time period. At any time step the price or rate direction can be upward, neutral, or downward. See also <b>binomial model</b> on page Glossary-0 .
<b>under-determined system</b>	A set of simultaneous equations in which the number of independent variables exceeds the number of equations in the set, leading to an infinite number of solutions.
<b>up-and-in</b>	A type of <b>barrier option</b> on page Glossary-0 that becomes active if the barrier is reached from below. See also <b>knock-in</b> on page Glossary-0 .

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<b>up-and-out</b>	A type of <b>barrier option</b> on page Glossary-0 that becomes deactivated if the barrier is reached from below. See also <b>knock-out</b> on page Glossary-0 .
<b>vanilla option</b>	A common option, such as a put or call. Opposite of <b>exotic option</b> on page Glossary-0 .
<b>vanilla swap</b>	A <b>swap</b> on page Glossary-0 agreement to exchange a fixed rate for a floating rate.
<b>Vega</b>	Vega measures the rate of change in the price of a derivative security relative to the volatility of the underlying security. When Vega is large, the security is sensitive to small changes in volatility.
<b>volatility specification</b>	A MATLAB structure that specifies the forward rate volatility process.
<b>yields</b>	The zero coupon rate.
<b>yield curve</b>	The zero curve.
<b>yield volatility</b>	The zero coupon volatilities.
<b>zero curve, zero-coupon yield curve</b>	A yield curve for zero-coupon bonds; zero rates versus maturity dates. Since the maturity and duration (Macaulay duration) are identical for zeros, the zero curve is a pure depiction of supply/demand conditions for loanable funds across a continuum of durations and maturities. Also known as spot curve or spot yield curve.
<b>zero-coupon bond, or zero</b>	A bond that, instead of carrying a coupon, is sold at a discount from its face value, pays no interest during its life, and pays the principal only at maturity.

